Innovative and Sustainable Groundwater Management in the Mediterranean

D6.1 Report on the Development of the Innovative DSS Tool

VERSION 1.0

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Table of Contents

List of Figures

List of Tables

Executive Summary

The overall objective of the InTheMED project is to implement innovative and sustainable management tools and remediation strategies for MED aquifers (inland and coastal) in order to mitigate anthropogenic and climate-change threats by creating new long-lasting spaces of social learning among different interdependent stakeholders, NGOs, and scientific researchers in five field case studies. These are located at the two shores of the MED basin, namely in Spain, Greece, Portugal, Tunisia, and Turkey.

InTheMED will develop an inclusive process that will establish an ensemble of innovative assessment and management tools and methodologies including a high-resolution monitoring approach, smart modelling, a socio-economic assessment, web-based decision support systems (DSS) and new configurations for governance to validate efficient and sustainable integrated groundwater management in the MED considering both the quantitative and qualitative aspects.

The current document, Deliverable D6.1, is part of Task 6.1 "Initial development and testing of an innovative Decision Support System" (Lead TUC, participants: UPV, UFZ, IST-ID, CERTE and BU), (Month 1 – Month 18). The aim of D6.1 is to describe the operation of the algorithm that has been developed and will be applied to enable the decision support system (DSS), as well as to present the results of the methodology application for the Tympaki study area. The DSS will aid groundwater managers in the sustainable management of groundwater resources taking into consideration multiple criteria: socio-economic and environmental. The algorithm is based on the multi-criteria optimization approach, thus meaning the formation of more than one objective functions. These objective functions refer to the respective criteria: optimum socio-economic management, optimum environmental management in terms of groundwater quantity and optimum environmental management in terms of groundwater quality.

Since optimization processes are usually applied within iterative simulation runs of the study area model, they constitute a time-demanding procedure. Along with the non-linear nature of the problem, the model complexity and time needed are increased. Therefore, focus has been given on efforts to overcome these difficulties, in order to provide the model with as much automation as possible. The DSS will be in line with a database from which it can retrieve meteorological and hydrological data. The database is developed to function along with the DSS algorithm and its set up is also described in the current document.

1. The DSS tool

The aim of Work Package 6 is the development of a Decision Support System tool to aid decision-making for sustainable groundwater resources management. The decision-making tool will be based on a novel multi-criteria optimization methodology that will operate within a Fuzzy logic web-based Decision Support System. The multi-criteria optimization methodology will serve as the evaluation model for the assessment and recommendation of the optimum alternative solution(s). The fuzzy approach will be deployed in the development, training and testing of a Fuzzy Inference System (FIS), Figure 1. The Fuzzy Inference System (FIS) will be trained according to the data and the results achieved by the social, economic and environmental modelling and for the optimized results. In parallel, it will also be trained and tested under different scenarios, in order to retrieve a wide spectrum of fuzzy rules that will serve to finding the optimum alternative for all the considered criteria.

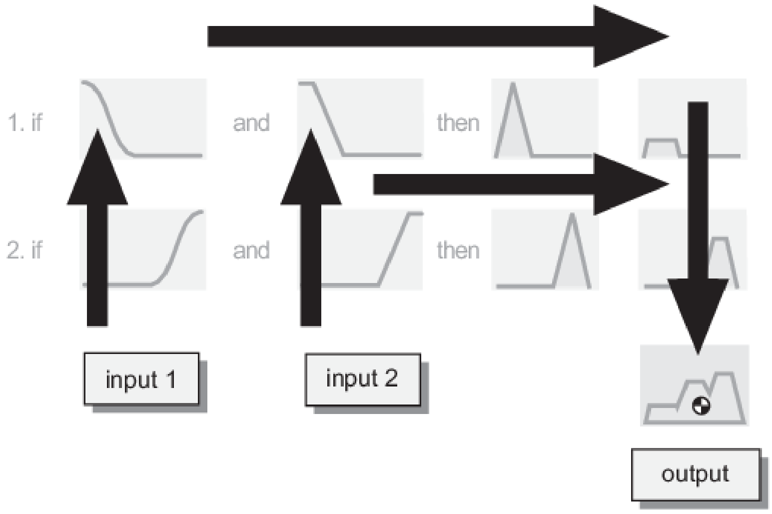


Figure 1: Schematic representation of a Fuzzy Inference System (FIS)

As the criteria are of different nature, the quantitative values of the optimum results cannot be in the same scale. Therefore, fuzzy sets are used to provide a means of normalization and overlay of the criteria. The optimum results for each criterion and scenario are used to train a Fuzzy Inference System. Each criterion consists of a different fuzzy rule, taking as input the problem variables. Through the FIS, the different fuzzy rules are overlayed and give as output the optimum result. Therefore, different alternatives can be evaluated under different scenarios through the FIS. This way, the FIS will serve as a Decision Support System.

2. The Simulation – Optimization Procedure

The simulation – optimization procedure refers to the separate simulation and optimization of each criterion that is taken into consideration. For each criterion, a different analysis is conducted according to its nature. This way, the optimum value for the examined scenario is obtained for each criterion and will be used for the training of the FIS, as presented in Figure 2. The objective functions (criteria) and constraints were selected so as to be applicable in all five case study areas of InTheMED project by addressing to the environmental issues that these areas have in common: overexploitation (except Castro Verde) and groundwater pollution (except Requena – Utiel), along with the socio-economic criteria.

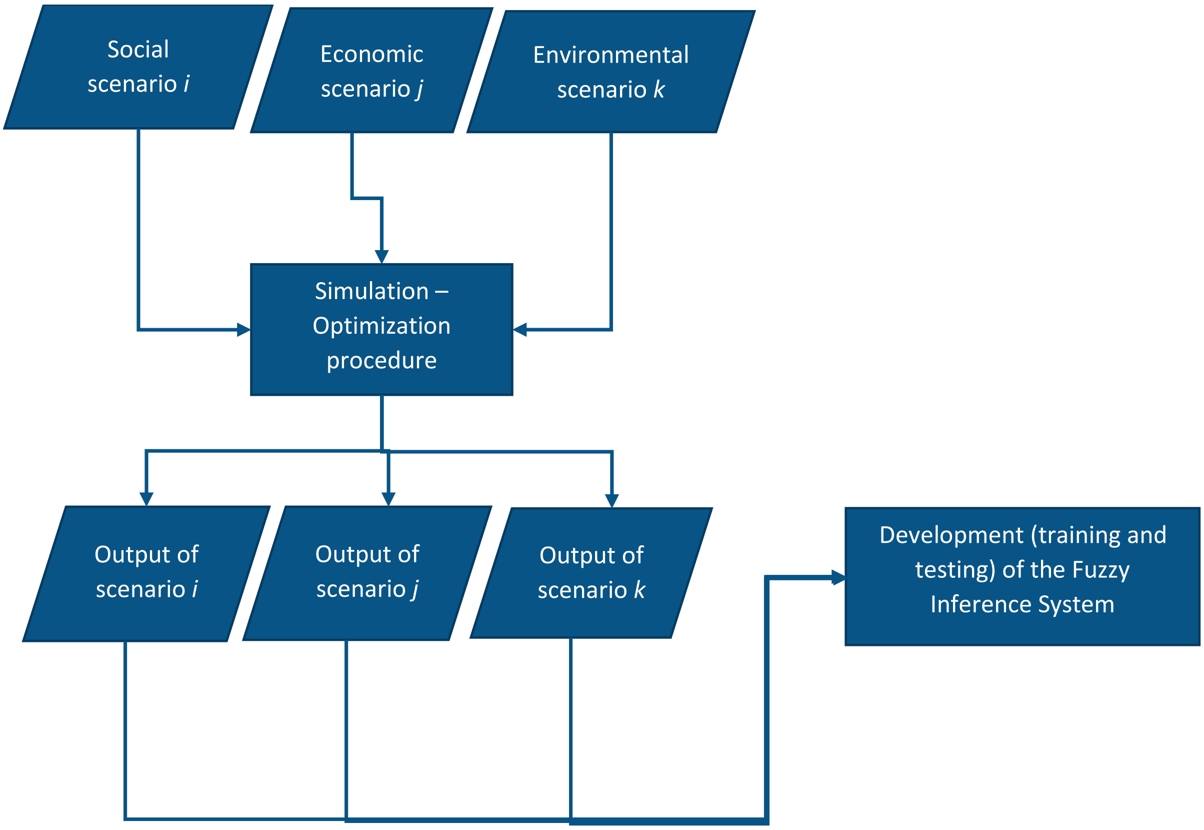


Figure 2. Graphic representation of simulation - optimization procedure and FIS training

In the following sections, the simulation model and the optimization procedure for the selected criteria are described.

2.1. Socio-economic criteria

A socioeconomic approach was established for the area considering socioeconomic data analysis and Shared Socioeconomic Pathways (SSP) scenarios using a statistical model and the WEAP model to identify the dynamics of social factors (population, economy) and agriculture (production, efficiency) in terms of water availability and use. Basic socioeconomic indicators (population, GDP, agriculture production) in the region are explored and the future trends are determined considering three different climate change RCP scenarios and relative Shared Socioeconomic Pathways (SSP) scenarios (IIASA - International Institute for Applied Systems Analysis) e.g., Figure 3. The development of more specific socioeconomic indicators such as agricultural intensity, crops, and land use is also investigated. The indicators' projected development is examined combining climate change RCP scenarios with SSP and the associated projected water availability to provide the expected climate change effect in the socioeconomic context. Another significant indicator that was investigated under green socioeconomic development was the capacity of the reservoir located in the area of study. The task considered the climatic scenarios effect in the reservoir's capacity. The WEAP model and a statistical model, is used to provide the projections on socioeconomic factors and water availability (basin, reservoir) Figures 4 and 5.

In addition, an assessment of adaptation measures in agricultural activities was conducted to determine their effect on the water availability considering the three different climate change RCP scenarios. Figure 6 provides the average results of the assessment based on the three scenarios. Furthermore, the current and future dependence on agriculture was estimated based on the socioeconomic status of the area and the expected rural investments (Figure 7). The irrigation water status was as well examined (Figure 8) assessing the previous and current situation and estimating the future patterns based on the available water sources in the area. It is obtained that surface water from the reservoir and the expected connection with a nearby new one will provide the majority of irrigation water around 67%. Finally, a swot analysis (Figure 9) was performed for the basin in order to study the strengths, weaknesses, opportunities and threats of water use and availability in the area considering current practises and expected policy transformations. All the presented results were obtained through the WEAP model considering a prior collection, statistical analysis and interpretation of the data.

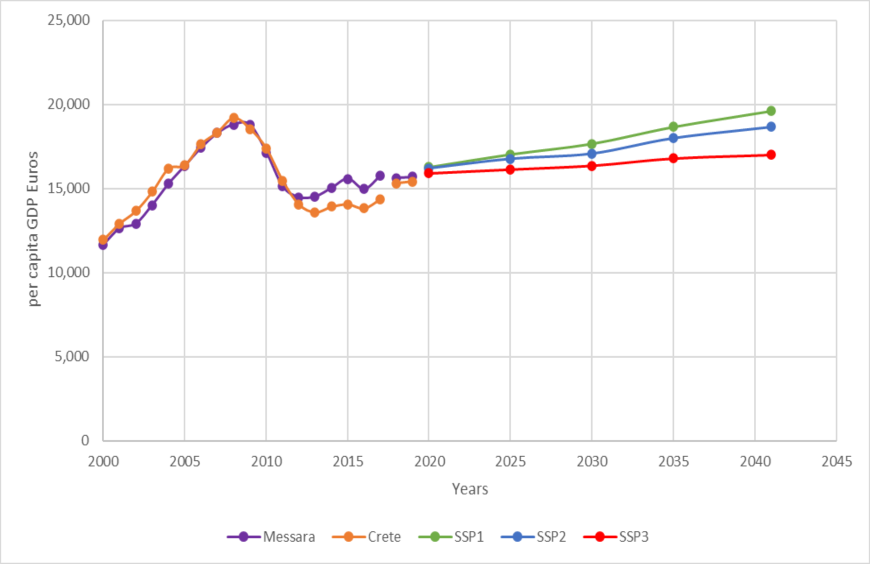


Figure 3 : GDP projection under Shared Socioeconomic Pathways (SSP) scenarios

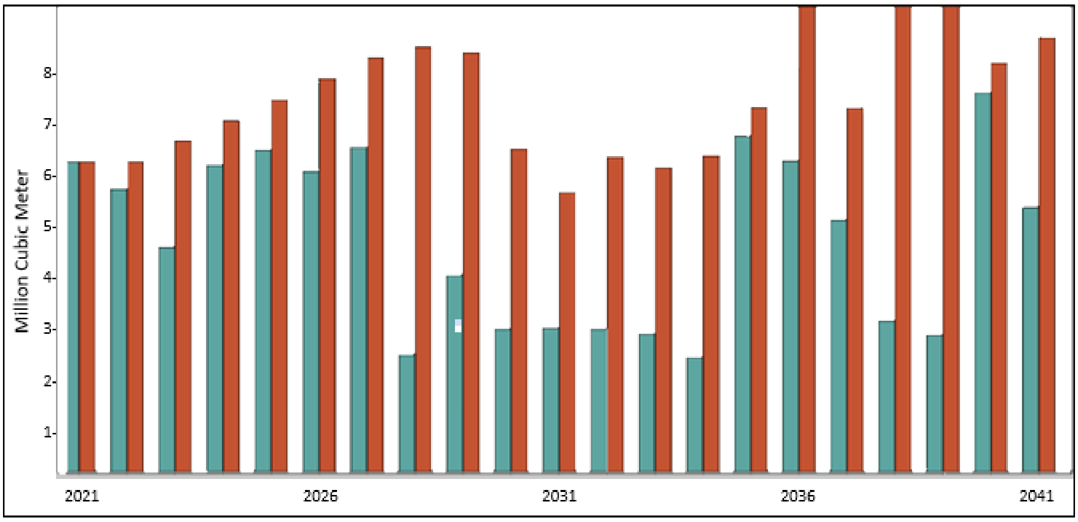


Figure 4: Projected Irrigation demand and water allocation from groundwater (green) and Faneromeni reservoir (red) using climate change scenarios, population variation, agricultural water demand and projected water availability through WEAP model

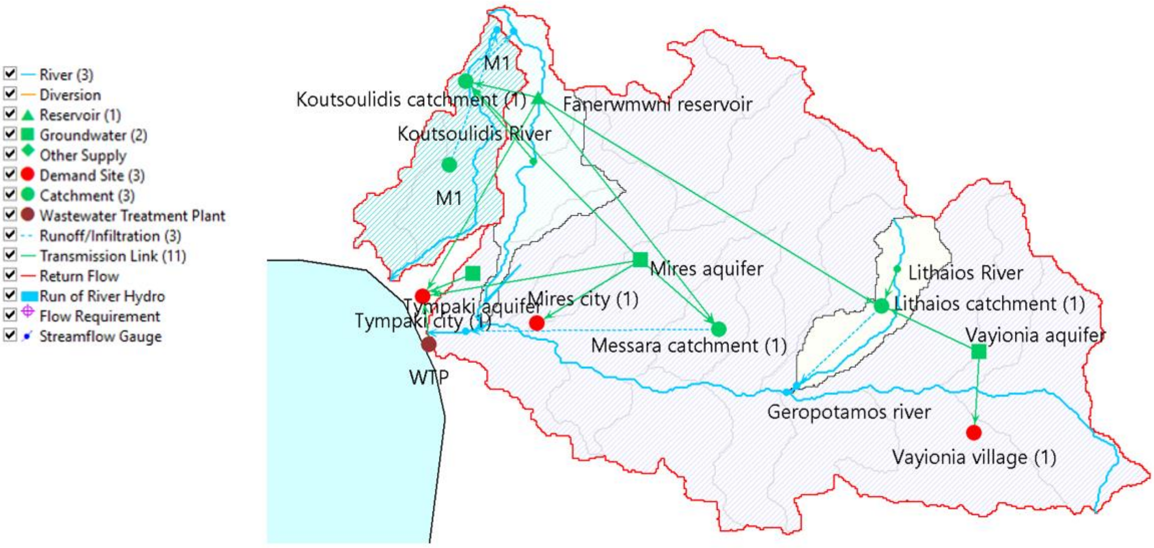


Figure 5: Schematic approach of the main demand sites and water sources, and their interconnections in Tympaki and upstream Messara basin using WEAP model

2041	Yield change %	Adaptation Measure	Water consumption Impact	Total impact
Olives	8	deficit Iriigation	-20%	
	20	efficient/smart irrigation systems and scheduling	-18%	-20%
	20	organic farming	-15%	
Grapes	5	deficit Iriigation	-20%	
	20	efficient/smart irrigation systems and scheduling	-21%	-22%
	20	organic farming	-18%	
Vegatables	35	organic farming	-18%	-25%
	35	crops diversification	-21%	
	35	efficient/smart irrigation systems and scheduling	-23%	

Figure 6: Assessment of adaptation measures in agricultures and their effect on the water availability

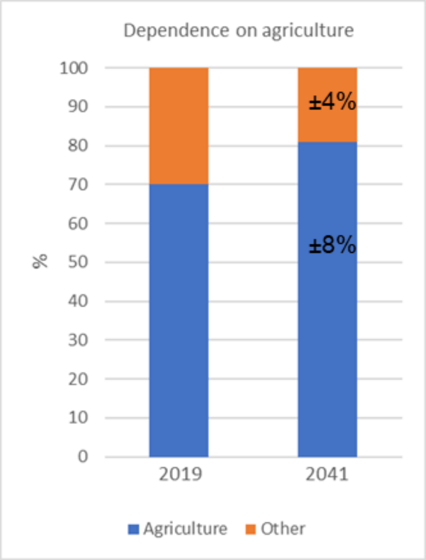


Figure 7: Current and future dependence on agriculture

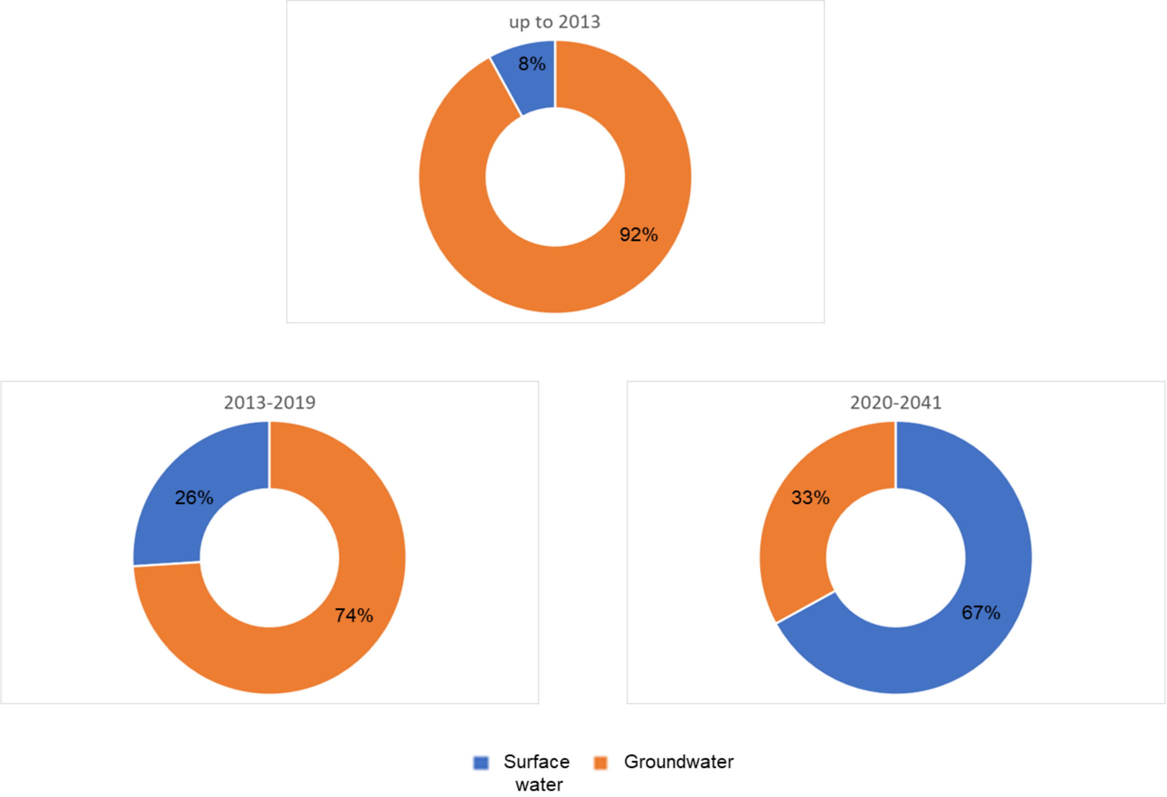
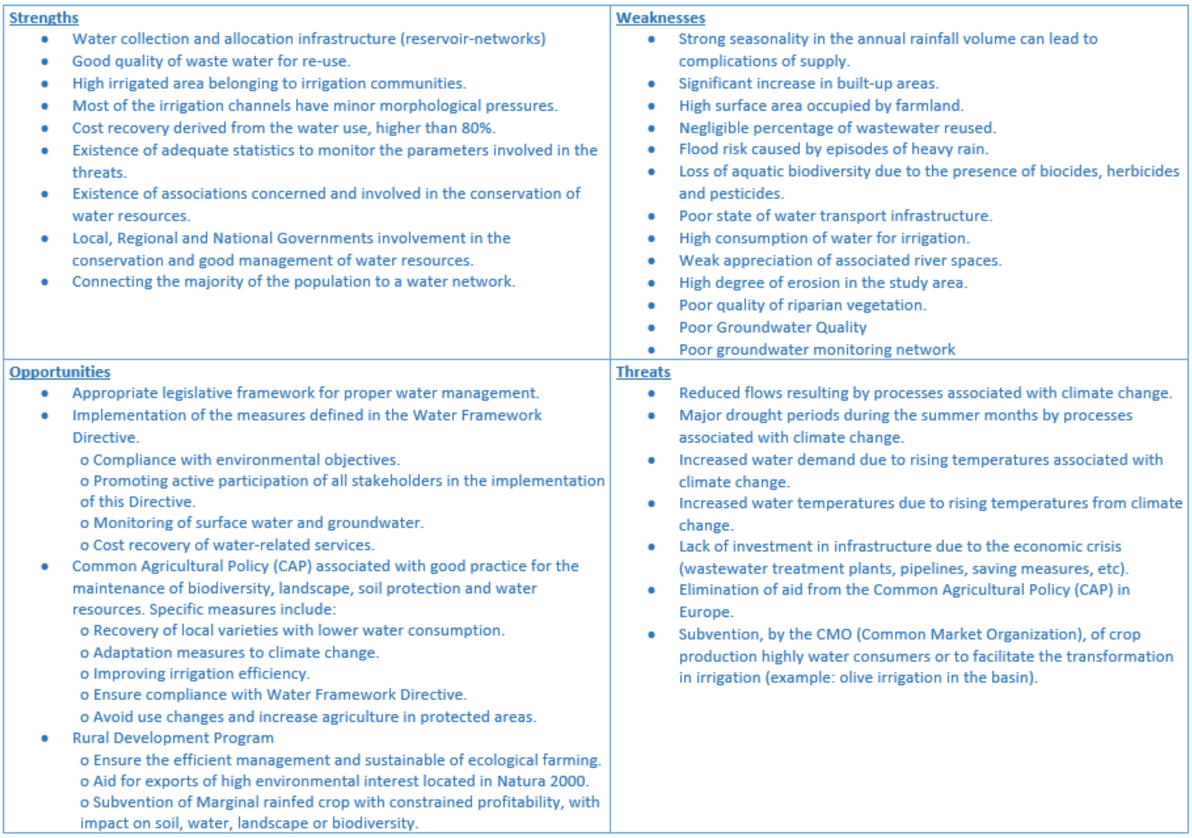


Figure 8: Irrigation water status analysis and prediction



Strengths
- Water collection and allocation infrastructure (reservoir-networks)
- Good quality of waste water for re-use.
- High irrigated area belonging to irrigation communities.
- Most of the irrigation channels have minor morphological pressures.
- Cost recovery derived from the water use, higher than 80%.
- Existence of adequate statistics to monitor the parameters involved in the threats.
- Existence of associations concerned and involved in the conservation of water resources.
- Local, Regional and National Governments involvement in the conservation and good management of water resources.
- Connecting the majority of the population to a water network.

Weaknesses
- Strong seasonality in the annual rainfall volume can lead to complications of supply.
- Significant increase in built-up areas.
- High surface area occupied by farmland.
- Negligible percentage of wastewater reused.
- Flood risk caused by episodes of heavy rain.
- Loss of aquatic biodiversity due to the presence of biocides, herbicides and pesticides.
- Poor state of water transport infrastructure.
- High consumption of water for irrigation.
- Weak appreciation of associated river spaces.
- High degree of erosion in the study area.
- Poor quality of riparian vegetation.
- Poor Groundwater Quality
- Poor groundwater monitoring network

Opportunities
- Appropriate legislative framework for proper water management.
- Implementation of the measures defined in the Water Framework Directive.
  - o Compliance with environmental objectives.
  - o Promoting active participation of all stakeholders in the implementation of this Directive.
  - o Monitoring of surface water and groundwater.
  - o Cost recovery of water-related services.
- Common Agricultural Policy (CAP) associated with good practice for the maintenance of biodiversity, landscape, soil protection and water resources. Specific measures include:
  - o Recovery of local varieties with lower water consumption.
  - o Adaptation measures to climate change.
  - o Improving irrigation efficiency.
  - o Ensure compliance with Water Framework Directive.
  - o Avoid use changes and increase agriculture in protected areas.
- Rural Development Program
  - o Ensure the efficient management and sustainable of ecological farming.
  - o Aid for exports of high environmental interest located in Natura 2000.
  - o Subvention of Marginal rainfed crop with constrained profitability, with impact on soil, water, landscape or biodiversity.

Threats
- Reduced flows resulting by processes associated with climate change.
- Major drought periods during the summer months by processes associated with climate change.
- Increased water demand due to rising temperatures associated with climate change.
- Increased water temperatures due to rising temperatures from climate change.
- Lack of investment in infrastructure due to the economic crisis (wastewater treatment plants, pipelines, saving measures, etc).
- Elimination of aid from the Common Agricultural Policy (CAP) in Europe.
- Subvention, by the CMO (Common Market Organization), of crop production highly water consumers or to facilitate the transformation in irrigation (example: olive irrigation in the basin).

Figure 9: Swot analysis for the Tympaki basin regarding water use and availability

2.1.1. Cost – Benefit Analysis (CBA)

Bayesian decision analysis is usually employed to make decisions in the presence of uncertainty. A common cost-benefit analysis approach coupled with Bayesian decision analysis are applied to aid the decision making of implementing mitigation measures for groundwater resources management. To this end, the full cost-benefit methodology has been established and the optimal options for managed use of groundwater. The mapping of those options is a dynamical process and will be adjusted over the course of the project, where key stakeholders can be added at a different stage to enrich the multi-stakeholder partnership process. The provided information will give insights into assessing the sustainability of the current and future groundwater management strategies. This analysis will identify the impact of the adequate mitigation option considering sustainable water resources availability, management and potential pollution risks.

The decision-making process involves two stages: state estimation and decision making. For state estimation, firstly, all the state parameters θ_i are defined. However, in the Bayesian approach, a state parameter is an unknown quantity and is considered a random variable that must be determined. The procedure of estimating each θ_i involves previous knowledge on the examined issue and the use of the subjective prior distribution $\pi(\theta_i)$ that expresses the prior information for each state parameter. Next, the Bayesian risk function is obtained to estimate the optimal decision or the decision with the minimum expected risk. The latter also applies in terms of a cost-benefit analysis procedure and denotes the preferable action. The Bayesian decision-making process follows these four steps, while a detailed approach is presented in the flowchart of Figure 10:

Set up the decision-making problem by introducing the possible actions set A and the parametric space Θ.

Action A(0): Use only groundwater

Action A(1): Surface water – Aquifer recharge

$$L\left(A(0),Y\right)=\begin{cases} K_1Y^2+GC+LGV, & 0\leq Y\leq n_1 \\ K_2Y^2+GC+LGV, & n_1<Y\leq n_2 \\ K_3Y^2+GC+LGV, & Y>n_2 \end{cases} \; ; \; K_1<K_2<K_3$$

where *GC* denotes the groundwater cost (pumping and volume) and *LGV* the lost value of groundwater as a sustainable source as soon as it is removed from the aquifer.

Whereas for the action *A(1)* the following applies,

$$L\left(A(1),\theta_1\right)=C+AC+M\theta_1$$

where *C* is the mitigation measure cost and *AC* its annual operational cost for the examined auditing period. In case there is a risk (probability) ϑ_1 water needs not to be covered from available water resources of the study area, an additional cost *M* is applied denoting a supplementary water supply (i.e., water transport) that should be considered. The condition, that shows which action is riskier, is the expression *R=R(A(1))- R(A(0)).*

Establish the expected loss function for each decision A(*i*), and provide the state of the goal function. If at this step, the parameters θ_i are considered known, then the decision process is called a cost-benefit analysis, and Step 4 is directly applied. If not, then both Steps 3 and 4 apply.

The goal function is the expected value of the loss function. Thus, for action *A(0)* the goal function is expressed as follows:

$$G\left(A(0),\theta_0\right)=\mathrm{E}\left[L\left(A(0),Y\right)\right]=GC+LCV+\left[K_2\mathrm{E}\left[\mathrm{Y}^2\right]+K_{1,2}\sum_{Y=0}^{n_1}Y^2 f\left(Y\right)+K_{3,2}\sum_{n_2+1}^{N}Y^2 f\left(Y\right)\right]$$

Where $\mathrm{K}_{1,2}=\mathrm{K}_1-\mathrm{K}_2$ and $\mathrm{K}_{3,2}=\mathrm{K}_3-\mathrm{K}_2$

$$G\left(A(1),\theta_1\right)=E\left[C+AC+M\theta_1\right]=C+AC+ME[\theta_1]=C+AC+M\theta_1$$

If θ and θ_1 the state parameters considered known from hydrological information then the cost benefit approach applies by means of Expected Net Loss Present Value (*ENLPV*).

$$\mathrm{ENLPV}{=}(C+AC+M\theta_1)-L\ (\mathrm{A}(0),\theta_0)$$

Positive ENPLV leads to decision *A(0*), while negative in decision *A(1)*.

Develop the subjective prior distributions for each θ_i quantifying the previous information.

If *Y* and Y_1 the state parameters considered unknown then Bayesian analysis is applied in the terms of the Bayesian Risk function that considers prior information in terms of probability density functions to determine *Y* and Y_1.

$$R(A(0))=E^{\pi}\left[G\left(A(0),\theta_0\right)\right]=\int_0^1 G\left(A(0),\theta_0\right)\pi\left(\theta\right)d\theta$$

$$R(A(1))=\mathrm{E}\left[G\left(A(1),\theta_1\right)\right]=\mathrm{E}\left[C+AC+M\theta_1\right]=C+AC+M\mathrm{E}\left[\theta_1\right]=C+AC+M\theta_1$$

Where *π(ϑ)* denotes the conjugative prior distribution in each case that depends on the fitted probability density function to the data. The probability that over-pumping or a drought year would occur or the necessary surface water would not be available is denoted as a “success” and as a “failure”

The condition, that shows which action is riskier, is the expression

$R=R(A(1))-R(A(0))$

Combine Steps 1, 2, and 3 via the risk function. The decision with the minimum expected risk is the optimal decision.

If R is positive, then the decision $A(1)$ is more risky than the decision $A(0)$, and thus, we need to redesign the mitigation measure. On the other hand, for negative values of R, we estimate the volume of ground water needed to cover the water demands. However, the appropriate volume, G, must not exceed the groundwater threshold ($GW_{threshold}$). If Δh is greater than $GW_{threshold}$, then either a water supply demand rebalance is required or an additional water volume WT need to be supplied occasionally in the area as an extra water source to cover the needs. Then, the decision-making process is re-examined to obtain the least-cost approach.

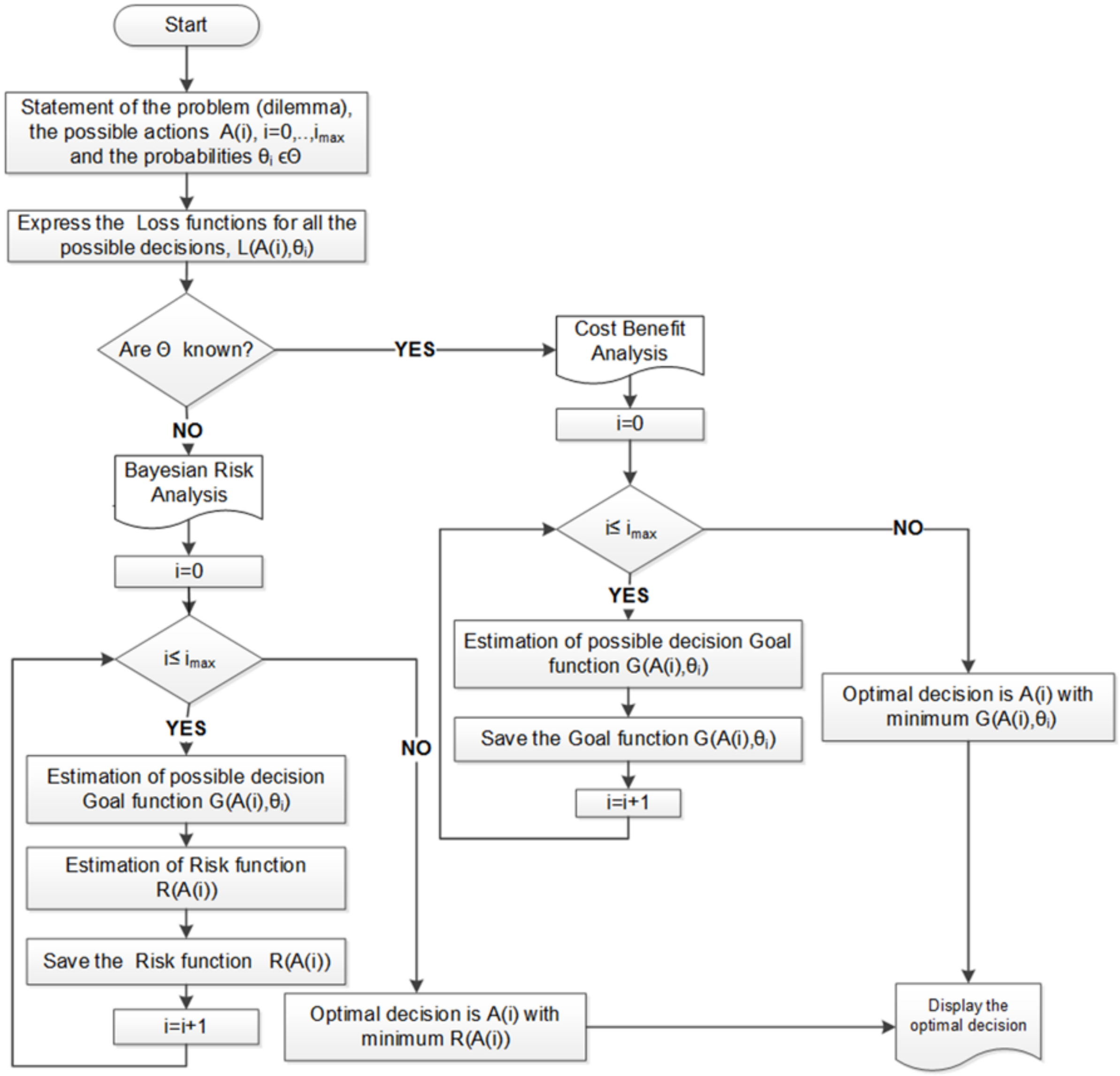


Figure 10: Flowchart of Bayesian Risk and cost-benefit risk analysis

Considering the available information for Tympaki basin and applying the proposed methodology it is obtained that for up to 15 overpumping violations using scaled cost effects, action *A(0)* is more affordable compared to action *A(1)* (groundwater only) involving aquifer recharge of 0.9 Mm^3 (**18%**) plus 2 Mm^3 (**40%**) from the reservoir to cover the needs, 1.1 Mm^3 (**22%**) groundwater and 1 Mm^3 (**20%**) waste water treatment plant effluent for irrigation. For more violations the financial and environmental costs of the mitigation measure: aquifer recharge and surface water use are lower compared to groundwater use only. According to the decision-making flowchart an assessment follows for the impact of the groundwater level decline in the aquifer considering the withdraw amount to cover the demand. The study area *A* and the storativity coefficient *S* are considered. Therefore, the expected aquifer level decline was calculated, $\Delta h = G/\left(A \times S_y\right)$, equal to 0.38 m/yr, less than 2.0 m/yr that may affect the set aquifer level threshold which is 10 meters above sea level according to the coastal area average. Therefore, considering the sustainable water resources policy that the local authorities desire to follow and the history of aquifer overpumping in the area, the investment to an integrated aquifer recharge scheme with balanced use of the available water resources of the basin is suggested. A similar and more detailed approach incorporating more information will be applied in the next steps of the project according to the climate change scenarios.

2.2. Environmental criteria

2.2.1. Groundwater quantity

The simulation – optimization procedure consists of 3 components/sub-routines:

1. The groundwater modelling
2. The construction of a response matrix based on iterative simulation runs of the groundwater model.
3. The optimization of the pumping rates by using the response matrix.

2.2.1.1. Groundwater modelling with FEFLOW

The study area has a width of about 12.5 km and a length of about 9.1 km. There are 371 pumping wells which were grouped into 20 by using the K-nearest neighbors and each group was represented as one pumping well by using the Median Center method, which Identifies the location that minimizes overall Euclidean distance to the features in a dataset (Figure 11). The pumping rate of each group is the total pumping rate of all the wells constituting the group in m^3/d. The pumping rates are imported in FEFLOW as timeseries with the values being different for the wet and dry season.

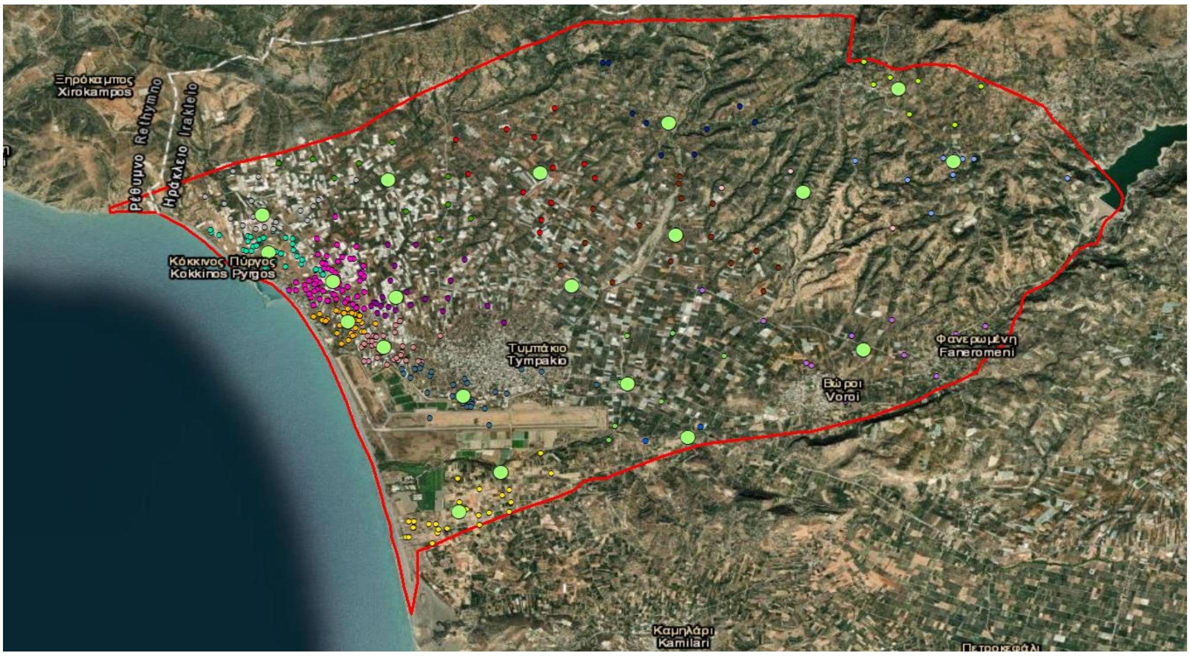


Figure 11: Tympaki study site with all the wells along with the representative ones for each group

The discretization of the unconfined aquifer was applied using a triangular finite element mesh which consists of 11877 nodes and 15340 elements. There are 2 layers and each one of them has 7670 elements and 3 slices with 3959 nodes for each slice. The depth of the model comprises 130 m deep from the sea level for the unconfined aquifer. The data for the hydraulic conductivity were extracted from the Decentralized Administration of Crete and they were set for the axis x'x and y'y. For z'z the hydraulic conductivity was 10% of the value of x'x. The pumping wells as considered in the model are depicted in Figure12.

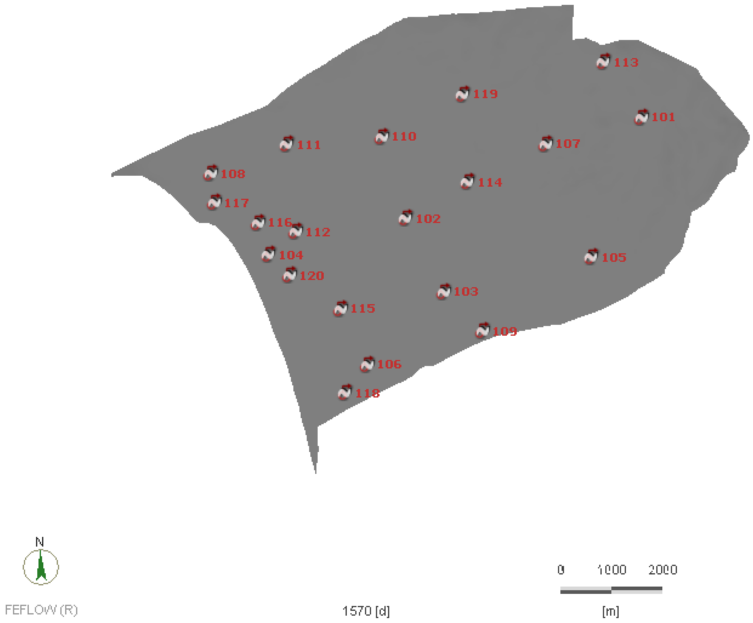


Figure 12: Pumping wells as introduced in the FEFLOW model

The calibration – validation process was conducted as follows. In the area of Tympaki, there are 6 observation wells that are scattered in the study area, as shown in Figure13. The hydraulic head data are used as timeseries and the period for each well is different. The timeseries period for the precipitation, observation wells and pumping wells are available for the years 2004-2014. For the precipitation, the data of 1 station is used. In sake of computational time, a mean precipitation was used with a step of 10 days and the input timeseries is in units of m/d. The data from 04/2004 until 04/2009 are used for calibration and from the rest of the period are used for validation. During the validation period, the field measurements in the observation wells were conducted in random dates so there are periods with no data available in the observation wells.

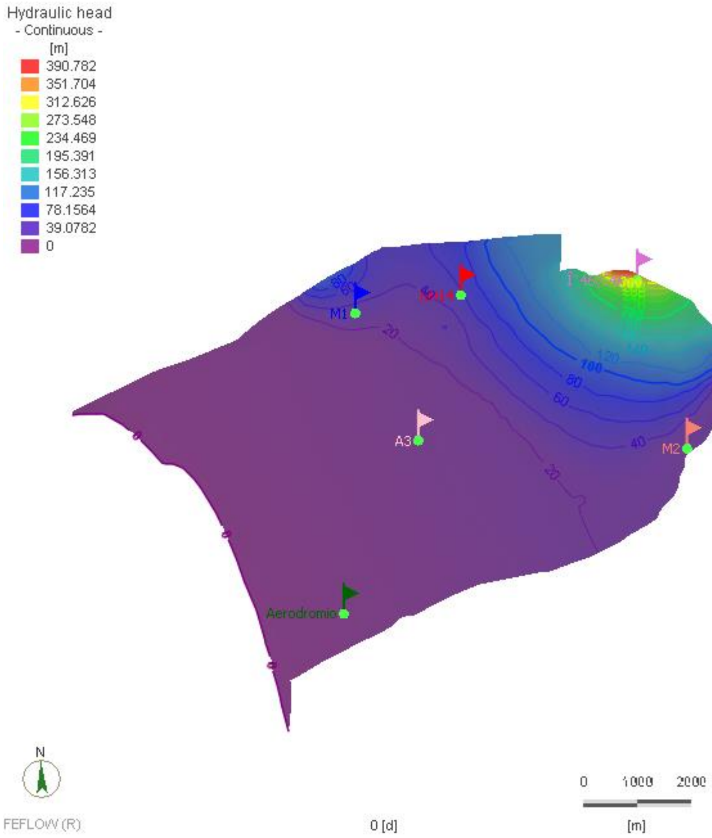


Figure 13: Observation wells with calibration – validation data available.

For the model of the Tympaki area, a first-type flow boundary condition was used along the coastline, where the hydraulic head was set at 0 m. Also, a second-type boundary conditions were set at the north boundaries of the study area. Those boundary conditions are connected to the precipitation so there is a fluctuation to the hydraulic head that is relative to the rainfall. The observation wells that are used in the pumping scenarios were set to monitor the saltwater intrusion zone. As it appears in Figure 14, the intrusion zone is calculated at 5.65 meters above the sea level (at the end of the calibration period), by using the Ghyben – Herzberg relation and considering the depth of the deepest pumping well in the area (226 m from the sea level). The simulation runs are applied for a ten-year period from 01/04/2010 until 31/03/2020, by using the available precipitation data, as well as timeseries for the pumping rates. For the values of pumping rates obtained from the licences of water use, the results of the model are depicted in Figures 15 and 16 and the values of the hydraulic heads in the observation wells in Figure 17.

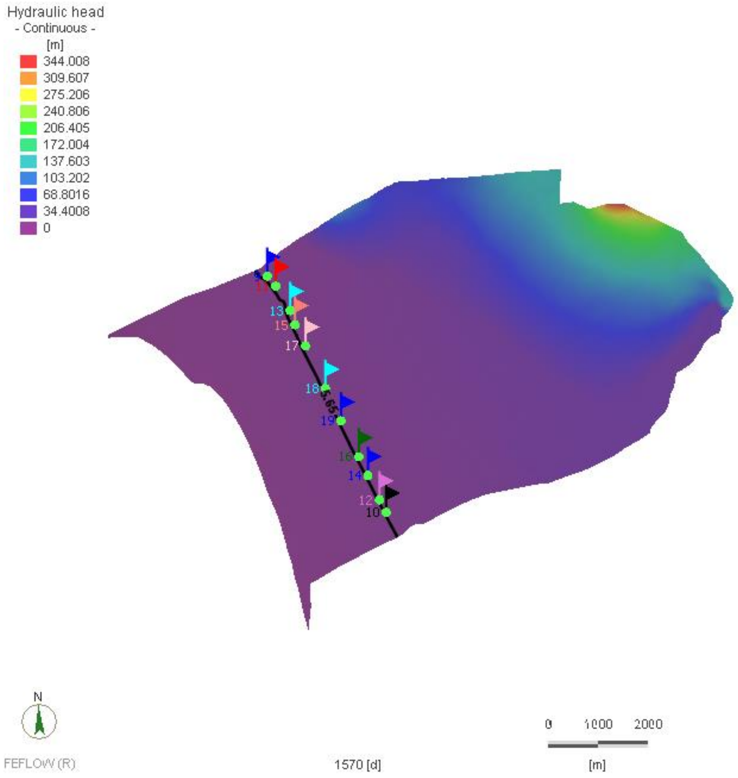


Figure 14: Saltwater intrusion zone at the end of calibration period

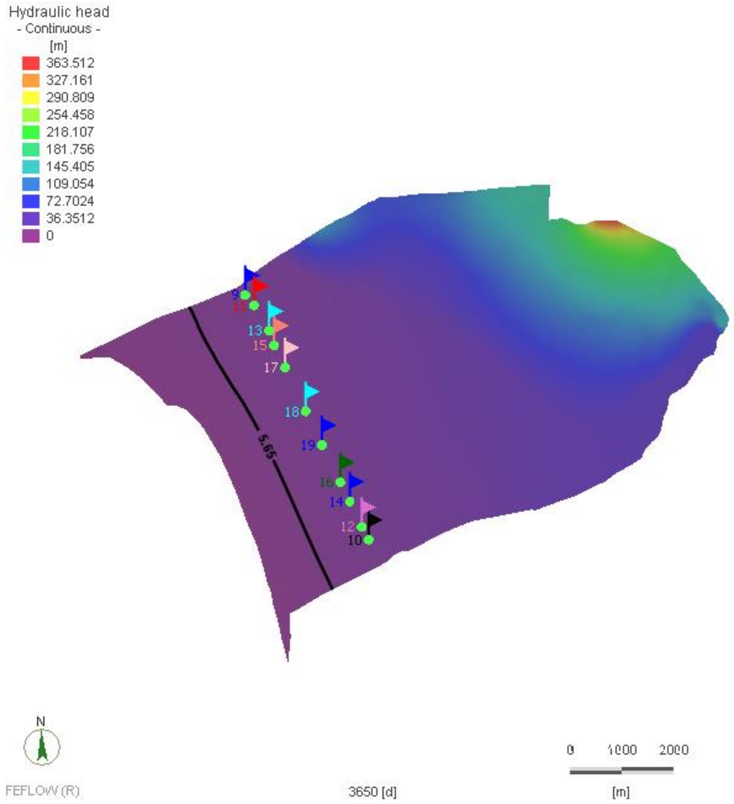


Figure 15: Current situation at the end of the wet period of the 10-year simulation

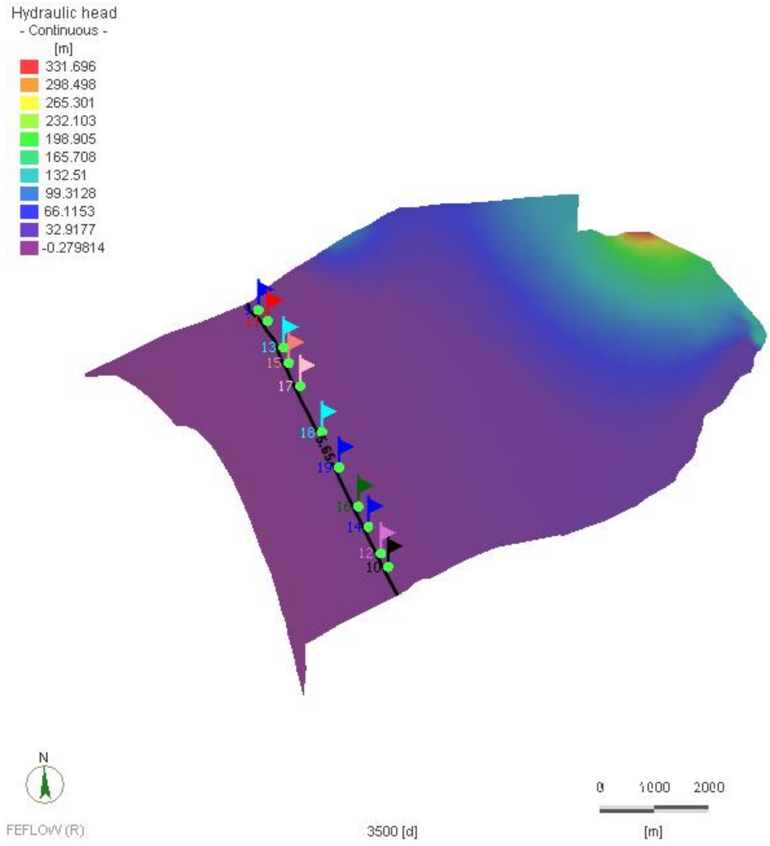


Figure 16: Current situation at the end of the dry period of the 10-year simulation

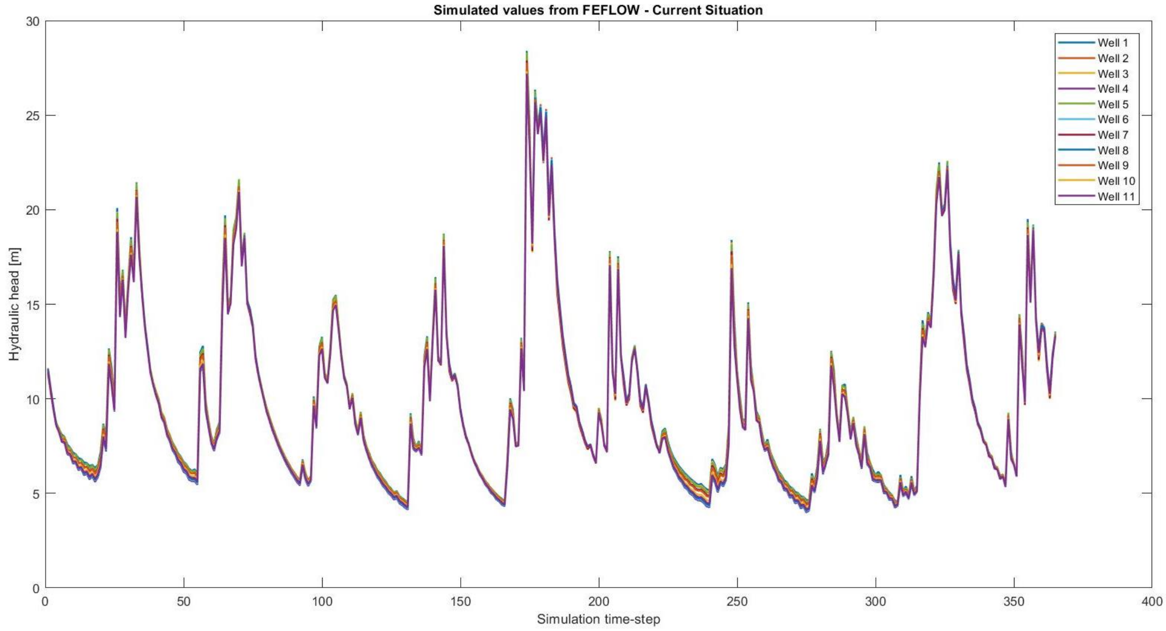


Figure 17: Hydraulic heads in the observation wells according to the current situation

2.2.1.2. Creation of the Response Matrix

The response matrix is created by sequentially disturbing the initial pumping rates of the *m* pumping wells, then running the simulation model and obtaining the hydraulic heads in *n* observation wells. The response matrix A is the *n x m* matrix, with its elements consisting of the values of hydraulic head's response to the disturbance of the pumping rate: $\frac{\partial H}{\partial Q}$.

In order to overcome the difficulties of the time-demanding runs of the groundwater simulation model, the response matrices can be constructed by using the results of the hydraulic heads and pumping rate values retrieved from a corresponding surrogate model. The ANN groundwater model that is being developed in WP3 of the project will be taken into consideration for integration into the simulation – optimization procedure. This way, the procedure can be fully automated, with no dependencies on the manual handling of a mainstream computational groundwater model.

2.2.1.3. Optimization of Pumping Rates

The optimization problem is set as:

$$maxQ$$

$$s.t.: H \geq H_{ref}$$

Where: $H = H_o + \partial H$ and $\partial H = A * \partial Q$

Therefore, the constraint:

$$H_o + \partial H \ \geq H_{ref} \ \rightarrow$$

$$H_o + A * \partial Q \geq \ H_{ref} \rightarrow$$

$$H_o + A * (Q - Q_o) \geq H_{ref} \ \rightarrow$$

$$A * Q \geq H_{ref} - H_o + A * Q_o$$

The problem is transformed to be in line with the requirements of Matlab optimization tool as:

$$\min(-Q)$$

$$s.t.: -A * Q \leq H_o - H_{ref} - A * Q_o$$

$-A$, the constraint coefficients matrix and

$b = H_o - H_{ref} - A * Q_o$, the vector of constraints of the linear problem.

Although the problem is not linear, it is solved by using the Simplex method in the frame of the piece-wise linear technique. Thus, after the first simulation – optimization cycle, the results for the optimum pumping rates are used in order to run again the simulation procedure, create the response matrix and apply the optimization procedure. The cycle is repeated until the results of two consecutive cycles converge to the same values for the individual wells and the total pumping rates as well. The results of the Tympaki study area are presented in the next section.

2.2.2. Results of the Simulation – Optimization Algorithm

At first, the simulation model was run for zero pumping rates. The saltwater intrusion zone is depicted in Figures 18 and 19, while the hydraulic heads in the observation wells in Figure 20. Then, the disturbance was set to 800 m^3/day for one well at a time and the model was run separately for each disturbance in the corresponding pumping well. The results of the model for the hydraulic heads in the observation wells are in the 350th simulation step for the end of the dry period and in the 365th simulation step for the wet one. These results are recorded in Tables 1 and 2 for each simulation run and respective disturbance in the pumping wells. In this set of runs, the first response matrix *A* was retrieved as follows in Table 3 for the Dry Season and in Table 4 for the Wet Season.

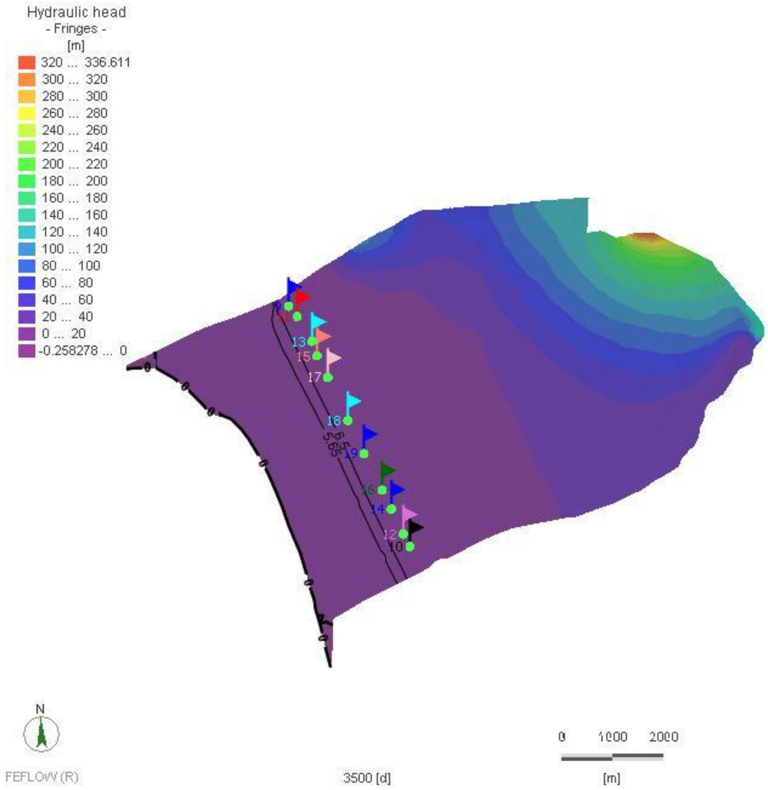


Figure 18: Saltwater intrusion zone at the end of the dry period considering zero pumping rates

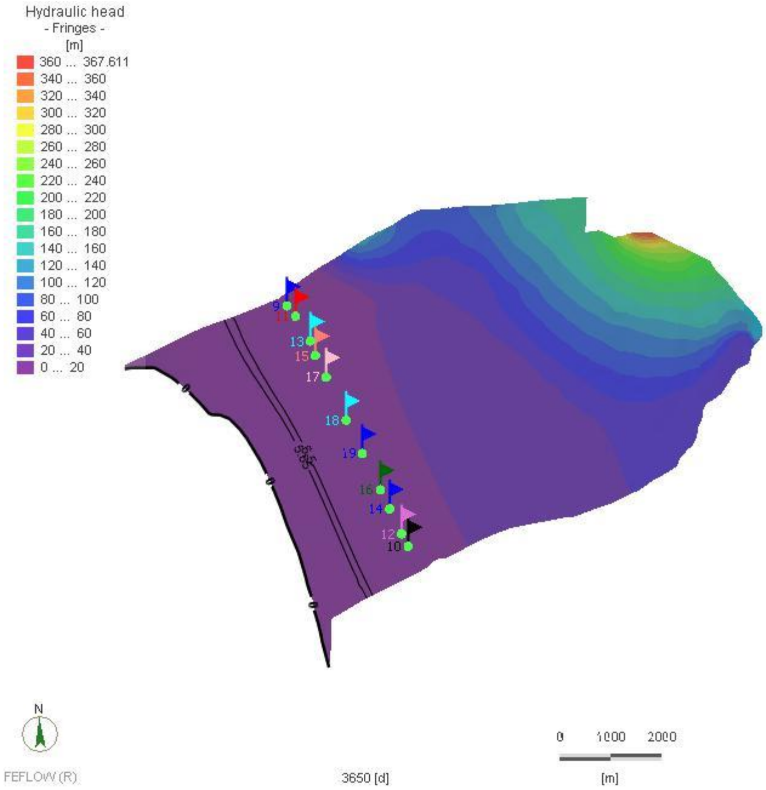


Figure 19: Saltwater intrusion zone at the end of the wet period considering zero pumping rates

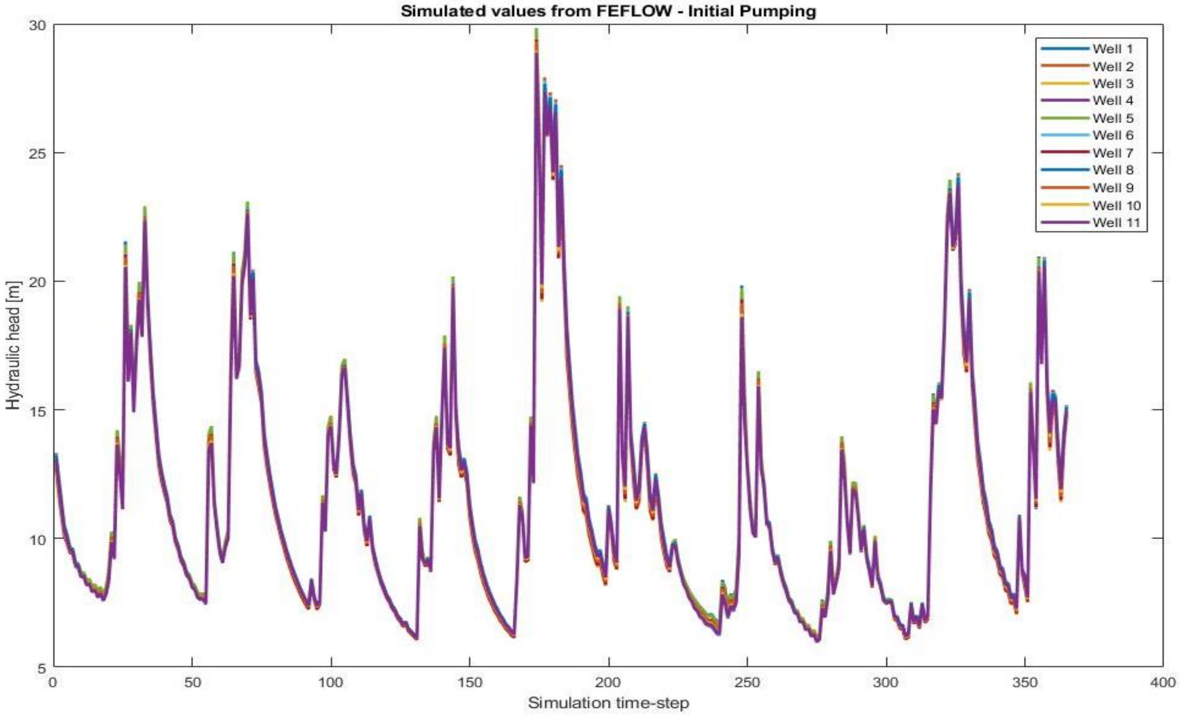


Figure 20: Hydraulic head in the observation wells considering zero pumping rates

Table 1: Results of the simulation model – Hydraulic heads in observation wells at the end of the dry season when considering zero pumping rates

Simulation Run	Hydraulic Head in Observation well										
	1	2	3	4	5	6	7	8	9	10	11
R0	8.062363	8.515735	8.09704	8.500163	8.246379	8.492439	8.136183	8.490928	8.247511	8.327616	8.392923
R1	8.028379	8.470545	8.063345	8.45532	8.215493	8.448152	8.100444	8.447421	8.210038	8.287694	8.351131
R2	8.025531	8.476084	8.059861	8.460445	8.203752	8.450493	8.096794	8.447096	8.20567	8.281288	8.346124
R3	8.034139	8.459457	8.069227	8.444508	8.227885	8.43866	8.105735	8.44021	8.215087	8.289915	8.349379
R4	8.050494	8.507137	8.084746	8.4917	8.231376	8.482782	8.122806	8.480166	8.232749	8.311556	8.379094
R5	8.029436	8.466444	8.064596	8.451501	8.221748	8.44482	8.101194	8.444665	8.210867	8.287461	8.349762
R6	8.048526	8.453117	8.082819	8.442191	8.230791	8.447671	8.121219	8.454485	8.231178	8.307245	8.3669
R7	8.026039	8.471146	8.061229	8.455801	8.217177	8.448094	8.098081	8.446973	8.20811	8.286104	8.35007
R8	8.037851	8.509672	8.072611	8.49435	8.225151	8.48591	8.116352	8.483896	8.230663	8.316098	8.384129
R9	8.034556	8.455279	8.069661	8.44194	8.22469	8.438912	8.106259	8.441619	8.215785	8.291066	8.351004
R10	8.008265	8.484565	8.042866	8.469055	8.192627	8.4596	8.082998	8.456449	8.195878	8.283065	8.354017
R11	7.98478	8.497638	8.014678	8.481674	8.162338	8.472528	8.06684	8.469537	8.195908	8.293935	8.366803
R12	8.035878	8.498156	8.070279	8.482597	8.218075	8.472811	8.105415	8.469048	8.212077	8.288769	8.363705
R13	8.026825	8.471462	8.061786	8.456063	8.212435	8.448546	8.098983	8.447556	8.209042	8.287046	8.350911
R14	8.024205	8.472842	8.058551	8.457806	8.214351	8.449282	8.09624	8.447444	8.206124	8.284437	8.349258
R15	8.045973	8.480261	8.081069	8.462491	8.228095	8.448141	8.117967	8.442773	8.227444	8.301043	8.356462
R16	8.042262	8.506376	8.076554	8.491107	8.234863	8.48227	8.114219	8.479866	8.225028	8.308772	8.378465
R17	8.049289	8.512084	8.084343	8.496338	8.232813	8.488302	8.124327	8.486537	8.237051	8.320172	8.38725
R18	8.054364	8.481212	8.089056	8.468851	8.24757	8.467113	8.12756	8.469849	8.23803	8.315737	8.37769
R19	8.017876	8.477733	8.052768	8.461985	8.199913	8.453336	8.091242	8.451046	8.202478	8.284152	8.351302
R20	8.049884	8.501994	8.084652	8.486351	8.241845	8.476638	8.122024	8.473316	8.231538	8.307011	8.371512

Table 2: Results of the simulation model – Hydraulic heads in observation wells at the end of the wet season when considering zero pumping rates

Simulation Run	Hydraulic Head in Observation well										
	1	2	3	4	5	6	7	8	9	10	11
R0	14.949	15.18338	14.81602	15.11287	15.03934	15.13988	14.78852	15.10108	14.85455	14.87993	14.94165
R1	14.91422	15.13834	14.78079	15.06821	14.99795	15.09553	14.75209	15.05722	14.81706	14.83994	14.89983
R2	14.91237	15.14419	14.77946	15.07277	15.004	15.09795	14.74903	15.05718	14.81273	14.83363	14.89485
R3	14.92063	15.12714	14.78701	15.05722	14.9978	15.0861	14.75808	15.05014	14.82211	14.8422	14.8981
R4	14.93728	15.17524	14.80446	15.10404	15.02874	15.13024	14.77499	15.09023	14.83981	14.86389	14.92781
R5	14.91597	15.13415	14.78233	15.0642	14.9945	15.09226	14.75359	15.05461	14.8179	14.83975	14.89849
R6	14.93528	15.12143	14.80254	15.05468	15.02546	15.09511	14.77336	15.06454	14.83823	14.85958	14.91562
R7	14.91259	15.1387	14.779	15.06842	14.99244	15.09554	14.75051	15.05693	14.81514	14.8384	14.8988
R8	14.92459	15.17762	14.79226	15.10662	15.01751	15.13337	14.76847	15.09394	14.83771	14.86843	14.93285
R9	14.92113	15.123	14.78752	15.05461	15.00204	15.08635	14.7587	15.05159	14.82282	14.84337	14.89973
R10	14.89498	15.15281	14.76242	15.0815	14.98311	15.10705	14.73511	15.06648	14.80293	14.83539	14.90274
R11	14.87139	15.16506	14.73292	15.09426	14.93769	15.11997	14.71957	15.07952	14.80293	14.84624	14.91553
R12	14.92258	15.16659	14.7892	15.09503	15.00917	15.12026	14.7572	15.07907	14.81912	14.84108	14.91243
R13	14.9134	15.13881	14.78004	15.06871	14.99904	15.09599	14.75137	15.05754	14.81609	14.83938	14.89963
R14	14.91089	15.14127	14.77811	15.07017	14.99135	15.09673	14.7485	15.05746	14.81317	14.83675	14.89798
R15	14.9326	15.14766	14.79898	15.0752	15.02237	15.09559	14.77031	15.053	14.83449	14.85334	14.90517
R16	14.92894	15.17472	14.79591	15.10343	15.00827	15.12974	14.76636	15.08968	14.83207	14.86109	14.9272
R17	14.93591	15.17957	14.80268	15.10907	15.02832	15.13574	14.77676	15.09643	14.84409	14.87249	14.93598
R18	14.94105	15.14948	14.80796	15.08117	15.02112	15.11454	14.77968	15.07992	14.84507	14.86805	14.92642
R19	14.9045	15.1453	14.7715	15.07471	14.99448	15.10079	14.74365	15.06109	14.80952	14.83647	14.90002
R20	14.93656	15.17018	14.80324	15.09868	15.01677	15.12408	14.7742	15.0832	14.83858	14.85933	14.92024

Table 3: Response matrix for the dry season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-42.48	-46.04	-35.28	-14.84	-41.16	-17.30	-45.40	-30.64	-34.76	-67.62	-96.98	-33.11	-44.42	-47.70	-20.49	-25.13	-16.34	-10.00	-55.61	-15.60
-56.49	-49.56	-70.35	-10.75	-61.61	-78.27	-55.74	-7.58	-75.57	-38.96	-22.62	-21.97	-55.34	-53.62	-44.34	-11.70	-4.56	-43.15	-47.50	-17.18
-42.12	-46.47	-34.77	-15.37	-40.55	-17.78	-44.76	-30.54	-34.22	-67.72	-102.95	-33.45	-44.07	-48.11	-19.96	-25.61	-15.87	-9.98	-55.34	-15.48
-56.05	-49.65	-69.57	-10.58	-60.83	-72.47	-55.45	-7.27	-72.78	-38.88	-23.11	-21.96	-55.13	-52.95	-47.09	-11.32	-4.78	-39.14	-47.72	-17.26
-38.61	-53.28	-23.12	-18.75	-30.79	-19.48	-36.50	-26.53	-27.11	-67.19	-105.05	-35.38	-42.43	-40.03	-22.85	-14.39	-16.96	1.49	-58.08	-5.67
-55.36	-52.43	-67.22	-12.07	-59.52	-55.96	-55.43	-8.16	-66.91	-41.05	-24.89	-24.53	-54.87	-53.95	-55.37	-12.71	-5.17	-31.66	-48.88	-19.75
-44.67	-49.24	-38.06	-16.72	-43.74	-18.71	-47.63	-24.79	-37.41	-66.48	-86.68	-38.46	-46.50	-49.93	-22.77	-27.46	-14.82	-10.78	-56.18	-17.70
-54.38	-54.79	-63.40	-13.45	-57.83	-45.55	-54.94	-8.79	-61.64	-43.10	-26.74	-27.35	-54.21	-54.35	-60.19	-13.83	-5.49	-26.35	-49.85	-22.01
-46.84	-52.30	-40.53	-18.45	-45.80	-20.42	-49.25	-21.06	-39.66	-64.54	-64.50	-44.29	-48.09	-51.73	-25.08	-28.10	-13.08	-11.85	-56.29	-19.97
-49.90	-57.91	-47.13	-20.08	-50.19	-25.46	-51.89	-14.40	-45.69	-55.69	-42.10	-48.56	-50.71	-53.97	-33.22	-23.56	-9.31	-14.85	-54.33	-25.76
-52.24	-58.50	-54.43	-17.29	-53.95	-32.53	-53.57	-10.99	-52.40	-48.63	-32.65	-36.52	-52.51	-54.58	-45.58	-18.07	-7.09	-19.04	-52.03	-26.76

Table 4: Response matrix for the wet season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-43.47	-45.79	-35.46	-14.65	-41.29	-17.15	-45.51	-30.51	-34.84	-67.52	-97.01	-33.02	-44.50	-47.64	-20.50	-25.07	-16.36	-9.94	-55.62	-15.55
-56.30	-48.99	-70.30	-10.17	-61.54	-77.44	-55.85	-7.20	-75.48	-38.21	-22.90	-20.99	-55.71	-52.64	-44.65	-10.82	-4.76	-42.37	-47.60	-16.50
-44.04	-45.70	-36.26	-14.45	-42.11	-16.85	-46.28	-29.70	-35.62	-67.00	-103.88	-33.53	-44.98	-47.39	-21.30	-25.14	-16.67	-10.08	-55.65	-15.97
-55.82	-50.12	-69.56	-11.04	-60.84	-72.74	-55.56	-7.81	-72.82	-39.21	-23.26	-22.30	-55.20	-53.38	-47.09	-11.80	-4.75	-39.62	-47.70	-17.74
-51.74	-44.17	-51.92	-13.25	-56.05	-17.35	-58.62	-27.29	-46.63	-70.29	-127.06	-37.71	-50.37	-59.99	-21.21	-38.84	-13.77	-22.77	-56.07	-28.21
-55.44	-52.41	-67.22	-12.05	-59.53	-55.96	-55.43	-8.14	-66.91	-41.04	-24.89	-24.52	-54.86	-53.94	-55.36	-12.67	-5.17	-31.67	-48.86	-19.75
-45.54	-49.36	-38.05	-16.91	-43.66	-18.95	-47.51	-25.06	-37.28	-66.76	-86.19	-39.15	-46.44	-50.03	-22.76	-27.70	-14.70	-11.05	-56.09	-17.90
-54.83	-54.87	-63.67	-13.56	-58.09	-45.68	-55.19	-8.92	-61.86	-43.25	-26.95	-27.51	-54.43	-54.52	-60.10	-14.25	-5.81	-26.45	-49.99	-22.35
-46.86	-52.27	-40.55	-18.42	-45.81	-20.40	-49.26	-21.05	-39.66	-64.52	-64.52	-44.29	-48.07	-51.73	-25.07	-28.10	-13.08	-11.85	-56.29	-19.96
-49.99	-57.88	-47.16	-20.05	-50.22	-25.44	-51.91	-14.37	-45.70	-55.67	-42.11	-48.56	-50.69	-53.97	-33.24	-23.55	-9.30	-14.85	-54.32	-25.75
-52.27	-58.50	-54.44	-17.30	-53.95	-32.54	-53.56	-11.00	-52.40	-48.64	-32.65	-36.52	-52.52	-54.59	-45.60	-18.06	-7.09	-19.04	-52.04	-26.76

The optimum pumping rates were determined for four different values for the water table level of reference, H_{ref}: a) H_{ref}=6,25 m (upgrading water level up to 0,60 m), b) H_{ref}=6,5 m (upgrading water level up to 0,85 m), c) H_{ref} =6,75 m (upgrading water level up to 1,1 m) and d) H_{ref} =7 m (upgrading water level up to 1,35 m). The water table level in the observation wells is constrained to values greater than H_{ref}. Additionally, the lower bounds of the pumping rates were set to zero and the upper bounds were determined by the licenses held.

2.2.2.1. Optimum Pumping Rates for Href=6,25 m

The optimum pumping rates for H_{ref} =6,25m were found after 2 iterations of the procedure. The first optimization run was based on the first response matrix (Tables 3 and 4). The calculated *b* vector for the constraints' values was found as recorded in Table 5.

Table 5: b vector of constraints for the first run of H_{ref}=6.25m

Well	b for the dry season	Well	b for the wet season
1	1.8124	1	8.699
2	2.2657	2	8.9334
3	1.8470	3	8.5660
4	2.2502	4	8.8629
5	1.9964	5	8.7893
6	2.2424	6	8.8899
7	1.8862	7	8.5385
8	2.2409	8	8.8511
9	1.9975	9	8.6046
10	2.0776	10	8.6299
11	2.1429	11	8.6917

The optimum values of the pumping rates were found as shown in Table 6 for the wet and the dry season.

Table 6: Optimum Pumping Rates per Season for the first run of H_{ref}=6.25m

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.164
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.82
8	474.19	381.20
9	1497.99	815.75
10	1334.78	1295.06
11	1559.42	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.11	140.08
19	1561.13	267.90
20	162.72	92.96
Total pumping rate	$3.7486 \cdot 10^4$	$3.1254 \cdot 10^4$

For the second simulation – optimization procedure the optimum values found in the previous set were used as the initial ones. The model was run for the initial pumping rates as well as for the disturbed ones. For this set of runs, the disturbance was set to 10% of the initial pumping rates. The same procedure as in the previous step was followed and the results for the optimum pumping rates are recorded in Table 7. The second response matrix A and the calculated b vector are available in the Appendix.

Table 7: Optimum Pumping Rates per Season for the second run of H_{ref}=6.25m

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.164
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.82
8	474.19	381.20
9	1497.99	815.75
10	1334.78	1295.06
11	1559.42	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.11	140.08
19	1561.13	267.90
20	162.72	92.96
Total pumping rate	$3.7486 \cdot 10^4$	$3.1254 \cdot 10^4$

The values obtained from the second iteration are the same with these of the first one. Therefore, these pumping rates are the optimum ones for upgrading the water table level up to 0,60 m and for the dry season are graphically presented in Figure 21. The saltwater intrusion zone is shown in Figures 22 and 23 and the water level in the observation wells in Figure 24.

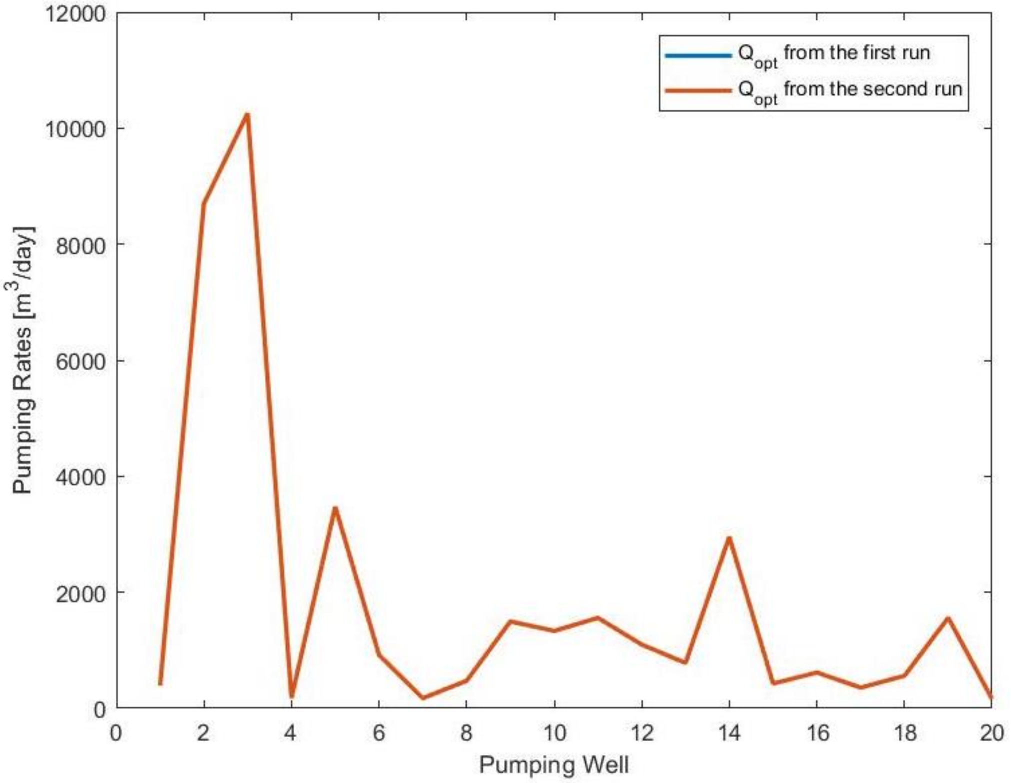


Figure 21: Optimum pumping rates for the dry season

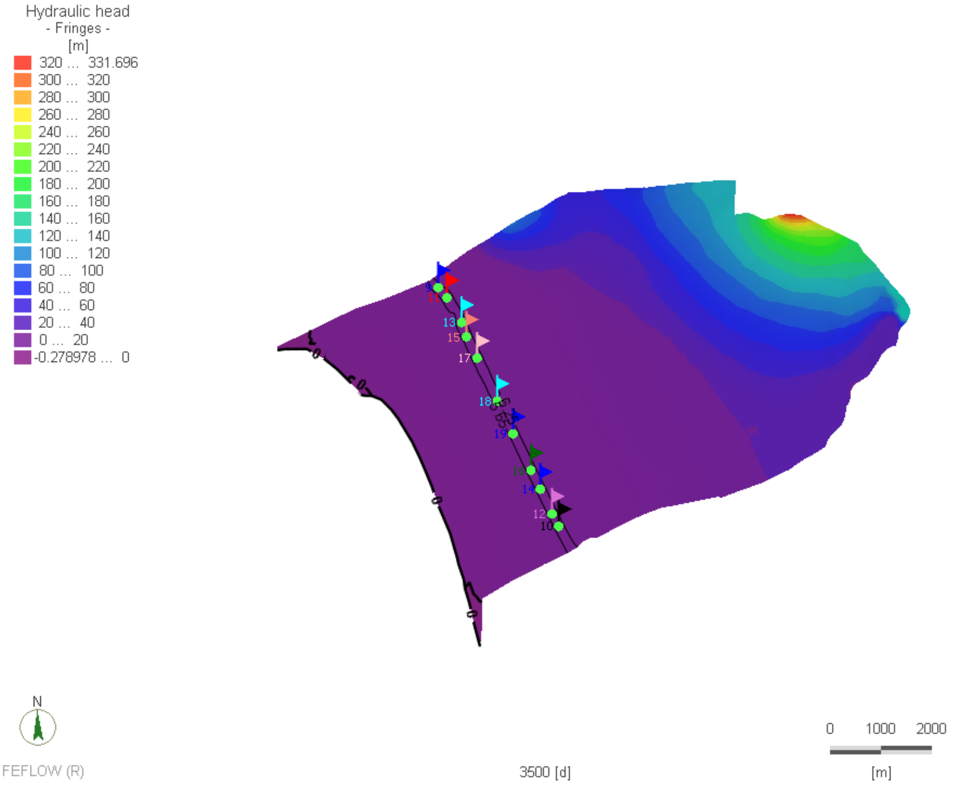


Figure 22: Saltwater intrusion at the end of the dry season considering the optimum pumping rates for H_{ref}=6.25m

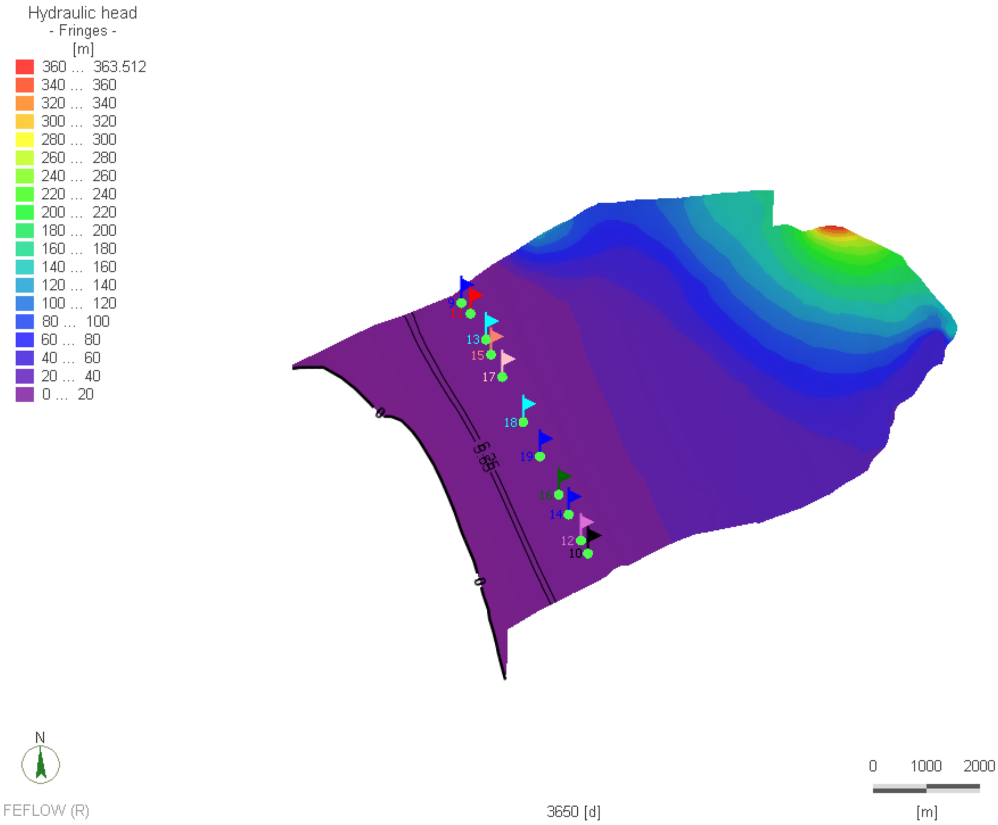


Figure 23: Saltwater intrusion at the end of the wet season considering the optimum pumping rates for H_{ref}=6.25m

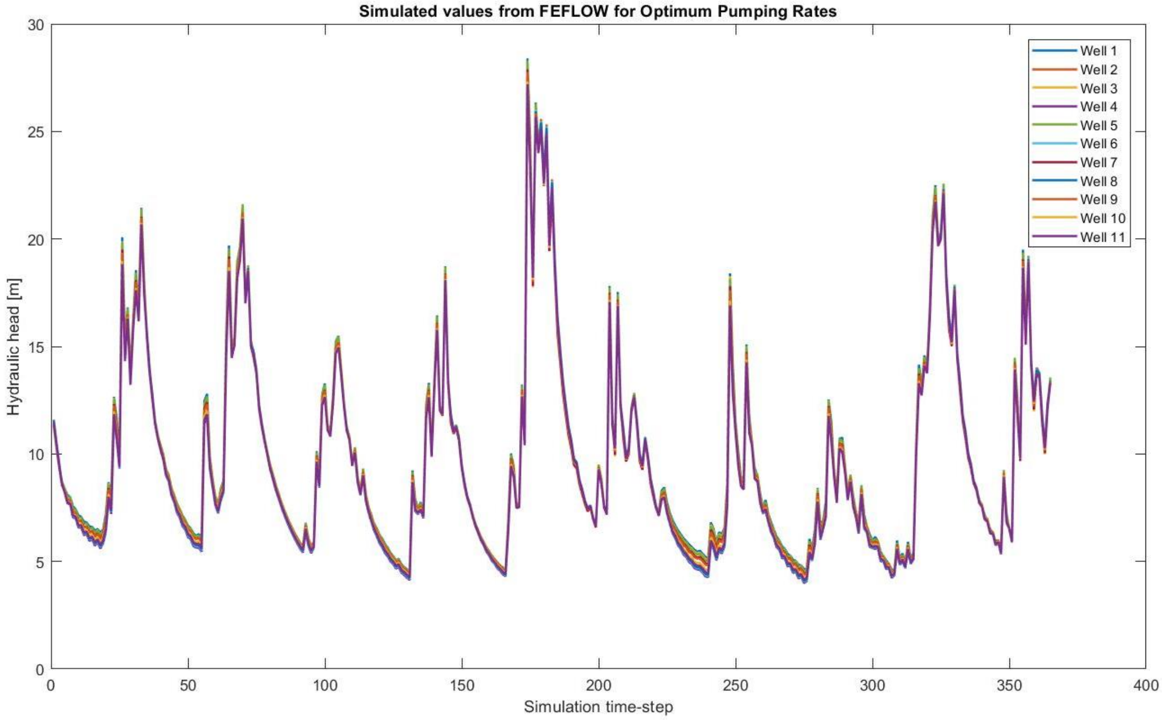


Figure 24: Hydraulic heads in the observation wells considering the optimum pumping rates for H_{ref}=6.25m

2.2.2.2. Optimum Pumping Rates for Href=6,5 m

The optimum pumping rates for H_{ref} =6,5m were found after 3 iterations of the procedure. The first optimization run was based on the first response matrix (Tables 3 and 4). The calculated *b* vector for the constraints' values was found as shown in Table 8.

Table 8: b vector of constraints for the first run of H_{ref}=6.5m

Well	b for the dry season	Well	b for the wet season
1	1.562363	1	8.449
2	2.015735	2	8.68338
3	1.59704	3	8.31602
4	2.000163	4	8.61287
5	1.746379	5	8.53934
6	1.992439	6	8.63988
7	1.636183	7	8.28852
8	1.990928	8	8.60108
9	1.747511	9	8.35455
10	1.827616	10	8.37993
11	1.892923	11	8.44165

The optimum values of the pumping rates were found as shown in Table 9 for the wet and the dry season.

Table 9: Optimum Pumping Rates per Season for the first run of H_{ref}=6.5m

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.16
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1371.23	815.75
10	1334.78	1295.06
11	1280.68	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7080 \cdot 10^4$	$3.1254 \cdot 10^4$

For the second simulation – optimization procedure the optimum values found in the previous set were used as the initial ones. The model was run for the initial pumping rates as well as for the disturbed ones. For this set of runs, the disturbance was set to 10% of the initial pumping rate. The same procedure as in the previous step was followed and the results for the optimum pumping rates are recorded in Table 10. The simulation results, the second response matrix *A* and the calculated *b* vector are available in the Appendix.

Table 10: Optimum Pumping Rates per Season for the second run of H_{ref}=6.5m

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.16

2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1498.00	815.75
10	1334.78	1295.06
11	1472.43	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7399 \cdot 10^4$	$3.1254 \cdot 10^4$

Concerning the wet season, the optimum values from the two consecutive runs are the same. However, for the dry season, there are two pumping wells that their pumping rates have a difference in their values, as shown in Figure 25. Therefore, the same procedure is repeated for a third time and optimum pumping rates are recorded in Table 11. For the third set of runs, the disturbance was set to 20% of the initial pumping rate. The simulation results, the third response matrix A and the calculated b vector are available in the Appendix. The optimum values of the third iteration converge with these from the second one, as depicted in Figure 26. Therefore, the problem has been linearized and the final optimum values are those of the third iteration.

Table 11: Optimum Pumping Rates per Season for the third run of Href=6.5m

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)

1	392.55	214.16
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1498.00	815.75
10	1334.78	1295.06
11	1464.07	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7390\cdot10^4$	$3.1254\cdot10^4$

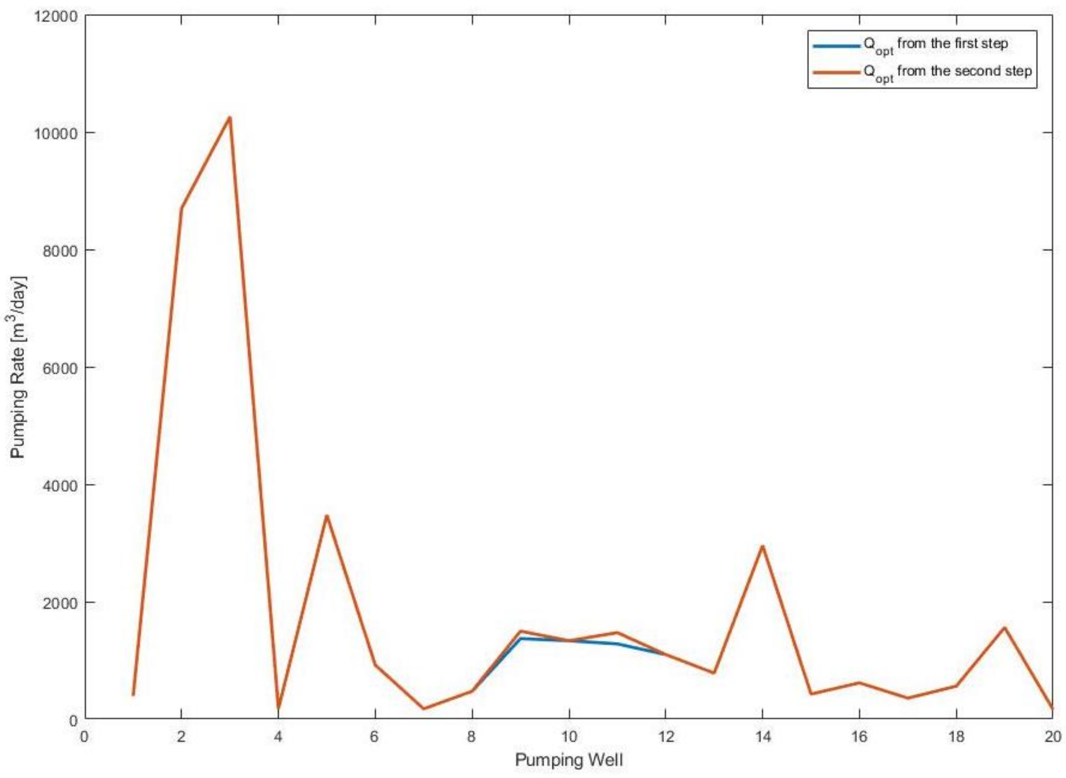


Figure 25: Optimum Pumping Rates for the Dry Season form the first and second S-O runs for H_{ref}=6.5m

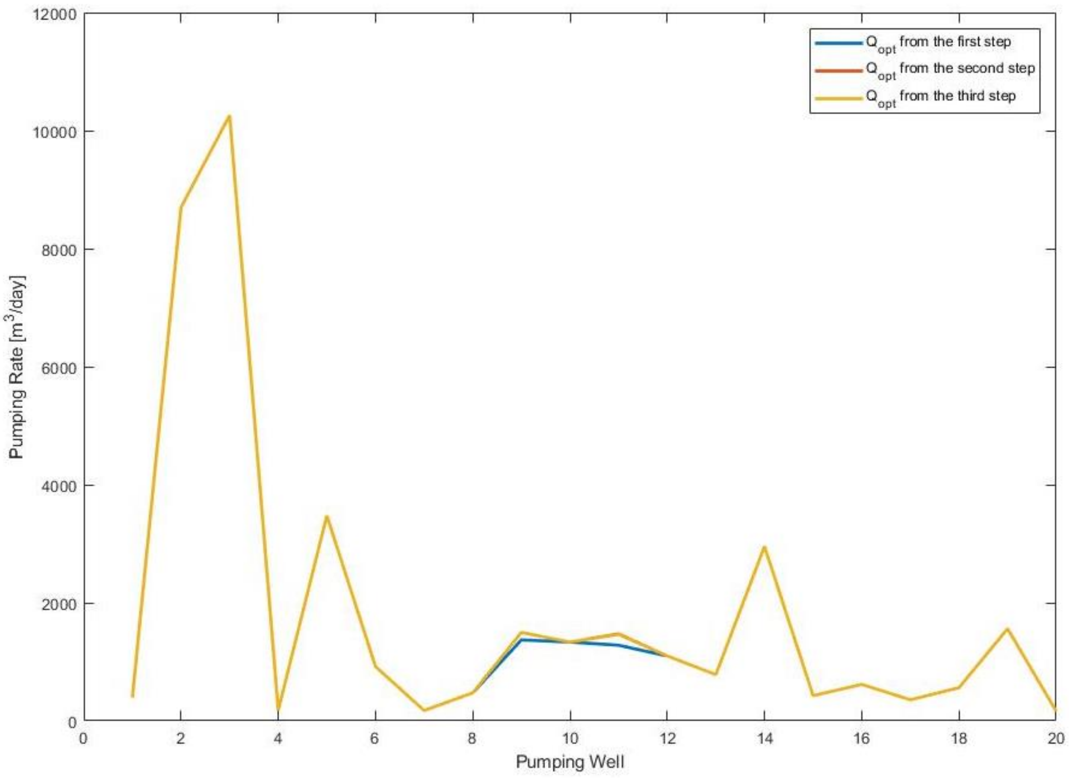


Figure 26: Optimum Pumping Rates for the Dry Season form the first, second and third S-O runs for H_{ref}=6.5m

The hydraulic head level in the observation wells is shown in the following Figure 29 and the saltwater intrusion front at the end of the dry and the wet period of the 10-year simulation in Figures 27 and 28.

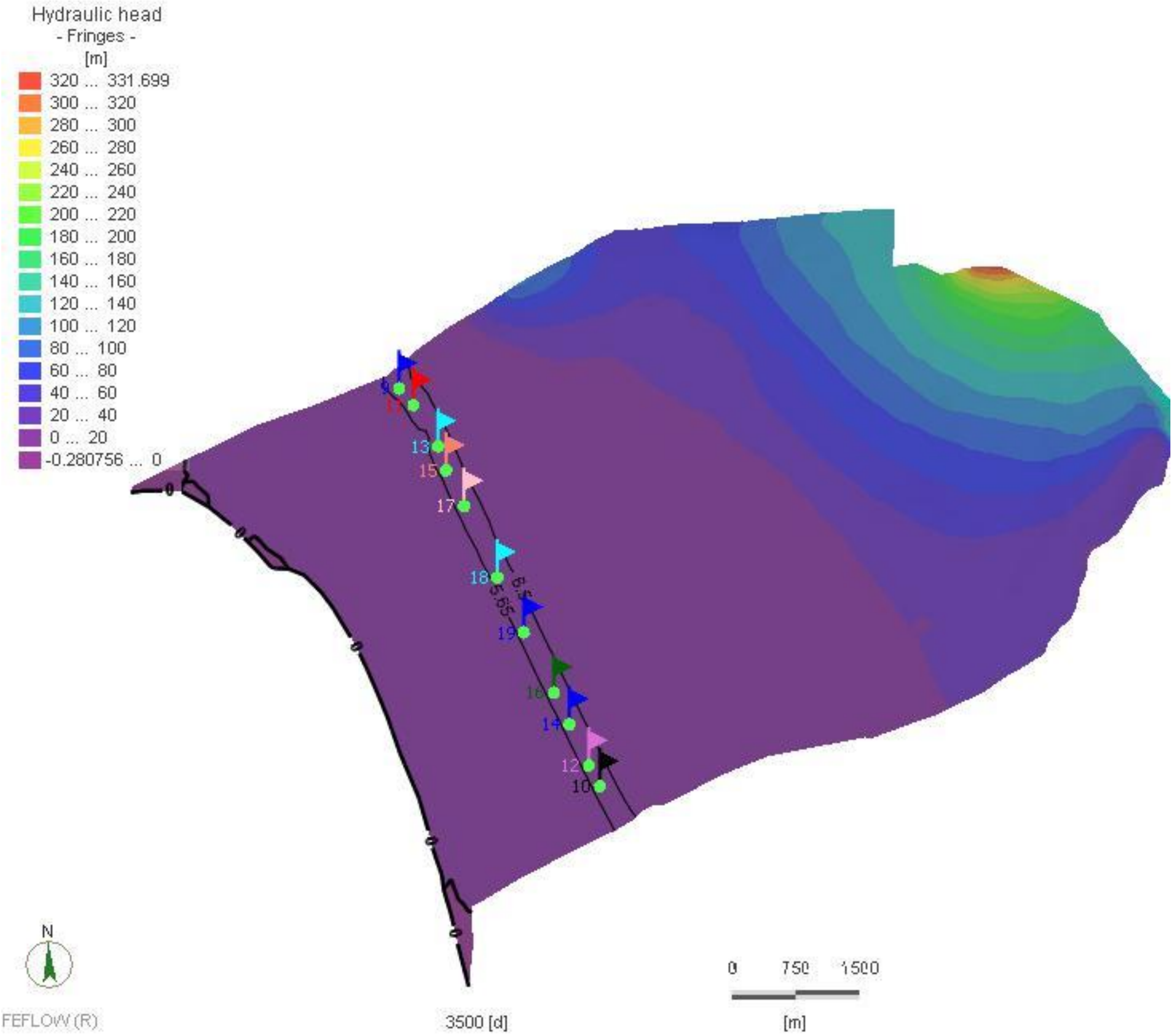


Figure 27: Saltwater intrusion at the end of the dry season considering the optimum pumping rates for H_{ref}=6.5m

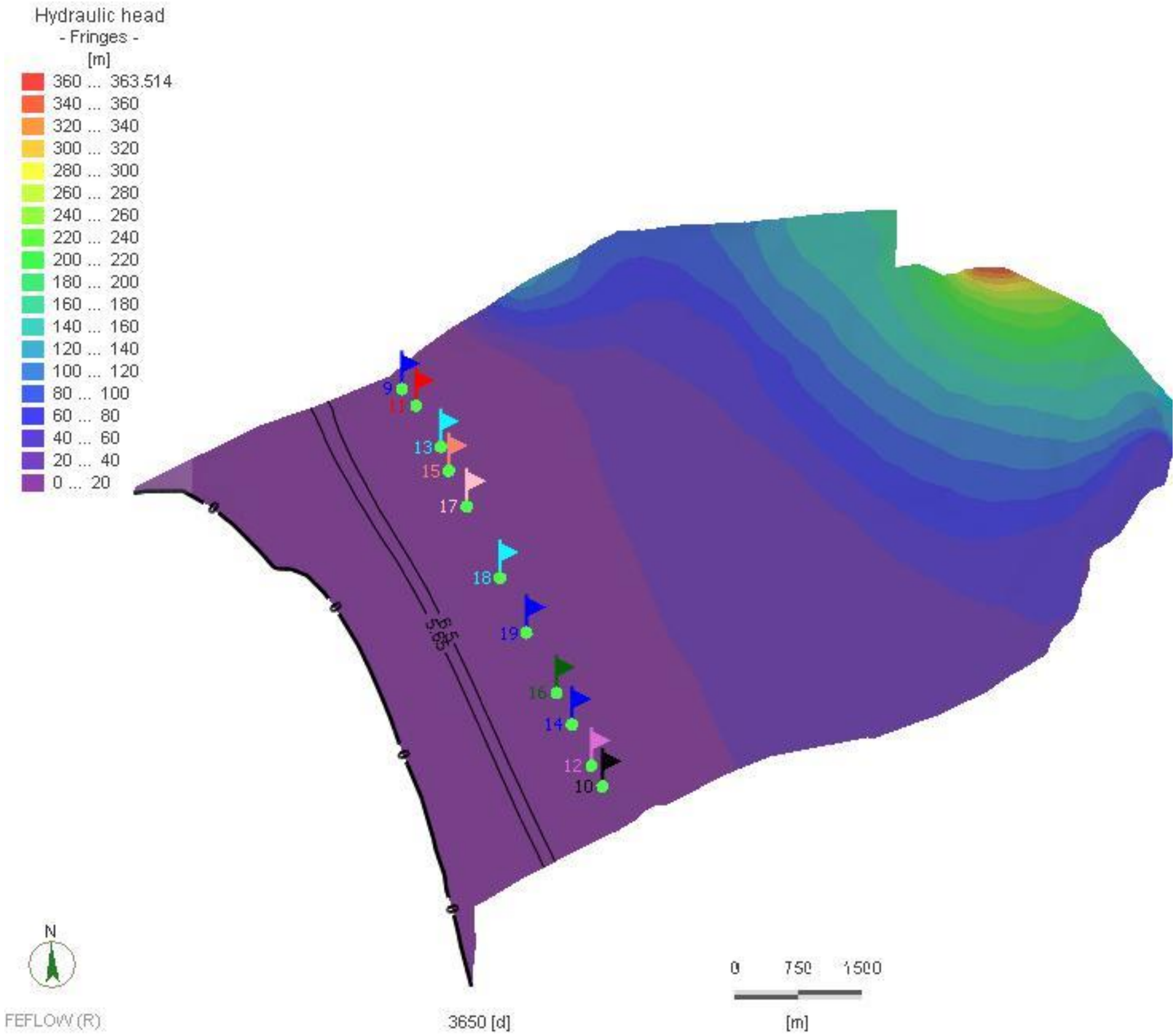


Figure 28: Saltwater intrusion at the end of the wet season considering the optimum pumping rates for H_{ref}=6.5m

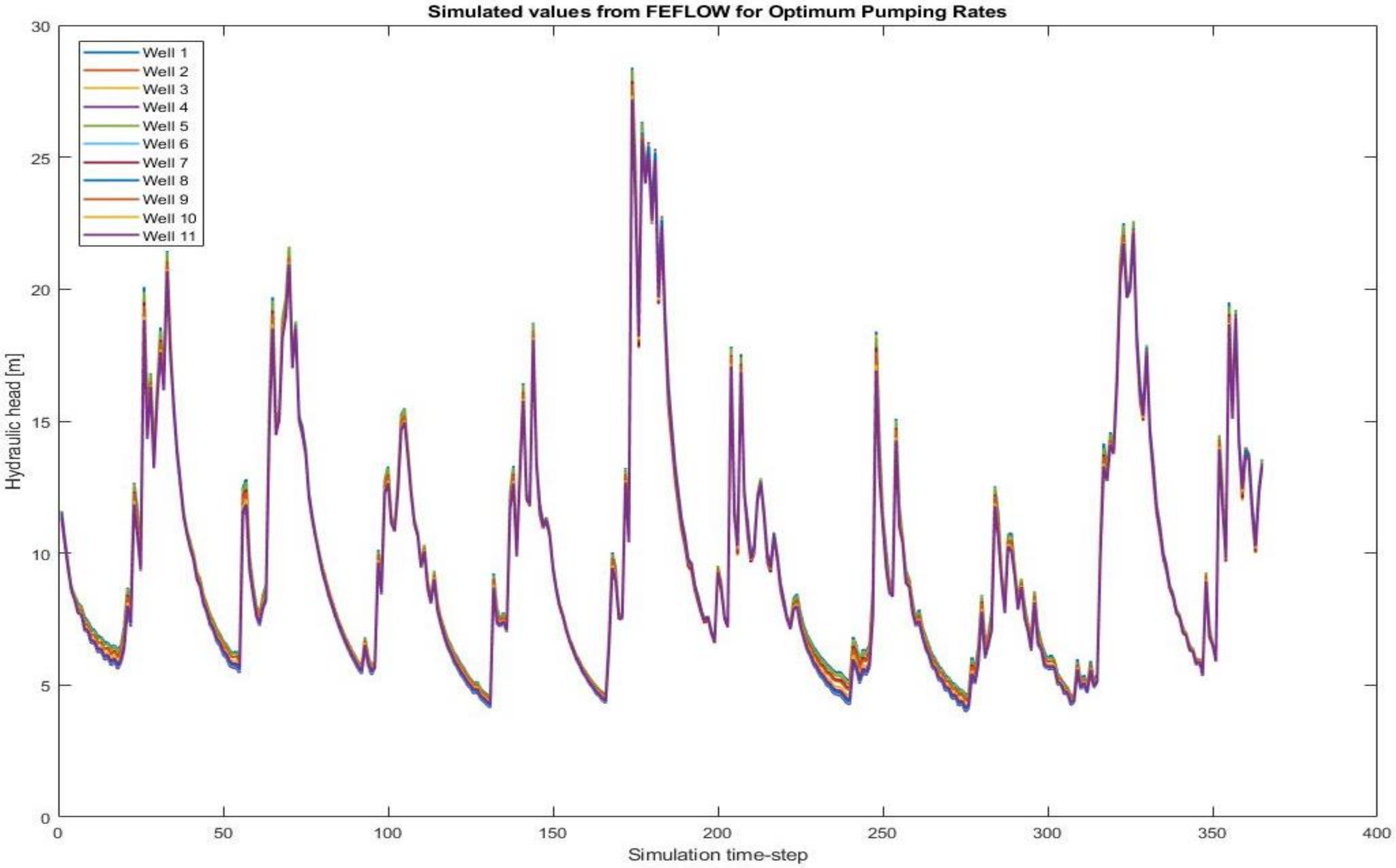


Figure 29: Hydraulic heads in observation wells when considering optimum pumping rates for Href=6.5m

2.2.2.3. Simulation and Optimization – Situation for the optimum pumping rates for Href=6.75 m

The simulation – optimization runs that were executed aiming at upgrading the water table level up to 1.1 m followed the pattern summarized in Table 12.

2.2.2.4. Simulation and Optimization – Situation for the optimum pumping rates for H_{ref}= 7 m

The simulation – optimization runs that were executed aiming at upgrading the water table level up to 1.35 m followed the pattern summarized in Table 13.

Table 12: Description of the simulation – optimization run parameters and results for H_{ref}=6.75m

Run	Initial Pumping Rates	Disturbances		Optimum Values for Pumping Rates	Number of Wells in which there is convergence with previous run's results	Total Pumping Rates (m^3/day)	
		If Qo = 0	If Qo ≠ 0			Dry Season	Wet Season
1	0	800	0.1 of Qo	Qopt1	-	$3.2451 \cdot 10^4$	$3.1254 \cdot 10^4$
2	Qopt1	800	0.1 of Qo	Qopt2	13	$3.7331 \cdot 10^4$	$3.1254 \cdot 10^4$
3	Qopt2	800	0.2 of Qo	Qopt3	14	$3.7357 \cdot 10^4$	$3.1254 \cdot 10^4$
4	Qopt3	800	0.3 of Qo	Qopt4	13	$3.2931 \cdot 10^4$	$3.1254 \cdot 10^4$
5	Qopt4	800	0.4 of Qo	Qopt5	13	$3.2860 \cdot 10^4$	$3.1254 \cdot 10^4$
6	Qopt5	800	0.5 of Qo	Qopt6	12	$3.3457 \cdot 10^4$	$3.1254 \cdot 10^4$
7	Qopt6	800	0.6 of Qo	Qopt7	14	$3.3008 \cdot 10^4$	$3.1254 \cdot 10^4$
8	Qopt7	800	0.7 of Qo	Qopt8	18	$3.7486 \cdot 10^4$	$3.1254 \cdot 10^4$
9a	Qopt8	800	0.8 of Qo	Qopt9a	14	$3.2886 \cdot 10^4$	$3.1254 \cdot 10^4$
9b	Qopt8	800	0.1 of Qo	Qopt9b	17	$3.2881 \cdot 10^4$	$3.1254 \cdot 10^4$
11	Qopt9b	800	0.2 of Qo	Qopt11	13	$3.3189 \cdot 10^4$	$3.1254 \cdot 10^4$

Table 13: Description of the simulation – optimization run parameters and results for H_{ref}=7 m

Run	Initial Pumping Rates	Disturbances		Optimum Values for Pumping Rates	Number of Wells in which there is convergence with previous run's results	Total Pumping Rates (m^3/day)	
		If Qo = 0	If Qo ≠ 0			Dry Season	Wet Season
1	0	800	0.1 of Qo	Qopt1	-	$2.7644 \cdot 10^4$	$3.1254 \cdot 10^4$
2	Qopt1	800	0.1 of Qo	Qopt2	11	$2.8303 \cdot 10^4$	$3.1254 \cdot 10^4$
3	Qopt2	800	0.2 of Qo	Qopt3	12	$3.5512 \cdot 10^4$	$3.1254 \cdot 10^4$
4	Qopt3	800	0.3 of Qo	Qopt4	13	$2.7824 \cdot 10^4$	$3.1254 \cdot 10^4$
5	Qopt4	800	0.4 of Qo	Qopt5	15	$2.8229 \cdot 10^4$	$3.1254 \cdot 10^4$
6a	Qopt5	800	0.5 of Qo	Qopt6a	12	$2.8425 \cdot 10^4$	$3.1254 \cdot 10^4$
6b	Qopt5	800	0.1 of Qo	Qopt6b	13	$3.3961 \cdot 10^4$	$3.1254 \cdot 10^4$

2.2.2.5. Discussion

The piece-wise linear technique was applied for four different values of the hydraulic head of reference (scenarios) and for the same 10-year period of simulation, with precipitation data available. As observed from the different scenarios, the higher the hydraulic head of reference, the more difficult to obtain the optimum values of the pumping rates. For the two first scenarios, the optimum values were found after two and three iterations, respectively. On the other hand, in the third and fourth scenarios, an optimal solution could not be found at all. In these scenarios, it was also observed that after a certain number of iterations, a maximum number of wells in which there is convergence between two consecutive runs exists, but does not reach the total number of the pumping wells. Therefore, it is assumed that the constraint is strict enough so as not to enable the finding of the optimum solution for the certain duration of the simulation period.

As the research progresses, new scenarios will be formed in order to find the optimum pumping rates for different hydraulic heads of reference, but in longer simulation periods. In addition, for the next steps, it is planned to perform the training of the Fuzzy Inference System that will combine the different optimization criteria (social, economic and environmental) in order to obtain the optimum solution based on these multiple criteria.

3. Environmental Data Management System

An environmental data management system has been created in order to gather and mine the environmental data. The design of the database includes the storage of the collected data in a coherent environmental database management system (DBMS) for further use within the project. A critical issue that was taken into is the existence of queries for automatic updating, importing data, exporting data, selecting specific subsets and grouping records. The database was connected with an open-source business intelligence tool, which helps users question the database and visualize answers in useful optical formats that can be shared.

3.1. Type of Database

The two most common types of databases are the relational and Not only SQL databases (NoSQL). The relational model, introduced by Edgar F. Codd in the early 1970’s, defines a methodology for organizing structured data into relations (tables with columns and rows) and for defining the relationships between those tables. Each table contains a primary key and must be indexed. A primary key is a field which is guaranteed to have a unique entry for each record in a table. Indexes provide a quick navigation through the database, in order to retrieve specific data subsets. The entries in the attribute chosen as index should be unique or rare, within that field. Usually, the primary key will be chosen as an index, although this is not a requirement. Relational databases can be normalised to improve performance of queries, reduce data duplication and make the database more elegant.

At the heart of the relational model is the Structured Query Language (SQL), a standards-based programming language used to define database schema and the relationships between tables. The language is also used to store, manipulate, and retrieve data from those tables. SQL has been adopted by both the American National Standards Institute (ANSI) and the International Organization for Standardization (ISO) and is well known and widely supported by developers around the globe.

Relational databases offer many important features that make them aptly suited to enterprise workloads, which is why organizations have been turning to them for so long. They’re optimized for handling highly structured data, and their inherent characteristics—such as normalization, atomicity, and consistency—ensure the integrity of that data throughout its lifespan. These features also contribute to better storage utilization, while providing flexible query support through standards-based SQL. However, relational databases are not appropriate for semi-structured or unstructured data, especially at scale. A relational database also requires a rigid schema that must be carefully planned and does not easily accommodate changing requirements, leaving little room for the dynamic nature of many of today's applications and development methodologies.

Due to the existence of highly structured data in the current project, the database schema is predefined, resulting in a rigid structure that helps to optimize storage and ensure data integrity. Furthermore, there is a crucial need for performing complex queries against the structured data. Finally, a high degree of data integrity, adhering to the principles of atomicity, consistency, isolation, and durability is also necessary, along with a mature database technology, that is supported by large developer communities. Thus, a relational database best suits the needs of the current project.

3.2. Relational Database Management System

MySQL is a popular, high performance, open-source, relational DBMS (Database Management System), paired with ongoing development and support from Oracle. MySQL can be used in conjunction with MySQL Workbench, a unified visual tool which provides data modeling, SQL development and comprehensive administration tools for server configuration, user administration, backup, and much more. It is platform agnostic and makes interactions with relational databases much easier. Often associated with internet applications or web services, MySQL was designed to be extensively compatible with other technologies and architectures. The RDBMS runs on all major computing platforms, including Unix-based operating systems, such as the myriad Linux distributions or Mac OS and Windows. MySQL's client-server architecture means it can support a variety of backends, as well as different programming interfaces. Third-party migration tools further allow MySQL to move data to and from a vast set of general storage systems, whether these are designed to be on-premises or cloud-based. MySQL can be deployed in virtualized environments, distributed or centralized, and even exists as portable standalone libraries for learning purposes, testing, or small applications. MySQL's wide compatibility with all these other systems and software makes it a particularly practical choice of RDBMS in most situations. Any individual or enterprise may freely use, modify, publish, and expand on Oracle’s open-source MySQL code base. The software is released under the GNU General Public License (GPL). The public and community-based nature of open-source releases enriches MySQL’s documentation and online support culture, while also ensuring that sustained or newly-developed capabilities never stray too far from current user needs.

Due to the features described above, MySQL is perfectly suited for the environmental data within this project, so it will be used for all interactions throughout it.

3.3. Design Features

Normalization is a database design technique that reduces data redundancy and eliminates undesirable characteristics like insertion, update and deletion anomalies. Normalization rules divide larger tables into smaller and link them using relationships. Thus, Normal Forms are used to eliminate or reduce redundancy in database tables. There are different levels of normalisation, such as:

- 1st Normal Form (1NF)

If a relation contains composite or multi-valued attribute, it violates first Normal Form or a relation is in first Normal Form if it does not contain any composite or multi-valued attribute. Thus, repeated sets of related data are removed and put into an individual table, where each set of related data has its own separate table and is identified with a primary key.

- 2nd Normal Form (2NF)

To be in second Normal Form, a relation must be in first normal form and relation must not contain any partial dependency. A relation is in 2NF if it has no partial dependency, i.e., no non-prime attribute (attributes which are not part of any candidate key) is dependent on any proper subset of any candidate key of the table.

- 3rd Normal Form (3NF)

A table is considered in third Normal Form if the table/entity is already in the second Normal Form and the columns of the table/entity are non-transitively dependent on the primary key.

- Boyce-Codd Normal Form (BCNF)

A relation is in Boyce-Codd Normal Form (BCNF) if every determinant is a candidate key. The difference between 3NF and BCNF is that for a functional dependency A-> B, 3NF allows this dependency in a relation if B is a primary-key attribute and A is not a candidate key, whereas BCNF insists that for this dependency to remain in a relation, A must be a candidate key.

In the context of the current project, every table was designed by means of describing one characteristic (i.e. weather station, temperature measurement, rain measurement) and the associated data which is necessary for describing that characteristic. 3NF normalization was used in order to eliminate or reduce redundancy in database tables and to improve the performance of queries. Thus, every table present on the database satisfies the criteria of 2NF, non-primary key columns shouldn’t depend on other non-primary key columns and there is absence of transitive functional dependencies.

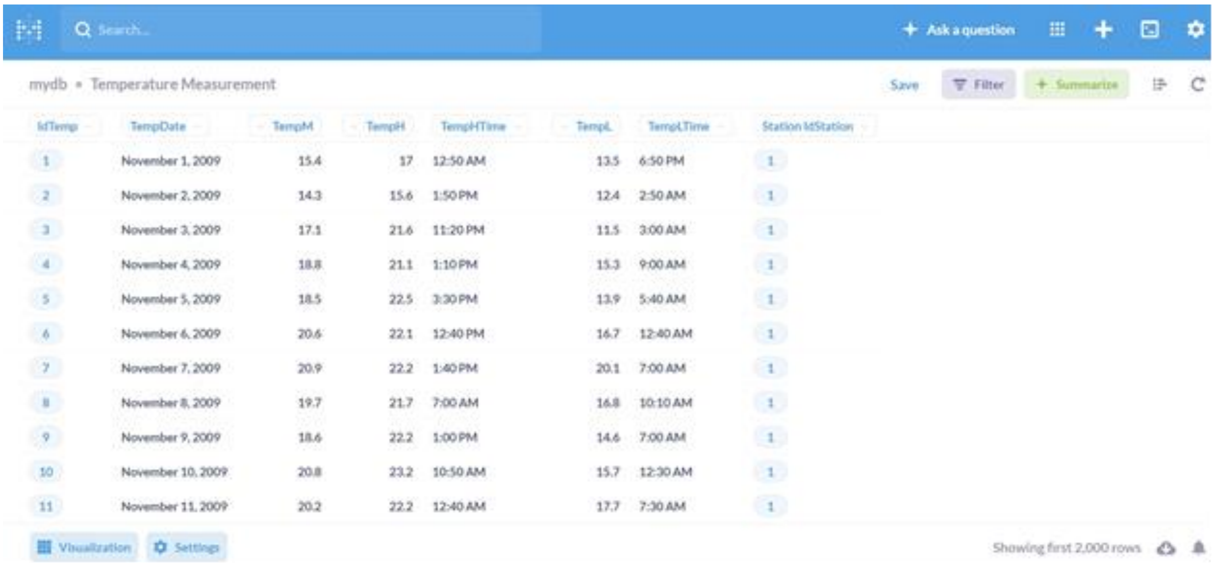
3.4. Metabase: a brief overview

Metabase is a rapidly growing open source business intelligence tool, crafted in such a way that all kinds of charts, plots and graphs with different designs can be positioned simultaneously for data visualization. Moreover, Metabase’s intuitive interface lets you ask questions on your data and displays answers in famous downloadable formats. Questions can be saved, making it easy to come back to them and they can also be grouped into dashboards in order to be shared with the rest of your team. Metabase uses the MariaDB connector to connect easily to MariaDB and MySQL servers. It sets up really quickly, connecting to your database and bringing its data to life in beautiful visualizations, opening data up for everyone, not just analysts and developers

Metabase home page allows some automatic explorations of the tables that a user can look at and save as a dashboard, an area where things that the teammates create will show up, along with a link to explore all the created dashboards and saved questions. Once some dashboards have been created, any of them that are pined in the main “Our analytics” collection will show up on the homepage for all of the teammates.

3.4.1. Browsing data

If a connection is finally established between Metabase and a database, the current database is listed at the bottom of the homepage along with the sample dataset that Metabase comes with. Users can easily click on a database to see its contents and can easily click on a table to see its rows. Moreover, by clicking on the bolt icon to x-ray a table, the user can see an automatic exploration of it. Metabase can present the data in a variety of ways, just by changing the visualization mode from the “Visualization” sidebar.



IdTemp	TempDate	TempM	TempH	TempHTime	TempL	TempLTime	Station IdStation
1	November 1, 2009	15.4	17	12:50 AM	13.5	6:50 PM	1
2	November 2, 2009	14.3	15.6	1:50 PM	12.4	2:50 AM	1
3	November 3, 2009	17.1	21.6	11:20 PM	11.5	3:00 AM	1
4	November 4, 2009	18.8	21.1	1:10 PM	15.3	9:00 AM	1
5	November 5, 2009	18.5	22.5	3:30 PM	13.9	5:40 AM	1
6	November 6, 2009	20.6	22.1	12:40 PM	16.7	12:40 AM	1
7	November 7, 2009	20.9	22.2	1:40 PM	20.1	7:00 AM	1
8	November 8, 2009	19.7	21.7	7:00 AM	16.8	10:10 AM	1
9	November 9, 2009	18.6	22.2	1:00 PM	14.6	7:00 AM	1
10	November 10, 2009	20.8	23.2	10:50 AM	15.7	12:30 AM	1
11	November 11, 2009	20.2	22.2	12:40 AM	17.7	7:30 AM	1

Figure 30: Sample visualization of temperature data as a table

Bubble size
Select a field

X-axis
TempDate

Y-axis
TempH

Figure 31: Sample visualization of temperature data as a scatter-plot

3.4.2. Asking Questions

Metabase's two core concepts are questions and their corresponding answers. Everything else is based around questions and answers. A user can ask a question in Metabase by clicking the "Ask a Question" button at the top of the screen. There are three ways to ask a specific question in Metabase:

- The simple question mode, which lets the user filter, summarize and visualize data.
- The custom question mode, which gives the user a powerful notebook-style editor to create more complex questions that require joins, multiple stages of filtering and aggregating, or custom columns.
- The SQL/native query editor.

The answers can be downloaded as an .xlsx, .csv, or .json file. The maximum download size is 1 million rows.

3.4.2.1. Simple Questions

After the user selects the Simple Question option, he will need to pick some data that has a question about. The Simple Question could be on a user's table or something like events, orders or downloads. Moreover, there are options for filtering data and/or summarizing them.

By clicking the "Filter" option, a list of all of the columns in the browsed table, as well as columns from tables that are related to the current table, appears. Depending on the column picked, there are slightly different options for the filter. Generally, there are three types of columns, each with their own set of filtering options:

- Numeric columns let the user add filters to only include table rows where the values of the attribute are between two specific values, greater or less than a specific value or exactly equal to a specific value.
- With text or category columns, users can specify that only want to include data where this column is or isn't a specific option or can exclude rows that don't have a value for this column.
- Date columns give a calendar or input box so that specific time ranges or all days before or after a certain date can be selected.

Metabase administrators own the option to create special named filters for the tables and if they do so, the filters appear at the top of the filter dropdown in purple text with a star next to them. These are called "segments" and they are shortcuts to a combination of filters that are commonly used by the members of the team sharing the database.

The "Summarize" option has two main parts. The metric desired kai the type of data group. By default, the "count of rows" metric will be selected, since it's super common, but it can easily be changed. The user can also select more than one metric to view. The available metrics are listed below:

- Count of rows: the total of number of rows in the table, after any filters have been applied.
- Sum of: the sum of all the values in a specific column.
- Average of: the average of all the values in a single column.
- Number of distinct values of: the number of unique values in all the cells of a single column.

- Cumulative sum of: a running total for a specific column. In order for this metric to be useful the user needs to group it by a date column to see it across time.
- Cumulative count of rows: a running total of the number of rows in the table over time. Just like "Cumulative sum of", the user needs to group this by a date column in order for it to be useful.
- Standard deviation of: a number which expresses how much the values of a column vary, plus or minus, from the average value of that column.
- Minimum of: the minimum value present in the selected field.
- Maximum of: the maximum value present in the selected field.

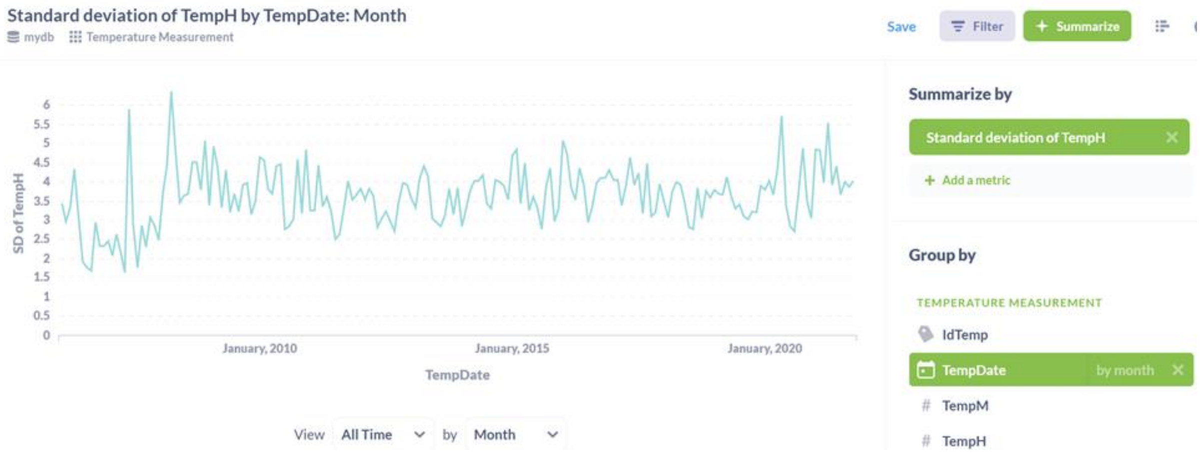


Figure 32: Sample of summarizing data in a simple question

3.4.2.2. Custom Questions with Notebook Editor

If the user has a question that's a bit more involved than a simple question, he can create a custom question using the notebook editor, by clicking the "Ask a Question" button in the top navigation bar and selecting "Custom Question".

The notebook is made up of a sequence of individual steps. Under each step, there are buttons to add more steps after the current one. To the right of each step is a preview button that shows the first 10 rows of the results of the question up to that step.

This first step is required and is where the user picks the data that wants to base the question on. In most cases, users pick one of the tables in the database created, but they can also choose a previously saved question's result as the starting point for the new question. What this means in practice is that the user can do things like use complex SQL queries to create new tables that can be used as starting data in a question just like any other table in your database.

The users can have most saved questions as source data, provided they have permission to view that question. They can even use questions that were saved as a chart rather than a table. Although, there are some kinds of saved questions that can't be used as source data:

- Druid questions,
- Google Analytics questions,
- Mongo questions,
- questions that use "Cumulative Sum" or "Cumulative Count" aggregations,
- questions that have columns that are named the same or similar thing, like "Count" and "Count 2".

During the next step of custom question design, the user has the option to select one or more columns to filter on. Adding a summarize step lets the user choose how to aggregate the data from the previous step. He can pick one or more metrics and optionally group those metrics by one or more columns. When picking the metrics, the user can choose from basic functions like sum, average and count, can pick a common metric that an admin has defined or can create a custom expression by writing a formula.

Furthermore, the user can join data to combine current table with others or even with a saved question. Currently joins are not available, if the starting data is from a Google Analytics or MongoDB database. After clicking on the 'Join Data' button to add a join step, the user needs to pick the data that he wants to join. It's important, that he can only pick tables and saved questions that are from the same database as your starting data. Next, the user needs to pick the columns he wants to join on. This means that he must pick a column from the first table and a column from the second table and the join will stitch rows together where the value from the first column is equal to the value in the second column. A very common example is to join on an ID column in each table, so if happened to pick a table to join on where there is a foreign key relationship between the tables, Metabase will automatically pick those corresponding ID columns. At the end of join step, there's a 'Columns' button that the user can click to choose which columns he wants to include from the joined data. By default, Metabase will do a left outer join, but there is a Venn diagram icon to select a different type of join. Metabase supports multiple stages of joins and joins on multiple conditions. Not all databases support all types of joins, so Metabase will only display the options supported by the database you're using.

Here are the basic types of joins:

- Left outer join: select all records from Table A, along with records from Table B that meet the join condition, if any.
- Right outer join: select all records from Table B, along with records from Table A that meet the join condition, if any.
- Inner join: only select the records from Table A and B where the join condition is met.
- Full outer join: select all records from both tables, whether or not the join condition is met.

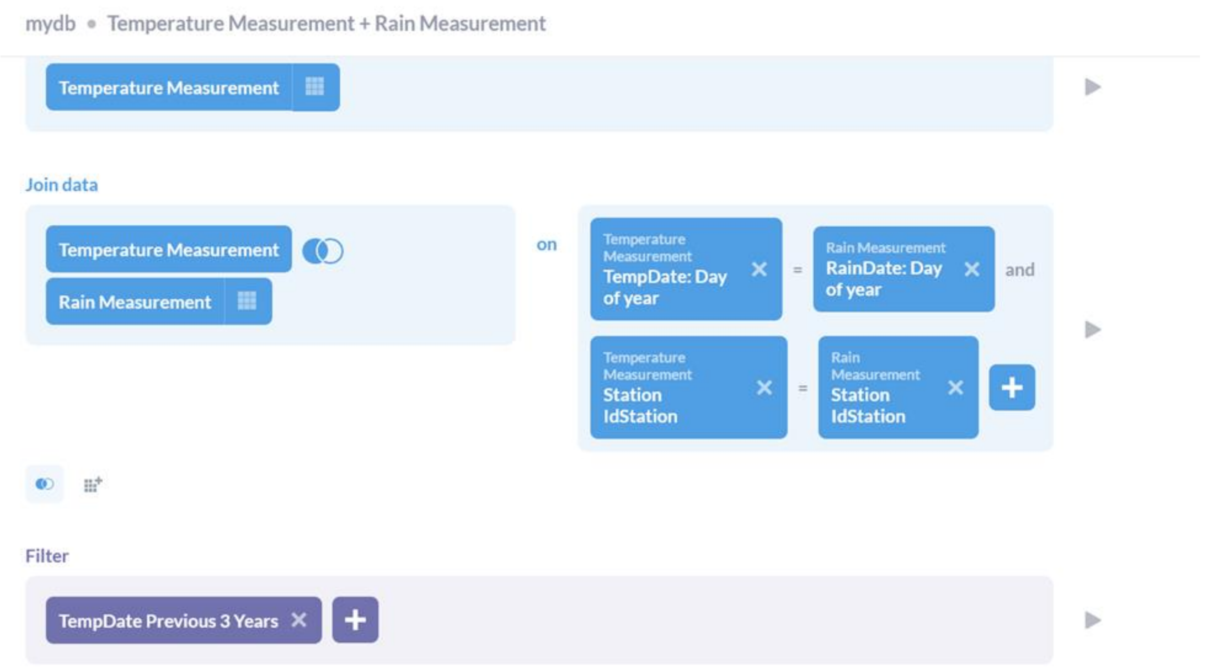


Figure 33: Sample of implementing a custom question (joining and filtering), through Notebook Editor

3.4.2.3 Advanced Questions in Native Query Editor

If a user has the permissions to use the Native Query Editor, he can directly write SQL or his database's native querying language. If any user writes a SQL query that includes variables, the question might have filter widgets at the top of the screen. Filter widgets let the user modify the SQL query before its run, changing the final results. Any team member can use SQL snippets to save, reuse and share SQL code across multiple questions that are composed using the SQL editor.

StationId: 1

```
    `w`.`windAVGSpeed` AS `windAVGSpeed`,
    `w`.`windH` AS `windH`,
    `w`.`windHTime` AS `windHTime`,
    `w`.`windDirection` AS `windDirection`
FROM
    (((`rain_measurement` `r`
    JOIN `temperature_measurement` `t` ON (((`r`.`rainDate` = `t`.`tempDate`)
        AND (`r`.`Station_idStation` = `t`.`Station_idStation`))))
    JOIN `wind_measurement` `w` ON (((`r`.`rainDate` = `w`.`windDate`)
        AND (`r`.`Station_idStation` = `w`.`Station_idStation`))))
    JOIN `station` `s` ON ((`r`.`Station_idStation` = `s`.`idStation`)))
WHERE
    (`r`.`Station_idStation` = {{stationId}})
ORDER BY Date ASC;
```

Figure 34: Sample of implementing an advanced question in Native Query Editor

3.4.3. Dashboards, Collections, Pulses

Dashboards group questions and present them on a single page. Dashboards are shareable reports that feature a set of related questions. Subscriptions to dashboards can be set via email or Slack to receive the exported results of the dashboard's questions. A dashboard comprises a set of cards arranged on a grid. These cards can be questions, such as tables, charts, maps or text boxes. Users can add filter widgets to dashboards that filter data identically across multiple questions and customize what happens when their teammates click on a chart or a table. In Metabase parlance, every chart on a dashboard is called a "question." Clicking on the title of a question on a dashboard will take the user to a detail view of that question. When you're looking at the detail view of a question, you can use all the same actions mentioned above. You can also click on the headings of tables to see more options, like summing the values of a column, or filtering based on that column. If the Metabase administrator has enabled public sharing on a dashboard, the user can go to that dashboard and click on the sharing icon to find its public links. Public links can be viewed by anyone, even if they don't have access to Metabase. The public embedding code can be used to embed the dashboard in a simple web page or a blog post.

Collections in Metabase are a lot like folders. They're where all team's dashboards and charts are kept. In order to explore a collection, a user must just click on one in the "Our analytics" section of the home page or click on "Browse all items".

The Pulses feature in Metabase gives users the ability to automatically send regular updates to their teammates to help everyone keep track of changes to the metrics that matter them most. Because of the space constraints of email and Slack, Metabase will automatically make some adjustments to the appearance of the saved question so that it looks great in the Pulse. For example, in order to save space, pie charts will automatically be transformed into bar charts. Users can also optionally include the results of a saved question in an emailed pulse as a .csv or .xls file attachment. Users can also set a different delivery schedule for email versus Slack. To deliver by email, the users have to type in the Metabase user names, or email addresses they want to send the pulse to, separated by commas. Then, they have to choose to either send it daily, weekly, or monthly and the time at which they want it to be sent. To send via Slack, the users need to choose which channel they want to post the pulse in, whether they want it to post hourly or daily and at what time. Again, the schedule for Slack can be different from the schedule for email.

The search bar is a quick way to find dashboards, questions, collections and pulses. The user can select from the typeahead's dropdown results or hit enter to view a search results page.

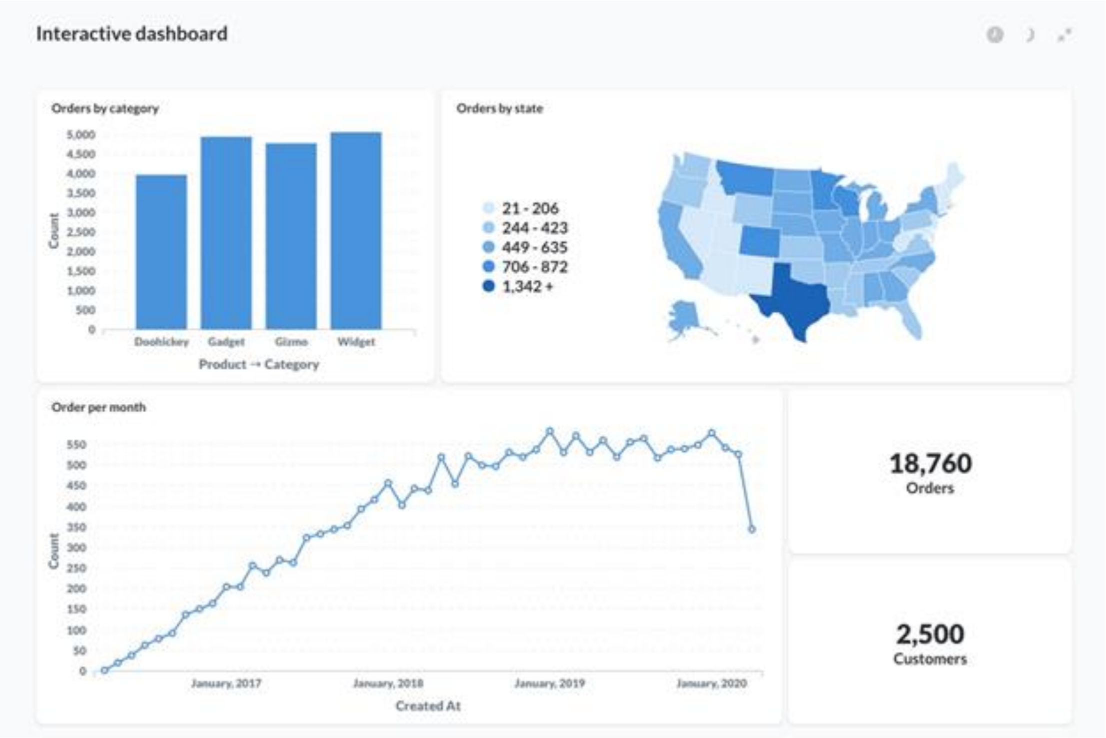


Figure 35: Sample dashboard

Appendix

The Appendix contains the tables for the response matrices, the constraints vectors and the results of the piece-wise linear optimization for the different selected values of the hydraulic head of reference. For the hydraulic heads of reference equal to 6.75 m and 7 m are provided only the results of the pumping rates through the different iterations in order to justify that an optimal solution could not be found.

For H_{ref}=6,25 m

Run 1

Response Matrix for the Dry Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-42.48	-46.04	-35.28	-14.84	-41.16	-17.30	-45.40	-30.64	-34.76	-67.62	-96.98	-33.11	-44.42	-47.70	-20.49	-25.13	-16.34	-10.00	-55.61	-15.60
-56.49	-49.56	-70.35	-10.75	-61.61	-78.27	-55.74	-7.58	-75.57	-38.96	-22.62	-21.97	-55.34	-53.62	-44.34	-11.70	-4.56	-43.15	-47.50	-17.18
-42.12	-46.47	-34.77	-15.37	-40.55	-17.78	-44.76	-30.54	-34.22	-67.72	-102.95	-33.45	-44.07	-48.11	-19.96	-25.61	-15.87	-9.98	-55.34	-15.48
-56.05	-49.65	-69.57	-10.58	-60.83	-72.47	-55.45	-7.27	-72.78	-38.88	-23.11	-21.96	-55.13	-52.95	-47.09	-11.32	-4.78	-39.14	-47.72	-17.26
-38.61	-53.28	-23.12	-18.75	-30.79	-19.48	-36.50	-26.53	-27.11	-67.19	-105.05	-35.38	-42.43	-40.03	-22.85	-14.39	-16.96	1.49	-58.08	-5.67
-55.36	-52.43	-67.22	-12.07	-59.52	-55.96	-55.43	-8.16	-66.91	-41.05	-24.89	-24.53	-54.87	-53.95	-55.37	-12.71	-5.17	-31.66	-48.88	-19.75
-44.67	-49.24	-38.06	-16.72	-43.74	-18.71	-47.63	-24.79	-37.41	-66.48	-86.68	-38.46	-46.50	-49.93	-22.77	-27.46	-14.82	-10.78	-56.18	-17.70
-54.38	-54.79	-63.40	-13.45	-57.83	-45.55	-54.94	-8.79	-61.64	-43.10	-26.74	-27.35	-54.21	-54.35	-60.19	-13.83	-5.49	-26.35	-49.85	-22.01
-46.84	-52.30	-40.53	-18.45	-45.80	-20.42	-49.25	-21.06	-39.66	-64.54	-64.50	-44.29	-48.09	-51.73	-25.08	-28.10	-13.08	-11.85	-56.29	-19.97
-49.90	-57.91	-47.13	-20.08	-50.19	-25.46	-51.89	-14.40	-45.69	-55.69	-42.10	-48.56	-50.71	-53.97	-33.22	-23.56	-9.31	-14.85	-54.33	-25.76
-52.24	-58.50	-54.43	-17.29	-53.95	-32.53	-53.57	-10.99	-52.40	-48.63	-32.65	-36.52	-52.51	-54.58	-45.58	-18.07	-7.09	-19.04	-52.03	-26.76

Response Matrix for the Wet Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-43.47	-45.79	-35.46	-14.65	-41.29	-17.15	-45.51	-30.51	-34.84	-67.52	-97.01	-33.02	-44.50	-47.64	-20.50	-25.07	-16.36	-9.94	-55.62	-15.55
-56.30	-48.99	-70.30	-10.17	-61.54	-77.44	-55.85	-7.20	-75.48	-38.21	-22.90	-20.99	-55.71	-52.64	-44.65	-10.82	-4.76	-42.37	-47.60	-16.50
-44.04	-45.70	-36.26	-14.45	-42.11	-16.85	-46.28	-29.70	-35.62	-67.00	-103.88	-33.53	-44.98	-47.39	-21.30	-25.14	-16.67	-10.08	-55.65	-15.97
-55.82	-50.12	-69.56	-11.04	-60.84	-72.74	-55.56	-7.81	-72.82	-39.21	-23.26	-22.30	-55.20	-53.38	-47.09	-11.80	-4.75	-39.62	-47.70	-17.74
-51.74	-44.17	-51.92	-13.25	-56.05	-17.35	-58.62	-27.29	-46.63	-70.29	-127.06	-37.71	-50.37	-59.99	-21.21	-38.84	-13.77	-22.77	-56.07	-28.21
-55.44	-52.41	-67.22	-12.05	-59.53	-55.96	-55.43	-8.14	-66.91	-41.04	-24.89	-24.52	-54.86	-53.94	-55.36	-12.67	-5.17	-31.67	-48.86	-19.75
-45.54	-49.36	-38.05	-16.91	-43.66	-18.95	-47.51	-25.06	-37.28	-66.76	-86.19	-39.15	-46.44	-50.03	-22.76	-27.70	-14.70	-11.05	-56.09	-17.90
-54.83	-54.87	-63.67	-13.56	-58.09	-45.68	-55.19	-8.92	-61.86	-43.25	-26.95	-27.51	-54.43	-54.52	-60.10	-14.25	-5.81	-26.45	-49.99	-22.35
-46.86	-52.27	-40.55	-18.42	-45.81	-20.40	-49.26	-21.05	-39.66	-64.52	-64.52	-44.29	-48.07	-51.73	-25.07	-28.10	-13.08	-11.85	-56.29	-19.96
-49.99	-57.88	-47.16	-20.05	-50.22	-25.44	-51.91	-14.37	-45.70	-55.67	-42.11	-48.56	-50.69	-53.97	-33.24	-23.55	-9.30	-14.85	-54.32	-25.75
-52.27	-58.50	-54.44	-17.30	-53.95	-32.54	-53.56	-11.00	-52.40	-48.64	-32.65	-36.52	-52.52	-54.59	-45.60	-18.06	-7.09	-19.04	-52.04	-26.76

Constraints Vectors

Well	b for the dry season	Well	b for the wet season
1	1.8124	1	8.6990
2	2.2657	2	8.9334
3	1.8470	3	8.5660
4	2.2502	4	8.8629
5	1.9964	5	8.7893
6	2.2424	6	8.8899
7	1.8862	7	8.5385
8	2.2409	8	8.8511
9	1.9975	9	8.6046
10	2.0776	10	8.6299
11	2.1429	11	8.6917

Optimum pumping Rates

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.164
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.82
8	474.19	381.20
9	1497.99	815.75
10	1334.78	1295.06
11	1559.42	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.11	140.08
19	1561.13	267.90
20	162.72	92.96
Total pumping rate	$3.7486 \cdot 10^4$	$3.1254 \cdot 10^4$

Run 2

Response Matrix for the Dry Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-37.70	-45.89	-35.07	-15.79	-41.16	-17.33	-44.63	-30.64	-33.86	-67.57	-97.05	-33.08	-36.22	-47.21	-20.37	-25.27	-16.28	-9.93	-51.12	-16.47
-51.87	-49.26	-70.05	-10.06	-61.58	-78.02	-54.69	-7.30	-74.63	-38.67	-22.73	-21.67	-45.75	-52.67	-44.11	-11.65	-4.37	-43.04	-42.66	-18.25
-32.91	-46.64	-35.47	-12.03	-42.28	-18.52	-51.21	-36.95	-33.60	-70.69	-102.73	-37.53	-34.48	-49.04	-16.29	-33.46	-12.89	-17.47	-50.67	-38.72
-49.90	-49.90	-69.23	-10.90	-60.71	-72.74	-52.64	-7.40	-71.69	-39.02	-23.09	-22.02	-45.47	-52.56	-46.74	-11.24	-4.65	-39.03	-42.87	-16.53
-211.26	-49.86	-33.33	-407.04	-23.68	-105.50	346.99	-183.87	3.99	-116.45	-82.78	-86.12	2.29	-60.20	14.53	-59.83	5.30	-36.02	-50.72	-83.76
-49.88	-52.42	-66.91	-11.75	-59.45	-55.96	-53.72	-8.10	-65.81	-40.98	-24.89	-24.43	-45.33	-53.37	-54.73	-12.55	-5.07	-31.53	-44.03	-19.60
-30.70	-49.26	-37.81	-8.77	-43.56	-18.38	-41.42	-24.29	-35.86	-66.16	-86.38	-37.66	-37.82	-49.15	-22.48	-25.69	-15.05	-8.43	-51.65	-10.75
-48.86	-54.84	-63.21	-13.49	-57.86	-45.64	-52.81	-8.79	-60.55	-43.10	-26.80	-27.33	-44.78	-53.86	-59.56	-13.73	-5.33	-26.11	-45.07	-21.57
-42.47	-52.29	-40.21	-18.32	-45.75	-20.41	-47.66	-21.00	-38.60	-64.48	-64.50	-44.03	-39.43	-51.19	-24.92	-27.97	-13.03	-11.46	-51.67	-19.85
-44.94	-57.90	-46.81	-19.95	-50.14	-25.46	-50.29	-14.34	-44.59	-55.63	-42.11	-48.14	-41.75	-53.43	-33.07	-23.50	-9.25	-14.45	-49.63	-25.63
-47.10	-58.50	-54.11	-17.25	-53.88	-32.53	-51.95	-10.92	-51.26	-48.57	-32.65	-36.40	-43.32	-54.02	-45.17	-17.98	-7.01	-18.62	-47.27	-26.49

Response Matrix for the Wet Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-58.83	-45.93	-36.05	-12.72	-41.34	-17.16	-47.86	-30.69	-38.12	-67.64	-96.96	-33.14	-98.56	-48.79	-20.91	-24.88	-16.54	-11.42	-104.52	-13.98
-72.84	-49.28	-71.04	-12.72	-61.64	-77.67	-58.91	-7.61	-79.07	-38.53	-22.76	-21.36	-120.36	-54.31	-45.26	-10.77	-5.36	-44.26	-99.29	-15.06
-70.04	-45.55	-35.87	-23.85	-40.44	-16.08	-39.76	-21.51	-38.98	-63.94	-104.08	-28.93	-105.67	-46.72	-26.07	-13.64	-21.91	18.56	-106.01	23.67
-75.18	-49.87	-70.35	-11.13	-61.02	-72.46	-61.11	-7.87	-76.74	-39.15	-23.28	-22.30	-119.41	-54.52	-47.55	-12.20	-4.92	-43.55	-99.29	-19.36
238.60	-47.36	-43.00	1096.91	-63.68	68.55	-556.66	167.63	-113.51	-19.84	-149.29	20.20	-250.19	-38.17	-66.45	21.53	-48.28	88.52	-117.21	97.89
-73.31	-52.42	-67.99	-12.72	-59.65	-55.94	-58.91	-8.13	-70.73	-41.16	-24.88	-24.72	-118.46	-55.23	-56.43	-12.92	-5.36	-33.55	-100.41	-19.36
-78.44	-49.35	-38.69	-39.74	-43.89	-19.23	-56.70	-25.71	-41.68	-67.10	-86.44	-40.19	-102.82	-51.53	-23.20	-30.62	-14.31	-22.13	-104.89	-30.12
-72.84	-54.82	-64.31	-14.31	-58.13	-45.62	-59.64	-8.92	-65.71	-43.32	-26.87	-27.67	-117.51	-55.76	-61.30	-14.59	-6.26	-28.55	-101.53	-23.67
-62.10	-52.28	-41.25	-19.08	-45.94	-20.42	-53.01	-21.25	-43.40	-64.63	-64.51	-44.71	-105.19	-52.94	-25.49	-28.71	-13.41	-14.99	-106.01	-20.44
-66.30	-57.88	-47.87	-20.67	-50.32	-25.42	-55.22	-14.43	-49.40	-55.75	-42.07	-49.23	-109.93	-55.18	-33.51	-23.92	-9.39	-17.85	-104.52	-25.82
-69.57	-58.50	-55.17	-19.08	-54.09	-32.59	-57.43	-11.28	-56.27	-48.80	-32.64	-36.82	-114.20	-55.89	-46.40	-18.42	-7.60	-22.13	-103.02	-27.97

Well	b for the dry season	Well	b for the wet season
1	1.812	1	8.6990
2	2.266	2	8.9334
3	1.847	3	8.5660
4	2.250	4	8.8629
5	1.996	5	8.7893
6	2.242	6	8.8899
7	1.886	7	8.5385
8	2.241	8	8.8511
9	1.998	9	8.6046
10	2.078	10	8.6299
11	2.143	11	8.6917

Optimum pumping Rates

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.164
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.82
8	474.19	381.20
9	1497.99	815.75
10	1334.78	1295.06
11	1559.42	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.11	140.08
19	1561.13	267.90
20	162.72	92.96
Total pumping rate	$3.7486 \cdot 10^4$	$3.1254 \cdot 10^4$

For H_{ref}=6,5 m

Run 1

Response Matrix for the Dry Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-42.48	-46.04	-35.28	-14.84	-41.16	-17.30	-45.40	-30.64	-34.76	-67.62	-96.98	-33.11	-44.42	-47.70	-20.49	-25.13	-16.34	-10.00	-55.61	-15.60
-56.49	-49.56	-70.35	-10.75	-61.61	-78.27	-55.74	-7.58	-75.57	-38.96	-22.62	-21.97	-55.34	-53.62	-44.34	-11.70	-4.56	-43.15	-47.50	-17.18
-42.12	-46.47	-34.77	-15.37	-40.55	-17.78	-44.76	-30.54	-34.22	-67.72	-102.95	-33.45	-44.07	-48.11	-19.96	-25.61	-15.87	-9.98	-55.34	-15.48
-56.05	-49.65	-69.57	-10.58	-60.83	-72.47	-55.45	-7.27	-72.78	-38.88	-23.11	-21.96	-55.13	-52.95	-47.09	-11.32	-4.78	-39.14	-47.72	-17.26
-38.61	-53.28	-23.12	-18.75	-30.79	-19.48	-36.50	-26.53	-27.11	-67.19	-105.05	-35.38	-42.43	-40.03	-22.85	-14.39	-16.96	1.49	-58.08	-5.67
-55.36	-52.43	-67.22	-12.07	-59.52	-55.96	-55.43	-8.16	-66.91	-41.05	-24.89	-24.53	-54.87	-53.95	-55.37	-12.71	-5.17	-31.66	-48.88	-19.75
-44.67	-49.24	-38.06	-16.72	-43.74	-18.71	-47.63	-24.79	-37.41	-66.48	-86.68	-38.46	-46.50	-49.93	-22.77	-27.46	-14.82	-10.78	-56.18	-17.70
-54.38	-54.79	-63.40	-13.45	-57.83	-45.55	-54.94	-8.79	-61.64	-43.10	-26.74	-27.35	-54.21	-54.35	-60.19	-13.83	-5.49	-26.35	-49.85	-22.01
-46.84	-52.30	-40.53	-18.45	-45.80	-20.42	-49.25	-21.06	-39.66	-64.54	-64.50	-44.29	-48.09	-51.73	-25.08	-28.10	-13.08	-11.85	-56.29	-19.97
-49.90	-57.91	-47.13	-20.08	-50.19	-25.46	-51.89	-14.40	-45.69	-55.69	-42.10	-48.56	-50.71	-53.97	-33.22	-23.56	-9.31	-14.85	-54.33	-25.76
-52.24	-58.50	-54.43	-17.29	-53.95	-32.53	-53.57	-10.99	-52.40	-48.63	-32.65	-36.52	-52.51	-54.58	-45.58	-18.07	-7.09	-19.04	-52.03	-26.76

Response Matrix for the Wet Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-43.47	-45.79	-35.46	-14.65	-41.29	-17.15	-45.51	-30.51	-34.84	-67.52	-97.01	-33.02	-44.50	-47.64	-20.50	-25.07	-16.36	-9.94	-55.62	-15.55
-56.30	-48.99	-70.30	-10.17	-61.54	-77.44	-55.85	-7.20	-75.48	-38.21	-22.90	-20.99	-55.71	-52.64	-44.65	-10.82	-4.76	-42.37	-47.60	-16.50
-44.04	-45.70	-36.26	-14.45	-42.11	-16.85	-46.28	-29.70	-35.62	-67.00	-103.88	-33.53	-44.98	-47.39	-21.30	-25.14	-16.67	-10.08	-55.65	-15.97
-55.82	-50.12	-69.56	-11.04	-60.84	-72.74	-55.56	-7.81	-72.82	-39.21	-23.26	-22.30	-55.20	-53.38	-47.09	-11.80	-4.75	-39.62	-47.70	-17.74
-51.74	-44.17	-51.92	-13.25	-56.05	-17.35	-58.62	-27.29	-46.63	-70.29	-127.06	-37.71	-50.37	-59.99	-21.21	-38.84	-13.77	-22.77	-56.07	-28.21
-55.44	-52.41	-67.22	-12.05	-59.53	-55.96	-55.43	-8.14	-66.91	-41.04	-24.89	-24.52	-54.86	-53.94	-55.36	-12.67	-5.17	-31.67	-48.86	-19.75
-45.54	-49.36	-38.05	-16.91	-43.66	-18.95	-47.51	-25.06	-37.28	-66.76	-86.19	-39.15	-46.44	-50.03	-22.76	-27.70	-14.70	-11.05	-56.09	-17.90
-54.83	-54.87	-63.67	-13.56	-58.09	-45.68	-55.19	-8.92	-61.86	-43.25	-26.95	-27.51	-54.43	-54.52	-60.10	-14.25	-5.81	-26.45	-49.99	-22.35
-46.86	-52.27	-40.55	-18.42	-45.81	-20.40	-49.26	-21.05	-39.66	-64.52	-64.52	-44.29	-48.07	-51.73	-25.07	-28.10	-13.08	-11.85	-56.29	-19.96
-49.99	-57.88	-47.16	-20.05	-50.22	-25.44	-51.91	-14.37	-45.70	-55.67	-42.11	-48.56	-50.69	-53.97	-33.24	-23.55	-9.30	-14.85	-54.32	-25.75
-52.27	-58.50	-54.44	-17.30	-53.95	-32.54	-53.56	-11.00	-52.40	-48.64	-32.65	-36.52	-52.52	-54.59	-45.60	-18.06	-7.09	-19.04	-52.04	-26.76

Well	b for the dry season	Well	b for the wet season
1	1.5624	1	8.4490
2	2.0157	2	8.6834
3	1.5970	3	8.3160
4	2.0002	4	8.6129
5	1.7464	5	8.5393
6	1.9924	6	8.6399
7	1.6362	7	8.2885
8	1.9909	8	8.6011
9	1.7475	9	8.3546
10	1.8276	10	8.3799
11	1.8929	11	8.4417

Optimum pumping Rates

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.16
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1371.23	815.75
10	1334.78	1295.06
11	1280.68	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7080 \cdot 10^4$	$3.1254 \cdot 10^4$

Run 2

Response Matrix for the Dry Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-37.80	-45.90	-35.06	-14.89	-41.16	-17.22	-44.40	-30.49	-33.95	-67.49	-87.57	-32.95	-36.36	-47.17	-20.27	-25.07	-16.34	-9.64	-51.14	-15.61
-45.57	-49.40	-70.00	-19.33	-61.19	-80.44	-45.88	-11.92	-73.77	-40.11	-21.06	-23.14	-44.78	-53.12	-42.94	-13.43	-3.56	-44.66	-42.57	-22.98
-44.02	-46.30	-35.64	-9.61	-42.67	-17.29	-64.42	-32.71	-35.84	-68.87	-100.22	-37.02	-37.64	-47.99	-25.24	-29.43	-19.48	-15.90	-51.26	-40.56
-49.11	-49.69	-69.30	-4.55	-60.75	-71.67	-53.95	-5.02	-72.05	-38.10	-20.46	-20.92	-45.91	-52.18	-47.39	-9.54	-5.16	-37.32	-42.92	-11.25
-279.30	-49.32	-40.43	-251.82	-47.02	-75.43	-91.54	-139.92	-42.52	-107.10	-134.85	-75.26	-36.84	-74.03	12.54	-150.58	21.44	-138.58	-48.00	-417.21
-49.80	-52.42	-66.91	-11.75	-59.47	-55.90	-54.12	-7.99	-65.97	-40.97	-22.31	-24.42	-45.35	-53.37	-54.75	-12.57	-5.07	-31.56	-44.03	-19.73
-35.66	-49.31	-37.76	-14.44	-43.54	-19.01	-42.28	-25.45	-35.98	-66.66	-81.74	-39.47	-37.79	-49.27	-22.64	-27.08	-14.18	-10.18	-51.49	-15.61
-49.14	-54.73	-63.31	-9.55	-57.98	-45.32	-53.84	-8.50	-60.74	-43.04	-24.31	-27.28	-44.83	-53.85	-59.61	-13.70	-5.35	-26.07	-45.07	-21.51
-42.59	-52.30	-40.22	-18.38	-45.75	-20.43	-47.66	-21.05	-38.72	-64.50	-59.25	-44.01	-39.42	-51.20	-24.94	-28.00	-13.06	-11.50	-51.67	-19.97
-44.68	-57.88	-46.80	-19.84	-50.13	-25.37	-50.23	-14.15	-44.70	-55.57	-40.10	-48.05	-41.73	-53.40	-33.07	-23.36	-9.25	-14.28	-49.63	-25.01
-46.95	-58.50	-54.11	-17.25	-53.88	-32.55	-51.83	-10.94	-51.38	-48.58	-30.17	-36.40	-43.32	-54.02	-45.17	-17.98	-7.04	-18.62	-47.27	-26.49

Response Matrix for the Wet Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-58.83	-45.92	-36.05	-14.31	-41.32	-17.16	-47.86	-30.69	-37.51	-67.64	-86.19	-33.24	-98.09	-48.79	-20.91	-25.12	-16.09	-12.14	-104.14	-15.06
-84.05	-49.15	-71.10	14.31	-62.05	-75.28	-70.69	-1.84	-80.17	-37.14	-19.49	-19.67	-124.15	-53.77	-46.69	-8.37	-6.71	-38.55	-100.04	-6.45
-49.49	-45.88	-35.68	-30.20	-40.06	-17.38	-23.56	-27.02	-34.81	-65.87	-97.98	-29.46	-93.35	-48.01	-15.18	-19.62	-11.62	12.14	-103.02	26.89
-76.11	-50.09	-70.27	-28.61	-60.99	-73.54	-59.64	-10.76	-75.64	-40.08	-19.11	-23.67	-117.99	-55.02	-46.98	-14.83	-4.47	-50.68	-98.92	-29.05
369.81	-48.01	-34.58	654.97	-39.97	38.56	6.63	113.33	-28.81	-29.34	-142.62	9.26	-109.93	-20.99	-65.02	155.99	-74.22	498.99	-132.88	680.95
-73.31	-52.42	-67.99	-12.72	-59.65	-55.94	-58.91	-8.39	-69.87	-41.16	-20.91	-24.72	-118.46	-55.23	-56.43	-12.92	-5.36	-33.55	-100.41	-19.36
-69.57	-49.31	-38.75	-25.44	-43.95	-18.68	-56.70	-24.40	-41.07	-66.72	-79.52	-38.29	-103.30	-51.41	-23.20	-28.71	-16.09	-15.71	-106.01	-22.59
-72.37	-54.95	-64.21	-25.44	-58.01	-45.95	-58.91	-9.44	-64.97	-43.40	-22.96	-27.77	-117.99	-55.81	-61.30	-14.83	-6.26	-29.27	-101.53	-24.74
-62.10	-52.28	-41.25	-19.08	-45.94	-20.42	-53.01	-21.25	-42.66	-64.63	-58.10	-44.71	-105.19	-52.94	-25.49	-28.47	-13.41	-14.99	-106.01	-20.44
-66.77	-57.91	-47.88	-22.26	-50.35	-25.53	-55.96	-14.69	-48.79	-55.83	-38.60	-49.34	-110.41	-55.27	-33.80	-24.16	-9.39	-18.56	-104.89	-27.97
-70.04	-58.50	-55.17	-19.08	-54.09	-32.59	-57.43	-11.28	-55.53	-48.80	-28.92	-36.82	-113.72	-55.85	-46.40	-18.42	-7.60	-22.13	-103.02	-27.97

Constraints Vectors

Well	b for the dry season	Well	b for the wet season
1	1.5492	1	8.4327
2	2.0219	2	8.6708
3	1.6354	3	8.2665
4	1.9858	4	8.6180
5	2.4301	5	7.9211
6	1.9888	6	8.6334
7	1.6254	7	8.2840
8	1.9860	8	8.5976
9	1.7405	9	8.3451
10	1.8237	10	8.3762
11	1.8894	11	8.4370

Optimum pumping Rates

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.16
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1498.00	815.75
10	1334.78	1295.06
11	1472.43	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7399\cdot10^4$	$3.1254\cdot10^4$

Run 3

Response Matrix for the Dry Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-38.34	-45.90	-35.06	-14.39	-41.17	-17.18	-44.14	-30.49	-33.80	-67.49	-97.27	-32.94	-36.22	-47.15	-20.38	-24.96	-16.31	-9.59	-51.13	-15.33
-51.80	-49.27	-70.04	-10.17	-61.57	-78.04	-54.61	-7.33	-74.63	-38.68	-22.80	-21.68	-45.74	-52.67	-44.11	-11.65	-4.37	-43.04	-42.66	-18.25
-39.14	-45.94	-35.27	-5.28	-41.46	-15.78	-45.83	-27.84	-33.93	-66.67	-103.52	-32.37	-36.13	-47.11	-19.03	-24.63	-15.19	-8.46	-50.93	-10.23
-49.83	-49.90	-69.23	-11.04	-60.70	-72.76	-52.52	-7.42	-71.68	-39.02	-23.14	-22.02	-45.45	-52.56	-46.74	-11.24	-4.64	-39.02	-42.86	-16.53
-66.54	-46.64	-36.96	39.79	-41.17	-10.91	8.12	-14.41	-29.20	-64.37	-106.25	-32.77	-32.46	-44.18	-26.25	3.09	-22.63	21.64	-53.48	84.77
-50.03	-52.42	-66.91	-11.75	-59.46	-55.95	-53.75	-8.09	-65.81	-40.98	-24.96	-24.42	-45.33	-53.36	-54.73	-12.55	-5.09	-31.53	-44.02	-19.60
-39.22	-49.33	-37.74	-17.56	-43.60	-19.10	-44.97	-25.42	-36.20	-66.72	-86.68	-38.60	-37.89	-49.53	-22.38	-27.65	-14.81	-10.59	-51.61	-17.94
-49.09	-54.83	-63.22	-13.18	-57.87	-45.59	-53.01	-8.72	-60.57	-43.09	-26.88	-27.31	-44.80	-53.85	-59.59	-13.73	-5.34	-26.10	-45.07	-21.57
-42.05	-52.29	-40.21	-18.38	-45.75	-20.41	-47.72	-21.01	-38.60	-64.48	-64.65	-44.04	-39.44	-51.20	-24.93	-27.98	-13.06	-11.46	-51.68	-19.85
-44.86	-57.90	-46.80	-20.03	-50.14	-25.47	-50.23	-14.36	-44.59	-55.63	-42.15	-48.14	-41.75	-53.43	-33.08	-23.51	-9.24	-14.45	-49.63	-25.60
-47.08	-58.50	-54.11	-17.22	-53.88	-32.53	-51.92	-10.92	-51.26	-48.57	-32.71	-36.39	-43.31	-54.02	-45.16	-17.98	-7.02	-18.63	-47.27	-26.49

Response Matrix for the Wet Season

$\frac{\partial H_i}{\partial Q_j} * 10^{-6}$																			
-58.13	-45.92	-36.06	-15.90	-41.33	-17.22	-48.60	-30.69	-38.25	-67.68	-96.61	-33.24	-98.56	-48.84	-20.77	-25.36	-16.32	-12.49	-104.33	-16.14
-73.07	-49.28	-71.05	-11.92	-61.65	-77.67	-59.27	-7.61	-79.19	-38.57	-22.67	-21.36	-120.59	-54.31	-45.26	-10.89	-5.36	-44.62	-99.48	-14.52
-57.20	-46.23	-36.12	-42.92	-41.27	-18.85	-46.76	-33.05	-38.43	-68.11	-103.24	-34.87	-99.51	-49.08	-22.77	-26.68	-18.11	-17.85	-104.70	-25.82
-75.41	-49.87	-70.35	-10.33	-61.02	-72.46	-61.11	-7.74	-76.80	-39.15	-23.18	-22.36	-119.64	-54.52	-47.69	-12.20	-5.14	-43.55	-99.29	-19.36
-14.01	-50.78	-38.60	-172.48	-45.83	-25.85	-120.76	-42.37	-52.65	-73.28	-125.69	-41.08	-121.78	-57.69	-16.90	-70.70	-3.80	-140.99	-101.16	-194.71
-73.54	-52.42	-68.01	-13.51	-59.66	-56.00	-59.64	-8.39	-70.79	-41.20	-24.78	-24.78	-118.70	-55.27	-56.57	-13.16	-5.59	-34.27	-100.41	-20.44
-63.04	-49.28	-38.77	-15.90	-43.87	-18.58	-52.65	-24.40	-41.13	-66.60	-86.12	-39.19	-102.82	-51.08	-23.49	-27.87	-14.98	-13.92	-105.26	-18.29
-72.84	-54.84	-64.30	-15.10	-58.11	-45.62	-59.64	-9.18	-65.71	-43.32	-26.77	-27.72	-117.75	-55.76	-61.30	-14.71	-6.26	-29.27	-101.53	-23.67
-62.80	-52.28	-41.26	-19.08	-45.94	-20.37	-52.65	-21.12	-43.33	-64.63	-64.32	-44.71	-105.19	-52.94	-25.35	-28.59	-13.19	-14.63	-106.01	-20.44
-66.77	-57.89	-47.87	-20.67	-50.34	-25.42	-55.59	-14.43	-49.46	-55.75	-42.00	-49.23	-110.17	-55.20	-33.51	-23.92	-9.39	-17.49	-104.70	-26.36
-69.57	-58.50	-55.17	-18.28	-54.08	-32.53	-57.43	-11.15	-56.27	-48.76	-32.54	-36.82	-113.96	-55.87	-46.40	-18.42	-7.38	-21.77	-102.84	-27.97

Well	b for the dry season	Well	b for the wet season
1	1.5622	1	8.4486
2	2.0183	2	8.6811
3	1.5883	3	8.3251
4	1.9988	4	8.6142
5	1.6023	5	8.6829
6	1.9924	6	8.6403
7	1.6365	7	8.2885
8	1.9901	8	8.6021
9	1.7475	9	8.3541
10	1.8277	10	8.3795
11	1.8929	11	8.4418

Optimum pumping Rates

Pumping Well	Q for the Dry Season (m^3/day)	Q for the Wet Season (m^3/day)
1	392.55	214.16
2	8698.63	8698.63
3	10257.63	8687.53
4	177.94	62.90
5	3474.58	3420.03
6	920.55	920.55
7	174.79	135.81
8	474.19	381.20
9	1498.00	815.75
10	1334.78	1295.06
11	1464.07	1559.42
12	1099.19	950.55
13	782.12	211.04
14	2955.60	2410.14
15	426.64	349.11
16	617.43	417.99
17	356.78	223.67
18	561.10	140.08
19	1561.13	267.90
20	162.73	92.96
Total pumping rate	$3.7390 \cdot 10^4$	$3.1254 \cdot 10^4$

For H_{ref}=6,75 m

Optimum Results for the Dry Season

Pumping Well	Qopt1	Qopt2	Qopt3	Qopt4	Qopt5	Qopt6	Qopt7	Qopt8	Qopt9b	Qopt9a	Qopt11
1	392.55	237.96	392.55	392.55	392.55	392.55	392.55	392.55	392.55	392.55	392.55
2	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63	8698.63
3	9683.17	10257.63	10257.63	7784.96	8161.91	10257.63	10257.63	10257.63	7804.13	7803.26	10257.63
4	177.94	177.94	177.94	177.94	177.94	177.94	177.94	177.94	177.94	177.94	177.94
5	3474.58	3474.58	3474.58	3474.58	3474.58	2435.08	168.90	3474.58	3474.58	3474.58	1226.12
6	920.55	920.55	920.55	920.55	920.55	920.55	920.55	920.55	920.55	920.55	920.55
7	174.79	174.79	45.54	174.79	174.79	0.00	174.79	174.79	174.79	174.79	174.79
8	474.19	474.19	474.19	474.19	474.19	474.19	474.19	474.19	474.19	474.19	474.19
9	0.00	1498.00	1498.00	1498.00	1498.00	276.49	1498.00	1498.00	1498.00	1498.00	816.13
10	0.00	1334.78	1334.78	279.19	1334.78	0.00	1334.78	1334.78	165.98	178.27	1334.78
11	874.37	1559.42	1559.42	532.47	94.24	1301.42	387.17	1559.42	581.69	565.06	193.18
12	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19
13	782.12	782.12	782.12	782.12	782.12	782.12	782.12	782.12	782.12	782.12	782.12
14	2955.60	2955.60	2955.60	2955.60	1890.88	2955.60	2955.60	2955.60	2955.60	2955.60	2955.60
15	426.64	426.64	426.64	426.64	426.64	426.64	426.64	426.64	426.64	426.64	426.64
16	617.43	617.43	617.43	617.43	617.43	617.43	617.43	617.43	617.43	617.43	617.43
17	356.78	356.78	356.78	356.78	356.78	356.78	356.78	356.78	356.78	356.78	356.78
18	561.10	561.10	561.10	561.10	561.10	561.10	561.10	561.10	561.10	561.10	561.10
19	618.71	1561.13	1561.13	1561.13	1561.13	1561.13	1561.13	1561.13	1561.13	1561.13	1561.13
20	162.73	162.73	162.73	162.73	162.73	162.73	162.73	162.73	162.73	162.73	162.73
Total pumping rate	$3.2451 \cdot 10^4$	$3.7331 \cdot 10^4$	$3.7357 \cdot 10^4$	$3.2931 \cdot 10^4$	$3.2860 \cdot 10^4$	$3.3457 \cdot 10^4$	$3.3008 \cdot 10^4$	$3.7486 \cdot 10^4$	$3.2886 \cdot 10^4$	$3.2881 \cdot 10^4$	$3.3189 \cdot 10^4$

For H_{ref}=7 m

Optimum Results for the Dry Season

Pumping Well	Qopt1	Qopt2	Qopt3	Qopt4	Qopt5	Qopt6a	Qopt6b
1	392.55	392.55	392.55	392.55	392.55	392.55	0.00
2	8654.54	7134.81	8698.63	8698.63	7671.64	8698.63	8698.63
3	8990.56	10257.63	8901.43	8091.44	6360.73	10257.63	10257.63
4	177.94	177.94	177.94	177.94	177.94	177.94	177.94
5	3474.58	0.00	3474.58	1828.31	3474.58	0.00	2422.00
6	920.55	920.55	920.55	920.55	920.55	920.55	920.55
7	174.79	0.00	174.79	174.79	174.79	174.79	0.00
8	474.19	474.19	474.19	474.19	474.19	474.19	474.19
9	0.00	422.38	1498.00	1498.00	1498.00	512.46	1498.00
10	0.00	0.00	1334.78	0.00	0.00	358.29	989.81
11	377.82	0.00	942.23	0.00	0.00	0.00	0.00
12	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19	1099.19
13	782.12	782.12	782.12	782.12	782.12	782.12	782.12
14	0.00	2955.60	2955.60	0.00	2955.60	890.43	2955.60
15	426.64	426.64	426.64	426.64	426.64	426.64	426.64
16	617.43	617.43	617.43	617.43	617.43	617.43	617.43
17	356.78	356.78	356.78	356.78	356.78	356.78	356.78
18	561.10	561.10	561.10	561.10	561.10	561.10	561.10
19	0.00	1561.13	1561.13	1561.13	122.13	1561.13	1561.13

20	162.73	162.73	162.73	162.73	162.73	162.73	162.73
Total pumping rate	$2.7644 \cdot 10^4$	$2.8303 \cdot 10^4$	$3.5512 \cdot 10^4$	$2.7824 \cdot 10^4$	$2.8229 \cdot 10^4$	$2.8425 \cdot 10^4$	$3.3961 \cdot 10^4$