DATA BASE OF MATROIDS ON EIGHT AND NINE ELEMENTS WITH THE HALF-PLANE PROPERTY AND SOS-RAYLEIGH

MARIO KUMMER AND BÜŞRA SERT

Using the classification given in [4], we provide the list of all simple matroids on 8 elements of rank 3 and 4 that

- have the half-plane property (HPP),
- don't have the half-plane property
- it is unknown whether they are SOS-Rayleigh (they have HPP),

and the list of some matroids on 8 elements that are not SOS-Rayleigh (in particular, they are not weakly determinantal).

Moreover, we provide the list of all simple and connected matroids on 9 elements of rank 3 that

- have the half-plane property,
- don't have the half-plane property,
- it is unknown whether they are SOS-Rayleigh.

We give the list of some simple and connected matroids on 9 elements of rank 4 that

- have the half-plane property,
- don't have the half-plane property,
- it is undetected whether they have the half-plane property,
- are candidates for having the half-plane property (numerical data suggest so).

The matroids are listed in an encoded format first given by Matsumoto *et. al.* in [5]. In this encoding, each line in the file keeps the fingerprint of the collection of the bases of a matroid M in the following way: for a fixed r, all size r subsets of $E = [8]$ are ordered increasingly in the reverse lexiographic order, and each subset S is represented with the character "$*$" if it appears in the collection of bases of M and with the character "0" if it doesn't appear in the collection. For example,

$$**0************************0$$

represents the matroid on 8 elements of rank 2 that has all size two subsets of $E = [8]$ as bases except the subsets $\{2,3\}$ and $\{7,8\}$ represented on the 3rd and 28th positions respectively. The formal definition of the reverse lexiographic order of sets is as follows.

Definition 0.1. Two distinct r element sets S_1, S_2 are called to have the relation $S_1 \prec S_2$ with respect to the reverse lexiographic order if $\max(S_1) < \max(S_2)$ or $\max(S_1) = \max(S_2) = a$ and $S_1 \setminus \{a\} \prec S_2 \setminus \{a\}$.

All matroids of rank $r \leq 2$ have the half-plane property, and taking the dual and direct sums preserve the half-plane property. The first examples of matroids that have the half-plane property that are not SOS-Rayleigh appear on rank 4 on 8 elements. Moreover, the half-plane property and SOS-Rayleigh are minor closed properties (see [4] for more information). Therefore, providing the list of simple

2010 *Mathematics Subject Classification.* Primary: 05B35; Secondary: 90C22.
Both authors have been supported by the DFG under Grant No.421473641.

matroids of rank 3 and 4 with or/and without the respective properties provide a complete database information to determine whether a given matroid on 8 elements has the desired property. Provided list of matroids on 9 elements of rank 3 gives a classification of those matroids with respect to the half-plane property.

For matroids on 9 elements of rank 4, we don't have a complete classification. However, we provide list of 4125 matroids that have the half-plane property, list of 1218 matroids that don't have the half-plane property all of whose proper minors have the half-plane property, list of 819 matroids that are candidates for the half-plane property and list of 556 matroids for which we couldn't detect whether they have the half-plane property.

The list of matroids on 9 elements of rank 4 that don't have the half-plane property consists of triples (M, J, X) where M is the fingerprint of the matroid, $J = (i, j)$ is an index, and $X \in \mathbb{R}^7$ such that $\Delta_{i,j} h_M(X) < 0$. This provides a counter example for the half-plane property of each such matroid.

Among the files provided, one can find Macaulay2 code for encoding and decoding matroids, and for sorting them with respect to the properties.

Acknowledgements. The authors thank Justin Chen and Lukas Kühne for communications on the data base and the conversion of the encoding.

References

[1] Justin Chen. Matroids: a package for computations with matroids. A *Macaulay2* package available at `https://github.com/Macaulay2/M2/tree/master/M2/Macaulay2/packages`.

[2] Justin Chen. Matroids: a *Macaulay2* package. *The Journal of Software for Algebra and Geometry*, 9, 2018.

[3] Daniel R. Grayson and Michael E. Stillman. Macaulay2, a software system for research in algebraic geometry. Available at `http://www.math.uiuc.edu/Macaulay2/`.

[4] Mario Kummer and Büşra Sert. Matroids on eight elements with the half-plane property and related concepts. *arXiv preprint arXiv:2111.09610*, 2021.

[5] Yoshitake Matsumoto, Sonoko Moriyama, Hiroshi Imai, and David Bremner. Matroid enumeration for incidence geometry. *Discrete & Computational Geometry*, 47(1):17–43, Nov 2011.

Technische Universität Dresden, Germany
Email address: `mario.kummer@tu-dresden.de`

Technische Universität Dresden, Germany
Email address: `buesra.sert@tu-dresden.de`