

The whetstone of witte,

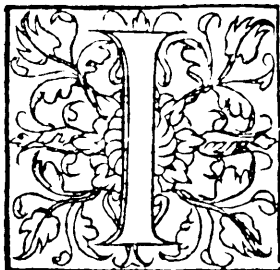
whiche is the seconde parte of
Arithmetike: containyng the extrac-
tion of Rootes: The Cosike praicise,
with the rule of Equation: and
the woorkes of Surde
Nombers.

*Though many stones doo beare greate price,
The whetstone is for exercise
As needefull, and in woork as straunge:
Dulle thinges and harde it will so change,
And make them sharpe, to right good vse:
All artesmen knowe, thei can not chuse,
But vse his helpe. yet as men see,
Noe sharpenesse semeth in it to bee.*

*The grounde of artes did brede this stone:
His vse is greate, and moare then one.
Here if you list your wittes to whette,
Noche sharpenesse therby shall you gette.
Dulle wittes hereby doe greatly mende,
Sharpe wittes are fined to their fulle ende,
No to proue, and praise, as you doe finde,
And to your self be not vnkinde.*

Chese Bookes are to bee solde, at
the Weste doore of Poules,
by Iohn Byngstone.

To the right worshipfull, the gou-
 uerners, Consulles, and the reste of the com-
 paigne of venturers into Moscouia, Robert Ke-
 cezde Phisitian, wissheth healtbe with
 continuall increase of commodi-
 tie, by their worthie and
 famous trauell.



I wil not, nother ought I
 so euilly to iudge of my
 countrie, that learnyng
 here can haue no liber-
 tie: but by aide of frende-
 shippe, oz strength of po-
 wer. For as Englande
 did neuer wante learned
 wittes, so at this tyme I doubt not, but there
 be a great multitude, that desirously embrace
 all kindes of knowledge, and frendely are af-
 fected toward the furtherance of it. And ther-
 fore I dare sate, thei can not malice me, whi-
 che am so willyng to helpe the ignoraunte, ac-
 cordyng to my gifte and simple talète. wher-
 by also this moche praise I mate iustly craue,
 to haue the commendation and rewarde of a
 solliciter in this cause. For though my trauell
 can not moche profite them, that be well lear-
 ned, yet doeth it excite the beste learned, to re-
 member their duetie to their countrie: and to
 be a shamed, that thei hauyng so greate habi-
 litie, shall be founde moare slacke to aide their
 coutrie, then he that hath smaller knowledge,

The Epistle

and lesse occasion otherwaies. Accoꝝdyng as men haue receiued, so are thei bounde to yeld. These excellent giftes are not lente vnto mē, to be hidden. And there are a great multitude that thirst, and long moche for soche aide. For bothe these causes I saie, that naturalle bōde to our countrie doeth chalenge it: and for that the honeste desires of so many good natures so moche requireth it, I exhorte them that be beste hable, to take from me this chargeable wooꝝke, and to further their countrie men, as equitie would. And in the meane ceason while I see them so slacke, let them not bee offended with me, for pꝛeuentynꝝ them. For better it is that a simple Cobe doe prepare thy bꝛekfast, then that thou shouldest goe a hungered to bedde. Yea better it is to haue some grosse repaste, then to sterue for hunger. And the common loꝝte will finde small faulte of wante, as long as thei see any man serue their expectation. So that for this cause also, that my paines for a time, doeth excusē other finer wittes, thei ought to render me some thanks again. But if thei staie for feare of tauntes, and barkynꝝ of curres, their cozage is small. If thei misdoubte the gratefull acceptation of their studies, thei doe iniurie to their countrie. For whoe cā doubt but so ciuile a coūtrie, will thankefully receiue, and moſte frendly recompense the trauelle, of soche as studie for their benefite,

Dedicatorie

benefite, and serueth their necessarie commodities. This perswasion maketh me so bolde, that I can not thinke it needefull, to seke any protector, for this or any like woork. Sith euery good man will offer hymself, to defende that, whereby his natie countrie is benefited. Excepte at some tyme, by excitation of the furies, some naughtie natures doe practice their fraude, to berefte the realme of some singulare commoditie. But as I feare no soche, so at this tyme I seke no soche aide against the. Yet for testefyng of frendshippe, and gratefulle remembraunce, I could doe noe lesse, but sende this Booke to soche as I thought, not onely to deserue it, but also would gladly receiue it. And if I maie perceiue, that you doe accepte it (as I doubt not) with as good a wille, as I dooe sende it, I will for your pleasure, to your counforste, and for your commoditie, shortly set forth the soche a booke of Nauigation, as I dare saie, shall partly satisfie and contente, not onely your expectation, but also the desire of a greate number beside. Wherein I will not forgett specially to touche, both the olde attempte for the Northlie Nauigations, and the later good aduenture, with the fortunate success in discoueryng that voyage, whiche noe men befoze you durste attempte, sith the tyme of Kyng Aluredes his reigne. I meane by the space of, 700, yere. Nothor euer
a. m. any

The Epistle

any before that tyme, had passed that boiage, excepte onely Ohchere, that dwelte in Halgolande: whose report that iorney to the noble King Alured: As it doeth yet remaine in aunciente recozde of the olde Saxon tongue. So that if you continue with corage, as you haue well begon, you shall not onely winne greate riches to your selues, and byng wonderfull commodities to your coutrie. But you shall purchase therewith immortall fame, and be praised for euer, as reason would: for opening that passage, that shall profite so many. In that Booke also I will shewe certain meanes, how without greate difficultie, you maie saile to the Northe Easte Indies. And so to Camul, Chinchital, and Baloz, whiche bee coutries of greate commodities. As for Chatai lieth so farre within the lande, toward the Southe Indian seas, that the iorneye is not to be attempted, vntill you be better acquainted with these countries, that you must first arriue at. But these thynges come in this place vntimely. I praie you accepte frendely in the meane reason this Booke, whiche will bee a greate aide to the well vnderstandyng of the rest that is behinde. And as I shall vnderstande your desire, so will I haste the other. God prospere well your endeouore, and sende you soche good successe, as so worthis aduenture doeth deserue: Whiche I doubt not will insue,

Dedicatorie.

insue, if cankered malice of some spitefull stomackes doe not preuaile, as thei can not cease to practice, to hinder your comoditie, and deface your trauel. But as it is euer seen, and therfoze commonly knowen, that enuie doeth still repine at glozie, so ought all honeste hartes, to prosecute their good attemptes, and contempne the ballynge of dogged currees. So fare you well. And loue hym againe, that delighteth and studieth to farther your comoditie.

At London the .xii. daie of
November, 1557.

THE PREFACE

to the gentle Reader.



Although number be infinite in increasynge: so that there is not in all the worlde, any thing that can exceede the quantitie of it: Neither the grasse on the ground, neither the droppes of water in the sea, no not the small graines of Sande though the whole masse of the earth: yet maie it seme by good reason, that noe man is so experte in *Arithmetike*, that can number the commodities of it. Wherefoze I maie truely saie, that if any imperfection bee in number, it is because that number, can scarcely number, the commodities of it self. For the moare that any experte man, doeth weighe in his mynde the benefites of it, the moze of them sha' he see to remain behinde. And so shall he well perceiue, that as number is infinite, so are the commodities of it as infinite. And if any thyng doe or maie exceede the whole worlde, it is number, whiche so farre surmounteth the measure of the worlde, that if there were infinite worlde, it would at the full comprehend them all. This number also hath other prerogatiues, about all naturalle thynges, for neither is there certaintie in any thyng without it, neither good agreemente where it wanteth. Whercof no man can doubt, that hath been accustomed in the Bookes of Plato, Aristotle, and other aunciente Philosophers, where he shall see, how they searche all secreete knowledge and hid misteries, by the aide of number. For not onely the constitution of the whole worlde, dooe they referre to number, but also the composition of
 The excellencie of number.

b. j. man,

mannie, yea and the whole substance of the soule. Of
 whiche thei professe to knowe no moare, then thei ca
 by the benefite of number attaine. Furthermoze, for
 knowledge and certaintie in any other thynge, that
 mannes witte can reche vnto, there is noe possibilitie
 without number. It is confessed euongeste all men,
 that knowe what learning moaereth, that beside the
 Mathematicke artes, there is noe infallible knowe-
 ledge, excepte it bee bozowed of them. And emongeste
 them, it is sufficiently knowen, and well declared by
Nicomachus, and diuerse other writers, that *Arithme-
 tike* is the fountaine of all the other, and ther ground
 and bonde, as he calleth it. If any man will saie, that
 Diuinitie, Lawe, and Physike, maie be had without
 it: or that thei take litle aide therby. Although I haue
 before this tyme answered thereto, yet now I saie
 again: that in Diuinitie there are greate hidde secre-
 tes in numbers. So that diuerse excellent Diuines,
 haue wrytten whole Bookes of the misteries of nom-
 bers. And some of their Bookes intituled: *The Diui-
 nitie of Numbers*. But what Chyssen manne is igno-
 raunte, that betwene *Trinitie* and *vnitie*, doeth consistte
 the full grounde of al Diuinitie: Wherefoze I neede
 not to allege the other hollie and sacred *Numbers*.
 Saue that. 7. will not permitte me to passe it with si-
 lence. In whiche is contained, not onely the secretes
 of the creation of all thynge: and the consummation
 of the whole worlde againe, with the state of eterni-
 tie: But also by it is the *Sabbthes* reste, and therby
 the full life and conuersation of godlie persones, re-
 presented and insinuate. In Lawe twoe kyndes of
Justice are the somme of the studie: *Iustice Distribu-
 tute*, and *Iustice Commutative*, whiche termes I vse,
 as beste knowen in that arte: But what is any of the
 bothe without *number*: I haue said in an other place
 (as I learned of that noble Philosopher *Aristotell*)
 that

Diuinitie.

Lawe.

that if the knowledg and distinction, of *Geometricall* and *Aritbmeticall* ppozition bee not well obserued, there can noe Justice well bee executed. And how often the ministers of the Law vse aide of *Number*. I meane not repete, bicause none but madde men doubt of it. And as soz *Physike*, without knowledg and aide of number is nothyng. Wee see that nature in generation, be the of manne and beastes, yea and of al thynges els doeth obserue number exauly. As well in the tyme of formation, as in the monethes of quickenyng, and of birth. The misteries of the seuenth and nineth monethes are sufficiente testimonies therein. Beside that from the fourthe monethe til the seuenthy many thynges bee permitted, that els bee not conueniente. Soz the vse of the pulse, and soz criticall daies, beside the ppozition in degrers in simple medicines, and mixture of compounde medicines, and othre infinite maters, what number can doe and what aide it glucth, onely the ignozaunte doe doubt.

But where can there bee any better testimonie soz *Number*, then that the celestiale bodies doe kepe an vnfallible number, in all their wonderfulle motions: By meanes whereof, mannes witte is habled to attaine the knowledg of them. As by the *Aritbmeticall* tables, of their motions it is easily knowen. Therefore and soz that we see the yere, and all the distinction of times, beside the common vse of trafike betwene menne, to depende of number, wee muste needs not onely confesse it to bee, as it were the onely staic of all natures woorkes, and of all ciuilitie: but we must also honoure and reuerence it, as often as wee duely remember the excellencie and benefite of it. Was not *Number*, thinke you, wonderfullie honoured, when noe name was thought moare meter soz God, then the name of *Number*: I meane. 1. and. 3. the name of the *Trinitie*. But to come to moare familiare mat-

b. u. ters,

THE PREFACE.

Measure.
weighte.

fers, I will saie, as Plato saith in his Booke De summo bono. Take awaie Arithmetike, with measure and weightes, from all other artes, and the reste that remaineth is but base, and of noe estimation. Where although Plato dooe name thre thinges in apperaunce, that is Number, Measure, and Weighte. What are Measure and Weighte, but number applied to seuerall uses? For Measure is but the nombryng of the partes of lengthe, bredthe, or depthe. And so weighte (as here it is taken) is the nombryng of the heuynesse of any thyng. So that if number were withdrawen, no manne could either measure, or weigh any quantitie. And therfore it must followe: that number onely maketh all artes perfecte, and woorthle estimation: seying that without it, all artes are but base, and without commendation. This maie suffice for the iuste comendation of Arithmetike. But yet one commoditie moare, whiche all menne that studie that arte, dooe sele, I can not omitte. That is the flyng, sharpenyng, and quickenyng of the witte, that by practice of Arithmetike doeth insue. It teacheth menne and accustometh them, so certainly to remember thynges past: So circumspectly to consider thynges presente: And so prouidently to foresee thynges that followe: that it maie truelie bee called the *File of witte*. Yea it maie aptly bee named the *Scholehouse of reason*. The like iudgemente had Plato of it, as appeareth by his woordes in the seuenth booke Dere publica. Where he saith thus: *Thei that be apte of nature to Arithmetike, bee readie and quicke to attaine all kindes of learnyng. And thei that bee dalle witted, and yet bee instructed and exercised in it, though thei gette nothyng els, yet this shall thei all obtain, that thei shall bee moare sharpe witted, then thei were before.* What a benefite that onely thyng is, to haue the witte whetted and sharpened, I neede not trauell to declare, with all menne confesse it to be as greate as maie be. Excepte any witlesse persons

some thinke he maie bee to wise. But he that mosse
 feareth that, is leaste in daunger of it. Wherefoze to
 conclude, I see moare menne to acknowldge the be-
 nifite of number, then I can espie willyng to studie,
 to attaine the benifites of it. Many praise it, but fewe
 dooe greatly practise it: onlesse it bee for the bulgare
 practise, concernyng Merchandes trade. Wherein
 the desire and hope of gain, maketh many willyng to
 sustaine some trauell. For aide of whom, I did sette
 forth the firste parte of *Aritbmetike*. But if thei knewe
 how farre this seconde parte, dooeth excell the firste
 parte, thei would not accounte any tyme losse, that
 were imploted in it. Yea thei would not thinke any
 tyme well bestowed, till thei had gotten soche habilitie
 by it, that it might be their aide in al other studies.
 And if *Plato* doe require *Aritbmetike*, as a specialle and
 a necessarie qualitie in hym, whom he would admitte
 as a citezeln in his politike tounce: How maie wee
 thinke of our selues, that desire to gouerne other, and
 yet can scante skille of common number: So farre are
 many, yea mosse parte of vs from cunnyng in nomi-
 ber. *Plato* thinketh noe manne hable to bee a good ca-
 pitaine, excepte he bee skilfulle in this arte: And wee
 accounte it noe parte of those qualitties, that bee re-
 quired in any soche manne. Howbeit for the better
 triall thereof, I haue in this Booke framed some of
 the questions in soche sorte, as thei maie approue the
 vse of this arte, not onely good for capitaines, but al-
 so mosse necessarie for them. So that without it, thei
 can not Marshall their battaile, nother belve their e-
 nemies campe or sorte. And if I shall saie as I thinke,
 without it a capitaine is noe capitaine. In this booke
 what I haue written, for the aide of all menne, and
 namely soche of my countrie menne, that vnderstand
 nothyng but Englishe, I meade not to repete peticu-
 larly, but remitte them to the booke it self, to see it at

large. Onely this male I saie: that as I haue doon in other artes, so in this I am the first venturer, in these darke maters. Wherefoze I trust thei that be learned, and happen to reade this woꝝke, wil beare the moare with me, if thei finde any thyng, that thei doe mistike: Wherein if thei will vse this curtesie, either by wꝛytinge to admonishe me thereof, either theim selves to sette foꝝthe a moare perfecter woꝝke, I will thynke them pꝛaïse woꝝthie. But if any manne will be so haſtie, other to blame that, whiche he is not hable to amende, oꝝ to condempne that, whiche he did neuer vnderſtande: As some ofte tymes doe of a ſonde curioſitie, I will wiſſhe hym a better witte, and moare modeſtie. And to pꝛeuent all ſoche ſeuere Judges, I thought it good to admoniſſhe you befoze, that by occasion of trouble vpon trouble, I was hindered from accompliſhyng this woꝝke, as I did intende. But yet is here moare, then any manne might well looke foꝝ at my handes, if thei did knowe and conſider myne eſtate. And this moche moare I ſaie: that if I maie perceiue, that this Booke bee as well receiued, as the fiſte parte was, I will ſtrive moche, to ſtele from my troubles ſo moche tyme, as to ſet out the reſte of this arte, moare completely in Engliſhe, then euer I ſawe it in any tounge, hetherto doen: truſt thereto adſuredly. And wiſſhe hym good, that traueleth foꝝ thy benefite.

Of the rule of Case.

*One thyng is nothyng, the prouerbe is,
whiche in some cases doeth not misse.
Let here by woorkyng with one thyng,
Soche knowledge doeth from one roote spryng,
That one thyng maie with right good skille,
Compare with all thyng: And you will
The practice learn, you shall sone see,
What thynges by one thyng known maie bee.*

To the curiouse scanner.

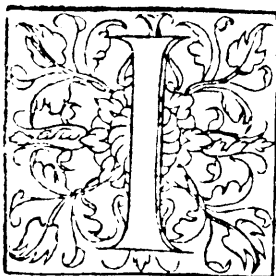
*If you ought finde, as some men maie,
That you can mende, I shall you praie,
To take some paine: so grace maie sende,
This worke to growe to perfecte ende.*

*But if you mende not that you blame,
I winne the praise, and you the shame.
Therefore be wise, and learne before,
Sith slaunder hurtes it self moſte ſore.*

The seconde parte of Arithmetike,
 containyng the extraction of Rootes in di-
 uerse kindes, with the Arte of Cossike
 numbers, and of Surdenombers
 also, in sondrie sortes.

The interlocutors, Master. Scholar.

The Master.



See your desire can not
 bee satisfied, neither your re-
 quest staied, vntill I maie tu-
 sly aunswere you, that I can
 teache you no moze : whiche
 aunswere maie staie your re-
 quest, although it content not
 your desire.

Scholar. I beseeche God of
 his mercie, to withstande all suche occasion: except it
 maie be moze to your owne contentation and profite,
 then it would be pleasaunt to the louers of learning.

Master. Yet a iuste excuse maie stande for my de-
 claration : As if ignozaunce doe inforce me to staie
 my trauell.

Scholar. Your owne ignozaunce, I trust, you will
 not allege: and as for the ignozaunce of other, it ought
 to bee no staie : sith the ignozaunte multitude doeth,
 but as it was euer wonte, enuise that knowledgce,
 whiche thei can not attaine, and wishe all men igno-
 raunt, like vnto themself, but all gentle natures, con-
 temneth suche malice : and despiseth theim as blinde
 woymes, whom nature doeth plague, to stay the poi-
 sone of their venemous slynge.

Master. We shall not nede to stande on this talke,
 but trauell with knowledgce to vanquish the ignozaunce:
 And beleue that the prick of knowledgce, is moze of

A. i.

Geometric,

The seconde parte

Geometrie, and the vnitie in *Arithmetike*, though bothe be vndiuifible, doe make greater woorkes, & increase greater multitudes, then the bruttish bande of ignorance is hable to withstande.

Vnitie.

Scholar. Our talke groweth well to our mater. I beseeke you therfore, with that vnitie beginne, and builde on it your worke, as a fozte against ignorance

Master. Vnitie is of it self vndiuifible, and yet is it in al partes of the woorld, and in euery thing. Yea, the woorld it self consisteth of vnitie, is named of vnitie, was made by vnitie, and is preserved by vnitie, and onely ignorance with her broode secluded from vnitie, so that of it to repete the fulle fozte, would occupie muche time, and make greate volumes.

Number.

Scholar. Sith vnitie is so mightie, and of suche fozte (as you saie) what maie bethought then of number, whiche containeth a multitude of vnties? And is nothyng els but a collection of vnties.

Master. Vnitie is the fountaine and originalle of number, yea vnitie by addition onely shall make a greater number, then any numbers can doe by multiplication. But this is marueilouse, that no number repineth against diuision, till it come to an vnitie: and then will it permit no farther diuision. And therfore it is said, that vnitie doeth neither multiplie nor diuide.

And as al numbers maie be moze or lesse, so the lesse is euer a *parte* or *partes* of the greater.

*A parte.
Partes.*

As 5 vnto 10 is a parte, named a halfe: but vnto 7.5. is not a parte, but partes, and is called $\frac{5}{7}$. So 8 to .24. is a parte that is $\frac{1}{3}$; but vnto .36. it is partes, that is $\frac{2}{9}$.

Scholar. I perceiue, you call it a parte, when the numeratoz in the fraction (reduced to the smalleste) is an vnitie. And when the numeratoz is a number, then that fractio betokeneth partes of a number.

But I praie you, what varieties of numbers bee there principally to be considered in this arte?

Master.

of Arithmetike.

Master. Number is divided into diuerse kinde, *The firste di-*
foz some are whole numbers, and thei onely of *Euclide*, *division of num-*
bers. *Boetius*, and other good wryters are called numbers.

Other are broken numbers, and are commonly called
fractions. Of these bothe I haue wrytten in the firste
and seconde partes of *Arithmetike*: So that I mighte
seme to curiouse, to repete any parte of it again.

But now in eche kinde of these, there are certaine *The seconde*
numbers named *Abstrakte*: and other called numbers *division of*
Contracte. *numbers.*

Abstrakte numbers are those, whiche haue no deno- *Abstrakte.*
mination annexed vnto them. And those that haue a-
ny denomination toynd to them, are called *Contracte*
Contracte,
numbers.

Scholar. This I see to be a reasonable distinctiō,
and agreeable to the signification of the names.

For as that number is contracte, from his generall
libertie of signification, whiche is boude to one deno-
mination, as in sayng. 10. grotes (where. 10. is re-
strained fro the libertie of valowng any other thing
but grotes) so if it had no denomination adioined, it
might then signifie the number of daies, or of miles,
or any like thyng, as well as of grotes. For when I
saie. 10. and doe not limite any denomination, then is
that. 10. abstracte and seuered fro all specialties, and
standeth free to any name of thing.

But this (me thinketh vnder your correccion) can *whether bro-*
not extend to broken numbers: whiche euer moze ear- *ken numbers*
ry with them their denomination: seyng thei consiste *be contracte,*
of a numerator and a denominator. *or not.*

Master. You seme to saie well. And the like iudge-
ment doeth appere to be in some wryters of this arte.
But yet seyng that fractions maie haue all other ar-
tificiall denominations, that whole numbers maie
receiue: and maie also bee without them: therefore
must wee either make a moze curiouse distinction of

The seconde parte

that name of denomination : or els wee must seclude fractions, fro the necessitie of that name: or els thirdly, to avoid contention, cal them numbers contracte improperly.

Scholar. I assente thereto as reason would.

*Why fractiōs
be not called
numbers pro-
perly.* Yet one thying moze I must demaunde of you, why Euclide, and the other learned men, refuse to accompte fractions emongest numbers.

Master. Because all numbers doe consist of a multitude of vnities : and euery proper fraction is lesse then an vnitie, and therefore can not fractions exactly be called numbers: but maie bee called rather fractions of numbers.

Scholar. In deede now that I doe waite the mater moze exactly, it appereth that a fractiō is not properly a number, but a connerion and conference of numbers, declaring the partes of an vnitie . For the numerator doeth signifie one nōber, and the denominator another: The denominator declaring into how many partes the vnitie is diuided , and the numerator signifyng that of those partes, not all, but so many onely are to be take, as the numerator importeth.

*The diuision
of numbers
Abstracte.
Numbers
Absolute.* Master. Well, then to procede, numbers abstracte are considered in .3. principall varieties: That is, first without comparison to any other number or figure. And that number maie well be called *number absolute*.

*Numbers
Relative.* Secondly, some numbers bee vsed onely in relation to other, and therefore ought to bee called *numbers relative*.

Thirdly, many numbers are referred to some figure, that maie rise by multiplicacion of their partes together, and that diuersly. And those numbers therefore maie bee called *figuralle numbers*.

*Numbers
Figuralle.* Scholar. If I conceiue your wordes rightly, this is your meauynge : that when I saie. 10. 25. 100. or 200. &c. these numbers stand absolute from all denomination

of Arithmetike.

minacion, and clere from all relatiō and comparifon.

But when I faie. 6. is halfe of. 12. or. 15. is triple to 5. here the numbers beeyng compared together, are aptly called *numbers relatiue*: So if I faie, that. 16. is a *square number*, bicause it is made of. 4. multiplied by. 4. then is. 16. here to be called a *figuralle number*.

Maſter. You take it well. Therfoze will I brieſly touche the membez of euery kinde.

First of absolute nombzes, ſome are *euen numbers*, and ſome are *odde*.

Scholar. All men knowe that. And farther, that *Numbers, euen numbers* are thoſe, whiche maie be diuided into *euen, & odde* qualle halſes: and ſo can not *odde numbers*, without a fraction.

Maſter. Of this plaine eaſie thyng, marke what ſoloweth: a greater doubtc diſſolued. For if an odde number (as. 7. for example) can not bee parted into. 2. equalle numbers, eche beeyng halfe of. 7. then. $3\frac{1}{2}$. whiche is commonly called the halfe of. 7. is no nōber

Scholar. It can not be denied. And ſo (I ſee now) no fraction can bee a number. This greate doubtc is plainly diſſolued, by a very certaine and moſte known principle.

Maſter. Now farther. Of bothe theſe kindes of *Numbers* *compounds*, ſome bee *compounde*, and ſome bee *ſimple* and *poude*, and *vncompounde*. *Compounde* numbers are made by multi-*ſimple* plication of. 2. nombzes together, and not by additiō, though the name might ſeme to ſerue to bothe.

Scholar. So I perceiue, that 5. is no *compounde* nōber, although it bee made by additiō of. 2. and. 3. but 6. whiche is made by multiplication of. 2. and. 3.

Likewaies. 9. is *compounde*, bicause that. 3. multiplied by. 3. doeth make. 9.

And. 15. alſo is *compounde* by multipliyng. 5. and. 3. together.

And hereby I ſe that. 1. is not to be called a number *number*.

One is no

The seconde parte

foz then all nöbers about it, must nedes be *compounde*, because thei consist all of vnities.

Master. But yet by multiplication of .1. no other number is *compounde*.

Scholar. By those wordes I am taught to knowe more, and speake better.

Two is vn:
compounde. Master. *Euen numbers* are yet diuersly to be considered in their diuisions. Foz although the greate multitude of *euen numbers* bee *compounde*, yet .2. is accounted truly an *euen number*, originall, and *vncompounde*. So that it maie make other numbers, & is made of no nöbers, but of vnities onely, as al *odde numbers* are.

Euen nom:
bers, euenly All other *euen numbers* are *compounde*, and are diuersly diuided, foz some are *euen numbers euenly*, and some are *euen numbers oddely*, and some are *euen numbers bothe euenly and oddely*. *Euen numbers euenly*, are suche numbers as maie bee parted continually into *euen halues*, till you come to an *vnitie*. As foz example. 32. first is diuided into. 16. as his *euen halfe*: and again, 16. into. 8. as his *halfe*: And. 8. againe by. 4. is parted into. 2. *euen partes*: Then. 4. also by. 2. And that. 2. is diuided into. 2. vnities, as his *iuste halues*.

Euen numbers
vneuenly. But *euen numbers vneuenly*, are suche numbers as maie bee diuided into. 2. *equalle partes*: whiche are *odde numbers*. As. 18. is diuided into. 9. and. 9. as his *halues*, and thei are *odde*. So. 10. is diuided by. 5. And 30. by. 15. with a greate number more of *suche sorte*.

Euen nom:
bers, euenly
and oddely. Numbers *euen euenly and oddely*, bee commonly called *suche numbers*, as maie bee diuided into. 2. *equalle and euen halues*: but befoze you come to an *vnitie*, the *halues* will be *odde numbers*. As. 60. maie be first parted into. 30. and. 30. whiche are *euen*: And thei againe diuided by. 15. whiche is *odde*.

Like waies. 24. is diuided first by 12. And that 12. by 6. & lastly. 6. is diuided by. 3. whiche is an *odde nöber*. So. 28. maie bee diuided into. 2. *equalle and euen halues*,

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halues, that is into 14. And that. 14. into. 7. whiche is the halfe of. 14. but is an odde number.

Scholar. This I perceiue well. And, as I iudge, the distinction into those. 3. kindes, is not onely reasonable, but also needfull. And yet you seeme to speake doubtfully, of this laste membre. Bicause I remember not that you vse this worde commonly, but where you giue place rather to custome, then to reason.

Master. Ouzels to custome of the common sorte of wylters, rather then to the iudgemente of the mooste aunciente wylters.

And so in this case *Euclide* doeth not seeme to admitte this thirde member. But accompteth it vnder the seconde kinde. As maie well appere in his. 9. boke, and 24. proposition, where he calleth suche a number, *euently euen, and euently odde* also, whiche place cōferred with the definitions in the same booke, doeth appone in many wise menues opinions, that *Euclide* minded but 2. onely kindes of those nōbers. And yet in this thing (I thinke) he did rather approue. 3. varieties by his proposition, then establishe onely. 2. sortes by his first definitions.

But herein I will spende no more tyme. But sale briefly that the distinctiō of. 3. kindes, serueth to good vse, and ease in teachyng.

And now for farther knowledge of numbers, some are called *numbers perfecte*, & some are *numbers imperfect*.

Perfekte numbers are suche ones, whose partes ioyned together, will make exactly the whole number. *Numbers perfecte.*

And therefore are. 6. and. 28. accompted perfecte nōbers: bicause the partes of eche of theim added together, doe make the ful and intere number, whose partes thei bee. As of. 6. the halfe is. 3. the thirde parte is 2. the sixte parte is. 1. As for a quarter, and fiftte parte it hath not in whole number. Now put together. 1. 2. and. 3. and thei make iuste. 6. whose partes thei bee.

And

The seconde parte

28. And therfoze is. 6. a perfecte number.
 Likelwaies. 28. hath foze his halfe. 14. foze his quar-
 ter. 7. foze his seventh parte. 4. and foze his sowertenth
 parte. 2. and foze his. 28. parte. 1. all whiche put toge-
 ther, that is. 1. 2. 4. 7. and. 14. doe make. 28. of this foze
 there are very fewe moze in cõparisõ. And foze an exã-
 ple, I sett here, as many as are vnder. 6000000000.
 and thei are these. 6. 28. 496. 8128. 130816. 2096128.
 33550336. 536854528.

*Numbers
imperfekte.*

But now of the contrary kind, *imperfekte numbers* be
 suche, whose partes added together, doe make either
 moze or lesse, then the whole number it self: whose
 partes thei be.

*Numbers
superfluouse.*

And if the partes make moze then the whole nom-
 ber, then is that nõber called *superfluouse*, or *abundaunt*.
 As 12. whose partes are. 1. 2. 3. 4. 6. whiche make 16.

So. 20. hath foze his partes. 1. 2. 4. 5. 10. whiche
 make. 22. Likelwaies. 120. hath these partes.

1. 2. 3. 4. 5. 6. 8. 10. } whiche make 240.
 60. 40. 30. 24. 20. 15. 12. }

And if the partes make lesse then the whole nom-
 ber, whose partes thei be, then is that number called
Diminute, or *Defektiue*. As. 8. hath these partes. 1. 2. 4.
 whiche make but. 7.

*Numbers
Diminute.*

So. 16. hath these partes. 8. 4. 2. 1. and thei make
 onely. 15.

Likelwaies. 32. whose partes are. 1. 2. 4. 8. 16. and
 make but. 31.

Scholar. In all these numbers I note that you re-
 ken one, foze a parte of eche one of theim: whiche be-
 foze I thought you had denied.

Maister. I canns neither multiplie noz deuide, and
 therfoze compoundeth no number. But one maie in-
 crease addition, and therfoze where partes be added
 together, there. 1. maie well be called a parte.

And this shall suffice foze the diuision of euen nom-
 bers

of Arithmetike.

bers Abstracte.

How to speake of *odde numbers*, some of the are com *Odde nōbers*
poude, & some *vncompoude*. Thei are *compoude*, whiche *Compoude*.
maie bee diuided into any other partes then vnities.
As. 9. whiche is compoude of. 3. And. 15. that is made
of. 5. and. 3. Also. 21. is compoude of. 7. and. 3. And
so furthe. But. 3. 5. 7. 11. 13. 17. 19. 23. 29. and suche
like, bee *odde numbers vncompoude*. For thei are not *Vncompoude*.
made of any other then of vnities.

Here must you vnderstande by *composition*, the mul
tiplication of the partes of numbers together, as you
remembre, before was declared.

Scholar. I consider it so. And I remembre all that
you haue taught me, for the diuisio of nōbers *abstracte*
and absolute. What saie you now of nōbers *relatiue*: *Numbers*

Master. Some tymes their *relation* hath regarde *Relatiue*.
to their partes, namely, whether these. 2. that bee so
compared, haue any common parte, that will diuide
them bothe. For if thei haue so, then are thei called
numbers commensurable. As. 12. and. 21. bee *numbers com-* *Commensu-*
mensurable: for. 3. will diuide eche of them. *able*.

Like waies. 20. and. 36. be *commensurable*, seying 4. is
a commo diuisor for them bothe. But if thei haue no
suche common diuisor, then are thei called *incommensu-*
rable. As 18 and 25. For 25 can bee diuided by no nou- *Incommen-*
ber more then by. 5. And. 18. can not be diuided by it. *surable*.

In like maner. 36. and. 49. are *incommensurable*: For
49. hath no diuisor but. 7. And 7. can not diuide. 36.

Scholar. Doe you meane then, that *incommensura-*
ble numbers, haue no cōparison nor *proportion* together?

Master. Naie, nothyng lesse. For any. 2. numbers
maie haue comparison & *proportion* together, although
thei be *incommensurable*. As. 3. and. 4. are *incommensu-*
rable, and yet are thei in a *proportion* together: as shall
appeare anon.

But first I will declare vnto you, the varieties of
B. i. *proportion*

The seconde parte

Proportion. *proportion*, wherein there maie be double conferre: \mathfrak{A} meane of the lesser to the greater, or of the greater to the lesser.

Of greater inequality. *Of greater inequality.* \mathfrak{A} he the greater is compared to the lesser, it is called a *Proportion of the greater inequality*. As 6 to 2, or 5 to 3.
Of lesser inequality. *Of lesser inequality.* And when the lesser is conferred to the greater, it is called a *proportion of the lesser inequality*. As. 3. to. 5. or. 2. to. 6.

Scholar. And what if \mathfrak{A} would compare two equalle numbers together?

Master. That is accounted also a proportion of many men: and is called the *proportion of equality*. And then ought the first diuision of proportion to be, thus

| | | |
|---------------|---|--------------|
| | } | Equality. |
| Proportion of | | |
| | } | The greater. |
| | | The lesser. |

Multiplex. So proportion of the greater inequality, is diuided into. 5. severall kindes: whereof. 3. be *simple*, and. 2. other *compounde*. The firste kinde is, when a greater number containeth the lesser diuerse times: as twice, or thise, or oftener. So. 6. containeth. 3. twice: and it containeth. 2. thise. This proportion is called generally, *multiplex*, that is to saie, many folde: but specially it is named, accordyng to the tymes that it containeth the lesser. So that if it contain hym twice, then is it named *dupla*, or double. As 2 to 1 and 4 to 2.

And if it containe it thise, As. 3. to. 1. and. 6. to. 2. it is called *tripla*, or triple.

If it containe it. 4. tymes, then is it *quadrupla*, or quadruple.

Of these and of diuerse other sortes in this kind also, here are the names byiesly set doune with examples.

Dupla

of Arithmetike.

| | | | |
|--------------------|--|----|--------------------|
| <i>Dupla.</i> | 4. to. 2: 6. to. 3: 10. to. 5: 18. to. 9. | 1 | Double. |
| <i>Tripla.</i> | 6. to. 2: 9. to. 3: 12. to. 4: 18. to. 6. | 2 | Triple. |
| <i>Quadrupla.</i> | 4. to. 1: 8. to. 2: 12. to. 3: 16. to. 4. | 3 | Four folde |
| <i>Quintupla.</i> | 5. to. 1: 10. to. 2: 15. to. 3: 20. to. 4. | 4 | Fivefolde. |
| <i>Sextupla.</i> | 6. to. 1: 12. to. 2: 18. to. 3: 24. to. 4. | 5 | Sixfolde. |
| <i>Septupla.</i> | 7. to. 1: 14. to. 2: 21. to. 3: 28. to. 4. | 6 | Seuenfolde. |
| <i>Octupla.</i> | 8. to. 1: 16. to. 2: 24. to. 3: 32. to. 4. | 7 | Eightfolde. |
| <i>Noncupla.</i> | 9. to. 1: 18. to. 2: 27. to. 3: 36. to. 4. | 8 | Ninefolde. |
| <i>Decupla.</i> | 10 to 1: 20. to 2: 30. to 3: 40. to 4. | 9 | Ten folde. |
| <i>Vndecupla.</i> | 11. to. 1: 22. to. 2: 33. to. 3. | 10 | Eleuenfolde |
| <i>Duodecupla.</i> | 12. to. 1: 24. to. 2: 36. to. 3. | 11 | Tweluefold. |
| And so infinitely. | | | |

Beside this there is an other kinde of proportion, when the greater containeth the lesser, more then ones, and not twise: and that maie bein 2 sortes. For if the greater containe the lesser, and any one parte of hym, that proportion is called *Superparticulare.*

For example, take 5. to. 4. With 5. doeth containe 4. Superparticulare. and his quarter. Like waies. 6. to. 5. is in the same kinde of proportion: although, not of the same speciall sorte. For 6. comprehendeth 5. and his fifte parte.

So that for a more speciall distinction, eche of these and many other, haue their seueral names, according to that parte, whiche they do containe. As if it containe the halfe more, it is named *Sesquialtera.* In whiche proportion are these numbers following. Sesquialtera

3. to. 2: 6. to. 4: 9. to. 6: 12. to. 8: 15. to. 10. | 11.

But if the greater comprehend the lesser, and his thirde parte, then is that named *Sesquitertia* proportion. As in these. Sesquitertia.

4. to. 3: 8. to. 6: 12. to. 9: 16. to. 12: 20. to. 15. | 14.

And when the fifte, sixte, seuenth, or eight part doeth make the proportion, or any other part els, the name is taken of that same parte. As for briefnesse I will here sette examples.

15. ii. *Sesquiquarta.*

The seconde parte

| | | | |
|-------------------------|-------------------------------------|-----------------|-----------------|
| <i>Sesquiquarta.</i> | 5. to. 4: 10. to. 8: 15. to. 12. | $1\frac{1}{4}$ | A quarter moze. |
| <i>Sesquiquinta.</i> | 6. to. 5: 12. to. 10: 18. to. 15. | $1\frac{1}{5}$ | a fiftie moze. |
| <i>Sesquisexta.</i> | 7. to. 6: 14. to. 12: 21. to. 18. | $1\frac{1}{6}$ | a sixtie moze. |
| <i>Sesquisiptima.</i> | 8. to. 7: 16. to. 14: 24. to. 21. | $1\frac{1}{7}$ | a seuenth moze. |
| <i>Sesquioc̄taua.</i> | 9. to. 8: 18. to. 16: 27. to. 24. | $1\frac{1}{8}$ | an eight moze. |
| <i>Sesquinona.</i> | 10. to. 9: 20. to. 18: 30. to. 27. | $1\frac{1}{9}$ | a ninth moze. |
| <i>Sesquidecima.</i> | 11. to. 10: 22. to. 20: 33. to. 30. | $1\frac{1}{10}$ | a tenth moze. |
| <i>Sesquiundecima.</i> | 12. to. 11: 24. to. 22: 36. to. 33. | $1\frac{1}{11}$ | a twelfth moze. |
| <i>Sesquiduodecima.</i> | 13. to. 12: 26. to. 24: 39. to. 36. | $1\frac{1}{12}$ | a twelueh moze. |

And so as farre as you liste to procede in suche proportion: where one parte of the lesser, is the iuste difference and excessse, betwene it and the greater.

But if the difference be. 2. partes. 3. partes, or moze *Superparties* partes: the proportiō is named *superpartiente*. As. 5. to 3. And. 10. to. 6. For as. 5. containeth. 3. and. $\frac{2}{5}$. of it: so 10. holdeth. 6. and. $\frac{4}{5}$. of it.

Scholar. Now I perceiue some vse also, of the distinction betwene a parte and partes in number: Of whiche at the beginning you did speake. But how many kindes are there of this sorte?

Master. There are infinite kindes in this sorte of proportion, as well as in the other. But for examples sake, I will set furthe some of the moste common numbers: that therby you maie gather the formes of the reste. And these be they, with their names.

| | | | | |
|-------------------------|---|-------------------|---|----------------|
| <i>Superbipartiens.</i> | { | <i>Tertias.</i> | 5. to. 3: 10. to. 6: 15. to. 9: 20. to. 12. | $\frac{2}{3}$ |
| | | <i>Quintas.</i> | 7 to 5: 14. to 10: 21. to 15: 28. to. 20 | $\frac{2}{5}$ |
| | | <i>Septimas.</i> | 9 to 7: 18 to 14: 27. to 21: 36. to 28. | $\frac{2}{7}$ |
| | | <i>Nonas.</i> | 11 to 9: 22 to 18: 33. to 27: 44. to 36 | $\frac{2}{9}$ |
| | | <i>Undecimas.</i> | 13. to 11: 26 to 22: 39 to 33: 52 to 44 | $\frac{2}{11}$ |

of Arithmetike.

| | | | |
|----------------------|--------------------|--|----|
| Suptriparties | Quartas. | 7. to. 4: 14. to. 8: 21. to. 12: 28. to. 16. | 1 |
| | Quintas. | 8. to. 5: 16. to. 10: 24. to. 15: 32. to. 20. | 2 |
| | Septimas. | 10. to. 7: 20. to. 14: 30. to. 21: 40. to. 28. | 3 |
| | Octauas. | 11. to. 8: 22. to. 16: 33. to. 24. | 4 |
| | Decimas. | 13. to. 10: 26. to. 20: 39. to. 30. | 5 |
| | Vndecimas. | 14. to. 11: 28. to. 22: 42. to. 33. | 6 |
| | Decimas tertias. | 16. to. 13: 32. to. 26: 48. to. 39. | 7 |
| | Decimas quartas. | 17. to. 14: 34. to. 28: 51. to. 42. | 8 |
| Superquadrupartiens. | Quintas. | 9. to. 5: 18. to. 12: 27. to. 15: 36. to. 20. | 9 |
| | Septimas. | 11. to. 7: 22. to. 14: 33. to. 21: 44. to. 28. | 10 |
| | Nonas. | 13. to. 9: 26. to. 18: 39. to. 27: 52. to. 36. | 11 |
| | Vndecimas. | 15. to. 11: 30. to. 22: 45. to. 33. | 12 |
| | Decimas tertias. | 17. to. 13: 34. to. 26: 51. to. 39. | 13 |
| | Decimas quintas. | 19. to. 15: 38. to. 30: 57. to. 45. | 14 |
| Superquintupartiens. | Sextas. | 11. to. 6: 22. to. 12: 33. to. 18: 44. to. 24. | 15 |
| | Septimas. | 12. to. 7: 24. to. 14: 36. to. 21. | 16 |
| | Octauas. | 13. to. 8: 26. to. 16: 39. to. 24. | 17 |
| | Nonas. | 14. to. 9: 28. to. 18: 42. to. 27. | 18 |
| | Vndecimas. | 16. to. 11: 32. to. 22: 48. to. 33. | 19 |
| | Duodecimas. | 17. to. 12: 34. to. 24: 51. to. 36. | 20 |
| | Decimas tertias. | 18. to. 13: 36. to. 26: 54. to. 39. | 21 |
| | Decimas quartas. | 19. to. 14: 38. to. 28: 57. to. 42. | 22 |
| | Decimas sextas. | 21. to. 16: 42. to. 32: 63. to. 48. | 23 |
| Supersextupartiens. | Septimas. | 13. to. 7: 26. to. 14: 39. to. 21. | 24 |
| | Vndecimas. | 17. to. 11: 34. to. 22: 51. to. 33. | 25 |
| | Decimas tertias. | 19. to. 13: 38. to. 26: 57. to. 39. | 26 |
| | Decimas septimas. | 23. to. 17: 46. to. 34: 69. to. 51. | 27 |
| | Decimas nonas. | 25. to. 19: 50. to. 38: 75. to. 57. | 28 |
| | Vicesimas tertias. | 29. to. 23: 58. to. 46: 78. to. 69. | 29 |

Scholar. I vnderstande by these examples, some what of their reasons: but I perceiue, you doe not followe their naturalle order, without interruption, in these

The seconde parte

these of the lasse kinde.

Maister. To thintente you maie the better vnderstande good ground in that omission, I wil set furthe here those omitted numbers: That you maie see how thei would expresse some other propoziti^o here named And therfoze thei doe seme rather to be omitted, then in deede so to be.

Marke theim well.

| | | | | | |
|------------------|---|-----------|-------------|-------------|----------------|
| Superbipartiens. | { | Secundas. | 4. to. 2: | 8. to. 4. | $2\frac{2}{3}$ |
| | | Quartas. | 6. to. 4: | 12. to. 8. | $1\frac{1}{2}$ |
| | | Sextas. | 8. to. 6: | 16. to. 12. | $1\frac{1}{3}$ |
| | | Octauas. | 10. to. 8: | 20. to. 16. | $1\frac{1}{4}$ |
| | | Decimas. | 12. to. 10: | 24. to. 20. | $1\frac{1}{5}$ |

Scholar. In deede here I see, the firste is double propozition. The seconde *sesquialtera*, the thirde *sesquitercia*, the folwerth *sesquiquarta*, & the fiftc *sesquiquinta*.

Maister. So marke these other.

| | | | | | |
|------------------|---|-------------|-------------|-------------|----------------|
| Supertripartiens | { | Secundas. | 5. to. 2: | 10. to. 4. | $2\frac{2}{3}$ |
| | | Tertias. | 6. to. 3: | 12. to. 6. | $\frac{7}{3}$ |
| | | Sextas. | 9. to. 6: | 18. to. 12. | $1\frac{1}{2}$ |
| | | Nonas. | 12. to. 9: | 24. to. 18. | $1\frac{1}{3}$ |
| | | Duodecimas. | 15. to. 12: | 30. to. 24. | $1\frac{1}{4}$ |

Scholar. The firste of these I knewe not, but all the other are named before.

Maister. The firste is a compounde propozition (as anon I will declare) and is named *dupla sesquialtera*.

But now will I sette furthe all the other omitted names.

Secundas.

of Arithmetike.

| | | | | | |
|---|-------------------------|-------------------------|------------------------------------|----------------------|---------------------------------------|
| <i>superquadru-</i> <i>partiens.</i> | { | <i>Secundas.</i> | 6. to. 2: 12. to. 4. | $\frac{3}{2}$ | <i>Tripla.</i> |
| | | <i>Tertias.</i> | 7. to. 3: 14. to. 6. | $2\frac{2}{3}$ | <i>Dupla sesquitertia.</i> |
| | | <i>Quartas.</i> | 8. to. 4: 16. to. 8. | $2\frac{1}{2}$ | <i>Dupla.</i> |
| | | <i>Sextas.</i> | 10. to. 6: 20. to. 12. | $1\frac{2}{3}$ | <i>superbiparties tertias</i> |
| | | <i>Octauas.</i> | 12. to. 8: 24. to. 16. | $1\frac{1}{2}$ | <i>sesquialtera.</i> |
| | | <i>Decimas.</i> | 14. to 10: 28. to 20 | $1\frac{2}{5}$ | <i>superbiparties quintas</i> |
| | | <i>Duodecimas.</i> | 16. to. 12: 32. to. 24 | $1\frac{1}{3}$ | <i>Sesquitertia.</i> |
| | | <i>Decimas quartas.</i> | 18. to. 14: 36. to. 28 | $1\frac{3}{7}$ | <i>superbiparties septimas</i> |
| | <i>Decimas sextas.</i> | 20. to 16: 40 to 32. | $1\frac{1}{4}$ | <i>Sesquiquarta.</i> | |
| <i>superquin-</i> <i>supartiens.</i> | { | <i>Secundas.</i> | 7 to 2: 14 to 4. | $3\frac{1}{2}$ | <i>Tripla sesquialtera.</i> |
| | | <i>Tertias.</i> | 8 to 3: 16 to 6. | $2\frac{2}{3}$ | <i>Dupla superbipartiens tertias.</i> |
| | | <i>Quartas.</i> | 9 to 4: 18 to 8. | $2\frac{1}{4}$ | <i>Dupla sesquiquarta.</i> |
| | | <i>Quintas.</i> | 10 to 5: 20 to 10. | $2\frac{2}{5}$ | <i>Dupla.</i> |
| | | <i>Decimas.</i> | 15 to 10: 30 to 20. | $1\frac{1}{2}$ | <i>Sesquialtera.</i> |
| | <i>Decimas quintas.</i> | 20 to 15: 40 to 30. | $1\frac{1}{3}$ | <i>Sesquitertia.</i> | |
| <i>supersextu-</i> <i>partiens.</i> | { | <i>Secundas.</i> | 8 to 2: 16 to 4. | $\frac{4}{1}$ | <i>Quadrupla.</i> |
| | | <i>Tertias.</i> | 9 to 3: 18 to 6. | $3\frac{1}{3}$ | <i>Tripla.</i> |
| | | <i>Quartas.</i> | 10 to 4: 20 to 8. | $2\frac{1}{2}$ | <i>Dupla sesquialtera</i> |
| | | <i>Quintas.</i> | 11 to 5: 22 to 10. | $2\frac{1}{5}$ | <i>dupla sesquiquinta.</i> |
| | | <i>Sextas.</i> | 12 to 6: 24 to 12. | $2\frac{1}{3}$ | <i>Dupla.</i> |
| | | <i>Octauas.</i> | 14 to 8: 28 to 16. | $1\frac{2}{3}$ | <i>supertripartiens quartas.</i> |
| | | <i>Novas.</i> | 15 to 9: 30 to 18. | $1\frac{2}{3}$ | <i>superbipartiens tertias.</i> |
| | | <i>Decimas.</i> | 16 to 10: 32. to 20. | $1\frac{2}{5}$ | <i>supertripartiens quintas.</i> |
| | | <i>Duodecimas.</i> | 18 to 12: 36 to 24. | $1\frac{1}{2}$ | <i>sesquialtera.</i> |
| | | <i>Decimas quartas.</i> | 20 to 14: 40 to 28 | $1\frac{3}{7}$ | <i>supertripartiens septimas.</i> |
| | | <i>Decimas quintas.</i> | 21 to 15: 42 to 30. | $1\frac{2}{5}$ | <i>superbipartiens quintas.</i> |
| | | <i>Decimas sextas.</i> | 22 to 16: 44 to 32. | $1\frac{2}{7}$ | <i>supertripartiens octauas.</i> |
| | | <i>Decimas octauas.</i> | 24 to 18: 48 to 36. | $1\frac{1}{3}$ | <i>sesquitertia.</i> |
| <i>Vicesimas.</i> | 26 to 20: 52 to 40. | $1\frac{3}{5}$ | <i>supertripartiens decimas.</i> | | |
| <i>vicesimas secundas.</i> | 28 to 22: 56 to 44. | $1\frac{3}{7}$ | <i>supertripartiens vndecimas.</i> | | |

Scholar. I see well that these proportions, bee agreeable with some other name: and therefore might seme superfluous in this place,

Master.

The seconde parte

Master. Not onely superfluously, but also falsely should they be placed here: seynge they doe belong to other places of right.

Scholar. Why doe you not name them all by Englishe names?

Master. Because there are no soche names in the Englishe tongue. And if I should giue them new names, many would make a quarrelle against me, for obscuryng the olde Arte with new names: as some in other cases all redy haue doon.

Scholar. Yet I praye you declare those doubtfull names of compounde ppozitions.

Master. As there is one kinde of ppozition, that is named *multiplex*, or manyfolde, whiche doth containe the lesser diuers times exactly. And two other whiche doe containe the lesser ones, and some parte or partes of the same: So those kindes may be compounded together. As when the greater number containeth the lesser, twice, or thise, or oftener: and yet more ouer some parte or partes of the same. So .8. containeth 3 twice, and his $\frac{2}{3}$. And 10 comprehendeth 3. thise and his $\frac{1}{3}$.

The firste example is generally called *multiplex superpartiens*: because the greater containeth the lesser certayne tymes, and some partes of it besides. But more specially it is called *dupla superbipartiens tertias*, that is, double with $\frac{2}{3}$ more.

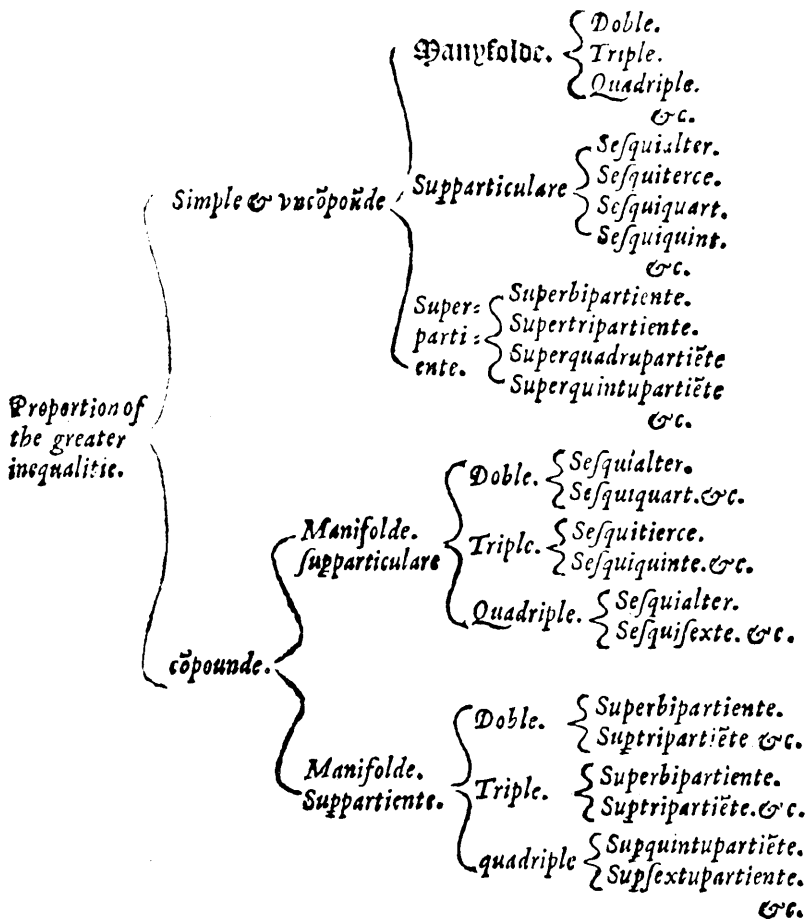
The seconde example is generally referred to *multiplex superparticularis*: because in it the greater comprehendeth the lesser oftentimes, (as here thise) and his $\frac{1}{3}$ more. And therfore specially it is called *tripla sesquitertia*.

But as I doe intende hereby to ouer runne this parte: so will I by tables set forth the kindes of the with their examples.

The

of Arithmetike.

The table of proportion of the greater inequality.



C. j. Examples

The seconde parte

Examples of eche compounde kinde,
mentioned in the former table.

| | | | | | |
|------------------------------------|---|-----------|---|-----------------------------------|----------|
| <i>Manifolde Superparticulare.</i> | } | Double. | { | <i>Sesquialter.</i> | 5 to 2. |
| | | | | <i>Sesquiquarte.</i> | 9 to 4. |
| | } | Triple. | { | <i>Sesquiterce.</i> | 16 to 3. |
| | | | | <i>Sesquiquinte.</i> | 16 to 5. |
| | } | quadriple | { | <i>Sesquialter.</i> | 9 to 2. |
| | | | | <i>Sesquisexte.</i> | 25 to 6. |
| <i>Manifolde fuppartiēte</i> | } | Double. | { | <i>Superbipartientetierces.</i> | 8 to 3. |
| | | | | <i>Superipartiente quartas</i> | 11 to 4. |
| | } | Triple. | { | <i>Superbipartiente tierces.</i> | 11 to 3. |
| | | | | <i>Suptripartiente quartes.</i> | 15 to 4. |
| | } | quadriple | { | <i>Supquītupartiēte quartas</i> | 29 to 6 |
| | | | | <i>supfextupartiēte septimas.</i> | 34 to 7 |

Scholar. What more is there to bee learned of these proportions: For by these formes, I maie easily gather the value or rate of any proportion.

Maſter. This maie ſtande for their numeration: ſaue that moſte aptly they ought to bee ſette as fractions, in their leaſte tearmes: as you haue here diuerſe examples.

Scholar. You meane that double *ſesquialter* muſt be written thus $\frac{2}{1}$, and ſo of the reſte.

Maſter. Or els thus $2\frac{1}{1}$, and ſo *triple ſesquiquinte* in this ſorte: $3\frac{1}{5}$, or thus $\frac{16}{5}$ and ſo of all other.

And for farther worke, you ſhall vnderſtande that proportions maie bee added, ſubtracted, multiplied and diuided: and verie ſtraunge workes thereby achieved:

of Arithmetike.

achieved. For of the Arte of Proportions, dependeth all the subtilties, and fine workes, not onely of *Arithmetike*, but also of *Geometrie*: besides farther matter that as now I will not touche. But as for the workes of *Proportions*, I will omitte them til an other tyme: considering not onely the troublesome condition, of my unquiete estate: but also the convenient order of teaching, whereby it is required that the extraction of rootes, should go orderly before the arte of *Proportions*: whiche without those other, can not be wrought.

Therefore will I now onely declare these kindes of proportion, whiche yet are not spoken of: to the intent that you maye haue here, the generall diuision of numbers, somewhat sufficiently touched.

As you see that betwene any two numbers, there maye be a conference of proportion: so if any one proportion be continued in moze then .2. numbers, there maye be then a conference also of these proportions, in their severall termes: and that conference or comparison is named *Analogie*: whiche some delight to call proportionalitie: As in this example, where 3 numbers beare like proportion in their progression: 4. 6. 9. You see that 6. to 4. is in proportion *sesquialter*: and so is 9. to 6. and therefore is there a like proportion betwene the .2. laste, as there is betwene the .2. firste.

Scholar. This I consider well by progression in *Arithmetike*.

Master. Like wales where soever termes or moze be set in order of proportion, as here 2. 6. 18. 54.

Scholar. I perceiue this wel: for here the proportion is triple. But what saie you to this forme of comparison in Proportion: As 6. is to .2: So, 30. is to .20. Is it not all one kinde of *Analogie*?

Master. It is one kinde of *Analogie* generalle, whiche maye be called *directe Analogie*: because the first *Direkte* one is compared to him that doeth folowe nexte: & so eche

C. 11.

other

The seconde parte

other is still referred to that, that foloweth nexte. But this is the difference: that in the firste, there is a continuance of collation: and one terme is compared with twoo numbers: But in that forme of example, whiche you put, there is no number compared twise: For the first is referred to the seconde, and the seconde to the thirde. And so haue thei seueralle names to distinge them a sonder.

*Continuall
Proportion.*

Wherefore whē the first number is referred to the seconde, and that seconde to the thirde: the proportion is called *continuall*: and it maie consist betweene 3. termes. As 5. 15. 45. doe procede in a continuall triple proportion. For as 5. is to 15: so is 15. to 45. as you doe see. But when I saie thus: as 5. is to 15. so 6. is to 18. Here is a triple proportion, but not continuall. For the seconde terme beyng 15. is not compared with the thirde terme, that is 6. And therefore is it called a proportion *discontinuall*.

*Discontinuall
Proportion.*

Scholar. Now I perceiue certainly their distinction: For in twoo pointes these examples doe agree, and differ in a thirde pointe.

Firste they agree in that (as you saied) that the firste is referred to the other that foloweth it nexte: And secondly, they agree in this also, that bothe are compared in a triple proportion. But in this they differ, that the seconde terme, doeth not beare like proportion to the thirde, as the thirde doeth to the fourth or the firste to the seconde.

Master. Farther more there is to bee noted, that in discontinuall proportion, there can bee no fewer then fower termes, or numbers: and so by euen termes still, as 6. or 8. and so forth. Where as in continuall proportion, your termes maie bee of any number, euen or odde: aboue 2.

And although I might saie more of the diuersities of proportion: as of *Proportion conuersed* or *indirecte*, *Proportion*,

of Arithmetike.

portion interchaunged, compounde Proportion, parted Proportion, reuerſed Proportion, and Proportion by equalitie. Yet I thinke better to procede for this tyme, to the other kinde of number, and to reſerue the explication of proportions to their peculiare place.

Scholar. As you knowe the beſt order, ſo it ſhalbe mete that you doe vſe your owne iudgement therein.

Of figuralle numbers.

Maſter.



TH E nexte kinde of numbers are called *Figuralle numbers*: becauſe thei doe, or maie repreſente ſome figure: And are euer conſidered in relation to thoſe formes.

Some of them haue a comparison and relation to length onely, and therefore are named *Linearie numbers*: whiche name, although it maie be referred moſte aptly to ſuche numbers, as will make no other forme duely, yet maie it alſo be applied to any number abſtract. Sith all ſuche numbers maie be conſidered as the ſides of other figuralle numbers.

Secondly, numbers maie be conſidered, according to ſuche formes as thei make other in progression, or in multiplication: And thoſe maie well be named *Superſiciall numbers*, or *Flatte numbers*. Whereof there be as many varieties, as there be diuerſities of figures in Geometrie. As numbers *Triangulare*, *Quadrate*, *Cinke-angeled*, *Sixe-angeled* and ſo forth. Alſo numbers *circulare*, *diametralle*, & *like flattes*, all whiche numbers haue bothe lengthe and breadthe: and thereof be named *ſuperſiciall numbers*.

The seconde parte

Sounde
numbers.

Beside these there are other numbers, whiche are made of many multiplications, and thei are called *sounde numbers*: because that as by the firste multiplication, thei take lengthe and breadthe, like flatte numbers, so by the second multiplication, thei take depthe also: And thei are thei named *bodily numbers*, or *sound numbers*.

The lease of them all is commonly called a *Cube*, or a *Cubike number*: And the other in their degrees severally named, as thei bee made by severalle multiplications. For accordynge to the number of their multiplications, thei take their names. And all that haue like number of multiplications, are of one kinde, and bere one name: as well in flatte numbers, as in sounde.

But considerynge the infinite multitude of those figuralle numbers, I thinke beste to speake of them onely in this place, whiche haue muche profitable vse in this arte. And, of those, among infinite flatte numbers, I will take onely sower. That is to saie, *square numbers*, *longesquares*, *diametralle numbers*, and *likeflattes*.

Square
numbers.

Square numbers are those, whiche may be diuided by some one number, and haue the same number for the quotient: that is to saie, that a square number is made by the multiplication of any number into it self, as 10 multiplied by 10, maketh 100. What 100, is a square number: whiche, 100, if I doe diuide by 10, the quotient will be 10, also.

Scholar. So, 4, multiplied by 4, maketh 16: and that must be a square number by like reason.

Master. So it is.

Scholar. And if I multiplie 9, by 4, is not that a square number? Seyng sower semeth to make all numbers square by multiplication.

Master. Consider this well, that a square number doeth make suche a square in number, as a *iuste square* doeth make in *Geometrie*: That is suche a one whose

of Arithmetike.

Whose sides are equalle. For if the one side be longer then the other, that figure in *Geometrie* is called a *long square*, and so it is named in number, a *long square* also.

Now if I sette downe the figure of your number, as you termed it, and sette .4. for the one side, and .9. for the other, this will the figure she be.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Here you se a plain longsquare: | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| yet is the whole number that amounteth of this multiplication: truly named a square number, as here you may see. But then is the side or roote of it, neither .4. nor .9. but .6. | <table style="border-collapse: collapse;"> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> <tr><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td><td>•</td></tr> </table> | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • |
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Scholar. Now I vnderstande it: and the better by this figurall example. And here also I haue learned what a *Roote* is: for you seeme to expounde it, to bee the side of a figurall number. *A roote.*

Master. Every flatte number, and euery sounde number also haue their sides: But no flatte number, saue onely squares haue a roote: because a roote in flatte numbers, is a number multiplied by it self.

And in sounde numbers, thei onely haue rootes, whiche bee made by many multiplications, of some one nuber by it self: other by that, whiche riseth of it.

As when I saie, twoo tymes, twoo twise, maketh 8. that number is a sounde number: and is named a *Cube*. And so, 3. tymes, 3. thise, doeth make, 27. whiche is also a *Cube*.

And generally, any number that is made by suche 2. multiplications, is called a *Cube*, or *Cubike* number. *A cube.*
 And the number of that multiplication, whiche commonly is named the multiplier, is in this pointe called the *Cubike roote* of that number. *A cubike roote.*

Wherfore, thus also maye you define a *Cubike* nō: *A cubike ber, number.*

The seconde parte

ber: to bee suche a number, as beeyng diuided by his roote, shall haue foꝛ the quotiente the square of the same roote.

Scholar. Hereby I perceiue, that one multiplication, of any number by it selfe, doeth make a square number. And twoo multiplications in that sorte, doe make a Cubike number.

What if I doe multiplie any number thysise, oꝛ foꝛwer tymes, oꝛ oftener in that sorte, are there proper names foꝛ suche numbers?

Master. Yes in deede: as I will declare anon.

But firste befoꝛe we attempte the other sounde nōbers, it shall bee mete, that we doe declare those twoo sortes of flatte numbers, whiche I named befoꝛe: that is diametralle numbers, and like flattes.

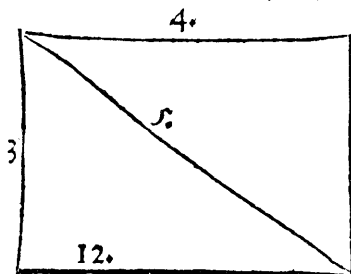
A diametral number. A *Diametralle number*, is suche a number as hath twoo partes of that nature: that if thei bee multiplied together, thei will make thesaied *diametralle number*:

And the squares of those twoo partes, beeyng added together, will make a square nōber also: whose roote

A diameter. is the *diameter* to that *diametralle number*.

As 12 is named a *diametralle number*, foꝛ that he hath twoo partes, that is. 3. and. 4, whiche beeyng multiplied together, doe make 12. that is the firste number. And if their squares be added together, thei wil make a thirde square: and the roote of that number will bee the *diameter* to that platte forme of 12. As in this example you see.

The one side is. 4. and the other side is 3 whiche bothe multiplied together, doe make 12. When take the square of foꝛwer whiche is 16 and the square of. 3, whiche is. 9. and put them



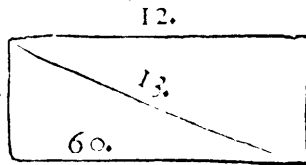
together

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together, and they will make .25. whose roote, beyng 5. is the *diameter* of that platte forme.

Scholar. That doe I perceiue well, bicause it is confirmed by the .33. theozeme of the pathe waie.

Master. Yet take an other example. In this platte forme of .60. you see the one side to bee .5. and the other side to bee 12. Now take the square number of .12. whiche is .144. and the square of .5. whiche is .25. and put them together : so will it make 169. whiche is a square number: and hath .13. for his roote.



Like waies .120. is to be accounted a *diametralle number*. For so muche as it hath two partes: that is 8. and .15. whiche beyng multiplied together, doe make the firste number .120. And the square of those two partes (that is .64. for .8. and .225. for .15.) beyng bothe added together, doe make .289. whiche is a square nōber: and hath for his roote .17. And therfore that .17. is the *diameter* to that *diametralle number* .120.

Like examples infinite might I giue you. But these for explication of the name, maie suffice.

Scholar. I doe well vnderstande the examples: saue that I knowe not how to finde the roote of the laste square number, whiche amounteth by the addition of the former two squares together.

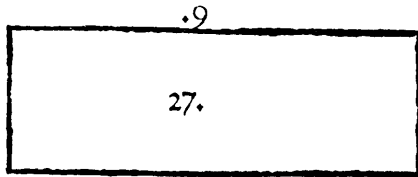
Master. That arte will I teache you anon. But we maie not forgette firste to ende all the definitions of soche names, as I minde to write of.

Whercof yet there resteth *like flattes*: whiche maie be as well taken for *trianguler figures*, as for *quadrate figures*. *Like flattes.*

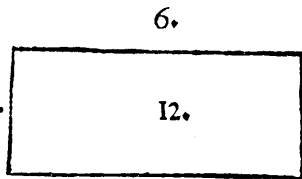
So that of any of them, when the sides of one platte forme, beareth like proportion together, as the sides

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of any other flatte forme of the same kinde doeth, then are those formes called *like flattes*. As in these. 2. longe 3.

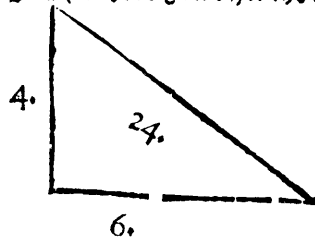
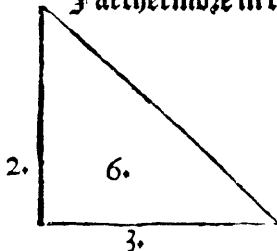


Squares: because the sides of them bothe, are in one proportion (for. 6. is triple to. 2: as well as. 9. is triple to 3.) Therefore are 2. the whole figures called *like flattes*.



And so of due conueniencie, their numbers (that expresse their quantities, whiche here are. 27. and 12) be called by the like names, *like flattes*.

Farthermoze in triangles (as here you se) if the si-



des of the one beare like proportion together, as the sides of the other doe: then are they called *like flattes* also. And their numbers, that declare their quantities, in like sorte are named *like flattes*.

Scholar. I perceiue here: As 4 is to 2: so. 6. is to 3, bothe being in a double proportion. And therefore 6 and. 24. are to be called *like flattes*.

Master. You vnderstande it well.

And thus haue we briefly ouer runne the diuision of number, into his principalle kindes: And haue set forth the the definitiōs of eche of them, with examples.

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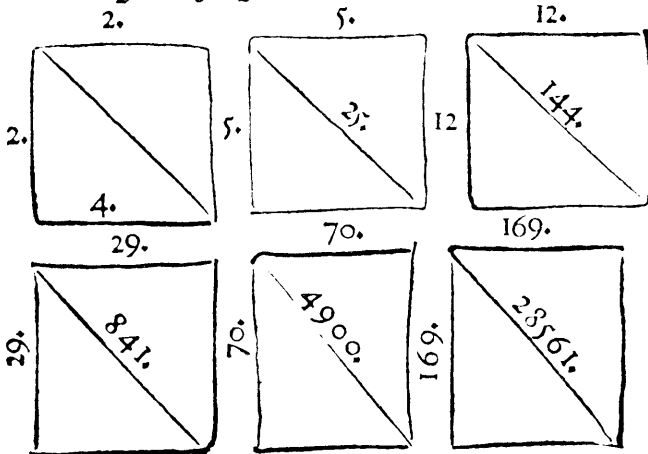
The vse of them you shall see largely in the practise of this arte.

• But to the intent you maie the better obserue and regarde these twoo laste kindes of numbers : whiche are commonly neglected of artes men , I will shew you some vse of them, with their properties.

Firste, all *diametralle numbers* doe sette fothe a tri-
 angle, hauyng all thzee sides knowen: whiche thing *Of diametralle numbers.*
 as it doeth serue to many and wonderfull purposes: so can it be found in no other numbers, then onely in *diametrall numbers*.

For although in figures *Geometricalle*, you maie euer moze vnfallibly finde one line, that will make a square, equall to the twoo squares of any other twoo lines (as in the pathe waie you doe see it taught) yet the measure certaine of those sides, are not knowen.

Wherfoze in number that is not possible alwaies to be doen : neither can it be doen with any other numbers, then onely *diametrical numbers*. Yet maie other numbers go very nigh. As namely in these examples

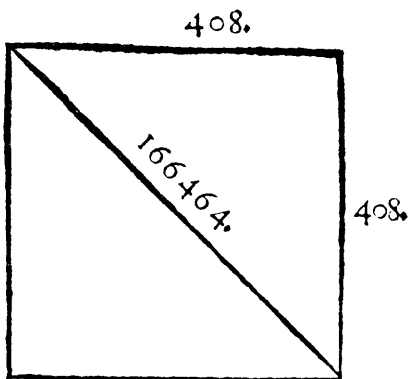


of square numbers: whose double, I take for the squares
 D. y. res

The seconde parte

res of the sides,
bicause thei are
equall: and thei
make. 8. 50. 288.
1682 . 9800.
57122. &. 332928.
All whiche dif-
fer onely by an
unitie, from a
square number.

For nine is a
square number
and so are these
other folowynge.



49. 289. 1681. 9801. 57121. &. 332929.
whose rootes be. 7. 17. 41. 99. 239. 577.

Whiche examples if you doe consider well hereaf-
ter, thei will helpe you to gesse at the nigheste rootes
of numbers that be not square. And also for doblving
of squares, in a square forme: within an vnspreak-
able nere nesse.

For as in doblving of this greater square. 166464.
there riseth. 332928. whiche wanteth one of a iuste
square. You se easely, that as that one is but a smalle
portion to the whole square: So yet, that one wan-
teth not in the roote, but in the whole square: where
by you maie perceiue, that it is a very smalle and vn-
sensible parte of one, that wanteth in the roote.

Scholar. It must seme by reason of multiplicati-
on: that it is scarce the. 10000. parte of one.

Master. You saie truthe.

Scholar. But how shall I finde the diameter of
soche numbers?

Master. That is easly doen, if you knowe firste
certainly that your number is a diametrall number.

And secondarily, if you knowe the true partes of
it:

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it: whiche you should vse in this case.

Scholar. Will not any twoo soche partes serue, whiche by multiplication will make the whole number?

Maister. You maie by the fozymer examples, easily se the contrary. For 12 is a *diametrall number*: and hath these partes (as it is sone perceiued). 2. 3. 4. 6. Yet if you take .2. and .6. for the sides of it, thei will not make a *diameter* in knowen number.

Scholar. What I vnderstande: for the square of 2. beyng .4. added to .36. whiche is the square of 6. doeth make .40. whose roote must bee greater then .6. and lesse then .7. And therfoze. 40. can haue no roote in whole number.

Maister. Neither yet in broken numbers: for that is a generalle rule: that if any whole number haue a roote, that roote shall be a whole number. So that if the roote can not bee founde in whole number: you shall neuer finde it in broken numbers.

And for moze certaintie of that I saied befoze, that all partes be not apte for the sides of a *diametrall number*, to finde out the *diameter*: marke well the seconde example, whiche is .60. and hath these partes.

2. 3. 4. 5. 6. 10. 12. 15. 20. 30.

So that beginnyng with the two extreme, that is .2. and .30. thei will by multiplication make .60.

And likewise any two numbers, equally distant from those extremes: As .3. and 20. Likewise .4. and 15: other .5. and .12. And in like maner .6. and .10. All those couples by multiplication doe make .60. Yet none of them are apte sides to finde the *diameter* by, but onely 5 and .12. For of the other sides beyng multiplied squarely (that is by the selves) and those squares beyng added together, there wil not rise a square number. As you shall better vnderstande, when you

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haue learned to knowe square numbers, by extractiō of their rootes.

¶ Yet in the meane reason I will set forth the certaine notes, to knowe the *diameter*, and the apte sides, in all *diametralle numbers*.

1. And firste I saie: that as thei are thzee numbers in all (I meane the twoo sides, and the *diameter*) so all waies if the firste or leaste side bee odde, then shall there be twoo of them odde numbers. And the *diameter* shall euer bee the other of the odde numbers: that is to saie, the greateste of them.

2. Secondly. It is true that all *diametralle numbers* are euen numbers. And no odde number can bee a *diametralle number*.

3. Thirdly. I saie, that all odde numbers aboue one, maie be the lesser side in soche *diametralle numbers*.

But euen numbers doe not serue so generally: for thei onely maie stand in soche place, whiche be greater then. 4: As. 6. 8. 10. 12. 14. 16. 18. 20. &c. And none other euen numbers then soche as maie be diuided by 4. maie be the greater side in any *diametralle number*.

4. Fourthly. If the lesser side bee an odde number, then ordinarily, the square of it is iuste equalle with that that amounteth by the addition of the *diameter*, to the greater number. As in the firste erample, 3. is the lesser number, and. 4. is the greater: vnto them bothe the *diameter* is .5. Now. 3. hath for his square 9. and so moche is made by the addition of. 4. and. 5.

Again in the seconde erample, the lesser number is 5. and his square is 25. The greater number is 12. and the *diameter*. 13. Put. 12. and. 13. together, and thei make. 25. whiche is equalle with the square of the lesser.

Like waies. 7. and 24. multiplied together maketh 168. whiche is a *diametralle number*. And because the square of the lesser side (whiche here is. 49.) must bee equalle

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equalle to the greater side, and the *diameter* added together: therfore seying. 25. added to. 24. maketh. 49. that. 25. must nedes bee the *diameter* to the foresaid number.

By these rules (if you doe marke them well) you maie sone perceiue, how to make any *diametralle number* : if the lesser side bee giuen vnto you, and bee an odde number. Yet for your ease, I will giue you this plaine rule.

When any odde number is propounded : as the lesser side of a *diametralle number*, and you would finde the other side, and the *diameter* also : or els the *diametralle number*, that maie haue soche a side: multiplie that propounded number by it selfe, and it will make a square number, and will be an odde number : so that of it you shall finde no iuste halfe. Therfore take you those twoo numbers, that are nexte vnto the halfe of it: The lesser shall alwaies bee an euen number, and shall be the seconde side of the *diametralle number*: The other number whiche is the greater, shall alwaies be an odde number: and shall bee the *diameter* of that number whiche you desire. For example marke wel these formes that doe folowe.

If three bee propounded as the one side of a *diametralle number*: And you would knowe, what maie bee the other side: and what is the *diametralle number*: And thirdly, what is the *diameter* to that number : Doe, as I saied before: multiply. 3. by it selfe, and it will make 9. whiche is a square number, and an odde number: and therfore hath no iuste halfe. But the highest numbers to the halfe, are. 4. and. 5.

Therfore I saie, that. 4. whiche is the lesser of the twoo, is the seconde side of the *diametralle number*: and 5. being the greater of them, is the *diameter* it selfe.

Scholar. Now is it light inough to perceiue that the *diametralle number* is. 12 : seying. 3. multiplied by
folwer

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4. maketh. 12.

Master. So is it.

Again, if. 5. be assigned for one side of a *diametralle* number, and you obserue the former worke you maie easily finde the other side, and the *diameter*.

First you see, that the square of 5. is. 25. and it hath no halfe. But. 12. and. 13. are the. 2. numbers nighest his halfe: wherfoze. 12. shall bee the seconde side: and 13. must be the *diameter*. And the *diametralle nōber* is. 60.

Like waies, if. 7. be set for the lesser side, the greater side shall be. 24. and the *diameter*. 25.

Scholar. Touching this I nede no moze instruction: the thynge is so manifeste.

Master. Then shewe your knowlege by an example, or twoo.

And first I appoinde 9 for the lesser side of a *diametralle* number, whereunto I would haue you to assigne the other side, and the *diameter*. &c.

Scholar. I folowe your precepte, and multiplie 9. by it self, whereof commeth. 81. whose halfe is betwene. 40. and. 41. Therfoze must. 40. be the other side: and 41. the *diameter*. And here the *diametralle number* is. 360.

Master. Wroue the like: where. 15. is the lesser number.

Scholar. 15. multiplied square maketh. 225: whose nighest halfe is. 112. and. 113. of whiche the first is the seconde side, and the later is the *diameter*: and the *diametralle number* is. 1680.

Master. What shall be the other numbers: where 21. is the lesser side?

Scholar. 21. yeldeth in square. 441. whose portions nighest his halfe, are. 220. and. 221: And so appereth their offices, and the *diametralle number* is 4620

Master. So maie you saie that vnto. 27. being the lesser side; the greater side shall be. 364. and the *diameter*

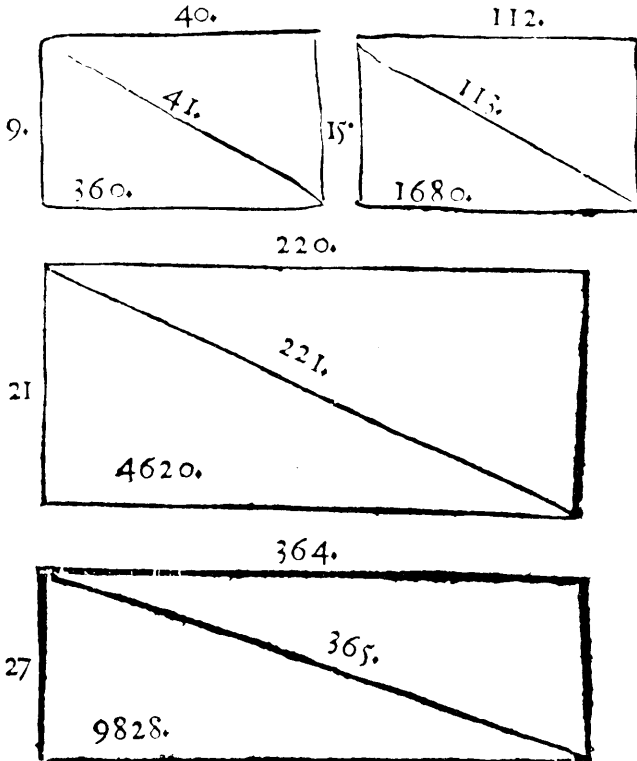
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ser. 365. because the square of. 27. is. 729. And the *diametralle* number is. 9828.

Scholar. So must it be, by your rule.

Master. Not onely the rule doth teache you that it is so, but also the nature and figure of soche *flatte* numbers. As here you see.

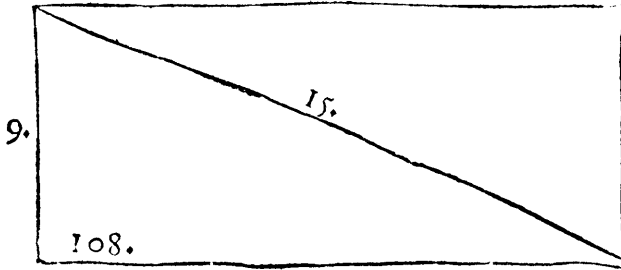


But to the Intente you make the better vnderstand the nature of these numbers: I wil set forth here the like sides with other numbers: Whereby you may knowe, that one side maye serue to diuerse *diametralle*

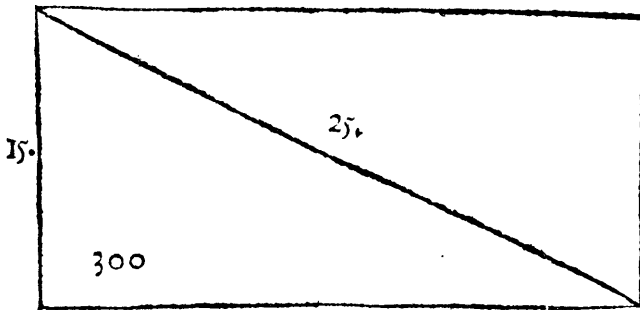
C. j. numbers

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numbers. Therfoze marke these formes well.

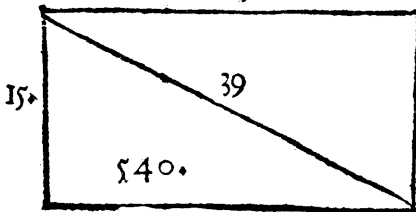
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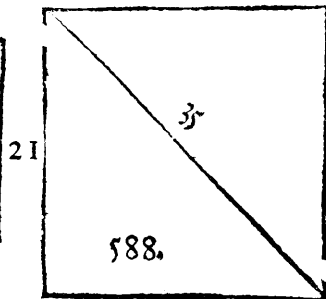
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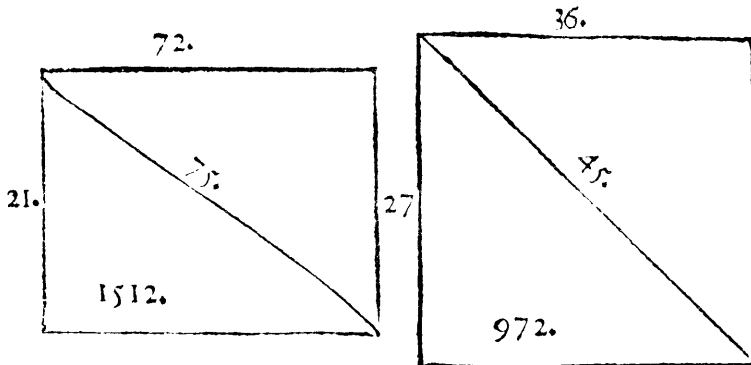
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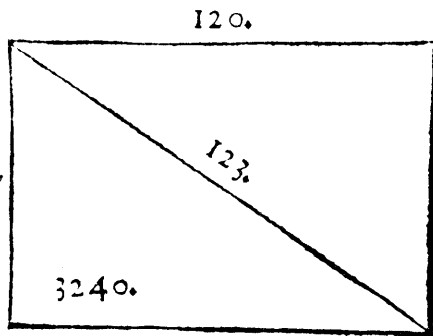
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Scholar. Here I see the same 4 numbers. 9. 15. 21. and. 27. set as the lesser sides: And their greater sides are soche as disagree fro the former rule. And in. 15. 21. and. 27. I see two varieties, unlike to the former example. But seeing the sides doe disagree, I doe not maruel that the *diametralle numbers* are diuerse from the former.



Master. Examine these numbers, whether they be true.

Scholar. I must multiply eche side by it self, and then adde the together: and if they make as moche tally, as the *diameter* beyng multiplied square, then are they true numbers. So I see, that. 9. maketh. 81. and 12 doeth yelde 144 whiche bothe added doe make. 225. And so moche doth 15 make, being multiplied square.

Likewises, for the second figure 15. byngeth forth

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225. and . 20. giueth. 400. that is by addition. 625. whiche somme doeth amounte also, when. 25. is multiplied square.

The thirde figure hath . 15. also for the one side, whose square is. 225. and for the other side. 36. whiche maketh in square. 1296. And thei bothe together giue 1521. And so many commeth of 39 multiplied by it self in square.

Again for the fourthe figure. 21. maketh. 441. and 28. doeth yelde. 784. whiche bothe beyng added, doe amounte vnto. 1225. And so moche doeth there arise by. 35. multiplied into it self.

The fiftte figure hath . 21. also, and his square is 441. and the seconde side beyng. 72. maketh in square 5184. So that bothe those squares doe make. 5625. And the like number is made by. 75. multiplied in square forme.

Now in the sixt figure 27 beyng multiplied square byngeth for the. 729. And. 36. like wates multiplied doeth make. 1296. and that with the other will make by addition . 2025. whiche somme (as is well seen) doeth come of the multiplication of. 45. by it self.

In the seuenth figure. 27. multiplied square, doeth giue. 729; and the other side (whiche is. 120.) doeth byng for the. 14400. These bothe ioyned together doe make. 15129. And the like somme is gathered by the multiplication of. 123. squarely.

So that all those figures doe appere true.

But how thei maie agree with your former rule, I can not see.

Walter. That rule did I make for nōbers vncompounde. For numbers compounde haue not onely in their owne name, the vse of that rule, but also thei folowe the forme of those numbers, of whiche thei bee compounde.

So. 9. beyng compounde of. 3. foloweth the forme
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of. 3. And therefore as. 3. hath. 4. for to make the second side with hym, so. 9. (being thise. 3.) shall haue. 12. (whiche is thise. 4.) for a matche side with hym.

Likewates. 15. being compoude of. 5. and. 3. shall haue their formes in the making of the *diametralle numbers*. For as. 3. hath. 4. so. 15. (being five tymes. 3.) shall haue. 20. (whiche is five tymes. 4.) for the seconde side.

Again, as. 5. hath. 12. so shall. 15. being three tymes. 5. haue. 36. (that is three tymes. 12.) for his seconde side.

Likewates. 21. being compoude of. 3. and. 7. shall haue bothe their formes.

And. 27. whiche is compoude of. 3. and. 9. shall haue all the varieties of their formes.

Scholar. I see it is euen so, and that in the *diameter*, as well as in the seconde side. But the *diametralle number* doeth varie moche in them.

Maister. Yet doe those numbers agree in a maruellous good proportion. For if you doe consider the proportion of bothe the sides in one figure, to bothe the sides in an other figure; and adde those two proportions together, the addition of them doeth make the number that representeth the proportio betwene their two *diametralle numbers*. Whiche thynge I will now onely touche, as briefly as maie bee, to giue you occasion to marke it better hereafter: With this place doeth not fully serue for it. As. 3. and. 4. being the two sides of a *Diametralle number*, doe make. 12. So if 9. and 12 be the sides of a *diametralle number*, that number must be. 9. tymes. 12. that is. 108. For. 9. is triple to. 3. and. 12. is triple to. 4. And because the addition of proportions, is like the multiplication of fractiōs, I must multiplie. 3. by. 3. or els $\frac{3}{4}$ by $\frac{3}{4}$, whiche is all one, and that will maie. 9.

Likewates, if 3. and. 4. be taken for the sides of the

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lesser number *diametralle*, and. 15. and. 36. for the sides of the greater number: As the lesser number shall bee 12. so the greater must be. 540. that is. 45. tymes. 12.

For. 15. vnto. 3. is in a *quintuple* proportion, and is written thus. $\frac{5}{1}$; and. 36. vnto 4 is a *noncaple* proportion, and is written thus $\frac{9}{1}$. Now if you multiplie these numbers together, they will make 45: whiche declareth the proportions of the twoo *diametralle* numbers. And so of all the rest, as you maie easily consider.

Scholar. I praye you, let me examine one or twoo of the, in comparison to that firste *diametralle* number. 12.

I see that 15 being the lesser side, and 20. the greater side, doe make. 300. as their *diametralle* number; and that. 300. is. 25. tymes so moche as. 12. is. Therefore by your saying the proportion of 15. to. 3. and of. 20. to 4. must make. 25. And so it doeth. For eche of them is a *quintuple* proportion. And it is quickly gotten, that 5. multiplied by. 5. doeth make. 25.

For farther prooffe, I take the *diametralle* number 1680. whose sides are. 15. and. 112. First I see, that. 15. to. 3. beareth a *quintuple* proportion; and. 112. to. 4. is as. 28. to. 1. Therefore I multiplie. 28. by. 5. and it maketh. 140. Then if I multiplie that number by. 12. it will make. 1680.

This is a sufficiente trialle for these numbers.

Of even sides But of soche *diametralle* numbers, as haue euen numbers for their lesser side, you haue giuen no rule, neither examples, saue onely of. 8. wherfore I praye you tell me, how shall I finde out the *diametralle* number, with his other side, and the *diameter* in soche euen numbers.

Master. You shall make it square, as you did in the other numbers, that were odde: And of that square you shall take twoo quarters, whiche you shall alter in soche sorte, that you shall abate. 1. fro the one quarter, and put it to the other quarter. And so haue you
twoo

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twoo numbers, differing onely by .2. and bothe being odde. The lesser of them twoo, is the greater side of the *diametralle number*: and the other is the *diameter* to it. As .8. being your lesser side, the square of it is 64. whose quarter is .16. from whiche I abate .1. and there resteth .15. and that is the seconde side. Also I adde .1. to 16. and it maketh .17: whiche is the *diameter*.

Scholar. This is no thyng harde. As by example I will proue. If .12. bee the lesser side: his square is 144. and the quarter of it is .36. Then abatynge .1. I see there will bee .35. for the other side of the *diametralle number*. And addynge .1. to .36. it maketh .37. to be the *diameter*. And if I multiplie .35. by .12. it byngeth forthe .420. whiche is the *diametralle number*.

Now for prooffe of these numbers, I multiplie .12. by it self, and it maketh .144. Then I multiplie the other side, that is .35. by it self, and it yeldeth .1225. Those bothe together doe make .1369. And seynge 37 multiplied by it selfe, doeth make the same number. Therefore are thei all true numbers.

An other example .10. being set for the lesser side, I doe multiplie it squarely: and there riseth .100. whose quarter is .25. For whiche I take (as you taught me) .24. and .26. And so the whole *diametralle number* is .240. For prooffe of the other numbers, I take .100. whiche commeth of .10. multiplied square, and to it I adde .576. whiche is the square to .24. and thei bothe doe make .676. And so muche amounteth by the multiplication of .26. squarely.

Master. This maie suffice for this presente: if you marke that the euē numbers haue not onely one generalle forme, whiche I did expresse in the forme rule, but also soche as be compounde of any other numbers, euē or odde: haue the like numbers in proportion, for the greater side, and for their *diameter* as the numbers haue, of whiche thei bee compounde. And because

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bicause I will not staie to long on this matter, I will here set foꝛthe diuerse varieties of *diametrall numbers*, whereby you maie gather not onely the true vnderstanding of the foꝛmer rules: But also in them you maie see other notable cōclusions: and straunge woꝛkes of the natures of numbers.

Marke well this table foꝛme, with the titles ouer it: whiche declare the true meanyng of it.

And where you see one number in the firſte columpne againſt twoo, thzee, oꝛ ſower in the other columpnes, you ſhall vnderſtande that that number is the ſide to ſo many ſeueralles numbers *diametralle*.

The table of diametrall numbers.

| The lesser side. | The greater side. | The diameter. | The number diametral. |
|------------------|-------------------|---------------|-----------------------|
| 3. | 4. | 5. | 12. |
| 5. | 12. | 13. | 60. |
| 6. | 8. | 10. | 48. |
| 7. | 24. | 25. | 168. |
| 8. | 15. | 17. | 120. |
| 9. | 12. | 15. | 108. |
| | 40. | 41. | 360. |
| 10. | 24. | 25. | 240. |
| 11. | 60. | 61. | 660. |
| 12. | 16. | 20. | 192. |
| | 35. | 37. | 420. |
| 13. | 84. | 85. | 1092. |
| 14. | 48. | 50. | 672. |
| 15. | 20. | 25. | 300. |
| | 36. | 39. | 540. |
| | 112. | 113. | 1680. |
| 16. | 30. | 34. | 480. |
| | 63. | 65. | 1008. |
| 17. | 144. | 145. | 2448. |
| 18. | 24. | 30. | 432. |
| | 80. | 82. | 1440. |
| 19. | 180. | 181. | 3420. |
| 20. | 48. | 52. | 960. |
| | 99. | 101. | 1980. |
| | 28. | 35. | 588. |
| 21. | 72. | 75. | 1512. |
| | 220. | 221. | 4620. |
| | 120. | 122. | 2640. |
| 22. | 120. | 122. | 2640. |
| 23. | 264. | 265. | 6072. |
| 24. | 32. | 40. | 768. |
| | 45. | 51. | 1080. |
| | 70. | 74. | 1680. |
| | 143. | 145. | 3432. |
| 25. | 60. | 65. | 1500. |
| | 312. | 313. | 7800. |

| The lesser side. | The greater side. | The diameter. | The number diametral. |
|------------------|-------------------|---------------|-----------------------|
| 26. | 168. | 170. | 4200. |
| 27. | 36. | 45. | 972. |
| | 120. | 123. | 3240. |
| | 364. | 365. | 9928. |
| 28. | 96. | 100. | 2688. |
| | 195. | 197. | 5460. |
| 29. | 420. | 421. | 12180. |
| 30. | 40. | 50. | 1200. |
| | 72. | 78. | 2160. |
| 31. | 480. | 481. | 14880. |
| 32. | 60. | 68. | 1920. |
| | 126. | 130. | 4032. |
| | 255. | 257. | 8160. |
| 33. | 44. | 55. | 1452. |
| | 180. | 183. | 5940. |
| | 544. | 545. | 17662. |
| 34. | 288. | 290. | 9792. |
| 35. | 84. | 91. | 2940. |
| | 120. | 125. | 4200. |
| | 612. | 613. | 21420. |
| 36. | 48. | 60. | 1728. |
| | 105. | 111. | 3780. |
| | 160. | 164. | 5760. |
| 37. | 223. | 225. | 11628. |
| | 684. | 685. | 25308. |
| 38. | 360. | 362. | 13680. |
| 39. | 52. | 65. | 2028. |
| | 252. | 255. | 9828. |
| | 760. | 761. | 29640. |
| 40. | 75. | 85. | 3000. |
| | 96. | 104. | 3840. |
| | 198. | 202. | 7920. |
| | 399. | 401. | 15960. |

The seconde parte

This table maie you extende infinitely. And these thinges maie you se, as thinges of greate admiratiō.

1. There is no *diametralle number*, but it maie be diuided by. 1 2. Wherfoze thei be all euen numbers euenly and oddely.
2. Again, there is no *diametralle number*, but it endeth in. 0. III. 2. 02 in. 8.
3. Thirddely, there is no *diametralle number*, that can haue any moze *diameters* then one.
4. Yet maie one number bee the *diameter* to diuerse other.
As you se 25. is the *diameter* to. 168. and also to. 300.
So. 65. is the *diameter* to. 1008. and also to. 1500.
Likewates. 145. is the *diameter* to. 2448. and to 3432.
5. Fiftely: No square number can bee a *diametralle number*.

Scholar. These properties be notable.

To knowe a *diametralle number*.
But how shall I knowe, when a number is proposed, whether it be a *diametralle number*, or not?

Maister. In that thyng I finde a tediousse trauell, by any rules, in those that write of it. But I wil ease you of moche paine therein.

Firste remember the properties of those numbers.

And if you haue any other figure in the first place, then. 0. 2. 02. 8. it is no *diametralle number*.

Secondarily, if it maie not bee diuided by. 1 2. although it ende in one of those. 3. figures, it is no *diametralle number*.

Wherfoze if it haue bothe those twoo properties (whiche an infinite multitude of numbers doe want) and be no square number (as none be that ende in. 2. 02. 8. 02 with odde cyphers) then sette out all the partes of it, in soche sorte, that the lesser parte doe stande directly ouer those greater partes, which beyng multiplied together, will make the whole number.

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And then examine those partes, whiche seme to haue any likelihod: accoꝝdyng to the foꝝmer doctrine.

As foꝝ example: if. 72. be pꝝoponed to be examined in that soꝝte, I sette his partes in oꝝder thus.

2. 3. 4. 6. 8.

36. 24. 18. 12. 9.

Howbeit I neded not to set doune. 2. nother. 4. foꝝ lesser partes, nother those other greater partes, that aunswere to them: Foꝝ, as I said befoꝝe, thei can not bee the lesser side in any *diametralle number*. Wherfoꝝe thei nede no examination.

Farthermoꝝe, foꝝ them that you shall nede to examine, if the lesser number bee an odde number, the square of it must contain double to that greater number (that is coupled with it) and one moꝝe.

And if the lesser be an euen number (of them twoo that you would examine) then must the square of it containe the greater number (that standeth by it). 4. tymes, and. 4. moꝝe. And this is not onely a shoꝝter waie, then I see to be taughte by other artes menne: but it is also moꝝe certaine, foꝝ all numbers not compounded of other *diametralle numbers*.

Scholar. By this doctrine it appeareth quickely, that. 72. is no *diametralle number*.

Foꝝ although it doeth ende in. 2. and maie be diuided by. 12. yet no couple of numbers here haue those properties that is required.

Foꝝ vnder. 3. is. 24. whiche is to greate: and vnder 6. there is. 12. whiche is to greate also.

But vnder. 8. standeth. 9: whiche is to litle, by a greate deale.

Master. Then proue in this other number. 132.

Scholar. His partes will stande thus.

F. ij. 3.

The seconde parte

| | | |
|-----|-----|-----|
| 3. | 6. | 11. |
| 44. | 22. | 12. |

Where I see quickly that it can not bee a *diametral number*. For the numbers vnder. 3. and. 6. be to greate: sith no number that should bee sette vnder. 3. maie be aboue. 4.

Neither vnder. 6. maie any number bee set greater then. 8. As it doeth sufficiently appeare by that that is taughte before.

And vnder. 11. there can bee no lesse number placed then. 60: and therfore. 12. is to smalle.

And herein I perceiue greate helpe by this table, whiche you haue set forth.

Master. It is well marked of you. But yet trie this other crample. $6 \circ 72$.

Scholar. I set doune his partes in order, thus.

| | | | | | | | |
|-------|-------|------|------|------|------|------|------|
| 3. | 6. | 8. | 11. | 12. | 22. | 23. | 24. |
| 2024. | 1012. | 759. | 552. | 506. | 276. | 264. | 253. |

| | | | | |
|------|------|------|-----|-----|
| 33. | 44. | 46. | 66. | 69. |
| 184. | 138. | 152. | 92. | 88. |

And here I see a greate sorte of numbers, whiche can not serue to my purpose, because those that bee euen, and are lesse then. 44. make to litle a square, to be 4. times so moche as the number vnder any of the.

And. 44. maketh to greate a square: wherfore it can be none of the euen numbers.

Again, those that be odde vnder. 23. doe make to litle a square, to bee double to the greater number vnder it. And those that bee odde aboue. 23. doe make to greate a square. So that. 23. doeth remain to bee the true nuber for the lesser side: and 264 the greater side.

Master. Because exercise is the beste instrument

of Arithmetike.

in learning : therfore will I propounde to you one example more.

What saie you of. 5460 ? Is it a *diametralle number* or no?

Scholar. I will trie it, by setting doune his partes thus.

| | | | | | | | | |
|-------|-------|------|------|------|------|------|------|------|
| 3. | 5. | 6. | 7. | 10. | 12. | 13. | 14. | 15. |
| 1820. | 1092. | 910. | 780. | 546. | 455. | 420. | 390. | 364. |

| | | | | | | | | |
|------|------|------|------|------|------|------|-----|-----|
| 20. | 21. | 28. | 30. | 35. | 42. | 52. | 60. | 70. |
| 273. | 260. | 195. | 182. | 156. | 130. | 150. | 91. | 78. |

And here I see diuerse and many numbers, whiche at the firste sighte, appere nothyng mete for this purpose. For. 20 . is to smalle a number, as I maie sone iudge : and therfore all other numbers vnder it, must nedes be to smalle, of force.

Againe, I see that. 30 . is to greate a number, and therfore, of necessitie, all other numbers aboute it, must nedes be to greate. So that. 21 . other. 28 . must be the true number, or els none.

Therfore I examine first. 21 . whose square is 441 whiche should bee one more then double, to the number vnder it, that is to saie, it should bee. 521 . And so it is not: Therfore I refuse it, and examine. 28 . whose square is. 784 . And that should bee fower tymes so moche as. 195 . (whiche is the number vnder it) and 4 . more. Therfore I doe *quadruple*. 195 . and it maketh. 780 . And then I see that it wanteth, but fower of the other square: wherfore I take those two numbers, I meane. 28 . and. 195 . for the true sides of. 5460 . whiche I finde to be a *diametralle number*.

Waster. By the waie, remember that you could easily perceiue, that all numbers vnder. 20 . were to small for your purpose: and contrary waies, all about. 50 .

The seconde parte

A shorke
meane in
working.

to be to greate. So that you need not to sette doune so many partes of your firste number.

Wherfoze if your number bee soche a one, as hath many partes, you maie chose one by gesse, which you thinke will go nigh to serue your purpose: and if you finde it to smalle, then set theim doune onely that bee greater then it, til you finde one other iuste: and then haue you your purpose. Or if you finde any to great, after that whiche was to smalle, and betwene them none iuste. then is not your number a *diametrall nōber*.

But and if the parte whiche you tooke by gesse, be to great, you shall refuse all partes aboue it, and take onely lesser partes, til you finde a iuste parte foꝛ your purpose: or els one that is to litle.

And if in descendynge orderly, you finde no iuste parte, befoze you come to one that is to litle, then is your number no *diametralle number*.

Scholar. This is a greate ease in shortenyng of woꝛke: whiche I will pꝛoue in this number. 9786.

Master. If you remembꝛed well your foꝛmer rules, you would not admitte this to be examined foꝛ a *diametralle number*: bicause it endeth in none of the thꝛe peccultare terminations: that is. 0. 2. 02. 8.

Scholar. I cōfesse my faulte. And therfoze I take this number. 9780. whose. 20. parte is. 489. But seꝛyng. 20. doeth make in square but. 400. therfoze is it very moche to litle.

Then I take the. 30. parte of it, whiche is. 326. and finde it also to litle.

Thirdely, I take the. 40. parte of it, whiche is 244½: and seꝛyng. 40. maketh in square. 1600. I see that it is almoste. 7. tymes so moche as. 244½: and therfoze is it to greate.

So must the true number be betwene. 30. and. 40: or els there is none at all.

Therfoze firste I take. 35. whiche is the middelle number,

of Arithmetike.

number (as the moſte apte for a coniecture) and it yeldeth. $279\frac{3}{4}$. And the ſquare of. 35 . is. 1225 . whiche is farre moze then the double of. $279\frac{3}{4}$.

Therefore, againe I proue with, 32 . whiche giueth $305\frac{5}{8}$. And ſeyng the ſquare of. 32 . is. 1024 . it is not 4 . tymes ſo moche as. $305\frac{5}{8}$. for that is. $1222\frac{1}{2}$.

Wherefore I take a greater number, betwene it and. 35 . And firſt I take. 33 . whiche bringeth forth the $296\frac{1}{4}$. wherby I maie ſee that. 33 . is to greate. And ſeyng there is no number leſte betwene. 32 . and. 33 . therefore I iudge that firſte number. 9780 . to bee no *diametralle number*.

Maſter. Examine this number. 43200 .

Scholar. Bicauſe I ſee it to be a greate number, I will begin with a greate parte of it. And therefore, I take. 100 . whiche yeldeth. 432 . And conſidering that the ſquare of. 100 . is. 10000 . whiche is farre to greate, I muſt ſeke a leſſer number.

Maſter. I will eaſe you of your paines in that.

For bicauſe here is moze to bee conſidered. You remember that I tolde you befoze, in making of *diametralle numbers*, how that ſome numbers doe followe the rules of other, of whiche thei be compounde. And farthermoze, that ſoche compounde *diametralle numbers*, did beare propoztion to the leſſer, as the propoztion was of bothe their ſides added together.

Scholar. That is true.

Maſter. Of like reaſon all ſoche *diametralle numbers*, muſt bee excluded from theſe rules, whiche bee made peculiarly for numbers that haue their owne proper formes, and depende not of other.

And yet ſome common rule muſt bee giuen, that maie extende as well to them, as to any other.

Wherefore let this be it.

That the two ſides of all *diametralle numbers*, haue ſoche a propoztion together, as here you ſee expreſſed
in

The seconde parte

in some one of these formes : if thei bee continued as
here thei be begon .

¶ The firste order.

$$\frac{3}{4} : \frac{5}{12} : \frac{7}{24} : \frac{9}{40} : \frac{11}{60} : \frac{13}{84} : \frac{15}{112} : \frac{17}{144} : \frac{19}{180} : \frac{21}{224} :$$

$$\frac{23}{264} : \frac{25}{312} : \frac{27}{360} : \frac{29}{420} : \frac{31}{480} : \frac{33}{544} : \frac{35}{612} : \frac{37}{684} : \frac{39}{760} \text{ ¶}$$

¶ The seconde order.

$$\frac{8}{15} : \frac{13}{35} : \frac{16}{63} : \frac{20}{99} : \frac{24}{143} : \frac{28}{195} : \frac{32}{255} : \frac{36}{323} : \frac{40}{399} : \frac{44}{483} :$$

$$\frac{48}{575} : \frac{52}{670} : \text{¶}$$

Here haue I sette the lesser side as the numerator,
and the greater side as the denominator. ¶ hereby
you maie perceiue the cause of their distinction.

For the first order is, when the lesser side, or nomi-
ner, is odde.

The seconde order is, when that lesser side is an
euen number.

Stifelius doeth set them so, that the numerator standeth
for the seconde, or greater side: and the denomi-
nator for the firste number, or lesser side. And for the
more delectable contemplation, to behold their forme
of progression, he setteth doune as many whole nomi-
ners, as the fraction will giue.

And this is his forme.

¶ The firste order.

$$1\frac{1}{3} : 2\frac{2}{5} : 3\frac{3}{7} : 4\frac{4}{9} : 5\frac{5}{11} : 6\frac{6}{13} : 7\frac{7}{15} \text{ ¶}$$

¶ The seconde order.

$$1\frac{7}{8} : 2\frac{11}{12} : 3\frac{15}{16} : 4\frac{19}{20} : 5\frac{23}{24} : 6\frac{27}{28} : 7\frac{31}{32} \text{ ¶}$$

¶ here

of Arithmetike.

Where in the first order, you see bothe in the whole numbers, and also in the numeratoꝛs of the fraction, the naturalle order of numbers. And in the denominatoꝛs, the naturalle progression of odde numbers.

But in the seconde order, you see that the whole numbers go in their naturalle order, and the numeratoꝛs and denominatoꝛs, kepe an *Arithmeticall* progression, by equalle distaunce of .4. saue that in the numeratoꝛs, all the numbers bee odde: and in the denominatoꝛs, they be all euen.

Now by this generall rule, if you finde any twoo partes of any number, in one of these former proportions, you maie bee sure that it is a *diametralle number*. But foꝛ the more apte conference of the partes, you shall doe beste to reduce them to their least numbers: as you haue learned in the firste parte of *Arithmetike*.

So in your last number, whiche was 43200 . you shall finde his .180. parte, to bee .240. whiche beynge reduced to their smallest numbers, will bee $\frac{4}{3}$: wherfoꝛe I am assured, that it is a *diametralle number*.

Let one thyng more shall you marke.

If any number ende in Ciphers, abate euen Ciphers, as often as you can (I meane .2. .4. .02. .6. &c. and if the reste be a *diametralle number*, so was the first. And therfoꝛe in this laste example. 432 . is a *diametralle number*, as well as. 43200 .

Also if any number beynge diuided by any square number, doe make a *diametralle number* in the *quotiente*, then was the firste number a *diametralle number* also.

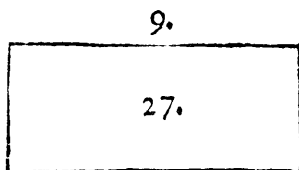
And this, foꝛ this tyme, shall suffice foꝛ *diametralle numbers*.

Now will I speake somewhat briefly of *like flattes*: *Of like flattes.*
and then procede to other *figurall numbers*. *flattes.*

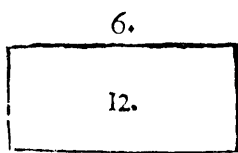
Scholar. I remember you defined them befoꝛe, to bee soche flatte numbers, as had one forme of proportion betwene their sides.

The seconde parte

As here 27. and 12. be
like flattes: because their
sides be in one proporti-
on. For as 9. is to 3. so 6
is to. 2. both becyng in
triple proportion.



Walter You saie well.
And that is the cause why they
be called like: for the likenesse
in the proportio of their sides.



Squarelike
figures.

Although some menne delite
more to call them *squarelike figures*: because they haue
some properties agreeable with square numbers (for
as *Euclide* saith in his. 8. booke, and. 18. proposition:
*Euery twoo numbers, beeyng like flattes, haue
one meane number betwene them in proporti-
on. And the one flatte number beareth vnto
the other flatte double that proportion, that
their sides doe.*

For declaration of whiche proposition, marke the
twoo flatte numbers before: I meane. 27. and. 12.
whose sides are in proportion *Sesquialter*: And the flat
numbers themselves be as $\frac{3}{2}$. or. 9. to. 4: that is double
Sesquiquarte. Now doe you double the proportion *Ses-
quialter*, and it will make double *Sesquiquarte*.

Scholar. Thus doe I sette them in order. $\frac{3}{2}$: $\frac{3}{2}$.
And I multiplie the numeratozs together, and the
denominatozs also. (For I remember, you tolde me
before, that proportions are added, as fractions are
multiplied) and then will it be. $\frac{9}{4}$: euen as you saied.

Walter. Again *Euclide* saith in the twentieth pro-
position of the same booke.

*If any number stande as a middle number in
proportion,*

of Arithmetike.

proportion, betwene other two numbers, those two are like flattes.

That is to saie: if any two numbers, beyng multiplied together, doe make a square number (for none but soche can haue a middle number betwene them) then are thei *like flattes*.

As. 3. and. 12. multiplied together doe make. 36. whiche is a square number: and. 6. therby appeareth to bee the middell number betwene them. And therfore are. 3. and. 12. *like flattes*

Likewales. 3. and. 27. for thei make. 81. whiche is a square: and their nuddle number is. 9.

And so are. 2. and. 8: 2. and. 18: 2. and. 50. 2. 7. 72
3. and. 48: 3. and. 75: 4. and. 9. 4. and. 16: 4. and
25. 5. and. 20. 5. and. 45: 6. and. 24: 6. and. 54.

And so of infinite other.

This exposition is confirmed by the firste and seconde proposition of the ninth booke of *Euclide*, where he saiech thus.

If two numbers beyng like flattes, bee multiplied together, the number that thei make, shall be a square number.

And if. 2. numbers beyng multiplied together, do make a square nōber, then are thei like flattes.

By whiche rules it doeth appere, that you cā haue no *progreſſiō Geometricalle*, but it must be made either of square numbers, or els of *like flattes*, wherby there appeareth a greate agreableness, betwene *like flattes*, and square numbers. And therfore saiech *Euclide* also in the. 26. proposition of the eight booke.

Numbers that bee like flattes, haue soche proportion together, as one square number bea-

The seconde parte

reth to an other.

This maie you proue by any of the former exam-
ples. For 12. to. 3. is in like proportion, as. 16. to. 4.
or. 36. to. 9.

Also. 27. to. 3. hath like proportion as. 36. to. 4: or
144. to. 16. other. 81. to. 9.

And farther, if you deuide the one of theim by the
other, the *quotiente* will be a square number.

Scholar. What doeth appeare euidentely at the
firste betwe.

For 12. diuided by. 3. doeth make. 4. And. 75. diui-
ded by. 3. giueth. 25.

So. 54. by. 6. maketh. 9. And. 72. by. 2. yeldeth. 36.
And so I see in the reste, that all the *quotientes* will be
square numbers.

how like flat-
tes be made.

But I desire moche to knowe, how those numbers
be produced. For that I knowe not yet.

Master. Take any twoo square numbers, what
so euer thei bee, and multiplie them by any one num-
ber, that you liste: and thei will make. 2. *like flattes*.

So. 4. and. 9. multiplied by. 2. doe make. 8. and. 18:
whiche bee *like flattes*.

Again, if you multiplie them by. 5. thei make. 20.
and. 45. whiche be also *like flattes*.

Scholar. I am perfect inough in this, if that be al.

Master. An other waie you maie make them al-
so: If you take any twoo square numbers, that will
admitte one diuisor, and diuide them bothe by it.

As for example. Seyng 9. and. 36. will be bothe di-
uided by. 3. I doe so diuide theim: and their *quotientes*
are, 3. and. 12. whiche are *diametralle numbers*.

So in like maner, if I diuide 196 and 49 (whiche
bothe are square numbers) by. 7. the *quotientes* will be
28. and. 7.

Again, 16. and. 100. beyng bothe square numbers
and

of Arithmetike.

and diuided by. 4. doe make. 4. and. 25. as their *quotiente*, and thei be *like flattes*.

Scholar. And in these I see an other straunge worke: that if those twoo *like flattes* bee multiplied together: thei will make the greater square, of whiche thei canre.

For 3. tymes. 12. maketh. 36: and. 7. tymes. 28. giueth. 196: And so. 4. tymes. 25. byngeth forthe. 100.

Passer. It doeth so happen often times: but it is not allwaies so.

For if you diuide. 16. and. 100. by. 2. the *quotientes* will be. 8. and. 50. whiche twoo numbers multiplied together, doe make. 400. farre differing from. 100. So. 36. and. 196. be yng bothe square numbers, and diuided by. 2. doe make. 18. and. 98. whiche be *like flattes*: and those *like flattes* multiplied together, doe yelde 1764. whiche is a square number, but it is. 9. tymes so greate as is. 196.

Scholar. Yet one doubt I haue: whether all square numbers be *like flattes*, and so bee not distinct from them?

For although in the diuision of figurall numbers you did distinct them, yet in the examples of *like flattes*, you put certain square numbers emongest other.

Passer. All square numbers are *like flattes*, be yng compared together: and els not. For as any. 2. square numbers maie be compared together: so maie thei be referred to their rootes, without comparison together. Or els thei maie be compared to other numbers that bee not square.

Therefore marke these twoo rules well. that no one number can bee called a *like flatte*: but in comparison to some other. For. 2. by hymself is not called a *like flatte*, excepte he bee compared to. 8. or to. 18. other to 32. or. 50. or some other soche.

So like waies. 4. whiche by nature is a square nō

The seconde parte

ber, and allwales shall bee so: yet is it not accepted as a *like flatte*, onles it bee referred to some other square number.

Scholar. What if it be compared with .12. which you named befoze to be a *like flatte*?

Maister. You remember: one of *Euclide* his rules (whiche I repeated befoze) is, that *like flattes* beeyng multiplied together, will make a square nōber. And sodoeth not. 12. beeyng multiplied by. 4.

Scholar. Now I doe vnderstande your woordes better. So. 3. and. 8. compared together, bee not *like flattes*: yet eche of them compared to other numbers, maie be *like flattes*. As. 3. compared to. 12. or to. 27: and 8. compared to. 18. or to. 50.

Of rooted
numbers.

Maister. Now will we lette these *like flattes* alone for a tyme: And intreate moze of rooted nōbers. And first I will tell you somewhat of the names and natures of soche numbers as haue rootes: Then secondarily I will teache you the order to extract their rootes: And afterwarde will I shewe some parte of the vse of them.

A roote.

Wherfoze to begin, where we lette a litle befoze, the explicatiō of rootes: I saie, that the roote of number, is a number also: and is of soche sorte, that by sondrie multiplications of it, by it self, or by the number resultyng thereof, it doeth produce that nōber, whose rooe it is. And accoꝝdyng to the number of times that it is multiplied, the number that resulteth thereof, taketh his name.

So that one multiplication maketh a *square number* And two multiplications doe make a *Cubike number*.

Likewaies. 3. multiplications, doe giue a *square of squares*. And. 4. multiplications doe yelde a *surfolide*.

And so infinitely.

For as multiplication hath no ende, so the numbers amountyng of them be innumerable, and their
rootes

of Arithmetike.

rootes as infinite. But their names thei take certainly, of the numbers that thei doe make.

So the roote of a square number, is called a *Square roote*: and the roote of Cubike number, is named a *Cubike roote*: In like sorte that roote is called a *Squared square roote*, whiche maketh a square of squares in number: And that roote is a *Surfolide roote*, that yeldeth a *Surfolide number*: in whiche sorte of multiplication, you may procede infinitely, as I saied.

A square roote.

A cubike roote.

A squared square roote.

A surfolide roote.

Notwithstanding for your ease, I haue set forth here in a table, certain of the most notable kindes of rooted numbers.

And to the intente you may partly conceiue the reason of their names, I will after the table, set forth a brief explication of their names, with the picture of the figures, that thei doe resemble in multiplications *Geometrically*: where pointes, lines, platte formes, or soundformes be multiplied: and bynne forth the other formes agreeable to suche multiplications.

But first marke the table well: And it will giue you greate lighte, and aptnesse to vnderstande all that foloweth, moche the better.

For examples are the
lighte of tea-
ching.

The

The vulgar names.

The table of rooted numbers.

The authors names.

| | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | |
|-----------------------------------|------|-------|---------|----------|----------|-----------|-----------|-----------|-------------|-----|------------------------------|
| 1. Rooters. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | Rooters. |
| 2. Squares. | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | | Squares. |
| 3. Cubikes. | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 | | Cubikes. |
| 4. Squares of Squares. | 16 | 81 | 256 | 625 | 1296 | 2401 | 4096 | 6561 | 10000 | | Longe Cubes. |
| 5. Squafolides. | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 | 100000 | | Squares of cubes |
| 6. Squares of cubes | 64 | 729 | 4096 | 15035 | 46656 | 117049 | 262144 | 531441 | 1000000 | | Cubike Cubes. |
| 7. Seconde Squafolides. | 128 | 1187 | 16384 | 78125 | 279936 | 833543 | 2097152 | 4783969 | 10000000 | | Longe Cubike Cubes. |
| 8. Squares of square Squafolides. | 256 | 6561 | 65536 | 390625 | 1679616 | 5764801 | 16277219 | 43945721 | 100000000 | | Squares of Cube bike Cubes. |
| 9. Cubes of Cubes. | 512 | 19683 | 363144 | 19738125 | 10077696 | 40313607 | 144217728 | 387410489 | 1000000000 | | Cubes of Cubike Cubes. |
| 10. Squares of Squafolides. | 1024 | 59049 | 1028576 | 9765625 | 60466176 | 281475249 | 107374824 | 348678401 | 10000000000 | | Longe Cubes of Cubike Cubes. |

of Arithmetike.

Here you see diuerse rowes of numbers, and against euery rowe twoo names wrytten: one on the right hande, and the other on the lefte hande, whiche serue for all the numbers in that rowe.

The names on the lefte hande bee those names, whiche bee commonly vsed, and attributed to those numbers.

The names on the righte hande, are names of my addition, whiche doe aptly expresse the very natures of the numbers, vnto whiche thei bee assigned: as a none I will declare.

And now concernyng the numbers, you see firste in the hedde of the table, a rowe of numbers set in order, as thei followe in common nombryng, from one forward. And thei bee called rootes, for that the multiplication of eche of them, by thei selfes, or by that, that therof amounteth, byngeth for the all thother, that bee set vnder them. Of the whiche, the seconde rowe is called *Square numbers*: bicause that their length *Square* and their bredth (whiche I vnderstand by the .2. numbers of their multiplication) is equalle.

As .2. tymes .2. doeth make .4. whiche is a square number, and maie bee figured thus. ♦♦

Like waies .3. tymes .3. maketh .9. whiche is a square number, and is represented thus. ♦♦♦

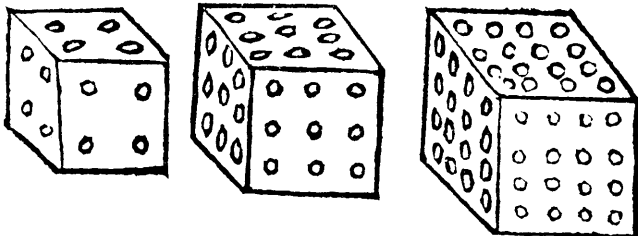
And here you se, that if you diuide the *Square number* by his roote, the *quotiente* will be the same nōber also.

Scholar. That must nedes be so.

Master. Then in the thirde rowe are placed *Cu Cubike* *bike numbers*: whiche are produced by triple multi- *numbers.* cation. As .2. tymes .2. twise, maketh .8. And .3. tymes .3. thise, yeldeth .27. So .4. tymes .4. sower tymes, giueth .64. These numbers can not be exprest aptly in flatte, but prospectiuelly, as Dice maie be made in portraiture.

The seconde parte

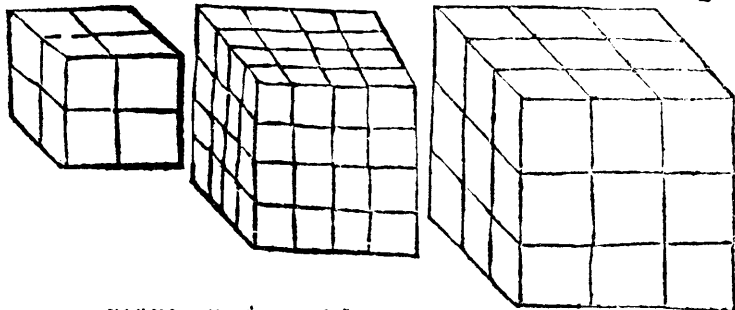
And these are their formes.



In the firste figure you see . 2 . expressed in lengthe bredthe, and depth. And in the second forme, . 3 . is represented in all those . 3 . dimensions. In the . 4 . figure . 4 . is the roote, and is drawn agreeably to that forme.

Scholar. This is manifeste inough to sight.

Master. Yet reason ought to waigh it more exactly, then sight can comprehend it. For as their triple multiplication doeth resemble the nature of sounde bodies, so it might appeare more iuste expressing of their figures, agreeably as sounde bodies ought: in whiche euery parte can not appeare to sight, sith diuerse of them loke inwardly. As by these . 3 . laste figu-



res you maie partely coniecture. Of whiche at this tyme and in this place, some men will thinke it an ouersight to speake, and moche more ouersight to write of them any thyng largely. Saue that we maie vse them for the apter explication of that triple multiplication,

of Arithmetike.

Application, wherby they be made.

So that as it is multiplied thise, so the number that doeth amounte thereof, hath gotten. 3. diuensions, whiche properly belongeth to a bodie, or sound forme. And therefore is it called a *Cube*, or *Cubike number*. Whiche number if you diuide by the roote, the *quotient* wil be the square of the same roote. As I said afore.

But to procede, if you doe multiplie that *Cubike number* by his roote, the number that riseth of it, is called a *Square of Squares* commonly: because that not *Squares of* onely it is a *Square number*, but the roote of it also is *Squares*. a *Square number*. As you maie perceiue by examination, of all those numbers that be in the fourth rewe, whiche numbers I doe call *longe Cubes*: because they *Long Cubes*. make a line of Cubes. And hath in lengthe so many Cubes, as the firste roote doeth containe vnties.

This line of Cubes, although it haue for his bredthe, and depthe also, the thickenesse of one Cube, yet because it hath no number of Cubes, in bredthe, nor in depthe (or generally no number of that thyng, whercof it is called a line) therefore maie it tollerably beare the similitude and name of a line. And so doe we commonly call lines, those smalle cordes, whiche are onely long, and haue litle bredthe to their length. But yet are they not without all bredthe.

Scholar. And thereof (I thinke men call a line of *Backes*, and a line of *Assclers stones*, when many be laied in a rowe, in lengthe: and but one (or fewe) in bredthe.

Master. You saie truth. And that name doeth continue still, amongest all our countrie menne: saue that moste menne doe not call it sharply a line, but moze broder (after tholde Englishe language) a *laine*. And so men vse to saie, a *laine* of wine buttes, and a *laine* of brode clothes: and soche other like.

And vse hath so largely applied this name, that it

The seconde parte

make seme no greate absurditie, to name any thyng
a line or laine, that hath moche moze lengthe then
brydthe: and is made by often addition, or multipli-
cation of any one quantitie. But yet for auoidyng of
erroure, it ought to bee limited, whereof that line is
named. As in our mater to saie, a line of vnitie: a line of
Cubes: a line of Cubike Cubes: and a line of Cubike Cubes Cu-
bikely: and so forth.

In likewates must we iudge of platte formes, that
they haue no depthe or thickenesse. When one num-
ber is multiplied by an other, onely twise: that is to
saie, in brydthe and lengthe onely: and is not multi-
plied the thirde time by any number, to make it beare
depthe.

And this must be considered generally, though the
number so multiplied bee a Cube, or any other sounde
nōber. For in soche case, that Cube, or sounde number,
what so euer it be, standeth but as an vnitie.

Scholar. Sir, I doe very well vnderstande the
meanynge, and reasonablenesse of those names, line,
and square, in any thing. But I knowe not those ter-
mes, Cubike Cubes, and Cubike Cubes Cubikely. Although
I se them set in the table, whiche you haue giuen me.

Master. No more then doe you vnderstande di-
uerse other names there, whiche I will therfore de-
clare vnto you.

If you agree to the vse of the name, of a line and a
square, in that sorte that you haue consented vnto:
then if I multiplie a Cubike number by his roote.

As to saie. 8. by. 2. or. 27. by. 3. other. 64. by. 4. then
shall I haue a line of Cubes: whiche I doe therfore
call longe Cubes: but commonly they bee called Squared
Squares, or Squares of Squares: and of some men they are
named Zenzizenzikes, as square numbers are called
Zenzikes. Whiche name although in sounde bodies,
it hath no vse, yet in practice of sounde numbers, it

maie

Squares of
Squares.

of Arithmetike.

maie and doeth expresse some properties aptly. As namely that all those numbers, whiche rise of 4 multiplications, maie be as well made by twoo multiplications. But then the roote of that multiplication shal be a square number also.

Scholar. So I vnderstande that. 16. is a number of that sorte, which here is called *Square of Squares*. And yet maie it bee called a square number: and is so in deede, in comparison to. 4. And therefore, I perceiue, it is set twise in the table: ones emongest square numbers, vnder 4 whiche then is his square roote: And again it is set emongest *squares of squares*, vnder 2 which in that place standeth as his squared square roote.

Likewates. 64. is twise set in thesame table, ones emongest *squares*, vnder 8. whiche is his square roote: And again emongest *Cubike numbers*, vnder. 4. whiche is his Cubike roote.

Master. You saie truthc. Although the lasse example be not to your purpose, concerning *Squared Squares* or *Zenzizenzikes*. And if you did note it onely, for this cause it is twise set in the table: then maie you see it thise sette in thesame table, soz it is in the sixte rewe vnder. 2.

Scholar. So I see, wherfoze I might rather haue take. 81. whiche is a *Zenzizenzike* number, and so hath for his roote. 3: And also it is a square number, and hath. 9. for his roote.

Master. Farther to procede, if I multiplie those *squares of squares* by their roote, thei will make *Surso: Surfolidcs. lide numbers*.

Scholar. I perceiue by the numbers in the table, that you meane the leaste roote of the twoo: bicause vnder. 16. I see. 32. in the rewe of *Surfolidcs*.

Master. Reason maie driue you to thinke so. For the number and his roote, muste beare allwaies one name. So that if I name. 16. as a square number, I

H. ij. must

The seconde parte

must referre it to his square roote. And if I name it as a *Zenzizenzike* number: it muste bee referred to his *Zenzizenzike* roote. And in like sort of al other names.

As when I call. 64. a square number, & demaunde what is his roote: you muste nedes aunswere by his *Square roote*, whiche is. 8. But if I name. 64. as a *Cube*, and doe then seke for his roote: you must vnderstande his *Cubike roote*, and that is. 4. But if I name it to bee a *Square of Cubes*, or *Zenzicube*: then is. 2. his roote. As you maie by the table perceiue. And also by the orderly multiplication of euery rewe, or order of numbers by their roote. For therby amounteth the nexte rewe.

And so maie you increase the numbers of those rewes, or orders, according to the tymes of your multiplication, as moche as you list. And euery order shall beare soche names, as agreeth to the nature of their rootes.

Wherfoze thei appeare to bee ouersene, that call those formers numbers *Surdesolides*, seing thei are not any waies *Surde numbers*, but haue their rootes. And yet, to confesse the truthe, I cannot well tell you the true *etymologie* of their name: except thei be so named, as it were *solide* vpon *solide*. And that interpretation were to streightly racked. But the name beyng receiued and well knowen, wee maie moze easly with libertie vse it, then with scrupulositie, curiously scã it.

These numbers are simple numbers in their kind. For thei rise of. 5. multiplications. And if their roote bee a digite number, then is it the same number, that standeth in their firste place. And if their roote be an article, then hath that *Sursolide*. 5. tymes so many *Cyphers* together in the firste places, as his roote hath: and the nexte figure after those *Cyphers*, is the firste figure significatiue of his roote.

Scholar. I see it so in all these numbers, that bee
in

of Arithmetike.

in the table.

Maſter. And ſo ſhall you finde it in all others.

And farther if the roote bee a number mixte, then the firſte number of the *ſurſolide*, is the firſt number of the roote. And this I doe tell you for ſome helpe, in getting at their rootes.

This name therfore of them, I meane *Surſolides*, in *Arithmetike*, maie ſerue to admoniſhe you of their roote. But in *Geometrie*, or in compoſition of ſounde bodies, it ſerueth to no uſe: and therfore I doe call the agreeable to their figure, *Squares of Cubes*: becauſe they make a ſquare forme: but ſo that euery unitie of that ſquare, is in it ſelf a *Cube*: As by the figures that fol-
Squares of Cubes.

lowe, you maie well coniecture.
And alſo they are made by multiplication of a *Cubike number*, and a *Square number* together, bothe hauyng one roote: and the *Surſolide* hauyng the ſame roote. Wherefore reaſon with the nature of their ſounde figure, inſoꝛceth me to call the *Squares of cubes*.

Yet other menne attendyng more to the nature of their rootes, then to their olone formes and nature, doe giue that name to the nexte rewe of numbers, becauſe they maie be made of multiplication, of any *Cubike number* by it ſelf, that is to ſaie ſquarely.

Scholar. It is ſo. For 8. whiche is a *Cubike number* multiplied ſquarely maketh 64. And that 64. is ſet amongeſt the *Squares of Cubes*.

Maſter. And this commoditie commeth by that name: that it putteth menne in remembraunce of the ſpedie and eaſie extraction of their roote: As you ſhall learne hereafter.

But I conſideryng their olone nature and mixtynge, as ſounde numbers or bodies: doe call them *Cubes of Cubes*, or *Cubike Cubes*.

After theſe numbers in the ſeuenth rewe, there do followe thoſe numbers, whiche commonly are called
Surſolides,

The seconde parte

Seconde
surfolides.

*b*surfolides, or *bissurfolides*, that is, *seconde surfolides*, or *dou-
ble surfolides*. But I maie call them *seconde squares of cus-
bes*, alludng at the same name. Howbeit if I looke to
their forme and nature, I shall be enforced to call the,
longe cubes of cubes, or *longe cubike cubes*.

Squares of
squared
squares.
Cubes of
Cubes.

And so by like reason, doe I cal the nexte numbers
square cubes of cubes, or *square cubike cubes*: whiche other
men doe cal *zenzizenzizenzikes*, that is *squares of squa-
red squares*.

The ninth rewe of numbers, is commonly called
Cubike Cubes, or *Cubes of Cubes*: bicause the *Cubike* rootes
of those numbers are *Cubike* numbers also. But I af-
ter their true nature, doe call them *Cubes of Cubes* *Cubi-
kely*: or *Cubes of Cubike Cubes*.

Squares of
Surfolides.

The tenth rewe of numbers is named vulgarely,
Squares of surfolides, bicause thei haue a *Square* roote,
whiche is of it self a *surfolide* number. And for their fi-
gure *Grometricalle*, I name the *long cubes of cubike cubes*.

So that I consideryng their nature, that thei be fi-
guralle numbers, am constrained to name them, ac-
cordng to their figure, I meane in this place, where
I doe make explication of their natures and names.

But other men for aide of woork, in extraction of
rootes, haue giuen them soche names, as maie beste
put menne in remembrance of redy woork therein.
Whiche names I will vse also hereafter, in my wri-
tynges, bicause I will not bee an authoz of vnnede-
full singularitie. And yet bicause truth in nature is
as well to be regarded, as ease in woorkng, and ra-
ther moze, I could not omitte in this place, the decla-
ration of their true nature and very formes.

And so bothe of vs hauyng good reasons, for those
names, neither maie contempne other, neither con-
tende together.

A generale
reason for na

And although the names that I doe giue, maie
seme to some menne (whiche are scarce apte iudges)
moze

of Arithmetike.

more obiousse, for the newe Invention (as thei maie ^{mes of these} think) then needfull to the practise of tharte: yet thal ^{numbers.} you see in theim a naturall sequele, and orderly propagation.

For all those numbers are considered, in one of .2. soymes firste. That is to saie, other thei bee taken as numbers absolute, without any consideration of multiplication: And so thei maie be named numbers onely, without name of relation. Or els thei bee considered as numbers multiplied, and that can be but in .3. varieties.

If thei be multiplied but ones, then doe thei make a line of numbers, or a lincarie number. And that number hath onely lengthe, without bredthe, or depth: And therfoze maie be the roote to a *Square*, or a *Cube*. But is of it self, in that consideration, nother *Square* nor *Cube*.

Secondarily, it maie bee multiplied twise, the one number stadyng for the lengthe, and the other for the bredthe: and so is it a *Square number*, and therfoze a *flat number*.

Thirdly, it maie bee multiplied thrise, and therby gette lengthe, bredthe, and depth: wherby it is made a *sounde number*. And bicause the sides bee equalle, it is specially a *Cube* or *Cubike number*.

Now can there be no sowerth waie, that any multiplication maie increase: for there are no more dimensions in nature.

But if any manne doe multiple the fourth time, then must he accoumpte that he maketh a *line of Cubes*: and the fifth multiplication maketh a *Square*, in which the every unitie is a *Cube*: So the sixte multiplication maketh a *Cube of Cubes*, accoumptyng every lesser *Cube* for an unitie. And there is a staie again.

Wherfoze if any man multiple the seventh time, he retourneth againe to the firste nature of numbers

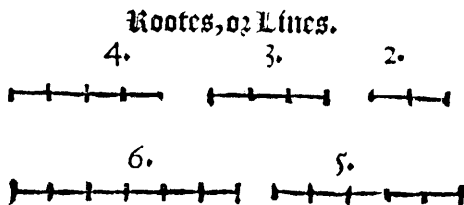
The seconde parte

multiplied, whiche are *liniarie numbers*: And the 8. multiplication, woorketh as the seconde did, and maketh *flatte numbers*. The ninth multiplication agreeably with the thirde, doeth make *Cubes*.

And so infinitely these. 3. woorkes maie bee reiterate, but a fourthe foyme can neuer be deuised.

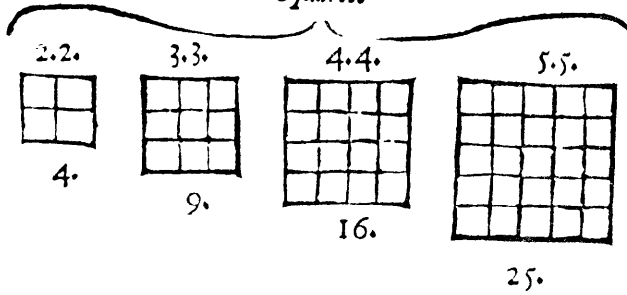
And therefore doe I, as reason doeth compell me, reduce all numbers to those. 3. foymes, as their verie originalle springes and fountaines.

But to the intente that you maie the more aptly iudge of them, and their natures, I haue here sette foorth the foymes, whiche they make in figures *Geometricalle*, or sounde quantities. Admonishyng you to remember this well. That after any number is become a sounde number, it is against reason, to reduce him to an absolute flatte number again, and mosse of all by multiplication. But now marke these figures.

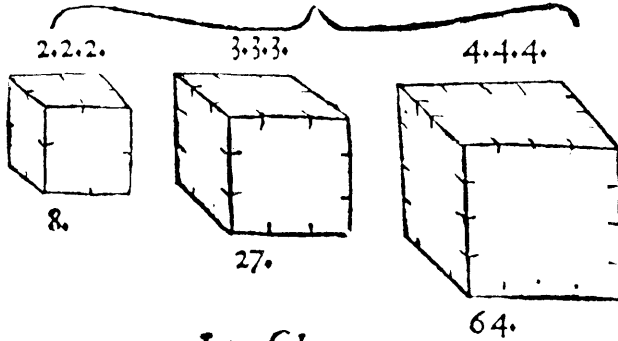


of Arithmetike.

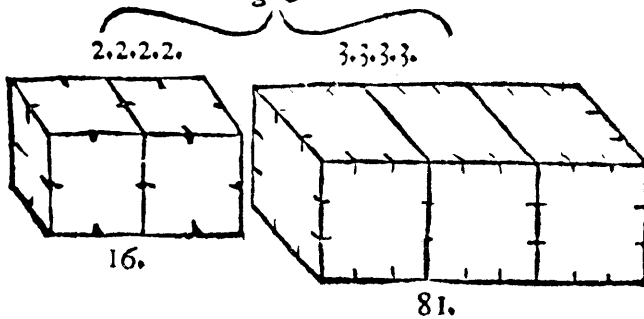
Squares.



Cubes.



Longe Cubes.

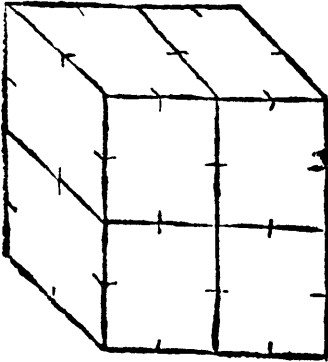


3.4.

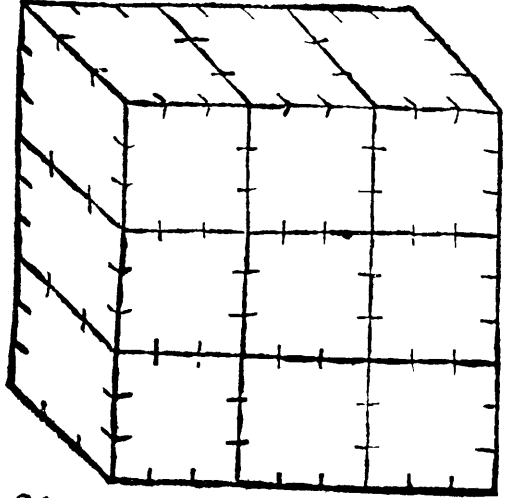
Squares

Squares of Cubes.

2.2.2.2.2.

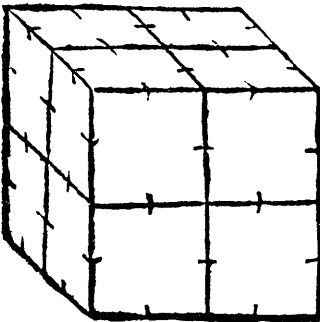


3.3.3.3.3.

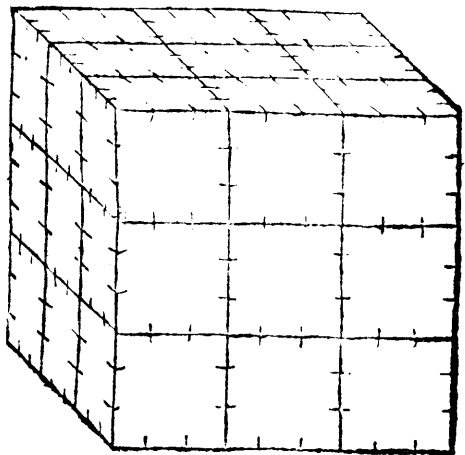


Cubike Cubes.

2.2.2.2.2.2.



3.3.3.3.3.3.



Here

of Arithmetike.

Here, as you see, I haue set first certaine lines, containyng soche partes as thei bee made of by multiplication: that is to saie, 2. 3. 4. 02. 5. And these bee produced by the first multiplication, where an vnitie of any thyng is multiplied by a number.

And so an ynche multiplied by 3. maketh 3. ynches: And a foote multiplied by 6. maketh 6. foote: and so of other measures and quantities, in like sorte. All whiche multiplications, doe make onely longe lines, 02 measures in lengthe onely, without bredth 02 thicknesse.

And in this multiplication, nother the number, nother yet the vnitie, is accounted 02 called a roote. But the line that is made therby: maie bee a roote to any of all the other kinde of numbers befoze reherfed, and sette forth in the table. For if you multiplye the same line, by the number that his lengthe doeth include, then there will be made thereof, by this seconde multiplication, a square figure, containyng a square number in it: As you see emongest those figures, the firste sower to be, whiche are marked with these numbers. 4. 9. 16. and. 25.

Scholar. I perceiue well in eche of the, that their lengthe is agreable with their bredthe, and so thei make square figures, but I knowe not what those numbers doe mean, that be set ouer their heddes.

Master. The quantitie of the number, doeth betoken the valewe of their roote. And the multitude of the same number repeted, doeth declare the number of multiplications, for eche figure.

And therefore the lines, whiche are made by one multiplication, haue eche of them their number simply set, ones onely.

The squares haue their numbers double: in token that thei haue. 2. multiplications. That is, one in lengthe, and an other in bredthe.

The seconde parte

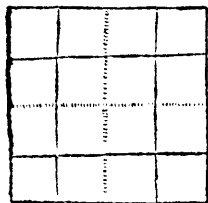
The third formes, whiche be *Cubes*, and are made of. 3. multiplications, haue their roote repeted thriſe.

And the like numbers did I ſette, in the ſide of the former table, againſt the like quãtities. Whiche ſhall helpe you ſomewhat in the extraction of rootes.

Scholar. Now doe I perceiue not onely their names, and multiplications, moche better then I did befoze: but alſo I vnderſtande better the difference of your names, and their reaſons. For by thoſe figures, whiche you haue ſet in the ſowerth place, and doe call them *longe Cubes*, I ſee their forme doeth agree to that name. For they are longer, then they are other brode or depe. And ſaue for their depth, I might liken them to *longe Squares* in *Geometrie*. Howbeit, other men negleatyng their forme, and looking onely to their rootes, doe call them, *Squared Squares*.

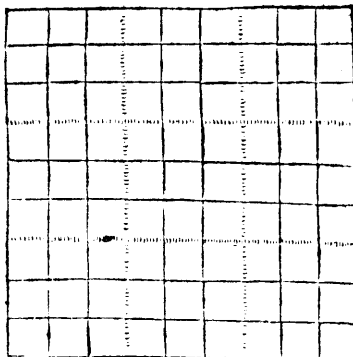
But if you will permitte me, to ſpeake in the defence of them, as a ſimple ſcholar maie ſpeake for affection, in the defence of his maſter, it appereth to me, that they maie well bee called *Squared Squares*: and might be figured thus.

2.2.2.2.



16.

3.3.3.3.



81.

Where the ſmalleſt ſquares, whiche be contained within the picked lines, beyng taken as rootes, and multiplied

of Arithmetike.

multiplied by the same number again, whiche they do containe (other els twice by their rootes) will make the whole greater squares.

And by this figuring of them, there doeth appere no inconuenience nor absurditie, in their vulgare names: but rather a iuste expressing of their naturalle formes.

For in the first figure, 2. standing as the side of the lesser square, and multiplied by it self, doeth make, 4. whiche is the quantitie of the lesser square. When if I multiply that lesser square, 4: by his owne number, it maketh 16. whiche is the greate and whole square: and is a *Square of squares*.

So in the seconde figure, 3. standeth for the roote of the lesser square, contained within the pricked lines, and if it bee multiplied by it self, it maketh, 9. whiche is the quantitie of the same lesser square. When if I multiplye that, 9. by it self, it will make, 81. whiche is the quantitie of the greate Square, and is a *Square of squares*.

Master. I commende you well: not onely for so diligente exercising of them, whiche for their honeste trauell, deserue moche thankes, but also for that you seke to bring manifest reason, and some shewe, at the least, of linearie demonstration for your purpose. So that you will not seme to speake, without some good grounde.

But as in dedde, your figure doeth truly expresse a square of squares, so it doeth suppose the other number, whiche by order of multiplication, doeth go next before it, to be a flatte number also. For it is not possible that a sounde number (as a *Cube* is alwaies) being multiplied by any other number, maie lose the nature of a sounde number: But shall continue a sounde number still. And therefore seeing the next number, before a *Square of squares* was a *Cube*, it is not possible

The seconde parte

possible that a *Square of squares* can be a mere *flatte number*, as you haue drawn it.

Wherfoze if thei had intended, that a *flatte number* should occupie the .4. place, then should thei haue set some *plat forme* in the third place also. Whiche might haue been made in this sorte.

And then will it be a *longe Square*, and not a *Cube*.

3.3.3.

| | | | | | |
|--|--|--|--|--|--|
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| | | | | | |
| | | | | | |

But in as moche as thei doe not admittle this *longe Square* (whiche by that name hath no roote) therfoze maie not the number that soloweth it, bee any other then a *founde number*. For euery *Cubike forme*, beeing multiplied by his roote, doeth make a *Square pillar*. Whose length beareth vnto his bredth the same proportion, that his roote doeth vnto an vnitie.

Scholar. I am very well satisfied now: concerning the names and formes of those numbers. And by this that you haue saied, I doe farther perceiue, that .5. multiplications doeth make the *square of Cubes*, whiche be set in the fiftie place, emongest the former figures. And also I vnderstande by the former table, that thei be called *Surfolides*.

Likevvaies I see in the sixte place of the foresaied figures, *Cubike Cubes*, made by .6. multiplications. But commonly the numbers of those quantities, be named *Squares of Cubes*. So that for their names, thus farre I am perfecte inough.

The

The extraction

of Rootes.

Passer.



Nowe will I shewe you, *The extraction of rootes* how you shal extract the roote of any soche number.

And first I must admonishe you, that you shal alwaies understande, soche a roote, as the number doeth admit. So that in a square number, you shall seke a *Square roote* onely, and no *Cubike roote*, nother any other kinde.

Likewates a *Cubike number* hath no other roote, but a *Cubike roote*. Excepte the namebee compounde, as *zenzicubike*, or *Squared Cube*. For in soche there are 2. sortes of rootes, accordyng to the 2. names that they beare. That is bothe *Square* and *Cubike* roote: as I will anon shewe you. But firste I will beginne with *Square numbers*, and their rootes.

And this generall order muste you obserue, befoze all other: That you shall haue by harte, in readie memorie all soche numbers, whose rootes are digites. For as it is superfluous to seke rules for them, so must they helpe in all greater numbers, whose rootes are aboue 9. And for your ease in remembraunce, I haue here sette foorth a table for square numbers. *The table of Square rootes vncompounde.*

Where in the firste columpne, you se the rootes set, and in the seconde pillar, right against eche roote, there is set his square. Touchyng whiche I nede to saie no more, but that you be not in any vncertaintie of them, whē

| Rootes. | Squares. |
|---------|----------|
| 1. | 1. |
| 2. | 4. |
| 3. | 9. |
| 4. | 16. |
| 5. | 25. |
| 6. | 36. |
| 7. | 49. |
| 8. | 64. |
| 9. | 81. |

The extraction

you shall neede their aied, whiche shall be continually in vse of searching for other greater rootes.

Now for greater numbers, this is the order.

1. First set doune the number as it is. Then sette a prickke vnder euery odde place, I meane the firste, the thirde, the fiftte, the seuenth, and so forth: and so shall euery prickke haue .2. numbers, excepte the laste, whiche some tymes hath but one.
2. Secondly, marke the numbers that belong vnto the laste prickke, toward the leftte hande: And whether he haue belongyng to it one number, or two, looke what the roote maie be of that number, if it bee square. And that roote sette by a crooked line, as you place the *quotiente* in diuision: & cancell all that square number, belongyng to that prickke.
3. But and if the number belongyng to that prickke, bee not a square number, then take the roote of the greatestte square, whiche is contained in it, and place the roote as I saied before. And the square of it shall you abate from the number, that belongeth to that laste prickke, and let the rest be set ouer those numbers cancelled, as you doe in diuision. And so haue you ended your worke for that prickke.

Scholar. This moche is easie inough, if I vnderstande you rightly.

Master. Then proue it in a number, or two. And first worke with this number. 5152900.

Scholar. I muste marke euery odde place with a prickke, thus.

And here I perceiue that vnto the first
5152900. prickke, there belongeth 2 Cyphers onely, and to eche of the other .2. prickkes following, there are appointed. 2. figures. But the fourth prickke hath but one number, and that is .5.

Now according to the second rule, I seke the roote of 5. (for because there belongeth no moze numbers to
that

The extraction

that pycke) and I see, it is no square number. Wherefore accordyng to the thirde rule, I take the greatest square in it, whiche is . 4. and the roote of . 4. is. 2. Therefore I doe subtracte . 4. out of . 5. | 1
and cancell that . 5. and the . 1. that remaineth, I set ouer . 5. as here you see. | 5 152900 (2.
And the roote. 2. I sette behinde the *quotiente* line, as you taught me, and then the nōbers stand, as you se.

Master. You haue doen wel. Youe again in this number. 18766224.

Scholar. First I set them doune | 18766224.
and pycke them, as here doeth appear. And now I see, that the laste pycke hath twoo numbers belongyng to it, that is. 18. with whiche I must begin. And seying it is no square number, I find 16. to be the greatest square in it: wherefore I subtract 16. out of 18. and set . 2. ouer the . 8. | 2
And the roote of . 16. whiche is . 4. | 4 18766224 (4.
I sette behinde the *quotiente* line, as here is seen.

Master. This maie suffice for the first wooyke.

Now to procede, you shall double your roote, and put that double vnder the nexte space, towarde the right hand, that is behinde the nexte pycke. Alwaies forseyng, that if the double doe contain moze figures then one, that the first shall be sette vnder that place, and the seconde vnder the nexte figure, towarde the left hande. 4.

Then seke a *quotiente*, as you doe in diuision, whiche shall shewe how often that double number maie be found in that, that is ouer it, appertainyng to that place: whiche *quotiente*, you shall set before the firste roote, within the *quotiente* line. 5.

But this regarde muste you haue here specially, that you maie leaue ouer the nexte pycke, towarde the right hande, as moche as the square of that *quotiente*,

The extraction

With which you worke, soz out of that rest, the square of that *quotiente* muste bee abated. And then make bothe subtractions, and note the remainder, if any be, and place your *quotient*, and then haue you doen with that prick also.

For the moze plaines, I will giue you an example in your firste number, whiche stood thus, after your worke was ended.

Here I see ouer the laste prick
saue one. 1 1 5. vnder the middell fi-
gure of whiche I must set the dou-
ble of the former roote. 2. that is. 4. And then I seke
how often. 4. is to bee founde in. 11. And I finde that
I maie haue it two tymes, and. 3. remainyng. Whi-
che. 3. with. 5. ouer the nexte prick, doe make. 35. and
that is moze then the square of my *quotiente*. 2. Ther-
foze am I bolde to sette doune that

quotiente: And accoꝝdyng to it, to a-
bate twise. 4. (whiche is. 8.) out of
11. and there resteth. 3. Therfoze I

cancell. 1 1. and sette. 3. ouer it. Then doe I multiplie
the laste *quotiente* squarely: and it maketh. 4. whiche
4. I subtracte out of the number ouer the prick, that
is. 35. where. 5. maie suffice for this number. Ther-
foze I abate. 4. out of. 5. and cancell that. 5. and set. 1.

whiche remaineth, ouer the. 5:
And then will the whole number
stande thus.

This worke, whiche I haue
wrought now, must be repeted as often as there bee
any prickes, or pricked numbers remainyng. Wher-
by you maie easily gesse, that it must bee twise moze
repeated in this example, bicanse there resteth yet. 2.
prickes vntouched.

Scholar. Although I thinke, I could doe, as I
haue marked you to doe, yet soz moze certaintie I
pate

of Rootes.

praie you worke out this example.

Master. Then marke it well.

I shall begin againe with doubling of all, that is within the *quotiente* line. And that double is 44. whiche I must set vnder. 312. that remaineth of the laste worke. And then will the numbers stande, as here you see.

Then I loke how often tymes maie I finde. 44. in. 312. And I

see it will be abated 7 times, and 4 remain: whiche 4 with the. 9. ouer the next prick doeth make. 49. And that will suffice to extracte the square of my *quotiente*.

7. For. 7. tymes. 7. maketh iuste. 49. Thus seying I maie take. 7. for my *quotiente*, I worke with it, as the rule teacheth: abating first. 7. times. 44. (that is. 308) out of. 312. and there resteth. 4. ouer the space before the nexte prick. Whiche. 4. with. 9. ouer the prick doe make. 49. out of whiche I

abate the square of my *quotiente*. 7. (that is. 49.) and so resteth nothing, but. 2. Cyphers. And the number standeth thus.

And seying there remaineth one prick vntouched, I should repeat the same order of worke againe, by doubling all the *quotiente*, whiche would bee. 454. and setting it so that. 4. whiche is in the firste place, should be sette vnder the Cypher, that is without the prick, and the other figures in order, toward the left hand. But all this worke were in vaine, seying there is nothing lefte, to serue for the subtraction.

Yet because there is lefte one pricked place vntouched, I must set for it a Cypher in the *quotiente*.

For this rule is generall: that how many prickes so euer your square number doeth containe, your *quotiente*, or roote shall haue so many numbers. Wherefore this roote must be made vp thus. 2270.

The extraction

The prooffe.

And so it appeareth that your number. 5152900. is a iuste square number. Whiche you maie proue by the orderly prooffe of extraction of rootes. That is to multiplie that *quotiente*, or roote (whiche you haue founde) by it self. And if it doe make the first number exactly, then haue you wrought well.

Scholar. What prooffe is as certaine, as can be. And therfoze I will proue, whether it will agree with this woork. Wherfoze multipliyng 2270. by it self, I see that it yeldeth the firste somme. As here it doeth appere. So is this woork approucd good.

$$\begin{array}{r}
 2270. \\
 2270. \\
 \hline
 158900. \\
 454 \\
 \hline
 454 \\
 \hline
 5152900
 \end{array}$$

And now will I attempte the like woork in the seconde example. Whiche was. 18766224.

But after the firste woork was ended, and the greatest square subtracted out of 18. it did remain in this forme.

$$\begin{array}{r}
 2 \\
 18766224 \quad (4.
 \end{array}$$

Now to continue the woork as you did, and as the rule doeth teache, I must double. 4. whiche is the roote, and standeth by the *quotiente* line: and must set it vnder. 7. that standeth in the space, betwene the laste picke (whose woork is ended) and the nexte picke towarde the right hande. And then will it stande thus as you see.

$$\begin{array}{r}
 2 \\
 18766224 \quad (4. \\
 8
 \end{array}$$

What doen, I must seke a *quotiente*, that maie declare how often 8. maie bee subtracted out of. 27. and that *quotiente* I finde to be. 3: bicause that after I haue taken. 3. tymes 8. (that is. 24. out of. 27. there will remain. 3. whiche 3. with. 6. that standeth ouer the picke, doe make. 36. And I see that number to bee greate enough, for the abatements of the square of my *quotiente*: whiche is but. 3. tymes. 3. that is, 9.

Wherfoze

of Rootes.

Therefore I sette doune .3.
foz my *quotiente*, before . 4. in
the *quotiente* line. And multi-
plyng 8. by that .3. there riseth
24. whiche I doe subtract out

$$\begin{array}{r} 2 \\ 237 \\ 1876224 \quad (43) \\ 8 \end{array}$$

of .27. that is ouer .8. and there will remain .3. That
.3. with .6. ouer the pücke, maketh .36. out of whiche
I must abate .9: whiche is the square of my *quotient* .3.
and so will there reste .27. ouer that pücke.

And thus haue I ended .2. pückes, and yet .2. more
doe remain: in whiche bothe I must repeate the same
foz me of woork.

Therefore I double the whole *quotiente* , and it ma-
keth .86: whiche I set vnder .276.

And then I seke the *quotiente*, declaring how many
tymes .86. maie be abated out of .276. whiche maie
be .3. tymes. And foz that cause I set .3. in the *quotiente*
before the .43.

Then doe I firste multi-
plic .86. by that .3. sayng .3.
tymes .8. maketh .24. which
I abate out of .27. and there
resteth .3. And again I saie,
3. tymes .6. is .18. whiche I
abate out of .36. and there doeth remain .18.

$$\begin{array}{r} 1 \\ 237 \\ 23783 \\ 1876224 \quad (433) \\ 86 \end{array}$$

That doen , I take the square of my *quotiente*, that
is .9. whiche I doe subtract out of .12. (foz the .2. ouer
the pücke must borowe .1. of .8.) and then will there
remain ouer that pücke .173.

And thus is that pücke ended.

Now, foz the laste pücke in woork, though he be
firste in place. The double of my *quotiente* is .866.
whiche I muste sette vnder
1732. As here is doen, where
I leaue out many cancelled
figures , as superfluous in

$$\begin{array}{r} 173 \\ 1876224 \quad (433) \\ 866 \end{array}$$

The extraction

this place.

And then seeking for a newe *quotiente*, I finde it to be. 2. whiche I set with the other numbers in the *quotiente*. And by it I multiplie and subtract the 866. sayng; 2. tymes. 8. is. 16. whiche I abate out of. 17. and there resteth. 1. Again. 2. times 6 is. 12 that I subtract out of. 13. and there remaineth. 1. Thirdly, I saie. 2. times. 6. gtueth. 12. whiche I abate from. 12. and there is left nothyng. Saue that ouer the prycke there standeth 4 whiche is equall with the square of my *quotient*.

$$\begin{array}{r}
 \text{I} \text{ I} \\
 \text{I} \text{ 7} \text{ 3} \\
 \text{I} \text{ 8} \text{ 7} \text{ 6} \text{ 6} \text{ 2} \text{ 2} \text{ 4} \text{ (4332.} \\
 \underline{ \text{ 8} \text{ 6} \text{ 6}} \\
 \text{ 8} \text{ 6} \text{ 6}
 \end{array}$$

Wherfore abatynge the square of my *quotiente* out of it, there resteth nothyng at all.

And therby I see that. 18766224. is a iuste square number. And his roote is. 4332.

The prooffe.

Master. Although I knowe it to bee so, yet for your better exercise, and full perswasion: I would haue you trie it, by square multiplication.

Scholar. What maie I sone doe.

And so I finde it to be true.

For. 4332. multiplied by it self, doeth make. 18766224. As this woork here set, doeth shewe.

$$\begin{array}{r}
 4332, \\
 \underline{4332.} \\
 8664. \\
 12996. \\
 12996. \\
 \underline{17328.} \\
 18766224.
 \end{array}$$

Master. Yet because some other small doubttes, maie happen in working, that maie trouble a yong practiser, I will propounde to you one or

two examples more. Wherin you shall finde some varietie, as well in the number propounded. as also in the *quotiente*.

And firste to begin, I will you to extract the roote of this number. 22071204.

Scholar. I must set doune the number, and note it with pryckes in euery odde place: For that rule I perceiue

The extraction

perceiue neuer falleth.

Master. No more doeth any of the other, although the woork maie varie in some small pointes: whiche yet maie be greate enough to trouble a young learner.

Scholar. Then accordyng to the firste rule, I seke out the greatest square in. 22. (so I see it is no square number it self) and it appereth to be 16. And his roote 4. wherfore I doe sette doune. 4. in the *quotiente*, and then I doe abate. 16. out of. 22. and the remainder is. 6. whiche I sette ouer the pycke, and cancell the. 22. as here is seen.

$$\begin{array}{r} 6 \\ 22 \overline{) 0712 \cdot 04} (4 \end{array}$$

Now goyng on with the nexte pycke, I shall double the former roote in the *quotiente*, and sette it vnder the Cypher, betwene the. 2. pyckes.

Then do I seke how ofte that 8 (whiche is the double of the *quotiente*) maie be found in 60 and I finde it to be 7 times, and 4 remainyng to be set ouer the Cypher. So that for the pycke there remaineth. 47. out of whiche I should abate the square of my *quotient*. But seeing that. 49 (whiche is the square of 7) can not be taken out of. 47. there is a newe *quotiente* to be sought.

Wherfore I take 6. And see that it will serue. So I set. 6. in the *quotiente*: and by it I multiplie 8 whereof commeth 48 That. 48. abated out of. 60. leaueh. 12. Therefore I cancell the 60. and set. 12. ouer it.

$$\begin{array}{r} 19 \\ 621 \\ 22 \overline{) 712 \cdot 04} (46 \\ \quad 8 \end{array}$$

Then doe I multiplie the *quotiente*. 6. by it selfe: whereof riseth. 36. And that abated out of. 127. leaueh. 91. And so haue I ended the seconde woork.

Now for the thirde woork, I double. 46. and it doeth yelde. 92. to bee sette vnder. 911. as I haue put it here.

And then seeking for a *quotient*: I see that I maie take

The extraction

9. Wherefore I set that 9 in the *quotiente* with. 46. and by it I multiply 92 and subtract that, that riseth, in this forme.

| | |
|-----------------|-----------------------|
| 7 | 7 |
| x 8 | x 8 |
| x 9 8 5 | x 9 8 5 |
| 6 2 1 3 1 | 6 2 1 3 1 |
| 2 2 0 7 1 2 0 4 | 2 2 0 7 1 2 0 4 (469. |
| 9 2 | 9 2 |

Nine tymes. 9. maketh .81. whiche I abate out of. 91. and there resteth 10. Then 9 tymes 2 giueth 18. whiche I must abate out of. 10. and there will remain. 83.

And now muste I multiple that laste *quotiente*. 9. squarely, wherby will amounte. 81. that shall I subtract out of. 832. and there will remain. 751. and so that prycke with his woork is ended.

Wherefore procedyng to the fourth prycke, I double all the *quotiente*, whiche will be 938. And I set it vnder 7510.

| | |
|-------------------|------------------------|
| 7 5 1 | 7 5 1 |
| x 2 2 0 7 1 2 0 4 | x 2 2 0 7 1 2 0 4 (469 |
| 9 3 8 | 9 3 8 |

Then doe I seke a newe *quotiente*, whiche I finde to bee. 8. For 8. times. 9. giueth. 72. whiche I abate out of. 75. and there remaineth. 3. Again. 8. tymes. 3. is. 24. and that I deduce out of. 31. and so resteth. 7. Then saie I. 8. times. 8. is. 64. whiche beeyng subtracted from. 70. doeth leaue. 6. And that. 6. with the 4. ouer the prycke maketh. 64. out of whiche I muste withdrawe the square of. 8. that is my *quotient*, and it beeyng 64. there resteth nothing. And the whole woork standeth thus.

| | |
|-----------------|-----------------------|
| 3 7 | 3 7 |
| 7 5 1 8 | 7 5 1 8 |
| 2 2 0 7 1 2 0 4 | 2 2 0 7 1 2 0 4 (4698 |
| 9 3 8 | 9 3 8 |

Wherefore I saie that the first nuber 22071204. is a square nuber: and hath for his roote. 4698 As I maie pzoone also, by square multiplicati- on. For, as in this example you see: 4698. multiplied by it self, doeth byyng forth, 22071204.

The prooffe.

| | |
|------------------|------------------|
| 4698 | 4698 |
| 4698 | 4698 |
| 3 7 5 8 4 | 3 7 5 8 4 |
| 4 2 2 8 2 | 4 2 2 8 2 |
| 2 8 1 8 8 | 2 8 1 8 8 |
| 1 8 7 9 2 | 1 8 7 9 2 |
| 2 2 0 7 1 2 0 4. | 2 2 0 7 1 2 0 4. |
| Master. | Master. |

of Rootes.

Maſter. Yet one crample more ſhall you proue: *Another example.*
and that is this. $9 \circ 174 \circ 841$.

Scholar. I ſet it downe, and prick it accoꝝdyng to the rule: And then I ſee over the laſte prick, one onely number, that is. 9. whiche hath. 3. foꝝ his ſquare roote. That. 3. I ſet within the *quotiente* line, and therfoꝝe I cancell. 9.

After this I ſhould proceade with doublynge the roote. 3. and that double ſhould I ſet in the next ſpace, over whiche remaineth no number, foꝝ. 9. beyng cancelled, the Cyppher is nothyng. And ſo am I at a ſtaie.

Maſter. Seeyng that you can not ſet the double of your *quotiente* downe there, where no number is (oꝝ if it ſo chaunce, as ſome times it doeth, that the number over it, is leſſer then the double) then ſet a Cyppher in the *quotiente*, and ſo haue you doen with that prick. Foꝝ in ſoche caſe there nedeth no multiplication, noꝝ ſubtraction.

Scholar. Then am I instructed ſully foꝝ that pointe: The woꝝke is ſo eaſie. I muſt therfoꝝe ſet my numbers thus.

Maſter. And doe you not ſee, that the double of the *quotiente*, is greater then the number over it?

Scholar. I was ſo mindfull of the one halfe of the rule, that I foꝝgate the other halfe.

But now I ſee, I muſt ſet an other Cyppher yet in the *quotient*. And then ſhall I ſet the double of all that, in the thirde ſpace, after this foꝝte.

And now we proceadyng to ſearche foꝝ anſwe *quotiente*, I ſee that. 2. ſhall ſerue me.

Wherfoꝝe I ſette. 2. in the *quotiente* line, with. 300. And by it ſhall I multiplye the double afoꝝeſaied: ſaiyng. 2. tymes. 6. maketh. 12.

The extraction

to bee abated out of. 17. and the remainder will bee. 5.

Then shall I ouerpasse the twoo Cyphers, bicause thei make nothing by multiplication: and so comyng to the picke, I bate the

$$\begin{array}{r} 5 \quad 4 \\ 9 \overline{) 1740841} \cdot 3002. \\ \underline{600} \end{array}$$

square of my *quotiente*: whiche is 4 out of. 8. and there resteth. 4. Therfoze I cancell. 8. and set doune. 4. and so haue I ended that picke. And haue but one woꝝke moze behinde.

Therfoze I set doune the numbers, with the double of al the *quotiente*, thus.

And then I loke foꝝ a new *quotiente*, whiche I finde to be. 9. by it therfoze I multiplie, first 6 and it maketh

$$\begin{array}{r} 5 \quad 4 \\ 9 \overline{) 1740841} \cdot 3002. \\ \underline{6004} \end{array}$$

54. that doeth abate the 54. ouer it. Then omit I the 2 Cyphers, and multiplie 4. by 9 whereof there cometh. 36. whiche I abate out of. 44. beyng ouer it, and there remaineth. 8. That. 8. with. 1. ouer the picke maketh. 81. out of whiche I muste abate the square of. 9. beyng also. 81. And so is nothyng lefte, wherby it appeareth, that. 901740841. is a square number, and his roote is. 30029. The pꝛoofe of it doeth confirme the same. Foꝝ 30029 multiplied by it self, doeth bynge foꝝ the. 901740841.

$$\begin{array}{r} 30029. \\ \hline 30029. \\ \hline 270261. \\ 60058. \\ \hline 90087. \\ \hline 901740841. \end{array}$$

The nigheste
roote of vn:
square nom:
bers.

Wasser. This shall suffice foꝝ soche numbers as bee fully square. Other numbers there bee infinite, whiche be not square, and therfoze haue thei no square rootes. Yet of ten tymes it happeneth, that we shall bee occasioned to searche foꝝ the nigheste number, that maie resemble their rootes.

Therfoze in soche case, this shall you doe. Firste
extracte

of Rootes.

extract the roote, as if it wer a square nōber. And that roote wil serue for the greatest square, that is in your former number: and there will be a remainder beside. Of whiche remainder with the *quotient*, you shal make a fraction, in this sorte.

Set the remainder ouer the line, for the numerator, and the double of the roote (that you haue founde) set vnder the line, for the denominator. And this shall be a sufficiente precisenesse in greate numbers, for any common woork.

Scholar. I will by an example, taken by chaunce, proue this rule. For it semeth to haue no difficultie. Wherefore I take. 296882.

And this, I am assured, can be no square number. For, I remember you told me before, that no soche number might be a square, which had 2 for his first figure.

Then to searche his nighest roote, I place it, and picke it thus.

| | |
|---|--|
| And vnder .29. I finde the greatestte roote to bee .5. whiche I set in the <i>quotiente</i> line, and cancell 29 settyn 4 | $\begin{array}{r} 4 \\ 296882 \end{array} \begin{array}{l} 5 \\ 4 \end{array}$ |
| ouer it. After that I double it, and there cometh 10. & that double I set in the nexte space vnder 46. Then finde I a newe <i>quotiente</i> , whiche is 4 and by it I multiplie. 10. whereof amounteth 40. to be abated out of 46. And so remaineth .6. Again I multiplie. 4. by it self squarely, and there riseth. 16. whiche I abate from 18. (seyng .8. is to small) and the remainder will be .2. So standeth the whole number, as you se. Wherefore I double the <i>quotiente</i> , whiche is .54. And it yeldeth. 108. that must be set vnder 528 as I haue here doen. | $\begin{array}{r} 452 \\ 296882 \end{array} \begin{array}{l} 54 \\ 4 \\ 18 \\ 2 \end{array}$ |

| | |
|---|---|
| Then I looke for a <i>quotiente</i> , how often I may abate. 108. out of .528. And I see it will be but .4. | $\begin{array}{r} 452 \\ 296882 \end{array} \begin{array}{l} 54 \\ 108 \end{array}$ |
|---|---|

The extraction

tymes. Wherfore I set .4. in the *quotiente*, with the other numbers, and then doe I woork with it: Firſt multiplying .4. and .1. together, whereof cometh but 4. whiche I abate out of .5. And there remaineth .1.

Again I multiplic. ^o. by .4. whereof cometh .32. that doe I ſubtract out of .128. and there will remain 96. When ſhall I take the ſquare of my *quotiente* . 4. whiche is 16. And that muſt I abate out of 962. And ſo remaineth . 946. of whiche number ſet as the numeratoz, with the double of the roote, ſet for the denominatoz, I ſhall make a fraction in this ſorte.

$$\begin{array}{r} 494 \\ 8266 \\ 296882544 \\ 188 \end{array}$$

$\frac{946}{1088}$. whiche is almoſte. $\frac{7}{8}$.

Maſter. You have doen wel. And ſo you perceiue that the nigheſte roote of your former number is $54\frac{473}{144}$. For thoſe fractions are all one.

And hereby alſo you maie vnderſtande, that if the remainder ouer your number bee euen, you maie take halfe of it for the numeratoz, and the whole *quotiente* for the denominatoz.

So maie you take the quarter of the remainder (if it will ſo bee parted) for the numeratoz, and the halfe of the roote for the denominatoz.

And in like maner generally, if the remainder and the roote in the *quotiente*, bee numbers *communicante*, diuide them ſo, that the diuiſor of the remainder, be euen double to the diuiſor of the *quotiente* roote. And ſo maie you eaſily reduce that fraction, to his leaſt termes.

But now for prooſe of this woork, there be two waies: the one is certain, and the other but in a neceſſe. For as the roote of ſoche numbers, is not a precise roote: So if you multiplic that roote by it ſelf, it will make a number, very nighe to that former number, but not exactly the ſame.

Whiche faulte ſome men thinke to redreſſe, by adding

The firſt
prooſe.

of Rootes.

dyng of. 1. to the denominatoꝝ: and yet that amende-
mente sometymes increaseth the erreure.

But because you shall not wante a sure prooffe, doe
thus: Multiplie the *quotiente*, or *Roote* of whole nomi- *The seconde*
bers by it self, and vnto the number that amounteth *prooffe.*
thereof, adde the whole remainer. And if then it make
your firste number, your woꝝke was well doen: els
haue you missed.

Scholar. That maie I proue here quickly. The
quotiente in whole numbers was. 544. whiche bring
multiplied squarely, doeth yelde. 295936. vnto whi-
che number, if I doe adde. 946. that did
remain, it will amounte to. 296882.
and that was the number proponed to
me: wherfore it appereth that the woꝝke
was well doen.

$$\begin{array}{r}
 544. \\
 544. \\
 \hline
 2176. \\
 2176. \\
 \hline
 2720. \\
 \hline
 295936.
 \end{array}$$

Master. You shall neede no more
exampls, for this forme of woꝝke.

But one other waie wil I shewe you,
hoꝝ you shall gesse verie nigh vnto the roote. And *An other*
you shall go as nigh as you will desire, in any prac- *waie to finde*
tike woꝝke. If you desire to gesse within lesse then $\frac{1}{2}$. *the nigheste*
of one, then set before your number. 2. Cyphers. And *roote.*
if you would not erre $\frac{1}{2}$. then set doune 4. Cyphers:
But and if you liste to sette doune. 6. Cyphers before
your number, you shall not misse $\frac{1}{1000}$ of an vnitie fro
the true roote. And if you list to go any higher in pre-
cisenesse of partes, adde still euen Cyphers.

Scholar. I would faine proue this forme, in the
same example, whiche I wroughte laste: Because I
would se the agremente betwene the bothe woꝝkes.

Master. Go to. Your consideration is reasonable
And because the partes maie the better agree, sette
doune. 6. Cyphers. And then shall your roote expresse
thousande partes of the whole number.

Scholar. I sette doune the number, and picke it
thus,

The extraction

thus. Whereby I perceiue that I shall haue the same order of woorkes, and the selfe same numbers that I had before, till that I come to the Cyphers and their prickes.

$$296882000000($$

Master. Truthe it is. And therfore maie you in soche a case sette doune onely the remainer, with the Cyphers. Or els cancell all the numbers, saue the remainer, and the Cyphers: and set the former whole roote, without the fraction, in the *quotiente*.

Scholar. Then will it stande thus.

$$\begin{array}{r} 946 \\ 296882000000(544 \end{array}$$

Now accordyng to the rule I will proceede: as if this whole nōber wer the first number p̄poned vnto me. And therfore I doe double al the *quotiente*, whiche maketh. 188. and that doe I set vnder. 9460. And then shall I seke a *quotiente*, that maie declare how often tymes, that double is cōtained in the number ouer it. And I se it will bee. 8. wherfore I set doune. 8. in the *quotiente*,

$$\begin{array}{r} 4 \\ 75 \\ 1829 \\ 94666 \\ 296882000000(5448 \\ 1088 \end{array}$$

and by it I multiplie the double, and subtrakte it, in this sorte: sayng 8. tymes. 1. out of 9. leaueh. 1. remainyng. Again. 8. times. 8. (that is. 64.) out of 146 will leaue. 82. Then farther I abate. 8. tymes. 8. out of. 820. and there resteth. 756. And last of all, I take the square of the *quotiente*, whiche is also. 64. out of 7560. and there will remain. 7496. And so haue I doen with the firste prick of the Cyphers.

A notable consideration.

Master. Consider now that by those. 2. Cyphers you haue gotten 8 into the *quotient* more then you had before. And all your former number of the roote, remoued by it into one place higher, then it was before

of Rootes.

So that, where by the first woꝝke, your roote was 544. and almoste $\frac{2}{3}$: by this woꝝke you haue founde it to bee $\frac{544}{3}$, and $\frac{1748}{135}$ of $\frac{1}{15}$: whiche is verie nigh the same number, that you had before.

Scholar. In deede, if I reduce the fractions, it wil bee . 544. $\frac{8}{15}$ and $\frac{97}{135}$ of $\frac{1}{15}$: whiche is in one fraction, $\frac{1163}{135}$ about. 544.

Maſter. Marke this triall. And vse the like after euery twoo Cyphers are ended: And you shall see a goodly agremente of the woꝝkes together.

Scholar. In the meane tyme, to procede with the former woꝝke, I set doune the number with the remainder, and the double of the *quotiente*, as here appeareth.

| | | |
|------|-------|--------|
| | 7496 | |
| 2968 | 8200 | (5448. |
| | 10896 | |

And searchyng for a newe *quotiente*, I finde that it will be. 6.

Therefore I sette doune. 6. in the *quotiente* with the other numbers. And by that . 6. I doe multiplic the double of the whole *quotiente*, and subtract it orderly, sayyng: 6. times. 1. be- yng abated out of. 7. leueth. 1.

| | | |
|----------|--------|--------|
| | 5 | |
| | 968 | |
| 18120 | 749644 | (54486 |
| 29688200 | 18996 | |

Likewiſe, 6. ty- mes. 8. maketh. 48, whiche I shall abate out of. 49. and so re- steth. 1. Then 6. times. 9. (whiche is. 54.) must be sub- tracted out of. 1016. and there will remaine. 962. Again I shall abate. 6. tymes. 6. (that is. 36.) out of 9620. and there is left. 9584. Then take I the square of my *quotiente*, whiche is also 6 times 6, or 36. and that I must abate out of. 40. and there resteth. 4. And thus is the seconde picke of the Cyphers ended.

And now I finde in the *quotiente* not $\frac{2}{3}$ as I did in
M. 1. the

The extraction

the lasse woork befoze this. But I finde $\frac{86}{10}$: whiche goeth moze nighe to $\frac{9}{10}$. For $\frac{90}{100}$ would be $\frac{9}{10}$: and $\frac{80}{100}$ is equalle with $\frac{8}{10}$. And I maie easily se, that $\frac{86}{100}$ is moze nigher to $\frac{90}{100}$ then to $\frac{80}{100}$: beside the remainer, whiche will make $\frac{47962}{54486}$ of $\frac{1}{100}$. or els $\frac{47962}{5448600}$ of one.

Passer. I see, a well willing mynde can marke diligently, and learne speedily: wherfoze go forwarde with your woork.

Scholar. I muste sette doune the double of all my *quotiente*, whiche will be. 108972. And it will stande thus.

| | |
|---|---|
| Wherfoze I doe seke for a newe <i>quo-</i> <i>tiende</i> , and I finde it to be. 8. whiche. 8. I | $\begin{array}{r} 95802 \\ 29688 \times 200000000 \\ \hline 108972 \end{array}$ |
|---|---|

set in the *quotiente*, with the other numbers, and by it I woork after my rule, sayng: 8. tyme. 1. is. 8. whiche I abate from. 9. and there resteth. 1. Then take 3. 8. tymes. 8. (that is. 64.) out of. 158. and the remainer will be. 94.

Again I subtract. 8. tymes. 9. (beeyng. 72.) from. 940. and there is left. 868. Farthermoze I take. 8. times. 7. (whiche is. 56) out of. 82. and there resteth. 26. Then doe I withdraue. 8. tymes. 2. or. 16. out of. 60. And there remaineth. 44.

| | |
|---|---|
| out of. 82. and there resteth. 26. Then doe I withdraue. 8. tymes. 2. or. 16. out of. 60. And there remaineth. 44. | $\begin{array}{r} 8624 \\ 19488 \\ 978024 \\ 29688 \times 200000000 \\ \hline 108972 \end{array}$ |
|---|---|

Last of al I take 8 times 80264 (whiche is the square of my last *quotiente*) out of 862440 and the remainer will be. 862376. And so have I ended all my woork.

And now I haue for the roote $\frac{544868}{1000}$ that is. 544. and $\frac{868}{1000}$ beside $\frac{431188}{144868}$ of $\frac{1}{1000}$ or in lesser termes $\frac{107797}{136217000}$ of one: whiche beynge reduced into one fraction with the $\frac{868}{1000}$ will make $\frac{118344153}{136217000}$.

Passer. You haue doen well.

And

of Rootes.

And here you see, that you drawe nigher & nigher still, to the very roote, if it might haue any. For 10 is a nigher number to $\frac{2}{10}$, then is $\frac{80}{100}$ as that was nigher then $\frac{8}{10}$.

And if you would worke with more Cyphers, you should perceiue still, that it would drawe nigher and nigher. But this maie suffice for examples sake.

Scholar. When I praye you tell me, what is the chief vse of this rule: and for what maters it serueth.

Master. One yere will not suffice, to expresse the commodities of it. It serueth so many waies, in building: in proiection of plattes, for measuring of ground Timber, or stone: And also in warre, for framing of battailes, for making of diuerse engines, and generally for all woorkes of *Geometrie* and *Astronomie*. But for to satisfie you partly, I will sette forth the two or three questions, that depende of this worke of extraction of square rootes.

And firste of a battaile: because it seemeth to serue leaste for that purpose.

*A question
of an armie.*

A capitaine generall hauyng three greate armies, would castte them into three square battailes, but he knoweth not how many men, he shall set in the fronte of eche battaile.

The numbers of the three armies, are for the firste 5625: For the second 9216: And for the third 15129

Scholar. I dooc perceiue easly, that for eche of these numbers, I muste searche out the square roote, and then haue I the fronte, or flanke. Sith bothe are equalle in a square battaile.

Wherefore I set doune the first number thus, with his pyckes. And then vnder the first pycke towarde the lefte hande, I finde the greatestte roote to bee . 7. seeyng the greatestte square is. 49. What roote doe I set within the *quote* line: and his square doe I abate from. 56. and so

P. ij.

remaineth

The extraction

remaineth. 7.

When doe I double that roote, and sette the double vnder. 72. and see that the newe *quotient* will bee. 5. And there will remaine. 25. whiche is the iuste square of the last *quod* *tiente*.

$$\begin{array}{r} 7 \\ 5 \overline{) 725} \\ \underline{35} \\ 375 \\ \underline{350} \\ 25 \end{array}$$

Wherby it is euident, that his first armie contained a square number, and the roote, or side of it is 75. And so many menne shall be in the fronte of the firste battaile, and as many in the flanke.

Now for the seconde battaile, I seke the square of 9216. and finde it to bee. 96. As in this example I haue wrought it.

$$\begin{array}{r} 96 \\ 9 \overline{) 9216} \\ \underline{81} \\ 1116 \\ \underline{1080} \\ 36 \end{array}$$

For the firste number is. 9. seying it is the greatestte square roote, that can bee founde in. 92. And so is the double of it. 18. and the *quotiente* for it. 6. as it appeareth manifestly inough.

Wherfore I saie that the second battaile shall haue in euery ranke. 96. men.

And now for the thirde battaile, I sette downe the number, accordyng to this rule: and I finde the firste roote to be. 1. bicause. 1. tymes. 1. maketh. 1. And his double is. 2. whiche I abate twice from the number ouer it: and after double those bothe numbers, whiche make. 24. And finde that to be abated. 3. tymes.

$$\begin{array}{r} 123 \\ 1 \overline{) 123} \\ \underline{1} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

And so haue I gathered that the number is square and the roote 123. Accordyng to whiche number, that thirde battaile must be marshalled.

Master. Seyng you are so redy in this pointe so sone. Tell me how many menne, shall be sette in the fronte, if all these. 3. armies be ioined into one square battaile.

Scholar. Firste I must adde all. 3. numbers together.

of Rootes.

ther. And so will thei make. 29960. as here by crample doeth appere.

| | |
|--------|--|
| 5625 | |
| 9216 | |
| 15129 | |
| 29960. | |

But this number can bee no square number, bicause it hath one odde Cypther in the firste place: for I remember your sayng, that square numbers can not begin with odde Cypthers. Wherefore this number will not make a square battaile.

Yet wil I proue, what mate be the frôt of the greatestte square battaile, that mate be made of that nôber.

And for that purpose I picke the numbers, and finde the greatestte roote in. 2. to be. 1

and the same nôber to bee the square also. Then double I that roote, and place his double vnder. 9. that is vnpricked: and serchng for a *quotiente*,

| | |
|------|------|
| 113 | |
| 1541 | |
| 2960 | (173 |
| 234 | |

I finde it to be. 7. with whiche I woork by the rule, and so doeth remaine so: the nexte picke. 10.

Then doe I double that. 17. whereby cometh 34 whiche I set vnder. 106. And for it I finde. 3. to be the meteste *quotiente*: with whiche if I woork accordingly, there will remaine. 31. as the excelle aboute the greatestte square.

Whereby it appeareth that. 29929. is a square nôber: and hath. 173. for his roote. And that should bee the fronte of this greate battaile.

Master. Now will I proue you with an other question of like sorte.

A Prince hath an armie verie greate. With whiche *The seconde* che he passeth in a Vallie, so that in marchynge the *question of* fronte can be but. 18. menne. And by that meanes the *an armie.* flanke containeth. 449352.

After that the armie is passed that valie, the kynge myndng to occupie all the beste grounde, willethe the battaile to be set square. How would you doe it?

Scholar. first I multiple the flanke, by the front.

M. ij. And

The extraction

And so I finde the whole number to be. 8088336 .

That number doe I picke
as my rule teacheth me, and
I finde the first roote to be. 2.
and his square. 4. whiche first
I subtrac out of. 8. and so re-
steth. 4. Then doe I double
that *quotiente*, and finde that double. 8. tymes in the
somme ouer it.

| | | | | |
|---|---|---|---|----|
| 2 | 2 | 2 | 3 | |
| A | 8 | 4 | 4 | 7 |
| 8 | 0 | 8 | 8 | 3 |
| | | | | 6 |
| | | | | (2 |
| | | | | 8 |
| | | | | 4 |
| | | | | 4 |
| | | | | . |
| | | | | 8 |
| | | | | 8 |
| | | | | 8 |

And so doe I procede till I haue founde out all the
4. figures, accordyng to the. 4. picke under that nō-
ber. And then the roote appeareth to be. 2844 .

*The thirde
question of
an armie.*

Master. Yet one question more, so; to exercise
your penne, will I propounde of a like mater.

A generalle hath thzee armies, to the number of
 28289 . men: and none of those thzee armies is apte
to make a square battaile, yet he is appointed by his
soueraigne, to sette theim in thzee square battailes.

These be the. 3. numbers of the. 3. armies. In the
firste there are. 10296 . men: In the seconde. 9493 :
and in the thirde. 8500 . Now let me see how you can
cast them into thzee square battailes.

Scholar. I thinke it reasonable, to take the grea-
teste squares of the first and second numbers, and the
excesse of them bothe, to put to the thirde number.

Master. So are you not sure that the thirde nom-
ber, will be a true square.

Scholar. Then knowe I not how to doe it.

Master. Take the greateste square in the thirde
number also. And note those thzee excesses, and their
rootes also.

Then put one to euery roote, and marke the squa-
res that will rise of them.

Thirdly, subtract the firste 3. numbers, out of those
3. newe squares, and note the difference of eche of the
firste numbers, from those squares: and so haue you. 3
numbers

of Rootes.

numbers of excesse, and. 3. other of wante.

Now compare those excessees and wantes well together: and you shall easily see from whiche you shall take any number, and to whiche you shall adde any.

Scholar. In the firste nōber the greatest square is 10201. and therby the excesse is. 95. and the roote 101.

In the second number the greatest square is. 9409 and his roote 97. So is the excesse. 84.

And in the thirde number, the greatest square is 8464: and the roote of it. 92. Wherefore the excesse appeareth to be. 36.

And thus haue I founde the. 3. excessees.

Now for to finde the 3 defaultes or wantes, I adde one to eche roote, and multiplie them square: and so of. 102. I finde the square to bee. 10404. and if I subtracte the firste number, whiche is. 10296. out of it, there will remain. 108. for the firste wante.

Then for the seconde roote. 97. I take. 98. whose square will bee. 9604. out of whiche I abate the seconde number, whiche is. 9493. and there is left 111 as the wante of the seconde number.

Thirdly, I take 93 for the newe roote, next aboue 92. and I finde his square to bee. 8649. from whiche when the thirde number. 8500. is abated, the defaulte appeareth to bee. 149. And thus haue I the. 3. defaultes or wantes, and also the. 3. excessees. Whiche for ease of comparng, I set in order thus.

| | A. | B. | C. | A. B. and C. beto- |
|------------|------|------|------|---------------------|
| Excessees. | 95. | 84. | 36. | ken the order of |
| Wantes. | 108. | 111. | 149. | the 3 first nōbers. |

And here I compare the excessees with the wantes, to see if any. 2. excessees will make up the others want. And I see by a lighte pproofe, it will not serue.

As for the wantes, I doe not compare them to the excessees,

The extraction

excesses, so I see that every one want, is greater then any one excesse. And therefore. 2. wantes are farre to greate aboute any one excesse. And so am I at a staie.

Master. Therefore although that rule bee generall, yet where it faileth, this shall you doe.

Take the. 2. wantes, of any. 2. numbers, and adde them firste together, and then abate them from the thirde number: and if the remainer be a square number, then haue you gotten your purpose.

Scholar. What will I proue here. And first I take the wantes, of the. 2. firste numbers, whiche make 219. And that doe I abate from the thirde number 8500. and there remaineth. 8281. whiche as I see, maie be a square number. And therefore I proue it, in my tables, and I finde it so to bee. And. 91. to see the roote of it.

Therefore I saie to the question, that these shall be the numbers of the 3 battailes, as here I haue set the.

The firste battaile. 10404. and his fronte. 102.

The second battaile. 9604. and his fronte. 98.

The third battaile. 8281. and his fronte. 91.

The somme of all
the. 3. battailes. } 28289.

And bicause these nōbers are not onely square, but also their whole somme doeth agree, with the somme of the 3 seuerall armies, you maie be sure that they are well parted, accoꝝdyng to the intente of the question.

But bicause soche questions, haue moze difficultie then commoditie, to them that are not mete, to be trauelled in soche marshall affaires, I wil leaue that matter to marshall men, and will come to lower matters in warre.

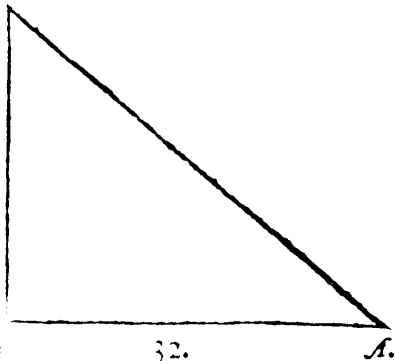
*A question
of scalyng.*

A citie should bee scaled, beyng double ditched. And the inner dicke. 32. foote broade. And the walle. 21. foote high. The capitaine commaundeth ladders to be made

of Rootes.

made of that iuste lengthe, that maie reche from the vnder brow of the inner ditch, to the toppe of the wal- as in this figure C. is partly expofed.

where the line *A B.* standeth for the breadth of the ditch. And the line *B. C.* for the heighte of the walle. Nowe I demaunde, what shall be the length *B* of the line *A. C.* whiche here doeth represente the ladder:



Scholar. This figure doth occasiō me to remēber the 33. theozeme of the pathewate, whiche saith thus.

In all righte angled triangles, the square of that side, whiche lieth against the righte angle, is equalle to the two squares of bothe the other sides.

Wherby I vnderstand, that I must multiply those two sides squarely, that is, eche of them by it selfe. And then adding those. 2. squares together, I muste extract the roote of that whole number: whiche roote shall be the true lengthe of the slope line.

| | | | | | | | | | | | | | | |
|---|-----|----------------------------------|-------|----|----|--------------------------------|----|----|----------------------------------|-----|-----|--------------------------------|--|--------------------------------|
| <table style="border-collapse: collapse; width: 100%;"> <tr><td style="text-align: right; padding-right: 5px;">21</td><td style="border-right: 1px solid black; padding-right: 5px;">21</td><td style="padding-left: 5px;">self,</td></tr> <tr><td style="text-align: right; padding-right: 5px;">21</td><td style="border-right: 1px solid black; padding-right: 5px;">21</td><td style="padding-left: 5px;">Againē, I multiplie. 21. by it</td></tr> <tr><td style="text-align: right; padding-right: 5px;">42</td><td style="border-right: 1px solid black; padding-right: 5px;">42</td><td style="padding-left: 5px;">self, and it yeldeth, 441. These</td></tr> <tr><td style="text-align: right; padding-right: 5px;">441</td><td style="border-right: 1px solid black; padding-right: 5px;">441</td><td style="padding-left: 5px;">bothe sommes, beynge added to-</td></tr> </table> | 21 | 21 | self, | 21 | 21 | Againē, I multiplie. 21. by it | 42 | 42 | self, and it yeldeth, 441. These | 441 | 441 | bothe sommes, beynge added to- | <table style="border-collapse: collapse; width: 100%;"> <tr><td style="padding-left: 5px;">gether, doe make. 1465. whiche</td></tr> </table> | gether, doe make. 1465. whiche |
| 21 | 21 | self, | | | | | | | | | | | | |
| 21 | 21 | Againē, I multiplie. 21. by it | | | | | | | | | | | | |
| 42 | 42 | self, and it yeldeth, 441. These | | | | | | | | | | | | |
| 441 | 441 | bothe sommes, beynge added to- | | | | | | | | | | | | |
| gether, doe make. 1465. whiche | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | |
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| 32 | 32 | self, | | | | | | | | | | | | | | | |
| 32 | 32 | and there riseth of it 1024. | | | | | | | | | | | | | | | |
| 64 | 64 | Againē, I multiplie. 21. by it | | | | | | | | | | | | | | | |
| 96 | 96 | self, and it yeldeth, 441. These | | | | | | | | | | | | | | | |
| 1024 | 1024 | bothe sommes, beynge added to- | | | | | | | | | | | | | | | |
| gether, doe make. 1465. whiche | | | | | | | | | | | | | | | | | |

number maie bee square, bicause it beginneth

The extraction

neth with. 5.

Maſter. It is no ſquare number, as it appeareth at the firſte ſighte. For although the firſte number be 5. yet in ſoche numbers it is requiſite. that the ſeconde figure ſhould be. 2. els can it not be ſquare: and here, you ſee, that the ſeconde figure is. 6. ſo that it can not be a ſquare number.

¶ Herefoze you ſhall ſeke the nightheſte roote, that you can finde in it, and take that for your purpoſe.

Scholar. Here is my woork ſet for the.

| | |
|--|--|
| And ſo it appeareth well that the nightheſte roote is . 38. $\frac{2}{3}$, whiche is leſſe then a quarter of a foote, aboue 38. foote and that muſt be the lengthe of the ladder. | $\begin{array}{r} 2 \\ 5 \overline{) 81} \\ \underline{x 40} \\ 65 \\ \underline{16} \\ 6 \end{array}$ |
|--|--|

Maſter. Yet one queſtion moze will I propound agreeable to the firſte forme.

A queſtio of encampyng.

A capitaine generalle hauynge thre armies, in thre ſeueralle battailes, in the firſte. 4900. menne, in the ſeconde. 2401. And in the thirde. 2500. (ſo that the greateſte armie, is as moche as bothe the other, excepte one manne) is inſoyced to ioine all thre battailes in one. But is in doubtte, whether he maie haue good and conueniente grounde to encampe the, in battaile forme. ¶ Herefoze conſideryng, that all. 3. battailes together, are but double to the greateſte of the. 3. alone. The capitaine deſiryng a mete grounde for his armie, ſo ioined in one ſquare battaile, is in doubtte, what ſquare of grounde will ſerue his purpoſe. But ſure he is, that it muſte bee double to the grounde, that the greateſte armie of the 3. did occupie and that was ſquare euery waies. 210. foote. ¶ Herefoze his demaunde is, how many foote ſquare, ſhall the ſide of that grounde bee, that is double to the former ſquare platte, whoſe ſide was. 210. foote euery waie:

Scholar.

of Rootes.

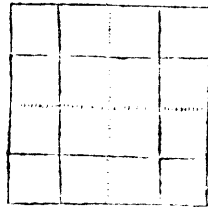
Scholar. Firſte I muſt multiplie. 210. by it ſelf, and ſo haue I the iuſt platte of grounde, of. 44100. foote, that muſt I double, and it will be. 88200. And out of this number, ſhall I ſeke the nightheſt ſquare roote. For a iuſte ſquare, I ſe, it is not: by reaſon that after the euen Cyphers, there foloweth. 2, whiche is one of thoſe figures, that can not beginne any ſquare number.

Therefore, ſekyng for the nightheſt roote, I finde it to bee 296. $\frac{1}{2}$ that is almoſt. 297. foote euery waies ſquare. And ſo moche muſte the ſquare ſide of that grounde bee, whiche ſhould ſerue for that whole ar-

$$\begin{array}{r}
 14 \\
 14 \ 58 \\
 42 \ 124 \\
 88 \ 200 \ (296 \frac{1}{2}) \\
 \cdot 4 \\
 \hline
 58
 \end{array}$$

And hereby I doe perceiue, the ouerſight of many men: whiche being required to double a ſquare platte do double the ſide of it, thinking the matter eaſily doen

But if they marke it well, they may perceiue, that they doe make, by that meanes, a ſquare ſolwer times ſo bigge as their firſt ſquare was. As by this figure, any man may ſee.



For if 2. be the ſide of the ſquare then is the ſquare 4. But if I double the ſide, and make it. 4. the ſquare thereof will be 16. whiche is. 4. tymes. 4. and not onely double.

So that the roote of the double platte, ſhould bee the roote of. 8. whiche is ſomewhat leſſe then. 3. and therefore moche leſſe then. 4.

Maſter. You may perceiue theſame, with the reaſon of it, by the 18. propoſition of the. 8. booke of Euclide, as it is before alleged.

But now for to ſhewe the larger uſe of this rule,

The extraction

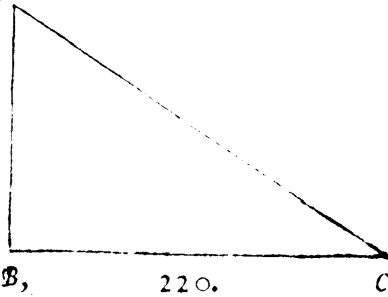
*A question
geographical*

I Demaunde this question.

Where be. 2. townes, as *Chichester* and *Yorke*. whiche
lye Southe and Nothe, and betwene them. 220. mi-
les. A thirde towne as *Excester*, lieth plaine Weste fro
Chichester. 120. miles. I desire to knowe the iuste di-
staunce of *Yorke* from *Excester*.

Scholar. I must set those. 3. townes, in forme of a
Triangle, with *A*

their distaunces:
As here is repre-
sented. Where
A. standeth for *Ex-
cester*, *B*. for *Chiche-
ster*, & *C*. for *Yorke*.



And then accor-
dyng to the rule,

I multiplye. 120. *B*, 220. *C*.
squarely: and it maketh. 14400. Likewise I dooe
multiplye. 220. and it yeldeth. 48400.

These bothe numbers I shall toyne in one, and so
haue I. 62800. whose roote is very nigh. 250. miles
and $\frac{2}{3}$ of a mile.

And that is the true distaunce
of *Yorke* and *Excester*.

By this example I gather, that this rule doeth helpe to *Geo-
graphic*, for to draue the true platte of any countrie.

Master. If I should stande in propounding ex-
amples of this rule unto you, byng but one for euery
arte and science. and for euery different kinde of com-
modious practise: it would make a greate booke.

And therefore omittynge that, till occasion serue o-
therwaies, I will proceade to the extraction of *Cubike
rootes*.

$$\left. \begin{array}{l} 250 \\ 62800 \end{array} \right| \begin{array}{l} 25 \\ 2500 \\ 25000 \end{array} (25000) \\ A$$

of Rootes.
Of Cubike rootes.



When any *Cubike number* is propou-
ned, whose *roote* you should extract
After the number is written doune
orderly: you shall set a *pricke* vnder
the firste figure: and vnder the .4.
and so vnder every third figure, o-
mittyng still .2. figures vnprieked.

And looke how many *prickes*, your number hath,
so many figures shall the *roote* of your n^ober contain

Then to begin the searche, for the firste figure of
the *roote* (in this order) you shall looke what mate be
the *roote* of the number, belonging to the last *pricke*
toward the lefte hande. And that *roote* shall you sette
by a *quotiente* line, as you did in *square rootes*.

And if the whole number ouer that *pricke*, be a *Cu-
bike number*, you shall cancell it all. But if it bee no *Cu-
bike number*, then subtracte out of it, the greateste *Cube*
in it, and cancell the whole number, and set the reste
ouer it: as you did in *square rootes*.

But consideryng, that you ought to haue in ready
remembraunce, all those *Cubike rootes*, whiche be digi-
tes, with the *Cubes* that they make: for without them
you can not procede in this woork. I thinke it good
to set forth herein a table, all those *rootes* with their
Cubes, that they by you mate be the more

adured in tyme of your woork. For els
a litle mistakynge, might be the occasion
of a greate erroure.

And now for this first rule I saie, as
I saied of *square rootes*, this shall be euer
more the firste woork, and shall not be
repeted in any one *Cubike n^ober*. All here-
as all the other rules solowynge, shall be
so often repeated, as there are *prickes* in

| | |
|----|------|
| 1. | 1. |
| 2. | 8. |
| 3. | 27. |
| 4. | 64. |
| 5. | 125. |
| 6. | 216. |
| 7. | 343. |
| 8. | 512. |
| 9. | 729. |

sty.

your

The extraction

your number.

2. And of theim this is the firste: that you shall triple the firste roote. And that triple shall you set vnder the nexte number, toward the righte hande, before that picke, whiche you did laste ende.

3. Then multiplie that triple, by thesame *quotiente*. And set it doune vnder the first triple: and that number shall be called your diuisor.

4. Thirdly, loke out a *quotient*, that maie declare how often the diuisor is in the number ouer it.

In whiche doynge, you must haue this regard, that betwene that picke that is ended, and the nexte that standeth toward the right hande, you must subtrate 2. other numbers. That is to saie, the square of the laste *quotiente*, multiplied by the former triple. 10. tymes: and the *Cube* of thesame *quotiente*.

Scholar. This rule is very obscure in woordes.

Master. Then will I terme it thus.

2. 4. 3. Take the square of your whole *quotiente*, 300. tymes: and that shall be your diuisor. Then seke a newe *quotiente*, declarynge how often that diuisor, maie be founde in the number, that doeth belong to the nexte picke. But so that the square of that newe *quotiente*, multiplied by the last *quotiente*, 30. tymes: and also the *Cube* of that newe *quotiente*, ioyned all in one somme, maie be taken out of thesame number. And if you vnderstande this, there resteth no more difficultie.

Scholar. I trust by exāple, to vnderstand it better

Master. Then take you this exāple. 26463592 whiche I shall set doune and picke, as I taught you before: and as you maie here see. ¶ here the .3. picke declare vnto me, that the roote will haue .3. figures,

And then vnder the picke that is nexte the leste hande, whose number is .26. I finde the greatestte *Cubike* number to bee .8. and his roote .2.

| | |
|----------|---|
| 26463592 | (|
|----------|---|

of Rootes.

For 27. whiche is the nexte *Cube*, is to greate.

Therefore I set. 2. in the *quotiente*, and his *Cube*, be yng. 8. I doe abate out of 26. and so remaineth. 18.

That. 18. I doe sette ouer. 26.

whiche I muste cancell: and then standeth the number, as here you doe see.

$$\begin{array}{r} 18 \\ 26 \ 46 \ 3592 \ 2 \end{array}$$

This is that firste woork, whiche is not repeted.

Then to procede forward, I doe triple the *quotiente* 2, and so haue 3.6. whiche I shall set vnder. 4. beyng the nexte number, on the righte hande of the prick that is ended.

And that triple must I multiplie, by the first *quotiente*, wherby amoüteth that number, that must be the diuisor: and it is in this woork 12. whiche must be set vnder the same triple: as here I haue placed it.

$$\begin{array}{r} 18 \\ 26 \ 46 \ 3592 \ 2 \\ 6 \end{array}$$

Then shall I take for a newe *quotiente*, declaring how often ty-

mes. 12. maie be founde in the number ouer it, that is 184. And I see it maie be in apperaunce. 15. tymes, but more then. 9. you shall neuer take for a *quotiente*: wherefoze it appeareth, that I maie boldly take. 9. whiche I shall sette in the *quotiente* with the firste. 2. And then shall I multiplie. 12. whiche is the diuisor, by. 9. and thereof cometh. 108. to bee sette vnder 184. benethe the line, whiche shall euermore be drawen vnder the diuisor.

$$\begin{array}{r} 12 \\ 26 \ 46 \ 3592 \ 2 \end{array}$$

Now muste I take the square of my laste *quotiente*. 9. (whiche is 81.) and multiplie it by the triple of the former *quotiente* (that is by 6.) and so haue 3.486. to be sette one place more toward the right hande.

$$\begin{array}{r} 2 \\ 18 \ 07 \ 4 \\ 26 \ 46 \ 3592 \ 29 \\ 6 \\ 12 \\ \hline 108 \\ 486 \\ 729 \\ \hline 16 \ 389 \end{array}$$

Last

The extraction

Laſte of all, I ſhall multiplie the laſte *quotiente* *Cu-
bikely*: and that maketh. 729. whiche muſt be ſet, yet
one place more toward the right hand, that is to ſaie,
vnder the nexte prick. And then ſhall I adde thoſe 3
ſommes into one: wherby will riſe. 16389. to be ſub-
tracted out of. 18463. and ſo will remaine ouer that
pricke. 2074.

And the woork of that prick is doen.

This order of woork, if you marke well, you haue
learned the whole arte of extraction of *Cubike rootes*.

For how greate ſo euer your number be: you ſhall
not haue any newe kinde of woork.

But yet becauſe I did teache you befoze, the ſame
woork in other woordes, I will woork the ſame ex-
ample again, accoꝝdyng to theſe woordes.

And firſt, after that the number is ſet doune, and
the firſt *Cubike roote* taken, and the *Cube* abated. Then
take the ſquare of that roote. 300. tymes, that is in
this example. 4. tymes. 300, whiche maketh. 1200.
and that ſhall be your diuiſor. This number, and all
other in this woork, ſhall you ſet doune ſo, that the
firſt number, ſhall be vnder the nexte prick, toward
the righte hande.

Then ſeke your *quotiente*, with the former cautele,
and it will be. 9. Wherefoze
multipliyng. 1200. by. 9. there
will amounte 10800. to be ſet
vnder the line.

After this, I ſhall take the
ſquare of. 9. (whiche is the new
quotiente) and multiplie it by. 2.
(whiche was the laſte *quotiente*
befoze) 30. tymes. So muſt I
multiplie 81. by. 60. and it will make. 4860. whiche
I place orderly.

| | |
|----------|-----|
| 18 | |
| 26463592 | (29 |
| 1200 | |
| | |
| 10800 | |
| 4860 | |
| 729 | |
| | |
| 16389. | |

Then ſet I doune the *Cube* of the *quotiente*, whiche
maketh

of Rootes.

maketh. 729. And so are the .3. numbers placed, and agree with the former worke, in all things, saue in 2. pointes. For here the triple of the *quotiente*, is not set doune, but kepte in memorie. And again, here are diuerse cyphers, whiche are not in the former worke.

Scholar. Sir, I perceiue, that the Cyphers doo nothing els, but set the numbers in their due places. And the triple of the *quotiente*, is supplied in worke by 2. multiplications. First by. 300. and then by. 30. So that it is all one in effecte.

And by the one worke, I vnderstande the other the better: when I compare them bothe together. But yet I praie you, ende the worke that you began.

Master. To continue that worke, firste I must set doune the numbers, as thei should remaine, after 16389. is abated out of. 18463. and then will thei stande thus.

When shall I repeat the former worke, by setting doune the triple of all the *quotiente*, whiche will be. 87. and that must be placed vnder. 45.

Perce that I shall multiplie that. 87. by . 29. and there will come. 2523. whiche must be the diuisor.

Wherefore I seke for a new *quotiente*, that maie shewe me how often. 2523. is contained in. 20745. And it will be. 8. That 8 doe I set in the *quotient* and by it I multiplie. 2523. and it giueth. 20184 whiche I sette doune, as here you see.

When doe I multiplie that *quotient* squarely, and that will be 64. Whiche I shall multiplie by the triple, that is 87, and there will amounte. 5568. to be set one place more toward the righte hande.

| | |
|--------------|----------|
| 2074 | |
| 28463592 (29 | |
| | 2074 |
| | 87 |
| | 2523 |
| | 20184 |
| | 5568 |
| | 512 |
| | 2074592. |

The extraction

Last of all, I must take the *Cube* of. 8. that is. 512, and it shall bee sette yet one place more towarde the righte hande.

And then by additiō, I shall byyng the all into one number: and it will bee. 2074592. whiche is equall wth the whole number aboue, that is vncancelled. And therfoze if I abate the one out of the other, there will remain nothyng.

Wherefoze I see, that the firste number, is a iuste *Cubike* number. And his roote is. 298.

Scholar. I haue marked you so well, that I trust to doe the like, without erreure.

But I praye you woozke this laste parte also, by your seconde rule, as you did woozke the other: that I maie see the due agremente of thein bothe: and also perceiue the righte vse of this woozke, the better by that other foyme.

The seconde
woorke.

Master. I must in that case sette doune the numbers, as thei were set in the other woozke. And then I shall multiplie al the *quotient*, whiche is. 29. by it self squarely, and it will make. 841. whiche must be multiplied by. 300. And so there amounteth. 252300. to be sette doune, as here you see.

$$\begin{array}{r} 2074 \\ 28463592 \end{array} (29.$$

Then I shall seke out a *quotiente*, declarynge how often 252300. maie bee founde in 2074592. And that *quotiente* will bee. 8: whiche I set in the *quotient* roome, with the other numbers.

$$\begin{array}{r} 2074 \\ 28463592 \end{array} (298.$$

$$\begin{array}{r} 252300 \\ \hline 2018400 \\ 55680 \\ 512 \\ \hline 2074592 \end{array}$$

And then I dooe multiplie the diuisor by the *quotiente*, and thereof riseth 2018400 whiche I set vnder a line, as you maie see.

Perce that, I doe multiplie the newe *quotient*, by it self,

of Rootes.

self squarely, whereof commeth. 64. and that square of the last *quotient*, I shall multiply by. 870. whiche is. 10. times the triple of the former *quotiente*. 29: and thereof commeth. 55680. whiche I set doune also orderly.

| | |
|-------|-------|
| 8 | 8 |
| 64 | 870 |
| 4480 | 512 |
| 55680 | 55680 |

Lasste of all, I multiply. 8. (that is the lasste *quotiente*) Cubikely, and it maketh. 512. whiche also I set doune in cōueniēt order.

And then shall I adde them all together. And so haue I thesame somme, that I had befoze in the other former woork, and it is. 2074592.

Scholar. I neede no more instruction for this: I thinke my self so cunnynge, by occasion of your examptes, whiche you haue wroughte so in double forme.

Master. That maie you proue, by this number 47832147.

Scholar. Firste I shall pricke it, as you taughte me, omittyng still. 2. numbers. *An other example.*

And then out of the number ouer the lasste pricke, I shall seke out the *Cubike* roote, and abate the *Cube* thereof, out of thesame number, and set the remainder ouer it, cancellynge the reste.

And so in this number, I finde in. 47. the greateste *Cube* to bee. 27. and the roote of it 3. Wherfoze I abate. 27. out of. 47. and finde the reste to be. 20. therfoze I cancell. 47. and set. 20. ouer it. And the. 3. whiche is the roote, I set in the *quotient*. And so is the first woork ended.

Then doe I triple that *quotiente*, and it maketh. 9. whiche I set doune vnder. 8.

Again I multiply that. 9. by. 3. and it yeldeth. 27. whiche I set vnder the triple, and take it for my diuisor.

Wherfoze I shall now seke a *quotiente*, that maie

The extraction

declare how often. 27. is in. 208
 and I see, it will bee. 7. tymes.
 Therfore I sette doune. 7. in the
quotiente: and by it I multiplie 27
 and it maketh. 189. whiche I set
 vnder the line: and then I dooe
 multiplie. 7. by it self, whiche
 maketh. 49. & that square doe I
 multiplie by the triple of the foze
mer quotiente, that is, by. 9. and it yeldeth. 441. whi-
 che I set one place moze toward the righte hande.

$$\begin{array}{r}
 20 \\
 47 \overline{) 832147} (37. \\
 \underline{9} \\
 27 \\
 \hline
 189 \\
 \underline{441} \\
 343 \\
 \hline
 23653
 \end{array}$$

Last of all, I take the *Cube* of. 7. whiche is. 343. and
 that doe I sette doune, yet one place moze toward the
 righte hande.

These. 3. *somme*s beyng added together, doe make
 23653.

Master. That will be hardely abated out of a les-
 ser *somme*.

Scholar. I see now my errour. I must take a lesse
quotient: whiche thyng I might haue perceiued by the
 seconde number. For thei twoo wer to greate, befoze
 the thirde was added.

So that I should haue taken but. 6. for the *quotiente*
 And then would the firste number haue been but 162
 and the seconde. 324. and the
 thirde. 216. but that their pla-
 cyng would make them to be of
 other values, saue the last of the.

$$\begin{array}{r}
 1 \\
 2 \overline{) 176} \\
 47 \overline{) 832147} (36. \\
 \underline{9} \\
 27 \\
 \hline
 162 \\
 \underline{324} \\
 216 \\
 \hline
 19656
 \end{array}$$

Therfore, I set euery one in
 his due roome: and adde them
 together, and there amounteth
 19656. to bee subtracted out of
 20832. and the remainer will
 be 1176. And thus is that picke
 with his woozke ended.

Then for the nexte picke, I repeat the same very
 foyme

of *R*ootes.

forme of worke again. First setting downe the triple of the whole *quotiente*, whiche is. 108. so that it shall stande vnder. 11761.02 vnder. 761. accomptyng figure for figure.

That triple must I multiplie againe by the whole *quotiente*. 36. and it will make. 3888. whiche number I muste take for my diuisor.

Wherefore I seke how many times, I maye finde that diuisor in. 11761. and I see, it will bee. 3. tymes. Wherefore I set. 3. as my *quotiente*, in his due place: and by that *quotient* I do multiplie. 3888. and so haue I for my firste number. 11664.

| | | | | |
|---|---|---|---------|------|
| 1 | 1 | 7 | 6 | |
| A | 7 | 3 | 2 | 147 |
| | | | | (363 |
| | | | 108 | |
| | | | 3888 | |
| | | | 11664 | |
| | | | 972 | |
| | | | 27 | |
| | | | 1176147 | |

Againe I doe multiplie the laste *quotiente*. 3. squarely, and so haue I. 9. whiche I shall multiplie by the triple of the former *quotient*, and it yeldeth. 972. that shall be set more nigher the right hande, by one place.

Thirdly, I take the *Cube* of. 3. whiche is. 27. and that doe I set yet one place more towarde the righte hande.

Then doe I adde those 3 sommes into one, and they make. 1176147. whiche is equalle somme, with all the numbers ouer it, that be vncancelled.

Wherefore I saie that. 47832147. is a *Cubike number*, and the *Cubike* roote of it is. 363.

Master. Now doeth the order of teachynge require, that I should instructe you, how to extracte the *root* in any nigheste *Cube* roote, out of any number, that is not a *ber* not *Cu* true *Cube*. As this number for example maye serue. *bike*.
694582951.

Where firste I muste extracte the nigheste roote, as I taughte you, for the nigheste *Square* rootes, in any numbers that are not square: and then shall I note the re-

D. ij. manner:

The extraction

mainer: whiche I shall set for the numerator. And his denominator shall be founde, as I will tell you anon. But firste doe you worke the example, to his nigheste roote in whole numbers.

Scholar. I set it doune, and picke it, and finde the greateste Cube ouer the laste picke to bee 512. and the roote of it is. 8.

| | | |
|---|---|---|
| 1 | 8 | 2 |
| 8 | 9 | 4 |
| 5 | 8 | 2 |
| 9 | 5 | 1 |
| 8 | 1 | 2 |

Wherfore I set doune. 8. in the *quotiente*. And I abate. 512. out of. 694. and so resteth 182. and the former. 694. cancelled.

Then to procede, I must triple that roote. 8. and it maketh. 24. whiche. 24. I set vnder. 1825. And then I doe multiplie that again, by the *quotiente* or roote. 8. and it maketh 192. to be set vnder the said triple. 24: as the diuisor. For whiche I seke a new *quotient*, and it will be 8. That. 8. I set in the *quotiente* place, and by it I multiplie the diuisor. 192. and there riseth. 1536. to be set vnder the line, in conueniente order.

Nexte I multiplie the *quotiente* squarely: whiche yeldeth. 46. and that square I multiplie again by the triple, and so haue I. 1536. also. But this must stand more forwardly by one place.

Last of all I take the Cube of the *quotient*. 8. and that is. 512. whiche I set vnder the other two sommes, and that by one place more forwardly.

Now gatherng all these. 3. somes into one, they will make 169472 whiche I shall abate out of. 182582. and so remaineth there. 13110. And that picke with his worke ended.

Wherfore hauing one other space to worke, I must repeat the same order of worke again. by tripling the whole

| | | | | | | | | |
|---|---|---|---|---|---|---|---|------|
| 1 | 3 | | | | | | | |
| 1 | 8 | 2 | 1 | 1 | 0 | | | |
| 8 | 9 | 4 | 8 | 2 | 9 | 5 | 1 | (88. |
| 2 | 4 | | | | | | | |
| 1 | 9 | 2 | | | | | | |
| 1 | 5 | 3 | 6 | | | | | |
| 1 | 5 | 3 | 6 | | | | | |
| | 5 | 1 | 2 | | | | | |
| 1 | 6 | 9 | 4 | 7 | 2 | | | |

quotiente

of Rootes.

quotiente. 88. and that will bee. 264. And againe I must multiplie that triplede number, by thesaied quoz-
tiente, and it will make . 23232. whicbe shall bee the
deuifoz.

Wherefore I seke a newe
quotiente, whicbe is easly per
ceued to be. 5. That. 5. doe I
set in the quotiente, and by it
I dooe multiplie the deuifoz
23232. and there amounteth
116160. as the firste nom-
ber, to bee set vnder the line.

Againe I shall multiplie
the quotient squarely, whicbe
giueth. 25. and that square

shall I multiplie by the triple. 264. and so will there
rise. 6600. to bee sette, as the seconde number vn-
der the line: and one place more forwardly, towarde
the righte hande.

Last of all, I shall sette vnder them bothe, and one
place more towarde the righte hande, the Cube of. 5.
whicbe is. 125.

And then shall I adde all those. 3. sommes together
of whicbe cometh. 11682125. to bee abated out of
13110951. and so the remainer will bee. 1428826.
Wherby I see, that the firste number that was pro-
poned, I meane 694582951 is no Cubike number, but
the greateste Cube in it is. 693154125. and his roote
is. 885.

And so, I see, all other numbers of like kinde must
bee wroughte.

But now for the remainer, how shall I dooe to
brynge it vnto a fraction, that maie aptly expresse the
reiheste roote in that sorte:

Answer. There bee as many waies, as there bee
writers almoste, for euery manne deuifeth, how to
bryng

| | |
|--|-----------|
| | 1 4 2 8 |
| x 3 x x 8 2 6 | |
| 6 9 4 5 8 2 9 5 1 | (885.) |
| | 2 6 4 |
| | 2 3 2 3 2 |
| <hr style="border: none; border-top: 1px solid black;"/> | |
| 1 1 6 1 6 0 | |
| | 6 6 0 0 |
| | 1 2 5 |
| <hr style="border: none; border-top: 1px solid black;"/> | |
| 1 1 6 8 2 1 2 5 | |

The extraction

Cardane,

bryng it mosse nigheste to a true roote, if any soche were: whereof *Cardane* his rule is this.

Multiplie the roote squarely, and againe by 3, and that number shall be the diuisor vnto the remainer,

¶ Here he might haue vsed moze plainesse in wordes, if he had saied: and that number shall be the denominator, to the remainer. ¶ Herefoze as here your roote is. 885 so is the square of it 783225 and the triple of that is. 2349675. So would that fraction bee

$\frac{1428826}{2349675}$

But how nigh this doeth go to the truth, I leaue it till an other tyme.

Scheubell.

Scheubelius doeth allege an other reason, and inferreth an other order, diuerse frō this, and soche as impugneth this, sayng:

Triple the roote, and the square of it also, and adde bothe those numbers together, and .1. more: And so haue you a denominator for your numerator.

The numerator euermoze is vnderstād to be the remainer. By whiche meanes the fractiō in this worke would bee $\frac{1428826}{2349675}$: whiche is a lesser fractiō by a good deale, then is the former fractiō, after *Cardanes* forme.

But bicause at this presente, I maie not spende so moche time, to scan their seueralle opinions, wherein eche of them, pleaseth hymself well: the one alleging demonstration (whiche scarcely serueth) and the other namynge it a secrete, as it is worthie to bee: I will procede to a thirde waie, moze certain then ether of these bothe. And that is by addition of certain Cyphers, to the remainer, in soche sorte, that thei muste all waies bee ternaries, as. 3. 6. 9. 12. &c. And then searche

of Rootes.

searche foꝝward with the like oꝝder of woꝝke, as you
used befoꝝe.

In this maner of pꝛactise, looke how many pꝛickes
yeur cyphers hath (oꝝ els how many ternaries of Cy-
phers, there bee set to your number) so many figures
shall the numerator of your fraction contain. And the
denominator shall euer moꝝe, contain 1. moꝝe. The last
of the laste onely shall bee an vntie, and all the other
shall bee Cyphers.

That is to saie, that if I adde but 3. Cyphers to the
number, the fraction shall contain certain. 10. partes
And if I adde. 6. Cyphers, it shall crycelle. 100. par-
tes. So. 9. Cyphers maketh the denominator to bee
1000. partes: And 12. Cyphers geueth 10000 partes.

For example. I will adde to our laste number that
remained. 12. Cyphers. And then will the number be
1428826.000.000.000.000. vnto whiche I set
no moꝝe pꝛickes, then serueth foꝝ the cyphers, because
I haue passed all the other pꝛickes, in my former
woꝝke.

And now to continue my woꝝke, I shall triple all
the former *quotiente*, and it will be 2655. whiche num-
ber I shall place, as $1428826 \overline{0000000000000000}$
here you see it set. And $\begin{array}{r} 2655 \\ 2349675 \end{array}$
then shall I multiplie that triple, by the foꝝ-
mer *quotiente*. 885. whiche will yelde. 2349675. to be
set vnder the saied triple: as I haue sette it here also.
And this number shall be the diuisoꝝ.

Then shall I seeke foꝝ a *quotiente*, whiche can bee
none other then. 6: wherefoꝝe I sette. 6. in a *quotiente*
line, and by that. 6. I dooc multiplie the saied diuisoꝝ
2349675. and it giueth. 14098050. to be the firste
number vnder the line.

After that, I take the square of the saied *quotiente*,
whiche is. 36. and by it I multiplie the triple. 2655:

P. J. wherby

The extraction

| | | | | | | | | | | | | | |
|---|---|----------|----------------------------|------|---------|--|--|----------|-------|-----|--|--|------------|
| wherby is made 95580. to be the seconde number vnder the line: & set, as it ought, one place moze toward the righte hande. | <table style="border-collapse: collapse; width: 100%;"> <tr><td style="text-align: right;">18064984</td></tr> <tr><td style="text-align: right;">14288260000000000000(8856.</td></tr> <tr><td style="text-align: right;">2655</td></tr> <tr><td style="text-align: right;">2349675</td></tr> <tr><td colspan="2" style="border-top: 1px solid black;"></td></tr> <tr><td style="text-align: right;">14098050</td></tr> <tr><td style="text-align: right;">95580</td></tr> <tr><td style="text-align: right;">216</td></tr> <tr><td colspan="2" style="border-top: 1px solid black;"></td></tr> <tr><td style="text-align: right;">1410761016</td></tr> </table> | 18064984 | 14288260000000000000(8856. | 2655 | 2349675 | | | 14098050 | 95580 | 216 | | | 1410761016 |
| 18064984 | | | | | | | | | | | | | |
| 14288260000000000000(8856. | | | | | | | | | | | | | |
| 2655 | | | | | | | | | | | | | |
| 2349675 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 14098050 | | | | | | | | | | | | | |
| 95580 | | | | | | | | | | | | | |
| 216 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
| 1410761016 | | | | | | | | | | | | | |

Last of all, for the thirde number I take the *Cube* of the sated *quotiente* whiche is. 216. and place it as you see, with his firste figure vnder the pycke.

Then doe I adde those. 3. numbers into one, whiche maketh. 1410761016. And that beyng subtracted out of 1428826000. doeth leaue 18064984. And so is the woork of the firste pycke ended.

Wherby it appeareth, that the fraction is somewhat moze then $\frac{6}{10}$ or $\frac{3}{5}$: as it shall bee tried better, by the woorkes that shall ensue.

Therefore I procede to the nexte pycke. And firste I triple that whole *quotiente*, whiche yeldeth. 26568. to bee set, as it is often befoze repeated, and therefore nedeth not hereafter to bee tediously rehearsed.

That triple shall I multiple again, by the whole *quotiente* (as here I haue sette it in woork, bicause the number is

| | | | | |
|--------------------------------|--|----------------------------|-------|-----------|
| 18064984 26568 235286208 | <table style="border-collapse: collapse; width: 100%;"> <tr><td style="text-align: right;">18064984000000000000(88560</td></tr> <tr><td style="text-align: right;">26568</td></tr> <tr><td style="text-align: right;">235286208</td></tr> </table> | 18064984000000000000(88560 | 26568 | 235286208 |
| 18064984000000000000(88560 | | | | |
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| 26568 | |
| 8856 | |
| | |
| 159408 | |
| 132840 | |
| 212544 | |
| 212544 | |
| | |
| 235286208 | |

greate, and not easily wrought by memorie) and it doe I set in his due place, as you see.

But then seeing that diuisor is greater then all the number ouer it, I shall set a Cypher in the *quotiente*: in token that the diuisor, can not be abated ones out of the number ouer it.

of Rootes.

it. And so is the woork of that ptecke ended, without any moze trauell.

Wherfore to go forward, I triple all that *quotiente* and set it dounc, as the rule would, & as here is seen.

| | |
|-----------------------------|----------|
| 1594819256457 | |
| 1808498400000000 | (885607. |
| 265680 | |
| 23528620800 | |
| 164700345600 | |
| 13018320 | |
| | 343 |
| 16470164743543 | |

Then dooe I multiplie that triple, by the whole *quotiente*, wherof cometh. 23528620800. and that shall bee the diuisor. And the *quotiente* for it will be. 7.

So then if I multiplie that diuisor by. 7. there will amounte. 164700345600. for the first number to be set vnder the line.

And for the next woork, I shall multiplie. 49. (whiche is the square of the newe *quotiente*) with the triple of the former *quotiente*, and it will bring forth. 13018320. whiche shall bee the seconde number, to be set vnder the line.

The thirde number shall bee the Cube of. 7. whiche is. 343.

And those . 3. sommes added together, will make 16470164743543. whiche is to be abated out of 18064984000000. and then shall there remain 1594819256457. And so haue I ended. 3. pteckes of the Cyphers. And thereby make saie, that the fraction is $\frac{667}{1350}$ and somewhat moze: That is somewhat moze then $\frac{1}{2}$.

Scholar. I see by the fraction, that it is $\frac{1}{2}$ and $\frac{7}{1350}$,
p. y. beside

The extraction

beside the quantitie of the remainer. But I praye you cande the woork of that other prycke, whiche dooeth remaine.

Wasser. I muste triple all the *quotiente*: whereby will rise. 2656821. whiche muste bee multiplied by the said *quotiente*: and thereof will procede the diuisor, beyng 2352899275347. And his *quotiente* will bee. 6.

Wherefore firste I set. 6. in *quotiente* line, with the other numbers: and then doe I multiplye the diuisor by that *quotiente*, and it byngeth forth the firste number to be sette vnder the line.

$$\begin{array}{r}
 2656821 \\
 885607 \\
 \hline
 18597747 \\
 159409260 \\
 13284105 \\
 21254568 \\
 21254568 \\
 \hline
 2352899275347
 \end{array}$$

$$\begin{array}{r}
 183078734793024 \\
 \cancel{1894819286457000} \quad (8856076. \\
 \quad 2656821 \\
 2352899275347 \\
 \hline
 14117395652082 \\
 \quad 95645556 \\
 \quad \quad 216 \\
 \hline
 1411740521663976
 \end{array}$$

And again the square of 6. beyng multiplied by the triple, will yelde. 95645556: whiche shall bee the seconde number vnder the line.

The thirde number shall be. 216. because it is the *Cube* of. 6. And those 3. numbers beeyng added together, doe make. 1411740521663976. to be abated out of. 1594819256457000. And so doeth there remaine. 183078734793024.

Wherefore

$$\begin{array}{r}
 2656821 \\
 \quad 36 \\
 \hline
 15940926 \\
 7970463 \\
 \hline
 95645556
 \end{array}$$

of Rootes.

Therefore it dooth appeare, that beside the first 3
 numbers of the roote, that is. 885. the rest (that is
 6076.) standeth for the numerator of a fraction, and
 the denominator vnto it is. 10000.

So that the higheste roote is. $885\frac{6076}{10000}$. beside the
 fraction that doeth remaine: whiche would make but
 $\frac{1}{10}$ of $\frac{1}{1000}$.

Scholar. This is a sufficiente precisenes. And so
 I iudge it sufficiently taughte.

Therefore I praye you propounde some questions,
 that doe require this arte, for their solution.

Master. I am contente. And let this be the firste.

The Grecians giuen to idle banketting, and soche
 like wantonnesse, did procure thereby soche mortalle *A question*
 sicknesses: that the quicke were scarce hable to burie *of doubling*
 the dedde. Therefore consultancye with their God-
 des, for redresse thereof, they receiued answer, that

when they would double the Altare, whiche was of
 Cubike forme, they should bee deliuered from that pla-
 gue. Meanyng that learning is a due meane, to de-
 liuer realmes from plagues and enomyties. But to
 the question, what saie you? If the side of a Cube be. 2.
 foote (as that altare might bee) how many foote shall
 the side be of that Cube, whiche must be double vnto it.

Scholar. This I consider. That firste I must finde
 the quantitie of the Cube, that is proponed. And then
 shall I double that quantitie. Thirdly, I must ex-
 tracte the Cubike roote, of that double number.

So in this question, the side of the knowen Cube is
 3. and therefore the whole Cube is. 27. whose double is
 54. And the Cubike roote is. 3. and $\frac{27}{27}$ by Cardanes rule:
 That is. 4. whiche is plainly false, for. 4. is the roote
 of. 64. and not of. 54. But by Scheubelius rule, it wil
 be. $3\frac{27}{27}$ that is. $3\frac{1}{3}$ almoste: whiche is moche nigher the
 truth. For. $3\frac{1}{3}$ multiplied Cubikely, doeth make. 52.
 $\frac{27}{27}$. whiche is to litle by a good deale, that is by. $1\frac{1}{3}$.

W. Iij. Whereas

The extraction

whereas $3\frac{27}{27}$ doeth make a lesser somme: that is to say but $51\frac{25929}{15551}$ and so wanteth. $2\frac{4694}{15551}$. And although bothe these sommes goe nigher to the truth, then *Cardanes* rule, whiche misleth. 10. Wholy: Yet maie it be easily seen, that *Scheubelius* rule is not so good, as he would it were. And the worse here, for the aduynge of that one more.

Master. You are lepte verie sodenly from a scholar, to a cōptroller. And yet I can not but praise your diligente obseruyng of soche thynges.

Woue now by the *Cyphers*, how it will frame.

Scholar. I sette doune the number with .6. *Cyphers*, and picke them thus.

Then dooe I take the greatestte
Cubike number in. 54. whiche is. 27
 and that I doe abate from 54. and
 so resteth. 27. the roote of the *Cube* is. 3. whiche I sette
 in the *quotiente* line.

And then I triple. 3. whiche maketh. 9. that muste
 be multiplied by the *quotiente* againe, and so commeth
 27. to be the diuisor. And his *quotiente* semeth to be. 9.

Wherfoze woorkyng with it,
 the firste number is. 243. and the
 seconde is. 729. that is. 81. mul-
 tiplied by 9. whiche is the triple.

Againe, the *Cube* of. 9. is. 729.
 And all thei together, dooe make
 32319 whiche seme is to greate,
 and therfoze I must take a lesser
quotiente. As I mighte haue per-

| |
|--------------|
| 27 |
| 84000000(38. |
| 9 |
| 27 |
| 216 |
| 576 |

ceiued well inough by the second
 nōber, if I had marked it in time.

But now amendyng my ouer
 sighte, I take. 8. for the *quotiente*.
 And woorkyng with it I see, the
 firste number vnder the line, will

be

of Rootes.

bec. 216. and the seconde. 576. And here all ready I espie my ouersight again.

Wherfore I take .7. to be the *quotiente*. And by it I multiplie the diuisor, and so haue 3.189. for the firste number.

And for the seconde number, I doe worke with .49. whiche is the square of the *quotiente*, multiplied by .9. that is the triple: and it yieldeth. 441.

Thirdly, I take the *Cube* of .7. whiche is. 343. And then addyunge al. 3. numbers together, I finde the somme to bee. 23653. whiche is to bee abated out of 27000. and so resteth 3347. Wherby I see, that .37. with somewhat more is the roote that I should finde.

But for farther triall, I triple all the *quotiente*, and finde thereby. 111. whiche I multiplie by the same *quotiente* again, and so commeth 4107. to bee the diuisor. And his *quotiente* will bee .8. as it semeth: and so the first number will bee. 32856. And the seconde shall bee. 7104. but those .2. are to greate, as it is manifeste all readie.

Wherfore I take 7 for the *quotiente*. And by it multiplying the diuisor, there riseth 28749.

And for the seconde somme, there is founde. 5439.

And for the thirde some. 343.

All whiche. 3. sommes ioined in one, dooe make. 2929633. And that beeyng abated out of the higher somme. 3347000. dooth leaue. 417367.

| | |
|---------|-------|
| 3 | |
| 27347 | |
| 84000 | (37. |
| 9 | |
| 27 | |
| 189 | |
| 441 | |
| 343 | |
| 23653 | |
| 3347000 | (378. |
| 111 | |
| 4107 | |
| 32856 | |
| 7104 | |
| 417367 | |
| 3347000 | (377. |
| 111 | |
| 4107 | |
| 28749 | |
| 5439 | |
| 343 | |
| 2929633 | |

Wherfore

The extraction

Wherefore I maie boldly saie, that the fraction is $\frac{27}{100}$ and more, by the portion of the remainer, whiche is nigh $\frac{1}{700}$.

And it is sone seen that $\frac{27}{100}$ are equalle to $\frac{3}{7}$: wherefore $\frac{27}{100}$ shall be more then $\frac{3}{7}$.

And so dooeth *Scheubelius* rule erre more, then I thought befoze.

So is your question answered, that the side of the double *Cube*, shall be, 3. foote and $\frac{27}{100}$ and $\frac{1}{7}$ of $\frac{1}{100}$.

*Of the rootes
of fractions.*

Master. For the rootes of fractions, I shall neede to saie no more but this: that if the numerator and denominator bothe be *Squares*, or *Cubes*, &c. then maie you finde in that fractiō the like roote. But if any of bothe doe swarue from that name, then hath that fraction no soche roote.

As $\frac{16}{9}$ is nother *Cubike* nor *Square*, bicause his partes doe not agree in *Square* name, nor in *Cubike* name: although the numerator bee a *Square*, and the denominator a *Cube*.

Scholar. That doeth appeare reasonable, at the firste sighte.

Master. Then seeyng you are so readie in learning: aunswere me to this question.

*A question of
a Gonne.*

A *Gonne* of sixe inches diameter in the mouth, doeth shotte a bollet of twentieth pound weight: what weighte shall that bollette haue, that setueth soz a gonne of 14. inches in the mouth?

But to helpe you in this question, and in all soche like, you shall marke well *Euclide* his sayng, in the 18 proposition of his. 12. booke, whiche is this.

All Globes bere together triple that proportion, that their diameters doe

So in this example, the proportion of the diameters beyng as. 14. to. 6. Or as. 7. to. 3. I shall triple it, and then haue I the proportion of their Globes.

Wherefore

of Rootes.

Wherefoze I sette the 3. fractions thus. $\frac{7}{7} \frac{7}{7} \frac{7}{7}$ and thei make $\frac{343}{343}$. that is. 1 2. $\frac{19}{37}$. And so is the ppozition of the Globes, as well in weighte, as in bignesse.

Wherefoze I must multiplie. 20. that is the weight of the lesser bollette, by the numeratoz of the ppozition, and diuide it by the denominatoz.

And so shall I haue. 254 $\frac{1}{37}$ foz the weighte of the greater bollete.

Now pzooue you the like
 woꝝke. Remembꝝyng that
 Cubes also, as well as Glo-
 bes, doe beare triple ppoz-
 ition, in comparison of their
 sides. As you learned befoze by the. 19. ppozition,
 of the. 8. booke of *Euclide*.

| | |
|------|----------------------------|
| 343 | 113 |
| 20 | 2412 |
| 6860 | 6860 (254 $\frac{1}{37}$) |
| | 2777 |
| | 22 |

A Cube of *Wrasle* of. 4. inches square, doeth weighe 7. pounce weighte, what shall a Cube of *Wrasle* of. 9. inches square, wale? *A question of. 2. Cubes.*

Scholar. The ppozition of the sides is as $\frac{9}{4}$ whiche I must set doune thise, and multiplie them together, as fractions should bee. And so will it bee thus. $\frac{9}{4} \frac{9}{4} \frac{9}{4}$. that maketh. $\frac{729}{64}$.

Wherefoze I multiplie the weighte of the lesser Cube, beyng. 7. by. 729. and it maketh. 5103. and that doe I diuide by. 64. and so finde I. 79. $\frac{27}{64}$, whereby I maie knowe, that the weighte of the greater Cube, is 79. pounce weighte, and very nigh $\frac{1}{4}$.

Master. These. 2. questions dooc teache you, rather the ppozition of Cubes, then the vse of the rule: wherfoze to make the questiōs moze agreable to this rule, I ppozounde them thus, in backer order.

A bollette of yron of. 7. inches diameter, doeth wale 27. pounce weighte: what shall be the diameter to that bollette that shall wale. 125. pounce weighte?

Scholar. I praie you aunswer to it your self, that I maie see the apte forme of applyng soche questions

The extraction

to this rule.

Master. As the *Cubes* are in triple proportion to the sides, so are the proportions of the sides, to be founde by triple division: that is to saie, by seeking the *Cubike rootes*, of the 2. termes of the proportion.

¶ Herefore I doe firste set doune the termes of the proportion of the bollettes, thus: $\frac{125}{27}$. And I see, that the *Cubike roote* of. 125. is. 5. and the like roote of. 27. is. 3. whiche numbers I shall set in the roome of the 2. others, thus: $\frac{5}{3}$ And thei declare the proportion, betwene the *diameters* of the. 2. bollettes. **¶** Hereof one that is the lesser, is knowen to be. 7. Therefore I multiplie that. 7. by. 5. whereof commeth. 35. and that. 35. doe I diuide by. 3. whiche giueth. $11\frac{2}{3}$.

¶ Herefore I saie, that if. 7. inches be the *diameter* to a bollette of. 27. pounce weighte, then. 11. inches and $\frac{2}{3}$ shall be the *diameter* to the bollete of. 125. pounde weighte.

Scholar. The prooffe of this had neede be certain, seeing the woork is obscure, to the common iudgemente.

The prooffe.

Master. You saie well. And this is the very order of prooffe for it. Multiplie bothe these rootes *Cubikely*. And if their *Cubes* be in soche proportiō as their weightes be (that is to saie in this exaple as $\frac{125}{27}$) then is the woork good: els not.

Scholar. What must needes be so. And therefore will I proue it so in these numbers.

And for that eande, firste I multiplie. 7. *Cubikely*, and it giueth. 343. Then I multiplie. $11\frac{2}{3}$. *Cubikely*, and it maketh $41\frac{4}{27}$. But now seeing the one number is a fraction, I will for ease tourne the other into a fraction of the same denomination: and it will be $\frac{2261}{27}$ in whiche. 2. fractions, the proportion muste consist betwene the numeratours. So that thei bothe beeing diuided by one common number, muste come to this fraction

of Rootes.

fraction $\frac{115}{37}$.

And so I see it will be: for the lesser being divided by .343. will yelde 27. And the greater divided by the same. 343. will giue. 125. So that by triall, that worke is approued good.

Master. I will now proue your cunnynge, in a newe question, whiche Bassiers often tymes, haue occasion to vse: as thus.

I haue a dice of Masse of. 64. vnces of Troye weighte, whose side is. 3. inches and $\frac{1}{2}$ and would haue an other dice of the same mettall of. 18. pounde weighte. *A question of weightes.*

My demaunde is: what shall be the side of the dice?

Scholar. This question must firste bee reduced to one kinde of denomination in the weightes, and then will it be moze apte to be answered.

¶ Therefore I shall tourne. 18. pounde into vnces, multiplying it by. 12. and it will be. 216.

And then I consider the proportion, that is betwene those. 2. numbers of weighte. 64. and. 216. and it is certainly. $3\frac{1}{2}$, or $2\frac{7}{2}$ out of whiche proportion, I must extracte the Cubike roote, as I made easly doo, seying bothe the numerator and the denominator, are Cubike numbers.

And so is their roote $\frac{1}{2}$: whiche is the proportion of the sides of the two dice.

And seying the side of the lesser die, is knowen to be 3. inches and $\frac{1}{2}$, the other his side must be in *Sesquialter* proportion to it, that is. $5\frac{1}{4}$: whiche is wroughte also thus. I multiplie. $3\frac{1}{2}$ by. 3. and it maketh. $10\frac{1}{2}$ whiche I shall diuide by. 2. and there commeth. $5\frac{1}{4}$.

Master. Yet one question moze I will prounde to giue you occasion, to vnderstande the apte conference of masses, of diuerse stufte.

And so for that purpose, I suppose this proportion in weighte, to bee betwene masses of one biggenesse.

Q. y.

What

The extraction

Examples of
rates for
weightes.

That if I compare
Woodde and Stone of one
quantitie together, the
stone shall weighe moze
then the woodde by $\frac{2}{3}$.

Like wates yron to be
heurier then stone by $\frac{1}{2}$.

And Masse to bee he-
uier then yron by $\frac{1}{3}$.

Ledde to be heurier then Masse by $\frac{2}{3}$.

All whiche rates, although thei be taken for exam-
ples, and not of truthe, yet thereby mate you learne,
how to woozke with true rates, set in a like table.

And now for the vse of this table, take this questiō.

A questiō
of weighte.

I would haue .5. weightes of Cubike forme, made
of these .5. stufes.

The weighte of the woodde shall be. 28. pounce.

The stone. 56. pounce.

The yron. 112. pounce.

The Masse. 224. pounce.

The Ledde. 448. pounce.

Of all these I haue but the yron weighte: whose
side, or Cubike roote is. 12. inches $\frac{1}{2}$.

And my desire is to knowe, of what quantitie the
sides of all the other weightes shall be.

Scholar. The questiō is pleasaunt: and yet some
what harder then the other.

Master. The table will helpe you fully, so that
you cōferre it well, with that you haue learned before

But bicause I haue litle leiser, to spende moche
tyme with you (saue that zeale to your furtheraunce
doeth make me partly to sozgette my owne businesse)
therefoze will I leaue this questiō to your self, to be
answered at your laisure.

And so in all the rest, I must posse it ouer: and giue
an eye to sache maters, that touche me moze nighe:
and

| Stoffe. | Weighte. |
|---------|--|
| Woodde. | 60. 1 |
| Stone. | 100. $\frac{2}{3}$ 1 |
| Yron. | 150. $\frac{2}{3}$ $\frac{2}{3}$ 1 |
| Masse. | 200. $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 |
| Ledde. | 280. $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ 1 |

of Rootes.

and weighe more heully, then all soche weightes, by 20. folde.

¶ herfoze, touchyng all the rootes of compounde numbers, you shall at my hand now, haue no priuate declaration. But soche as you haue learned all reddie.

Of compounde rootes.



If the number bee compounde, other of Square numbers, or of Cubike numbers, then accordyngly as the composition is, so shall you draw the roote: and without one of these two there can bee no composition.

¶ herfoze to begin with the smallest compounde number in that sorte, whiche is a Square of squares, you shall firste extracte the square roote, as you haue learned before. And out of that roote (whiche must nedes bee a Square number) you shall extracte his square roote also: and that roote is the *zenzizenzike* roote, of the firste Square of squares or *zenzizenzike* number.

For example take .14641. whose Square roote is 121. and that same roote is it self, a Square number: and hath for his roote. 11.

¶ herfoze I make saie, that .11. is the Squared square roote, or the *zenzizenzike* roote of 14641.

Again 8503056. is a Square of squares, and therfoze a Square number. And his Square roote is 42916. whiche is a Square number also, and hath .54. for

$$\begin{array}{r} 2 \\ 14641 \end{array} \begin{array}{l} (2916 \\ 224 \end{array}$$

$$\begin{array}{r} 342 \\ 499893 \\ 883856 \end{array} \begin{array}{l} (2916 \\ 48882 \\ 8 \end{array}$$

¶ .ij.

his.

The extraction

his roote.

So that .54. may well be called the *zenzizenzike* roote of .8503056.

And so shall you woork, with all of that name.

Zenzizenzikes But and if the number be compounde, of .3. *zenzizenzikes* .02.3. Squares, as a Square of squared squares, or a *zenzizenzizenzike* (whiche some men for shortnesse, call *zenzizenzizenzike*). Then shall you drawe firste the Square roote, and then the Square roote of that roote, and thirdly the Square roote of that laste roote.

As for example .6561. is a Square of squared squares. And his firste roote is .81. whiche is also a Square number, and hath 9. for his roote. That .9. likewaies is a Square number, and hath .3. for his roote.

$$\begin{array}{r} 1 \\ 6561 \text{ (81)} \\ 16 \end{array}$$

So that the *zenzizenzizenzike* roote of .6561. is .3.

And for these formes of numbers, I shall not neede to state for any more explication, or examples: seeinge the matter is plaine.

Now for compounde Cubike numbers, you shall vnderstande the like forme.

Cubes of cubes.

If the number be a Cube of Cubes, you shall firste extracte the Cubike roote. And because that roote is a Cubike number also, therefore shall you seke the Cubike roote of it. And that seconde roote shall be the Cubicubike roote of the firste number.

As for example .512. is a Cubike number, or a Cube of Cubes. And his Cubike roote is .8. whiche .8. againe is a Cubike number and hath .2. for his roote.

So that .2. is the Cubicubike roote of .512.

Likewaies .10077696. is a Cubicubike number, and his firste Cubike roote is .216. as you may easily perceiue by these woorkes: where I haue sette forthe the order of extraction of his Cubike roote, whiche is .216. And that .216. is a Cubike number, you neede not to doubt,

of Rootes.

$$\begin{array}{r}
 816 \\
 \times 77696 (216. \\
 \quad 63 \\
 \quad 1323 \\
 \hline
 \quad 7938 \\
 \quad \quad 2268 \\
 \quad \quad \quad 216 \\
 \hline
 810696
 \end{array}$$

doubte, for that it is one of
 the, which you
 haue, I dare
 saie, in perfecte
 maner: Si
 cause his roote
 is a digite, and
 that is, 6.

| |
|-----------|
| 2816 |
| 77696 (21 |
| 6 |
| 12 |
| 1261 |
| 1261 |

By this you may iudge of *Cubicubikes* *Cubikely*, or *Cubes of Cubicubes*. that in the
 you shall firste seke their *Cubike* roote: And then the *Cu-*
bike roote of that roote. And thirdly the *Cubike* roote of
 that roote againe. And so haue you the *Cubicubikike*
 roote of that firste number.

The thirde waie of composition is, when *Squares* The thirde
 and *Cubes* be compounde together: as *Zenzicubes*, *Zen-* composition.
zizenzicubes, *Zenzicubicubes*, or soche like, as it happ-
 neth diuersely.

In all these you shall as often abate the *Zenzike*
 roote, as that name is in the composition, and so haue
 waies of the *Cubike* roote.

So that in a *Zenzicubike*, you shall extracte firste the *Zenzicubike*.
Square roote: and out of that *Square* roote, you shall ex-
 tracte the *Cubike* roote.

As 64. is a *Zenzicubike* number, whose *Square* roote is
 8. and that, 8. is a *Cubike* number, and hath, 2. for his
 roote.

So, 531441. is a *Zenzenzicube*: whose firste *Square*
 roote is, 729. whiche number is *Zenzizenz-*
 a *Zenzicube*, & hath for his *Square* *zicube*.
 roote, 27. And that no-
 ber is a *Cube*, and hath
 for his roote, 3. where-
 fore I may iustly saie,

| |
|-------------|
| 144 |
| 430 |
| 531441 (729 |
| 1444 |
| 1 |

that, 3. is the *Zenzizenzicubike* roote of, 531441.

But as I saied before, that I might not staie long
 at

The extraction

at this presente, so the vse of these greate numbers is rare in practise; and therefore I will ouerpasse them, for this tyme.

And yet for your aied in the meane season, I haue here drawn a table, whiche may bee called the table of ease: in whiche you haue greate plentie of these numbers, with their rootes in diuerse kindes.

The table it self is so manifeste, that it needeth no declaration: if you haue not forgotten, what you learned before.

And if you liste to enlarge this table, you may easily doe it, multiplying the numbers still by their rootes, whiche bee set ouer them, in the hedde of the table. And so may you make it to extende infinitely: whiche shall ease you wonderfully, in the extraction of any kinde of rootes. For which at some other time if my leisure serue me better, with quietnesse, I will giue you more specialle rules.

And also I counsell you, well to examine this table, and trust not to my casting. For haste and other troubles, may often times cause erreure in supputation.

The

The frutefull table, whiche maie be called the table of ease.

| | | | | | | | | | | | | | | | | | | | | | | | | |
|----|-----------------------------|----------|------------|------------|------------|---------------------|------------|------------|------------|-------------|-------------|-------------|-------------|------------|------------|------------|------------|-------------|-------------|------------|------------|------------|------------|------------|
| 1 | Rootes. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 2 | Squares. | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | 169 | 196 | 225 | 256 | 289 | 324 | 361 | 400 | 441 | 484 | 529 | 576 |
| 3 | Cubes. | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 | 1331 | 1728 | 2197 | 2744 | 3375 | 4096 | 4913 | 5832 | 6859 | 8000 | 9261 | 10648 | 12167 | 13824 |
| 4 | Squares of Squares. | 16 | 81 | 256 | 625 | 1296 | 2401 | 4096 | 6561 | 10000 | 14641 | 20736 | 28561 | 38416 | 50625 | 65536 | 83521 | 104976 | 130321 | 160000 | 194481 | 234256 | 279841 | 331776 |
| 5 | Surfolides. | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 | 100000 | 161051 | 248832 | 371293 | 537824 | 759375 | 1048556 | 1419857 | 1889568 | 2476099 | 3600000 | 484101 | 5153632 | 6436343 | 7962624 |
| 6 | Squares of Cubes. | 64 | 729 | 4096 | 15625 | 46656 | 117649 | 207144 | 351441 | 1000000 | 1771561 | 2985984 | 4826809 | 7529536 | 11390625 | 16777216 | 2437569 | 34012224 | 47045881 | 64000000 | 85766121 | 113379904 | 148035889 | 191002976 |
| 7 | Seconde Surfolides. | 128 | 2187 | 16384 | 78125 | 279936 | 823543 | 2097152 | 4782969 | 10000000 | 19497171 | 35831808 | 62748517 | 105413504 | 170859375 | 268435456 | 410338073 | 611220032 | 893871739 | 1280000000 | 1801088541 | 2494357888 | 3404825447 | 4586471424 |
| 8 | Squares of squared Squares. | 256 | 6561 | 65536 | 390625 | 1679616 | 5764801 | 16777216 | 43046721 | 100000000 | 214488881 | 429981696 | 815730721 | 1475789056 | 2542820625 | 4294967296 | 695787441 | 11019960576 | 16983563041 | | | | | |
| 9 | Cubes of Cubes. | 512 | 19683 | 261144 | 1953125 | 10077696 | 40353607 | 134217728 | 387420489 | 1000000000 | 2359157691 | 5159780352 | 10604499373 | | | | | | | | | | | |
| 10 | Squares of Surfolides. | 1024 | 59049 | 1048576 | 9765625 | 60466176 | 282475249 | 1073741824 | 3486784401 | | | | | | | | | | | | | | | |
| 11 | C. Surfolides. | 2048 | 177147 | 4194304 | 48828125 | 362777056 | 1977326743 | 8589934592 | | | | | | | | | | | | | | | | |
| 12 | Squares of Zenx cubes. | 4096 | 531441 | 16777216 | 244140625 | 217666236 | | | | | | | | | | | | | | | | | | |
| 13 | D. Surfolides. | 8192 | 1594323 | 67108864 | 1220703125 | 13059974016 | | | | | | | | | | | | | | | | | | |
| 14 | Squares of B surfolides. | 16384 | 4782969 | 268435456 | 6103515625 | | | | | | | | | | | | | | | | | | | |
| 15 | Cubes of Surfolides. | 32768 | 14348907 | 1073741824 | | | | | | | | | | | | | | | | | | | | |
| 16 | Zenzizenzizēzēzikes | 65536 | 43046721 | 4294967296 | | | | | | | | | | | | | | | | | | | | |
| 17 | B surfolides. | 131072 | 129140163 | | | | | | | | | | | | | | | | | | | | | |
| 18 | Squares of Cubicubes. | 262144 | 387420489 | | 1. | Rootes. | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | | |
| 19 | B surfolides. | 524288 | 1162261467 | | 2. | Squares. | 625 | 676 | 729 | 784 | 841 | 900 | 961 | 1024 | 1089 | 1156 | 1225 | 1296 | 1369 | 1444 | 1521 | 1600 | | |
| 20 | Zenzizenzizēzēzikes | 1048576 | 3586784401 | | 3. | Cubes. | 15625 | 17576 | 19683 | 21952 | 24389 | 27000 | 29791 | 32768 | 35937 | 39304 | 42875 | 46656 | 50625 | 54872 | 59319 | 64000 | | |
| 21 | Cubes of B surfolides. | 2097152 | | | 4. | Squares of Squares. | 390625 | 456976 | 531441 | 614656 | 707281 | 810000 | 923521 | 1048576 | 1185921 | 1336336 | 1500625 | 1679016 | 1874101 | 2085136 | 2313441 | 2560000 | | |
| 22 | Squares of C surfolides. | 4194304 | | | 5. | Surfolides | 9765625 | 11881376 | 12448907 | 17210368 | 20511149 | 24300000 | 28229151 | 33554432 | 39235395 | 45435424 | 52522875 | 60466176 | 69343957 | 79235168 | 90224199 | 102400000 | | |
| 23 | B surfolides. | 8388608 | | | 6. | Zenzicubes. | 244140625 | 308915776 | 336120489 | 481890304 | 594823321 | 729000000 | 887503681 | 1073741824 | 1291467969 | 1544804416 | 1838165725 | 2175782336 | 2565720009 | 3010936384 | 3518743761 | 4096000000 | | |
| 24 | Zenzizenzizēzēzicubes | 16777216 | | | 7. | B surfolides. | 6103515625 | 8031810176 | 9075253103 | 13492928512 | 17249876309 | 21870090000 | 27512614111 | | | | | | | | | | | |

of Coslike numbers.
Of numbers denominate.



Thus haue I lightly ouer run the moſte *Numbers* common kindes of numbers *Abſtraete*, *contracte*. And now reſteth the treatiſſe of numbers *Contracte*, or *Denominate*. Of whiche kinde there bee ſome called numbers *denominate vulgarely*: and other bee called numbers *denominate Coſikely*. And a thirde ſorte there is of numbers *radicalle*, whiche commonly bee called numbers *irrationalle*: becauſe many of them are ſoche, as can not bee expreſſed, by common numbers *Abſtraete*, nother by any certain *rationalle* number. Other men call them more aptly *Surde numbers*.

And although many menne would not accounte them, with numbers *denominate*, yet I maie juſtly doe it, for that thei require a reduction to one denomination, if thei haue ſeueralle ſignes of quantities, as you ſhall heare hereafter. And thoſe numbers in euer goe alone, without ſome other ſigne, and name of rooted quantity, annexed to them.

Of the firſt kinde of numbers *denominate*, whiche are *vulgarely denominate*, as. 10. ſhillings. 10. men 20. ſhippes, 100. ſhepe. 1000. yerres, and ſoche like, I will ſpeake nothyng in this treatiſſe. But of the other twoo kindes I will ſomewhat wryte, for youre learning and contentation.

Scholar. Sir, I am moche bounde vnto you: And therefore remit all to your owne diſcretion and good will. Truſtyng ſo to applye my ſtudie, and employe my knowlege, that it ſhall neuer repente you, of your eurteliſſe in this behalfe.

Maſter. When marke well my wordes, and you ſhall perceiue, that I will vſe as moche plainneſſe, as I maie, in teachyng: And therefore will beginne with *Coſike numbers* firſt.

The Arte Of Cossike numbers.



Numbers Cossike, are soche as bee contracte vnto a deno- mination of some Cossike signe as 1. number. 1. roote. 1. square 1. Cube. &c.

But as for cōpendiousnesse in the vse of them, there bee certain figures set for to signi- fic them: so I thinke it good to expresse vnto you those figures, before wee enter any farther, to thintente we maie procede alwaies in cer- tentie, and knowe the thynges that wee intermedle withall: so; thei are the signes of all the arte, that fo- loweth here to be taught.

And although there be many kindes of irrationall numbers, yet those figures that serue in Cossike nōbers, bee the figures also of all irrtrionalle numbers, and therfore being ones well knowen, thei serue in bothe places commodiously.

These therfore be their signes, and significations brie- fly touched: for their nature is partly declared be- fore.

- ¶. Betokeneth number absolute: as if it had no signe.
- ℥. Signifieth the roote of any number.
- ☉. Representeth a square number.
- ∪. Expresseth a Cubike number.
- ☉☉. Is the signe of a square of squares, or Zenzicubike.
- ℥℥. Standeth for a Surfolide.
- ☉℥. Doeth signifie a Zenzicubike, or a square of Cubes.
- ℥☉. Doeth betoken a seconde Surfolide.
- ☉☉☉. Doeth represent a square of squares squared

of Cosike numbers.

- ly, 02 a Zenxixenzixenzike.
- $\text{C} \text{C}$. Signifieth a *Cube of Cubes*.
 $\text{S} \text{S}$. Expresseth a *Square of Surfolides*.
 $\text{C} \text{S}$. Betokeneth a *thirde Surfolide*.
 $\text{S} \text{S} \text{C}$. Representeth a *Square of Squared Cubes*: 02 a *Zenxixenzixcubike*.
 $\text{D} \text{S}$. Standeth for a *fourthe Surfolide*.
 $\text{S} \text{S} \text{S}$. Is the signe of a *square of seconde Surfolides*.
 $\text{C} \text{S}$. Signifieth a *Cube of Surfolides*.
 $\text{S} \text{S} \text{S} \text{S}$. Betokeneth a *Square of Squares*, squaredly squared.
 $\text{E} \text{S}$. Is the *firste Surfolide*.
 $\text{S} \text{C} \text{C}$. Expresseth a *square of Cubike Cubes*.
 $\text{E} \text{S}$. Is the *sixte Surfolide*.
 $\text{S} \text{S} \text{S}$. Doeth represente a *square of squared surfolides*.
 $\text{C} \text{S} \text{S}$. Standeth for a *Cube of seconde Surfolides*.
 $\text{S} \text{C} \text{S}$. Is a *square of thirde Surfolides*.
 $\text{E} \text{S}$. Doeth betoken the *seuenthe Surfolide*.
 $\text{S} \text{S} \text{S} \text{C}$. Signifieth a *square of squares*, of squared Cubes.

And though I maie proceade infinitely in this sorte, yet I thinke it shall be a rare chaunce, that you shall nede this moche: and therfore this maie suffice. Notwithstandynge, I will anon tell you, how you maie continue these numbers, by progression, as farre as you liste.

And farther you shal vnderstande, that many men doe ruer more call square numbers *Zenxikes*, as a short and apter name, other men call those squares the *firste quantities*, and the cubes thei call *seconde quantities*: squares of squares thei call *thirde quantities*, and surfolides *fourthe quantities*. And so namynge them all quantities (excepte numbers and rootes) thei doe adde to them for a difference, an ordinall name of number, as thei doe goe in order successiuelly.

The Arte

As here foloweth in example.

| | | | |
|---|--|---|-------------|
| \mathfrak{z} . \mathfrak{c} . $\mathfrak{z} \mathfrak{d}$. $\mathfrak{z} \mathfrak{d}$. $\mathfrak{z} \mathfrak{c}$. $\mathfrak{z} \mathfrak{d} \mathfrak{d}$. $\mathfrak{c} \mathfrak{c}$. $\mathfrak{z} \mathfrak{z}$. $\mathfrak{z} \mathfrak{z}$. $\mathfrak{z} \mathfrak{d} \mathfrak{c}$. $\mathfrak{z} \mathfrak{z}$. | Firſte. Seconde. Thirde. Fourthe. Fiſte. Sixte Seuenthe. Eighthe. Nincth. Tenth. Eleuenthe Twelfth. | } | Quantities. |
|---|--|---|-------------|

And ſo forth, of as many as maie bee reckened.

But althoughe ſome men accompte this the moze eaſie waie: becauſe the other names be comberouſe, yet thoſe other names befoze, do expreſſe the qualitye of the number,

better then theſe later names doe.

Scholar. I thank you double, ſith you are contente to teach me double names: ſo ſo ſhall I be acquainted with bothe formes, as I ſhall chaunce on them in other mennes bookes.

Therefore now you maie procede to numeration: whiche I thinke it nexte.

Maſter. There be other .2. ſignes in oſten uſe, of whiche the firſte is made thus —|— and betokeneth moze: the other is thus made —|— and betokeneth leſſe.

And where theſe come in any number *Coſike*, or other, that number is called a compounde number, becauſe it conſiſteth of .2. numbers. And where neither of theſe is, the number is called vncompounde, althoughe the ſigne be compounde. For the compounde ſigne, maketh not a compounde number. And now I will procede to numeration.

of Cossike numbers.
Of Numeration in numbers
Cossike, vncompounde.

Master.



Numbers Cossike vncompounde, haue no *Numeration* difficultie in their numeration: for euer more the nōber representeth, so many of that Cossike denominatiō (be thei nōbers, rootes, squares, Cubes, squares of squares, or any other like) as ther be vnities in that nōber
So. 6. 9. is. 6. numbers: And. 6. 2. is. 6. rootes:
20. 5. is. 20. squares: 30. C. betokeneth. 30. Cubes.
Scholar. I see it well. For by this nōber. 20. 5. is not appointed any nōber absolute, of one certaintie, but onely so many quantities of that kinde: whiche maie bee. 80. if. 4. be one square. And if. 9. bee one square, then 20. squares make 180. And if. 25. be one of those squares thereby represented, then. 20. squares make 500. And as for the signes, you taught me the be foze.

Of Addition.

Master.



This numeration is so plaine, that wee *Addition of* maie passe from it vnto addition: whiche *like signes.* is as easie also, if the quantities be of one denomination. For then needeth no more, but to adde the numbers together, and to put that same common Cossike denomination, to the totall thereof.

Scholar. I take it thus, 20. 2. added to. 30. 2. will make. 50. 2. And. 12. 3. added to. 16. 3. b) together for the. 28 3.

Master. As you doe easily see al the mater of this addition, so maie you as easily conceiue, all the worke *Subtraction* of subtraatiō. For it is wrought as in vulgare nōbers *of like signes*

S. iy.

Scholar.

The Arte

Scholar. When if I abate. 6. ℄. out of. 10. ℄. there will reste 4. ℄. And so. 9. ʒ. ʒ. out of. 25. ʒ. ʒ. doeth leaue. 16. ʒ. ʒ.

Master. This is all for numbers of like signes *Cosike*.

Scholar. What then if I would adde. 10. ʒ. to 6. ʒ. where the signes bee vnlike: maie it be doth: scyng thei be not of one denominatio, nor signe *Cosike*.

Addition of vnlike signes

Master. As well as shillynges maie bee added with poudes, or pennes: and in like forme.

For thei shall stand still as thei wer, with the signe of addition, whiche is this. —+— . & betokeneth more.

So that. 10. ʒ. put to. 6. ʒ. maketh. 6. ʒ. —+— 10. ʒ. that is. 6. ʒ. more. 10. ʒ. or. 6. ʒ. and. 10. ʒ.

Scholar. And why not. 10. ʒ. —+— 6. ʒ.?

Master. Because it is moſte orderly, to sette the greateſte ſigne *Cosike*, for moſte in order.

As you ſaie. 20. shillynges, and. 6. pennies: rather then. 6. pennies and. 20. shillynges.

Scholar. When I ſe. if. 15. ℄. be added to. 18. ʒ. ʒ. it will make. 18. ʒ. ʒ. —+— 15. ℄. An ſo. 12. ʒ. ʒ. toynd with. 20. ʒ. ℄. dooe make. 20. ʒ. ℄. —+— 12. ʒ. ʒ.

Of Subtraction.

Master.

Subtraction of vnlike signes



*S*ubtraction is as caſte: for it doeth depend onely of the ſigne of abatemente, whiche is this. —, and ſignifieth leſſe, or abatynge. And therefore if I would abate

6. ʒ. out of. 10. ʒ. I muſt ſette it thus 10. ʒ. —. 6. ʒ. that is to ſaie. 10. ʒ. leſſe. 6. ʒ. or abatynge. 6. ʒ.

Scholar. When if I haue 30. ℄. and would abate out of the. 12. ʒ. I muſt ſet it thus. 30. ℄. —. 12. ʒ. that is. 30. cubes ſaue. 12. numbers. And if multiplicati

tion

of Coslike numbers.

tion and diuision, bee as easie, thei shall neede no greate studie.

Of Multiplication.

Master.



Some what more labour is there *Multiplication.* in multiplication and diuision, to haue out the newe signes as I will tell you anon. But for finding of the numbers, the common multiplication and diuision doeth serue. So that when. $12 \cdot 3$. is multiplied by. $6 \cdot 2$. it maketh. $72 \cdot 2$. And if. $24 \cdot 2$. bee multiplied by. $5 \cdot 3$. there riseth. $120 \cdot 3$.

Scholar. This passeth my cunningge, for the finding of the newe signe: although the multiplication of the numbers, be as easie as can be.

Master. If you did well remeiber, what you haue learned before: the mater would not seme so harde.

Doe not you knowe, that a roote multiplied by a roote, doeth make a square: And a square multiplied by his roote, doeth bring forth a cube?

Scholar. That I knowe right well: and therefore a *Square of Squares* multiplied by his roote, will yelde a *Surfolide*.

Master. Then by like reason, a *Cube* multiplied by a *Square*, shall make a *Surfolide*.

Scholar. In deece it is all one, to multiplie a *Cube* by a *Square*, and a *Square of Squares* by a roote.

Master. Then for a generall rule, I will sette forth here a presidente for you: whereby you maie knowe the newe signe, in all multiplication or diuision: not onely by sight very speedily, but that you maie also commit it aptly to memorie.

Wherefore marke wel this table folowing: where you see in the higher rowe, a line of numbers, set in naturall

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naturall progression : and vnder them you see the signes of *coslike* numbers.

The table of *Coslike* signes, and their peculiernumbers.

| | | | | | | |
|------|---------|------|------|------|-------|------|
| 0. | 1. | 2. | 3. | 4. | 5. | 6. |
| 9. | ℥. | ∫. | ℄. | ∫∫. | ∫∫. | ∫∫℄ |
| 7. | 8. | 9. | 10. | 11. | 12. | 13. |
| b/∫. | ∫∫∫.℄℄. | ∫∫∫. | ∫∫∫. | ∫∫∫. | ∫∫∫℄. | d/∫. |

This table is largely set forth, in the title of progression, whereunto you maie haue recourse, if your number be to greate for this table.

By this table maie you easily knowe, the signe that shall serue for your newe somme, in multiplication.

As for example, if I dooe multiple squares by rootes : I looke in the table, what numbers stande ouer them bothe, and puttyng those .2. numbers together, I seeke the totall in the same line, and vnder it I finde the newe denomination *coslike*, whiche I should haue

Scholar. I perceiue ouer ℥. the number of 1. and ouer ∫. the number . 2. whiche bothe added together make . 3. And bicause vnder . 3. I find the figure or signe of. ℄. I muste take that for the newe denomination.

Master. You saie truthe.

Scholar. Then if I multiplie . 12. ∫. ℄. by . 8. ℄. the somme will be . 96. ℄ ℄. For ouer. ℄. I finde 3. and ouer. ∫. ℄. standeth . 6. whiche bothe together doe make . 9. and vnder . 9. I see. ℄ ℄. whiche I take for the denominatoz.

And if the same rule bee generall, I am cunnynge
inoughe

of Cossike numbers.

trouge in it.

Master. It is generall, for multiplication in this kinde.

Of Division.

Division. At for diuision, you muste abate the one number out of the other, to finde a new denomination.

Wherefore if you would diuide 96. ℥ by 8. ℥ . the *quotiente* will be 12. ℥ . because that ouer the signe of your diuident, standeth 9. And ouer the diuisors signe is set 3. Wherefore abating 3. from 9. there resteth 6. vnder whiche is the signe 3. ℥ . that I must take, to put to my *quotiente*.

Scholar. Then for an other triall, if I would diuide. 260. ℥ by 53. the *quotient* will be 52. ℥ . For because that ouer 53. I finde. 17. and ouer 53. standeth 5. then subtractyng 5. fro. 17. there resteth 12 vnder whiche in the table I finde. 3. ℥ .

So diuidyng. 20. ℥ . by 4. ℥ . the *quotiente* will be 5. ℥ : and so of other.

Master. But and if you would diuide. 12. ℥ . by 5. ℥ . that must be set in forme of fraction, thus. $\frac{12}{5}$.

So. 18. ℥ . by 7. ℥ . maketh. $\frac{18}{7}$ and 6. ℥ . by 2. ℥ . yeldeth. $\frac{6}{2}$. of whiche fractions, wee will speake amongst the fractions of Cossikes compoude. For they degenerate out of this kinde.

Wherefore this maie suffice briefly, for the custorable woorkes of whole Cossike numbers.

Of Fractions in Cossike numbers,

As for fractions, the woorkyng is like *Offractions* in euery pointe, vnto the woork of numbers *Abstracte*: remembryng onely that as those broken numbers, haue a Cossike denomination annexed with them, so must

E. 1. that

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that denomination followe the rules, now lasse declared.

Wherefore I shall not neede to doe any more, but to set forth the onely certain examples, of euery kinde of woorkes in them.

Examples of Numeration.

- $\frac{2}{3}\mathcal{C}$. Signifieth $\frac{2}{3}$ of a *Roote*.
 $\frac{8}{9}\mathcal{S}$. Betokeneth $\frac{8}{9}$ of a *Square*.
 $\frac{12}{17}\mathcal{C}$. Representeth $\frac{12}{17}$ of a *Cube*.

And so of all other formes of *Cosike* signes: where by is intended, that the *Cosike quantitie*, is diuided into so many partes, as the denominator containeth, and there is here represented onely so many of them, as the numerator doeth impoete.

Scholar. Hereby I dooe perceiue, that a fraction *Cosike*, maie signifie a number, and not onely a parte of an vnitie, as it did in numbers *Abstrakte*.

For when I saie $\frac{2}{3}\mathcal{S}$, if that *Square* be. 9. then that fraction signifieth. 6. But if the *Square* be. 4. then that fraction doeth represente. $2\frac{2}{3}$.

Likewises $\frac{3}{4}\mathcal{C}$. if the *Cube* be. 8. then that fraction doeth signifie. 6. But if the *Cube* be. 27. then that fraction is equalle to. $20\frac{3}{4}$.

Master. You doe consider it well.

Of Addition.

Addition.

Now for addition, take these cramples.

$\frac{2}{3}\mathcal{S}$, added to $\frac{1}{4}\mathcal{S}$, doe make $\frac{11}{12}\mathcal{S}$, or. $1\mathcal{S}\frac{1}{12}$.

$\frac{1}{2}\mathcal{C}$ ioined with $\frac{2}{7}\mathcal{C}$, doe make $\frac{9}{14}\mathcal{C}$, or. $1\mathcal{C}\frac{11}{14}$.

And in vnlike signes,

$\frac{3}{4}\mathcal{S}$, added to $\frac{1}{7}\mathcal{C}$, doe make $\frac{1}{4}\mathcal{C}$, — $1\mathcal{S}\frac{1}{4}$, or els thus by one common denominator.

| | | |
|-------------------|---|---------------------|
| 16. \mathcal{C} | — | 15. \mathcal{S} . |
| 20. | | |

¶

of Cossike numbers.

Of whiche I will speake moze in the *Binomialles*, and therefore will omitte it, till we come to them.

Scholar. As for the reste, I see it well: For the woork is all one with fractions *Abstraete*.

And here the denominatioⁿ of *Cossike* signe is not varied, although here be vsed diuerse multiplications.

Maister. And good reason: for the whole *quotiente* whiche is represented by that *Cossike* signe, is not multiplied, but certaine partes of it: and therefore oughte that *Cossike* signe, to stand vnaltered, as the quantitie represented by it, is not multiplied nor altered.

Examples of Subtraction.

$\frac{1}{2} \text{℥}$. abated out of $\frac{3}{4} \text{℥}$. doe leaue $\frac{1}{4} \text{℥}$.

$\frac{1}{2} \text{ʒ}$. out of $\frac{3}{4} \text{ʒ}$. there resteth $\frac{1}{4} \text{ʒ}$.

$\frac{1}{2} \text{ʒ}$. subtracted fro^m $\frac{3}{4} \text{ʒ}$ doe leaue $\frac{1}{4} \text{ʒ}$. or $\frac{1}{4} \text{ʒ}$.

And in unlike signes.

$\frac{1}{2} \text{℥}$ abated fro^m $\frac{1}{4} \text{℥}$ doe leue $\frac{1}{4} \text{℥}$ — $\frac{1}{2} \text{℥}$.

$\frac{1}{2} \text{ʒ}$ taken out of $\frac{1}{4} \text{℥}$. the reste is $\frac{1}{4} \text{℥}$ — $\frac{1}{2} \text{ʒ}$.

Like waies as in additioⁿ, so in this sorte of subtraction, there maie be an other kinde of woork, whiche I will remit to the treatise of *Binomialles*.

Examples of Multiplication.

$\frac{1}{2} \text{ʒ}$ multiplied by $\frac{1}{4} \text{ʒ}$. doe make $\frac{1}{8} \text{ʒ}$.

$\frac{1}{2} \text{℥}$ multiplied by $\frac{1}{4} \text{℥}$. byngeth fo^r the $\frac{1}{8} \text{℥}$.

$\frac{1}{2} \text{ʒ}$. multiplied by $\frac{1}{4} \text{ʒ}$. doe yelde $\frac{1}{8} \text{ʒ}$. or $\frac{1}{8} \text{ʒ}$.

Here the signes doe alter, as in the multiplication of whole *Cossike* numbers.

℥. y.

Scholar.

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Scholar. This doeth some what trouble me: that the *Coslike* signes should chaunge here, rather then in addition, or subtraction: Seyng there was as moche multiplication, in any of them bothe, as there is here.

Master. Marke the mater well, and you shall bee some satisfied.

For in addition and subtraction, the multiplicatio serueth onely for the reduction, of the 2. fractions, vnto one denomination: And therefore in them, you neuer multiplie the numeratoꝝ together: but you multiplie crosse waies, the numeratoꝝ of the one, by the denominatoꝝ of the other, where as in multiplicatio, you vse no reduction, but doe make a plaine multiplicatio.

And so like waies in diuision, there is vsed no meane of reduction: and therefore in it the signes must alter, as befoze is declared.

Examples of Diuision.

$\frac{5}{7} \div \frac{3}{8}$. diuided by $\frac{6}{11} \div$. doe make in the *quotiente*
 $\frac{44}{21} \div$. or $\frac{11}{7} \div$.
 $\frac{2}{9} \div$. diuided by $\frac{3}{15} \div$. doeth yelde $\frac{10}{21} \div$. or els $\frac{10}{15}$.

For seyng I shall diuide. \mathcal{C} . by. \mathcal{C} . I must therefore abate. 3. from. 3. and so resteth nothing, whiche is signified by this Cypher. 0. and that standeth ouer the signe of number: therefore the fraction, that is as the *quotiente*, must be taken as a number *Abstracte*.

Like waies $\frac{3}{7} \div \frac{3}{8}$. diuided by $\frac{5}{9} \div \frac{3}{8}$. doeth make $\frac{24}{35} \div$. that is to saie. 3. And so $\frac{3}{11} \div \mathcal{C}$. diuided by $\frac{2}{11} \div$ doeth byyng for the $\frac{30}{11} \div \mathcal{C}$. or $\frac{15}{5} \div \mathcal{C}$.

Scholar. This is sufficiente for diuision. Now if you thinke good to speake of progression, I can not but remember you of your promise.

of *Coslike* numbers. Of Reduction.

Maſter.



Although *Reduction* ſhould go in order be *Reduction*,
foze *Progreſſion*, yet ſecyng this *Reduction*,
conſiſteth in the onely numbers, and not
in the ſignes: and therefore agreeth with
vulgare reduction of fractions (as here
you maie ſee befoze in diuerſe examles) therfoze will
we omitte it, and go in hande with *Progreſſion*: whiche
is moze ſtraunge.

Scholar. I praie you ſo: For I ſee this reduction,
is but to reduce the greater fraction, to a leſſer in nō-
ber: as I learned long a gone by your other booke.

Of *Progreſſion* in *Coslike* ſignes.

Maſter.



Progreſſion is thus wroughte: Firſte ſette
doun as many vulgare nōbers, in their
naturall pprogreſſion, as you liſte to haue
Coslike ſignes, that by them you maie the
better know, the true places of the *Coslike*
ſignes: ſo that you ſet in the firſte place a Cipher, and
vnder it. 9. And then vnder. 1. ſet. 2. vnder. 2. put. 3.
and vnder. 3. write. 4. As you ſee in the table ſolow:
yng. And by theſe ſhall you ſet, as many as you liſte.

For all the vulgare numbers, whiche you haue ſet
in the higher rewe, be other compounde numbers, or
els vncompounde: and if the place, where you would
ſet any *Coslike* ſigne, be noted with a number vncom-
pounde, then muſt there be ſet one of the *Surſolides*.

For vnder the firſt nōber vncompounde, you muſt
ſet the firſt *Surſolide*, and the ſecond vnder the ſecond
number vncompounde: and the thirde vnder the thirde,

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and so forth.

The numbers Uncompoude, are these in th en p^{ro}gression.

5. 7. 11. 13. 17. 19. 23. 29. 31. 37. 41.
43. 47. 53. 59. 61. 67. &c.

Under nethe .5. must you set . ζ . and vnder .7. $b\zeta$. vnder .11. $c\zeta$. and vnder .13. $d\zeta$. and so forth, til you come to .67. vnder whiche you must set . $f\zeta$. and vnder 71 you must set $g\zeta$. and so as farre as you list.

But for any other place, becaufe the vulgare number is compoude, that is set (as the pecuhare number, in the higher rewe) therefore the *Coslike* signe must nedes be compoude, other of .2. or of .3. or els of bothe. And if it be compoude of .2. then set doune . ζ . so often tymes, as .2. is in the composition of that number.

As for example: 16. is compoude of .2. to lwer tymes (not by addition, but by multiplication, as in sayng, twice. 2. two tymes, twice.

Scholar. I perceiue twice. 2. to bee. 4. and twice that to be. 8. and twice that to make. 16.

Master. So mate you worke backe warde, in sayng. 16. diuided by .2. maketh .8. that is ones: then .8. by .2. yeldeth .4. that is twice. Again. 4. by .2. maketh 2. that is thise: and .2. for hirsself, is the fourth: wherfore vnder. 16. I must set doune. ζ ζ ζ ζ .

And so vnder. 32. I muste sette. ζ ζ . in one thus. ζ ζ ζ ζ ζ ζ .

And vnder. 64. I shall sette it. 6. tymes, thus. ζ ζ ζ ζ ζ ζ . Bicaufe. 64. is made of .6. multipl^{it}ations by .2.

Scholar. Were by I see, that vnder .8. I muste put 3. tymes that signe: and vnder .4. twice the same.

Master. So must you in deede.

And now for other places, if their numbers bee co^{mp}ounde

of Cossike numbers.

pounde of .3. onely, then must you set downe the signe of *Cube*, as oftentimes as .3. is multiplied, to make that number.

As for example. 27. is compounde onely of .3. and not of .2. (for of all other compounde numbers here in then of soche as be cōpounde of .2. or .3. we take no regard.) And. 3. multiplied thise, doeth make .27. in sayng. 3. tymes. 3. thise. And therefore vnder. 27. I shall set this signe of. \mathcal{C} . three times, thus. $\mathcal{C}\mathcal{C}\mathcal{C}$. whiche betokeneth a *Cube of Cubes Cubikely*.

But and if the number bee compounde bothe of .2. and. 3. then for euery tyme that. 2. is multiplied, so that composition, I shall sette. \mathcal{F} . and for euery tyme that. 3. is multiplied, I shall set. \mathcal{C} . remembryng well to set. \mathcal{F} . before. \mathcal{C} . and not after hym.

As for example. Vnder. 24. I shall set. $\mathcal{F}\mathcal{F}\mathcal{C}$. bicause that. 2.2.3. that is to saie. 2. tymes. 2. twise thise, doeth make. 24. Or by resolution, thus. 24. diuided by. 2. giueth. 12. For that firste. 2. set. \mathcal{F} . Again 12. diuided by. 2. yeldeth. 6: for this seconde. 2. set. \mathcal{F} . also. Then diuide. 6. by. 2. and it maketh. 3. For the. 2. I must set. \mathcal{F} . and for. 3. I must put. \mathcal{C} . and so all together maketh. $\mathcal{F}\mathcal{F}\mathcal{C}$. in the. 24. place.

Like waies vnder. 36. I must sette. $\mathcal{F}\mathcal{C}\mathcal{C}$. bicause that. 2.2.3.3. doeth make it, that is. 2. tymes. 2. thise, thise. And by resolution, thus. 36. diuided by 2. giueth. 18. For that. 2. I set. \mathcal{F} . Again. 18. diuided by. 2. maketh. 9. For that. 2. I sette doune againe. \mathcal{F} . Thirdly, for bicause. 9. can not bee diuided by. 2. but by. 3. 3. tymes: therefore I muste sette doune. for those twoo. 3. twise. \mathcal{C} . so the whole signe is. $\mathcal{F}\mathcal{C}\mathcal{C}$.

Now if the number of the place, or peculiere number, bee compounde of one of theim twoo, with some other number vncōpounde, then must we toyne their signes together.

As. 10. is compounde of. 2. and. 5. therefore must I set

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set vnder. 10. the signe that is in the fifth place, whiche is $\text{f}\bar{\text{z}}$, and befoze it I muste set the signe of $\bar{\text{z}}$. soz 2. So must that signe be. $\bar{\text{z}}\text{f}\bar{\text{z}}$.

Likewaies, bicause. 15. is compounde of. 3. and. 5. I shall ioine together the signe of \mathcal{C} . and of $\text{f}\bar{\text{z}}$. and make it. $\mathcal{C}\text{f}\bar{\text{z}}$.

Scholar. So I vnderstande it now, that I cannot misse it. Haue that soz lacke of vnderstande, and throughte soz getfulnesse, when I heare the name of composition in nombers, I doe mistake it sometimes soz addition, els here can be no erroure. For when I doe consider, that. 20. is compounde of. 2. 2. 5. that is twise. 2. and. 5 (sith. 2. tymes. 2. maketh. 4: and. 5. tymes. 4. maketh 20.) I maie sone consider, to set. $\bar{\text{z}}$. twise befoze. $\text{f}\bar{\text{z}}$. and then it will be. $\bar{\text{z}}\bar{\text{z}}\text{f}\bar{\text{z}}$. to be put in the. 20. place.

Likewaies in the. 21. place, I set. $\mathcal{C}\text{b}\bar{\text{z}}$. seying 21 is compounde of. 3. and. 7. and. \mathcal{C} . is the signe to the thirde place, as $\text{b}\bar{\text{z}}$. serueth soz the. 7. place.

Master. What shall you set in the. 84. place?

Scholar. 84. is compounde of. 2. 2. 3. 7. thercoze his signe must be. $\bar{\text{z}}\bar{\text{z}}\mathcal{C}\text{b}\bar{\text{z}}$.

Master. Now I see, you are cunnyng inough in this, and thercoze take here this table, soz a patron: and then will we procede to the woꝝke of *Cossike nombers* compounde,

The table for progression Cossike,

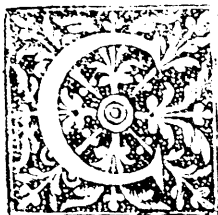
whiche maie increase it self infinitely,

without any difficultie.

| | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0. | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10. | 11. |
| ၅. | ၆. | ၇. | ၈. | ၉. | ၁၀. | ၁၁. | ၁၂. | ၁၃. | ၁၄. | ၁၅. | ၁၆. |
| 12. | 13. | 14. | 15. | 16. | 17. | 18. | 19. | 20. | | | |
| ၁၇. | ၁၈. | ၁၉. | ၂၀. | ၂၁. | ၂၂. | ၂၃. | ၂၄. | ၂၅. | ၂၆. | ၂၇. | ၂၈. |
| 21. | 22. | 23. | 24. | 25. | 26. | 27. | 28. | | | | |
| ၂၉. | ၃၀. | ၃၁. | ၃၂. | ၃၃. | ၃၄. | ၃၅. | ၃၆. | | | | |
| ၃၇. | ၃၈. | ၃၉. | ၄၀. | ၄၁. | ၄၂. | ၄၃. | ၄၄. | | | | |
| 45. | 46. | 47. | 48. | 49. | 50. | 51. | | | | | |
| ၅၂. | ၅၃. | ၅၄. | ၅၅. | ၅၆. | ၅၇. | ၅၈. | | | | | |
| 59. | 60. | 61. | 62. | 63. | 64. | 65. | | | | | |
| 66. | 67. | 68. | 69. | 70. | 71. | 72. | 73. | | | | |
| 74. | 75. | 76. | 77. | 78. | 79. | 80. | | | | | |

In this table, ဃ, င, and ဇ, are the groundes:
of all the reste aboue them. For of these
three, all those other bee made.

The Arte Of Coslike numbers compounde.



*C*oslike numbers compounde, are made by addition of .2. or moze simple *Coslike* numbers together :

As. 6. ze . — | — . 5. z . . 02.

12. C . — | — . 4. ze . — | — . 3. 9. and

so for the in diuerse formes, whiche be infinite. Ho wbeit for brieftnesse, we maie comprehend, vnder thesame name (bicause of the like worke) all other *residualles Coslike*, whiche be made by subtraction : as. 3. C . — | — . 4. z . And also those that bee made by addition and subtraction, bothe together: As. 9. z z . — | — . 4. ze . — | — . 6. z . In whose numeration is no hardnesse.

Scholar. When your rules maie be the shorter.

Of Numeration.

Master.



*T*his Numeration is easily vnderstande by addition of simple *Coslikes*. For this is the forme. 6. z — | — . 10. 9. that is. 6. *Squares*, moze. 10. *nöbers*. Likewates. 8. C . — | — . 11. ze . is 8. *Cubes* and. 11. ze .

Now for *residualles*, take these examples. 9. z z . — | — . 12. C , whiche is. 9. *Squares* of *Squares*, saue. 12. *Cubes*. Also. 4. z z . — | — . 15. z . that is. 4. *surfolides*, abatung. 15. *Squares*.

And for bothe together, this is the forme.

10. z z — | — . 6. C . — | — . 30. ze . whiche signifieth 10. *Squares* of *Squares*, and. 6. *Cubes*, abatung. 30. *rootes*.

Scholar. This is plaine. For so maie I vnderstande of all other As. 9. z C . — | — . 3. z z . — | — . 8. 9 that is. 9. *Squares* of *Cubes*, lesse 3. *Squarss*, moze. 8. *nöbers*.

Master.

of Cossike numbers.

Master. It were moze orderly, to kepe the signes of moze and lesse in order, then to followe the order of the *cossike* signes: bicause that addition, is orderly placed before subtraction. So were it better to set them thus 9. 3. 2. — | — 8. 9. ———. 3. 3. Hobbit in deede all is one in these kinde of numbers, but not so in other *Surde numbers*, where the order foloweth of necessitie, as shall be declared in their place moze largely.

Of Addition.



In addition, you must haue consideration of the *Cossike* signes: for noe other number, maye bee added into one, then soche as appertain to one signe *Cossike*.

As in vulgare denominations, you doe not adde the numbers of shyllynges, to the numbers of pennies: but you toime shyllynges to shillings, and pennies to pennies: & poudes to poudes, so in *Cossike* numbers, *Cubes* muste bee ioined to *Cubes*, *Squares* to *Squares*, and generally, like to like.

Scholar. If this be al, I cā marke it well inough. Master. There is somewhat moze to be considered, that if there bee any signe in the one number, whiche is not in the other, that severall signe with his number, muste bee sette downe with his figure of — | . 02. ———. as it standeth there.

And farther, touchyng those twoo signes. — | — . whiche bee the figures of moze and lesse, you muste glue regarde, whether they bee like or unlike, in those numbers that muste be added: For if they be like in numbers, of one denomination, then muste they remaine as they be. But if they be unlike, euermoze abate the smaller number of them, that followe those
A. y. unlike

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will the signes, out of the greater: and sette doune the reste, with the signe of the greater number.

Scholar. By examples, I shall better conceiue those rules.

Master. Take these examples.

| | |
|--|--|
| $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 20. \text{q} \cdot \end{array}$ | $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 20. \text{q} \cdot \end{array}$ |
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| | |
|--|--|
| $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 20. \text{q} \cdot \end{array}$ | $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 20. \text{q} \cdot \end{array}$ |
|--|--|

| | |
|---|---|
| $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 4. \text{q} \cdot \end{array}$ | $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 4. \text{q} \cdot \end{array}$ |
|---|---|

| | |
|---|---|
| $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 4. \text{q} \cdot \end{array}$ | $\begin{array}{r} 10. \text{z} \cdot \text{—} \cdot 8. \text{q} \cdot \\ 4. \text{z} \cdot \text{—} \cdot 12. \text{q} \cdot \\ \hline 14. \text{z} \cdot \text{—} \cdot 4. \text{q} \cdot \end{array}$ |
|---|---|

Here haue I varied one example diuersly, to the u-
sente you maie marke the vse of your rules in them.
And for the reason of those rules, you shall marke
those

fo Cossike numbers.

those examples well.

For where in the first example, bothe signes are —|—, it must nedes be, that after the addition of the first numbers, the seconde muste bee added with the signe. —|—.

In the seconde example, where bothe the signes be —|—, because there wanteth. 21. 9. of the first. 10. 3. Therfore is it reason, that bothe those wantes should be sette doune with the signe of. —|—; and so in the thirde and fourthe examples.

In the fifth example, the seconde somme is not fully. 4. 3. but there wanteth of it. 8. 9. and therefore if you put donne the. 4. 3. fully, you must abate. 8. out of the. 12. 9. in the higher somme: and so of the other examples.

But for more practise, and better declaration of the vse of them, here are other examples, of more varietie.

$$\begin{array}{r}
 20. \text{ 3. } \text{℥} \text{.} \text{---|---} . 9. \text{ 3. } \text{---} \text{---} . 120. \text{ 3. } \text{℥} \text{.} \\
 15. \text{ 3. } \text{℥} \text{.} \text{---|---} . 5. \text{ 3. } \text{---|---} . 16. \text{ 3. } \text{℥} \text{.} \\
 \hline
 35. \text{ 3. } \text{℥} \text{.} \text{---|---} . 14. \text{ 3. } \text{---} \text{---} . 104. \text{ 3. } \text{℥} \text{.}
 \end{array}$$

$$\begin{array}{r}
 16. \text{ 3. } \text{3. } \text{---|---} . 28. \text{ 9. } \text{---} \text{---} . 16. \text{ 3. } \\
 12. \text{ 3. } \text{3. } \text{---|---} . 12. \text{ 3. } \text{---} \text{---} . 19. \text{ 9. } \\
 \hline
 28. \text{ 3. } \text{3. } \text{---|---} . 9. \text{ 9. } \text{---} \text{---} . 4. \text{ 3. }
 \end{array}$$

In the first example of these. 2. you see. 120. 3. ℥. with the signe of lesse, to bee added with. 16. 3. ℥. with the signe of more: and therefore, seeing the signes of one *Cossike* denomination disagree, I dooe subtracte the lesser, out of the greater: and that. 104. whiche remaineth, I doe set doune with the signe of

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lette, bicause the remainer is of that umber, that bare that signe.

And in the seconde exāple, the placynge of the signe
 —+— befoze ——— maketh numbers to bee sette be-
 foze squares: and so the like denominations, dooc not
 stande one ouer an other. Yet is the wooꝝke dooen as
 if they did stande eche ouer his like.

Scholar. I pꝛaie you lette me trie my cunnynge,
 with an example oꝝ twoo.

$$17. \text{ʒ} \text{ʒ} \cdot \text{—+—} \cdot 10. \text{℥} \cdot \text{—} \cdot 2. \text{ʒ} \text{℥} \cdot$$

$$16. \text{ʒ} \text{℥} \cdot \text{—+—} \cdot 12. \text{ʒ} \cdot \text{—} \cdot 6. \text{ʒ} \text{℥} \cdot$$

$$16. \text{ʒ} \text{℥} \cdot \text{—+—} \cdot 17. \text{ʒ} \text{ʒ} \cdot \text{—+—} \cdot 10. \text{℥} \cdot$$

$$\text{—+—} \cdot 12. \text{ʒ} \cdot \text{—} \cdot 8. \text{ʒ} \text{℥} \cdot$$

I set the example, as numbers came to my mynde:
 but I had almoste set my self on ground: saue that I
 called to remembꝛaunce, the comparison that you
 made, to bulgare denominations of poundes, shillin-
 ges, and pennies: and so was instructed to place eue-
 ry seueralle denomination seuerally. And to sette the
 greateste denominatiō first, & eche other in his order.

Now will I pꝛoue an other example, oꝝ twoo.

$$3. \text{ʒ} \text{ʒ} \cdot \text{—+—} \cdot 4. \text{℥} \cdot \text{—} \cdot 20. \text{ʒ} \cdot$$

$$20. \text{℥} \cdot \text{—} \cdot 8. \text{ʒ} \cdot \text{—} \cdot 16. \text{ʒ} \cdot$$

$$3. \text{ʒ} \text{ʒ} \cdot \text{—+—} \cdot 24. \text{℥} \cdot \text{—} \cdot 8. \text{ʒ} \cdot \text{—} \cdot 36. \text{ʒ} \cdot$$

$$13. \text{ʒ} \text{℥} \cdot \text{—+—} \cdot 8. \text{℥} \cdot \text{—} \cdot 4. \text{ʒ} \text{℥} \cdot$$

$$7. \text{ʒ} \text{℥} \cdot \text{—} \cdot 6. \text{℥} \cdot \text{—} \cdot 7. \text{ʒ} \cdot$$

$$20. \text{ʒ} \text{℥} \cdot \text{—+—} \cdot 2. \text{℥} \cdot \text{—} \cdot 4. \text{ʒ} \text{℥} \cdot \text{—} \cdot 7. \text{ʒ} \cdot$$

6. ʒ ·

of Coslike numbers.

$$\begin{array}{l}
 6. \text{ ʒ. } + \text{ 10. } \text{ ʒ. } = 8. \text{ ʒ. } \\
 4. \text{ ʒ. } + \text{ 17. } \text{ ʒ. } = 7. \text{ ʒ. } \\
 \hline
 10. + 3. \text{ ʒ. } = 9. \text{ ʒ. }
 \end{array}$$

$$\begin{array}{l}
 4. \text{ ʒ. } \text{ ʒ. } + 5. \text{ ʒ. } = 6. \text{ ʒ. } \\
 8. \text{ ʒ. } = 8. \text{ ʒ. } = 10 \text{ ʒ. }
 \end{array}$$

$$4. \text{ ʒ. } \text{ ʒ. } + 8. \text{ ʒ. } = 3. \text{ ʒ. } = 4. \text{ ʒ. }$$

Matter. You haue doen well : And for prooffe of your worke, you maie in this arte not onely proue it, by the contrary kynde, as you did in nôbers *Abſtraſte*, but alſo by the reſolution of all thoſe *Coslike* numbers into nôbers *Abſtraſte*, takyng any number for a roote and then the *Squares* and *Cubes*. &c. accordingly. As here in this table, you maie by clefly ſee, but moze largely in the table at the cande of numbers figurall.

A table for trialle by reſolution, of any worke in this arte.

| ʒ | ʒ | ʒ | ʒʒ | ʒʒ | ʒʒʒ | ʒʒʒʒ |
|---|----|----|-----|------|------|-------|
| 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 46384 |

| ʒʒʒʒ | ʒʒʒʒ | ʒʒʒʒ |
|-------|--------|---------|
| 256 | 121 | 1024 |
| 6561 | 19683 | 59049 |
| 65536 | 262144 | 1048576 |

And if this table in any parte, ſeme to ſhozte or to little:

The Arte

title: you maie haue recourse to the table, at the ende of figuralle numbers, whiche therfoze is made large and generalle: so that it maie well be called the frute: full table, or table of ease.

But now for triall of the lasse example: firste there is. 4. $\sqrt[4]{\text{C}}$: for whose roote I take 2. and therfoze thole. 4. $\sqrt[4]{\text{C}}$. make. 256. 256.
 whiche I sette doune in number *Abstracte*. 20.
 Nexte is. 5. squares, whiche accordyng to that roote, must nedes be. 20. and that. 20. I sette doune also: and then. 6. rootes, whiche make 12. And all thei yelde. 288. and that is all the firste somme.

Then for the seconde somme, I see firste. 8. Cubes, whiche make. 64. to bee added. 32.
 Then foloweth. 8. squares lesse, that is. 32. to bee abated, and also. 6. rootes lesse, that is. 20. also to bee abated: So must I abate. 52. (for theim bothe) out of. 64. and then there resteth but 64.
 12. whiche added vnto 288. of the first somme doe yelde. 300. 52.

Now if the totall agree with this, then is the woork good. 288

For triall whereof, I resolute. 4. $\sqrt[4]{\text{C}}$. in to number *Abstracte*, and thei will make. 256. 12.
 then. 8. $\sqrt[4]{\text{C}}$. maketh. 64: whiche bothe yelde 320. 300
 When foloweth in the same somme. 3. $\sqrt[3]{\text{Z}}$ and. 4. $\sqrt[4]{\text{Z}}$. to be abated. The. 3. $\sqrt[3]{\text{Z}}$. make. 12. 256
 and the 4. rootes yelde. 8. whiche together do amounte to. 20. and that must bee abated fro the said somme of 320 and then there remaineth onely 300. agreeable to the former somme above the line. 64
320.

Scholar. This prooffe I like well: And I perceiue that if I would woork the like, takyng for the roote 3, or any other number, the prooffe will succede a like.

Master. Now to make an eande of Addition, because

of Cossike numbers.

cause you shall the better remmember the rules of it, I will giue you them in this bryfe forme.

*In greatenesse like and signes also,
Adde like to like there nedes no mo:
And where the greatenesse disagree,
Place eche by other seuerally.*

*with signe of eche, as doeth require,
But if the signes vnlke appere
Then from the more abate the lesse,
The greater his signe with the excesse.
will make the somme,
Of that addition.*

*The prooffe is by resoluyng,
Eche number into his reukenyng.*

This lesson doeth containe the former rules onely in bryef, and therefore needeth no declaration: but the greatenesse doeth betoken the *Cossike* denomination, and signes betoken specially, $+$ and $.$ the signes of more and lesse, and no other signes.

Scholar. This bryef lesson will helpe memoize moche: and shall suffice for the rules of Addition.

Of Subtraction.

Master.



Then for subtraction, this shall you marke in especiall: that when your numbers are sette doune, after the comon maner, firste the totall, and then the deduction: you shall consider well, whether the signes be $+$ or $.$ For in the deduction, if you haue $+$ then must that be subtracted from the like aboute.

And if that somme in the deduction, that hath the $.$ signe

I. j.

1. Rule.

The Arte

signe $—|—$, bee greater then the number of the like quantitie ouer hym, with the like signe $—|—$. then abate the higher out of the lower, and write the reste with this signe $—$.

3. Rule. But if the like quantitie in the totall, haue the signe $—$, then adde bothe numbers together and set them vnder the line with that signe $—$.

4. Rule. And if the seconde somme (that is the deduction or abatement) with any number, haue this signe of lesse $—$, it must be accounted for more, and must be added to the like number ouer it, excepte the ouer number haue the signe of lesse also: For then must you abate the lesser, out of the greater, and sette doune the reste, with the signe of the greater number: whiche thei haue at this conferre: I meane to regarde what the signe of the seconde somme is by estimation, and not by writing, for thei are contrary.

Scholar. I see good reason in this: For in any abatemente, the more is abated, the lesse by so moche shall remain: and the lesse is abated, the more doeth remain by so moche.

5. Rule. Master. Yet one thyng more is to bee marked, that if there be some denominations, in the one some that are not in the other, you shall marke in whiche somme thei bee. For if thei bee in the firste, then shall thei kepe still their owne signe. And if thei bee in the seconde somme, whiche is the deduction, then shall thei chaunge their signe to the contrary: But where soeuer thei be, thei must be set in the remainer.

Scholar. I can better vnderstande you, then remembre those rules.

Master. Then take this bysel lesson, apter to bee remembred, then to bee vnderstande, but by the letter befoze, and by the examples solowng. But memorie liketh well soche aide.

of Cosikeuomers.

A brief rule of Subtraction.

1. When signes and greatnesse bot be agree,
Your woorke procedeth forthe commonly.
2. But if thabatemente greater bee,
The excessse shall chaunge his signe therby.
3. And where the signes doe disagree,
The higher signe must rest duely:
And though the batemente be the greater,
The reste still ioyneth bot be sommes together.
4. If quantities doe disagree,
Place them with signes all severallie:
The totall kepeth the signe he had,
The batemente still, to chaunge is glad.

Scholar. Now some examples, will lighten these rules well.

Master. I will propounde the like, as I did in addition, to the intete you maie iudge the likenesse, and diuersities in bothe woorkes.

| | |
|-------------------|------------------|
| 10. 8. — 12. 9. | 10. 8. — 8. 9. |
| 4. 8. — 8. 9. | 4. 8. — 12. 9. |
| 6. 8. — 4. 9. | 6. 8. — 4. 9. |

| | |
|-------------------|-------------------|
| 10. 8. — 12. 9. | 10. 8. — 8. 9. |
| 4. 8. — 8. 9. | 4. 8. — 12. 9. |
| 6. 8. — 4. | 6. 8. — 4. |
| | £. 4. 10. 8. |

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| | |
|--|--|
| $\begin{array}{r} 10. \text{z} . - + . 12. \text{q} . \\ 4. \text{z} . - - - . 8. \text{q} . \\ \hline 6. \text{z} . - + . 20. \text{q} . \end{array}$ | $\begin{array}{r} 10. \text{z} . - + . 8. \text{q} . \\ 4. \text{z} . - - - . 12. \text{q} . \\ \hline 6. \text{z} . - + . 20. \text{q} . \end{array}$ |
|--|--|

| | |
|--|--|
| $\begin{array}{r} 10. \text{z} . - - - . 12. \text{q} . \\ 4. \text{z} . - + . 8. \text{q} . \\ \hline 6. \text{z} . - - - . 20. \text{q} . \end{array}$ | $\begin{array}{r} 10. \text{z} . - - - . 8. \text{q} . \\ 4. \text{z} . - + . 12. \text{q} . \\ \hline 6. \text{z} . - - - . 20. \text{q} . \end{array}$ |
|--|--|

The firste and thirde examles be very plaine: and in the seconde where . 12. should bee abated out of . 8. there is . 4. to fewe: and therefore I abate the higher, out of the lower, and I set doune . 4. with the signe of wantyng, or abatements.

In the fourth example: because the higher number is the lesser, I doe subtrakte him out of the nether, and sette doune the reste . 4. with a contrary signe of $- +$.

But in the . 4. later examles, where the signes do disagree, the numbers that followe the signes, are not subtraced one from an other, but are added together: and they take still the higher signe. Because in value, the signe of abatements is contrary, to that it appeareth to be.

And for your exercise, to make you full prompt in this arte, I haue set forth the more examles.

| | |
|--|--|
| $\begin{array}{r} 6. \text{c} . - + . 120. \text{q} . \\ 9. \text{c} . - - - . 40. \text{q} . \\ \hline 160. \text{q} . - - - . 3. \text{c} . \end{array}$ | $\begin{array}{r} 8. \text{z} \text{c} . \\ 9. \text{z} \text{c} . - - - 89. \text{q} . \\ \hline 89. \text{q} . - - - . 1. \text{z} \text{c} . \end{array}$ |
|--|--|

of Coflike numbers.

| | |
|--|--|
| $3. \text{z} \cdot - + - 18. \text{ze} \cdot$ $12. \text{ze} \cdot - - - 3. \text{z} \cdot$ | $18. \text{ze} \cdot - + - 3. \text{z} \cdot$ $12. \text{ze} \cdot - - - 3. \text{z} \cdot$ |
| $6. \text{z} \cdot - + - 6. \text{ze} \cdot$ | $6. \text{ze} \cdot - + - 6. \text{z} \cdot$ |

$$3. \text{z} \cdot - + - 18. \text{ze} \cdot - - - 10. \text{q} \cdot$$

$$12. \text{ze} \cdot - + - 8. \text{q} \cdot$$

$$3. \text{z} \cdot - + - 6. \text{ze} \cdot - - - 18. \text{q} \cdot$$

$$4. \text{f} \text{z} \cdot - + - 10. \text{cc} \cdot - - - 6. \text{z} \cdot$$

$$5. \text{z} \cdot \text{z} \cdot - + - 12. \text{z} \cdot - - + - 3. \text{q} \cdot$$

$$4. \text{f} \text{z} \cdot - + - 10. \text{cc} \cdot - - - 5. \text{z} \cdot \text{z} \cdot - - - 18. \text{z} \cdot - - - 3. \text{q} \cdot$$

Here in the firste example, where I would abate 9 cc . out of 6. cc . I may easily perceiue, that there are 3. cc . to seue. And therefore doe I sette doune 3. cc . with this signe —, whiche signifieth wante or abatements: and the 2. numbers that followe the vnlike signes, I set doune bothe added into one: and put therto the signe of the totall or ouermosse somme.

In the seconde example, there is the like woork: For in abatynge 9. out of 8. I finde 1. to seue: that 1. doe I set doune with his denomination of $\text{z} \cdot \text{cc}$: and the signe. —.

And the number 89 that soloweth the signe — in the seconde somme, standeth in force as — + —, for the lesse is abated, the more must remain: therefore in the remainer, I set not the signe of more, before that number of 89. but I put it in the firste place of the somme: whiche place of it self, signifieth still more.

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And bicause ouer that number 89, there are no nū-
bers in the total, therfore I muste putte doune that
somme as it is, without addyng to it, or abatyng fro
it, in it self.

Scholar. Those. 2. exampls might be set thus, as
I thinke, bicause the places doe so require.

$$\begin{array}{r}
 6. \text{C.} \text{---} + \text{---} . 120. \text{9.} \\
 9. \text{C.} \text{---} . 40. \text{9.} \\
 \hline
 \text{---} . 3. \text{C.} \text{---} + \text{---} . 160. \text{9.} \\
 \\
 8. \text{9.} \text{C.} \\
 9. \text{9.} \text{C.} \text{---} . 8. \text{9.} \\
 \hline
 \text{---} . 1. \text{9.} \text{C.} \text{---} + \text{---} . 8. \text{9.}
 \end{array}$$

Master. Remember your self well, and marke
the remainder how it is wrytten.

Scholar. I see my owne oversighte: For no nom-
ber maie begin, with signe of lesse: and therfore must
their places be altered of necessitie, and set in order as
they were before.

Master. Then for all the reste of the exampls, or
any other like, I shall not neede to giue you any far-
ther instruction: sith that by these former, you maie
iudge of all other.

Prooffe.

And for the examination of your worke, the trialle
by resolution doeth serue here, as well as els where:
remembryng onely (as the order of subtraction maie
admonishe you) that the somme of the totale, whiche
is the firste somme, must counteruaile the other bothe
somes: that is of the deduction, and of the remainder.

So to trie the firste exampl, takyng. 3. for a roote:
6. C. make. 162. whiche I put to. 120. and it yeldeth
282. Then in the seconde somme. 9. C. are. 243.
whereof. 40. must bee abated for the signe ---, so
is

of Cossike numbers.

ts that somme. 203. Again in the remainer. 3. \mathcal{E} . are 81. whiche must bee abated out of. 160. and so resteth 79. whiche with. 203. doe make. 282. agreable with the firste somme.

Scholar. This doo I well vnderstande, and prae you to procede to multiplication.

Of Multiplication.

Master.



In multiplication, there is no difficultie, so that you dooe well marke the signes $+$ and $-$, whiche beyng bothe like, will haue the signe $+$ sette in the totalle. and beyng vnlike, thet will haue in the totalle the signe $-$.

And like waies in diuision $+$ diuided by $-$ or cotrary waies $-$ by $+$ will alwaies haue in the totalle $-$: but $-$ diuided by $+$, or $+$ by $-$, will make alwaie $+$.

Whiche rule for ready remembraunce, I haue giuen you here in meter.

*Who that will multiplie,
Or yet diuide trulie:
Shall like still to haue more,
And mislike lesse in store.
Their quantities doe kepe soche rate,
That M. doeth adde: and D. abate.*

Scholar. So meane you, that like signes multiplied together, doe make more, or $+$: And vnlike signes multiplied together, doe yelde lesse, or $-$.

Master. So is the rule. But to go forward now: of the next difficultie, as touchyng Cossike quantities that chaunge their denomination, here is no more to be

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bee saied, then was taught in multiplication of numbers *Coſlike* vncompounde, and in the table ſet foꝛ the foꝛ the chaunge of their names.

Scholar. I vnderſtande, that in multiplication (that is, *M.*) their figures muſt bee added. And in *D.* (oꝛ diuſion) thei muſt bee abated. Wherefoꝛe a ſeue examples ſhall ſuffice foꝛ the reſte.

Maſter. Take theſe foꝛ a preſidente, of all that woꝛke: by whiche you maie iudge of all other like.

$$\begin{array}{r}
 10. \text{℥.} \quad + \quad 9. \text{ſ.} \quad + \quad 20. \text{d.} \\
 5. \text{ſ.} \quad + \quad 7. \text{d.} \quad + \quad 8. \text{q.} \\
 \hline
 80. \text{℥.} \quad + \quad 72. \text{ſ.} \quad + \quad 160. \text{d.} \\
 70. \text{ſ.ſ.} \quad + \quad 63. \text{℥.} \quad + \quad 140. \text{ſ.} \\
 50 \text{ſ.} \quad + \quad 45 \text{ſ.ſ.} \quad + \quad 100. \text{℥.} \\
 50 \text{ſ.} \quad + \quad 115 \text{ſ.ſ.} \quad + \quad 83. \text{℥.} \quad + \quad 68 \text{ſ.} \quad + \quad 160. \text{℥.}
 \end{array}$$

$$\begin{array}{r}
 15. \text{ſ.} \quad + \quad 12. \text{ſ.} \\
 14. \text{ſ.} \quad + \quad 2. \text{d.} \quad + \quad 5. \text{q.} \\
 \hline
 \quad \quad \quad + \quad 75. \text{ſ.} \quad + \quad 60. \text{ſ.} \\
 \quad \quad \quad 30. \text{ſ.ſ.} \quad + \quad 24. \text{℥.} \\
 210. \text{ſ.ſ.ſ.} \quad + \quad 168. \text{ſ.ſ.} \\
 210. \text{ſ.ſ.ſ.} \quad + \quad 30. \text{ſ.ſ.} \quad + \quad 75. \text{ſ.} \quad + \quad 168. \text{ſ.ſ.} \\
 \quad \quad \quad + \quad 24. \text{℥.} \quad + \quad 60. \text{ſ.}
 \end{array}$$

Scholar. I perceiue, that theſe woꝛkes doe appere moꝛe hard, then thei bee in deede, and that becauſe of their ſtraunge formes: but by uſe I truſte to bee acquainted with them well inough: and thefoꝛe I will begin with moꝛe eaſie examples. As theſe bee, that folowc

of Cossike numbers.

followe here.

$$\begin{array}{r}
 18. \text{z.} \text{---} | \text{---} 20. \text{q.} \\
 15. \text{ze.} \text{---} | \text{---} 4. \text{q.} \\
 \hline
 \text{---} 72. \text{z.} \text{---} | \text{---} 80. \text{q.} \\
 270. \text{cl.} \text{---} | \text{---} 300. \text{ze.} \\
 \hline
 270. \text{cl.} \text{---} | \text{---} 300. \text{ze.} \text{---} 72. \text{z.} \text{---} | \text{---} 80. \text{q.}
 \end{array}$$

$$\begin{array}{r}
 16. \text{z.} \text{---} | \text{---} 14. \text{ze.} \\
 8. \text{cl.} \text{---} | \text{---} 7. \text{q.} \\
 \hline
 \text{---} 112. \text{z.} \text{---} | \text{---} 98. \text{ze.} \\
 128. \text{fz.} \text{---} | \text{---} 112. \text{z.} \text{z.} \\
 \hline
 128. \text{fz.} \text{---} | \text{---} 112. \text{z.} \text{z.} \text{---} 112. \text{z.} \text{---} | \text{---} 98. \text{ze.}
 \end{array}$$

And this I see farther now, that these woorkes seme moze difficulte to looke on, then they be in practise, if a manne giue good hede to the signes, and the quantitties.

Master. Befoze we go any farther, I will shewe you somewhat of the reason, why the signes ought to chaunge. And that by twoo plaine woorkes, in numbers *Abstraite*. As here foloweth.

Where you see, that when I had multiplied. 16 — | — 12 by 20 it made. 320 — | — 240 that is in all. 560.

But bicause the multipli- are ought not to be so moche by 4 therfoze it is reason, that I shall multiplie the higher somme by . 4. and abate that out of the former totall.

| |
|----------------|
| 16. — — 12. |
| 20 — — 4. |
| — 64 — — 48. |
| 320 — — 240 |
| 560 — — 121 |
| that is. 448. |

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Whiche thyng you see here doen by, ———. 64.
 ———. 48. whiche bothe make. 112. to bee deducted
 out of 560. and so remaineth 448. The iuste somme
 that commeth of that multiplication.

Scholar. This I vnderstande well: and
 maie proue it in this sozte. 16. ———. 12.
 maketh. 28: and. 20. ———. 4. is. 16.
 Then if I multiplie. 28. by. 16. it will yelde
 448. as the woozke here declareth.

| | |
|------|--|
| 28. | |
| 16. | |
| 168. | |
| 28. | |
| 442 | |

And hereby maie I iudge, of *Coslike* nom-
 bers like waies.

Master. Yet one exrample moze will I propound
 bicause I would put you out of all doubt. Wherfoze
 marke this sozme of woozke.

Here you maie see, that if
 the firste somme of 24 ——— 3
 wer multiplied by 15 it would ——— 48. ———. 6.
 make. 360. ———. 45. that is
 315. But it ought not to bee so ——— 360 ———. 45.
 moche, but lesse by. 2. tymes ——— 366 ———. 93.
 24 ——— 3. that is. 48 ——— 6: that is. 273.
 bicause the multiplier doeth wante. 2. of. 15.

And so abatynge. 42. of. 48. ———. 6. out of. 315.
 there resteth. 273. whiche is the iuste totall, when. 21
 is multiplied by. 13. wherby the multiplication is de-
 clared to bee good.

And soz bicause that ——— multiplied with ———
 doeth make ———: marke here, that you maie not a-
 bate fully. 48. but 48. ——— 6.

Then seeyng in abatemente, the signes in figure
 are contrary to their owne estimation and foze: ther-
 foze that. 48. must be made ———. and the ——— be-
 foze. 6. tourned into ———.

Scholar. I see it well, it must nedes be so.

For if thei were set, to bee subtrated, then should
 thei stande so. 48. ———. 6: whiche declareth that 42
 should

of *Coslike* numbers.

Should bee abated.

But when the same numbers, are set amongst themselves to be added: as it is here in working of multiplication, then must they be written thus. — 48 — + — 6
 declaring that if you abate. 48. you muste adde. 6. again, because you abated. 6. more then you ought.

Master. You vnderstand it well. Wherefore here will wee make an eande of multiplication: sith there resteth nothing but the prooffe of it: whiche maie bee wrought by resolution, of all the *Coslike* numbers, into numbers *Abstrakte*, as in other kindes before. Duly considering that the resolutions of the first and seconde sommes, must be added together.

The prooffe of multiplication.

And therfore if you liste to proue the firste example taking. 2. for the roote, you shall finde the firste idine 80. — + — 36 — + — 40. that is. 156. And the seconde somme is, 20. — + — 14. — . 8. that is. 26. The thirde somme is. 1600. — + — 1840. — . 664. — + — 272. — . 320. whiche maketh. 4056. And so voeth. 156. multiplied by. 26.

Scholar. This maie I proue at any tyme: so that you shall not neede to staie aboute it.

Of Diuision.

Master.



Diuision is nexte in order, and agreeable in the generall rules: and hath noe more speciall, then the very nature of the woork dooeth require. For as concerninge the signes of — + — and — — —, the same order is here, as is in multiplication. And touchyng the *Coslike* signes, it is all one with that I saied in diuision of numbers *Coslike* vncompounde.

Scholar. When a fewe examples maie supplie the
 D. v. declaration

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Declaration of the vse of the rules, with the practike
woozke.

Master. Take these for your purpose.

An example of the firste woozke.

$$\begin{array}{r}
 60. \\
 12. \cancel{3} \cancel{3} \text{---} | \text{---} 76. \text{C.} \text{---} | \text{---} 80. \cancel{3}. (2. \cancel{3}. \\
 6. \cancel{3}. \text{---} | \text{---} 8. 20.
 \end{array}$$

The removing of the diuisor,
for the seconde woozke.

$$\begin{array}{r}
 60. \\
 12. \cancel{3} \cancel{3} \text{---} | \text{---} 76. \text{C.} \text{---} | \text{---} 80. \cancel{3}. (2 \cancel{3} \text{---} | \text{---} 10 20 \\
 6. \cancel{3}. \text{---} | \text{---} 8. 20.
 \end{array}$$

The prooffe in numbers *Abstrakte*,
accountptyng. 2. for roots.

$$\begin{array}{r}
 3 \qquad \qquad 480. \\
 12. \text{---} | \text{---} 608. \text{---} | \text{---} 320. (8. \\
 24. \text{---} | \text{---} 16.
 \end{array}$$

$$\begin{array}{r}
 480 \\
 12 \text{---} | \text{---} 608 \text{---} | \text{---} 320. (8 \text{---} | \text{---} 20. \\
 24. \text{---} | \text{---} 16.
 \end{array}$$

The same woozke in
vulgare forme.

$$\begin{array}{r}
 \cancel{3} \\
 120 (28. \\
 440
 \end{array}$$

Here I haue not onely parted
the woozke, for your ease in vn-
derstanding: but I haue also put
against it, the declaration of the
same, by resoluyng the *Cosike*
nōbers, into numbers *Abstrakte*.

And finally, I haue putte one example of the same
numbers,

of Coslike numbers.

numbers, after the vulgare forme: all whiche. 3. agree together: and haue one an other.

Scholar. Yet I praye you worke, one example more.

Master. Here is an other.

¶ The firste extraction
of the diuisio.

$$\begin{array}{cccc} 48. \text{z} . \text{c} . & - | & 48. \text{z} . \text{z} . & - | & 20 \text{c} . & - | & 24. \text{z} . & (8. \text{z} . \text{z} . \\ 8. \text{z} . & - | & 6. \text{z} . & & & & & \end{array}$$

¶ The remouynge for-
ward of the diuisio.

$$\begin{array}{cccc} 48 \text{z} . \text{c} . & - | & 48 \text{z} . \text{z} . & - | & 20 \text{c} . & - | & 24 \text{c} . & (8 \text{z} . \text{z} . & - | & 4 \text{z} . \\ & & & & 5. \text{z} . & - | & . \text{z} . & & & . \text{z} . \end{array}$$

¶ The comprobation of the same by resolu-
tion, accomptyng still. 2. for a roote.

$$\begin{array}{cccc} 2868 & - & 768 & - & 160 & - & 48. & (128. \\ 28 & - & 8. & & & & & \end{array}$$

¶ The setting forward of the diuisio.

$$\begin{array}{cccc} 2868 & - | & 768 & - | & 168 & - | & 48. & (128 & - | & 8. \\ & & & & 28 & - | & 8. & & & \end{array}$$

Scholar. Yet ones again, I praye you worke the like.

For although I perswade my self, that I perceiue the worke: yet would I see more confirmation of it, before I would be to constant in my persuasion.

Master. Good aduise mēte is euer sure: but if you doubt, your counselloure is not farre absente.

Scholar. I maie iustly reioice thereof: But for e-
uery mater to require aied, and neuer to trauell my
owne witte, it might seme mere dastardinesse. And

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so were it plaine babithenesse, to couet enery mozell,
to be chaſed befoze hande, and put into my mouth.

Maſter. When take this other example, in one
platte complete: But with a caueat, to beware of to
moche confidence, while you ſeeme to ſee doubtfull
daſterdincſſe.

$$\begin{array}{cccccccc}
 16. & & & & & & & \\
 14\cancel{3}c - | - 3\cancel{0}\cancel{3}\cancel{3} - | - 16\cancel{3} - | - 6\cancel{3} - | - 8c (7c - | - 8z - | - 3\cancel{3} \\
 2.c. - | - 2.zc. & & & & & & & \\
 2.c. - | - 2.zc. & 2c - | - 2zc
 \end{array}$$

Scholar. Now haue I, that I looked for.

Maſter. Softe, lette vs trie this woozke, as wee
haue doen the other: befoze we goe from it.

Scholar. I praye you let me doe it.

Maſter. With a good will.

| | | |
|-----|-----|--|
| 64 | 16 | Scholar. I kepe ſtill the old roots 2. Then is the. $3c$. 64: whiche be- ing multiplied by. 14. maketh. 896. And ſo. 30.33 . doe yelde. 480. And 16. ſquares make. 64. All thei toge- ther yelde. 1440. |
| 14 | 30 | |
| 256 | 480 | |
| 64 | | |
| 896 | | |

The reſte of the numbers, muſt be abated, 896
becauſe of the ſignes. ———. and thei make 480

| | | |
|------|----|---|
| 32 | 8 | 240. For euery. $3z$. is. 32. and then. 6. times that, that makeh 192. whereunto I put. 48. for 6. Cubes: and ſo haue I. 240. to be ab- ated out of. 1440. and then remaineth. 1200. for the diuidende. The diuiſor is but. 20. ſith. 2. c. |
| 6 | 6 | |
| 192. | 48 | |

are. 16. and. 2. rootes make. 4, 1440

If I diuide now. 1200. by. 20. 240
the *quotiente* will be. 60. agreeably 1200
to the former *quotiente*. For 7. c. make. 56
And

of Coslike numbers.

And. 8. rootes yelde. 16. that is. 72. From whiche I must abate. 3. 8. that is. 12. And then it is iuste. 60.

Master. This is well doen.

Scholar. Yea sure, I am perfecte inough, in this feate of diuision, I tro we.

Master. You doe well to doubt.

Scholar. I thinke my self sure without doubt: As by one or twoo examples, I will declare.

And first I take this nōber 322 b/8 — + — 115 8. ℄
 — 42. ℄ — + — 69. 8. — + — 30. 7e. to be diuided
 by. 14. 8. — + — 5. 7e. wherefoze I sette them doune
 thus.

$$\begin{array}{r}
 322\text{ b/8} \text{ — + — } 115\text{ 8. } \text{℄} \text{ — } 42\text{ ℄} \text{ — + — } 69\text{ 8.} \text{ — + — } 30\text{ 7e} \text{ (23/8 — + — } 3\text{ 7e} \\
 14\text{ 8.} \text{ — + — } 5\text{ 7e.} \quad | \quad 14\text{ 8.} \text{ — + — } 5\text{ 7e.} \\
 \hline
 32\text{ b/8} \text{ — + — } 115\text{ 8. } \text{℄} \quad | \quad 42\text{ ℄} \text{ — + — } 15\text{ 8.}
 \end{array}$$

And finde the firste *quotiente* to bee . 23. 8. by whiche I multiplie the diuisor, and it taketh awaie all the numbers ouer it: Wherefoze I set the diuisor forward, & finde 37e. for the *quotiente*, whiche I multiplie into the diuisor, & it maketh 42 ℄ — + — 15 8. Whereby I am at a stale. For although I see in the diuidende, the like numbers, yet the signe of ——— declareth, that it is not possible, to abate this newe nōber thens: sepng ——— 42. ℄. is lesse then naughte.

Master. Wherefoze consider it, in chosyng your *quotiente*: and giue your *quotiente* the like signe.

Scholar. But then riseth an other doubt. For there will be ———. 15. 8. whiche disagreeeth in signe from the number ouer it.

Master. Yet maie you subtracte it well inoughe, if you haue not so gotten, your rules of subtraction.

Scholar. Now I dooe better remember my self: that by good reason, I must leaue as a remainer, not onely the whole number ouer it, whiche is . 69. 8. but

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but I must adde thereto. 15. 3. more.

So shall I cancell the. 69. and set ouer it. 84. And then doe I remoue the diuisor forward, setting 14 3 vnder. 84. 3. and the reste in order, wherebp I perceiue, that the newe *quotiente* will be. —. 6. 9.

$$\begin{array}{r}
 84. \\
 322.6/3 \text{ --- } 1153 \text{ --- } 42 \text{ --- } 683 \text{ --- } 30. \text{ --- } 23/3 \text{ --- } 7e \text{ --- } 69 \\
 \hline
 14. 3. \text{ --- } .8. 7e \text{ --- } 143. \text{ --- } .8. 7e. \\
 \hline
 322.6/3 \text{ --- } 1153 \text{ --- } 42 \text{ --- } 183. \\
 \hline
 84. 3. \text{ --- } 302e. \\
 \hline
 14. 3. \text{ --- } .8. 7e.
 \end{array}$$

Whiche *quotiente* I doe multiplie into the diuisor, and it doeth make. 84. 3. —. 30. 2e. agreeable to the somme ouer it. And so there remaineth nothyng.

Master. You haue dooen well. But in choysege your diuidende, and the diuisor, your lucke was better then your cunningg.

Scholar. What shall I proue againe, by an other example, takyn also at all aduentures.

I would diuide this somme.

$$16. 3. \text{ --- } 20. 3. \text{ --- } 12. 7e. \text{ --- } .8. 9. \text{ by } 4. 3. \text{ --- } 2. 7e. \text{ And therfore I set them doune in order thus.}$$

$$16. 3. \text{ --- } 203 \text{ --- } 127e \text{ --- } 89. (4. 3. 3. \\
 4. 3. \text{ --- } 2. 7e.$$

And firste I see, that. 4. is contained in. 16. sower tymes: and so maie I finde. 2. in any other numbers there. 4. tymes. Wherfore I set. 4. in the *quotiente*.

And bicause the. 4. in the diuisor are. 3. and the 16 to bee diuided, are. 3. C. accordyng to the former rules, I finde the newe denomination *Coslike* to be. 3. 3. whiche

of *Coslike* numbers.

whiche I set in the *quotient* with 4 and so is it. 4. $\frac{8}{2}$

Then saie I. 4. $\frac{8}{2}$. multiplied by. 4. $\frac{8}{2}$. do make 16 $\frac{8}{2}$. and therfore cleareth and consumeth al that some ouer it. Then farther saie I 4. $\frac{8}{2}$. multiplied by. 2. $\frac{8}{2}$. doe yelde. 8. $\frac{8}{2}$: But I see noe solche deno-
mination in the diuidente.

Master. Then maie you perceiue, that you haue missed.

Scholar. Why sir, I thinke I ought to doe as you did: that is to multiplie the *quotiente* into eucry parte of the diuisor.

Master. That is true: but I wil detecte the faute vnto you. And that is this.

That all numbers *Coslikes* compounde, can not bee diuided orderly, by diuisors compounde. And those that can bee diuided, will not receiue any other diuisor. of the same kinde, but one of. 2. numbers, by multiplication of whiche, it was made: and so the other of those. 2. shall be the *quotiente*: As it came to passe in all those. 3. examples, which I set forthe. And therfore it is losse labour, to goe aboute to diuide theim in that sorte.

Scholar. Then are there but selue numbers of *Coslikes* compounde, that maie be diuided.

Master. So many men saie. But I saie thereto, that though many of them can not be diuided, by like numbers *Coslikes* compounde, yet are there many thousandes, that maie be so diuided.

And again I saie, that all sortes of theim, maie bee diuided, by an *Abstract* number. And also any of them maie be diuided, by conuersion into a fraction: And so maie your example be set thus.

$$\begin{array}{r} 16. \frac{8}{2}. \text{c.} \quad | \quad 20. \frac{8}{2}. \quad | \quad 12. \text{z.} \quad | \quad 8. \text{g.} \\ \hline 4. \frac{8}{2}. \quad | \quad 2. \text{z.} \\ \text{Z. j.} \end{array}$$

And

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And in all other cases like, sette the diuidende ouer a line, and the diuisor vnder the same line, and so is your diuision randed: and this is the reddicste waie, and the moſte indifferencie, in all ſoche numbers.

Scholar. What is ſone learn'd. And theretoꝛe needeth no moare examplcs.

It is like in numbers *Abſtrakte*, when the greater number, doeth diuioe the leſſer. As. 6. diuided by. 11. maketh $\frac{6}{11}$.

Maſter. Somewhat like it is. Howbeit here is a woꝛke moare like therevnto, as when we ſhould diuide the leſſer *Coſiſke* number, by the greater, ſoꝛ then we muſt ſet them in that ſoꝛme. So. 6. ζ . diuided by 7. \mathcal{C} . ſhall be ſet thus: $\frac{6}{7\mathcal{C}}$. And. 20. \mathcal{C} . diuided by 5. ζ . muſt ſtande in this maner: $\frac{20\mathcal{C}}{5\zeta}$.

Scholar. Why? 20. maie be diuided by. 5.

Maſter. But. \mathcal{C} . can not be diuided by ζ . And in *Coſiſke* numbers, the chief regard is to be had, to the *Coſiſke* ſignes.

Scholar. Then, as ſoꝛ any other ſoꝛme, of regular diuiſion, here is none.

Maſter. Doe, excepte your diuiſoꝛ, bee a number *Abſtrakte*: Or at the leaſte, if it haue one onely *Coſiſke* ſigne, and be uncompounde, that ſigne muſt be other equalle, oꝛ leſſer then the leaſte *Coſiſke* ſigne, in the diuidende.

For ſo. 60. ζ . \mathcal{C} . — + — 48 \mathcal{C} . — + — 18. ζ . maie bee diuided by any number, hauyng one of theſe. 3. ſignes *Coſiſke*. ζ . \mathcal{C} . \mathcal{C} .

Scholar. I vnderſtand it well. For. ζ . is the laſte ſigne in the diuidende: And. \mathcal{C} . and. \mathcal{C} . arc not onely leſſe then it, but alſo. \mathcal{C} . leaueth the number, as if it were a number *Abſtrakte*.

So if I would diuide your number, aſſigned by 40. ζ . the *quotiente* would bee thus.

60. ζ .

of Coſſike numbers.

$$\begin{array}{r}
 60.8. \text{C} - | - 48 \text{C} - | - 188. (1\frac{1}{2}8. 8 - | - 1\frac{1}{2} \text{C} - | - 39. \\
 40.8. \qquad \quad 40.8. \qquad \quad 40.8.
 \end{array}$$

Maſter. Beſoze we cande this worke of diuiſion, I will admoniſhe you, of one caſſe alſo, in the diuiſio of diuerſe numbers. And that is, to conſider, whether your diuidende, doe omit any Coſſike denominations, betwene them, whiche it hath. For if it doe, you muſt yet ſupplie their roomes, with ſignes and Ciphers. As by example, you ſhall vnderſtande.

I require to haue this number. 8. C. — | — 64. 8.
 diuided by. 2. 8. — | — .4. 8.

Scholar. What will I doe quickly. For I ſee. 4. will be the firſt *quotiente* and his denomination will be. 8. ſith. C. diuided by. 8. doe make. 8.

But firſt I ſette downe the numbers orderly. And then I multiplie the diuiſor by the *quotiente*, & there riſeth. 8. C. — | — 16. 8.

Maſter. Stande you now amaſed, for all your greate confidence: You ſee that you can not finde any 8. in the diuidende. Therefore ſet downe the number as I told you beſoze, in this ſorte.

$$\begin{array}{r}
 \text{—————} 168. \\
 8. \text{C} - | - .0.8. - | - .0.8 \text{C} - | - 46.8. (4.8. \\
 2.8 \text{C} - | - .4.8. \\
 \hline
 8. \text{C} - | - .16.8.
 \end{array}$$

And then I take the ſame *quotient* that you did, and I finde the remainer to be. ———. 16. 8. Therefore I doe again ſette forward the diuiſor: And finde the *quotiente* to be ———. 8. 8. by whiche I multiplie the 2. 8. diuiſor,

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diuisor, and it maketh. $16. \text{z}$. ———. $32. \text{ze}$ so that a-
batyng the. $16. \text{z}$. the reste, that is, ———. $32. \text{ze}$.
shall be the remainder with the signe — — by the rule
of subtraction.

$$\begin{array}{r}
 \text{—————} 16\text{z} \text{ ———} + 32\text{ze} \\
 \text{s } \text{c} \text{ ———} + 2\text{z} \text{ ———} + 2\text{ze} \text{ ———} + 64\text{q} (4\text{z} \text{ ———} + 8\text{ze} \text{ ———} + 16\text{q} \\
 \phantom{\text{s } \text{c} \text{ ———}} 2\text{ze} \text{ ———} + 4\text{q} \\
 \phantom{\text{s } \text{c} \text{ ———}} 2\text{ze} \text{ ———} + 4\text{q}
 \end{array}$$

Then vnder that remainder, I remoue the diuisor,
and finde the newe *quotiente* to bee — — $16. \text{q}$. And
so is the number clerely consumed.

Scholar. If I forgette any parte of this, I am de-
ceiued to foule.

Master. Then haue you learned this parte, well
inough, for this tyme. And therfore will we go forth
vnto fractions, whiche partly were omitted befoze,
and partly are compoude of them self.

Of fractions, and their numeration.



Fractions of this kinde appere sim-
ple: and yet are scante so to bee iud-
ged: as $\frac{4}{3} \text{z}$ betokeneth $4. \text{z}$. to bee
diuided by. $3. \text{c}$. Likewates this
fractio $\frac{12}{5} \text{z}$ doeth import that $12. \text{z}$
muste bee diuided by. $5. \text{z}$ c. But
 $\frac{10}{19} \text{z}$ betokeneth. $10. \text{z}$. to bee parted
into. $19.$ portions.

And here shall you note, the doubtfull forme, that
many menne in this arte vse, whiche write that laste
fraction thus. $\frac{10}{19} \text{z}$. where as this fractio doeth repre-
sent $\frac{10}{19}$ of a square: and not $10. \text{z}$. to be diuided by. $19.$

Scholar. Because you saie, that some doe so vse it,
and

of Coslike numbers.

and I would gladly excuse all good wryters: I maie saie for them that as in bulgare numbers, when. 10. should be diuided by 19. And is set thus $\frac{10}{19}$ it doeth importe bothe that. 10. is diuided into. 19. and also that euery portion of those. 19. is $\frac{10}{19}$ of an vnitie: so that if 10. l. should be parted amongest. 19. men, euery man should haue $\frac{10}{19}$ of. 1. l.

Master. Your wordes haue so moche apperance that they maie persuade hym, that is not very precise in termes, especially seying there is no other *quotiente* there, but the same number. But as the somme of 10. l. being diuided by. 19. is farre more then $\frac{10}{19}$ of an vnitie: So. 10. s. to bee diuided by. 19. differ moche from $\frac{10}{19}$ of a square. For the one is 19. tymes so moche as the other. And therfore oughte to haue a distincte forme in wrytyng.

Scholar. When you would haue me to wryte the so, that $\frac{10}{19}$ of a square, should haue the signe against the line, as here is set $\frac{10}{19}\text{Z}$: and when I would represent. 10. s. diuided by. 19. I shall wryte it thus. $\frac{10}{19}\text{Z}$. With the signe about the liue.

Master. You maie see their agremente, and their difference by resolution, in this maner $\frac{10}{19}\text{Z}$ will make $\frac{10}{19}$ accountptyng. 2. for a roote, and $\frac{10}{19}\text{Z}$. maketh $\frac{10}{19}$ of 4. 9. $\frac{10}{19}$ of. 1.

Again, accountptyng. 3. for the roote, then $\frac{10}{19}\text{Z}$ yeldeth $\frac{30}{19}$: and $\frac{10}{19}\text{Z}$ maketh $\frac{20}{19}$ of an vnitie: so they appere to bee equall in valewe by reduction.

But now maie you see, that the one doeth betoken the firste namber, whiche is to be diuided: and the other doeth signifie the *quotiente* of the diuision: and so are they distincte in office and nature. But because by resolution, the one tourneth into the other, therfore many men account them as one. Howbeit, we stand to longe aboute this, consideryg the erreure, is not alwaies daungerous.

The Arte

But their ouersight is moze dangerous, whiche misplace the signe, when it should bee sette vnder the line: as a greate clerke doeth (except I shall fo: his excuse, impute the faulte to the printer) fo: he meaning to diuide. $\frac{3}{7}$. by. 7 . $\frac{3}{7}$. $\frac{3}{7}$. writeth it thus. $\frac{3}{7}$. $\frac{3}{7}$. where he should writc it thus. $\frac{3}{7}$. $\frac{3}{7}$: and againe, myndyng to diuide. 7 . by. $\frac{3}{7}$. he writeth it thus $\frac{7}{3}$. $\frac{3}{7}$. where he should writc. $\frac{7}{3}$.

Scholar. This faulte is manifeste, and detecteth the firste negligence: fo: $\frac{7}{3}$. doeth make in number, after the former resolution. $\frac{11}{3}$ and. $\frac{7}{3}$. dooeth make. $\frac{7}{48}$.

Maister. Well, seying you perceiue the faulte, we will stande no longer aboute it. Wherefoze to procede distinctly and certainly, whether that fraction be compounde, or simple, where the numerator is a *Cosike* number, and the denominator, a number absolute, yet maie you boldly thinke, that fraction to bee compounde, whose numerator is a number *Cosike* and the denominator an other *Cosike* of unlike signe: as. $\frac{3}{12}$ and $\frac{3}{12}$.

Yet as in numbers Abstracte, it maie seme moſte aptly to bee called a fraction, when the numerator, is lesser, then the denominator, so in numbers *Cosike*, moſte aptly the signe of the denominator, should bee the greater. Yet bothe formes come in vse.

And fo: because casinesse in wo:kyng, doeth oftentimes bying certaintie with it befoze we take in ha:de the addition of fractions, I thinke it good to speake somewhat of Reduction, to an other denomination. So that you fo: gette not, that any. 2. numbers *Cosike* compounde, with a line betwene them, maie be called a fraction. As thus. $\frac{3}{3} + \frac{3}{3} = 6$ that is, 3 . 3 . $+$. 3 . 3 . $=$. 6 . 3 . to bee diuided by 3 .

Examples of Numeration.

33. — † . 12. 9. and so of other like.

Of Reduction of fractions.



Fractions *Cosike*, not onely in their numbers, but also in their signes maye be reduced to other valuations, and namely to their leaste termes, and yet continue still in one proportion, betwene the numerator, and denominator.

So $\frac{12}{7}$ maye bee reduced to $\frac{36}{21}$: for so high as. C is above. 9. that is in the thirde place from it: So is 3. 3. in the thirde place about. 21.

Againe. $\frac{17}{8}$. by reduction doeth make $\frac{34}{16}$: And so $\frac{17}{8}$ will bee by reduction.

And so in all other fractions, where the numbers bee commensurable.

But if any one number, bee incōmensurable with the other, then can there be made no reduction in the numbers. Yet in the signes *Cosike*, there maye be a reduction, ether to greater, or to smaller signes: For those signes be ever commensurable.

And there is no exception, but they maye bee reduced to smaller quantities, excepte any one quantitie of them bee. 9. that is a number. For that can bee no smaller. And therefore none other maye be altered, with every one must be abated alike.

And looke how moche, the smalleste quantitie of that fraction, is above a number, so moche maye they all bee abated: for they are never reduced to the smalleste, till one of them be a number.

Scholar. And why maye not this reduction, serue to whole *Cosike* numbers?

Master. Because the whole number, doeth not cō

The Arte

list of a p^{ro}portion, as the fraction doeth, and so maie bee expressed in diuerse termes: but it impo^{ss}eth one somme certaine, whiche maie nother bee increased, no^r decreased, but it will chaunge his valewe, and alter his office.

And if I saie: a foote is $\frac{1}{2}$ of a yarde, I maie saie as trucly, increasynge bothe numbers, in the like p^{ro}portion, a foote is $\frac{4}{8}$ of a yarde: or in lesser termes: a foote is $\frac{1}{3}$ of a yarde.

But when I saie in whole number, a yarde is . 3. foote, or a foote is . 12. ynches, I saie trucly: and if I doe increasc or abate any of those numbers, my wo^ordes will be false.

So although in this number. 8. $\frac{3}{4}$. — | — . 6. $\frac{2}{3}$. — | — | — 10. $\frac{3}{4}$. by reason of bothe numbers and signes, there might bee a reduction, yet bicause it is a whole n^ober, it should therby bee abated moche: as here you maie see. 4. $\frac{2}{3}$. — | — | — 3. $\frac{2}{3}$. — | — | — 5. $\frac{4}{5}$. whiche by resolution into vulgare numbers, 2. beyng sette as the roote, doeth make. 32 — | — 6. — | — 5. that is. 33. and the other number befoze, doeth yelde by the like resolution. 256 — | — 48. — | — 40. that is. 264. and is 8. tymes so moche as the other.

Scholar. I perceine now good reason, why reduction scrueth for fractions onely. And if there bee noe moze difficultie in it, then you haue declared. I can wo^orke it easly.

Reduction in signes onely For other the reduction consisteth in the signes *Cosike* onely, as $\frac{10}{13} \frac{3}{4}$ where the numbers bee vncommensurable, and therfoze can not bee altered to any lesser termes. But the signes *Cosike* maie bee abated by . 3. denominations: seynge the smalleste of them, is so many in order aboue. 9. And therfoze it maie be reduced to $\frac{108}{139}$.

Reduction in n^obers onely Other els secondarily, the reduction consisteth in the numbers onely, when the numbers be communitate.

of Cossike numbers.

cante. And the signes *Cossike* bee all redy at the leaste: as when one of them is. $\frac{9}{11}$. So $\frac{100}{119}$ will bee reduced to $\frac{100}{119}$.

¶ els thirdly, the reduction maie bee wroughte, *Reduction in signes and numbers.* When all the signes be about. $\frac{9}{11}$. and the numbers be communicant

So $\frac{100}{119}$ maie be reduced well vnto. $\frac{100}{119}$.

Master. Yet one forme of reduction moze, I will shewe you, where not onely the like woorkie maie be, *An other reduction.* but also the number maie be broughte from his composition, to a moze simplicitie, by abatynge some of his partes.

As this number $\frac{100}{119}$ maie bee reduced, firste by his numbers to $\frac{100}{119}$.

Secondarily, by his signes it maie be altered thus.

Thirdely, by abatynge the numbers, that followe signe of composition (that is $\frac{9}{11}$) it maie be brought to $\frac{100}{119}$. or $\frac{100}{119}$. whiche fractions, kepe the self same proportion, that the firste fraction did.

Likewaies with the signe of $\frac{9}{11}$. numbers residuales, maie bee reduced. As $\frac{100}{119}$ will bee reduced, as the other was to $\frac{100}{119}$.

Scholar. This is vnto me a marvellouse mater, that those. 2. contrary numbers, should be reduced to one fraction.

Master. The like happeneth in vulgare numbers. For $\frac{100}{119}$ will bee reduced to $\frac{100}{119}$. For firste it maketh $\frac{100}{119}$ and then $\frac{100}{119}$. So likewaies $\frac{100}{119}$ will make firste $\frac{100}{119}$ and then $\frac{100}{119}$.

And the reason of it, doeth depende of the. 19. proposition, of the fifth booke of *Euclide*, where it is written thus.

Aaj. If

The Arte

If the proportion of the abatements vnto abatements be, as the whole is in proportion to the whole. Then shall the residue bee in like proportion to the residue, as the whole is to the whole.

That is in the laste example. As, 18. is vnto, 24. so is 6 vnto 8. Therefore shall 12 be to 16. as 18. is to 24.

And for to exercise you the better, loe, here are one or twoo examples more, of the like reduction.

$\frac{7c}{8f}$ maketh $\frac{7c}{8f}$ or $\frac{7g}{8g}$. Again $\frac{192g}{288g}$ yeldeth $\frac{192g}{288g}$ or $\frac{48g}{72g}$.

But this muste you farther marke, that in *Coslike* numbers, not onely the numbers, but also the *Coslike* signes must bee, accoꝝdyng to *Euclides* proposition.

Scholar. What doe I see.

For in the laste example: As, 9. is to, 3. so, 2. is to, 2.

And in the nexte example before: As, 2. is to, 3. so is, 3. to, 3.

Likewaises in the other examples, as 2. is to 3. so is, 3. to, 3.

All this is good and reasonable.

Master. How doe you see, bothe the maner of reduction, and also some reason for it. Therefore I will procede, to declare the woꝝke of Addition.

Of Addition and Subtraction.



Addition there is nothyng more, then you haue learned before: For as for the multiplications of the denominatoꝝ together, and then crosse waies with the numeratoꝝ of thother, is iuste agreable with the reductions of Abstracte fractions, to byng them to one common denominatoꝝ

of Cossike numbers.

nominatoꝝ.

And then the numeratoꝝ added together, dooe make the newe numeratoꝝ in addition.

And likewise the lesser numeratoꝝ, subtracted fro the other, doeth make the numeratoꝝ in subtraction: wherfoꝝe a fewe examples maie suffice.

Examples of Addition.

| | |
|--|---|
| 54.ʒ. — — .28.℥. | 40.ʒ. — — .42.ʒ℥. |
| $\frac{7}{7}$.ʒ. to $\frac{4}{2}$.℥. | $\frac{1}{2}$.ʒ. to $\frac{7}{8}$.ʒ℥. |
| 63. | 48. |

| | |
|---------------------------------|---------------------|
| That is in smal- ler termes. | 20.ʒ. — — .21.ʒ℥. |
| | 24. |

Here you see how the .2. fractions be sette betwene .2. lines: and vnder the nethermoste line, is sette the newe denominatoꝝ: and ouer the higher line, are set the .2. newe numeratoꝝ ioyned in one.

The firste of them, can not be reduced to any smaller termes, bicause the numbers be not all .3. commensurable: & the denominatoꝝ, also is a number *Abstract*.

The seconde hath also a number *Abstracte* foꝝ his denominatoꝝ, and therfoꝝe there can be noe reduction in signes: but the numbers all .3. beyng commensurable, & diuisible by .2. maie be reduced, as there you see.

More examples of Addition.

| | |
|---------------------------------------|---------|
| 16.ʒ. — — .4.℥. | |
| 12.ʒ. — — .9.℥. to 4.ʒ. — — .5.℥. | |
| 20.ʒ.℥. | 20.ʒ.℥. |
| 20.ʒ.℥. | |

An. y

That

The Arte

What is in final
ler termes.

$$\frac{4. \text{z} . - + - . 1. \text{q} .}{5. \text{c} .}$$

Here is noe multiplication wroughte , bicause the denominatozs are like.

Another Example of Addition.

$$\begin{array}{r} 5. \text{z} . \text{c} . - + - . 20. \text{c} . - - - . 3. \text{f} \text{z} . \\ \hline 5. \text{z} . \text{c} . - + - . 3. \text{f} \text{z} . \quad \text{to} \quad 20. \text{c} . - - - . 6. \text{f} \text{z} . \\ \hline 6. \text{c} . \text{c} . \quad \quad \quad 6. \text{c} . \text{c} . \\ \hline 6. \text{c} . \text{c} . \end{array}$$

What is in les
ser termes.

$$\frac{5. \text{c} . - + - . 20. \text{q} . - - - . 3. \text{z} .}{6. \text{z} . \text{c} .}$$

Here is noe multiplication , noz reduction to one common denominatoz: sith thei bee one all ready: noz ther can the numbers be reduced, to any other lesser: but the quantities onely be reduced as you see.

Scholar. I praye you let me proue.

Another Example.

$$\begin{array}{r} 80. \text{b} \text{f} \text{z} . - + - 90. \text{z} . \text{c} . - + - 60. \text{z} . \text{c} . - - - 30. \text{f} \text{z} . \\ \hline 8. \text{c} . - + - 9. \text{z} . \quad \text{to} \quad 6. \text{c} . - - - 3. \text{z} . \\ \hline 10. \text{c} . \quad \quad \quad 10. \text{z} . \text{z} . \\ \hline 110. \text{b} \text{f} \text{z} . \quad \quad \quad \text{What is} \end{array}$$

Master. Marke your woꝝke well, befoze you reduce it.

Scholar. I see my faulte: I haue sette .2. numbers seuerally, with one signe *Cosike*: by reason I did not foresee, that, *c*. multiplied with, *c*. doeth make the like

of *Cosike* numbers.

like quantitie, as, $\frac{80}{3}$ multiplied by $\frac{3}{1}$. Wherefore it should be thus.

$$\frac{80 \cdot b/\frac{3}{1} \cdot + \cdot 150 \cdot \frac{3}{1} \cdot \text{℥} \cdot \text{---} \cdot 30 \cdot \frac{3}{1} \cdot}{110 \cdot b/\frac{3}{1} \cdot}$$

Whiche maie bee reduced, by meane of the numbers, to this somme.

$$\frac{8 \cdot b/\frac{3}{1} \cdot + \cdot 15 \cdot \frac{3}{1} \cdot \text{℥} \cdot \text{---} \cdot 3 \cdot \frac{3}{1} \cdot}{11 \cdot b/\frac{3}{1} \cdot}$$

And now considering the *Cosike* signes, and woꝝ kyng as I haue marked you to dooe: What is to abate the leaste signe, out of them all: bicause, $\frac{3}{1}$ is here the leaste, I abate it out of $b/\frac{3}{1}$, and there resteth, $\frac{3}{1}$, and so doing with the other signe, $\frac{3}{1}$ ℥, there remaineth, $\frac{3}{1}$ ℥ then $\frac{3}{1}$ out of $\frac{3}{1}$ doeth leaue, $\frac{3}{1}$ or nôber: So will the fraction bee thus: $\frac{80}{11} = \frac{150}{11} - \frac{30}{11}$ by reduction in signes and numbers also.

After. Seyng you haue so well marked the reduction of the signes (whiche follo weth the forme, taught befoze in diuision) I thinke it not nedefull, to staie any longer aboute this.

Wherefore we will goe foꝝward to subtraction, after that I haue admonished you of fractions, in apperaunce simple, whiche in deede by addition, bee come compoude. As this $\frac{3}{1}$ ℥, added to $\frac{3}{1}$ ℥, maie firste be added by the common signe of addition, thus.

$$\frac{3}{1} \text{℥} \cdot + \cdot \frac{3}{1} \text{℥} \cdot \text{whiche by reduction, into one denomination, wil be thus witten, } \frac{6}{1} \text{℥} \cdot$$

But as this is easie inough to vnderstand, so maie it helpe often times, foꝝ speidie woꝝke, as well in additiõ, as in subtractiõ, by the onely addyng of the signe.

As if I would subtracte this fraction $\frac{3}{1}$ ℥, out of $\frac{3}{1}$ ℥.

The Arte

$\frac{1}{10} \div \frac{1}{7} = \frac{7}{10}$. I maie write it thus. $\frac{7}{10} \div \frac{1}{7} = \frac{49}{10}$.
 And so is the Subtraction wroughte.

Yet maie you reduce theim, to one denomination, if you will, after the same forme, as you did in addition. And then will it bee. $\frac{63}{70} - \frac{10}{70} = \frac{53}{70}$. Whiche can not bee reduced to any smaller termes, bicause the numbers are not commensurable: and one of them (that is to saie, the denominato^r) is a number *Abstract*

Scholar. I see in this, there is no difference from Addition, but in the signes. — + and. — —. wherefore I will proue an other example, by your leaue.

I would subtracte $\frac{1}{4} \div \frac{1}{7}$ out of $\frac{4}{7} \div \frac{1}{7}$. and it will bee at the firste $\frac{4}{7} \div \frac{1}{7} = \frac{28}{7}$. And by reduction $\frac{28}{7} - \frac{7}{7} = \frac{21}{7}$.

Master. Your woork is well doen, acco^rdyng to your firste meanyng: But as the numerato^r of this laste reduction doeth declare, it can not bee well, that $15 \div \frac{1}{7}$ maie bee abated out of $16 \div \frac{1}{7}$. For the greater absolutely, can not well be abated out of the lesser: and therfore you might rather haue abated $\frac{1}{4} \div \frac{1}{7}$ out of $\frac{1}{4} \div \frac{1}{7}$.

Scholar. I see it well now: for the $\frac{1}{7}$ is alwaies double or triple, or yet more tymes greater, then the $\frac{1}{7}$. Bicause the $\frac{1}{7}$ commeth by multiplication of the $\frac{1}{7}$ by his firste roote.

Master. Yet here in is discretion to be vsed, for in fractions, sometyme the number of the greater signe maie be the lesser. As for example $\frac{1}{10} \div \frac{1}{7}$ is lesser then $\frac{1}{4} \div \frac{1}{7}$. as by resolution you maie proue, accompting 2. for the common roote.

Scholar. 2. beynge the roote. 32. is the $\frac{1}{7}$. and his $\frac{1}{10}$ maketh 6. then $\frac{1}{4} \div \frac{1}{7}$ beynge 12. dooth appere double to it: and therfore greater by moche.

If I doe by the like resolutioⁿ, proue the other fractions before, $\frac{1}{4} \div \frac{1}{7}$ will bee 24: and $\frac{1}{7} \div \frac{1}{7}$ will bee $12 \frac{1}{7}$: whiche is lesser moche.

of *Cosike* numbers.

So, I perceiue the greatnesse and smalnesse of the fractions, must be considered, as well in the numbers as in the *Cosike* signes. And farther, if their fractions be nigh of one greatnesse, or the fraction of the lesser signe the greater, then can not the subtraction, appeare reasonable.

Master. That is true, if those .2. fractions stande alone: els beyng partes of other numbers, it maie appeare reasonable inough. As in this example of compounde fractions. $\frac{1}{10} \mathcal{C}$. $-$ $\frac{1}{4} \mathcal{Z}$. maie bee abated out $\frac{1}{7} \mathcal{C}$. $-$ $\frac{1}{3} \mathcal{Z}$. and yet in the abatemente after $-$ not onely the number $\frac{1}{4}$ is greater, then $\frac{1}{7}$ in the other, but also, the *Cosike* signe. \mathcal{Z} . is greater then the other *Cosike* signe. \mathcal{C} .

Scholar. I consider it to be so: and yet $\frac{1}{7} \mathcal{C}$. doeth so moche exceede $\frac{1}{10} \mathcal{C}$. that it supplieth sufficiently the other default: els could it not be well doen.

But for this woorde, I must craue your helpe: because I haue not seen the like.

Master. You maie doe in this, as I saied befoze, generally for all subtractions.

Set doune bothe numbers in due order, so that the abatemente dooe folowe in order: and putte betwene them the signe of subtraction: as thus.

$$\frac{1}{7} \mathcal{C} - \frac{1}{3} \mathcal{Z} . \quad \frac{1}{10} \mathcal{C} - \frac{1}{4} \mathcal{Z} .$$

Wholbeit, if you will firste reduce euery compounde fraction, into one fraction, it will seme moze apte. As thus. $\frac{1}{7} \mathcal{C}$. $-$ $\frac{1}{3} \mathcal{Z}$. beyng reduced by additiō will make $\frac{15 \mathcal{C} - 10 \mathcal{Z}}{21}$. and by farther reduction of numbers. $\frac{3 \mathcal{C} - 2 \mathcal{Z}}{7}$. Likewates $\frac{1}{10} \mathcal{C}$. $-$ $\frac{1}{4} \mathcal{Z}$. will make by the firste addition. $\frac{15 \mathcal{C} - 30 \mathcal{Z}}{40}$. and by farther reduction $\frac{3 \mathcal{C} - 6 \mathcal{Z}}{8}$.

Now toyne them together, with the signe of subtraction, and thei will stande thus.

$$\frac{3 \mathcal{C} - 6 \mathcal{Z}}{8} - \frac{3 \mathcal{C} - 2 \mathcal{Z}}{7}$$

Scholar.

The Arte

Scholar. This doeth appeare verie strange vn-
to me: but by vse I shall finde it moze familiare: See-
yng I see the reason of this woꝝke, to agree with the
woꝝke of common fractions.

The prooffe.

But foꝝ pꝛooꝝe of it, I will resolue eche woꝝke, in-
to numbers absolute, accoumptyng. 2. foꝝ a roote.

Maſter. So shall you finde it true: But foꝝ caſie
woꝝke, take rather. 10. foꝝ the roote.

Scholar. I thanke you foꝝ your aide.

Then if. 10. be the roote, the ſquare will be. 100.
and the Cube. 1000. Now $\frac{2}{3}$ C. that is $\frac{2}{3}$ of. 1000.
is. 600. And $\frac{1}{3}$ of. 10. whiche is the roote, will bee. 6.
whiche bothe put together, doe make. 606. and that
is the greater number.

Then foꝝ the leſſer $\frac{4}{10}$ C. arc in this example. 400
Foꝝ the Cube beeyng. 1000. his $\frac{1}{10}$ is. 100. Againe
the ſquare beeyng. 100. $\frac{3}{4}$ of. muſt nedes bee
75. whiche beeyng put vnto. 400. dooeth
make. 475.

| | |
|---|------|
| When doe I abate. 475. out of. 606. and | 475. |
| there will reſte. 131. How now. | 131. |

Maſter. I perceiue you ſaie, as beeyng aſtoniſ-
hed, bicauſe in the former woꝝke, there is not leſte a
remainer: But the. 2. firſte ſommes enclly altered by
reduction, and ioyned together, with the ſigne of ſub-
traction: where in if you had continued your woꝝke,
you ſhould haue ſounde the ſame numbers.

Foꝝ. 3. C. muſt nedes bee. 3000. ſeyng. 1. C. is a
1000. And alſo. 370. are. 30: whiche bothe added to
gether, make. 3030. Diuide them by. 5. (as the deno-
minatoꝝ would) and it will be. 606. as the valewe of
the firſte fraction.

Then come to the later number: and you maie ſome
thinke that. 8. C. arc. 8000. And. 15. Squares are
1500. adde them together, and thei will make
9500. whiche muſt bee diuided by. 20. (as the deno-
minatoꝝ

of Cossike numbers.

minatoꝝ (intoꝝ) and there will a
 mounte. 475. the valewe of the lesser 11
 fraction: whiche numbers appeare the 9800 (475
 same, that were befoꝝe: and thereby 2220
 the woꝝke is good.

But if you will byng it to a remainer, doe thus.
 Reduce these. 2. newe fractions, into one denomina-
 tion.

Scholar. What can I doe, by multiplyng the nu-
 meratoꝝ together: that is. 20. by. 5. and thereof coun-
 meth. 100. whiche shall be the common numeratoꝝ:
 then must I multiplie in crosse wales, the numeratoꝝ
 of the firste, by the denominatoꝝ of the seconde, and
 contrarily.

So foꝝe the firste numeratoꝝ $3. \text{℥} . - + . 3. \text{z} \text{e} .$
 I woꝝke thus. And thereby $20 .$
 dooeth amounte (as you see) $\frac{60. \text{℥} - + - 60. \text{z} \text{e} .}{60. \text{℥} - + - 60. \text{z} \text{e} .}$. And foꝝe
 the seconde numeratoꝝ, I multiplie. $8. \text{℥} - + - 15 \text{z} \text{e} .$
 by. 5. and there doeth rise. $40. \text{℥} - + - 75. \text{z} \text{e} .$. eche
 of them haung one common numeratoꝝ. 100.

Wherfoꝝe, seing bothe numbers, haue one deno-
 minatoꝝ, I shall abate the lesser numeratoꝝ, out of the
 greater, as here in exauple is set foꝝe: and then the

$$\begin{array}{r} 60. \text{℥} . - + . 60. \text{z} \text{e} . \\ 40. \text{℥} . - + . 75. \text{z} \text{e} . \\ \hline 20. \text{℥} . - + . 60. \text{z} \text{e} . \text{ — } . 75. \text{z} \text{e} . \end{array}$$

remainer will bee (as you see). $20. \text{℥} . - + - 60. \text{z} \text{e} .$
 $\text{ — } . 75. \text{z} \text{e} .$. into whiche I muste adde the common
 denominatoꝝ. 100. and it will be thus.

$$\begin{array}{r} 20. \text{℥} . - + . 60. \text{z} \text{e} . \text{ — } . 75. \text{z} \text{e} . \\ \hline 100 . \end{array}$$

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How youe whether this remainer, doe not agree to
thother remainer befoze, in your trial: which was 131

Scholar. 200℥ do make. 20000. & 60℥. yelde
600: those 2 sommes I must adde together, bicause of
the signe. — + —. and it will be. 20600. then. 75. 8.
are. 7500. whiche I must abate from the
fozmer somme of. 20600. and there will 20600.
remain. 13100. for the numeratoz, and $\frac{7500}{100}$
100. for the denominatoz, thus. $\frac{13100}{100}$. 13100

Master. And what doe you thinke of it?

Scholar. By that I learned in the vulgare fracti-
tions, I knowe that it is iuste. 131. and so doeth it a-
gree precisely, with the fozmer prooffe.

Master. Well yet for moare exactnesse in this
wozke, I will farther reduce that fractiō, by diuiding
the numeratoz by the denominatoz: wherfoze. 20.℥
diuided by. 100. doeth yelde. $\frac{1}{5}$ ℥. And. 60.℥. diui-
ded by. 100. doeth make $\frac{3}{5}$ ℥. And lastly. 75. 8. di-
uided by. 100. will yelde $\frac{3}{4}$ 8. so is thesame fraction
so reduced $\frac{1}{5}$ ℥ — + — $\frac{3}{5}$ ℥. — $\frac{3}{4}$ 8. And now trie
what that is, by the fozmer prooffe.

Scholar. I maie sone perceiue, that $\frac{1}{5}$ ℥. is. 200.
when the Cube is. 1000: And so $\frac{3}{5}$ ℥. is. 6. whiche I
must adde together, and it will be. 206. Then $\frac{3}{4}$ 8. is
75. whiche if I dooe abate from. 206. there will re-
main. 131. agreeably as befoze. And so is this woozke
fully examined.

Master. Yet will I propounde one or two exam-
ples moze, partly to practise your memorie, and part-
ly to admonishe you, if you happen to see any soche
misse wroughte, in some other bokes (as I haue doen)
how you maie amende the erreure, and not staie at it.

Firste take this example. I would subtrate.

$$\begin{array}{r} 48.9. \\ \hline 12.7\ell. \quad \text{---} \quad 3.8. \end{array} \quad \text{out of} \quad \begin{array}{r} 489. \\ \hline 7.8. \end{array}$$

Scholar.

of Coflike numbers.

Scholar. I must first multiplie the denominatoꝛs together, and so it will make, as here is sette foꝛthe

$$84.℥. \text{---} 21.ʒʒ.$$

Then I multiplie the numerator of the firſte, by the denominator of the ſeconde, and it will bring

$$\begin{array}{r} 48.ʒ. \\ \underline{7.ʒ.} \end{array}$$

foꝛthe. 336.ʒ: whiche is the numerator foꝛ the abatemente.

336.ʒ. Afterward I multiplie the numerator of the ſeconde, by the denominator of the firſte, and it will make

$$\begin{array}{r} 12.℥. \text{---} 3.ʒ. \\ \underline{7.ʒ.} \end{array}$$

$$84.℥. \text{---} 21.ʒʒ.$$

$$\begin{array}{r} 12.℥. \text{---} 3ʒ. \\ \underline{48.ʒ.} \end{array}$$

$$576.℥. \text{---} 144.ʒ$$

Now if I ſubſtrate that

$$336.ʒ. \text{ out of } 576.℥. \text{---} 144.ʒ. \text{ it will bee}$$

$$\begin{array}{r} 96 \\ \underline{48} \\ 576.℥. \text{---} 144.ʒ. \end{array}$$

576.℥. --- 480.ʒ. foꝛ the abatemente that ſhould be ſubſtracted now, is ſette after the ſigne --- with the ſoꝛmer ſomme of. 144.

Finally, to make the remainer complete, as that laſte number is the numerator, ſo vnto it I muſt adde the common denominator. 84. ℥. --- 21. ʒʒ.

and it will bee. $\frac{576℥}{84℥} \text{---} \frac{180ʒ}{21ʒʒ}$, that is in leſſer termes

$$\begin{array}{r} 192.ʒ. \text{---} 160.℥. \\ \underline{288.} \quad \underline{7℥.} \end{array}$$

Maſter. Now proue your cunning in this ſoꝛne, ſubſtractyng it out of. $\frac{232℥}{84ʒ} \text{---} \frac{176ʒ}{21℥}$

Scholar. Firſte I muſt reduce them, to one common denominator; by multiplyng bothe denomina:

$$\begin{array}{r} 84.ʒ. \text{---} 21.℥. \\ 12.℥. \text{---} 3.ʒ. \\ \hline 1008.℥. \text{---} 252.ʒʒ. \\ 63.ʒʒ. \text{---} 252.ʒʒ. \\ \hline 63.ʒʒ. \text{---} 1008.℥. \text{---} 504.ʒʒ. \end{array}$$

Bb. y. toꝛs

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tozs together. And so wil it be. $63\sqrt{3} \text{ --- } 1008\text{C}$
 $\text{---} 504.\text{z}.\text{z}.$ as by speciall woorkes, I haue here
 proued.

Then doe I multiplie the numeratoz of the totall,
 by the denominatoz of the abatements, as here also I
 haue perticularly set foz the in woorkes, foz my owne
 ease, and auoidyng of erreure: And so I finde it to be
 $1056.\text{z}.\text{---} 6912.\text{ze}.$ $\text{---} 696.\text{C}.$ whiche
 shall bee the numeratoz of the totalle.

$$\begin{array}{r}
 232.\text{ze} \text{ --- } 576.\text{z} \\
 12.\text{ze} \text{ --- } 3.\text{z} \\
 \hline
 2784\text{z} \text{ --- } 6912.\text{ze} \\
 \text{---} 696.\text{C} \text{ --- } 1728.\text{z} \\
 \hline
 1056.\text{z} \text{ --- } 6912.\text{ze} \text{ --- } 696.\text{C}
 \end{array}$$

Then doe I multiplie the numeratoz of the abate
 mēte, by the denominatoz of the totalle (whiche thing
 is easily dooen, bicause the one number, is a number
Abstrakte) and so haue I foz the numeratoz of the aba
 tements. $4032.\text{z} \text{ --- } 1008.\text{C}.$

And seyng these two numbers, haue one common
 denominatoz, I shall abate the lesser numeratoz, out

$$\begin{array}{r}
 1056.\text{z} \text{ --- } 6912.\text{ze} \text{ --- } 696.\text{C} \\
 4032.\text{z} \text{ --- } 1008.\text{C} \\
 \hline
 6912.\text{ze} \text{ --- } 312.\text{C} \text{ --- } 2976.\text{z}
 \end{array}$$

of the greater, & so wil there be left foz the numeratoz
 of the remainer $6912\text{ze} \text{ --- } 312\text{C} \text{ --- } 2976\text{z}$
 unto whiche, I shall adde the common denominatoz,
 and then wil it be.

$$\begin{array}{r}
 6912.\text{ze} \text{ --- } 312.\text{C} \text{ --- } 2976.\text{z} \\
 \hline
 63\sqrt{3} \text{ --- } 1008.\text{C} \text{ --- } 504.\text{z}.\text{z}
 \end{array}$$

That

fo Cosbike numbers.

What is in lesser termes.

$$\begin{array}{r} 2304.9. \text{---} | \text{---} | .104.8. \text{---} | \text{---} | .992.ze. \\ \hline 21.8.8. \text{---} | \text{---} | .336.8. \text{---} | \text{---} | .168.℥. \end{array}$$

Master. You haue wrought it well. And hereby I coniecture, that you are experte inough in subtrac-
tion. Wherefore now we will goe in hand, with mul-
tiplication and diuision.

Of Multiplication.



And firste, concernyng multiplie-
tion, here is no more to bee saied,
then hath been taughte before.

For the numbers shall bee mul-
tiplied, as common fractions are
wonte to bee: that is to saie, nume-
rator, by numerator, and denomi-
nator, by denominator.

And for the chaunge of their denominations Cos-
like, the rules giuen before shall suffice: so that a fewe
examples shall sufficiently instruct you, in the worke
of it.

As this for the firste.

$$\begin{array}{r} 20.8. \text{---} | \text{---} | 19.ze. \\ 6.℥. \text{---} | \text{---} | 3.9. \\ \hline 120.8. \text{---} | \text{---} | 114.8.8. \\ \text{---} | \text{---} | 60.8. \text{---} | \text{---} | 57.ze. \\ \hline 120.8. \text{---} | \text{---} | 114.8.8. \text{---} | \text{---} | 60.8. \text{---} | \text{---} | 57ze. \end{array}$$

Where I dooe multiplie.

$$\begin{array}{r} 20.8. \text{---} | \text{---} | 19.ze. \text{ by } 6.℥. \text{---} | \text{---} | .3.9. \\ \hline 31.℥. \qquad \qquad \qquad 4.8. \end{array}$$

And these I shall multiplie, numerator by nume-
rator;

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ratoꝝ: where. 20. z . multiplied by. 6. c . dose make 120. fz . as the former table of multiplication, so change of *Cosike* signes doeth declare. And so in all the reste, there is no difficultie, if you remember that, that you haue learned befoze.

Scholar. I perceiue it well. And so the whole newe numerator will bee. 120. fz . — 114. z . z .
 — 60. z . — 57. ze . And the denominator will be. 124. fz .

So will the whole fraction bee.

$$\frac{120.\text{fz} \text{---} 114.\text{z}\text{z} \text{---} 60.\text{z} \text{---} 57.\text{ze}}{124.\text{fz}}$$

That is not to be reduced to smaller termes of numbers, because they be vncommensurable, but in *Cosike* signes, it mighte bee broughte to one letter, as.

$$\frac{120.\text{z}\text{z} \text{---} 114.\text{c} \text{---} 60.\text{ze} \text{---} 57.\text{q}}{124.\text{z}\text{z}}$$

Now will I proue an other number, as fortune doeth offer it to mynde. What is $\frac{32.\text{c} \text{---} 28.\text{z}}{21.\text{z} \text{---} 17.\text{ze}}$ to be multiplied by. $\frac{12.\text{z} \text{---} 10.\text{z}}{36.\text{z}}$?

An Absurde Matter. It appeareth that you take them, at all number ex- adventures. For your firste number, semeth to be an *absurde* number. Seeyng his numerator, is lesse then *then naught* naughte, in apperaunce. And then maie it not be diuided by any number; and moche lesse by so greate a denominator.

Scholar. It is easie to see, now that I am admonished thereof. For it is not possible, that any *Surfolide* number, can be lesse then so fewe tymes so moche, as the *Cube* of the same nature. Seeyng every *Surfolide* is made, by multiplying the *Cube* by the *Square* of the like *Roote*, but lesse then. 4. is there no *Square*. And therefore every *Surfolide*, doeth excede his *Cube* so fewe tymes at the leaste.

of Cossike numbers.

So that $32. \mathcal{C}.$ — $8. \mathcal{f}\mathcal{z}.$ were nothyng, and so is an *Absurde* nöber. And therfore $32. \mathcal{C}.$ — $28. \mathcal{f}\mathcal{z}.$ is moche lesse then nothyng, and is therby an *Absurde* number also.

Master. Yet maie your example serue, to teache and practise multiplication by, as well as any other.

And farthermore, I will tell you by this occasion, that I spake to you, moze after the opinion of the common number of artes men, then after my owne iudmente.

Scholar. I might thinke so, by termynge of your sentence: but yet was your sayng truc.

Master. Yet maie that fraction stand well, if you take a brokē number *Abstracte* for the roote. Although in whole numbers, it bec an *Absurde* number.

Scholar. That will I proue, by setting $\frac{3}{4}$. for a Roote. When will the Square be $\frac{9}{16}$. and the Cube $\frac{27}{64}$. Also the Square of squares will bec $\frac{81}{256}$. And the Surfolide $\frac{243}{1024}$.

$\frac{3}{4}$ The Roote.

$\frac{9}{16}$ The Square.

$\frac{27}{64}$ The Cube.

$\frac{81}{256}$ Thezenzenzike

$\frac{243}{1024}$ The Surfolide.

And now to proue by resolution, how my number will rise, I take $32. \mathcal{C}.$ that is $\frac{64}{2}$, or $3. \frac{1}{2}$. whiche I note as the firste somme. When I take likewates $28. \mathcal{f}\mathcal{z}.$ whiche yeldeth $\frac{68}{16}$, that is $6. \frac{10}{16}$. And now I see that I maie abate it very well, out of $13. \frac{1}{2}$.

Master. So maie you see, that as in whole numbers, euer moare the greater *Cossike* signes, will haue the greatestte numbers: So in fractions resolued by *Cossike* signes, the greatestt fraction, aunswereth to the leaste signe: and the leaste fractiō, agreeth to the greatestte signe.

The reason of it is this. That the moare any fraction is multiplied by a fraction, the lesser it wareth. For as whole numbers by multiplication, maie increase infinitely: so fractions by multiplication, maie decrease

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decrease infinitely.

But before we passe from multiplication, I will proue you with one example more. I would haue $\frac{100}{78} - \frac{170}{69} = 9$ multiplied by $\frac{24}{53} = 70$.

Scholar. I am troubled with the multiplier. For I knowe not what to make of it?

Master. You doubt (I thinke) of the numeratio of it, because you had not the like example before: for it is a mixte number of a fraction, and a whole number. But seying the signe of abatemente is set against the whole fraction, and nother againste the numerator, nor denominator, therfore must that 40 be vnderstande, to be abated out of the full fraction.

Scholar. Now I perceiue the mater. For there might be 3 diuerse formes, to place that abatemente. As here I haue set them. $\frac{240}{53} - \frac{470}{69} = \frac{140}{17}$.

And as it was set by you, $\frac{24}{53} - 40 = 70$, whiche I will resolue into absolute numbers, to see their difference the better. And so, taking 3, for the roote, these will be their 3 formes.

The firste.

For the firste $\frac{648}{81} - 12$, or els $\frac{616}{81}$ that is $\frac{112}{17}$.

For the seconde $\frac{648}{81} - 12$, or els $\frac{648}{69}$ that is $\frac{116}{17}$.

And for the thirde number, whiche is our specialle number, $\frac{648}{81} - 12$, that is 8. ——— 12. and is an *Absurde* number. For it betokeneth lesse then naught by 4.

Master. If you would haue it no *Absurde* number, you must increase the proportion of the fraction, by augmentyng the numerator, or abatyng the denominator, or els thirdly, by abatyng the number, after the signe of abatemente. As $\frac{400}{53} - 40$; or els secondarily, thus, $\frac{240}{53} - 40$, or thirdely $\frac{240}{53} - 20$.

Howbeit for examples sake, you maye worke, as well with *Absurde* numbers, as with any other.

But

of Cossike numbers.

But for you ease, I will shewe you the woork of this example, in two formes.

First, you shall multiplie the firste whole number, by the fraction of the seconde number, that is.

$$\frac{100}{78} + \frac{120}{9} \text{ by } \frac{21}{25} \text{ and it will bee.}$$

$$\begin{array}{r} 456. \text{ ʒ. } \text{℥.} - + - 72. \text{ ʒ. } \text{ʒ.} - \text{---} .120. \text{ ℥.} \\ \hline 63. \text{ ʒ. } \text{ʒ.} - \text{---} .54. \text{ ʒ.} \end{array}$$

As here in woork you may see it plaine.

$$\begin{array}{r} 19. \text{ ℥.} - + - 3. \text{ ʒ.} - \text{---} .5. \text{ ʒ.} \\ 24. \text{ ℥.} \\ \hline 456. \text{ ʒ. } \text{℥.} - + - .72. \text{ ʒ. } \text{ʒ.} - \text{---} .120. \text{ ℥.} \end{array}$$

$$\begin{array}{r} 7. \text{ ʒ.} - \text{---} .6. \text{ ʒ.} \\ 9. \text{ ʒ.} \\ \hline 63. \text{ ʒ. } \text{ʒ.} - \text{---} 54. \text{ ʒ.} \end{array}$$

That is in lesser termes, bothe of numbers, and of signes *Cossike*.

$$\begin{array}{r} 152. \text{ ʒ. } \text{ʒ.} - + - .24. \text{ ʒ.} - \text{---} .40. \text{ ʒ.} \\ \hline 21. \text{ ʒ.} - \text{---} .18. \text{ ʒ.} \end{array}$$

And this is the firste parte of your somme.

Then for the nexte parte, multiplie your firste number, that is $\frac{100}{78} + \frac{120}{9}$ by the abatement of the seconde number, that is by $\frac{21}{25}$. and it will be.

$$\begin{array}{r} 20. \text{ ʒ.} - \text{---} .76. \text{ ʒ. } \text{ʒ.} - \text{---} .12. \text{ ʒ.} \\ \hline 7. \text{ ʒ.} - \text{---} .6. \text{ ʒ.} \end{array}$$

The Arte

As by this woork you maie see.

$$\begin{array}{r}
 19.\text{c}.\text{---} | \text{---} .3.\text{z}.\text{---} .\text{---} .5.\text{q}.\text{---} \\
 \text{---} .4.\text{z}.\text{---} \\
 \hline
 20.\text{z}.\text{---} .\text{---} .76.\text{z}.\text{z}.\text{---} .\text{---} .12.\text{z}.\text{---}
 \end{array}$$

whiche being reduced to the denomination of the former number, will be tripled (sith that denominatoz is triple to this) and so will it be

$$\begin{array}{r}
 60.\text{z}.\text{---} \text{---} 218.\text{z}.\text{z}.\text{---} \text{---} 36.\text{z}.\text{---} \\
 21.\text{z}.\text{---} \text{---} 18.\text{q}.\text{---} \\
 \hline
 \text{Now adde those two numbers together, by puttyng their bothe numeratozs in one, and it will be.}
 \end{array}$$

$$\begin{array}{r}
 20.\text{z}.\text{---} .\text{---} .76.\text{z}.\text{z}.\text{---} .\text{---} .12.\text{z}.\text{---} \\
 \hline
 21.\text{z}.\text{---} .\text{---} .18.\text{q}.\text{---}
 \end{array}$$

As here appeareth in woork.

$$\begin{array}{r}
 152.\text{z}.\text{z}.\text{---} | \text{---} .24.\text{z}.\text{---} .\text{---} .40.\text{z}.\text{---} \\
 60.\text{z}.\text{---} .\text{---} .228.\text{z}.\text{z}.\text{---} .\text{---} .36.\text{z}.\text{---} \\
 \hline
 20.\text{z}.\text{---} .\text{---} .76.\text{z}.\text{z}.\text{---} .\text{---} .12.\text{z}.\text{---}
 \end{array}$$

whiche will not bee reduced to any smaller fraction, bicause the numbers be incommensurable. and one of the *Coslike* signes is. $\frac{1}{9}$. And so is that the somme of the multiplication.

An other waie you maie woork it, and all soche like, by reducyng the multiplier, into one vniforme fraction. As here in $\frac{24}{9}$ $\frac{40}{9}$. you shall multiplie $\frac{40}{9}$ by $\frac{9}{9}$. whiche is the former denominator, and it will be $\frac{360}{9}$. Then putte that to. 24. over the line, and set the common denominator. 9. vnder the line, and it will bee in one fraction reduced $\frac{24 \times 40}{9 \times 9} = \frac{360}{9}$.

Scholar. Here I maie see at the firste belwe, that this fraction is an *Abfurde* number: for the abatement after the signe $\frac{1}{9}$ is greater then the number be-
foze

of Cossike numbers.

foze it.

Master. That was cōfessed befoze. But yet make you worke the example by it.

Scholar. That is true: and so will the numeratozs, beeyng multiplied together, make exactly, 60. ℥. ——— 228. ʒ. ℥. ———. 36. ʒ. ʒ. As here in example of woozke, I haue set it, foꝛ my owne case and certentic.

$$\begin{array}{r}
 19. \text{℥.} \text{ --- } + \text{ --- } 3. \text{ʒ.} \text{ --- } 5. \text{ʒ.} \\
 24. \text{℥.} \text{ --- } 36. \text{℥.} \\
 \hline
 456. \text{ʒ.} \text{ ℥.} \text{ --- } + \text{ --- } 72. \text{ʒ.} \text{ ʒ.} \text{ --- } 120. \text{℥.} \\
 \text{--- } 684. \text{ʒ.} \text{ ℥.} \text{ --- } 108 \text{ʒ.} \text{ ʒ.} \text{ --- } + \text{ --- } 180. \text{℥.} \\
 \hline
 60. \text{℥.} \text{ --- } 228. \text{ʒ.} \text{ ℥.} \text{ --- } 36 \text{ʒ.} \text{ ʒ.}
 \end{array}$$

And that is the newe numeratoz.

And then foꝛ the seconde number, if the firste denominatoz, 7 ʒ. ——— 6. ʒ. be multiplied by the seconde denominatoz, 9. ʒ. it is easily seen, that thei will make. 63 ʒ. ʒ. ——— 54 ʒ. whiche shall be the newe denominatoz.

And so the intere fraction shall bee.

$$\begin{array}{r}
 60. \text{℥.} \text{ --- } 228. \text{ʒ.} \text{ ℥.} \text{ --- } 36. \text{ʒ.} \text{ ʒ.} \\
 \hline
 63. \text{ʒ.} \text{ ʒ.} \text{ --- } 54. \text{ʒ.}
 \end{array}$$

That is in the smalleste numbers and figures *Cossike*.
 $\frac{2072}{118} = \frac{178}{118} \times \frac{178}{118} = \frac{178}{118}$: whiche somme, dooeth in all thyngez fully agree, with the former number that you wrought.

Master. Proue theim. bothe by resolution: And then shall you knowe, the reason of their agremente.

Scholar. I see that the woozke of the denominatozs, doeth agree. Wherfoze I will take. 3. foꝛ a roote to proue how the woꝛke of the numeratozs wil agree

And so foꝛ. 19. ℥. I shall haue. 513. And foꝛ. 3. ʒ.

Ct. y. I

The Arte

I shall haue .9. to be added to .513. And so haue I .522
 out of whiche somme I must abate .5.
 And then remaineth .517. to be multi-
 plied by .24.℥. that is by .648. And the
 totalle will bee (as here in woꝝke appea-
 reth). 335016. whiche somme must be a-
 bated to a smaller number, in like rate as
 the other was reduced, firste by partition
 into .3. And then will it be .111672. And

$$\begin{array}{r}
 648 \\
 \underline{517} \\
 4536 \\
 648 \\
 \underline{3240} \\
 335016
 \end{array}$$

again, it must bee diuided by .9. for that is the quan-
 titie of a square, by whiche the former reduction, was
 wroughte for the Cobike signes: and then will it bee.
 12408. And that is the firste parte of the first woꝝke.
 When for the seconde parte of that woꝝke, I shall
 multiply the firste numbers, that is 517 by the abate-
 mente of the fraction, that is by 47℥, 02 — 12.
 (sith .3. is the roote) and thereof will come — 6204.
 whiche somme I must triple, as I did his equalle (that
 is .207℥. — 76.3.3. — 12.3.) And so will
 it bee — 18612. Now shall I adde this somme,
 with the firste parte, whiche was .12408. and it will
 be .12408. — 18612. that is .6204. lesse then
 nothyng: and is the numerator of the firste woꝝke.

Wherfoze I procede to the seconde woꝝke, where
 the numerator of the fraction, beeyng reduced to the
 common denominator, is .24.℥. — 36.℥. whi-
 che is — 12.℥. and in numbers resolute (keeping
 3. still as a roote) it is — 324. by whiche if I mul-
 tiple .517. it will yelde .167508. And that somme
 beeyng abated, by diuision into .3. and .9. as the other
 was, or els diuided by .27. whiche is all one, it giueth
 6204. as the former woꝝke did.

Walter. Thus I see, you are experte inoughe in
 multiplication: Wherfoze I will shewe you now, the
 order and forme of diuision.

of Coslike numbers.
Of Diuision.



There is noe spectall rule to be giuen, for the woork of Diuision, other then soche as are all ready taughte in other woorkes of diuision befoze. Wherfoze I will by one or 2. examples, shewe you the woork of it.

The firste example of Diuision.

$$\begin{array}{r}
 14.\text{c} \text{---} 9.\text{z} \cdot \\
 \hline
 15.\text{q} \cdot
 \end{array}
 \text{ to be diuided by }
 \begin{array}{r}
 5.\text{z} \cdot \text{---} 2.\text{ze} \cdot \\
 \hline
 3.\text{ze} \cdot
 \end{array}$$

doeth yelde. $\frac{42.\text{z} \cdot \text{z} \cdot \text{---} 27.\text{c} \cdot}{75.\text{z} \cdot \text{---} 30.\text{ze} \cdot}$ that is in a les-
ser fraction, by bothe reductions of numbers & signes.

$$\begin{array}{r}
 14.\text{c} \text{---} 9.\text{z} \cdot \\
 \hline
 25.\text{ze} \text{---} 10.\text{q} \cdot
 \end{array}$$

An other example.

$$\begin{array}{r}
 12.\text{fz} \cdot \text{---} 16.\text{z} \cdot \\
 \hline
 2.\text{c} \cdot \text{---} 5.\text{ze} \cdot
 \end{array}
 \text{ diuided by }
 \begin{array}{r}
 19.\text{z} \cdot \text{---} 3.\text{q} \cdot \\
 \hline
 4.\text{z} \cdot \text{---} 5.\text{q} \cdot
 \end{array}$$

doeth make.

$$\begin{array}{r}
 48.\text{bfz} \cdot \text{---} 60.\text{fz} \cdot \text{---} 64.\text{z} \cdot \text{z} \cdot \text{---} 80.\text{z} \cdot \\
 \hline
 38.\text{fz} \cdot \text{---} 15.\text{ze} \cdot \text{---} 101.\text{c} \cdot
 \end{array}$$

whose numbers bee incommensurable, and therefore mate not bee reduced, but by abatynge one denomination Coslike. And so will it be.

$$\begin{array}{r}
 48.\text{z} \cdot \text{c} \cdot \text{---} 60.\text{z} \cdot \text{z} \cdot \text{---} 64.\text{c} \cdot \text{---} 80.\text{ze} \cdot \\
 \hline
 38.\text{z} \cdot \text{z} \cdot \text{---} 15.\text{q} \cdot \text{---} 101.\text{z} \cdot
 \end{array}$$

The Arte

Scholar. I see that you multiplie crosse wates (as in vulgare fractions) the numeratoz of the one nomber, by the denominatoz of the other. And so is diuision of noe difficultie, to hym that remembzeth the former rules.

Of the golden rule.

Master.



The golden rule, that is the rule of ppozition, should folowe now, by the commo order. But seyng there is no difficultie in it, nother any other forme of woork, then is in vulgare nombers, I will not staie any tyme aboute it. Saue that for your pleasure, I haue set here certaine exampls, as wel in whole numbers *Coslike*, as in broken.

$$\begin{array}{l}
 32.8. \quad \bigvee \quad 4.8. \\
 6. \text{c.} \quad \bigvee \quad \frac{3}{4}. \text{c.}
 \end{array}
 \qquad
 \begin{array}{l}
 250. \text{c.} \quad \bigvee \quad 20.2. \\
 26.8. \quad \bigvee \quad 2\frac{2}{3}. \text{c.}
 \end{array}$$

$$\begin{array}{l}
 5.8. \quad \text{---} \quad 3.2. \quad \bigvee \quad 4. \text{c.} \quad \text{---} \quad 5.9. \\
 15. \text{c.} \quad \text{---} \quad 9.9. \quad \bigvee \quad \frac{668. \text{c.}}{58. \text{---} \quad 111. \text{c.} \quad \text{---} \quad 45.9}
 \end{array}$$

$$\begin{array}{l}
 3. \text{c.} \quad \text{---} \quad 12.2. \quad \bigvee \quad 14.2. \quad \text{---} \quad 4.9. \\
 61.8. \quad 79 \quad \bigvee \quad \frac{1024888. \text{---} \quad 2928. \text{c.} \quad \text{---} \quad 11768. \quad \text{---} \quad 3162.}
 \end{array}$$

Scholar. These selwe exampls, dooe sufficiently teache the forme of the whole rule. So that here needeth noe farther explication.

Wherfoze, if in this arte, there be any forme of extraction of rootes, I praie you to pzoceede therto.

¶

of *Cosike* numbers.
Of extraction of rootes.

Maſter.



In numbers *Abſtraſte*, every number is not a rooted number, but ſome certaine onely emongest them, ſo in numbers *Cosike*, all numbers haue not rootes: but ſoche onely emongest ſimple *Cosike* numbers are rooted, whoſe number hath a roote, agreeable to the figure of his denomination.

So that. 16. \mathcal{C} . is not a Square number, nother hath any roote. For although. 16. bee a ſquare number, and hath. 4. for his roote, yet the denomination (whiche is. \mathcal{C} .) hath noe ſquare roote: but. 16. \mathcal{Z} . is a ſquare number: and hath. 4. \mathcal{Z} , for his roote.

Likewaies. 8. \mathcal{C} . is a *Cubike* number, and his roote is. 2. \mathcal{Z} : but. 8. \mathcal{Z} . hath noe roote. For becauſe. 8. hath no ſquare roote, agreeable to the ſigne. \mathcal{Z} . nother is it a *Cubike* number, although it haue a *Cubike* roote, becauſe the roote is diſagreeable from the ſigne. \mathcal{Z} .

Scholar. I perceiue that in theſe numbers, as well as in all other, the roote becyng multiplied by it ſelf, will make the number, whoſe roote it is. And therefore can no number be called ſquare, or *Cubike*, or any waies els a rooted number, excepte the roote of the number agree with his ſigne: Whereby I perceiue well, that. 32. \mathcal{Z} . is a rooted number, for becauſe that 32. hath a *Surſolide* roote, agreeable to the ſigne. So likewaies. 125. \mathcal{C} . is a rooted number, ſeyng 5. is the *Cubike* roote of. 125. But. 27. \mathcal{Z} . is no rooted number.

Maſter. Thus you vnderſtande ſufficiently, the iudgements of rooted numbers, and their knowlege, in ſimple *Cosike* numbers, that be utterly vncompounde.

Wherefore, for extraction of their rootes, take this brief order.

Extrade

The Arte

Extracte the roote of your number, as if it were absolute, and put to it. \mathcal{L} . for the denomination.

So. 27 . Cubes hath for his roote. 3 . \mathcal{L} .

And. 49 . hath. 7 . \mathcal{L} . for his roote.

Again, the roote of. 216 . \mathcal{L} . is. 6 . \mathcal{L} .

Scholar. This I perceiue. And by like reason, the roote of. 243 . \mathcal{L} . is. 3 . \mathcal{L} . But why dooe you name nōbers *Cosbike* vtterly vncoumounde? For as I vnderstande, that there bee numbers coumounde, in their signes, so I see that thei maie haue rootes also.

As. 16 . hath for his roote. 2 . \mathcal{L} . And like waies. 64 . hath. 2 . \mathcal{L} . for his roote.

Master. And dooe you not see, that those coumounde numbers, maie haue moare rootes then one? Sith. 16 . hath for his square roote. 4 . as well as it hath. 2 . for his *zenzizenzike* roote.

So. 4 . hath for his square roote. 2 . And hath no *zenzizenzike* agreable to his whole signe.

Like waies. 9 . hath no *zenzicubike* roote, according to his whole signe: but it hath a square roote agreable to parte of the signe, and that is. 3 .

Scholar. I see that also. And so hath. 8 . noe *zenzicubike* roote, but a *Cubike* roote: whiche is. 2 .

Master. Wherefore in coumounde signes, if the signe maie haue soche a roote, as the number will yelde, it is a rooted number, els not.

Whereby you maie perceiue, that if any number coumounde in signe, haue a roote agreable to his whole signe, then maie it haue also, as many rootes, as ther be partes in that coumounde signe.

So 4096 hath not onely a *zenzizenzicubike* roote, whiche is. 2 ; but it hath a square roote that is. 64 . And also it hath a *Cubike* roote, that is, 16 . Farther moze, it hath a *zenzizenzike* roote, whiche is. 8 . And fourthly, it hath a *zenzicubike* roote, that is. 4 .

And

of Cossike numbers.

And so shall you iudge, of all other like.

Scholar. This shall suffice, as I will practise the mater, at moare leiser. But and if the numbers bee compounde, with signes of addition, is there then any speciall order for their rootes? As in this erample. $81 \cdot 3 \cdot 3 \cdot + - 27 \cdot c$. where I haue made eche parte to be a rooted number.

Maister. In deede. $81 \cdot 3 \cdot 3$. hath bothe a Square roote, and also a *zenzizenzike* roote. But $27 \cdot c$. hath none of those twoo rootes, although it haue a *Cubike* roote, whiche the other number wanteth. And therefore is not that whole number, a rooted number.

But to the intente, that you maie be the moze certein of rooted numbers, I will tell you certein notes, how it maie bee knowen, whether your number be a rooted number.

Firste, if the number annexed to the greatestt signe of that compounde *Cossike* number, bee not a rooted number, the whole number can not be a rooted nōber

Secondarily, if the number that is toynd with the leaste *Cossike* signe, be not a rooted number, the whole number can not be a rooted number.

And eche of these bothe rootes (if thei haue any) are partes of the whole roote, for the compounde *Cossike* number.

Thirdly, if the number be a rooted number, euery parte of it, that is not a rooted number, is a meane number, betwene the greatestt and the leaste.

Fourthly, if $\cdot 20$. bee any denomination in it, then is $\cdot 9$. an other denomination in it also.

Fiftly, and generally, all rooted nōbers, other are specially framed, by orderly multiplication, or els are numbers equalle to some one rooted number *Abstract*.

Now specially framed are soche, as are made by multiplicatio of one number by it self, and litle or nothing altered from that very forme.

Dd.j. Example

The Arte

Of square
rootes.

Exāple of. $529\text{z}\text{c} \text{---} 184\text{z}\text{z} \text{---} 16\text{z}$
whiche is a Square number, made by multiplication
of. $23\text{c} \text{---} 4\text{z}$. by it self. This number maie
haue his Roorē orderly extracted thus.

$$\begin{array}{r} 529\text{z}\text{c} \text{---} 184\text{z}\text{z} \text{---} 16\text{z} \\ 23 \qquad \qquad 46\text{c}. \end{array}$$

In the firste number, I finde the Square roote to bee
 23 . And for his denomination, I take halfe the *Cosike*
signe zc , and that is. c . For as. c . multiplied by
 c . doeth make. zc . So in diuision by. 2 . and in ex-
traction of Square rootes, I shall take the. c . for the
halfe of zc and the denomination of his roote: and
so set it doune in the *quotiente*.

Then I shall double the number *Abstrakte* of that
quotiente (kepyng his *Cosike* signe vnaltered) and that
double shall I set euermore vnder the nexte number,
toward the righte hande. As here, you see, I haue set
 46 (whiche is the double of 23) with his signe c . vn-
der the seconde number. And there I perceiue, I maie
haue it. 4 . tymes, if I doe diuide (as I ought) 184 . by
 46 . And that. 4 . I sette in the *quotiente*, with the signe
 --- , and the denomination. z : seying. zz . diui-
ded by. c . doeth yelde. z .

Laste of all, I maie multiplie that parte of the *quo-*
tientie. 4z . by it self, and it will yelde. 16z . whiche
beyng subtracted also (as it should) leaucth nothyng
remainyng of the Square number.

This order must you kepe in all Square numbers,
how greate so euer thei be. As in this seconde exāple.

$$\begin{array}{r} \text{---} 90\text{z}\text{z}. \\ 25\text{z}\text{c} \text{---} 80\text{z}\text{z} \text{---} 26\text{z}\text{z} \text{---} 144\text{c} \text{---} 81\text{z}\text{z} \text{---} 9\text{z} \\ 5\text{c}. \qquad 10\text{c} \text{---} 64\text{z}\text{z}. \\ \text{---} 10\text{c} \text{---} 16\text{z}. \text{---} 9\text{z}. \end{array}$$

The

of Coſſike numbers.

The roote of the firſt number is. 5℥ , whiche I ſet in a *quotiente*.

Then doe I double that. 5 , and it maketh. 10 , to be ſette vnder. 8 . with his denomination, whiche is. 2℥ . like to the roote.

That. 10℥ . maie be founde in. 80ſ . 8 . times, & therfoze I ſet. 8 . in the *quotiente*, with the ſigne $+$ and the denomination. 3 . And then dooe I multiplie that. 8ſ . ſquaredly, whiche giueth. $+$ 64ſſ . to be ſubtracted out of 26ſſ . and ſo remaineth 90ſſ .

After this I double all the *quotiente* again, whereof commeth $+$ 10℥ . $+$ 16ſ . And bicauſe there is a remainer, ouer the number that I wrought laſte, I muſt ſet. 10℥ . vnder the remainer, and the other number in order, as you ſee it ſet.

Then ſeke I how often tymes maie. 10℥ . diuide 90ſſ , and I finde the *quotiente* to be 9℥ . And like waies $+$ 16ſ . multiplied by 9℥ doeth make 144℥ . equalle to the ſomme ouer it: And ſo ſubtracteth it cleane. Wherefoze to ende that worke, I multiplie the laſte *quotiente*, by it ſelf ſquare, and it yeldeth. $+$ 81ſ . whiche is to bee ſubtracted out of the like ſomme, in the ſquare number: and ſo reſteth nothyng. Wherefoze I iuſtly affirme, that the firſte number is a ſquare number, and hath for his roote. 5℥ . $+$ 8ſ . 9℥ .

Scholar. What maie I ſone proue, if I multiplie

$$\begin{array}{r} 5\text{℥}. \quad + \quad 8\text{ſ}. \quad \text{---} \quad 9\text{℥}. \\ 5\text{℥}. \quad + \quad 8\text{ſ}. \quad \text{---} \quad 9\text{℥}. \end{array}$$

$$\begin{array}{r} 25\text{ſſ}\text{℥} \quad + \quad 40\text{ſſ} \quad \text{---} \quad 45\text{ſſ} \\ \quad \quad + \quad 40\text{ſſ} \quad + \quad 64\text{ſſ} \\ 81\text{ſſ}. \quad \text{---} \quad 72\text{℥} \quad \text{---} \quad 45\text{ſſ} \\ \quad \quad \text{---} \quad 72\text{℥}. \end{array}$$

$$25\text{ſſ}\text{℥} \quad + \quad 80\text{ſſ} \quad \text{---} \quad 26\text{ſſ} \quad \text{---} \quad 144\text{℥} \quad + \quad 81\text{ſſ}. \quad \text{that}$$

Dd. y.

The Arte

that roote by it self, as here I haue doen it. Wherby I haue not onely confirmed it to be a square number: but also I haue espied, that you vsed the number not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the . $3 \cdot 3$. where the particulare multiplication hath $\text{---} \text{---} \text{---} 64 \cdot 3 \cdot 3$. and $\text{---} \text{---} 90 \cdot 3 \cdot 3$. For whiche twoo numbers you sette one, that resulteth of the bothe, that is $\text{---} \text{---} 26 \cdot 3 \cdot 3$.

Master. But if you would take the nōber in that sorte, the woorkke would be not onely all one: but also somewhat plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forth here, the example of that woorkke. And loe, here it is.

$$\begin{array}{r}
 25 \cdot 3 \cdot 3 + 80 \cdot 3 \cdot 3 + 64 \cdot 3 \cdot 3 \text{---} 90 \cdot 3 \cdot 3 \text{---} 144 \cdot 3 \cdot 3 + 81 \cdot 3 \cdot 3 \text{---} 90 \cdot 3 \cdot 3 \\
 5 \cdot 3 \cdot 3 \quad 10 \cdot 3 \cdot 3 + 64 \cdot 3 \cdot 3 \quad 10 \cdot 3 \cdot 3 \text{---} \text{---} 16.
 \end{array}$$

Scholar. By comparynge these bothe formes of woorkke together, I dooe better vnderstande, the reason of the firste woorkke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generall patrone, for all the varieties, in this sorte of rooted numbers. And if you examine it diligently, and marke it well, you shall neede fewe other examples, for this kinde of square numbers.

The Square number, with the
woorkke of extraction
of his roote so:
lo weth
here.

The

The Arte

that roote by it self, as here I haue doen it. Wherby I haue not onely confirmed it to be a square number: but also I haue espied, that you vsed the number not so plainly set doune, as the particulare multiplication did make it: but rather as a reasonable reduction would expresse it. I meane in the. $3 \cdot 3$. where the particulare multiplication hath $—+—64 \cdot 3 \cdot 3$. and $—90 \cdot 3 \cdot 3$. For whiche twoo numbers you sette one, that resulteth of the bothe, that is $—26 \cdot 3 \cdot 3$

Master. But if you would take the nōber in that sorte, the woorkke would be not onely all one: but also somewhat plainer to bee perceiued of a learner. And therefore for your pleasure, I will set forth here, the example of that woorkke. And loe, here it is.

$$25 \cdot 3 \cdot 3 + 80 \cdot 3 \cdot 3 + 64 \cdot 3 \cdot 3 = 90 \cdot 3 \cdot 3 = 144 \cdot 3 \cdot 3 + 81 \cdot 3 \cdot 3$$

$$5 \cdot 3 \cdot 3 + 10 \cdot 3 \cdot 3 + 64 \cdot 3 \cdot 3 = 10 \cdot 3 \cdot 3 + 16 \cdot 3 \cdot 3$$

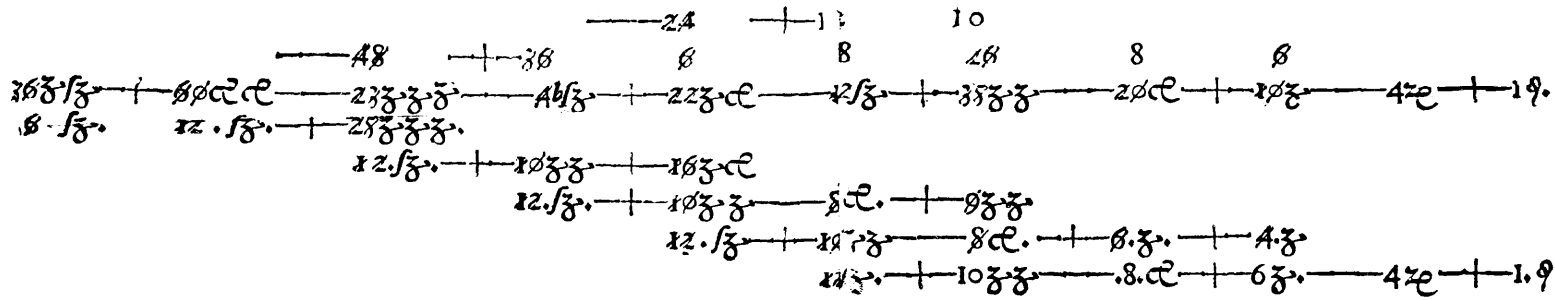
Scholar. By comparynge these bothe formes of woorkke together, I dooe better vnderstande, the reason of the firste woorkke.

Master. One example moare of this kinde of extraction of rootes, will I set doune, that maie be a generalle patrone, for all the varieties, in this sorte of rooted numbers. And if you examine it diligently, and marke it well, you shall neede fewe other examples, for this kinde of square numbers.

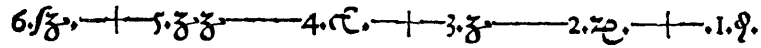
The Square number, with the
woorkke of extraction
of his roote so:
lo weth
here.

The

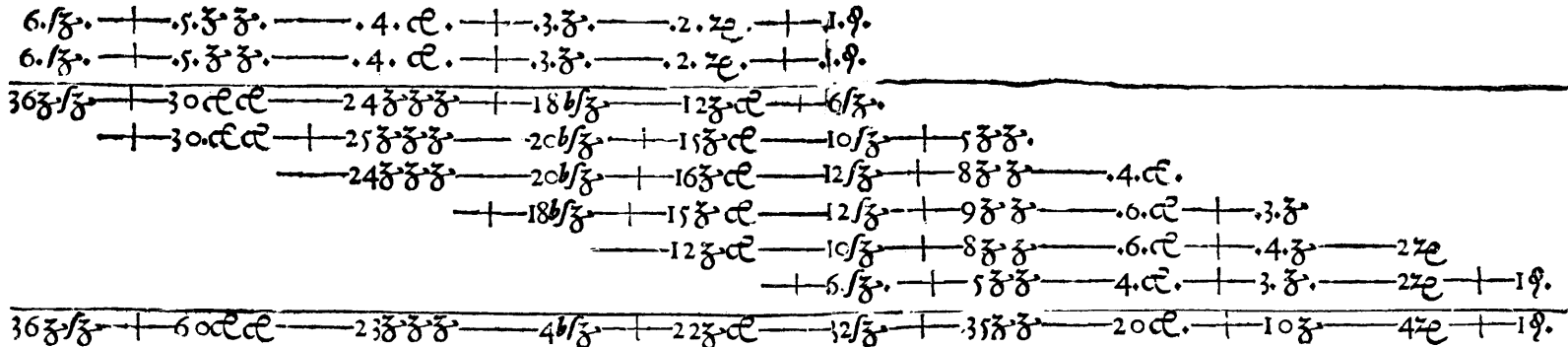
The square nomber, with the woorke of extraction of his roote.



The Roote.



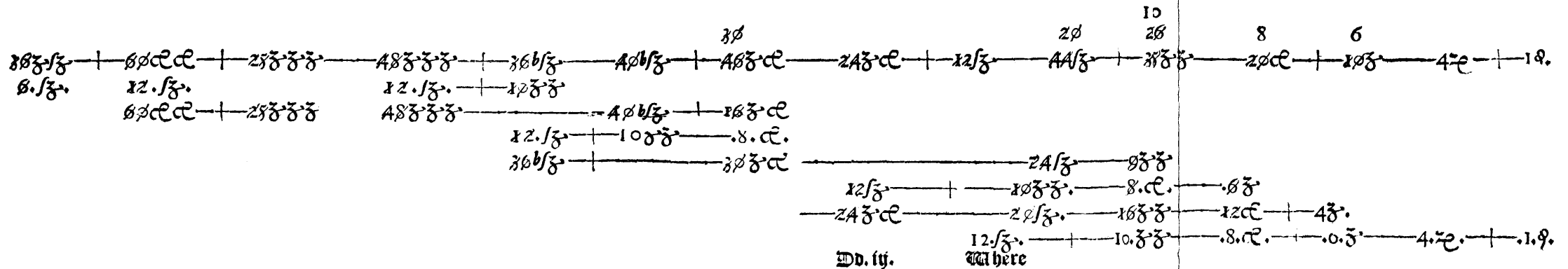
The prooffe by Multiplication.



Scholar. It maie appeare easily, that this example serueth foꝛ many other, it doeth contain so many varieties of signes *Coslike*, multiplied so diuersely.

And in this number also, as well as in the other, I see that many numbers be omitted, by reduction: namely in the thirde, fourth, fift, and sixthe orders of numbers. For in the 2. fiftie orders, and in the 5. laste, there is no varietie of the signes $+$ and $-$.

Wherefoꝛe to see the varietie of woorke, I will sette doune the numbers, as they rise in particulare multiplication, and in it will I make an experimete of my cunningg. As here foloweth.



of Coſike numbers.

Where for myne owne ease, and aied of memoꝛie, I haue set vnder euery doubling of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the numbers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in numbers *Abſtraete*. But sometime I must take .2. places nexte together, and at an other tyme, I shall scippe .2. or .3. places.

Paſſer. You marke it well. And yet that is a good and true rule, that some menne teache: that in these *Coſike* numbers, as well as in other *Abſtraete* numbers, you shall marke euery odde place, and vnder eche of them to finde a Square roote. But that is to be vnderſtande, when the numbers are sette, in their beſeſte and exacteſte order.

These fewe examples maie suffice, for a declaratiō of extractyng the roote of Square numbers, made by multiplication. And now touchyng those numbers, that bee equalle to some rooted number, and namely soche as be equalle to a square number, I will teache you how their roote maie be extracted. *The rootes of numbers equal to be ſqrts.*

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and numbers, the rootes maie bee coupled with the numbers onely, in .3. formes. That is, $20 \text{ --- } 10$ (whiche is all one with $9 \text{ --- } 30$) or els thus, $9 \text{ --- } 20$. Or thirdly, $20 \text{ --- } 9$. And for eche of these .3. formes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where $20 \text{ --- } 10$ is equalle to 20 take these exāples 10 is equall to 4 , $20 \text{ --- } 21$ or 10 is equalle to 35 , $9 \text{ --- } 20$. Like waies 10 is equalle to 10 , $20 \text{ --- } 75$, or 10 is equalle to 105 , $9 \text{ --- } 8$.

of Coſike numbers.

Where for myne owne ease, and aied of memoꝛie, I haue set vnder euery doubling of the *quotiente*: And the somme that amounteth, by the multiplication of thesame, into the newe *quotiente*, with the Square of thesame newe *quotiente*.

Whereby I perceiue that the numbers, doe not go in soche order, that euery odde place, maketh a newe roote, as it doeth in numbers *Abſtraete*. But sometime I must take .2. places nexte together, and at an other tyme, I shall scippe .2. or .3. places.

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These fewe examples maie suffice, for a declaratiō of extractyng the roote of Square numbers, made by multiplication. And now touchyng those numbers, that bee equalle to some rooted number, and namely soche as be equalle to a square number, I will teache you how their roote maie be extracted. *The rootes of numbers equal to be ſqrts.*

But firste you shall marke, that a Square beeyng compared, as equalle to rootes and numbers, the rootes maie bee coupled with the numbers onely, in .3. formes. That is. $20 \text{ --- } 1 \text{ --- } 9$ (whiche is all one with $9 \text{ --- } 1 \text{ --- } 20$) or els thus. $9 \text{ --- } \text{---} 20$. Or thirdly, $20 \text{ ---} 9$. And for eche of these .3. formes, there is some varietie, in the extraction of the roote. And in them all moche agremente.

For the first forme, where $20 \text{ --- } 1 \text{ --- } 9$ is equalle to *The firste* 7 take these exāples 17 is equall to. $4 \text{ ---} 20 \text{ ---} 21$ forme. *forme.*
 02 . 17 . is equalle to $35 \text{ ---} 9 \text{ ---} 2 \text{ ---} 20$. Like waies 17 is equalle to. $10 \text{ ---} 20 \text{ ---} 75 \text{ ---} 9 \text{ ---} 02$. 17 is equalle to $105 \text{ ---} 9 \text{ ---} 8 \text{ ---} 20$.

The Arte

In all these exāples, and other soche like, you must first consider the number annexed with the signe. \mathcal{Z} . (whiche is the middell quantitie) and the halfe of it shall you note, for with it shall you worke twice. First you shall multiplie halfe of that number by it self, and this is the firste worke, and to it shall you adde the other whole number, that is ioyned with. \mathcal{Q} . And thei will euer more make a square number: out of whiche square you shall extracte the roote. And to that roote shall you adde halfe the number, that was annexed with the signe of. \mathcal{Z} . (whiche was the number that I bade you to mark). And this is the seconde worke. The totall that commeth of this addition, is the roote of the compounde Cobike number.

An example

Example of the firste. $4.\mathcal{Z} \text{ --- } 21.\mathcal{Q}$. halfe the number annexed with. \mathcal{Z} . is. 2. whose square is. 4. that shall I put to. 21. and there riseth. 25. beeyng a square number, and haupng. 5. for his roote. To that 5. I loyne halfe the number annexed with. \mathcal{Z} . and it maketh. 7. whiche is the number that I seke for: and is the roote to. $4.\mathcal{Z} \text{ --- } 21.\mathcal{Q}$.

The prooffe.

For triall whereof take. 4. rootes, that is. 28. and putte to it. 21. and thereof commeth. 49. whiche is a square number, and hath. 7. for his roote.

An other example.

Scholar. When can I doe the like with the second exampl. $35.\mathcal{Q} \text{ --- } 2.\mathcal{Z}$. And firste the halfe of. 2. is 1. and the square of it is. 1. whiche I put to. 35. and it maketh. 36. a square number: whose roote is. 6. To that. 6. if I adde. 1. that was the halfe before reserved, it will make. 7. whiche is the roote that I doe seke.

The prooffe.

The prooffe is this: 2. rootes maketh. 14. and. 35. giueth. 49. whose roote is. 7.

The thirde example.

Like wates for the thirde example $100.\mathcal{Z} \text{ --- } 75.\mathcal{Q}$ I worke thus. Halfe. 100. is. 50. and his square is. 2500. that dooe I adde to. 75. and there riseth. 1000. whose roote is. 100. to whiche roote I add. 50. and there cometh

of Coslike numbers.

meth. 15. that is the roote whiche I would haue.

And that I maie proue by triall in this sorte. 10. rootes giue. 150. vnto whiche if I adde. 75. there will amounte. 225. whiche is a Square number: and hath 15. for his roote.

The fourthe example is. 105. $9 - + - 8.20$. where *The fourthe*
I take firste the halfe of. 8. that is. 4. and it in Square *example.*

giueth. 16. whiche I adde to. 105. and there amounteth. 121. beyng a Square number, and the roote of it 11. vnto whiche I shall adde. 4. for halfe the number of rootes: and so there riseth. 15. as the roote that I seke for. And to approue it I take. 8. times. 15. whiche *The prooffe.*

is. 120. and adde it vnto. 105. and so commeth. 225.

For the square, and the roote of it is. 15.

Walter. The like order of worke shall you vse, in *Other for-
all numbers Coslike compounde, whē any. 2. numbers mes in like
with immediate denominatiōs Coslike, are equalle to sorte.*
one of the nexte denomination, in order aboue them.

As. 1. \mathcal{C} . is equalle to. 3. \mathcal{Z} . $3 - + - 10.20$.

And again. 1. \mathcal{Z} . equalle to. 6. \mathcal{Z} . $3 - + - 40.20$.

Like waies. 1. \mathcal{Z} . \mathcal{C} . equalle to. 3. \mathcal{Z} . $3 - + - 28.3.3$.

But in al these the roote shall beare name of the greater quantie.

Scholar. By the former order of worke, I shall in *The fiftle
the firste of these. 3. examples, take halfe. 3. (bicause it example.*
is the number of the middell quantie). And that is $\frac{3}{2}$.

and that shall I multiplie squarely, and so will there rise $\frac{9}{4}$, vnto whiche I shall adde 10 or $2\frac{10}{4}$. And that maketh $\frac{19}{4}$. whiche is a square number, and his roote is $\frac{7}{2}$. vnto whiche I must put the firste halfe, that is $\frac{3}{2}$, and then will it be $\frac{17}{2}$, or els. 5. whiche is the Cubike roote of that number. 3. \mathcal{Z} . $3 - + - 10.20$. beyng equalle to 1 \mathcal{C} .

For prooffe whereof, I multiplie. 5. Cubikely, and it *The prooffe.*
maketh. 125. Then doe I multiplie it squarely, and it will be. 25. Now. 3. \mathcal{Z} . is. 75. and. 10. 20 . maketh. 50 whiche bothe added together, giue. 125.

The Arte

The seconde example.

In the seconde example, where $1.\sqrt[3]{8}$. is equalle to $6.\sqrt[3]{8}.\sqrt[3]{8}.\sqrt[3]{8}$. — + —. $40.\mathcal{C}$. I shall take halfe. 6 . (whiche is the number of the middell quantitie) and that is. 3 . and the square of it is. 9 . whiche I must adde vnto 40 and thereof commeth. 49 . whiche is a square number and hath. 7 . for his roote, vnto whiche I adde 3 . and so haue I 10 for the *Surfolide* roote, of $6.\sqrt[3]{8}.\sqrt[3]{8}.\sqrt[3]{8}$. — + —. $40.\mathcal{C}$

The prooffe.

And for prooffe I saie, if. 10 . bee the roote, then is 100 . the square, & 1000 . the *Cube*, the $\sqrt[3]{8}.\sqrt[3]{8}$. is 10000 . And the *Surfolide*. 100000 . Wherfore. $6.\sqrt[3]{8}.\sqrt[3]{8}$. make 60000 . and. $40.\mathcal{C}$. yelde. 40000 . And bothe thei together doe make. 100000 . whiche is the quantitie of the *Surfolide*.

The thirde example.

In the thirde example. $1.\sqrt[3]{8}.\mathcal{C}$. is equalle to $3.\sqrt[3]{8}.$ — + —. 28 . $\sqrt[3]{8}.\sqrt[3]{8}$. whose *zenzicubike* roote, I seke in this sozte.

Firste I take halfe. 3 (as the number of the middell quantitie) that is $\frac{3}{2}$, & that maketh in square $\frac{9}{4}$. whiche I adde vnto 28 (that maketh $\frac{112}{4}$) & it yeldeth $\frac{121}{4}$. whiche is a square number, and his roote is $\frac{11}{2}$. vnto whiche I adde $\frac{3}{2}$, and it will be $\frac{14}{2}$, or. 7 . whiche is the *zenzicubike* roote vnto the foresaid number. $3.\sqrt[3]{8}$. — + —. 28 . $\sqrt[3]{8}.\sqrt[3]{8}$.

The prooffe.

For prooffe whereof I multiplie. 7 . *zenzicubikely*, and it maketh 117649 . When must the $\sqrt[3]{8}$. be 16807 and. $3.\sqrt[3]{8}.$ 50421 . Again the $\sqrt[3]{8}.\sqrt[3]{8}$. is. 2401 . and so $28.\sqrt[3]{8}.\sqrt[3]{8}$. shall bee. 67228 . And those bothe together yelde. 117649 .

A thirde forme.

Master. Yet one other forme is there, that in all things, saue in one poinde onely: followeth the same rule. And that is whē the 3 denominations doe not go immediatly together, but yet are equally distante. As $\sqrt[3]{8}.\sqrt[3]{8}.\sqrt[3]{8}$. and. 9 . where the distaunce is one onely quantitie. Likewise. $\sqrt[3]{8}.\mathcal{C}.\mathcal{C}$. and. 9 . whiche differ by. 2 . quantities. And in like sozte. $\mathcal{C}.\mathcal{C}.\sqrt[3]{8}$. and $9\mathcal{C}$. are distante by. 3 . quantities. And so of other, how many so cuer bee omitted, so that the difference bee equalle

of Coſſike numbers.

equalle. In all whiche you ſhall worke, as you did in the former rule, till you haue eanded all that worke. But then haue you here, one thing moze to bee conſidered. For the laſte number, whiche you haue ſounde, is not the roote, but a rooted quantitie: And his roote is the roote that you ſeke for.

Scholar. Doe you meane the ſquare roote of that quantitie, or ſome other?

Maſter. It maie be any kinde of roote, in diuerſe numbers, but not in one number. Wherefoze for your certeintie marke this rule.

If the denominations of your numbers, differ one by one, then is it a ſquare nōber, that you doe finde by the practice of the laſte rule. And therfoze ſhall you take his ſquare roote, for the roote of your number.

But if the denomination differ by . 2 . quantities, then ſhall you extracte a *Cubike* roote, out of your laſte number. And if the diſtaunce bee . 3 . quantities, the roote muſt bee a *zenzizenzike* roote: and for . 4 . quantities diſtante, a *Surſolide* roote, and ſo forth.

As for example. $1. \text{z} \text{z}$. is equalle to $80. \text{z} \text{z}$. *An example*
 $2000. \text{z}$. Now for to finde the roote of $80. \text{z} \text{z}$.
 $2000. \text{z}$. I worke thus. Firſte I take the halfe of $80.$
 (becauſe it is the number of the middle quantitie) and
 that halfe is. $40.$ whiche I multiplie ſquare, and it
 maketh. $1600.$ to it I adde. $2000.$ and it will bee
 $3600.$ whiche is a ſquare number, $60.$ is his roote:
 to that. $60.$ I ſhall adde the ſoꛛeſaid. $40.$ and then
 will it bee. $100.$ whiche number in the firſte rule, had
 been the true roote. But here conſidering the diſtāce
 is of one quantitie, I muſte extracte his ſquare roote,
 whiche is. $10.$ And that is the *zenzizenzike* roote,
 that my number containeth.

An other example. $1. \text{z} \text{c}$. is equalle to $400. \text{c}$. *The ſeconde*
 $—+—57344. \text{z}$. I take $200.$ for the halfe of the mid:
 dell quantities number, and multiplying it ſquare, I

Ce. i. finde

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finde. 40000. whiche I put to. 57; 44. and then I haue. 97344: whiche is a Square number, and his roote is 312 vnto whiche I shall adde the halfe of 400 and so will it bee. 512. But now must I take the *Cubike* roote of this number (that is. 8) for my roote, that I desire: Because the denominations in the number, differ by. 2. quantities.

Scholar. I see very well the order of this worke: And the prooue is in like sorte, whiche I maie practise by my self at any tyme. Wherefore I praie you, proceede forthe to other rules.

*The seconde
sort of equal
numbers.*

Master. This is sufficiente for the firste sorte. Now for the seconde sorte, in numbers *diminute* or *residuale* where. 3. is equalle to. 9. ——— 20. the forme of worke is like vnto the other, in all pointes saue in one. For in steedes of the laste addition, you shall vse in these numbers, Subtraction. As here for example, when I saie. 1. 3. is equalle to. 60. 9. ——— 4. 20. to finde the roote, firste I take the halfe of. 4. (because it is the number of the middell signe) and that halfe being. 2. doeth make in square. 4. whiche I put to 60 and so is it. 64. a square number, and hath. 8. for his roote. From whiche roote (by the order of this rule) I must abate. 2. that is the halfe of the firste number of rootes. And then will there remaine. 6. for the verie roote of. 60. 9. ——— 4. 20. being equalle to. 1. 3.

Example.

The prooue.

Scholar. What is sone proued. For. 6. being the roote, then. 4. 20. maketh. 24. whiche being abated out of. 60. leaueth 36 and that is the iuste square vnto. 6. as the equation saith.

*The seconde
example.*

Master. An other example is this. 1. 3. is equall to. 162. 2. ——— 9. 3. 3.

Scholar. What can I worke, thus: Firste I take the halfe of. 9. (because it is the number of the middell signe) and it is $\frac{9}{2}$, whiche I multiplie squarely, and it will be $\frac{81}{4}$, that must bee added to. 162. or $162\frac{1}{4}$, and then will

of Coſike numbers.

Will there amounte $\frac{729}{4}$. whiche is a Square number, and hath for his roote $\frac{27}{2}$ out of whiche, by this rule, I muſt abate $\frac{7}{2}$, and then riſeth $\frac{1}{2}$, that is. 9. whiche is the very roote to 162. $\overline{162} \text{---} 9 \cdot \overline{9} \cdot \overline{9}$. being equall to. 1. $\overline{9}$.

And for the prooffe, I multiplie. 9. Cubikely, and it giueth. 729. ſo that. 162. $\overline{162}$. doe make. 118098. out of whiche I muſt abate. 9. $\overline{9} \cdot \overline{9}$. that is. 59049. (by the ſame roote, ſith. 1 $\overline{9} \cdot \overline{9}$. is. 6561). And then will there remaine. 59049. whiche is the iuſte quantitie of. 1. $\overline{9}$.

Maſter. Yet one example moze ſhall you haue of a thirde ſorte. *The thirde example.*

When. 1 $\overline{9} \cdot \overline{1}$ is equalle to. 275456. $\overline{9} \text{---} 26 \overline{1}$ I demaunde of you, what is the valewe of. 1. $\overline{9}$?

Scholar. I ſearche it thus. The number of the middell ſigne is. 26. whoſe halfe I muſt take, and firſt multiplie it ſquarely, and there will riſe 169. whiche I adde to. 275456. and it will bee. 275625. whiche is a ſquare number, and hath for his roote. 525. from whiche number I muſt abate halfe the number, of the middell ſigne, that is. 13. and ſo there will remaine 512. whoſe Cubike roote I muſt extract, becauſe the denominations differ by. 2. quantities, and that roote will be. 8. whiche is the Cubike roote to. 512. but to the number propounded, it is the *zenzicubike* roote.

Maſter. This is enough to the worke of the ſeconde ſorte. Now for the thirde ſorte of equation, where. $\overline{9}$. is equalle to. $\overline{9}$. --- . $\overline{9}$. I will giue you a brief admonition encl^y, though it differ from bothe the ether. 2. rules, in ſorme of worke. For as the equalitie maie be in diuerſe ſortes, ſo ſome tymes you maie uſe the worke of the firſt ſorte, by Addition of halfe the number of the middle ſigne: and ſome times you ſhall worke by ſubtraction. Wherein this is the difference, from the ſeconde rule. That there you doe
Ec. v. ſubtrate

The Arte

subtracte halfe the number of the middell signe, from the roote whiche you fonde. And in this thirde rule, you shall subtracte the roote from the halfe, and not the halfe from the roote. For because that that roote, is ever lesser then that halfe.

And in this rule, this is specially to bee obserued: that the Square of halfe the number, of the middell signe, will euer moze bee greater, then the number of the lesser signe: And therfore shall the number of the lesser signe, bee abated out of that square. And the remainer will bee a Square number, with whiche you shall worke, as I haue taught you before.

And farther in this rule, it is commonly seen, that euery soche equalle number, hath. 2. valuations for his roote. I meane that any of those. 2. numbers, will bee as the roote in this equation. For otherwates no number can haue. 2. rootes of one denomination.

Scholar. I vnderstande you thus. That no number can haue. 2. square rootes, or. 2. Cubike rootes, and so forth: Els one number maie haue. 3. or. 4. rootes. As. 64. hath. 8. for his Square roote: 4. for his Cubike roote: and. 2. for his *zenzicubike* roote.

Master. You take it well. And farther for the easie knowledg of those. 2. numbers, or rootes: They must bee soche, as beeyng added together, will make the nōber of the middell signe: and beeyng multiplied together, wil make the number of the least signe. And so maie you finde them without farther multiplication, or extraction of rootes.

*The firste
example.*

For example, I sette firste. 1. 3. equalle to. 16. 7. ———. 63. 9. where I maie espie quickely, that. 63. can haue no moze partes to his composition, but. 3. 7. 9. 21 And if I take. 3. and. 21. then their addition will bee greater then. 16. but 7. and 9. maketh iuste 16. by addition, and. 63. by multiplication. And therfore they shall be the. 2. rootes.

Scholar.

of Cossike numbers.

Scholar. I will proue that by examination, thus. If 7. be the roote, then is. 49. the square. And. 16. ζ make. 112. out of whiche I must abate. 63. and there resteth. 49. equalle with the square: so is that a true roote. When for. 9: his square is. 81. And. 16. ζ . doe yelde 144 frō whiche I shal abate 63. And the remainder will be. 81. equalle to the square. And so is that al so a true roote.

Master. Now worke it by the other rules, that I taught you.

Scholar. Firste I take. 8. as halfe the number of the middell signe, and that multiplied square, doeth giue 64 from whiche I shall abate 63 and then doeth there remain but. 1. whiche is counted as a square number, and his roote to be. 1. also, whiche if I adde to. 8. it will make. 9. that is one of the rootes: And if I abate it from. 8. it will leaue. 7. whiche is the other roote. And thus I see one worke confirmeth the other.

Master. Take this for the seconde exāple. $1\zeta\mathcal{C}$ is equalle to. $8\sqrt{3}$. ———. $12.\zeta.\zeta$. what is the roote saie you? *The seconde example.*

Scholar. To finde it, firste I loke for the partes of 12. And thei be. 2. 3. 4. 6. of whiche. 2. and. 6. doe serue my purpose, for their addition maketh. 8. and so doeth not. 3. and. 4. Therefore I saie, that. 2. maie be the roote, and so maie. 6. But for farther trialle of it: I worke it by the other rule, sayng halfe. 8. is. 4. and his square is. 16. From whiche I abate. 12. and there remaineth. 4. whose roote is. 2. that I maie adde to. 4 and so haue I. 6. for one roote: or els abating it from 4. I shall haue. 2. for the other roote.

The proofe is manifeste for. 6. beeyng a roote, the *zenzicube* is. 46656. The *Surfolide* is. 7776. And the *zenzizenzike* is 1296. So that $8\sqrt{3}$. doe make 62208 And. $12.\zeta.\zeta$. are. 15552. whiche being abated out of 62208 do leaue 46656. the true quantitie of $1\zeta\mathcal{C}$ *The proofe.*

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And so is that woꝝke good, 6. beyng a roote.

Now if, 2. be sette for a roote: then is the. $\sqrt[3]{8}$. 16. the. $\sqrt[3]{27}$. 32, and the. $\sqrt[3]{64}$. And so are. 8. $\sqrt[3]{8}$. equall to. 256. And. 12. $\sqrt[3]{8}$. yelde. 192. Wherfore atating 192. out of. 256. thire resteth. 64, the iuste quantitie of. 1. $\sqrt[3]{64}$. And so is that woꝝke also good, and. 2. a true roote.

The thirde example.

Master. Now pꝛoue this thirde exaruple, where 1. $\sqrt[3]{470016}$ is equalle to. 2000. $\sqrt[3]{470016}$.

Scholar. Halfe the number of the middell sign is 1000. And the square of it is. 1000000. From whiche I shall abate. 470016. and there will remaine 529984. whose square roote by trialle of extraction, I finde to be 728. whiche I maie other adde to. 1000 and so there riseth. 1728. whiche I finde to bee (as it ought) a Cubike number. And his roote to be. 12.

But and if I abate 728. from 1000, there will remain. 272. whiche is no Cubike number.

Master. So that here semeth to be but one roote. And yet these. 2. numbers. 1728. and. 272. kepe soche a rate, that beeyng multiplied together, thei make 470016. whiche is one of the numbers, and beeyng added together, thei make 2000. whiche is the other number of the same *Cubike residuall*.

But now pꝛoue in other like nōbers, whiche haue some distaunce, betwene their Denominations, whether it will so happen still. As namely in this, where 1. $\sqrt[3]{12}$ is equalle to. 12. $\sqrt[3]{12}$.

The fourth example.

Scholar. Halfe. 12. is. 6. and his Square. 36. from whiche abatyng. 32. there is leftte. 4. whose roote is. 2. And if I adde that 2. to. 6. it maketh. 8. whiche is a Cubike number, and hath. 2. for his roote. But if I abate 2. from. 6. there remaineth. 4. whiche is no Cubike nōber, and therfore hath no soche roote. And yet these. 2. numbers. 4. and. 8. by addition, make the middell nōber, and by multiplication, thei make the lastte nōber.

Master.

of Cossike numbers.

Master. Prove yet ones againe in a number, *The fyste* where one quantitie onely is omitted. As when $1\sqrt{3}$ *example.* is equalle to. 24 . ——— 135 .

Scholar. 12 . maketh in square. 144 . from whiche I shall deducte. 135 . and then resteth. 9 . whose square roote is. 3 . whiche if I adde to. 12 . it will bee. 15 . and hath no square roote, as here is required. But if I abate. 3 . from 12 . then remaineth 9 whose square roote is. 3 . and seructh to the number, as I haue here proceud in my tables. And. 9 . and. 15 . kepe the customa- ble rate. For by addition thei make. 24 . And by mul- tiplication, thei yelde. 135 .

But in all these eramples, where the denominati- ons be are a distaunce, I can finde but one roote, and not. 2 . As it was in the other eramples of thesame rule.

And in some of theim, the greater number conta- neth the roote: but in other, the lesser number hath the roote.

Master. Because I can not stele now, about this varietie, I will remitte it till an other tyme. But this by the waie, I must admonishe you, that I doe solowe here, the common forme of writers, in calling these rootes, that rise in equatio, where as thei are not the rootes of those numbers, but are the value of a roote. For of a *Cossike* number, the roote must neades bee a *Cossike* number also. And soche as by multiplication will make the rooted number: But so can not those numbers doe.

And here will I make an eande, of the woorkes of *Cossike* numbers. And now will I ap- plie them to praicise in the rule of equation, that is com- monly called *Al- gebers* rule.

The Arte
The rule of equation, commonly
called Algebers Rule.

The rule of equation.



Ether to haue I taughte you, the
 common formes of woꝝke, in nom-
 bers *Denominate*. Whiche rules are
 vsed also in nōbers *Abstratte*, & like-
 waies in *Surde* numbers. Although
 the formes of these woꝝkes be scue-
 ralle, in eche kinde of number. But
 now will I teache you that rule, that is the pꝛincipall
 in *Coslike* woꝝkes: and soꝝ whiche all the other dooe
 serue.

This Rule is called the Rule of *Algeber*, after the
 name of the inuentoure, as some men thinke: oꝝ by a
 name of singular excellencie, as other iudge. But of
 his vse it is rightly called, the rule of *equation*: bicause
 that by *equation* of numbers, it doeth dissolue doubt-
 full questions: And vnfolde intricate riddles. And this
 is the order of it.

The somme of the rule of equation:



WHen any question is propounded,
 apperteinyng to this rule, you
 shall imagin a name for the nom-
 ber, that is to bee soughte, as you
 remember, that you learned in
 the rule of *false position*. And with that number
 shall you procede, accordyng to the question, vntil
 you finde a *Coslike* number, equalle to that nom-
 ber, that the question expresseth, whiche you shal
 reduce

of Cossike numbers.

reduce euer more to the leaſte numbers. And then diuide the number of the leſſer denomination, by the number of the greateſte denomination, and the quotient doeth aunſwere to the queſtion. Except the greater denominatiō, doe beare the ſigne of ſome rooted nōber. For then muſt you extract the roote of that quotiente, accordyng to that ſigne of denomination.

Scholar. It ſemeth that this rule is all one, with the rule of falſe poſition: and therefore mighte ſo be called: ſeyng it taketh a falſe nōber, to worke with al.

Maſter. This rule doeth farre excell that other. And dooeth not take a falſe number, but a true number for his poſition, as it ſhall be declared anon. Wherby it maie be thoughte, to be a rule of wonderfull inuention, that teacheth a manne at the firſte worde, to name a true number, befoze he knoweth reſolutely, what he hath named.

But bicauſe that name is common to many numbers (although not in one queſtion) and therefore the name is obſcure, till the worke doe detect it, I thinke this rule might well be called, the rule of darke poſition, or of ſtraunge poſition: but not of falſe poſition.

And for the moze eaſie and apte worke in this arte wee dooe commonly name that darke poſition. i. \mathcal{R} . And with it doe we worke, as the queſtion intendeth, till we come to the equation.

This rule of equation, is diuided by ſome men, into diuerſe partes. As namely *Scheubelius* dooeth make. 3. rules of it. And in the ſeconde rule, he putteth. 3. ſeueralle cannōs. Some other men make a greater nōber of diſtinctiōs in this rule. But I intende (as I thinke beſte for this treatice, whiche maie ſerue as farre

*The partes
of the rule.*

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as their woorkes doe extende) to distincte it onely into two partes. Whereof the firste is, when one number is equalle vnto one other. And the seconde is, when one number is compared as equalle vnto .2. other numbers.

Alwaies willyng you to remēber, that you reduce your numbers , to their leaste denominations , and smalleste formes, befoze you procede any farther.

And again, if your equation be soche, that the greatestte denomination Cobike, be ioined to any parte of a compounde number , you shall tourne it so , that the number of the greatestte signe alone , maie stande as equalle to the reste.

And this is all that needeth to be taughte , concernyng this woorkes.

Howbeit, for easie alteratiō of equations. I will propose a fewe exāples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tediousse repetitiō of these woorkes : is equalle to : I will sette as I doe often in woorkes use, a paire of paralleles, or Remoūe lines of one lengthe, thus: =====, bicause noe. 2. thynges, can be moare equalle. And now marke these numbers.

1. 14.ze. — | . 15.9. ===== 71.9.
 2. 20.ze. ——— . 18.9. =====. 102.9.
 3. 26.8. — | — 10ze ===== 9.8. — 10ze — | — 213.9.
 4. 19.ze — | — 192.9. ===== 108. — | — 1089 — 19ze
 5. 18.ze — | — 24.9. ===== 8.8. — | — 2.ze.
 6. 348. ——— 12ze ——— 40ze — | — 4809 — 9.8.
1. In the firste there appeareth, 2. numbers , that is
14.ze.

of Cossike numbers.

14.ze. - 15.9. equalle to one number, whiche is 71.9. But if you marke them well, you maie see one denominatiō, on bothe sides of the *equation*, which neuer ought to stand. Wherefoze abating the lesser, that is. 15.9. out of bothe the numbers, there will remain.

14.ze. = 56.9. that is, by reduction, 1ze. = 4.9.

Scholar. I see, you abate. 15.9. from them bothe. And then are thei equalle still, seying thei wer equalle befoze. Accordyng to the thirde common sentence, in the patthewaite:

If you abate euen portions, from thynge that bee equalle, the partes that remain shall be equall also.

Master. You doe well remeber, the firste groundes of this arte. For all springeth of those principles Geometricalle. Wherefoze call to your minde likewaies the secende common sentence, in the same booke, and then haue you another reason, whiche will helpe you not onely, in the other formes of woork here, but also very often in the practise of this arte.

Scholar. What is this.

If you adde equalle portions, to thynge that bee equalle, what so amounteth of them shall be equalle.

Master. These twoo sentences doe instructe you that when you see on bothe the sides of the *equation*, any one denominatiō Cossike, you shall marke the signe that is annexed to the lesser of them bothe: and if it be the signe of addition. - + -. then shall you abate that lesser number, from bothe the partes of the *equation*. As I did in this firste example. But if the signe be of abatemente —, then shall you adde that lesser number, to bothe partes. And so shall you doe, till there be noe one denomination on bothe partes, but diuerse and distincte.

So the secende number will be. 20.ze. = 1209
and in the leasse termes. 1.ze. = 6.9.

Scholar. I see that you adde. 18.9. to bothe partes. tes

ff. ij.

The Arte

3. tes of the equation. But by that reason, I doubt in the thirde somme, because. $10.ze.$ is in bothe partes of the equation: in the firste parte with $+$, and in the seconde parte with $-$, whether I shall adde $10.ze.$, or abate them.

Master. In soche a case, you maie dooe either of bothe, at your libertie: and all will be to one endc.

Scholar. If I adde. $10.ze.$ then will it be. $26.ze.$
 $10.ze. + 9ze. = 213.ze.$

Master. And doe you not see. $ze.$ on bothe sides of the equation:

Scholar. I did loke but for one alteration onely.

Master. If there were twentie like denominations, you should alter them all. For that is the principalle and peculiarre reduction, that belongeth to equations.

Scholar. Then must I abate. $9.ze.$ on bothe partes, and so will there remaine. $17.ze.$
 $17.ze. - 20.ze. = 213.ze.$

Master. Now reduce it by abating. $10.ze.$

Scholar. So it will be. $17.ze.$
 $17.ze. - 20.ze. = 213.ze.$

And now I remeber, that this is the better forme of reduction. Because the greater denomination, that is. $ze.$ is alone with his number on the one side of the equation, and the. 2. lesser denominations, on the other side.

Master. How doe you reduce the other equations, to their smalleste formes:

4. Scholar. In the fourth example, there is noe denomination, before the signe of equation, or in the first parte, but the like is in the seconde parte also, after the signe of equation. Wherefore firste, because I see $19.ze.$ on bothe sides, I will abate it on bothe sides. And then will it be thus.

$192.ze. = 10.ze. + 108.ze. - 38.ze.$
But

of Coslike numbers.

But bicause I see φ . yet remainyng on bothe partes, I abate the lesser, that is . 108 φ . from bothe partes, and it will be. 84. φ . ———. 10. ζ . ———. 38. ζ .

Master. This equation would bee better, if the greater denomination, did stande as one parte of the equation alone. Whiche thyng you maie easily doe, by addyng. 38. ζ . to bothe partes: bicause so moche foloweth ———, on the one parte.

And euermoze when occasion serueth, to translate *Translations* numbers compoude, ——— on the one side is equalle *of numbers.* to ——— on the other side.

Scholar. When it will be thus.

$$84.\varphi. - + 38.\zeta. = 10.\zeta.$$

Master. It were better thus.

$$10.\zeta. = 38.\zeta. - + 84.\varphi.$$

And in smaller termes.

$$5.\zeta. = 19.\zeta. - + 42.\varphi.$$

But now procede with the examples.

Scholar. The fifth is easily reduced, by abatyng 5. ζ . on bothe sides: for so will it bee.

$$8.\zeta. = 16.\zeta. - + 24.\varphi.$$

The sixth equation will be, by addyng. 12. ζ . on bothe sides. 34. ζ . ——— 52. ζ . ——— 480. φ . ——— 98. φ .

But yet I must reduce it farther, by addyng. 9. ζ . on bothe sides. And then it will stande thus.

$$43.\zeta. = 52.\zeta. - + 480.\varphi.$$

Master. Now will I shewe you the varieties of equations, taught by Scheubelius, bicause you maie perceiue, how they bee contained in those. 2. formes, named by me. As for the manyfolde varieties, that some other doe teache, I accoumpte it but an idle bablyng, or (to speake moare fauourably of them) an vnnecessary
Varieties of equations.

The Arte

The firste
equation.

distinction.

The first equatio after *Scheubelius*, & after my meanyng also, is, when one number is equall to an other: meanyng that thei bothe must be simple numbers *Cosfiske*, and vncompounde. As. 6. \mathcal{Z} . equalle to. 18. \mathcal{Q} :

$$4. \mathcal{Z}. \text{---} = 12. \mathcal{Z}.$$

$$14. \mathcal{C}. \text{---} = 70. \mathcal{Z}:$$

$$15. \mathcal{Z}. \text{---} = 90. \mathcal{Z} \mathcal{Z}:$$

$$20. \mathcal{Z}. \mathcal{C} \text{---} = 180. \mathcal{Z} \mathcal{Z}:$$

$$26. \mathcal{Z}. \mathcal{Z}. \text{---} = 117. \mathcal{C} \mathcal{C}.$$

In all these examples, as you see but one number, compared to an other: so to finde the quantitie of one roote, you shall diuide the number of the lesser Character, by the number of the greater Character, and so shall the *quotiente* byyng forth the quantitie of. 1. \mathcal{Z} .

Scholar. It semeth at the firste belve, that it is against reason, to diuide the number of the lesser signe, by the number of the greater. But when I consider, that if I compare a number of crownes, or any like denomination, to a number of shillynges in equaltie, the number of crownes, or other soche like, must needs be lesser, then the nōber of shillinges. And so diuiding the nōber of the shillinges (or other lesser name) by the number of crownes (or other greater name) the *quotiente* will shewe, how many shillynges make a crowne: and generally, how many of the lesser, dooe make one of the greater.

As if. 20. crownes bee equalle to. 100. shillynges, then. 5. shillynges dooeth make a crowne. So when 6. \mathcal{Z} . bee equall to. 18. \mathcal{Q} . then. 3. \mathcal{Q} . doeth make. 1. \mathcal{Z} . And. 4. \mathcal{Z} . --- = 12. \mathcal{Z} . dooeth cause that. 3. \mathcal{Q} . must be a roote.

Master. As your examplarie profe is good, so reduction will be a sufficiente profe in this.

Scholar. I see it manifestly. For if. 14. \mathcal{C} . bee equalle to. 70. \mathcal{Z} . then. 1. \mathcal{C} . is equalle to. 5. \mathcal{Z} . by that
reduction

of Coslike numbers.

reduction in numbers. And again by reduction in *U*gnes. 1. \mathcal{Z} . is equalle to. 5. \mathcal{G} .

Likewais. 15. \mathcal{F} . beynge equalle to. 90. \mathcal{Z} . \mathcal{Z} . reduction by signes and numbers also, will make 1. \mathcal{Z} = 6. \mathcal{G} . So shall. 20. \mathcal{Z} . \mathcal{C} . = 180. \mathcal{F} . be reduced to. 1. \mathcal{Z} . = 9. \mathcal{G} . And. 26. \mathcal{Z} . \mathcal{F} . = 104. \mathcal{C} . \mathcal{C} . will make. 1. \mathcal{Z} . = 4. \mathcal{G} .

Master. And so generally, when there is no denomination omitted, betwene those. 2. that bee compared in equalitie, still the diuision of the number, of the lesser denomination, by the number of the greater denomination, will bynng forth in the *quotiente*, the quantitie of. 1. \mathcal{Z} .

But if there bee any denominations omitted, betwene those. 2. whiche be compared together in equalitie: loke how many denominations are omitted, and so many in order is the rooted quantitie, whose roote you must extract, for the answer to the questiō. For in soche a case, euer moze you shall extracte the roote of your laste number.

As for example, when. 6. \mathcal{C} . be equalle to. 24. \mathcal{Z} . by the former rule, you shall finde. 4. in the *quotiente*. But here that. 4. is not the quantitie of a roote, but is a rooted number, whose roote I shall extracte. And seynge betwene. \mathcal{C} . and. \mathcal{Z} . there is no quantitie omitted, but one, that is. \mathcal{Z} . Therefore I shall accounte. 4. the firste quantitie, that is to saie, a *Square* number, and so take his *Square* roote, beynge. 2. for the quantitie of a roote.

Again if. 7. \mathcal{F} . be equalle to. 567. \mathcal{Z} . the *quotiente* will be. 81. and declareth a *zenzizenzike* number, because there are omitted betwene. \mathcal{F} . and. \mathcal{Z} . three numbers: and *zenzizenzike* is the thirde quantitie: as you did learne in the beginning of this treatise, of numbers denominate.

Scholar. I perceiue it. And therefore I must take
the

The Arte

the *zenzizenzike* root of 81. whiche is. 3. and that is the true roote, where. $7\sqrt{3}$. be equalle to. $567.ze$.

Master. And if those. $7\sqrt{3}$. were accepted equalle to. 56.3 . the *quotiente* will be. 8. And because there are omitted. 2. quantities, that is. \mathcal{C} . and. 33 . therefore you shall accompte that. 8. to be 1 \mathcal{C} . or a seconde quantitie. And his roote *Cubike* is. 2. whiche standeth as the valewe of a roote, in the former equation.

And it is not possible that any other number, maie be placed as a roote, in that equation: or in any other forme of this firste kinde. Whobecit in one sorte of equation, of the seconde kinde, there maie be. 2. diuerse rootes, when one number hath. 2. rootes in valewe. As I taught you before in the extraction of rootes.

The seconde kinde of equation.

The second kinde of equatio, after *Scheubelius* minde and myne also, is, when one simple number *Cosike*, is compared as equalle to. 2. other simple numbers *Cosike*, of seueralle denominations, and like distaunce.

And in soche equation, being reduced as is taught before, the roote of those. 2. numbers compounded, as in one (or rather the valewe thereof) shall be extracted: As I haue before taughte also. And that roote doeth aunswere to the question.

The seconde forme of the second kinde

Whobecit, here is the like obseruation, as was in the seconde forme of the firste kinde. For if those. 3. denominations be not immediate, but doe omit some other betwene them, then shall you extracte the roote of that laste number, in all pointes, as you did in the firste equation.

Examples of the firste sorte.

$$4.3. = 6.ze - 4.9.$$

whiche being reduced, will bee:

$$1.3. = 1.ze - 1.9. \text{ And the roote will be. } 2$$

$$\text{And. } 6.\sqrt{3}. = 12.33. - 18.\mathcal{C}.$$

That

of Cossike numbers.

What is by reduction.

$$1. \text{fz} \cdot \text{---} 2. \text{z} \cdot \text{z} \cdot \text{---} 3. \text{c} \cdot \text{oz}$$

$$1. \text{z} \cdot \text{---} 2. \text{ze} \cdot \text{---} 3. \text{q} \cdot \text{And the roote. z}$$

$$5. \text{fz} \text{---} 25. \text{z} \cdot \text{z} \text{---} 30. \text{c} \cdot \text{Dz by reduction.}$$

$$1. \text{fz} \cdot \text{---} 5. \text{z} \cdot \text{z} \cdot \text{---} 6. \text{c} \cdot \text{Dz}$$

$$1. \text{z} \cdot \text{---} 5. \text{ze} \text{---} 6. \text{q} \cdot \text{whose roote is. 3. oz. 2.}$$

$$\text{Like waies. } 2. \text{z} \text{---} 120. \text{q} \cdot \text{---} 8. \text{ze}$$

$$\text{Dz by reduction. } 1. \text{z} \text{---} 60. \text{q} \text{---} 4. \text{ze} \cdot \text{whose roote is. 6.}$$

Examples of the seconde sorte.

$$5. \text{z} \cdot \text{z} \text{---} 60. \text{z} \cdot \text{---} 320. \text{q}$$

What maketh by reduction.

$$1. \text{z} \cdot \text{z} \cdot \text{---} 12. \text{z} \cdot \text{---} 64. \text{q}$$

And the square roote. 4.

$$\text{Like waies. } 8 \text{z} \cdot \text{c} \cdot \text{---} 40. \text{c} \cdot \text{---} 30208. \text{q}$$

Dz by the orderly reduction.

$$1. \text{z} \cdot \text{c} \text{---} 5. \text{c} \cdot \text{---} 3776. \text{q} \cdot \text{whose Cubike roote is. 4.}$$

Again in residualles.

$$8. \text{z} \cdot \text{c} \text{---} 864. \text{z} \cdot \text{---} 24. \text{z} \cdot \text{z} \cdot$$

What maketh by reduction.

$$1. \text{z} \cdot \text{c} \text{---} 108. \text{z} \cdot \text{---} 3. \text{z} \cdot \text{z} \cdot \text{Dz els}$$

$$1. \text{z} \cdot \text{z} \text{---} 108. \text{q} \text{---} 3. \text{z} \cdot \text{whose roote is. 3}$$

$$30. 9. \text{bz} \text{---} 90. \text{z} \cdot \text{z} \cdot \text{---} 144. \text{ze}$$

$$\text{Dz by reduction. } 1. \text{z} \cdot \text{c} \text{---} 10. \text{c} \cdot \text{---} 16. \text{q} \cdot \text{whose roote is. 8. oz. 2.}$$

But now because *Scheubelius* dooth make. 2. severall equations of these. 2. formes: And giueth. 3. diuerse rules, or canons for eche of them, I will declare his. 6. canons to be all contained in this seconde kind of equation.

The Arte

He maketh his diuision thus. When .1. number is compared as equalle to .2. other, other that one number is of the smalleste denomination. And then is it of the firste Canon. As. $1.3 \text{ --- } + \text{ --- } 8.ze \text{ ---} = 65.9.02$ els that one number, is of the greateste denomination: As. $3.ze \text{ --- } + \text{ --- } 4.9 \text{ ---} = 1.3$. And then is it of the seconde Canon: Or els thirdely, the alone number is of the middle denomination: and then is it of the thirde Canon. As. $1.3 \text{ --- } + \text{ --- } 12.9 \text{ ---} = 8.ze$.

The like forme he useth, for the numbers of denominations distaunte.

Wherby you maie perceiue, that in my rule there is noe forme of numbers, like the of the firste Canon, nother yet of the thirde: but onely of the seconde. What then again in my rule, there are .2. sortes of examples whiche he hath not. And if you compare them well together, you shall perceiue, that thei be agreeable together.

As for exaple. In his firste canon, this is the forme $1.3 \text{ --- } + \text{ --- } 6.ze \text{ ---} = 27.9$. whiche equation in my rule, by translation, is expressed thus.

$1.3 \text{ ---} = 27.9 \text{ ---} - 6.ze$. because I doe still set the greateste denomination alone.

Again in his thirde Canon, this is an example.

$1.3 \text{ --- } + \text{ --- } 13.9 \text{ ---} = 8.ze$ and that number doe I translate into this forme $1.3 \text{ ---} = 8.ze \text{ ---} - 13.9$.

Now where as he giueth generalle rules, for euery Canon, I sawe for them all: extracte the roote of that composide number. For all his rules doe teache nothing els.

Scholar. I doe vnderstande the diuersitie, and agreemente of your rules and his. But for my exercise, I doe couette some apte questions, appertaining to these equations.

*A question
of ages.*

Master. Take this for the firste question.

Alexander being asked how olde he was, I am .2. yeres

of Coslike numbers.

peres elder (quod he) then Ephestio. Wea, saied Ephe-
stio. And my father was as olde as we bothe, and. 4.
peres moare. And my father hauyng all those yeres,
saied Alexander, was. 96. peres of age. I demaunde
now of you, how olde was eche of them.

Scholar. I praie you awiswere the question your
self, to teache me the forme.

Master. I will begin with the yongeste mannes
age, and that will I call x . whiche is the common
supposition in all soche questions. Then is Alexan-
ders age. 2. yeres moare, that is. $x + 2$. And
those bothe together dooe make. $2x + 2$.
Wherunto if you put. 4. moare, then haue you the age
of Ephestio his father, that will be. $2x + 6$.
And all these put together, that is. $4x + 8$.
will make 96 whiche is the equation that shall open
the question.

Wherfore I set downe the equation thus.

$4x + 8 = 96$. And bicause I see on
bothe sides, one denomination of. 4. I doe abate. 8.
fro bothe sides: & then there remaineth. $4x = 88$
And by reduction or diuision, $x = 22$.

Scholar. Then maie I easily saie, that Ephestio
was. 22. yeres olde, scyng you did putte. 1. for his
age: and now. 1. is founde to be. 22. And therby all
the other yeres be manifeste. For Alexander beyng. 2
peres elder, must be. 24. And Ephestio his fa-
ther had in age. 22. and. 24: and. 4. moare, that
is. 50. yeres. All whiche make. 96. So is that
question fully answered.

Master. An other question is this. I had
a soume of monee owyng vnto me: whereof I did re-
ceiue at one tyme $\frac{1}{2}$ and after ward I receiued $\frac{1}{3}$ of that
residue, whiche remained vnpaied. And so remained
the reste of the debte 27. l. I would knowe what was
the first debte, & what wer the. 2. generall paymentes

C. ij. Scholar

The Arte

Scholar. This muste I obserue still, to name the firste doubtfull thyng. $1. \frac{2}{3}$. wherefoze I saie that the firste debte was $1. \frac{2}{3}$. whercof I receiued $\frac{1}{3}$. And so did there remain. $\frac{1}{3}$. of whiche reste, againe I receiued $\frac{1}{3}$. that is $\frac{2}{9}$. of the whole somme, or $\frac{2}{9}$. And that be- yng abated also, then did there remaine $\frac{1}{9}$. whiche you named to be. $27. \text{li}$. Then if $\frac{2}{9}$. be equalle to $27. \text{li}$. diuide. $27. \text{li}$. by $\frac{2}{9}$. and the *quotiente* will bee 108 . that is. 60 . whiche was the whole debte: And then is it plaine, that $\frac{1}{3}$. of it is. 15 . and $\frac{2}{3}$. of the residue is. 18 . whiche maketh. 33 . and then remaineth. 27 .

Master. There is nothyng better then exercise, in attainyng any kynde of knowlege: And therfoze I will proue you with diuerse questions, to make you the moare experte in this rule. And this is one.

*A question of
sayng.*

There is a flooze paved with Square Brickes, the lengthe of that flooze be yng longer then the bredth, by $\frac{1}{7}$. and the whole pavemente containeth. 3584 . byckes: I require to knowe the bredthe and lengthe.

Scholar. The lesser quatitie, whiche is the bredth I doe name. $1. \frac{2}{7}$. And then the lengthe will bee, by your proportion. $1 \frac{1}{7}$. Now must I multiplie the bredthe by the lengthe (soz that is the orderly worke in all flatte formes, to finde out the whole platte) that is here. $1. \frac{2}{7}$. by $1 \frac{1}{7}$. and there will amounte the whole platte. 3584 . whiche by your supposition is equalle to. 3584 .

Wherefoze accordyng to your rule, I diuide. 3584 . by $\frac{2}{7}$. and the *quotiente* will be. 3136 . whiche is a Square number, bicause there is one denomination omitted in this equatio. For betwene 7 and 9 . there is omitted. 20 . And therfoze must I extracte the square roote of. 3136 . and it will bee the quantitie of. $1. \frac{2}{7}$. that I woork in my tables, and finde it. 56 . whiche must be the bredthe: soz that I named. $1. \frac{2}{7}$. Then the length must be moare by $\frac{1}{7}$. of it: and so shall it be. 64 .

Now

of Coslike numbers.

Now for to confirme my woorkes, I multiplie .56. by .64. and it will make .3584. whiche is the number that you old name.

Master. What question is well aunswered: And if you had put .1.20. for the lengthe, as you might do, then the bredthe will be 720. and the square 720. and so .1.20. would bee .64. as you maie proue at letter: but in the meane time, what saie you to this questiō?

An other woorkes of that questiō

There is a capitain, whiche hath a greate armie, & would gladly Marshall thē, into a square battaile, as large as mighte bee. Wherefore in his firste prooffe of square foume, he had remainyng .284. to many. And prouyng again by puttynge .1. moare in the fronte, he founde wante of .25. men. How many souldiars had he, as you gesse:

A questiō of an armie.

Scholar. I call the firste fronte .1.20. and then multipliyng it squarely: I shall haue for the whole battaile .i. 3. and so by your sayng, there was leste 284. men, wherefore the whole number of men, was 1.3. — 284.9.

Now for the seconde prooffe, when the fronte was increased by .1. man: I shall set the former fronte, and 1. manne moare, that is 1.20. — 1.9. And multipliyngz that number, squarely: there will arise for the whole armie.

$$1.20. - 1.9.$$

$$1.20. - 1.9.$$

$$1.3. - 1.20.$$

$$1.20. - 1.9$$

$$1.3. - 220. - 1.9.$$

out of whiche I muste abate 25 that, you saie, did

$$1.3. - 220. - 1.9.$$

wante, to make hy that square battaile. And then it

will be .1.3. — 220. — 24.9.

Now haue I one number of menne, expressed by 2 Coslike numbers: Of necessitie therefore must these 2. numbers be equalle: seyng they represente one armie.

Wherefore I set them thus.

Og. ij. 1.3.

The Arte

$$1. \text{z} \text{---} | \text{---} 284. \text{q} \text{---} \text{---} 1. \text{z} \text{---} | \text{---} 2. \text{ze} \text{---} \text{---} 24. \text{q}.$$

And findyng. 1. z. on bothe partes of the equation, I doe abate it, & then standeth. 284 q --- 2 ze --- 24 q. Yet again I see. q. on bothe sides of the equation, and therfore, seing the lesser of them hath the signe of subtraction, I doe adde. 24: to bothe numbers, and then will there be. 308. --- 2. ze. that is. 154. --- 1. ze

So that seing. ze was set for the first fronte: the same front must be. 154. whose Square is. 23716. unto whiche I muste adde the. 284. that did abounde, and then will the whole number be. 24000.

| | |
|--------|------|
| 154. | 154. |
| 154 | 154 |
| 616. | |
| 770 | 770 |
| 154 | 154 |
| 23716. | |
| 284. | 284. |
| 24000. | |

For farther trialle whercof, I take the seconde fronte to be. 155. that is. 1. moare then the firste: and his Square will bee 24025. And so is there. 25. moare then the iuste number of the armie, as the question supposed.

*An other
woorke of
that questio.*

Master. That question maye be wrought also, by namyng the seconde fronte. 1. ze. and then will his square bee. 1. z. but seing there wanteth. 25. menne, to make that Square battaile, the number shall bee 1. z. --- 25. q.

Then for the firste front, you must set. 1. man lesse, as the question importeth, & that will be. 1. ze --- 1. q whose square will be 1. z --- 1. q. --- 2. ze.

$$1. \text{ze} \text{---} \text{---} 1. \text{q}.$$

$$1. \text{ze} \text{---} \text{---} 1. \text{q}.$$

$$1. \text{z} \text{---} \text{---} 1. \text{ze}.$$

$$\text{---} \text{---} 1. \text{ze} \text{---} | \text{---} 1. \text{q}.$$

$$1. \text{z} \text{---} | \text{---} 1. \text{q} \text{---} \text{---} 2. \text{ze}.$$

unto whiche I must adde the. 284. menne that did abounde, whē that battaile was framed, and then will the

of Coslike numbers.

the number be. $1. \text{z}.$ — $285. \text{q}.$ — $2. \text{ze}.$ And
it must bee equalle to. $1. \text{z}.$ — $25. \text{q}.$ Wh. toze to
reduce that equation, strike z adde on bothe sides $25. \text{q}$
& then resteth. $1. \text{z}.$ equalle to. $1. \text{z}.$ — $310.$ — $2. \text{ze}$
Then z adde. $2. \text{ze}.$ because z will haue noe — in
the equation: and it will be,
 $1. \text{z}.$ — $2. \text{ze}.$ — $1. \text{z}.$ — $310. \text{q}.$ Throvely z
abate. $1. \text{z}.$ on bothe sides of the equation: and then
remaineth. $2. \text{ze}.$ — $310. \text{q}.$ that is. $1. \text{ze}.$ — $155. \text{q}.$
wherby it appeareth that the seconde fronte was. 155
and the firste fronte. $154.$ & so forthe, as you wrought
it before.

An other question is this.

Where is a kyng with a greate armie: And his ad: *A question*
uerfarie corrupteth one of his heraultes with gistes, *of an armie.*
and maketh hym swere, that he will tell hym, how
many Dukes, Erles and other souldiars there are in
that armie. The heraulte lothe to leafe those gistes,
and as lothe to bee vnttrue to his Prince, diuiseth his
aunswere, whiche was true, but yet not so plain, that
the aduersarie coulde therby vnderstande that, whiche
he desired. And that aunswere was this.

Looke how many Dukes there are, and for eche of
them, there are twise so many Erles. And vnder eue-
ry Erle, there are sower tymes so many soldiars, as
there be Dukes in the fiede. And when the muster of
the soldiars was taken, the. $200.$ parte of them, was
 $9.$ tymes so many as the number of the Dukes.

This is a true declaratiō of eche number, quod the
heraute: and z haue discharged my othe. Now gesse
you how many of eche sozte there was.

Scholar. Although the question seme harde, z see
many tymes, that diligence maketh harde thynges
easie, and therfore z will attempte the worke of it.

And firste for the number of Dukes, z sette. $1. \text{ze}.$
then will the number of Erles bee. $2. \text{z}.$ that is. $1. \text{ze}.$
by

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by 1. \mathcal{C} multiplied twice: And the number of soldiars are 8. \mathcal{C} . that is. 2. \mathcal{Z} . multiplied by 1. \mathcal{C} . sover ty mes, but bicause the. 200. parte of the soldiars is. 9. tymes so moche as the number of the Dukes, therfoze must the. 200. parte of 8. \mathcal{C} be equalle to. 9. \mathcal{Z} . And so consequently. 8. \mathcal{C} \longleftarrow 1800. \mathcal{Z} and 1. \mathcal{C} \longleftarrow 225. \mathcal{Z} . or. 1. \mathcal{Z} . \longleftarrow 225. \mathcal{Z} .

For if I set $\frac{1}{100}$ and 9. as equalle together, & would by the arte of fractions, bynge the same ppozition in whole numbers, I shall haue for. 9. this fraction $\frac{1^{100}}{300}$. And seyng the denominato^rs, be all one in $\frac{1}{100}$ and in $\frac{1^{100}}{300}$ the ppozition consisteth betwene the numera to^rs.

When to procede, if. 225. be equalle to. 1. \mathcal{Z} . I shall take the square roote of. 225. for. 1. \mathcal{Z} . and that is. 15 whiche must be the number of Dukes.

And so haue I the firste number, wherfoze the se conde number, that is the number of Ceres, must bee 15. tymes. 15. twice: that is. 450. And the number of soldiars shall be. 4. tymes. 15. multiplied by. 450. that is. 27000. And for iuste trialle of this woork, I take the. 200. parte of the soldiars that is. 1350. and I

| | |
|-------|--|
| 450. | |
| 60 | |
| 27000 | |

finde it to bee. 9. tymes. 15. that is. 9. tymes so moche as the number of the Dukes. And so is that question solued, and tried.

*An other
question of
walles.*

Master. This is an other qucition. There is a grounde inclosed with. 4. walles, beyng like iambes and of one heigthe. The longest. 2. walles are in ppozition to the shorteste, as. 5. to. 3. And vnto the height thei bee double *Sequialter*. Now multiptyng the longeste by the shorteste, and that totalle by the heighte, there will rise. 39936. foote. I demaunde then, what is the lengthe and the heighte of eche walle?

Scholar. The least quantitie is the heighte, whiche I call. 1. \mathcal{C} . and vnto it the longeste walle is double *Sequialter*:

of Coslike numbers.

Sesquialter: that is. $2\frac{1}{2}$. ℥ . Now that same longeste is in propozition *Superbipartiente quintas*, to the shortestte walle. So must the shorter wall be $1\frac{1}{2}$ ℥ . Then must I multiplie all those. 3. nöbers together, that is $1\frac{1}{2}$. ℥ . by $1\frac{1}{2}$ ℥ . whereof doeth come. $\frac{3}{2}$ ℥ . then shall I multiplie that totalle, by $\frac{3}{2}$ ℥ . and it will be $\frac{9}{4}$ ℥ . or $3\frac{3}{4}$ ℥ whiche must be equalle, by the woordes of the questi-
on, to. 39930 .

So by reducyng them to one denomination. $\frac{9}{4}$ ℥ . shall be equalle to $\frac{159720}{4}$ that is. $15. \text{℥} = 159720. \text{℥}$. and. $1. \text{℥} = 10648$. wherefore I shall extracte the *Cubike* roote out of. 10648 . and that is the quantitie of. $1. \text{℥}$. or the heighte of the walle.

In my Tables I woorkke that extraction of *Cubike* roote, and finde it to be. 22 . And therfore must the longeste walle bee double *Sesquialter* to it, that is. 55 . And the shortestte walle will be. 33 .

For prooffe whereof I dooe multiplie. 22 . with. 55 . *The prooffe.* and it maketh. 1210 . whiche number I shall multiplie by. 33 . and it will be. 39930 . according to the sup-
position of the question.

Master. You doe chose still the leasest number, to be equalle to. $1\frac{1}{2}$. as the easiest forme. Howbeit you maie put. $1\frac{1}{2}$. for the lengthe of any of the walles.

And if you sette it for the longeste walle, then the shortestte walle will be $\frac{2}{3}$ ℥ . and the heighte $\frac{2}{3}$ ℥ . and all those. 3. numbers will make, by multiplication together $\frac{4}{27}$ ℥ . equalle to. 39930 . And so will. $6. \text{℥}$. be equalle to. 998250 . ℥ . and. $1. \text{℥} = 166375. \text{℥}$. whereof the *Cubike* roote is 55 . and aunswereth to the quantitie of. $1\frac{1}{2}$. *An other forme of woorkke.*

But if. $1. \text{℥}$. be set for the measure of the shortestte walle, then the longeste walle will be $\frac{1}{3}$ ℥ . and the heighte $\frac{1}{3}$ ℥ . And so all. 3. numbers multiplied together will make $\frac{1}{27}$ ℥ . =. 39930 . So shall. $10. \text{℥}$. be equall to. 359370 . And. $1. \text{℥} = 35937$. where

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of the *Cubike* roote is. 33. and is the baluc of. 1. 720. in this position.

Scholar. This varietie of woorkes, is not onely pleasaunte, but it maketh the reason of the woorkes to appeare moare plainly. So that I could neuer be we- ric to heare soche questions.

Master. Then will I propounde one or 2. moare befoze we passe from this kinde of equation. Wher- of one shall be somewhat like that last. And this it is.

A question
of Bricke.

A Brickelair had a pile of Bricke, whiche he sold by the yarde. The lengthe of it was 7 to the bredthe, that is *Triplafesquialtera*. And the heighte was five ty- mes so moche as the legthe. This pile the owner sold for. 980. crounes. By soche rate that he had for every yarde so many Crounes, as the Pile had yardes in bredthe. Now is the question, what was the lengthe, bredthe, and heighte of this pile?

Scholar. I suppose the bredthe to bee. 1. 20. then was the length 3 1/2 20. and the heighte. 17 1/2 20. These 3. foumes dooe I multiplie together, and thei make 245 000. whiche standeth as equalle to all the yarde in the whole pile. But yet what that is, I knowe not.

Wherfoze to proccede farther, I consider that eue- ry yarde coste as many crounes, as the bredthe contain- ed yarde. Now the bredthe being 1. 20. I must saie, that every yarde did coste. 1. 20. of crounes. And then by the Golden rule: if. 1. yarde cosse. 1. 20. of Crounes, what shall 245 000. cosse?

$$1. \quad \begin{array}{l} 1. 20. \\ \hline 245 \text{ } 000. \end{array}$$

Woorkyng by the rule, I finde that it shall cost 245 000. And the question doeth suppose that it cosse. 980. crounes. Wherfoze I must saie, that. 980. crounes, are equalle to 245 000. And consequently. 245 000. = 3920. 0. wherfoze di- uidinge the number of the lesser name, by the other, the *quotiente* will be 16. whose *zenzizenzike* roote is 2

And

of Cossike numbers.

And that therfoze must be the value of a roote, and the bredthe of the pile. So shall the lengthe be .7. yardes, and the heighte. 35. yardes.

For trialle of it, I multiplie the lengthe, by the bredthe, and that totalle by the heighte, and so haue I *The prooffe.*
 490. for all the yardes of Wicke. Then consideryng that euery yarde coste .2. crownes, bicause .2. yardes is the bredthe of the pile: the number of crownes must be twice .490. that is .980. And so is the woorkie good.

Master. Now woorkie that question, by settinge *Another forme of woorkie.*
 1. ℥. for the lengthe.

Scholar. If the lengthe be .1. ℥. the bredthe must bee $\frac{3}{7}$ ℥. that is *Subtriplesesquialtera* to .1. ℥. And the heighte must bee .5. ℥. All whiche sommes make by multiplication $\frac{15}{7}$ ℥.

Then farther, if 1. yarde coste $\frac{3}{7}$ ℥. shall coste 980. ℥. whiche is equalle to 980. And so is .20. ℥. equal to .48020. and by diuision $\frac{15}{7}$ ℥. $\frac{1}{7}$ ℥. $\frac{1}{35}$ ℥. $\frac{1}{49}$ ℥.
 1. ℥. $\frac{3}{7}$ ℥. = .2401. whose *zenzenzike* roote is .7. And that is the lengthe of the walle, and is the value of .1. ℥.

The reste of this woorkie, is like as befoze.

Master. Yet proue the thirde waie.

Scholar. The heighte beeyng .1. ℥. the lengthe must be the first part of it, that is $\frac{1}{7}$ ℥. And the bredth $\frac{3}{7}$ ℥. All these make by multiplication $\frac{3}{49}$ ℥. Then for the price, if 1. yarde coste $\frac{3}{7}$ ℥. what shall $\frac{3}{49}$ ℥. coste? By the *A thirde forme of woorkie.*
 Golden Rule there is founde, $\frac{1}{49}$ ℥. $\frac{1}{35}$ ℥. $\frac{1}{49}$ ℥.

$\frac{1}{49}$ ℥. $\frac{3}{7}$ ℥., whiche is equalle to 980. And so shall .4. ℥. be equalle to .6002500. And .1. ℥. = 1500625. whose *zenzenzike* roote is .35. And that is the value of .1. ℥. and the heighte of the pile.

Master. One question more will I propounde,
 14. v. and

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and so eande wltth this equation.

*A question of
a Testament.*

A pooze man died, whiche had solwer childzen, and all his goodes came to. 72. crounes: whiche he would haue parted so, that the seconde & thirde childe should haue. 7. times so moche as the firste. And that the portions of the thirde and fourthe childe should bee. 5. tymes so moche as the secondes parte: And that the first and the fourthe, should haue twise as moche as the thirde. If you worke the solution wel, you maie seme wortthy to be master of those wardes.

Scholar. I trust to obtaine moare benefite by the question, then by that office. Wherefore I will giue good hede vnto it. And so; the firste nōber I set. 1. \mathcal{Z} then muste the seconde and thirde portions make together. $7. \mathcal{Z}$. And the fourthe must bee all the reste of the. 72. that is. $72 - 8 \mathcal{Z}$. Now the thirde must be halfe the firste & the fourthe, that is. $36 - 3 \frac{1}{2} \mathcal{Z}$. And the thirde & fourthe, is. 5. tymes the second. wherefore the seconde shall be the. 5. part of. $108 - 11 \frac{1}{2} \mathcal{Z}$ that is. $21 \frac{1}{2} - 2 \frac{1}{2} \mathcal{Z}$, whiche number I shall set in order with Letters, as here I haue dooen so; my owne case, and aide of memory. And then shal I adde them all together. Whereof there commeth.

$129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$, whiche is equalle to 72. First ther

| | |
|---|--|
| A | 1. \mathcal{Z} . |
| B | $21 \frac{1}{2} - 2 \frac{1}{2} \mathcal{Z}$. |
| C | 36. ———. $3 \frac{1}{2} \mathcal{Z}$. |
| D | 27. ———. $8. \mathcal{Z}$. |
| | |
| | $129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$. |

fore I do adde all that solo weth — to bothe partes of the equatiō. And so haue I $129 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z} - 72$. But bicause there are numbers Absolute on bothe sides, I shall abate the lesser somme, that is. 72. from bothe partes, and then will there bee lefte, $57 \frac{1}{2} - 12 \frac{1}{2} \mathcal{Z}$. that is. $288 - 64 \mathcal{Z}$. And by diuision $4 \frac{1}{2} = 1. \mathcal{Z}$.

The prooffe.

So shall the firste mannes portion bee $4 \frac{1}{2}$. And the seconde and thirde mannes portion. 7. times so moche that

of Coflike numbers.

that is. $31\frac{1}{2}$. Whereby it follo weth, A $4\frac{1}{2}$.
 that the fourthe manne, shall haue B $11\frac{1}{2}$ } $31\frac{1}{2}$.
 the reste of 72. that is. 36. C $20\frac{1}{2}$ }

Then seeing the thirde manne, D 36 .
 hath halfe so moche as the first and
 the fourthe, his portio shall be $20\frac{1}{2}$.
 And then by diuerse reasons, the seconde mannes part
 shall bee. $11\frac{1}{2}$. And all these partes added together, doe
 make iuste 72. Wherefoze the woork is good.

Master. You haue wroughte it well. And yet *Another*
 maie you woork it thus. Firste sette doune. $1.ze$. for *forme of*
 the firste mannes parte. And then for the seconde and *woork.*
 thirde ioyntly. $7.ze$. so shall the fourthe manne haue
 72.9 . ——— $8.ze$. And because the seconde mannes
 parte is $\frac{1}{2}$. of the thirde and fourthe mannes portio,
 if you ioyne all their. 3. partes together, the seconde
 mannes portio will be $\frac{1}{2}$ of that totalle. But therfoze
 $7.ze$, whiche is the partes of the second and the thirde
 into. 72 ——— $8ze$, whiche is the fourthe mannes
 parte, and the totalle will be. 72.9 ——— $1.ze$. whose
 firste parte is 12.9 . ——— $\frac{1}{2}ze$, for the seconde man-
 nes share. Whiche somme if you abate out of. $7.ze$.
 there wil remain for the thirde
 mannes parte $7\frac{1}{2}ze$ ——— 12.9 .

And so haue you euey man-
 nes portio allotted to hym due-
 ly. As I haue here set it for the
 for you. And all thei added to-
 gether, doe make. 72.

| | | |
|---|------------------|---------------------|
| A | $1.ze$. | |
| B | 12.9 . | ——— $\frac{1}{2}ze$ |
| C | $7\frac{1}{2}ze$ | ——— 12.9 |
| D | 72 . | ——— $8.ze$. |
| | | 72 . |

Scholar. But here is noe equatio yet, though the
 partes be diuided iustly.

Master. Now shall you see it.

The question salet, that the thirde mannes portio
 is halfe the portions of the firste and fourthe man-
 wherefoze seeing the firste and fourthe mannes portio-
 ns doe make. 72 ——— $7ze$. the thirde mannes portio
th. iij. tion

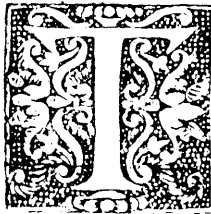
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tion being doubled, shall make as moche. But the double of the thirde manes parte, is $14\frac{1}{2}\text{℥}$ — 24℥ . and therefore I saie, that those .2. numbers be equalle; that is, 72℥ . — 7℥ . — $14\frac{1}{2}\text{℥}$. — 24℥ . Firste adde. 7℥ . to eche parte, and it will bee 72℥ . — $21\frac{1}{2}\text{℥}$. — 24℥ . Then adde. 24℥ . on bothe sides, and there will be. 96℥ . — $21\frac{1}{2}\text{℥}$. that is by reduction. 288 . — 64℥ . as you made it. And then all agreeth.

Like wates for the equation, you may set the thirde mannes portion, with the halfe of the firste & fourth mannes partes. And so will. $7\frac{1}{2}\text{℥}$. — 12℥ . be equalle to. 36℥ . — $3\frac{1}{2}\text{℥}$. And by reduction, $10\frac{1}{2}\text{℥}$. — 48℥ . That is in other termes of whole number. 32℥ . — 144 . And by diuision it will bee 1℥ . — $4\frac{1}{2}$. And thus will we eade the examplis of the firste equation, for this tyme. And will shewe you some questions of the seconde equation.

Examples of the seconde equation, by questions propounded.

A question
of silkes.



There are two men that haue silke to sell. The one hath. 40 . elnes, and the other. 90 . And the firste man his silke is not so fine as the seconde man his silke. So that he selleth in euery angell, price more by $\frac{2}{3}$ of an elne, then the seconde ma doeth. And at the eande, bothe their monies made but 42 . angelles. Now I demaunde of you, how moche eche man solde for an angell?

Scholar. I will sololue my olde forme, in putting 1℥ . for the leaste quantitie, whiche is the seconde mannes somme, and then shall the firste mannes somme be. $1\frac{1}{2}\text{℥}$.

Paster. You are deceiued all readle. For you set 1℥ .

of Costike numbers.

1. Lxx . for an elne. Seyng you name $\frac{1}{3}$ of an elne, to be $\frac{1}{3}$. Lxx . And so were the position neadelesse, and likewise all the woorkes.

Scholar. I see my faulte: but I knowe not how to amende it. For that 1. Lxx . maie bee a parte or partes of an elne: and so maie it be moare then 1. or 2. elnes so that I ought not to haue set $\frac{1}{3}$ (whiche is certainly referred, in this question, to an elne) as the parte of a doubtful quantitie, but rather as the parte of a quantitie certaine. Whereas 1. Lxx . is euer put for a number unknowen.

Master. To helpe you herein, I will set the firste numbers, as you began them. The seconde man his numbers of elnes, shall bee 1. Lxx . as you did name it, and then shall the firste mannes portion be as moche, and $\frac{1}{3}$ of an elne moare: whiche $\frac{1}{3}$ I maie best call $\frac{1}{3}$. And so shall it bee disfaunte from 1. Lxx . clerely in all woorkes *Arithmetically*.

But now to procede, I shall diuide eche mannes number of elnes, whiche he had, by the number of elnes, whiche he solde for an angelle, and the *quotiente* will declare how many angelles eche man had receiued. So that the firste mannes number of elnes, being 40. shall bee the numeratoz, and the somme of measure, whiche he solde for an Angelle, that is 1 Lxx $\frac{1}{3}$. shall bee the denominatoz. And so is the diuision ended. And that fraction is the *quotiente*.

Scholar. Now I perceiue the woorkes. And by like reason: the seconde mannes somme of elnes being 90. shall bee the numeratoz, and 1. Lxx . being the somme of measure, solde for one Angelle, shall be the denominatoz, that is in one fraction $\frac{90}{170}$: accordingly as I haue sette bothe numbers

$$\begin{array}{r} 40. \\ \hline 170 \overline{) 1 \text{Lxx} \frac{1}{3}} \\ \underline{90.} \\ 170. \end{array}$$

here

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here distantly.

Master. It were moare ease for you in woorkyng, if you did tourne that fractiō of $\frac{1}{3}$ into an intere vnitis.

Scholar. That wil easily be doon, by multiplieng every number, of that whole fraction by. 3. And then will it be $\frac{110}{300}$, whiche is all one in valne with

40. And this I consider farther, that as
 $1. \frac{70}{100} + \frac{1}{3} \frac{9}{10}$. these. 2. fractions, severally dooe expresse the sommes of angelles, that eche of them receiued, so toynaly bothe together, dooe declare the full somme, of all their angelles. Wherefore I shall adde them bothe together. And they will make.

$\frac{390}{100} + \frac{90}{100}$ As here in woork I haue expressed.

$$\begin{array}{r}
 390. \frac{70}{100} + \frac{90}{100} \\
 \hline
 120. \qquad \qquad \qquad 90. \\
 \hline
 3. \frac{70}{100} + \frac{1}{10} \frac{9}{10} \qquad \qquad \qquad 1. \frac{70}{100} \\
 \hline
 3. \frac{70}{100} + \frac{1}{10} \frac{9}{10}
 \end{array}$$

And by your supposition, their bothe sommes of Angelles made. 42. So that those. 2. sommes are equalle: and therefore am I come to an equation. In whiche I see a number absolute, equalle to a fraction *Coslike* compoude.

Master. When so euer that, or the like dooeth chaunce, you shall reduce the whole nōber, to the like denomination: and then their numerators will bee equalle.

Scholar. When shall I multiplie. 42. by the denominator $3 \frac{70}{100} + \frac{1}{10} \frac{9}{10}$ & it wil be $126 \frac{70}{100} + 42 \frac{90}{100}$ whiche muste bee equalle to. $390. \frac{70}{100} + \frac{90}{100}$. That is in lesser termes.

$21 \frac{70}{100} + \frac{70}{100} = 65. \frac{70}{100} + \frac{15}{100}$.
 Where firste I dooe abate. $7. \frac{70}{100}$. on bothe sides: and there remaineth then. $21 \frac{70}{100} = 58. \frac{70}{100} + \frac{15}{100}$.
 But

of Coslike numbers.

But now I remeber your admonitic, that because the number annexed to the greatestte signe, is moare then. 1. I shall diuide all the numbers by it, and sette their *quotientes* in their stede, with their signes. And so will the number of the greatestte signe, euermoze be 1. And this equation will be $1. \frac{3}{4} = \frac{1}{4} \frac{2}{3} = \frac{1}{4} \frac{2}{3}$. Where I must extracte the square root, of the latter part, according to your doctrine, and it will be. 3. As it appereth in this worke following, whiche I did frame in my tables.

$\frac{3}{4}$. in square doeth make $\frac{9}{16}$, vnto whiche I muste adde $\frac{1}{16}$, whiche is all one with $\frac{1}{4}$, by reduction to one denominatio. So is the full additio $\frac{10}{16}$. whose square root is $\frac{5}{4}$. vnto whiche I shall adde $\frac{1}{4}$, and it will bee $\frac{3}{2}$, that is. 3.

Master. This is well deen. Now worke the same questiō, as it was proponed, and you shall easily finde all the other numbers to bee true, and agreable to the question.

Scholar. Seyng the seconde manne sold. 3. elnes *The prooffe.* for an angell, the firste manne did sell. 3. elnes and $\frac{1}{4}$. So. 40. whiche is the somme of elnes of the first man his silke) diuided by. $\frac{3}{4}$. doeth yelde. 12. and she weth how many angelles that man receiued.

Again for the seconde man, whiche had. 90. elnes, diuide that. 90. by. 3. and so shall you finde. 30. for the number of his Angelles. And that. 30. and. 12. dooe make. 42, it needeth not to be proued.

Master. Now againe for your exercise, suppose the firste mannes somme to be. 1. 20. *An other forme of worke.*

Scholar. Then muste the seconde manne sell for an angelle. $1. \frac{2}{3} = \frac{1}{3} \frac{2}{3}$. And their numbers of elnes, diuided by those numbers will make $\frac{12}{3}$. and $\frac{20}{3}$. whiche bothe added together, will bee $\frac{32}{3}$. equalle to. 42. 9. That is by reduction. $390. \frac{2}{3} = 409. \frac{1}{3} = 126. \frac{2}{3} = 42. \frac{2}{3}$. And

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And by addition of. $42.ze$. on bothe partes.

$432.ze$ ——— $40.9.$ = = = $126.8.$ And by diuision it will be. $\frac{24}{7}ze$ ——— $\frac{20}{63}9.$ = = = $1.8.$

So that now I must extracte the roote of that com-
pounde *Cosike* fraction, thus. $\frac{12}{7}$ squarely, dooe make
 $\frac{144}{49}$ out of whiche I shall abate $\frac{20}{63}$. And therfoze, firste of
all I doe reduce the to one denomination, & ther make
 $\frac{9072}{3087}$. and $\frac{980}{3087}$. wherefoze if I abate the lesser out of the
greater: there will remaine $\frac{8092}{3087}$. that is in lesser ter-
mes $\frac{1160}{441}$ and is a square number, whose roote is. $\frac{34}{21}bn$.
to whiche if I adde $\frac{12}{7}$ that is $\frac{36}{21}$. it will make $\frac{70}{21}$, or $\frac{10}{3}$.
that is the vale we of. $1.ze$. And is the firste mannes
number of elnes, agreably as I tried it befoze. And
so doe bothe woekes agree.

But now cometh to my remembraunce, that this
number, whose roote I did extract, in this laste woike
is of that sorte, where. ze ——— $9.$ is equalle to. $8.$.
And therfoze hath in it. 2. rootes: thone by addition,
as this, whiche I now founde: And the other by sub-
traction, whiche in this example, by abatynge $\frac{11}{21}$ out of
 $\frac{10}{3}$, will bee $\frac{2}{21}$. But how I maie frame that roote, to a-
gree to this question, I doe not see.

Master. That varietie of rootes dooeth declare,
that one equation in number, maie serue for. 2. seuer-
alle questions. But the forme of the question, maie
easily instruct you, whiche of those. 2. rootes, you shall
take for your purpose. Howbeit sometymes you shall
take bothe. As for example again, marke this que-
stion.

*A question
of money.*

A gentelman, willing to proue the cunnynge, of a
bragging *Aritmetician*, saied thus: I haue in bothe
my handes. 8. crownes: But and if I accounte the
somme of eche hande by it self seuerally, and put ther
to the squares and the *Cubes* of the bothe, it will make
in number. 194. Now tell me (quod he) what is in
eche hande; and I will giue you all for your labour.

Scholar.

of Coslike numbers.

Scholar. Soche incoragements, would make me studie harde, and trauell very willyngly in learned exercises: though learning bee mosse to be loucd, for knowledges sake. But for to finde the true aunswere thus I doe p[ro]ccade.

Firste I suppose the one number in one hand, to be 1. ze . And then must the other nedes be 8. y . — 1. ze . Then doe I make theim bothe Squares. And for the firste I haue. 1 z . and for the seconde. 1. z . — 64 y — 16. ze . Whirdeley I multiplie theim bothe Cubi- kely: and so haue I for the firste. 1. ce and for the other 24. z . — 512. y . — 1. ce . — 192. ze . Then must I adde bothe the n[um]bers, with their squares, and their Cubes, into one somme. As here in work

$$\begin{array}{r}
 1. \text{ze} . - | . 1. \text{z} . - | . 1. \text{ce} . \\
 \phantom{1. \text{ze} . - | . } 8. \text{y} . - | . 1. \text{ze} . \\
 1. \text{z} . - | . 64. \text{y} . - | . 16. \text{ze} . \\
 24. \text{z} . - | . 512. \text{y} . - | 1. \text{ce} . - | 192. \text{ze} . \\
 \hline
 26. \text{z} . - | . 584. \text{y} . - | . 208. \text{ze} .
 \end{array}$$

It is set for the. Where for ease I haue set. 1. ze , 1. z . and. 1. ce (whiche is the k[ro]ote, the Square and the Cube of one number) all in one line: and the other k[ro]ote, Square, and Cube, I haue set seuerally. And so all thei doe make. 26 z . — 584 y — 208 ze whiche is equalle to . 194. by the intente of the question. Wherefore I adde firste. 208. ze . to bothe partes, and there remaineth.

26. z . — 584. y — 208. ze — 194. y .
 Then I abate. 194. from bothe sides, and so resteth the equati[on] thus. 26. z . — 390. y — 208. ze
 That is by diuision. 1. z . — 15. y . — 8. ze .
 And by translation of. 15. y . to sette. 1. z . alone, it wil be. 1. z . — 8. ze — 15. y . And now haue I the craxe and complete equation, where I must seke for

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the value of. 1. \mathcal{Z} . by extractyng the roote. Therefore firste \mathcal{I} take halfe of. 8 and that is. 4. whose square is. 16. out of whiche \mathcal{I} abate. 15. and the remainder is 1. whiche \mathcal{I} maie either adde to. 4. and so haue \mathcal{I} . 5. or ther, \mathcal{I} maie abate it from 4 and so haue \mathcal{I} 3. Whiche numbers also accordyng to thesame rule, beyng added together dooe make. 8. that is the number of the middell denomination. And beyng multiplied together, thei dooe make. 15. that is the other parte of thesame compounde *Cosike* number.

Master. And if you had marked that firste, you might easly haue found bothe those numbers, by the partes of. 15. whiche can be none other, but. 5. and 3.

And farther, seying thei. 2. doo make. 8. and. 8. is the number (named in the questio) that thei should make, therfoze you shall take them bothe. And name whiche of them you liste to be. 1. \mathcal{Z} . And the other shall be of necessitie, the reste of. 8.

The prooffe.

Scholar. To examine them, by the order of the question, \mathcal{I} doe proceade thus. 3. with his Square. 9. and his *Cube*, 27. dooeth make. 39. And. 5. with his square 25 and his *Cube*. 125. doo yelde 155. And bothe thei together doe byng fozthe. 194. accordyng to the sayng of the question: and therfoze it is certain, that the woork is good.

An other woork for equations.

Master. Befoze you passe any farther, \mathcal{I} will admonishe you of one waie, whiche \mathcal{I} ofte vse in reduction of soche equations, as this is, when there is noe denomination on the one side, but the like is on the other side, with a greater number annexed to it. Then maie you abate all the lesser nöbers, out of their greater, and the reste shall bee accountped equalle to nothyng. Whiche chaunce can neuer happen: excepte there bee some numbers on the greater side, with the signe of abatmente. ———.

As here you had.

of Cossike numbers.

26 \bar{z} . — + — 584 \bar{q} . — — — 208 \bar{ze} . — — — 194 \bar{q} .
 Because on the one side, there is noe nōber but 194 \bar{q}
 and on the other side, there is. 584 \bar{q} . beeyng a grea-
 ter number, and of the same denomination: therefore
 maie you abate. 194. from bothe sides, and then re-
 maineth. 26 \bar{z} . — + — 390 \bar{q} . — — — 208 \bar{ze} . — — — 0
 Wherefore you maie well consider, that the numbers
 whiche be ioined wiche — + —. are equalle to the num-
 bers that bee set with — — —. And therefore the one a-
 batyng the other iustly, dooe remaine together as e-
 qualle to nothyng.

Wherefore it is reasonable, that seeyng the num-
 bers with — — — be equalle to the numbers with
 — + — that I maie translate the numbers with — — —
 from that side of the equation, and set them on the co-
 trary side, with the signe of — + —. And so in this crā-
 ple it will bee. 26 \bar{z} . — + —. 390 \bar{q} . — — — 208 \bar{ze} .
 And this forme shall ease you moche, in reducyng of
 equations.

Scholar. I thanke you moche. And I will not for-
 get to vse it, as occasiō shall happen. But I praie you
 propounde yet some moare questions, that I maie see
 their diuerse variettes.

Master. There were two seueralle men, which *A question*
 had certaine sommes of angelles, in soche rate, that *of money.*
 the seconde manne his somme, was triple sesquiquarta
 to the firste: and if their. 2. sommes were multiplied
 together, and to that totall the 2 firste sommes added,
 there would amounte. 142 $\frac{1}{2}$. I demaunde of you,
 what was eche of their sommes in angelles?

Scholar. The firste mannes somme I call. 1. \bar{ze} .
 And the seconde mannes some shall be. 3 $\frac{1}{4}$ \bar{ze} . which
 2. sommes beeyng multiplied together, dooe make
 3 $\frac{1}{4}$ \bar{z} . vnto whiche I must adde bothe the firste nom-
 bers, that is 4 $\frac{1}{4}$ \bar{ze} . And it will be 3 $\frac{1}{4}$ \bar{z} . — + — 4 $\frac{1}{4}$ \bar{ze}
 equalle to. 142 $\frac{1}{2}$. All whiche numbers, I shal bring
 xi. iiij. into

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into whole numbers, if I multiplie theſe by. 4. And ſo will it be. $1\frac{3}{8}$. — $17\frac{7}{8}$ = 570 . And by reducyng the greatſte denomination *Coſike*, to an vñſſe. $1\frac{3}{8}$. — $\frac{17}{8}$ = $4\frac{11}{13}$. And laſte of all, by tranſlatyng the number of. $\frac{7}{8}$. to ſet. $1\frac{3}{8}$. alone, on one ſide of the equation, it will be. $1\frac{3}{8}$ = $4\frac{11}{13}$ $\frac{7}{8}$ = $\frac{17}{8}$. where I muſt extract the value of the roote thus. $\frac{17}{8}$. ſquarely dooe make $\frac{289}{64}$. vnto whiche I ſhall adde the. $4\frac{11}{13}$ (it beeyng firſte multiplied by. 52. to byyng it to the denomination of. 676. And ſo making $\frac{2964}{676}$) And it will be $\frac{29929}{676}$ whiche is a ſquare number (as I haue pꝛoued in my *Tables*) and his roote is $\frac{173}{26}$. from whiche roote I muſt abate $\frac{17}{26}$, and then will there remain $\frac{156}{26}$, that is. 6.

And that. 6. is the value of. $1\frac{7}{8}$, and ſtandeth foꝛ the firſte mannes number. So that the ſeconde mannes nōber muſt be as. $\frac{1}{2}$ to it: that is *tripla ſequiquarta*. And ſo ſhall it be. $19\frac{1}{2}$.

The prooffe.

Maſter. Now pꝛoue thoſe numbers, according to the queſtion.

Scholar. $19\frac{1}{2}$ multiplied by. 6. doeth make. 117. vnto whiche I ſhall adde. $25\frac{1}{2}$. amountyng of their. 2. additiōs, and all will be. $142\frac{1}{2}$, according to the purpoſe of the queſtion.

*An other
works of the
ſame queſtiō.*

Maſter. So is your woꝛke good. Yet woꝛke it again, by chaungyng the poſition.

Scholar. I maie put. $1\frac{7}{8}$. to betoken the ſeconde manne his ſomme. And then ſhall the firſte mannes ſomme bee $\frac{4}{13}$. $\frac{7}{8}$. whiche bothe multiplied together doe make $\frac{4}{13}$ $\frac{7}{8}$. And then addyng the. 2. firſte ſommes to it, it will bee $\frac{4}{13}$ $\frac{7}{8}$ — $1\frac{4}{13}$ $\frac{7}{8}$. And that is equalle to. $142\frac{1}{2}$. All whiche numbers will bee reduced to whole numbers, by multiplication conueniente. And ſo will it be. $8\frac{7}{8}$ — $34\frac{7}{8}$. equalle to. $370\frac{5}{8}$; that is by reduction, $1\frac{3}{8}$. — $4\frac{1}{2}$ = $463\frac{1}{8}$. and by tranſlation of the termes.

of Cosſike numbers.

18. ——— 463 $\frac{1}{2}$. 9. ——— 4 $\frac{1}{2}$. 20. out of whiche number I ſhall extract the value of the roote, in this ſorte.

Fiſt I ſaie $\frac{1}{2}$ multiplied Square, doeth make $\frac{1}{2}$, vnto whiche number I muſt adde. 463 $\frac{1}{2}$, reduced as it ought, and it will bee in all $\frac{1}{2}$. whiche is a ſquare number, and hath for his roote $\frac{1}{2}$. from whiche I muſt abate $\frac{1}{2}$. And then will there remain $\frac{1}{2}$, that is 19 $\frac{1}{2}$, for the value of. 1. 20. And ſo conſequently for the ſecond mannes nōber: whiche was named in this poſition, 1. 20. And this maie bee proued as the other was.

Maſter. What ſaie you then to this queſtion? *A queſtion of iorneyng.*
 There is a ſtraunge iorneye appointed to a manne. The fiſt daie he muſt goe 1 $\frac{1}{2}$ mile, and euery daie after the fiſt, he muſt ſtill augmente his iorney, by $\frac{1}{2}$ of a mile. So that his iorney ſhall procede by an *Arithmeticalle* progression. And he hath to trauell for his whole iorney. 2955. miles. I demaunde in what nōber of daies, ſhall he cande his iorney?

Scholar. I knowe not how to procede in this queſtion.

Maſter. Doe you not heare me name it, an *Arithmeticalle* progression: Wherby you might be adſured, that it doeth appertaine to that rule. And accoꝝdyng to the canons of that rule, muſt you wooꝝke this queſtion. But for your better inſtruction, I will helpe you in this wooꝝke.

Fiſt aunſwere to the queſtion, by the common poſition: and ſaie that the tyne of his iorney is. 1. 20. of daies. And then ſhall all the *exceſſes* (whiche maie alſo be called the *number of the ſpaces*) be. 1. 20 ——— 19
 The *common exceſſe* was ſuppoſed to be. $\frac{1}{2}$ of a mile. And therefore the *ſomme of all the exceſſes* muſt be $\frac{19 \times 1}{2}$ that is to ſaie, the number of all the *exceſſes* multiplied by $\frac{1}{2}$, that is here, the ſixe parte of the number

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number of the excesses.

And because that the firste number is $1\frac{1}{2}$. I must adde it vnto the somme of the excesses, and so haue I the laste number of that progression. Wherefore adding $1\frac{1}{2}$. (whiche is $\frac{3}{2}$, or in like denomination with the other, $\frac{2}{2}$) with $\frac{170}{6}$ it will make $\frac{173}{6}$. And that is the laste number of the progression.

Now you remember, that in progression Arithmetically, if you adde the firste number to the laste: and multiply that totalle, by the number of halfe the places, there doeth amounte the somme totalle of that progression.

And therefore in this example, if you adde $1\frac{1}{2}$ (whiche is the firste number in the progression) vnto $\frac{173}{6}$ (that is the laste number of the progression) there will amounte $\frac{175}{6}$, whiche being multiplied by the number of halfe the places, that is $\frac{1}{2} \times 2$. (For all the number of places is . 1 . 2) there will rise, $\frac{175}{12}$, whiche is the totalle somme of all the miles: and therefore is equalle to . 2955.

Scholar. All the reste, and this againe can I dooe now. Seing $\frac{175}{12}$ is equalle to . 2955. I will firste bring the whole number to the like denomination, with the fraction, and it will bee. $\frac{35460}{12}$. And then omitting the like denominations. $1750 = 35460$. What is by translation $1750 = 35460$. whose roote in value I shall finde out thus. I multiply $\frac{17}{2}$ squarely, and it will be $\frac{289}{4}$, vnto whiche I shall adde 35460 . & it will make $\frac{35749}{4}$, whiche is a square number, and hath for his roote $\frac{189}{2}$, from whiche I shall abate $\frac{17}{2}$, and then remaineth $\frac{181}{2}$, that is. 180. whiche is the value of. 120. And expresth the number of dayes, whiche the question requireth.

The prooffe.

Master. The prooffe in this, and the like questions, is, to set forth the progression with all his termes.

of Cossike numbers.

mes. Excepte you will for shortnesse, sette downe the firste terme, whiche in this example is. $1\frac{1}{2}$: and then by the number of the *excesses*, or distaunces (whiche is euer one lesse then the number of places) multiplye the quantitie of one *excesse*: and put to it the firste terme: and so haue you the laste terme. When hauyng the firste terme and the laste, with the number of *excesses* you knowe how to finde the totalle.

As in this example, the number of *excesses* beeyng 179. And the quantitie of one *excesse* beeyng $\frac{1}{2}$. their multiplication giueth $89\frac{1}{2}$. vnto whiche if you adde the firste number, that is $1\frac{1}{2}$, it will be 91 . And that is the laste number of that progression. When to trie the totalle somme of the miles, adde the firste number. $1\frac{1}{2}$ to the laste, and thei will make 92 , that you shall multiplye by halfe the number of the places, whiche in our example are. 90 (with the whole number is. 180) and there will amounte. 2955. accoꝝdyng as the question saith.

Scholar. This is sufficient for this question. And at some idle time, I will not sticke to trie it out, by setting the progression foꝝthe at large. In the meane tyme I praie you foꝝ better exercise, giue me some moare questions.

Master. There is a number, whiche I haue foꝝ gotten: and it is diuided into. 2. partes, whereof the one I haue foꝝgotten also, but the other was. 4. And yet this I remember, that if the parte, whiche I haue foꝝgotten, be multiplied by it self, and then also with 4. those. 2. sommes will make. 117. Now would I knowe what was the whole number, and also what is the parte, whiche I haue foꝝgotten. *Another question.*

Scholar. I suppose the whole number to be. 12. And bicause. 4. is his one parte, the other parte must needs bee. 12. ——— 4. When doe I accoꝝdyng to the question, multiplye. 12. ——— 4. firste by it self,

lik.), and

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and it will make. $1\ 3\ \text{---}\ 16\ 9\ \text{---}\ 8\ 7e$. Secondly, I doe multiplie it, that is. $1\ 7e\ \text{---}\ 4\ by\ 4$. And it giueth. $4\ 7e\ \text{---}\ 16$.

Then adde I bothe those numbers together, and it will be. $1\ 3\ \text{---}\ 4\ 7e$. whiche by the question shall be equalle to. 117 .

$$1\ 3\ \text{---}\ 16\ 9\ \text{---}\ 8\ 7e$$

$$4\ 7e\ \text{---}\ 16\ 9$$

$$1\ 3\ \text{---}\ 4\ 7e$$

But then must I vse the accustomed translation, to bring the greatestte quantitie in denomination, to stande alone. And so will it bee.

$$1\ 3\ \text{---}\ 4\ 7e\ \text{---}\ 117\ 9$$

Where I must sicke for the value of a roote. And therfore I multiplie. 2. by it self squarely, and so haue I. 4. vnto whiche I adde. 117 . and it maketh. 121 . whose roote is. 11 . vnto whiche I muste adde. 2. and there commeth. 13 . as the value of. $17e$ and the quantitie of the whole number.

The prooffe.

For prooffe of this worke, I abate. 4. out of. 13 . and there resteth. 9. as the other parte. Then doe I multiplie. 9. by it self, and therof riseth. 81 . Also I doe multiplie. 9. by. 4. and it maketh. 36 . whiche bothe together, doe make. 117 . as the question would.

Another worke.

Passer. Set. $17e$. for the unknowen parte, and then worke it, to see the diuersitie of the woorkes.

Scholar. If. 4. bee one parte, and. $17e$, the other parte, then will the whole number be. $17e\ \text{---}\ 4$

Wherefore firste I multiplie. $17e$. by it self, and it yeldeth. $1\ 3$. Then dooe I multiplie. $17e$. by. 4. and it giueth. $4\ 7e$. whiche bothe sommes together, dooe make. $1\ 3\ \text{---}\ 4\ 7e$. whiche is equalle to. 117

And by translatiō. $1\ 3\ \text{---}\ 117\ 9\ \text{---}\ 4\ 7e$.

Wherefore I doe multiplie. 2. squarely, and it giueth

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ucth. 4, whiche added to. 117. maketh. 121. and the roote of that is. 11. from whiche 3 shall abate. 2. and there will rise. 9. as the other parte of the number. This is verie plain, & the profe of it as it was before.

Master. Then aunswere to this question.

There are 3 numbers in proportion *Geometricall*. And *A question of proporti*
 one of the extremes is. $20\frac{1}{4}$. the other extreme, with the double of the middell terme, doeth make 22. Now would I knowe of you, what those. 2. numbers bee:

Scholar. For trialle, I name the other extreme, 1. 20. And because it, with the double of the middle terme dooeth make. 22. the middell terme shall bee 11. ———. $\frac{1}{4}$. 20. for his double is. 22. ——— 120. whiche with. 1. 20. doeth make. 22.

Then to proceede, I knowe the propertie of those numbers in proportion *Geometricall* to bee soche, that the multiplication of bothe the extremes is equalle to the square of the middell terme, wherefore I multiplie the. 2. extremes together, and there will rise. $\frac{1}{4}$ 20. Then dooe I multiplie. 11 ——— $\frac{1}{4}$ 20. by it self in Square, and it will bee. 121. 9. ———. $\frac{1}{4}$ 80. ———. 1120, whiche must bee equalle to $\frac{1}{4}$ 20. or. $20\frac{1}{4}$ 20. When to reduce it, I adde. 11. 20. on bothe sides, and it will be. $31\frac{1}{4}$ 20. ——— $\frac{1}{4}$ 80. ———. 121 9. and by translation. $\frac{1}{4}$ 80. ——— $31\frac{1}{4}$ 20. ———. 121. 9. That is 1. 80. ——— 125. 20. ——— 484. 9.

Now resteth nothing, but to searche the value of 1. 20. Therefore I take $\frac{1}{4}$ 20, and multiplie it Square, and so haue $\frac{1}{4}$ 80, from whiche I must abate. 484. that is $\frac{1}{4}$ 20. And there will remain $\frac{1}{4}$ 20 whose roote is $\frac{1}{4}$, whiche I shall abate from $\frac{1}{4}$, and there will remain $\frac{1}{4}$, that is. 4. for the other extreme.

Then for the middell terme, thus shall I doe. *The profe.*
 Multiplie. 4. and. $20\frac{1}{4}$ together, and there will rise. 81. whose roote is. 9. and is the middell number. That 9 doubled will make. 18. and 4. ioined thereto, giueth 22

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So are those .3. termes in progression *Geometricall*, according to the conditions limited in the question.

Maister. Youe the worke now, how it wil frame if. $1. \text{ze}$. be set for the middell number. For it wer folle, to trie whether this question, would admitte addition of the .2. laste numbers. Although the rule doe declare that in soche sorte of equations, there is double valuation to eche roote.

Scholar. Yet I beseeke you, let me examine it a litle, to see the cause, why I maie not adde them, and so take the roote.

Maister. I must here with you so moche. By addition you see, there will rise $\frac{21}{2}$, that is $12 \frac{1}{2}$. And then the middell number will be. $49 \frac{1}{4}$. And so the proportion is $\frac{2}{9}$. that is *Dupla superquadripartiens nonas*. Here as in the other .3. numbers. $4.9.20 \frac{1}{4}$. the proportion is *Dupla sesqui quarta*.

But in the question is one condition, that secludeth the roote, that riseth by additiō. For the double of the middell terme, with the other vnknown extreme, should make. 22. As in. 4. and. 9. it doeth. But in $49 \frac{1}{4}$ and $12 \frac{1}{2}$. it would be 220. that is 10. tymes so moche.

Scholar. And if you had said in the question, that the double of the middell number, with the other extreme, would haue made, 220. then I should haue taken this later roote by additiō, and not the firste roote by subtraction.

And so I perceiue the varietie of conditions in the question dooeth limite, whiche of the .2. rootes I shall of necessitie take, and leaue the other.

*An other
woorke.*

But now to varie that worke, I will set. $1. \text{ze}$. for the middell terme. And then the double of it, with the other terme, will make. 22. The double of. $1. \text{ze}$. is. $2. \text{ze}$. So must the other terme be $22 \text{ } \varnothing \text{---} 2. \text{ze}$.

Then to seke out an equation, I multiplie the .2. extremes together, that is. $22 \text{ } \varnothing \text{---} 2 \text{ze}$. by $20 \frac{1}{4}$.
And

of Coslike numbers.

And there riseth. $445\frac{1}{2}$. ——— $40\frac{1}{2}$. 20 . And the square of. 1.20 . being the middell terme, is some perceived to be. 1.3 . And so the firste equation is,

$$13 \text{ ——— } 445\frac{1}{2} 9. \text{ ——— } 40\frac{1}{2} 20.$$

Wherefore I take halfe. $40\frac{1}{2}$, that is. 20 , whose square is 400 . And vnto it I putte. $445\frac{1}{2}$. whereby there commeth 889 . whose roote is 29.8 . from whiche roote I must abate 20 , and so remaineth 8.8 . that is. 9 . As the value of. 1.20 . And for the middle number.

Then for the proofe: of. 9 . see the middell number, *The proofe.* the square of it, whiche is. 81 , shall bee equalle to the multiplications of the extremes. Wherefore if I diuide. 81 . by $20\frac{1}{2}$, the *quotiente* being. 4 . declareth the other extreme.

Master. You seme experte inough in this forme of worke. Therefore I will procede to other questions, that differ somewhat from these.

There are. 2. menne talking together of their monies, and nother of them willing to expresse plainly his somme, but in this sorte. The number of angelles in my purse, saith the firste manne, maie bee parted into soche 2. numbers, whiche being multiplied together, will make. 24 . And their *Cubes* being added together, will make. 280 . Then, quod the other man. And the like maie I saie of my money, saue that the *Cubes* of the. 2. partes, will make. 539 . Now I desire to knowe, what monie eche of them had. *A double question.*

Scholar. The firste mannes somme, I set to be 1.20 whiche I must parte into two soche partes, that thei bothe multiplied together, maie make. 24 .

Master. You erre verte moche. For it is not possible, that the partes of any *Coslike* number multiplied together, can make an absolute number. Wherefore in soche cases, where you perceiue that there is required, after the firste position, any multiplication to make an absolute number, you shall call the firste no-

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bers, by some other name of pleasure. As here you maie call the firste mannes somme. *A.* And the seconde mannes somme. *B.* and then in their partition, vñ the name of. 1. 7c.

And as they are twoo questions in one, so shall you make seueralle woorkes for them.

Scholar. Then shall I saie, that the firste mannes somme is. *A.* and it is diuided as he declared. Wherefore for one number of that diuision, I set. 1. 7c. And then the other shall be $\frac{24}{10}$. for as the one number multiplied by the other, doth make. 24. So. 24. 9. diuided by the one of them, must needs bying for the other.

Master. That is well remembred of you. For as 4. and. 5. by multiplication, doe make. 20. So. 20. diuided by. 5. byingeth for the 4. and diuided by. 4. it yieldeth. 5.

Scholar. So $\frac{1}{2}$ is but. 4. and $\frac{2}{3}$ is. 5.

Master. Go forth then with the rest of the worke.

Scholar. The Cube of. 1. 7c. is. 1. 7c. and the Cube of $\frac{2}{3}$ is $\frac{8}{27}$ whiche. 2. numbers I maie not adde together, vñ till I haue reduced them vnto one denomination: whiche thyng I shall doe, by setting. 1. 7c. as a fraction thus $\frac{100}{100}$. And then woorkyng after the rate of fractions, in the firste reduction they will stande thus. $\frac{100}{100} + \frac{13824}{100}$. And by farther addition thus.

And hether to the woork of bothe these. 2. mannes sommes, are indifferent and agreynge. So that this one woork serueth for them bothe. But now they will differ. For in the firste mannes woordes, and so in the woork for him $\frac{13824}{100}$ is equalle to 280: but in the seconde mannes woork, it must be accompted equalle to. 539.

But firste to goe so:ward with the firste man. Setyng $\frac{100}{100} + \frac{13824}{100}$ is equalle to. 280. Therefore by reduction

of Coslike numbers.

reduction to one d. nomination, $\frac{13824}{12} = 1152$ is e-
 qualle to $\frac{13824}{12}$. And remouyng the common denomi-
 nator, the numerators shal kepe thesame proportion:
 and therfore. $13824 \cdot 9$. Shall be equalle
 to. $280 \cdot 12$. And by translation, to haue the greatestte
 d. nomination alone, $13824 \cdot 9 = 280 \cdot 12$
 Here 9 shall seeke the value of. $1 \cdot 12$. whiche shall
 not be here accounted the square roote, but the Zen-
 zicubike roote, or the Cubike roote of the square roote,
 accordyng to the greatestte denomination.

Wherfore. 140 . in square, maketh 19600 . from
 whiche 9 must abate 13824 . And there docth remain
 5776 whose square roote is. 76 . whiche beyng added
 vnto. 140 . dooeth giue. 216 . and beyng abated from
 it, it leaueh. 64 . of whiche bothe 9 must extrate the
 Cubike roote, bicause in the equation there are. 2 . qua-
 tities omitted. So that of. 216 . the Cubike roote is 6 .
 And of. 64 . the Cubike roote is. 4 . Were 9 see bothe
 rootes serue so my purpose, that 9 shall take the both.

Master, And good reason. For as in setting 12
 for your position, you could not tell whether it were
 the greater parte, or the lesser, so maie you not now
 applie it to either of them bothe, but take bothe roo-
 tes for the. 2 . partes of your number.

Scholar. So doeth the firste mannes number ap-
 peare to be. 10 . seying the partes bee. 4 . and. 6 . whiche
 9 maie examine thus. What thei make. 24 . by multi-
 plication, it is easily seen. And that their Cubes added
 together, doe make. 280 . is sone perceiued: seying the
 Cube of. 4 . is. 64 : and the Cube of. 6 . is. 216 . whiche. 2 .
 numbers by addition, doe make. 280 . *The prooffe.*

Master. Now proue the seconde mannes worke. *The worke*

Scholar. In his worke $\frac{13824}{12} = 1152$ is equalle *of the second*
 to 539 . And by reduction to one denomination, it is *parte.*
 equalle to $\frac{13824}{12}$. So that. $13824 \cdot 9$. is
 equalle to. $539 \cdot 12$. and by translation.

$13824 \cdot 9$.

The Arte

1. z^{c} . ———. 539. c . ———. 13824. q . whose
zenzicubike roote I seke, thus: $\frac{539}{4}$ doth make in square
 $\frac{290521}{4}$, from whiche I must abate $\frac{539^2}{4}$, and then remaineth
 $\frac{23525}{4}$, whose roote is $\frac{153}{2}$ vnto whiche I maie adde
 $\frac{13}{2}$. and then will it bee $\frac{166}{2}$, that is. 512. whose *Cubike*
 roote is .8. And is one parte of the seconde mannes
 number. And for the other parte, I shall abate $\frac{25}{4}$ out
 of $\frac{539}{4}$, and there remaineth $\frac{514}{4}$. that is, 27. whose *Cubike*
 roote is .3. And is the other parte of the seconde man-
 nes number. As it maie sone be tried thus. For .3. ty-
 mes .8. maketh. 24. and. 27. whiche is the *Cube* to .3.
 added with. 512. whiche is the *Cube* to .8. dooeth make
 539, as the question intendeth.

The prooffe.

*A question
 of an armie.*

Master. One other question I will propounde,
 of .2. armies beyng bothe square, and of like nombcr.
 And if you abate .4. from the one armie, and adde .10.
 to the other armie, and then multiplie them bothe to-
 gether, there will amounte. 9853272. I demaunde
 of you, what is the fronte of those square battailes.

Scholar. I call the fronte 1z . And then must the
 battaile bee. 1. z . Now abatynge .4. from the one, it
 will bee. 1. z . ———. 4. q . Then addynge .10. to the o-
 ther, it wil make. 1. z . ———. 10. q . And if you mul-
 tiplie those .2. numbers together, there will amounte
 by it. 1. z z ———. 6. z . ———. 40. q . whiche somme
 must be equalle to. 9853272.

$$1. \text{z} . - - - - . 10. \text{q} .$$

$$1. \text{z} . - - - - . 4. \text{q} .$$

$$1. \text{z} \text{z} . - - - - . 10. \text{z} . - - - - . 4. \text{z} .$$

$$- - - - . 4. \text{z} . - - - - . 40. \text{q} .$$

$$1. \text{z} \text{z} . - - - - . 6. \text{z} . - - - - . 40. \text{q} .$$

And if you adde .40. q . to bothe partes of the equa-
 tion, it will be. 1. z z ———. 6. z . equalle to. 9853312
 And

of Cossike numbers.

And by translation. $1z \cdot z = 9853312$. — $6z$.
 out of whiche laste equation, z shall searche for the
 value of. $1z$. by multiplieng first z . squarely, where
 of commeth. 9 . and then addyng it to. 9853312 . And
 so commeth. 9853321 . whose roote is. 3139 . from
 whiche z must abate. 3 . And then remaineth. 3136 .
 whiche is the full number and square of the one ar-
 me. And hath for his roote. 56 .

For as here is one onely quantitie omitted, so the
 firste number, whiche in other questios of immediate
 equations, was the verie roote, in these interrupte e-
 quations is a rooted number, and is here a square no-
 ber: whose roote therfore, z haue drawen acco-
 dyngly. And for triall of this woork. 56 . in square maketh *The prooffe.*
 3136 . from whiche if you abate. 4 . there will reste
 3132 . Again if you adde. 10 . there will rise. 3146 .
 And those. 2 . numbers multiplied together, doe make
 9853272 , as the question intendeth.

Passer. This you see, what vse is in these equa-
 tions, yet are there many other equatiōs, whiche here
 be not spoken of: but here after you shall haue moare
 largely declared, if you shewe your self diligente in
 this parte.

And one question z will propounde, & assoyle with *A question*
 out woork for byescence, that you maie see there is *of Arange*
 moare behinde. There is a number whose square *equation.*
 abated by. 16 . and the firste number augmented by
 8 . and then bothe thei multiplied together, will byng
 for the. 2560 .

Scholar. z will proue the woork of it. And there-
 fore suppose the firste number to be. $1z$. Then is his
 square $1z^2$ whiche abated by 16 . leueth. $1z^2 - 16$.
 and the nōber augmented by. 8 . yeldeth $1z + 8$.
 These. 2 . numbers multiplied together, will make
 $1z^2 + 8z - 16$. — $16z$. — 128 . byng
 equalle to. 2560 .

ll. j. 1. z.

The Arte

$$1. \text{z.} \text{---} .16. \text{q.}$$

$$1. \text{z.} \text{---} + .8. \text{q.}$$

$$1. \text{c.} \text{---} .16. \text{z.}$$

$$8. \text{z.} \text{---} .128. \text{q.}$$

$$1. \text{c.} \text{---} + .8. \text{z.} \text{---} .16. \text{z.} \text{---} .128. \text{q.}$$

And addyng 128. q. on bothe sides of the equation, it will be. $1. \text{c.} \text{---} + 8 \text{z.} \text{---} = 16 \text{z.} \text{---} = 2688 \text{q.}$
 Againe addyng. 16. z. on bothe sides, it will bee $1. \text{c.} \text{---} + 8 \text{z.} \text{---} = 16 \text{z.} \text{---} + 2688 \text{q.}$

Master. Where at staie you now?

Scholar. I see no thiste, but other to leaue it, as it is, 2. numbers equalle to. 2: other els to make. 1. number equalle to. 3. And all that is aboue my cunningg. For hether to I haue learned noe rule for any of them bothe. So that I can not gesse, what the firste number might bee.

Master. The number is. 12. And his Square is 144. from whiche if you abate. 16. it will bee. 128. And if you adde. 8. to. 12. it will yelde. 20. Then multipliyng. 128. by. 20. the somme will be. 2560. as the question declared.

Of other equations.

But to put you out of doubtte, this equation is but a trifle, to other that bee vntouched. And yet I will tourne this equation a litle, to giue you some light in it, and other soche. As here.

$1. \text{c.} \text{---} = 16. \text{z.} \text{---} + 2688. \text{q.} \text{---} 8 \text{z.}$
 where you see. 1. c. equalle to. 3. other numbers. And is it not certaine to you. that this equation is true?

Scholar. Yes, I am adfured thereof.

Master. And yet to auoide doubtfulness the more trie it by resolution, accountpyng. 12. for. 1. z.

Scholar. Where. 12. is. 1. z., there. 1. z., is. 144. and. 1. c. is. 1728. whiche. 1728. must bee equalle to
 16. z.

of Cossike numbers.

16. \mathcal{Z} (that is. 192) and to. 2688. saue that you must abate. 8. \mathcal{Z} . that is 1152. Now if I adde 192. to 2688 it will make . 2880. out of whiche abatinge. 1152. there will remaine. 1728. wherby I see the equation is iuste.

Master. Then you see that the equation is true. And can you doubt, that any number, whiche is equalle to a Cubike number, hath in it a Cubike roote?

Scholar. It must needs be a Cubike number, that is equalle to a Cubike number: and therefore muste needs haue a Cubike roote: although I knowe not how to extracte that roote.

Master. Likewates, when I saie:

8. \mathcal{Z} . \mathcal{C} . \equiv 12 \mathcal{f} . \mathcal{Z} . \mathcal{C} . 128. \mathcal{f} . it is certaine, not onely that. 12. \mathcal{f} . \mathcal{Z} . \mathcal{C} . 128. \mathcal{f} . containeth in it as moche as. 8. \mathcal{Z} . \mathcal{C} . but that the. 8. parte of it is a \mathcal{Z} . \mathcal{C} . number, and hath a *zenzicubike* roote.

*Here the
roote is. 2.*

And farther it is manifeste, that as euery. \mathcal{Z} . \mathcal{C} . number, dooeth containe in it certaine. \mathcal{f} . numbers exactly, so if any number be annexed with those *Surfolides* (as here in this example are set 128) it is of necessitie, that that. 128. must containe in it certaine *Surfolides* exactly.

So if. 8. \mathcal{Z} . \mathcal{C} . bee equalle to

10. \mathcal{f} . \mathcal{Z} . \mathcal{C} . 20. \mathcal{f} . \mathcal{Z} . \mathcal{C} . 400. \mathcal{C} . \mathcal{C} . 3125. \mathcal{f} . \mathcal{f} . it must needs be that the. 8. parte of this compounde number shall bee a. \mathcal{Z} . \mathcal{C} . number. And also that the \mathcal{Z} . \mathcal{Z} . with the other numbers following dooeth containe a certain number of \mathcal{f} . \mathcal{Z} . numbers. And the. \mathcal{C} . in like sorte includeth a number of. \mathcal{Z} . \mathcal{Z} . numbers. And laste of all. 3125. \mathcal{f} . \mathcal{f} . dooeth comprehend certain Cubike numbers exactly.

The roote is 5

In like sorte, when we saie, that. 1. \mathcal{f} . \mathcal{Z} . is equalle to

6. \mathcal{C} . \mathcal{C} . 8. \mathcal{Z} . \mathcal{C} . 9. \mathcal{f} . All this compounde number is a *Surfolide*, and hath a. \mathcal{f} . \mathcal{Z} . roote. And 8. \mathcal{Z} . \mathcal{C} . 9. \mathcal{f} . includeth certaine Cubes. And so

*The roote
here is. 3.*

L. y. doeth

The Arte

doeth. 9. 9. containe exactly. 1. 7. 02 moare.

But of these and many other verie excellent and
wonderfulle woorkes of equation, at an other tyme I
will instructe you farther, if I see your diligence ap-
plied well in this, that I haue taughte you.

And therefore here will I make an
eande of *Cosike* numbers,
so2 this tyme .

Of Surde numbers, in diuerse sortes

And firste of Surde numbers
vncompounde.



Now that you haue somewhat learned the arte of *Cosike* numbers, with the rule of equation, it seemeth good time and apte place, to teache you the arte of *Surde* nōbers, whiche are diuerse in name, according as there are diuerse natures of rootes, whiche maie

giue them name.

For generally, a *Surde* number is nothing els, but soche a number set for a roote, as can not be expressed by any other number absolute. *A Surde number.*

As the *Square* roote of. 10. or of. 8. or of any number, that is not square. Likewise the *Cubike* roote of. 4. or of. 5. or of any number that is not *Cubike*. So the *zenzenzike* roote of. 8. 12. or. 20. or of any number that is no *zenzenzike*, is called a *Surde* number. And in like maner, any other roote of any number, that hath noe soche roote, doeth cause that number to be a *Surde* number.

For if you see those signes annexed with numbers, that hath soche rootes, those numbers are not *Surde* numbers properly, but sette like *Surdes*. As the *Square* roote of. 4. or of. 9. or. 25. &c. The *Cubike* roote of. 8. 27. or. 125. &c. whiche sometymes is vsed for apte worke, as you shall see here after.

Of Numeration.

The numeration of thē doeth consist, in knowledge of their figures, whiche partly be declared before. But their common and peculare signes are these. $\sqrt{\quad}$. $\sqrt[3]{\quad}$. $\sqrt[n]{\quad}$. Although there maie be moare
Ll. 19. varieties

The Arte

varieties: Yet these for this tyme maie suffice.

The firste, that is. $\sqrt{\quad}$. is customably set, to signifie a *Square roote*. As this. $\sqrt{5}$. betokeneth the *Square roote* of. 5. And. $\sqrt{12}$. is the *Square roote* of. 12. Whobert many tymes it hath with it, for the moare certieintie the *Cosike* signe. $\sqrt{\quad}$. And is written thus. $\sqrt{8 \cdot 20}$. the *Square roote* of. 20. And. $\sqrt{8 \cdot 56}$. the *Square roote* of. 56.

The seconde signe is annexed with *Surde Cubes*, to expresse their rootes. As this. $\sqrt[3]{16}$. Whiche signifieth the *Cubike roote* of. 16. And. $\sqrt[3]{20}$. betokeneth the *Cubike roote* of. 20. And so forth. But many tymes it hath the *Cosike* signe with it also: as $\sqrt[3]{25}$ the *Cubike roote* of. 25. And. $\sqrt[3]{32}$. the *Cubike roote* of. 32.

The thirde figure doeth represente a *zenzizenzike roote*. As. $\sqrt[4]{12}$. is the *zenzizenzike roote* of. 12. And $\sqrt[4]{35}$. is the *zenzizenzike roote* of. 35. And like waies if it haue with it the *Cosike* signe. $\sqrt[4]{8 \cdot 8}$. As $\sqrt[4]{8 \cdot 8 \cdot 2 \cdot 4}$ the *zenzizenzike roote* of. 2. 4. and so of other.

Scholar. It were againste reason, to seke reason for those signes, whiche be set voluntarily to signifie any thynge: although some tymes there bee a certaine apte conformance in soche thynge. And in these figures, the number of their minomes, seemeth disagreeable to their order.

Master. In that there is some reason to bee shewed: for as. $\sqrt{\quad}$. declareth the multiplication of a number, ones by it self: So. $\sqrt[3]{\quad}$. representeth that multiplication *Cubike*, in whiche the roote is represented thise. And. $\sqrt{\quad}$. standeth for. $\sqrt{\quad}$. that is. 2. figures of *Square* multiplication: and is not expresse with. 4. minomes. For so should it seme to expresse moare then. 2. *Square* multiplications. But of voluntarie signes, it is inoughe to knowe that this thei doe signifie. And if any manne can devise other, moare easie or apter in vse, thei maie well be refused.

of Surde numbers.

But concerning the numeration of *Surde* numbers this shal you marke: that when any compounde signe is putte before a number, whiche hath any roote, that maie bee expresseed by parte of that signe, that number is not absolutely so to bee expresseed, onlesse it bee for ease or aptnesse in woꝝke. As. $\sqrt{36}$. whiche betokeneth the *zenzizenzike* roote of .36. Seyng it is well knowen, that .36. hath .6. for his *Square* roote, it were moare apte expresseynge that number thus. $\sqrt{36}$. that is the square roote of .6.

Other waies, if the nōber that foloweth the signe, haue a roote agreable to that signe: it is noe *Surde* number. As. $\sqrt{16}$. is .4. and is noe *Surde* number. So. $\sqrt{27}$. is .3. and needeth not to bee written in *Surde* forme, excepte it bee for aptnesse of woꝝke. And by this maie you iudge of all other, as thei come in vse

Scholar. If this bee all that is requisite to numeration, I praye you proceed. to addition. For that is nexte in order.

Master. That is the common order. I dolebet in vulgare fractions, you remember that multiplication and diuision, are set before addition and subtraction: bicause of the easer formes of woꝝke, in multiplication and diuision. And so in these *Surde* numbers, bicause the woꝝkes of multiplication, and of diuision, be not onely moare easie, then the woꝝkes of addition, and of subtraction, but also be requisite to them, therefore will I begin with them, and so come to the other.

Of Multiplication.



Multiplicatiō in *Surde* numbers vncōpounde hath noe difficultie, if thei be of one denomination: els must thei be reduced to one denomination: and that by multiplication, accoꝝding to their signes.

But where noe reduction needeth, you shall mul-
tiple

The Arte

to multiply the numbers together, and sette their common
 signe before the number, that resulteth of that multi-
 plication.

Examples of square Surdes.



Of you will multiplie. $\sqrt{\cdot 3}$. 15. by. $\sqrt{\cdot 3}$.
 26. it will make. $\sqrt{\cdot 3}$. 390.
 So. $\sqrt{\cdot 3}$. 32. multiplied by. $\sqrt{\cdot 3}$. 48.
 dooeth make. $\sqrt{\cdot 3}$. 1536.
 And. $\sqrt{\cdot 3}$. 56. multiplied by. $\sqrt{\cdot 3}$ 21.
 doeth yelde. $\sqrt{\cdot 3}$. 1176.

Howbeit some tymes it happeneth, that the nom-
 ber, whiche is made by that multiplication, is a nom-
 ber absolute, and not a *Surde number*.

Examples of soche as make *numbers Absolute.*

$$\begin{array}{r} \sqrt{\cdot 12.} \\ \sqrt{\cdot 3.} \\ \hline \sqrt{\cdot 36.} \text{ that is } 6. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 48.} \\ \sqrt{\cdot 3.} \\ \hline \sqrt{\cdot 144.} \text{ that is } 12. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 12. \frac{1}{2}.} \\ \sqrt{\cdot 4. \frac{1}{2}.} \\ \hline \sqrt{\cdot 56. \frac{1}{4}} \text{ that is } 7 \frac{1}{2}. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 28 \frac{1}{4}.} \\ \sqrt{\cdot 7 \frac{1}{4}.} \\ \hline \sqrt{\cdot 207 \frac{9}{16}} \text{ that is } 14 \frac{3}{4}. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 240.} \\ \sqrt{\cdot 15.} \\ \hline \sqrt{\cdot 3600.} \text{ that is } 60. \end{array}$$

$$\begin{array}{r} \sqrt{\cdot 325.} \\ \sqrt{\cdot 13.} \\ \hline \sqrt{\cdot 4225.} \text{ that is } 65. \end{array}$$

And generally when any number is multiplied by
 an other, if the proportion betwene those 2. numbers
 bee represented by a Square number, as by. 4. 9. 16.
 25. &c. then dooe they make a square number by their
 multiplication.

Examples

of Surde numbers.

Examples of Cubike rootes.

| | | |
|----------------------------|--------------------------|-----------------------------|
| $\sqrt[3]{91}$ | $\sqrt[3]{7\frac{1}{3}}$ | $\sqrt[3]{256}$ |
| $\sqrt[3]{12}$ | $\sqrt[3]{\frac{1}{3}}$ | $\sqrt[3]{12}$ |
| $\sqrt[3]{109\frac{1}{3}}$ | $\sqrt[3]{5\frac{1}{3}}$ | $\sqrt[3]{190\frac{1}{11}}$ |

*Examples of soche as make
Absolute numbers.*

| | |
|-------------------------------|-----------------------------|
| $\sqrt[3]{54}$ | $\sqrt[3]{686}$ |
| $\sqrt[3]{32}$ | $\sqrt[3]{4}$ |
| $\sqrt[3]{1728}$ that is 12. | $\sqrt[3]{2744}$ that is 14 |
| $\sqrt[3]{486}$ | |
| $\sqrt[3]{96}$ | |
| $\sqrt[3]{46656}$ that is 36. | |

Examples of renzizenzike rootes.

| | | |
|---|------------------------------------|-------------------------------------|
| $\sqrt{15}$ | $\sqrt{204}$ | $\sqrt{162}$ |
| $\sqrt{7}$ | $\sqrt{26}$ | $\sqrt{32}$ |
| $\sqrt{105}$ | $\sqrt{5304}$ | $\sqrt{5184}$ that is $\sqrt{72}$. |
| $\sqrt{7\frac{1}{2}}$ | $\sqrt{27}$ | |
| $\sqrt{\frac{1}{3}}$ | $\sqrt{12}$ | |
| $\sqrt{5\frac{1}{3}}$ that is $\sqrt{2\frac{1}{3}}$. | $\sqrt{324}$ that is $\sqrt{18}$. | |

*Examples of renzizenzike rootes
that make absolute numbers.*

| | |
|--------------------------|---------------------------|
| $\sqrt{32}$ | $\sqrt{128}$ |
| $\sqrt{8}$ | $\sqrt{32}$ |
| $\sqrt{256}$ that is 16. | $\sqrt{4096}$ that is 64. |
| | Or. $\sqrt{288}$ |

The Arte

w/. 288.

w/. 72.

w/. 20736. that is. 12.

But here is to bee noted, that if you would multiplie any *Surde* number, by an absolute number, or any *Surde* number of one denominatiō, by a *Surde* number of an other denomination: you must firste reduce that Absolute number to the like denomination. And so must you reduce the.2. *Surde* numbers to one denomination.

And because that this woork doeth serue often in this arte, and that in diuerse woorkes, I will set here the arte of reduction.

Of reduction in Surdes.



Reduction in *Surdes*, is the bringyng of sundrie denominatiōs vnto one. Whiche in absolute nēbers is thus doen. You shall multiplie the absolute number, according to the signe of the *Surde*, and then sette befoze it the like signe. So that if you would double. $\sqrt{3} \cdot 8$. that is to saie, if you would multiplie it by.2. you must firste multiplie that.2. squarely, and then multiplie those numbers together. What is to saie, you shall multiplie. $\sqrt{3} \cdot 8$. by. $\sqrt{3} \cdot 4$. and so is it doubled.

Like wates, to triple any Square *Surde*, is to multiplie it by.9. And so to quadruple any square *Surde*, is to multiplie it by.16. And so forth.

But if you double any *Cubike* number, you shall multiplie it by.8. that is the *Cube* of.2. And so if you would triple a *Cubike* roots, you muste multiplie it by 27. And if you would quadruple it, you shall multiplie

Of Surde numbers.

it by .64. And so of other like woordes.

Again, if you will double any *zenzizenzike* roote, you must multiplie it by .16. And if you will triple it, you shall multiplie it by .81. And so if you will *quadruple* it, you must multiplie it by 256. And in like maner ever moare, for the number absolute, you shall set his *zenzizenzike* number. Like as in Squares, for any number absolute, you shall set his square. And in Cubes you shall take his *Cube*.

Scholar. This is plaine inoughe: yet I praye you put an example of twoo, of eche kinde.

Master. Take these examples for square rootes.

$$\begin{array}{r} \sqrt{} \quad 38. \\ \underline{ \quad 2.} \\ \sqrt{} \quad 152. \end{array} \quad \begin{array}{r} \sqrt{} \quad 7. \quad 128. \\ \underline{ \quad \quad 6.} \\ \sqrt{} \quad 7. \quad 4608. \end{array} \quad \begin{array}{r} \sqrt{} \quad 3264. \\ \underline{ \quad \quad 12.} \\ \sqrt{} \quad 469976. \end{array}$$

Examples in Cubike rootes.

$$\begin{array}{r} \sqrt[3]{} \quad 52. \\ \underline{ \quad \quad 2.} \\ \sqrt[3]{} \quad 416. \end{array} \quad \begin{array}{r} \sqrt[3]{} \quad 163. \\ \underline{ \quad \quad 5.} \\ \sqrt[3]{} \quad 20375. \end{array} \quad \begin{array}{r} \sqrt[3]{} \quad 4806. \\ \underline{ \quad \quad 8.} \\ \sqrt[3]{} \quad 2460672. \end{array}$$

Examples in zenzizenzike numbers.

$$\begin{array}{r} \sqrt{} \quad 69. \\ \underline{ \quad \quad 2.} \\ \sqrt{} \quad 1104. \end{array} \quad \begin{array}{r} \sqrt{} \quad 251. \\ \underline{ \quad \quad 4.} \\ \sqrt{} \quad 64256. \end{array} \quad \begin{array}{r} \sqrt{} \quad 1385. \\ \underline{ \quad \quad 5.} \\ \sqrt{} \quad 2250625. \end{array}$$

Scholar. This I perceiue well. But now in *Surde* numbers of diuerse denominations, what the order of reductiō is, I praye you to set forth with some examples

Master. These examples with their declaration, maie sufficiently serue for a shewe, if I would multiplie . $\sqrt{12}$. by . $\sqrt{5}$. I must firste multiplie the number of one signe, accoꝝyng to the signe of the other

An. y. number,

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number, and so alter them bothe. Whiche woorkie is like the reduction of fractions, to one common denomination. As here I make multiplie. $\sqrt[3]{5}$. Cubikely, and 12. must be multiplied squarely, and then shall I adde bothe signes in one, for their common signe. So shall I haue for them the $\sqrt[3]{\text{C}}$. roots of. 144. to be multiplied by the *zenzicubike* roote of 125. And so wth there come of that multiplication, the *zenzicubike* roote of. 18000. As here by example doeth appeare.

$$\begin{array}{r} \sqrt[3]{5} \cdot \text{C} \cdot 144. \\ \sqrt[3]{5} \cdot \text{C} \cdot 125. \\ \hline \sqrt[3]{5} \cdot \text{C} \cdot 18000. \end{array}$$

Like waies if I would multiplie. $\sqrt{34} \cdot 250$. by $\sqrt{34}$. I shall firste multiplie. 250. Cubikely, and it will bee. 15625000. And 34. must I multiplie *zenzicubikely*, and it will yelde. 1336336. Wherefore multiplying them together, and adding thereto the common denomination, it will bee the $\sqrt[3]{5} \cdot \text{C}$. roots of. 20880250000000.

This woorkie is aptly represented in figure, after this sorte. And then shall you multiplie crosse waies the number of the one, by the signe of the other. And so mate you dooe in all other like numbers, of diuerse denominations.

$$\begin{array}{r} \sqrt{34} \cdot 250. \\ \times \\ \sqrt{34} \end{array}$$

This reduction doeth serue for any other woorkie, as well as for multiplication.

Of Diuision.



Diuision is as easie as multiplication. For in it there is noe regard had to the signes. But the one number diuided by the other as if thei were numbers absolute. And then the firste signe added to the *quotiente*. For the more lighte and certaintie, I haue set here, examples of eche sorte.

And

of Surde numbers.

And first examples of square rootes.

$$\begin{array}{l} \sqrt{\cdot} 72. \\ \sqrt{\cdot} 8. \end{array} (\sqrt{\cdot} 9. \text{ that is. } 3.) \quad \begin{array}{l} \sqrt{\cdot} 128. \\ \sqrt{\cdot} 4. \end{array} (\sqrt{\cdot} 32.)$$

$$\begin{array}{l} \sqrt{\cdot} 457\frac{1}{2}. \\ \sqrt{\cdot} 21. \end{array} (\sqrt{\cdot} 21\frac{1}{2}.)$$

Examples of Cubike rootes.

$$\begin{array}{l} \sqrt[3]{\cdot} 96. \\ \sqrt[3]{\cdot} 4. \end{array} (\sqrt[3]{\cdot} 24.) \quad \begin{array}{l} \sqrt[3]{\cdot} 1664. \\ \sqrt[3]{\cdot} 32. \end{array} (\sqrt[3]{\cdot} 52.)$$

$$\begin{array}{l} \sqrt[3]{\cdot} 5624. \\ \sqrt[3]{\cdot} 76. \end{array} (\sqrt[3]{\cdot} 74\frac{1}{2}.)$$

Examples of zenzizenzike rootes.

$$\begin{array}{l} \sqrt[4]{\cdot} 54. \\ \sqrt[4]{\cdot} 6. \end{array} (\sqrt[4]{\cdot} 9. \text{ that is. } \sqrt{\cdot} 3.)$$

$$\begin{array}{l} \sqrt[4]{\cdot} 286. \\ \sqrt[4]{\cdot} 42. \end{array} (\sqrt[4]{\cdot} 6\frac{11}{12}.) \quad \begin{array}{l} \sqrt[4]{\cdot} 5892. \\ \sqrt[4]{\cdot} 54. \end{array} (\sqrt[4]{\cdot} 109\frac{1}{2}.)$$

And this maie suffice for Diuision. The profe of it is by the contrary kinde. For Multiplication proueth Diuision: and Diuision trieth Multiplication.

Scholar. All this is easie enough to remember.

Of Addition.

Halter.



ADDITION is not so easie, but hath diuerse *The firste* varieties of worke, as anon shall appere. *forme of* Whereof the firste is as easie as can bee. *Addition.* For it requireth onely the signe of addition. —. As if I would adde, $\sqrt{\cdot} 12.$ to $\sqrt[3]{\cdot} 26.$

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$\sqrt{.26}$. I shall set it thus. $\sqrt{.26}$. — | — $\sqrt{.12}$. And so
 $\sqrt{.20}$. put vnto. $\sqrt{.54}$. maketh. $\sqrt{.54}$. — | — $\sqrt{.20}$.
 This forme serueth chiefly for rootes of diuerse na-
 mes. As. $\sqrt{.7}$. $\sqrt{.20}$. — | — $\sqrt{.7}$. $\sqrt{.20}$. Where
 $\sqrt{.20}$. is added to $\sqrt{.7}$. $\sqrt{.20}$. And so of al other.

*The seconde
forme.*

The seconde forme is not so easie: and yet many ti-
mes it is moare certaine. And this is the order of it.

You shall sette doune your . 2. numbers , that you
would adde together , forseyng that thei be of one de-
nomination. Then shall you adde in plaine forme,
their numbers together, puttyng thereto the signe of
the roote. And kepe that as a parte of the addition.
Again you shall multiplie the . 2. firste numbers toge-
ther. And their totalle you shall multiplie by . 4. And
before that shall you sette the signe of the roote. And
it shall stande as the seconde parte of that addition.
So that those . 2. partes, shall be added with the signe
— | — . And then is the woork e canded. Example
hereof. I would adde the . 2. firste sommes , that is,

$\sqrt{.12}$. to. $\sqrt{.26}$. wherfoze I
 set them thus. And then doe
 I adde the bothe plainly to-
 gether, and thei make. $\sqrt{.38}$
 whiche I set by, as one part
 of the addition. Then doe I
 multiplie $\sqrt{.26}$. by. $\sqrt{.12}$. and
 there riseth. $\sqrt{.312}$. whiche
 I must double, or multiplie
 by . 2. And therfoze seyng the

$$\begin{array}{r|l}
 \sqrt{.26} \text{ — | — } \sqrt{.12} & \\
 \hline
 \sqrt{.26}. & \\
 \sqrt{.12}. & \\
 \hline
 \sqrt{.312}. & \\
 \sqrt{.4}. & \\
 \hline
 \sqrt{.1248}. &
 \end{array}$$

woork e is in square rootes, I set the square of 2. with
 the signe of. $\sqrt{.}$. for. 2. and then multipliyng them to-
 gether, I haue. $\sqrt{.1248}$. whiche is the seconde parte
 of the roote. Wherfoze addyng those . 2. partes toge-
 ther, with the signe. — | — . there commeth. $\sqrt{.38}$.
 — | — $\sqrt{.1248}$. as the totalle of that addition.

Scholar. As me thinketh, the firste forme of addi-
tion,

of Surde numbers.

tion serueth better for these numbers, then this se-
conde forme. For it is moare easie to vse, in any kinde
of woork, and moare speedily doen: and it serueth that
this laste number, is moare obscure then the firste.

Haile. Yet is this woork good, and very neces-
sarie. For in these numbers, and soche other like, it
serueth onely (as appereth) to alter the state of the nō-
bers, whereby they maie bee commensurable, with o-
ther, then they were befoze that alteration. But in
some numbers, and that very many, it reduceth them
to one simple forme of roote. As by the examles folo-
wyng you shall perceiue.

An example.

| | | | |
|---|---|---|---|
| $\begin{array}{r} \sqrt{.28.} + \sqrt{.7.} \\ \hline \sqrt{.28.} \\ \sqrt{.7.} \\ \hline \sqrt{.196.} \\ \sqrt{.4.} \\ \hline \sqrt{.784.} \end{array}$ | $\begin{array}{r} 28. \\ 7. \\ \hline 35. \\ 28. \\ 7. \\ \hline 35. \end{array}$ | <p>The same example other waies wrought.</p> $\begin{array}{r} \sqrt{.28.} + \sqrt{.7.} \\ \hline \sqrt{.28.} \\ \sqrt{.7.} \\ \hline \sqrt{.196.} \text{ whose roote is } 14 \\ 14. \\ \hline 2. \\ \sqrt{.35.} + \sqrt{.28.} \end{array}$ | <p><i>A thirde forme of ad- dition.</i></p> |
|---|---|---|---|

Where firste I haue set forthe. 2. examles of one
addition, that you maie see the agremente of the both

And firste I would adde. $\sqrt{.28.}$ with. $\sqrt{.7.}$ where-
foze I dooe toyne. 28. and. 7. in one somme, whiche I
set a parte, as the firste portion of the addition. Then
I doe multiplie. 28. by. 7. And thereof cometh. 196.
whiche is a square nōber, and hath. 14. for his roote.
So that I maie vse now. 2. woorkes. For other I maie
continue my woork, as I haue doon (agreable to the
firste example) in multipling that. $\sqrt{.196.}$ by. $\sqrt{.4.}$
(whiche is but doubling) and so there cometh. $\sqrt{.784}$
whiche

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whiche is a number absolut e: bicause it hath a roote,
accoyding to his signe, whiche roote is. 28. and maie
be set for. $\sqrt{784}$.

Now in the seconde woork, bicause the firste mul-
tiplication of. 28. by. 7. doeth make a square number,
I doe take the roote of that number for it: scyng it is
all one thyng to saie. $\sqrt{196}$, and. 14. for. 14. is the
roote of. 196. And then hauyng the roote, I make
double it, accoyding to the rule, or multiplye it by. 2.
and there of commeth. 28. whiche I shall adde with
35. And so haue I. 63. whose roote containeth the ad-
dition of. $\sqrt{28}$. and. $\sqrt{7}$.

Scholar. This woork seemeth straunge: and far-
thesse from common reason, of all other woorkes in
this arte.

Maister. I mighte easily by demonstration make
you, to perceiue as moche reason in this woork, as can
be in any: for it dependeth of the. 38. Theorome of the
pathewate. But haste of other businesse, maketh me
to omit the demonstration at this tyme, whiche shortly
you shall haue, for all the equations, and other
woorkes likewaies.

But for this presente tyme, it shall be sufficiente to
woork an example in *rationall* numbers, as if they wer
Surde numbers: that therby you maie perceiue the or-
der, and the truth of the woork.

Wherfore I take these twoo numbers. $\sqrt{36}$. and
 $\sqrt{49}$. to be added together. Where I doe firste adde
the twoo numbers plainely together: And then make
85. for the firste parte of the addition. Then dooe I
multiplye. 49 by. 36. and there riseth. 1764. whiche
is a square number. And therefore maie I vse. 2. woorkes,
as you see. In the firste I multiplye that square
number by. 2. or by. $\sqrt{4}$. whiche is all one: and there
doeth amounte. 7056. a square number also, whose
roote is. 84.

The

Of Surde numbers.

| The firste foyme. | The seconde foyme. |
|---|--|
| $\begin{array}{r} \sqrt{.36.} \text{---} \text{---} \sqrt{.49.} \\ \hline \sqrt{.} \quad 49 \\ \sqrt{.} \quad 36 \\ \hline 294 \\ 117 \\ \hline \sqrt{.} \quad 1764 \\ \sqrt{.} \quad 4 \\ \hline \sqrt{.} \quad 7056 \\ \hline \sqrt{.85.} \text{---} \text{---} \sqrt{.7056.} \\ \text{D.} \sqrt{.85.} \text{---} \text{---} .84 \end{array}$ | $\begin{array}{r} \sqrt{.36.} \text{---} \text{---} \sqrt{.49.} \\ \hline \sqrt{.} \quad 49. \\ \sqrt{.} \quad 36. \\ \hline 294. \\ 85. \\ \hline 147. \\ \hline \sqrt{.} \quad 1764. \\ \text{That is.} 42. \\ \quad 2. \\ \hline 84. \\ \hline \sqrt{.85.} \text{---} \text{---} 84. \end{array}$ |

That is. $\sqrt{.169.}$

D. 13.

In the seconde wooyke I take the roote of .1764. whiche is 42 and doubling it, I haue 84. agreable to the other wooyke. Then doe I sette those .2. numbers doune with ---, and putte to them the signe. $\sqrt{.}$ in token that I muste take the roote of that compounde number: and not of any one parte of it.

Scholar. That haue I marked well: For 85. hath no roote, nother 84. hath any roote. But $85 \text{---} | \text{---} 84$ that is. 169. hath. 13. for his roote.

And so I see, that the roote of. 36. whiche is. 6. And the roote of. 49. that is. 7, beeyng bothe added together will make. 13. that is the roote of. 169.

Maister. Yet one other foyme of casie wooyke, *Of numbers commensurable, a fourth forme.* I will shewe you, whiche is bothe pleasaunte and profitable: But is not generable, for it serueth onely for one common diuisor, male bee brought into Square numbers. With whiche numbers, you shall wooyke thus.

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Firste diuide them by the common diuifoz: and set for them their rootes. Then adde those .2. rootes together, and multiplie it squarely. And that square being multiplied by the common diuifoz, will byynge for the the Square of bothe the rootes. As here followeth in erample.

Where I would adde $\sqrt{384}$ vnto $\sqrt{150}$ which numbers I doe examin, til I maie finde their commo, and leaste diuifoz, whiche here is .6. When diuidyng them by that .6. I haue for 384. a square number. 64. And for .150. I haue another square, that is .25. Of

$$\begin{array}{r}
 \sqrt{384} \quad | \quad \sqrt{150} \\
 6.) \quad 64 \quad \quad 25 \\
 \quad \quad 8 \quad \quad \quad 5 \\
 \quad \quad \quad 13 \\
 \quad \quad \quad \underline{13} \\
 \quad \quad \quad 169 \\
 \quad \quad \quad \quad 6 \\
 \quad \quad \quad \quad \underline{6} \\
 \quad \quad \quad 1014
 \end{array}$$

whiche bothe squares I set doune the rootes: and the common diuifoz also. Then doe I adde bothe rootes together, and thereof commeth .13. whose Square is 169. that I doe multiplic by .6. whiche is the commo diuifoz, and it will bee .1014. whose roote doeth contain bothe the rootes befoze named. As you shall see it proued anon by Subtraction.

Scholar. In the meane season I consider, that one of these formes, maie confirme the other. And therefore if I woork this laste erample, by one of the other formes, and finde thesame totall, it must needs be that the woork is good. Whiche I proue thus.

Firste setting doune the numbers, in forme of the castelle Addition. And then addyng them together, I finde .534. whiche I sette a side, as one parte of the number, that I doe seke for.

Then dooe I multiplie the .2. numbers together, and thei make .57600. whiche I dooe multiplie again by 4. And there riseth .230400. being a square number, and hath .480. for his roote. Wherefoze I set

of Surde numbers.

| | |
|--|--|
| $\sqrt{.384} \text{---} \text{---} \sqrt{.150}.$ | |
| 384 | |
| 150 | |
| 19200 | |
| 384 | |
| 57600 | |
| 4 | |
| 230400 | |
| $\sqrt{.534} \text{---} \text{---} \sqrt{.230400}.$ | |
| $\text{D. } \sqrt{.534} \text{---} \text{---} .480.$ | |
| That is. $\sqrt{.1014}.$ | |

set. 534. and. 480. together. with the signe of Addition, thus.

$534 \text{---} | \text{---} 480,$ And the roote of that number, is equalle to bothe the firste rootes. But considering that bothe those numbers, which bee ioyned laste of all with $\text{---} | \text{---}$, are numbers rationally and absolutely, I made adde the in one, & so they make

1014. agreeably to the other woork. Wherefore I iudge them bothe to be good

Master. You might have wrought this woork either waies, because the firste number, that riseth of the multiplication is a square number.

Scholar. When I perceiue, I mighte haue taken the roote of it, whiche is. 240. and doubling it, I should haue. 480. As I had in the other woork. And so all doe agree in one.

But my chief doubt now is, how to knowe those numbers that bee *commensurable*: For if I shall stande long in searchyng for that, I might sooner woork the other forme of woork, then to make that trialle of *commensurableness*.

Master. The easieste waie is, to diuide the greater number, by the lesser, as if they were bothe numbers absolute: & the *quotiente* will declare their *Squares*. *The finding of numbers*

As if you doubt, whether. 384. and. 150. bee *commensurable*. diuide. 384. by. 150. and the *quotiente* will be $2\frac{1}{3}$, that is $\frac{7}{3}$. Then diuide whiche of the. 2. firste numbers you list, by his like number in the *quotiente*: And the common diuisor will amounte. So if you di-

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Use the greater number. 384. by the greater number in the *quotiente*, whiche is. 64, you shall finde the new *quotiente* 6. whiche. 6. is the common number. And if you diuide. 150. by 25. the common number. 6. will be the *quotiente*.

But and if the *quotiente* be a whole number, and no fraction, and be a Square number, then is it the lesser square. Wherefore if you diuide the lesser number of the. 2. by the *quotiente*, the common number will appere in the seconde *quotiente*. And then if you diuide the greater of the. 2. numbers, by that common number, his *quotiente* will shewe you the other Square.

And if so happen, that the *quotiente* of the firste diuision be not a square number, then are those numbers *incommensurable*.

So. $\sqrt{32}$. and. $\sqrt{128}$. be *commensurable*: and the *quotiente* of their diuision is. 4. whiche is the lesser square. And. 8. appeareth to be the common number. And the greater square is. 16.

Wherby this number it maie easily be espied, that some numbers maie be resolued, into moze squares then one. As these. 2. numbers, beyng diuided by. 2. dooe giue. 16. and. 64. And beyng diuided by. 8. the byng for the. 4. and. 16.

But for their addition, what Squares so euer you take, that redounde by one common diuisor, the triall will be like, and the roote one.

Scholar. I praye you let me proue that varietie.

Master. Then proue it in suche numbers, where you maie finde moare varietie. As these be. $\sqrt{288}$. and. $\sqrt{1152}$.

Scholar. If I diuide. 1152. by. 288. the *quotiente* will be. 4. whiche I must take for the leaste Square. Then by it I diuide. 288. and the *quotiente* will be. 72. as the common diuisor. By whiche if I diuide. 1152. there will rise. 16. as the seconde square. Then let I

the

of Surde numbers.

the nōbers in order thus. $\sqrt{.1152} - \sqrt{.288}$.

And vnder. 1152. I set the one Square. 16. And vnder. 288. I putte the other Square. 4. And vnder eche of thaim his roote. Then adde I the Rootes together, whiche maketh. 6. whose square is. 36. And that beyng multiplied by 72. the common number, doeth yelde. 2592. whose roote doeth containe bothe the other. 2. rootes by addition.

| | |
|-------|-------------------|
| 16 | 4. |
| 72) 4 | 2. |
| | 6. |
| | 6. |
| | 36. |
| | 72. |
| | 72. |
| | 252. |
| | $\sqrt{. 2592}$. |

But now how I shall finde any other Squares in those nōbers, to make any farther trial, I knowe not.

Master. Diuide alwaies one of the numbers, by some square nōber, that will parte it exactly, without any remainder. And marke the *quotiente*. For by it that you diuide the other nōber, and if the *quotiente* in that last diuision, be a square number, then haue you your purpose. Els muste you proue with an other Square number.

Scholar. I vnderstande you. And therfore in these numbers, I will make trialle with. 9. by whiche I diuide. 288. And finde the *quotient*. 32. Then by the same 32. I diuide 1152. and the *quotiente* is. 36. So haue I 9 and. 36. for the. 2. squares, and. 32. for the comon diuisor. Therfore I set the nōbers in order as they ought. And vnder them I place the. 2. square numbers with their rootes. Then addyng the rootes together, I finde. 9. whiche I multiplie square, and it yeldeth. 81. that. 81. I doe multiplie by the common number. 32. and there amounteth. 2592. As it did before in the other worke. Wherby I perceiue that these woordes doe confirme one an other.

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$$\begin{array}{r}
 \sqrt{1152} - \sqrt{288} \\
 \hline
 32) \quad 36 \quad 9 \\
 \quad \quad 6 \quad 3 \\
 \quad \quad \quad 9 \\
 \quad \quad \quad 9 \\
 \hline
 \quad \quad \quad 81 \\
 \quad \quad \quad 32 \\
 \hline
 \quad \quad 162 \\
 \quad 243 \\
 \hline
 \sqrt{2592}
 \end{array}$$

And therefore I will proue, how many varieties of this worke, I may finde in these numbers.

And soz my purpose, I will diuide the lesser of the .2. numbers, by as many Squares as I can, soz that seemeth to be the readiest waie. And firste I proue with .16. And so the *quotient* is .18. by whi-

che .18. I diuide .1152. and the *quotiente* is .64. whiche is a square n^ober. So that I haue that varietie moze.

Then againe I proue with .25. But I see, that will not frame. Wherefoze I assaie with .36. And finde the *quotiente* 8. by whiche I diuide the greater square, and the *quotiente* is .144. a square number also. And therfoze I note that soz an other varietie.

Thirdly, I proue with .49. but that wil not agree. Then attempte I with .64. And that serueth as euil. Perce that I assaie .81. 100. and .121. but none of them will diuide .288. wherefoze I passe vnto .144. whiche is twise contained in 288. by that .2. I diuide 1152. and finde the *quotiente* .576. whiche is a Square number also. And so haue I .3. other varieties beside the .2. former woorkes: whiche .3. varieties, soz my remembraunce I set doune, thus.

$$\sqrt{1152}.$$

of Surde numbers.

$$\begin{array}{r}
 \sqrt{.1152} \text{ --- } | \text{ --- } \sqrt{.288} \\
 18) \quad 64 \qquad 16 \\
 \quad \quad 8 \qquad \quad 4 \\
 \qquad \qquad \quad 12 \\
 \qquad \qquad \quad 12 \\
 \hline
 \qquad \qquad \quad 144 \\
 \qquad \qquad \quad 18 \\
 \hline
 \qquad \qquad 1152 \\
 \qquad \qquad 144 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.1152} \text{ --- } | \text{ --- } \sqrt{.288} \\
 8) \quad 144 \qquad 36 \\
 \quad \quad 12 \qquad \quad 6 \\
 \qquad \qquad \quad 18 \\
 \qquad \qquad \quad 18 \\
 \hline
 \qquad \qquad 324 \\
 \qquad \qquad \quad 8 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.1152} \text{ --- } | \text{ --- } \sqrt{.288} \\
 2) \quad 576 \qquad 144 \\
 \quad \quad 24 \qquad \quad 12 \\
 \qquad \qquad \quad 36 \\
 \qquad \qquad \quad 36 \\
 \hline
 \qquad \quad 1296 \\
 \qquad \quad \quad 2 \\
 \hline
 \sqrt{.} \quad 2592
 \end{array}$$

Master. Then for to gratifie you, I will sette doune .2. other numbers with 6 varieties. Whiche maie serue to suffice for this worke, without more exâples. And because you know the order to trie the I will sette them doune

without any explication, other declaration. As here you see.

| | |
|---|--|
| $ \begin{array}{r} \sqrt{.28800} \text{ --- } \text{ --- } \sqrt{.7200} \\ 2) \quad 14400 \qquad 3600 \\ \quad \quad 120 \qquad \quad 60 \\ \qquad \qquad \quad 180 \\ \qquad \qquad \quad 180 \\ \hline \qquad \quad 32400 \\ \qquad \quad \quad 2 \\ \hline \sqrt{.} \quad 64800 \end{array} $ | $ \begin{array}{r} \sqrt{.28800} \text{ --- } \text{ --- } \sqrt{.7200} \\ 3) \quad 3600 \qquad 900 \\ \quad \quad 60 \qquad \quad 30 \\ \qquad \qquad \quad 90 \\ \qquad \qquad \quad 90 \\ \hline \qquad \quad 8100 \\ \qquad \quad \quad 8 \\ \hline \sqrt{.} \quad 64800 \\ \qquad \qquad \qquad \sqrt{.28800} \end{array} $ |
|---|--|

The Arte

| | |
|---|---|
| $\sqrt{.28800} \quad \quad \sqrt{.7200}$ <hr/> <div style="display: flex; justify-content: space-between;"> 1600 400 </div> <div style="display: flex; justify-content: space-between;"> 18) 40 20 </div> <div style="margin-left: 100px;"> 60 60 <hr/> 3600 18 <hr/> 28800 36 <hr/> $\sqrt{.64800}$ </div> | $\sqrt{.28800} \quad \quad \sqrt{.7200}$ <hr/> <div style="display: flex; justify-content: space-between;"> 900 225 </div> <div style="display: flex; justify-content: space-between;"> 32) 30 15 </div> <div style="margin-left: 100px;"> 45 45 <hr/> 2025 32 <hr/> 4050 6075 <hr/> $\sqrt{.64800}$ </div> |
|---|---|

| | |
|---|--|
| $\sqrt{.28800} \quad \quad \sqrt{.7200}$ <hr/> <div style="display: flex; justify-content: space-between;"> 576 144 </div> <div style="display: flex; justify-content: space-between;"> 50) 24 12 </div> <div style="margin-left: 100px;"> 36 36 <hr/> 1296 50 <hr/> $\sqrt{.64800}$ </div> | $\sqrt{.28800} \quad \quad \sqrt{.7200}$ <hr/> <div style="display: flex; justify-content: space-between;"> 400 100 </div> <div style="display: flex; justify-content: space-between;"> 72) 20 10 </div> <div style="margin-left: 100px;"> 30 30 <hr/> 900 72 <hr/> $\sqrt{.64800}$ </div> |
|---|--|

Scholar. This varietie is pleasaunte.

Maister. I will satisfie your delite better at moze letfure. But yet one thynge moare will I saie, before we canse this sorte of Additio: that if you would adde any roote to it self. As. $\sqrt{.6.}$ to $\sqrt{.6.}$ or $\sqrt{.10.}$ to $\sqrt{.10.}$ &c. you shall oncly *quadriple* the number: and so haue you doen.

Scholar. I see good reason in that: For addition of any number to it self, is but doublyng that number or multiplication by 2. And that must be doen by that *quadriplation*, as you taught before.

Addition of cubike rootes

Maister. Now will I set for the some examples of addition in *Cubike rootes*. For the worke is like vnto this laste forme in *Square rootes*, saue that the *mul-*
tuplications,

Of Surde numbers.

uplications, whiche were Square in that woꝝke, must be *Cubike* in this woꝝke. And that onely in numbers *commensurable*. For numbers *incommensurable* be added with the signe. — + —. without moare woꝝke.

I call soche *Cubike* rootes *commensurable*, whiche being divided by any common number, will make *Cubike* numbers in their *quotiente*. As. $\sqrt[3]{24}$. and. $\sqrt[3]{81}$ whiche divided by. 3. doe make. 8. and. 27. bothe being *Cubike* numbers. So. $\sqrt[3]{320}$. and. $\sqrt[3]{135}$ being divided by. 5. doe make. 27. and. 64. bothe *Cubike* numbers. Likeways. $\sqrt[3]{2744}$. and. $\sqrt[3]{1000}$. be *commensurable*, bicause thei make. 343. and. 125. bothe *Cubike* numbers: If thei be divided by. 8.

Scho. I praye you make your examles with these.

Master. There nedeth noe woꝝdes in this woꝝke it is so like the Addition of square rootes. And therefore marke these examles well.

$$\begin{array}{r}
 \sqrt[3]{81} \text{ — + — } \sqrt[3]{24} \\
 27 \qquad \qquad \qquad 8 \\
 3 \) \ 3 \qquad \qquad \qquad 2 \\
 \qquad \qquad \qquad \qquad \qquad 5 \\
 \qquad \qquad \qquad \qquad \qquad 5 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad 125 \\
 \qquad \qquad \qquad \qquad \qquad 3 \\
 \hline
 \sqrt[3]{375}
 \end{array}$$

$$\begin{array}{r}
 \sqrt[3]{320} \text{ — + — } \sqrt[3]{135} \\
 64 \qquad \qquad \qquad 27 \\
 5 \) \ 4 \qquad \qquad \qquad 3 \\
 \qquad \qquad \qquad \qquad \qquad 7 \\
 \qquad \qquad \qquad \qquad \qquad 7 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad 343 \\
 \qquad \qquad \qquad \qquad \qquad 5 \\
 \hline
 \sqrt[3]{1715}
 \end{array}$$

$$\begin{array}{r}
 \sqrt[3]{2744} \text{ — + — } \sqrt[3]{1000} \\
 343 \qquad \qquad \qquad 125 \\
 8 \) \ 7 \qquad \qquad \qquad 5 \\
 \qquad \qquad \qquad \qquad \qquad 12 \\
 \qquad \qquad \qquad \qquad \qquad 12 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad 1728 \\
 \qquad \qquad \qquad \qquad \qquad 8 \\
 \hline
 \sqrt[3]{13824}
 \end{array}$$

Do. f. Scholar.

The Arte

Scholar. Here is noe diuerſitie, from the former woꝝkes, but in ſettyng the *Cubike* roote, foꝝ the ſquare roote. And in multiplyng the addition of the. 2. rootes *Cubikely*.

Maſter. That is all. And therefore will I ſtande noe longer aboute it: But will pꝛoccede to an other forme of addition, whiche ſerueth alſo foꝝ *Cubike* rootes commensurable. The rule is this. Set doune the *Cubike* rootes, with their common diuiſoꝝ, and the *Cubes* that riſe therby, and their rootes alſo. All this you did in this former woꝝke. But now peculiarly in this rule, you ſhall ſet doune. 3. other numbers orderly, vnder thoſe. 3. former numbers. The firſt is the ſquare of that laſt *Cubike* roote: the ſecõde is the *triple* of that ſquare: and the thirde is a number reſultyng of the multiplication of that triple by the other roote.

An other forme of addition.

Then take the. 4. extreme numbers, that is thoſe 2 laſt numbers, and the. 2. *Cubes*, and adde them together into one ſomme. And that ſomme beyng multiplied by the common diuiſoꝝ, will make a *Cubike* number, whoſe *Cubike* roote ſhall containe bothe the firſt rootes, whiche you intended to adde. Now marke theſe examles: and cõferre them well with the woꝝdes of the rule.

| | | |
|--|--|--|
| $\begin{array}{r} \sqrt{.384} \quad \quad \sqrt{.48} \\ \hline 64 \\ 6) \quad 4 \\ \quad 16 \\ \quad 48 \\ \hline 48 \\ \quad 216 \\ \quad \quad 6 \\ \hline \sqrt{.1296} \end{array}$ | $\begin{array}{r} \sqrt{.15972} \quad \quad \sqrt{.2592} \\ \hline 1331 \\ 8 \ 12) \quad 11 \\ \quad 121 \\ \quad 363 \\ \quad 1188 \\ \hline 4913 \\ \quad 12 \\ \hline 9826 \\ \quad 4913 \\ \hline \sqrt{.58956} \end{array}$ | $\begin{array}{r} \sqrt{.2592} \\ \hline 216 \\ \quad 36 \\ \quad 108 \\ \hline 2178 \\ \quad 4913 \\ \quad 12 \\ \hline 9826 \\ \quad 4913 \\ \hline \sqrt{.52488} \end{array}$ |
|--|--|--|

of Surde numbers.

$\sqrt[3]{52488.} - \sqrt[3]{24696.}$

| | |
|---------------|---------|
| 5832. | 2744. |
| 9) 18 | 14. |
| 324 | 196. |
| 972 | 588. |
| 10584 | 13608 |
| | 32768. |
| | 9. |
| $\sqrt[3]{.}$ | 294912. |

Scholar. In these examples I see, the woordes of your rule obserued. For vnder eche *Surde Cubike* roote, there is set a true *Cubike* number, whiche is founde by the common diuisor: then foloweth the roote of that true *Cube*: and beside it standeth the common diuisor. Then in the fourthe roome is the Square of the true *Cubike* roote. And vnder it his number tripled (as. 48 vnder .16, and .12. vnder .4) whiche triple being multiplied by the roote of the other side, dooeth make the loweste number in that rowe. So .48. multiplied by. 2. maketh.96. whiche is set vnder the roote.2. And.12. multiplied by.4. yeldeth.48. whiche is placed vnder that.4.

Then those.4. extreme numbers.64.and.48,8. & 96. doe make by addition 216. whiche somme is multiplied by.6. that is the common diuisor, and so riseth 1296. whose *Cubike* roote comprehendeth bothe the firste rootes.

Master. The like maie you iudge of the other.2. examples. But because you maie vnderstande the certaintie of this woork the better, I haue here sette forth the.2. examples of true *Cubike* rootes, so named like *Surde* numbers.

Do. ij. $\sqrt[3]{.4096}$

The Arte

| | | |
|-----------|------|----------|
| w/.4096. | + | w/.1728. |
| 512. | | 216. |
| 8) 8. | | 6. |
| 64. | | 36. |
| 192. | | 108. |
| 864. | | 1152. |
| | 2744 | |
| | 8 | |
| w/. 21952 | | |

| | | |
|-----------|-------|----------|
| w/.19683. | + | w/.3375. |
| 729 | | 125 |
| 27) 9 | | 5 |
| 81 | | 25 |
| 243 | | 75 |
| 675 | | 1215 |
| | 2744 | |
| | 27 | |
| | 19208 | |
| | 5488 | |
| w/. 74088 | | |

Scholar. I perceiue by craminati-
on of woork in my
Tables here, that
4096. is a *Cubike*
number, and hath
16 for his roote. So
1728 is a *Cubike* nō-
ber also, & his roote
is . 12. those bothe
rootes added toge-
ther, doe make. 28.
And that. 28. is the

Cubike roote to. 21952. as the firste example would.
And for the seconde example, I see likewise that
19683. hath. 27. for his *Cubike* roote. And. 3375. hath
15. for his roote. And thei bothe make. 41, whiche is
the *Cubike* roote to. 74088. according to the woork
of the seconde example.

Addition of
zenzike roots, in these numbers, wee will goe in hande with *zenzike*
zike roots, and their additiō: wherein is no ditte-
rence of woork, but onely for the multiplicatiō, whiche
must be agreable to the nature of the numbers, *zen-*
zike. And the reduction by the common diu-
sor,

of Surde numbers.

For, in like forme, into *zenzizenzike* numbers, whē the firste numbers bee *commensurable*. But if they be *incommensurable*. then must the addition be wrought by the signe. —, without any other businesse.

Examples of *zenzizenzikes* being *commensurable*.

| | |
|---|--|
| $\begin{array}{r} \sqrt{.648} \text{---} \sqrt{.5000} \\ 81 \qquad \qquad 625 \\ 8) \quad 3 \qquad \qquad 55 \\ \qquad \qquad \qquad 8 \\ \qquad \qquad \qquad \underline{8} \\ \qquad \qquad \qquad 4096 \\ \qquad \qquad \qquad \underline{8} \\ \sqrt{.32768} \end{array}$ | $\begin{array}{r} \sqrt{.1280} \text{---} \sqrt{.6480} \\ 256 \qquad \qquad 1296 \\ 5) \quad 4 \qquad \qquad 6 \\ \qquad \qquad \qquad 10 \\ \qquad \qquad \qquad \underline{10} \\ \qquad \qquad \qquad 10000 \\ \qquad \qquad \qquad \underline{5} \\ \sqrt{.50000} \end{array}$ |
|---|--|

$$\begin{array}{r} \sqrt{.38416} \text{---} | \sqrt{.65536} \\ 2401 \qquad \qquad 4096 \\ 16) \quad 7 \qquad \qquad 8 \\ \qquad \qquad \qquad 15 \\ \qquad \qquad \qquad \underline{15} \\ \qquad \qquad \qquad 50625 \\ \qquad \qquad \qquad \underline{16} \\ \qquad \qquad \qquad 303750 \\ \qquad \qquad \qquad \underline{50625} \\ \sqrt{.810000} \end{array}$$

In the firste and seconde examples the numbers are *Surdes*, but in the thirde example they are ratiouall numbers, framed like vnto *Surdes* to the intente that you mighte the better perceiue the forme of the worke. For 38416. is a *zenzizenzike* number, & hath. 14. for his roote

So. 65536. is a *zenzizenzike* number, and hath. 16. for his roote. And these. 2. rootes do make. 30. whiche is the *zenzizenzike* roote vnto. 810000. And therefore maie it bee truly saied, that. $\sqrt{.810000}$ doeth containe the twoo firste rootes.

Scholar. I praye you proceede to Subtraction. For all this I doe well perceiue.

The Arte Of Subtraction. Daster.



Subtraction doeth differ from addition, in little moare then the signe ———. whiche signe serueth generally, for all numbers incommensurable. And considering there is little difficultie in Subtraction: If you remember well the arte of Addition, I wil lightly passe it ouer in the same examples, that I haue wrought in Addition, bicause it maie bee a proofoe of that woork: and that woork also a confirmation of this.

Onely this shall you obserue in this rule peculiarly: that as in the seconde forme of Addition, you must adde the rootes together, befoze you multiplie them. So here you shall Subtracte the lesser roote, from the greater, befoze you doe multiplie them.

Example of Subtraction, with ———.

$\sqrt{12}$. abated out of $\sqrt{26}$. maketh $\sqrt{26}$ ——— $\sqrt{12}$. and so of other.

Examples of the seconde forme of Subtraction

| | | |
|---|---|--|
| $\sqrt{63}$. ——— $\sqrt{28}$. <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: right; padding-right: 10px;">63</div> <div style="text-align: right; padding-right: 10px;">28</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: right; padding-right: 10px;">504</div> <div style="text-align: right; padding-right: 10px;">126</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: right; padding-right: 10px;">$\sqrt{1764}$</div> <div style="text-align: right; padding-right: 10px;">$\sqrt{4}$</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: right; padding-right: 10px;">$\sqrt{7056}$</div> <div style="text-align: right; padding-right: 10px;">$\sqrt{91}$ ——— $\sqrt{7056}$</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: right; padding-right: 10px;">D^r. $\sqrt{91}$ ——— 84.</div> | <div style="padding: 5px 0;">63</div> <div style="padding: 5px 0;">28</div> <div style="padding: 5px 0;">91</div> | <p style="text-align: center;">The seconde forme of that woork.</p> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center; padding: 5px 0;">$\sqrt{63}$. ——— $\sqrt{28}$.</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center; padding: 5px 0;">63</div> <div style="text-align: center; padding: 5px 0;">28</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center; padding: 5px 0;">1764</div> <p style="text-align: center;">whose roote is. 42.</p> <div style="text-align: center; padding: 5px 0;">42</div> <div style="text-align: center; padding: 5px 0;">2</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center; padding: 5px 0;">84</div> <hr style="border: none; border-top: 1px solid black; margin: 5px 0;"/> <div style="text-align: center; padding: 5px 0;">$\sqrt{91}$ ——— 84.</div> |
|---|---|--|

That is. $\sqrt{7}$.

$\sqrt{169}$

of Surde numbers.

| | | |
|---|---|--|
| $\begin{array}{r} \sqrt{.169} \text{ --- } \sqrt{.36} \\ \hline 169 \\ \quad 36 \\ \hline 1014 \\ \quad 507 \\ \hline \sqrt{.} \quad 6084 \\ \sqrt{.} \quad \quad 4 \\ \hline 24336 \\ \sqrt{.205} \text{ --- } \sqrt{.24336}. \\ 02.\sqrt{.205} \text{ --- } \sqrt{.156}. \end{array}$ | $\begin{array}{r} 169 \\ \hline 36 \\ \hline 205 \end{array}$ | <p style="text-align: center;">An other forme of that woorkie.</p> $\begin{array}{r} \sqrt{.169} \text{ --- } \sqrt{.36}. \\ \hline 169 \\ \quad 36 \\ \hline \sqrt{.} \quad 6084 \\ \text{whose roote is.} 78. \\ \quad 78 \\ \quad \quad 2 \\ \hline 156 \\ \sqrt{.205} \text{ --- } 156. \end{array}$ |
|---|---|--|

That is. $\sqrt{.49}$.

Scholar. I see in all these examples, you take the same numbers, that you had before in Addition. And firste you set the totalle, out of whiche you abate one of the nōbers, that before were added, & the remainder bringeth forthe the other. For in the firste of these. 2. examples. $\sqrt{.28}$. is abated out of. $\sqrt{.63}$. and there remaineth. $\sqrt{.91}$. --- 84. that is. $\sqrt{.7}$. for. 84. taken out of. 91. leaueth. 7. And in the seconde exāple. $\sqrt{.39}$ abated out of. $\sqrt{.169}$. doeth leaue remainyng. $\sqrt{.49}$.

Master. The thirde forme of Subtraction, is like the thirde forme of Addition: saue that we set ---. for +. And here wee muste abate the lesser roote fro the greater (as I said) before we doe multiplie that number by it self. As by this exāple, you may perceiue Where I dooe Subtrate. $\sqrt{.105}$. out. $\sqrt{.1014}$. and the remainder is. $\sqrt{.384}$. Now marke the woorkie

| | |
|--|--|
| $\begin{array}{r} \sqrt{.1014} \text{ --- } \sqrt{.105}. \\ \hline 169 \qquad 25. \\ 6) \quad 13 \qquad \quad 5. \\ \qquad \quad 8 \\ \qquad \quad 8 \\ \hline \qquad \quad 64 \\ \qquad \quad 6 \\ \hline \sqrt{.} \quad 384 \end{array}$ | <p>Here you see all thinges agree, with the forme of Addition, saue ---. for +. and when I begin to gather the number, that standeth in the middle, whiche I multiplie by it selfe, and I dooe not make that number,</p> |
|--|--|

The Arte

number, by addyng bothe rootes together: for so. 13. and. 5. would make. 18, but I adate. 5. out of 13. and so there doeth remain. 8. with whiche I procede as I did in Addition. And then cometh forth the remainer.

$\sqrt{384}$.

Scholar. I vnderstande it very well. And I praye you that for a prooffe, I maie varie the other examles of addition. Partly for my exercise, and partly for examination of the former additions, by the contrary kind.

Master. With good will.

Scholar. When will I set them, and worke them, as here foloweth.

But firste I will begin, with the worke of this last example, after the seconde forme of Subtraction: for a double confirmation of it.

| | |
|---|--|
| $\begin{array}{r} \sqrt{1014} \text{ — } \sqrt{150} \\ \hline \begin{array}{r} 1104 \\ 150 \\ \hline 50700 \\ 1014 \\ \hline \end{array} \\ \sqrt{152100} \\ \sqrt{4} \\ \hline \sqrt{608400} \\ \sqrt{1164} \text{ — } \sqrt{608400} \\ \text{Or } \sqrt{1164} \text{ — } 780 \end{array}$ | <p style="text-align: center;">☐ An other forme of thesame worke.</p> $\begin{array}{r} \sqrt{1014} \text{ — } \sqrt{150} \\ \hline \begin{array}{r} 1014 \\ 150 \\ \hline 50700 \\ 1014 \\ \hline \end{array} \\ 152100 \\ \text{whose roote is } 390. \\ 390: \\ 2 \\ \hline \sqrt{1164} \text{ — } 780. \end{array}$ <p style="text-align: center;">That is. $\sqrt{384}$.</p> |
|---|--|

And now here are the variations of the other examles.

$\sqrt{2592}$.

Of Surde numbers.

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.288.} \\
 72) \begin{array}{r} 36 \\ 6 \end{array} \quad \begin{array}{r} 4. \\ 2. \end{array} \\
 \quad \quad \begin{array}{r} 4 \\ 4 \\ \hline 16 \\ 72 \\ \hline 32 \\ 112 \\ \hline \end{array} \\
 \sqrt{.} \quad 1152
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.288.} \\
 32) \begin{array}{r} 81 \\ 9 \end{array} \quad \begin{array}{r} 9. \\ 3. \end{array} \\
 \quad \quad \begin{array}{r} 6 \\ 6 \\ \hline 36 \\ 32 \\ \hline 72 \\ 108 \\ \hline \end{array} \\
 \sqrt{.} \quad 1152
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.288.} \\
 18) \begin{array}{r} 144 \\ 12 \end{array} \quad \begin{array}{r} 16. \\ 4. \end{array} \\
 \quad \quad \begin{array}{r} 8 \\ 8 \\ \hline 64 \\ 18 \\ \hline 512 \\ 64 \\ \hline \end{array} \\
 \sqrt{.} \quad 1152
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.288.} \\
 8) \begin{array}{r} 324 \\ 18 \end{array} \quad \begin{array}{r} 36. \\ 6. \end{array} \\
 \quad \quad \begin{array}{r} 12 \\ 12 \\ \hline 144 \\ 8 \\ \hline \end{array} \\
 \sqrt{.} \quad 1152
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.288.} \\
 2) \begin{array}{r} 1296 \\ 36 \end{array} \quad \begin{array}{r} 144. \\ 12. \end{array} \\
 \quad \quad \begin{array}{r} 24 \\ 24 \\ \hline 576 \\ 2 \\ \hline \end{array} \\
 \sqrt{.} \quad 1152
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.2592} \text{ --- } \sqrt{.1152.} \\
 2) \begin{array}{r} 1296 \\ 36 \end{array} \quad \begin{array}{r} 576. \\ 24. \end{array} \\
 \quad \quad \begin{array}{r} 12 \\ 12 \\ \hline 144 \\ 2 \\ \hline \end{array} \\
 \sqrt{.} \quad 288
 \end{array}$$

Other examples varied, for proofs of the like. 6. examples in Addition.

The Arte

| | |
|--|--|
| $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 32400 \quad 3600 \\ 2) \quad 180 \quad 60 \\ \quad \quad 120 \\ \quad \quad 120 \\ \hline \quad \quad 14400 \\ \quad \quad \quad 2 \\ \hline \sqrt{.28800} \end{array}$ | $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 8100 \quad 900 \\ 8) \quad 90 \quad 30 \\ \quad \quad 60 \\ \quad \quad 60 \\ \hline \quad \quad 3600 \\ \quad \quad \quad 8 \\ \hline \sqrt{.28800} \end{array}$ |
|--|--|

| | |
|--|---|
| $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 3600 \quad 400 \\ 18) \quad 60 \quad 20 \\ \quad \quad 40 \\ \quad \quad 40 \\ \hline \quad \quad 1600 \\ \quad \quad 18 \\ \hline \quad \quad 12800 \\ \quad \quad 16 \\ \hline \sqrt{.28800} \end{array}$ | $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 2025 \quad 225 \\ 32) \quad 45 \quad 15 \\ \quad \quad 30 \\ \quad \quad 30 \\ \hline \quad \quad 900 \\ \quad \quad 32 \\ \hline \sqrt{.28800} \end{array}$ |
|--|---|

| | |
|---|--|
| $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 1296 \quad 144 \\ 50) \quad 36 \quad 12 \\ \quad \quad 24 \\ \quad \quad 24 \\ \hline \quad \quad 576 \\ \quad \quad \quad 50 \\ \hline \sqrt{.28800} \end{array}$ | $\begin{array}{r} \sqrt{.64800} \text{ --- } \sqrt{.7200} \\ 900 \quad 100 \\ 72) \quad 30 \quad 10 \\ \quad \quad 20 \\ \quad \quad 20 \\ \hline \quad \quad 400 \\ \quad \quad \quad 72 \\ \hline \sqrt{.28800} \end{array}$ |
|---|--|

*Subtraction
of Cubike
rootes.*

Master. Like difference is there in subtraction of Cubike rootes commensurable. And therfore I set the examples onely, without any larger declaration.

of Surde numbers.

| | |
|---|---|
| $\begin{array}{r} \overline{w/.375} \quad \overline{w/.81.} \\ 125 \quad 27. \\ 3) \quad 5 \quad 3 \\ \quad \quad 2 \\ \quad \quad 2 \\ \quad \quad \hline \quad \quad 8 \\ \quad \quad 3 \\ \quad \quad \hline w/. \quad 24 \end{array}$ | $\begin{array}{r} \overline{w/.1715} \quad \overline{w/.135.} \\ 343 \quad 27. \\ 5(\quad 7 \quad 3 \\ \quad \quad 4 \\ \quad \quad 4 \\ \quad \quad \hline \quad \quad 64 \\ \quad \quad 5 \\ \quad \quad \hline w/. \quad 320 \end{array}$ |
|---|---|

$$\begin{array}{r} \overline{w/.13824} \quad \overline{w/.1000} \\ 1728 \quad 125 \\ 8) \quad 12 \quad 5 \\ \quad \quad 7 \\ \quad \quad 7 \\ \quad \quad \hline \quad \quad 343 \\ \quad \quad 8 \\ \quad \quad \hline w/. \quad 2744 \end{array}$$

In the seconde forme of *Surde Cubes*, you remember that you added 4 numbers together. But in *Subtraction*, you shall adde to eche roote seuerallie that, that commeth of his owne multiplication, with the other *triple*. And

An other woork of Subtraction for Surde Cubes.

then shall you *Subtracte* the lesser number, out of the greater. And the remainder you shall multiply by the common diuisor. And so shall you haue the roote that remaineth of the *Subtraction*. As in example,

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|---|---|
| $\begin{array}{r} \overline{w/.1296} \quad \overline{w/.48.} \\ 216 \quad 8 \\ 6) \quad 6 \quad 2 \\ \quad 36 \quad 4 \\ \quad 108 \quad 12 \\ \quad 72 \quad 216 \\ \quad \quad 64 \\ \quad \quad 6 \\ \quad \quad \hline w/. \quad 384 \end{array}$ | $\begin{array}{r} \overline{w/58956} \quad \overline{w/15972} \\ 4913 \quad 1331 \\ 12) \quad 17 \quad 11 \\ \quad 289 \quad 121 \\ \quad 867 \quad 363 \\ \quad 6171 \quad 5537 \\ \quad \quad 216 \\ \quad \quad 12 \\ \quad \quad \hline w/. \quad 2592 \end{array}$ |
|---|---|

The Arte

| | |
|--|---|
| $\begin{array}{r} \text{w}/.294912 \\ \hline 32768 \\ 9) \quad \quad 32 \\ \quad \quad 1024 \\ \quad \quad 3072 \\ \hline 18816 \end{array}$ | $\begin{array}{r} \text{w}/.24696 \\ \hline 2744 \\ \quad \quad 14 \\ \quad \quad 196 \\ \quad \quad 588 \\ \hline 43008 \\ 5832 \\ \quad \quad 9 \\ \hline \text{w}/. \quad 52488 \end{array}$ |
|--|---|

Scholar. In all these examples I see the confirmation of the former additio. And in these laste woorkes, this I see peccatare from additio, that the Cube is added with the loweste number in

that rowe (as in the firste example. 216. is added with 72. and maketh. 288: And. 8. is added with. 216. that yeldeth. 224.) And then is the lesser abated from the greater (as. 224. from 288.) And the remainder (whiche there is. 64) set in the middle vnder bothe the rewes of numbers. And then is multiplied by the common number, to make the remainder.

So in the firste example, the remainder is. w/. 384. where. w/. 48. is abated out of. w/. 1296. And in the seconde example where. w/. 15972. is subtracted out of. w/. 58956. the remainder is w/. 2592. Like wates in the thirde example. w/. 24696. is abated out of. w/. 294912 & leaueth remainyng. w/. 52488

Master. But now in addition there foloweth. 2. other examples, whiche by subtraction maie bee produced thus: as here you see.

| | |
|--|---|
| $\begin{array}{r} \text{w}/21952 \\ \hline 2744 \\ 8) \quad \quad 14 \\ \quad \quad 196 \\ \quad \quad 588 \\ \hline 2688 \\ \quad \quad \quad 216 \\ \quad \quad \quad \quad 8 \\ \hline \text{w}/. \quad 1728 \end{array}$ | $\begin{array}{r} \text{w}/4096 \\ \hline 512 \\ \quad \quad 8 \\ \quad \quad 64 \\ \quad \quad 192 \\ \hline 4704 \\ \quad \quad 216 \\ \quad \quad \quad 8 \\ \hline \text{w}/. \quad 1728 \end{array}$ |
|--|---|

| | |
|--|---|
| $\begin{array}{r} \text{w}/74088 \\ \hline 2744 \\ 27) \quad 14 \\ \quad \quad 196 \\ \quad \quad 588 \\ \hline 3402 \\ \quad \quad \quad 125 \\ \quad \quad \quad \quad 27 \\ \hline \text{w}/. \quad 3375 \end{array}$ | $\begin{array}{r} \text{w}/19683 \\ \hline 729 \\ \quad \quad 9 \\ \quad \quad 81 \\ \quad \quad 243 \\ \hline 5292 \\ \quad \quad 125 \\ \quad \quad \quad 27 \\ \hline \text{w}/. \quad 3375 \end{array}$ |
|--|---|

Scholar.

of Surde numbers.

Scholar. I see, in these examples of Subtraction: that the firste number is the totale, or laste number in addition. And the seconde number, whiche foloweth ——. is the number to be abated: and then laste and loweste of all, is the remainer, whiche was one of the firste sommes in addition.

And though there remaine. 3. other exaples of *zen- rizen zike* numbers, I see no difficultie in them, but that I can wooske them: As here I haue set the fozth.

| | | |
|---|---|--|
| $\begin{array}{r} \sqrt{32768} \\ 4096 \\ 8) \quad 8 \\ \quad \quad 5 \\ \quad \quad \quad 5 \\ \quad \quad \quad \quad 625 \\ \quad \quad \quad \quad \quad 8 \\ \hline \sqrt{5000} \end{array}$ | $\begin{array}{r} \sqrt{648} \\ 81 \\ 3) \quad 10 \\ \quad \quad 6 \\ \quad \quad \quad 6 \\ \quad \quad \quad \quad 1296 \\ \quad \quad \quad \quad \quad 5 \\ \hline \sqrt{6480} \end{array}$ | $\begin{array}{r} \sqrt{50000} \\ 10000 \\ 5) \quad 10 \\ \quad \quad 6 \\ \quad \quad \quad 6 \\ \quad \quad \quad \quad 1296 \\ \quad \quad \quad \quad \quad 5 \\ \hline \sqrt{6480} \end{array}$ |
|---|---|--|

$$\begin{array}{r} \sqrt{810000} \\ 50625 \\ 16) \quad 15 \\ \quad \quad 7 \\ \quad \quad \quad 7 \\ \quad \quad \quad \quad 2401 \\ \quad \quad \quad \quad \quad 16 \\ \hline \sqrt{38416} \end{array}$$

Master. Seeing you are experte inough in the 5. wooskes of these *Surdes* vncōpounde, I wil teache you the like wooskes in cōpounde *Surdes*.

Scholar. Is there the *Of reduction* noe reduction, nother *and extracti-* traction of rootes, to bee *on of rootes.*

taughte in these vncōpounde *Surdes*?

Master. As foz reduction, I haue taughte you all readie in multiplication, as moche as is required in these numbers.

And foz extraction of rootes, you maie sone vnder- stande, that here can be none. For then were they not *Surde* numbers. And therfoze I saied vnto you befoze,

Pp. iij. that

The Arte

that $\sqrt{8}$. $\sqrt{100}$. is not a *Surde* number, although it be w^ritten like a *Surde* number, bicause it hath a *Square* roote, acco^rdyng to his signe: and that is. $\sqrt{10}$. Likewa^yes $\sqrt{256}$. is no *Surde* number: fo^r his *Square* roote is knowen to be. 16.

Scholar. I might haue cōsidered as moche, by the definition of *Surde* numbers, that their rootes can not be assigned in numbers absolute. And therfo^re I see that. $\sqrt[3]{125}$. is no^t *Surde* nōber, sith his *Cubike* roote is. 5. And. $\sqrt[3]{256}$. is a number *rationalle*, and no *Surde* number: fo^r his *zenzizenzike* roote is. 4.

Master. But. $\sqrt[3]{64}$. is a *Surde* number, and yet hath. 64. a *Square* roote, and a *Cubike* roote also, but not a *zenzizenzike* roote, acco^rdyng to his signe. And therfo^re ought better to be w^ritten thus. $\sqrt{8}$.

Scholar. I p^raise you to p^rocede to *Surde* numbers compoⁿde.

Of *Surde* numbers compoⁿde.

Master.



Urde numbers compoⁿde, are made not onely of. 2. 02. 3. 02 moare *Surde* numbers trⁱ compoⁿde, but also of *rationalle* 02 *Abstra^tte* numbers ioyned with *Surde* numbers. As. $\sqrt{10}$
 $\sqrt{12}$. and. 8. — $\sqrt{6}$. like
 waies. $\sqrt{20}$. — 3. and. $\sqrt{40}$.

— $\sqrt{14}$. — .3.

Compoⁿde
Surdes.

But here shall you marke, that I call compoⁿde numbers, not onely soche as haue the signe of. — + —, but also soche as haue the signe of — — fo^r although in nature of the number $\sqrt{10}$ — $\sqrt{5}$. be not compoⁿde, but abated, yet in name he is compoⁿde, and augmented. Fo^r. — —. doeth as well augemented the

of Surde numbers.

the name, as — + — doeth.

Scholar. It seemeth reasonable. For when I saie, $\sqrt{.12.} \text{---} \sqrt{.7.}$ the name is compounde, an well as if I had saied. $\sqrt{.12.} \text{---} + \sqrt{.7.}$ although the quantitie bee not so greate. For — — doeth ener abate the quantitie of the nūber, though it do increase the name.

Maister. Yet for a difference, the numbers that be compounde with — + — be called *Bimedialles*: and those *Bimedialles*. that be compounde with — —, be named *Residualles*. *Residualles*. And if the *Bimedialles* haue all their numbers and partes of one denominations, then bee they called onely by their generalle name *Bimedialles*. But if their partes be of 2. denominations, then are they named *Binomiales* properly. Whobett, many vse to call *Binomialles* *Binomialles*. all compounde numbers that haue — + —. And so will I let the names passe.

Euclides definitions doe not very aptly agree to this place, as at an other tyme I will shewe you, and therefore I doe omitte them for this tyme.

But touchyng our principalle intent, whiche is to declare the practike woork of *Binomialles*, and *Residualles*, there is litle difficultie, if you marke well that whiche is taught before. For as *Binomialles* and *Residualles*, bee made of *Surdes*, or els of *rationalle* numbers with *Surdes*, so the woork of the compounde numbers dependeth of the woork of the simple numbers, and is all one with them. And concernyng the signes — + — and. — —. here is no moare to bee saied, then was taughte in *Cosike* numbers compounde.

Scholar. Yet of every kinde, it maie please you to set for the some examles.

Maister. I thinke that mete, without many woordes els. Not forgettyng by the waie, that vniuersalle rootes, are not accompted emongeste these compounde *Surdes*: but are reserued to their peculiare treatice, as rootes of compounde *Surdes*.

The Arte Of Numeration.

Numeration is moare plain, then that I neede to stande in declaring it, other waies then by examples; As here you see.

Examples of Binomialles.

6. — + √.8. That is 6 moze the Square roote of 8.
 √.20 — + .3. Is the Square roote of 20. moare .3.
 √.30 — + √.9. Signifieth the Cubike roote of .30.
 moze the zenzizenzike roote of .9.
 And so of other.

Examples of Residualles.

24. — — √ 96. That is 24. abating the roote of 96
 √.150. — — .9. Is the Square roote of 150. abating 9
 √5208 — — √35. The zenzizenzike roote of. 5208.
 saue the Square roote of. 35. And so
 foz the.

Scholar. So I see any Surdes maie bee compounde
 with other: And any nōbers *rationalle* ioined with the.

Of Addition.

Master. Addition is as plaine. For as the partes
 bee, so shall the Addition bee, accozdyng as you haue
 learned before.

Examples of Binomialles.

| | | |
|---|---|--|
| $\begin{array}{r} \sqrt{.50} \text{ — + } 10 \\ \sqrt{.2} \text{ — + } .8 \\ \hline \sqrt{.72} \text{ — + } 18 \end{array}$ | $\begin{array}{r} 15 \text{ — + } \sqrt{.15} \\ 18 \text{ — + } \sqrt{.60} \\ \hline 33 \text{ — + } \sqrt{.135} \end{array}$ | $\begin{array}{r} \sqrt{.1264} \text{ — + } 8 \\ 28 \text{ — + } \sqrt{316} \\ \hline 36 \text{ — + } \sqrt{2844} \end{array}$ |
|---|---|--|

| | | |
|--|---|--------------|
| $\begin{array}{r} \sqrt{.48} \text{ — + } \sqrt{.5} \\ \sqrt{.243} \text{ — + } \sqrt{.45} \\ \hline \sqrt{.1875} \text{ — + } \sqrt{.80} \end{array}$ | $\begin{array}{r} \sqrt{.32} \text{ — + } \sqrt{10} \\ \sqrt{.4} \text{ — + } \sqrt{19} \\ \hline 108 \text{ — + } \sqrt{29} \end{array}$ | $\sqrt{760}$ |
|--|---|--------------|

Examples

Of Surde numbers. Examples of Residualles.

$$\begin{array}{r|l} \sqrt{.75} \text{ --- } .4 & 14 \text{ --- } \sqrt{.3} \\ \sqrt{.3} \text{ --- } 1 & 16 \text{ --- } \sqrt{.27} \\ \sqrt{.108} \text{ --- } 5 & 30 \text{ --- } \sqrt{.12} \end{array} \left| \begin{array}{l} 250 \text{ --- } \sqrt{.108} \\ \sqrt{.44} \text{ --- } 76 \\ 174 \text{ --- } \sqrt{.275} \end{array} \right.$$

$$\begin{array}{r|l} \sqrt{.72} \text{ --- } \sqrt{.96} & \sqrt{.32} \text{ --- } \sqrt{.5} \\ \sqrt{.9} \text{ --- } \sqrt{.6} & \sqrt{.32} \text{ --- } \sqrt{.24} \\ \sqrt{243} \text{ --- } \sqrt{162} & \sqrt{512} \text{ --- } \sqrt{29} \text{ --- } \sqrt{480} \end{array}$$

Examples of Binomialles with Residualles.

$$\begin{array}{r|l} \sqrt{.80} \text{ --- } 6 & 30 \text{ --- } \sqrt{.20} \\ \sqrt{.5} \text{ --- } 2 & 12 \text{ --- } \sqrt{.5} \\ \sqrt{.125} \text{ --- } 4 & 42 \text{ --- } \sqrt{.5} \end{array} \left| \begin{array}{l} 561 \text{ --- } \sqrt{512} \\ \sqrt{288} \text{ --- } 340 \\ 901 \text{ --- } \sqrt{1568} \end{array} \right.$$

$$\begin{array}{r|l} \sqrt{.63} \text{ --- } \sqrt{160} & \sqrt{.320} \text{ --- } \sqrt{.56} \\ \sqrt{.7} \text{ --- } \sqrt{.20} & \sqrt{.40} \text{ --- } \sqrt{.24} \\ \sqrt{.112} \text{ --- } \sqrt{684} & \sqrt{1680} \text{ --- } \sqrt{.80} \text{ --- } \sqrt{5376} \end{array}$$

Scholar. I see that you make severalle Additions in all these numbers. For you adde still like numbers with their matches. So that here is nothyng diverse from the bookes of simple Surdes. Although in every thirde example, there appeare moare difficultie, then there is in deede: when I consider the like transposition in Cosike numbers. For the wooske addeth like numbers together.

Of Subtraction.

Master. In Subtraction there is as litle diversitie. As these examples will sufficiently declare: whiche be set as trialles of the former Additions.

293. Examples

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Examples of Binomialles.

$$\begin{array}{r} \sqrt{.72} \text{---} | \text{---} 18 \\ \sqrt{.2} \text{---} | \text{---} 8 \\ \hline \sqrt{.50} \text{---} | \text{---} 10 \end{array}$$

$$\begin{array}{r} 36 \text{---} | \text{---} \sqrt{2844} \\ \sqrt{.1264} \text{---} | \text{---} 8 \\ \hline 28 \text{---} | \text{---} \sqrt{.316} \end{array}$$

$$\begin{array}{r} 33 \text{---} | \text{---} \sqrt{.135} \\ 15 \text{---} | \text{---} \sqrt{.15} \\ \hline 18 \text{---} | \text{---} \sqrt{.60} \end{array}$$

$$\begin{array}{r} \sqrt{.1875} \text{---} | \text{---} \sqrt{.80} \\ \sqrt{.48} \text{---} | \text{---} \sqrt{.5} \\ \hline \sqrt{.243} \text{---} | \text{---} \sqrt{.45} \end{array}$$

$$\sqrt{.108} \text{---} | \text{---} \sqrt{.29} \text{---} | \text{---} \sqrt{.760}$$

$$\sqrt{.4} \text{---} | \text{---} \sqrt{.19}$$

$$\sqrt{.32} \text{---} | \text{---} \sqrt{.10}$$

Examples of Residualles.

$$\begin{array}{r} \sqrt{.108} \text{---} | \text{---} 5 \\ \sqrt{.3} \text{---} | \text{---} 1 \\ \hline \sqrt{.75} \text{---} | \text{---} 4 \end{array}$$

$$\begin{array}{r} 174 \text{---} | \text{---} \sqrt{.275} \\ \sqrt{.44} \text{---} | \text{---} 76 \\ \hline 250 \text{---} | \text{---} \sqrt{.108} \end{array}$$

$$30 \text{---} | \text{---} \sqrt{.12}$$

$$14 \text{---} | \text{---} \sqrt{.3}$$

$$16 \text{---} | \text{---} \sqrt{.27}$$

$$\sqrt{.243} \text{---} | \text{---} \sqrt{.162}$$

$$\sqrt{.9} \text{---} | \text{---} \sqrt{.6}$$

$$\sqrt{.72} \text{---} | \text{---} \sqrt{.96}$$

$$\sqrt{.512} \text{---} | \text{---} \sqrt{.29} \text{---} | \text{---} \sqrt{.480}$$

$$\sqrt{.32} \text{---} | \text{---} \sqrt{.5}$$

$$\sqrt{.32} \text{---} | \text{---} \sqrt{.24}$$

Examples of bothe together.

$$\begin{array}{r} \sqrt{.125} \text{---} | \text{---} 4 \\ \sqrt{.5} \text{---} | \text{---} 2 \\ \hline \sqrt{.80} \text{---} | \text{---} 6 \end{array}$$

$$\begin{array}{r} 901 \text{---} | \text{---} \sqrt{1568} \\ \sqrt{.288} \text{---} | \text{---} 340 \\ \hline 561 \text{---} | \text{---} \sqrt{.512} \end{array}$$

of *Surde* numbers.

| | |
|---|---|
| $42 \text{---} + \text{---} \sqrt{.5}.$ $12 \text{---} \text{---} \sqrt{.5}.$ <hr style="width: 100%;"/> $30 \text{---} + \text{---} \sqrt{.20}.$ | $\sqrt{.112} \text{---} \text{---} \sqrt{.648}.$ $\sqrt{.7} \text{---} + \text{---} \sqrt{.20}.$ <hr style="width: 100%;"/> $\sqrt{.65} \text{---} \text{---} \sqrt{.160}.$ |
| $\sqrt{.1080} \text{---} \text{---} \sqrt{.80} \text{---} \text{---} \sqrt{.5376}.$ $\sqrt{.40} \text{---} + \text{---} \sqrt{.24}.$ | |
| $\sqrt{.320} \text{---} \text{---} \sqrt{.56}.$ | |

Scholar. This is as easie as Addition, saue for 3. exam- ples, whiche I vnderstande not. For although I see the laste exam- ple, of eche of the sortes of num- bers, to bee agreable with the like exam- ples in Addi- tion, yet I can not so well perceiue, the order of their Subtraction, as I doe knowe the maner of their Ad- ditiõ. For by the arte of simple *Surdes*, I see that $\sqrt{.10}$ and $\sqrt{.19}$ doe make $\sqrt{.29} \text{---} + \text{---} \sqrt{.760}$. But when $\sqrt{.29} \text{---} + \text{---} \sqrt{.760}$ is set as a totalle, and $\sqrt{.19}$ to be Subtracted out of it, how I shall woork that, and leaue $\sqrt{.10}$ for the remainer, I see not.

So in the *residualles*, I knowe how $\sqrt{.5}$ and $\sqrt{.24}$ doe make $\sqrt{.29} \text{---} + \text{---} \sqrt{.480}$. But I knowe not how $\sqrt{.5}$ abated out of $\sqrt{.29} \text{---} + \text{---} \sqrt{.480}$ doeth make for the remainer $\sqrt{.24}$.

And the like doubt is in the thirde sorte of *Surdes*, whiche are mixte numbers. For where I see in Addi- tion $\text{---} + \text{---} \sqrt{.24}$ added with $\text{---} \text{---} \sqrt{.56}$. And the totalle to bee $\text{---} \text{---} \sqrt{.80} \text{---} \text{---} \sqrt{.5376}$. I knowe the reason of the woork, for the signes $\text{---} + \text{---}$ and $\text{---} \text{---}$ by that I learned in *Cosike* numbers: And the reaste is manifeste by Addition of simple *Surdes*. For it is wrought by abatynge $\sqrt{.24}$ out of $\sqrt{.56}$. But then in Subtraction, how $\text{---} + \text{---} \sqrt{.24}$ being Subtracted from $\text{---} \text{---} \sqrt{.80} \text{---} \text{---} \sqrt{.5376}$ shall leaue $\text{---} \sqrt{.56}$ I can not iudice. And yet by the signes I gesse (as I learned in *Cosike* numbers) that it is doen by Additiõ, because the signes doe disagree.

Aq. 11. Master.

The Arte

Maſter. In that you remember the former rules, to conferre them aptly with theſe later woꝝkes, I can praiſe you well. But in that you can not vnderſtande the reaſon of that, whiche was not yet taughte you, I can not greatly blame you. Although I can not praiſe you, foꝛ that you thinke your ſelf to be cunnynge then you are. Foꝛ in thoſe Additions, that you thinke your ſelf to be experte inough, I'dare ſaie, that you bee diſciued, if you take them to bee numbers of any ſoche, as hetherto hath been taughte vnto you.

Scholar. I take them foꝛ compounde *Surdes*.

Maſter. Thei are not ſo: Nother is their woꝝke agreable, with the woꝝke of compounde *Surdes*. But thei are the rootes of compounde *Surdes*: And therfoꝛe are called *vniverſalle rootes* of *Surdes*. And accoꝛdyng to their pꝛoper nature, thei ought to bee called rootes of *Surdes*, and not *Surde* rootes. As I will tell you anon. When I will alſo diſcuſſe your doubt.

But befoꝛe I ſpeake any moare of theim, I will cande the woꝝkes of theſe compounde *Surdes*: Where of. 2. kindes yet remaine behinde.

Of Multiplication.



Multiplication of compounde *Surdes*, is as eaſie as can bee. And differeth in nothyng, frõ the woꝝke of ſimple *Surdes*. Onely this muſt you marke, as reaſon would, that you muſte multiplie euery parte of the one number, by euery parte of the other number: as you remember the woꝝke of compounde *Cofike* numbers.

Scholar. I praye you giue me ſome examles.

Maſter. That ſhall you haue. And that maie ſuffice foꝛ this woꝝke. Marke them well therfoꝛe.

Examles

of Surde numbers.

Examples of Binomialles.

$$\begin{array}{r}
 23 \text{ --- } \sqrt{.15.} \\
 6 \text{ --- } \sqrt{.8.} \\
 \hline
 138 \text{ --- } \sqrt{.120.} \\
 \text{--- } \sqrt{.540.} \text{ --- } \sqrt{.4232.} \\
 \hline
 138 \text{ --- } \sqrt{.4232} \text{ --- } \sqrt{.540} \text{ --- } \sqrt{.120.}
 \end{array}$$

$$\begin{array}{r}
 \sqrt{.120} \text{ --- } \sqrt{.12.} \\
 \sqrt{.12} \text{ --- } \sqrt{.7.} \\
 \hline
 \sqrt{1440} \text{ --- } \sqrt{.84.} \\
 \text{--- } \sqrt{.840} \text{ --- } 12. \\
 \hline
 12 \text{ --- } \sqrt{.1440} \text{ --- } \sqrt{.840} \text{ --- } \sqrt{.84.}
 \end{array}$$

Examples of Residualles.

$$\begin{array}{r}
 5. \text{ --- } \sqrt{.10.} \\
 5. \text{ --- } \sqrt{.10.} \\
 \hline
 25 \text{ --- } 10. \\
 \text{--- } \sqrt{.250} \text{ --- } 250. \\
 \hline
 35 \text{ --- } \sqrt{.1000.} \\
 \\
 \sqrt{.24} \text{ --- } \sqrt{.20.} \\
 \sqrt{.30} \text{ --- } \sqrt{.24.} \\
 \hline
 \sqrt{.720} \text{ --- } \sqrt{.480.} \\
 \text{--- } 24 \text{ --- } \sqrt{.600.} \\
 \hline
 \sqrt{.720} \text{ --- } \sqrt{.480} \text{ --- } 24 \text{ --- } \sqrt{.600.}
 \end{array}$$

Examples of bothe together.

$$\begin{array}{r}
 32 \text{ --- } \sqrt{.14.} \\
 \sqrt{.124} \text{ --- } 6. \\
 \hline
 \sqrt{.126976} \text{ --- } \sqrt{.1736.} \\
 \text{--- } .192. \text{ --- } \sqrt{.504.} \\
 \hline
 \sqrt{.126976} \text{ --- } \sqrt{.1736} \text{ --- } 192 \text{ --- } \sqrt{.504} \\
 \text{--- } \text{Rq. lll.} \quad \sqrt{.52.}
 \end{array}$$

The Arte

$$\begin{array}{r} \sqrt{}. \quad 52 \text{ --- } + \text{ --- } 17. \\ \quad \quad 17 \text{ --- } \text{ --- } \sqrt{.52.} \\ \hline \sqrt{.15028} \text{ --- } + \text{ --- } 289. \\ \quad \quad \quad \quad \quad 52 \text{ --- } \text{ --- } \sqrt{.15028.} \\ \hline 37. \end{array}$$

Scholar. Multiplication, as I see, is the easieſte woork of all the other. So that I dooc marke the reduction, in gatherynge the totalle: whiche is easie enough to vnderſtand, by that I haue learned in *Cosike* numbers. And Diuision be no harder, it maie ſone be learned.

Of Diuision.

Maſter.



Diuision by one ſimple number, is no moare difficulte: as theſe exam-
ples doe declare. Where the diuiſor
is a number vncompoude.

$\sqrt{.26} \text{ --- } + \text{ --- } 15$ diuided by .5. doeth
make $\sqrt{.1 \frac{1}{5}}$ --- + --- 3.

Again. $\sqrt{.56} \text{ --- } + \text{ --- } \sqrt{.24}$. diu-
ded by $\sqrt{.6}$. doeth yelde. $\sqrt{.9 \frac{1}{3}}$ --- + --- 2.

And ſo $\sqrt{.75} \text{ --- } + \text{ --- } \sqrt{.48}$. diuided by $\sqrt{.3}$. doeth
byng ſothe. 5. --- 4. that is. 1.

Like waies. $\sqrt{.3200}$. --- + --- $\sqrt{.1800}$. beynge parted by
 $\sqrt{.5}$. doeth make the *quotiente*. 14.

Scholar. I ſee it ſo. For at the firſt it is. $\sqrt{.64}$.
--- + --- $\sqrt{.36}$. that is. 8. --- + --- 6. whiche maketh. 14.

Maſter. So maie you worke all like diuisions.
But when the diuiſor is a compoude number, then
muſt you vſe an other meane: that is to reduce that
compoude nuber, to a ſimple number: whiche thing
you maie eaſily doe, by multiplyng any *Binomiale*, by
his *Reſiduale*, or contrary waies, the *Reſiduale* by his
Binomiale.

of Surde numbers.

As $6 \div \sqrt{10}$ multiplied by $6 \div \sqrt{10}$ doeth
make 26.

And so $\sqrt{8} \div \sqrt{5}$ multiplied by $\sqrt{8} \div \sqrt{5}$
doeth yelde 8. \div 5. that is 3.

Scholar. I perceiue a brief waie in this multipli-
cation: For I meade not in the firste example, to mul-
tiple 6. by $\sqrt{10}$. sith it would amounte to nothyng.
In so moche as at one multiplication, it would bee
 \div , and at an other. \div . And so the one would
abate the other, and leaue nothyng for them bothe.

Maister. That is well marked. And it is so gene-
rally. Wherefore (as you see) the diuisor by this mea-
nes, maie lightly be tourned into a simple number, or
a plaine absolute number.

And now to make the diuidende, in the same propo-
rtion, to this newe diuisor, that it was vnto the old di-
uisor, you shall multiplie it by the same number, by
whiche the diuisor was multiplied. For if any num-
bers bee multiplied, by one common number, their
newe totalles kepe the same proportion, that was be-
twene the firste numbers.

Scholar. That must needs be so. For as 3. is *ses-
quialtera* vnto 2. so if you multiplie them by 5. thei will
make 15. and 10. whiche be in *sesquialtera* proportion
and like waies will their proportion remain, by what
so euer number thei be multiplied. Wherefore it must
needs be reasonable, that if the diuidende and the di-
uisor, be multiplied by any one number, simple or co-
pounde, thei shall kepe the same proportion, that thei
had before.

Maister. For more certain vnderstandyng of this
rule, take these examples. The firste
is, where $\sqrt{58} \div \sqrt{54}$ is sette
to bee diuided by $\sqrt{6} \div \sqrt{3}$.

Here firste I multiplie the diuisor
by his contrarie, that is his *Binomi*:

| | |
|--------------------------|-------------|
| $\sqrt{6} \div \sqrt{3}$ | $\sqrt{3}$ |
| $\sqrt{6} \div \sqrt{3}$ | $\sqrt{3}$ |
| | 6 \div 3. |
| | What is 3. |

alke

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alle. $\sqrt{6}$ ——— 3. And there riseth. 6 ——— 3. that is. 3
whiche I shall kepe for the newe diuifoz.

Then doe I multiplie the diuidede $\sqrt{68}$ ——— $\sqrt{54}$
by the same Residuale.

$$\begin{array}{r} \sqrt{68} \text{ ——— } \sqrt{54} \\ \sqrt{6} \text{ ——— } \sqrt{3} \\ \hline \sqrt{408} \text{ ——— } \sqrt{324} \\ \qquad \qquad \qquad \sqrt{204} \text{ ——— } \sqrt{162} \\ \hline \sqrt{408} \text{ ——— } \sqrt{324} \text{ ——— } \sqrt{204} \text{ ——— } \sqrt{162} \end{array}$$

And there doth amoûte, as here in woꝝke is expꝛessed.

$$\sqrt{408} \text{ ——— } \sqrt{324} \text{ ——— } \sqrt{204} \text{ ——— } \sqrt{162}$$

whiche number shall be taken for the newe diuidede:
and must be diuided by. 3. that is the newe diuifoz. For
whose steẽ I set. $\sqrt{9}$. for moare redinesse in woꝝke.
Therefore I set the doune in order, as here foloweth.

$$\begin{array}{ccccccc} \sqrt{408} \text{ ——— } \sqrt{324} \text{ ——— } \sqrt{204} \text{ ——— } \sqrt{162} & (\sqrt{45\frac{1}{3}} \text{ ——— } 6 \text{ ——— } \sqrt{22\frac{2}{3}} \text{ ——— } \sqrt{18} \\ \sqrt{9} & \sqrt{9} & \sqrt{9} & \sqrt{9} & & & \end{array}$$

And then doe I seke how often. $\sqrt{9}$. maie bee founde
in. $\sqrt{408}$. whiche maie bee. $45\frac{1}{3}$ of tymes. Where-
foze I set. $\sqrt{45\frac{1}{3}}$ in the *quotiente*. And then doe I re-
terate the diuifoz, and sette it vnder. $\sqrt{324}$. In here I
finde it. 36. tymes: and therefore set 36. for it, because
the *quotiente* els would bee. $\sqrt{36}$. whiche is iustly. 6.
Thirdly, I remoue the diuifoz vnder $\sqrt{204}$. where
it maie bee founde. $22\frac{2}{3}$ tymes. For whiche I sette
 $\sqrt{22\frac{2}{3}}$ in the *quotiente*. And then set 3 the diuifoz last
of all vnder. 162. where it is founde. 18. tymes: and
for that cause I set $\sqrt{18}$. in the *quotiente*: And so is the
whole *quotiente* $\sqrt{45\frac{1}{3}} \text{ ——— } 6 \text{ ——— } \sqrt{22\frac{2}{3}} \text{ ——— } \sqrt{18}$.

Scholar. This diuifion is straunge to credite, al-
though it be not difficulte to woꝝke.

Maister. If you doubt of it, you maie vse the ac-
cusomable trialle by the contrary kinde.

Scholar.

of Surde numbers.

Scholar. So must it folowe, that if I dooe multiplie this *quotiente* by the firste diuisor, the firste diuidentide will resulte thereof.

And for the prooffe of that, I dooe multiplie,
 $\sqrt{.45\frac{1}{3}}$ ——— $\sqrt{.6}$ ——— $\sqrt{.22\frac{2}{3}}$ ——— $\sqrt{.18}$ by
 $\sqrt{.6}$ ——— $\sqrt{.3}$. But for the moare ease, I doe tourne all the mixte numbers into onely fractions. And then doe I multiplie them orderly.

$$\begin{array}{l} \sqrt{.136\frac{2}{3}} \text{ ——— } \sqrt{.6} \text{ ——— } \sqrt{.68\frac{2}{3}} \text{ ——— } \sqrt{.18} \\ \sqrt{.6} \text{ ——— } \sqrt{.3} \\ \hline \sqrt{.816} \text{ ——— } \sqrt{.216} \text{ ——— } \sqrt{.408\frac{2}{3}} \text{ ——— } \sqrt{.108} \\ \sqrt{.408\frac{2}{3}} \text{ ——— } \sqrt{.108} \text{ ——— } \sqrt{.204\frac{2}{3}} \text{ ——— } \sqrt{.54} \\ \hline \sqrt{.272} \text{ ——— } \sqrt{.216} \text{ ——— } \sqrt{.136} \text{ ——— } \sqrt{.108} \\ \sqrt{.68} \text{ ——— } \sqrt{.54} \text{ ——— } \sqrt{.136} \text{ ——— } \sqrt{.108} \\ \hline \sqrt{.68} \text{ ——— } \sqrt{.54} \end{array}$$

First I multiplie $\sqrt{.136\frac{2}{3}}$ by $\sqrt{.6}$. and there cometh $\sqrt{.816}$ that is. $\sqrt{.272}$. Again I doe multiplie $\sqrt{.6}$ by $\sqrt{.36}$ by $\sqrt{.6}$. and it maketh $\sqrt{.216}$. Then I multiplie $\sqrt{.68\frac{2}{3}}$ by $\sqrt{.6}$. & it giueth $\sqrt{.408\frac{2}{3}}$, whiche is. $\sqrt{.136}$. Fourthly $\sqrt{.18}$. multiplied by $\sqrt{.6}$. dooth make $\sqrt{.108}$. All whiche I set doune with their conueniente signes.

After that I multiplie $\sqrt{.136\frac{2}{3}}$ by $\sqrt{.3}$. and it yeldeth $\sqrt{.408\frac{2}{3}}$ that is. $\sqrt{.136}$. whiche I sette doune with his signe ———. Then $\sqrt{.36}$ by $\sqrt{.3}$. maketh $\sqrt{.108}$. Thirdly $\sqrt{.68\frac{2}{3}}$ by $\sqrt{.3}$. dooth giue $\sqrt{.68}$. and last of all, $\sqrt{.18}$. multiplied by $\sqrt{.3}$. byugeth forthe. $\sqrt{.54}$.

When all these be placed conueniently, I doe consider that ——— $\sqrt{.136}$. and ——— $\sqrt{.136}$. maie bee bothe cancelled, because the one doeth abate the other. And like waies, ——— $\sqrt{.108}$. and ——— $\sqrt{.108}$. eche abate other: so that thei must bothe be reiecte.

When I see, that $\sqrt{.68}$. being abated out of $\sqrt{.272}$ there will remain. $\sqrt{.68}$. And in like. $\sqrt{.54}$. being abated out of $\sqrt{.216}$. dooth leaue. $\sqrt{.54}$. So that the whole multiplicatio doth make iustly $\sqrt{.68}$ ——— $\sqrt{.54}$

R. j. whiche

The Arte

whiche is the firste diuident. And so is that diuision approued good.

An other example.

Master. Yet for you exercise, you shall haue some examples moare of diuision.

$\sqrt{.456}$ ——— $\sqrt{.72}$. is sette to bee diuided by $\sqrt{.18}$ ——— $\sqrt{.6}$.

Scholar. That diuisor must I multiplie by his contrarie, whiche is the *Residuale*. $\sqrt{.18}$ ——— $\sqrt{.6}$. And so, as you maie some perceiue, there will rise. $.18$ ——— $.6$. that is 12 . whiche must be kepte for the newe diuisor.

Then shall I multiplie the former diuident, that is $\sqrt{.456}$ ——— $\sqrt{.72}$ by the same *residuale* $\sqrt{.18}$ ——— $\sqrt{.6}$

$$\begin{array}{r} \sqrt{.456} \text{ ——— } \sqrt{.72} \\ \sqrt{.18} \text{ ——— } \sqrt{.6} \end{array}$$

$$\begin{array}{r} \sqrt{.8208} \text{ ——— } \sqrt{.1296} \\ \sqrt{.432} \text{ ——— } \sqrt{.2736} \end{array}$$

$$\sqrt{.8208} \text{ ——— } \sqrt{.432} \text{ ——— } \sqrt{.2736} \text{ ——— } \sqrt{.1296}$$

And there will rise of that multiplication, as here by example appereth $\sqrt{.8208}$ ——— $\sqrt{.432}$ ——— $\sqrt{.2736}$ ——— 1296 . whiche nōber I shall diuid by. 12 . that was founde for the newe diuisor. And then will the *quotiente* bee. $\sqrt{.57}$ ——— $\sqrt{.3}$ ——— $\sqrt{.19}$ ——— $\sqrt{.9}$. As here in woorkes doeth appeare.

$$\begin{array}{r} \sqrt{.8208} \text{ ——— } \sqrt{.432} \text{ ——— } \sqrt{.2736} \text{ ——— } \sqrt{.1296} (\sqrt{.57} + \sqrt{.3} \text{ ——— } \\ \sqrt{.144} \quad \sqrt{.144} \quad \sqrt{.144} \quad \sqrt{.144} \end{array}$$

Where I haue set. $\sqrt{.144}$. for. 12 . saying that be all one: but that. $\sqrt{.144}$. is moare apte for this woorkes. And I haue repeated it as often tymes, as the diuisor should be remoued.

The prooffe.

But now to trie this woorkes, whether it bee well wroughte, I shall multiplie this *quotiente* by the firste diuident, & then ought the firste diuident to amounte.

As

Of Surde numbers.

As here in example, you see wroughte.

$$\begin{array}{r}
 \sqrt{.57.} \text{---} | \sqrt{.3.} \text{---} | \sqrt{.19.} \text{---} | \sqrt{.9.} \\
 \sqrt{.18.} \text{---} | \sqrt{.6.} \\
 \hline
 \sqrt{.1026} \text{---} | \sqrt{.54} \text{---} | \sqrt{.342} \text{---} | \sqrt{.162.} \\
 \sqrt{.342} \text{---} | \sqrt{.18} \text{---} | \sqrt{.114} \text{---} | \sqrt{.54.} \\
 \hline
 \sqrt{.1026} \text{---} | \sqrt{.18} \text{---} | \sqrt{.114} \text{---} | \sqrt{.162.}
 \end{array}$$

Here $\sqrt{.54.}$ doeth cancell $\sqrt{.54.}$ and is cancelled by it.

So $\sqrt{.342.}$ and $\sqrt{.342.}$ exclude one another, and therefore must bee bothe relected. And then remaineth onely,

$\sqrt{.1026} \text{---} | \sqrt{.18.} \text{---} | \sqrt{.114} \text{---} | \sqrt{.162.}$
 Whiche numbers I dooe well examine: and finde that $\sqrt{.114.}$ beyng abated out of $\sqrt{.1026.}$ there will remaine $\sqrt{.456.}$ Again if $\sqrt{.18.}$ be subtracted out of $\sqrt{.162.}$ there will reste $\sqrt{.72.}$ And so is that whole multiplicatio onely $\sqrt{.456} \text{---} | \sqrt{.72}$ agreeable to the firste diuidende. Wherby it is manifeste, that the former diuision was good.

Master. How can you woork this example: Here $24.$ is set to be diuided by $3.$ $\text{---} | \sqrt{.8.}$

The thirde example.

Scholar. I must still obserue the generall rule. And multiplie bothe those numbers, by the contrarie of the diuisor, that is, by the *residuale*. $3 \text{---} | \sqrt{.8.}$ And

| | | |
|--------------------------------|----------------------------|--|
| $24.$ | $3 \text{---} \sqrt{.8}$ | of the firste multiplication of it, |
| $72 \text{---} \sqrt{.4608}$ | $3 \text{---} \sqrt{.8}$ | with the diuidende. $24.$ there resteth. $72 \text{---} \sqrt{.4608}$. Of the |
| | $3 \text{---} \sqrt{.8}$ | seconde multiplicatio, where the <i>Binomiale</i> is multiplied |
| | $3 \text{---} \sqrt{.8}$ | by the <i>Residuale</i> , that is his contrary, the |
| | $9 \text{---} \sqrt{.8}$ | totalle will be. $9 \text{---} \sqrt{.8}$. that is but. $1.$ That is. $1.$ |
| | $9 \text{---} \sqrt{.8}$ | And therefore seying. $1.$ doeth nother multiplie nor diuide, the former number. |

That is. $72 \text{---} | \sqrt{.4608}$. is the *quotiente*, when $24.$ is diuided by $3.$ $\text{---} | \sqrt{.8.}$

The Arte

The prooffe.

For p^{ro}ofe whereof, I mult^lplie 72 ——— √.4608
that is the qu^oti^ente, by. 3. ——— √.8. And there riseth
216 ——— √.41472 ——— √.41472 ——— √.36864.
whereof. 2. numbers differ^yng but by ——— & ———
mu^lte bothe bee re^lected, as numbers superfluous.

$$\begin{array}{r} 72 \text{ ——— } \sqrt{.4608}. \\ 3 \text{ ——— } \sqrt{.8}. \\ \hline 216 \text{ ——— } \sqrt{.41472} \\ \sqrt{.41472} \text{ ——— } \sqrt{.36864} \\ \hline 216 \text{ ——— } .192. \end{array}$$

That is. 24.

Then. 36864. is a square
number, and hath. 192
for his roote. Wherfoze
the whole number is,
216 — 192 that is (as
it is manⁱfeste inough)
24. And so is the whole
woozke p^{ro}ued good.

*The fourth
example.*

Master. You shall haue one ex^{am}ple moare, and
then will I make an ende of diuⁱsi^on.

When √.6570. ——— √.254. is p^{ro}ounded to
bee diuⁱded by √.54 ——— √.6. I would knowe the
qu^oti^ente.

Scholar. I see the newe diuⁱsi^on will be. 54 — 6.
that is. 48.

And then for to finde a diuⁱdeⁿde conueniente, I

$$\begin{array}{r} \sqrt{.6570.} \text{ ——— } \sqrt{.254.} \\ \sqrt{.54} \text{ ——— } \sqrt{.6.} \\ \hline \sqrt{.354780.} \text{ ——— } \sqrt{.13716.} \\ \sqrt{.39420.} \text{ ——— } \sqrt{.1524.} \end{array}$$

shall mult^lplie the firste
diuⁱdeⁿde, by the contra^r
ie of the firste diuⁱsi^on,
that is by √.54 ——— √6
And there will rise, as
you see. √.354780.

——— √.13716 ——— √.39420. ——— √.1524.
That diuⁱdeⁿde mu^lt be diuⁱded by. 48. or moare ap^{pr}
ty by. √.2304. And the qu^oti^ente will bee.

$$\sqrt{.153\frac{80}{192}} \text{ ——— } \sqrt{.5\frac{181}{192}} \text{ ——— } \sqrt{.17\frac{63}{176}} \text{ ——— } \sqrt{.1\frac{117}{192}}$$

As here appeareth in woozke.

$$\begin{array}{r} \sqrt{.354780.} \text{ ——— } \sqrt{.13716.} \text{ ——— } \sqrt{.39420.} \text{ ——— } \sqrt{.1524.} \text{ ——— } \sqrt{.153\frac{80}{192}} \text{ ——— } \sqrt{.5\frac{181}{192}} \text{ ——— } \sqrt{.17\frac{63}{176}} \text{ ——— } \sqrt{.1\frac{117}{192}} \\ \sqrt{.2304.} \quad \sqrt{.2304} \quad \sqrt{.2304} \quad \sqrt{.2304.} \end{array}$$

The prooffe.

And that this woozke is good, I will p^{ro}ue it by
multiplication.

of Surde numbers.

multiplication. As the example folowynge dooeth declare. $\sqrt{}$ here by the firste multiplication there cometh 8. numbers, that is . 4. with $\sqrt{}$. and . 4. with $\sqrt{}$.

$$\begin{array}{cccc} \sqrt{\frac{29565}{192}} & \sqrt{\frac{1141}{192}} & \sqrt{\frac{3285}{192}} & \sqrt{\frac{127}{192}} \\ \sqrt{.54.} & \sqrt{.6.} & & \\ \hline \sqrt{\frac{1596510}{192}} & \sqrt{\frac{61721}{192}} & \sqrt{\frac{177390}{192}} & \sqrt{\frac{6858}{192}} \\ \sqrt{\frac{177390}{192}} & \sqrt{\frac{6858}{192}} & \sqrt{\frac{19710}{192}} & \sqrt{\frac{761}{192}} \\ \sqrt{\frac{1596510}{192}} & \sqrt{\frac{1722}{192}} & \sqrt{\frac{19710}{192}} & \sqrt{\frac{761}{192}} \\ \hline \sqrt{.6570.} & \sqrt{.254.} & & \end{array}$$

And because the firste nōber with $\sqrt{}$, is equalle to the thirde with $\sqrt{}$, therfoze thei bothe must be reiected. Again in as moche as the seconde nōber with $\sqrt{}$ is equalle to the fourthe nōber with $\sqrt{}$, thei bothe shall bee cancelled. And then remaineth . 2. numbers with $\sqrt{}$, and other . 2. with $\sqrt{}$.

So if you abate the thirde $\sqrt{}$ out of the firste $\sqrt{}$, the *quotiente* will be $\sqrt{.6570.}$

Like waies if you abate the fourthe $\sqrt{}$ out of the seconde $\sqrt{}$, the *quotiente* will yelde $\sqrt{.254.}$

And thei bothe will make the firste diuidende . $\sqrt{.6570.}$ $\sqrt{}$. wherby the former diuision is approued good. *Palter.* This shall suffice for diuision.

Of extraction of rootes.



The nexte woork is extraction of rootes: whiche you maye very easilie woork, by puttynge the signe of the roote, that you desire, befoze the whole number. As if you would haue the square roote of $\sqrt{.10}$ $\sqrt{.5.}$ this is it $\sqrt{.10}$ $\sqrt{.5.}$ The *Cubike* roote of the same nobel is $\sqrt[3]{.10}$ $\sqrt[3]{.5.}$ And the *zenzizenzike* roote of it is $\sqrt[7]{.10}$ $\sqrt[7]{.5.}$ But if you will haue the square roote of $\sqrt{.10}$ $\sqrt{.5.}$

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it is. $\sqrt{10} - \sqrt{5}$. And his Cubike roote is. $\sqrt[3]{10}$
 $\sqrt{10} - \sqrt{5}$. Likewais his *zenzenzike* roote is
 $\sqrt[3]{10} - \sqrt{5}$.

So of. $\sqrt[3]{18}$ ——— 2. the Square roote is $\sqrt{18}$
 ——— 2. The Cubike roote is. $\sqrt[3]{18}$ ——— 2.
 And the *zenzenzike* roote is. $\sqrt[3]{18}$ ——— 2.

Scholar. Hereby I perceiue that the later parte of
 the cōposition, is not varied at all, but onely the firste
 parte taketh vnto it the signe of the roote. And that
 signe is referred to the whole compounde number.

*Vniuersalle
 rootes.*

Maſter. These rootes therefore bee called *vniuersalle
 rootes*, becauſe they are the rootes, not of the ſeu-
 ralle partes of the compounde nōber, but of the whole
 compounde number. And that is the difference, be-
 twene the common *Surde* numbers, and *vniuersalle roo-
 tes*. For if $\sqrt{24} - \sqrt{144}$, be ſette for a common
Surde number, then doeth it betoken, that I muſt take
 2. rootes, that is. $\sqrt{24}$. and $\sqrt{144}$, and ioyne them
 together. But if it ſtande for an *vniuersalle roote*, it re-
 preſenteth the roote of this whole number. $\sqrt{24 - 144}$.
 whiche is. 6. for the whole Square is. 36.

Scholar. I perceiue it well. For $\sqrt{144}$. beynge
 12, that. 12. with. 24. dooeth make. 36. And therefore
 muſt the *vniuersalle roote* of. $\sqrt{24 - 144}$. bee. 6.
 And ſo $\sqrt{24 - 144}$. is iuſt. 6.

But if. $\sqrt{24 - 144}$. doe ſtande for a com-
 mon *Surde* number compounde: then is it made of. 2.
 rootes, that is $\sqrt{24}$. whiche is almoſt. 5. and $\sqrt{144}$
 beynge. 12. And ſo the whole compounde roote, in that
 ſorte is almoſt. 17. And is nigh. 3. tymes ſo moche
 as the ſame number, beynge an *vniuersalle roote*.

Maſter. Becauſe you maie perceiue it the better,
 I will put an example in *Square* numbers, made like
Surdes. As this. $\sqrt{81} - \sqrt{36}$ if it be an *vniuersalle
 roote*, then it is equalle to 10. For I muſt take firſt the
 roote of the laſte number, whiche is. 19. And adde it
 with

of Surde numbers.

With 81. wherby there amounteth. 100. whose roote is. 10. But if it stand after the common sorte of *Surde numbers*, it betokeneth the roote of. 81. and the roote of. 361. (that is. 9. and. 19) to bee added together. And so they make. 28. whiche is farre aboue. 10.

But farther now, if it stande for a common *Surde number*: And I would haue the *Square roote* of it, then is that. $\sqrt{\sqrt{81}} + \sqrt{361}$. And betokeneth the *Square roote* of the *Square roote* of. 81. and the *Square roote* of. 361. added together, that is the *Square roote* of. 28. But moſte generally and moſte aptly, it betokeneth the roote of the *vnuerſalle roote* of. 81. & $\sqrt{361}$.

Scholar. Now I perceiue that in Addition, and Subtraction of *Surdes*, the last numbers that did result of that woork, were *vnuerſalle rootes*.

Maſter. You ſaie truth. But haſte what meaneth that haſtie knocking at the doore?

Scholar. It is a meſſenger.

Maſter. What is the meſſage: tel me in mine care

What ſir is that the mater: When is there noe remedie, but that I muſt neglect all ſtudies, and teaching, ſo; to withſtande thoſe daungers. My fortune is not ſo good, to haue quiete tyme to teache.

Scholar. But my fortune and my fellowes, is moche worſe, that your vnquietnes, ſo hindereth our knowledge. I praye God amende it.

Maſter. I am inforced to make an eande of this mater: But yet will I promiſe you, that whiche you ſhall chalenge of me, when you ſee me at better leiſure: That I will teache you the whole arte of *vnuerſalle rootes*. And the extraction of rootes in all *Square Surdes*: With the demonſtration of them, and all the former woorkes.

If I mighte haue been quietly permitted, to reſte but a litle while longer, I had determined not to haue ceaſed, till I had ended all theſe thinges at large. But

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now farewell. And applie your studie diligently in
this that you haue learned. And if I male gette any
quietnesse reasonable, I will not forget to perfoyme
my promise with an augmentation.

Scholar. My harte is so oppressed with pēisenes,
by this sodaine vnquietnesse, that I can not expresse
my grief. But I will praye, with all them that
loue honeste knowlodge, that God of his
mercie, will sone ende your troubles,
and graunte you soche reste, as
your trauell doeth merite.

And al that loue lear-
nyng: saie ther-
to. Amen.

Passer. Amen,
and Amen.



CImprinted at London,
by Iohn Kyngston.

Anno domini. 1557.