Office for National Statistics

United Kingdom Input-Output Analytical Tables 2010

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Office for National Statistics

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Contents

Preface

The 2010 United Kingdom Input-Output Analytical Tables (IOATs) are a National Accounts product of the Office for National Statistics. These tables are consistent with the 2013 editions of United Kingdom National Accounts: The Blue Book (ONS, 2013a) and United Kingdom Balance of Payments: The Pink Book (ONS, 2013b).

The 2010 IOATs are derived from the 2010 annual Supply and Use Tables (SUTs). The SUTs provide a picture of the flows of products and services in the economy for a single year and are used to set the level of annual current price Gross Domestic Product (GDP). They show the composition of uses and resources across institutional sectors and the inter-dependence of industries in order to reconcile the production, income and expenditure approaches to the measurement of GDP.

The IOATs show separately the consumption of domestically produced and imported goods and services, providing a theoretical framework for further analysis of the structure of the economy, its composition and the effect of changes in final demand on the economy. These tables form an essential tool for economic modelling.

The first IOATs for the United Kingdom covered the year 1954. Since then, they have been produced roughly every five years, see Mahajan (2006, pp306-308).

The presentation of the SUTs and the IOATs is based on the European System of Accounts 1995 (ESA 95: Eurostat, 1996), which itself is based on the United Nations System of National Accounts 1993 (SNA 93: United Nations, 1993). SNA 93 has been adopted worldwide and the production of ESA 95 based accounts is a legal requirement of European Union member states.

The previous set of IOATs covered the year 2005 and was published in August 2011 (ONS, 2011a). These were also produced on a ESA 95 basis but using Eurostat's Standard Industrial Classification 2003 (SIC 2002) and Classification of Products by Activity 2002 (CPA 2002), which were the industry and product classifications used by the UK National Accounts until the publication of *Blue Book 2011* (ONS, 2011b) in September of that year. From then onwards, the Accounts have been produced in accordance with SIC 2007 and CPA 2008 and these IOATs are the first UK tables to be produced on this basis. While this ensures comparability with the SUTs published since 2011, it limits comparison with previously published IOATs. In particular, care should be taken when comparing old and new industries and/or products, which may be similar in description but different in composition.

All the monetary value estimates in the tables are calculated as accurately as possible, however they cannot be regarded as being accurate to the number of digits shown. Several revisions of the underlying SUTs are produced after the first set of tables has been compiled.

Introduction

This publication describes the 2010 IOATs derived from the SUTs for the same year, as published in ONS (2013a). These tables, together with supplementary data and certain economic assumptions, have been combined to construct the input-output table. This table describes how products (and primary inputs) are used to produce further products and satisfy final demand. The input-output table and its derivative outputs collectively form the IOATs. Their derivation and construction are described in the following sections of this publication.

The 2010 IOATs have been derived from SUTs compiled using 114 industry input-output groups (IOGs) consistent with the UK's Standard Industrial Classification 2007 (SIC 2007) for industries and Eurostat's Classification of Products by Activity (CPA 2008) for products. As some IOGs are not explicitly measured and others are combined for quality purposes, in the published 2013 SUTs the economy is divided into 106 IOGs. However, for the IOATs 21 additional IOGs (giving 127 in total) have been created by separating components of the non-market output produced by general government and NPISHs (Non-Profit Institutions Serving Households) from the output produced by the market sectors, in order to allow for their different roles in the economy.

The close relationship between SUTs and IOATs has sometimes led to confusion despite being different products. This problem has been exacerbated by inconsistent use of terminology. This publication adheres to the terminology of Eurostat (2008).

This report contains a description of the theory and assumptions behind the 2010 IOATs and their practical implementation, together with summary IOATs. It is accompanied by spreadsheets containing both summary and detailed IOATs. It should be noted that some of the tables needed for the compilation of the IOATs cannot be published in order to adhere to the Code of Practice for Official Statistics (see UK Statistics Authority, 2009, Principle 5: Confidentiality).

This report focuses on product-by-product tables. Some users may wish to use industry-based analyses instead of the product-based analyses shown here. We therefore conclude this publication with a description of an alternative way of performing industrial analyses based on the product-by-product tables.

The tables presented in this product, including summary and detailed tables in Microsoft Excel format, are provided free of charge at:

http://www.ons.gov.uk/ons/rel/input-output/input-output-analytical-tables/2010/index.html

Symbols and conventions
In general, the following symbols are used throughout this publication:

"0" denotes values from nil to less than £0.5 million
£1 billion denotes £1,000 million

In the summary and detailed tables, the sum of components may not equal the total due to rounding differences. As a result of these rounding differences, totals in different tables may differ slightly.

Acknowledgements

The author would like to thank Markus Sova and Ellis Daniel (ONS); and Stevan Croasdale and Gary Campbell (Scottish Government) for their assistance with this publication. Particular thanks also go to Tim Butler (formerly of ONS) whose technical work in moving the IOAT compilation systems on to SIC 2007 / CPA 2008 underpins the majority of the figures presented.

Transforming Supply and Use Tables to Basic Prices

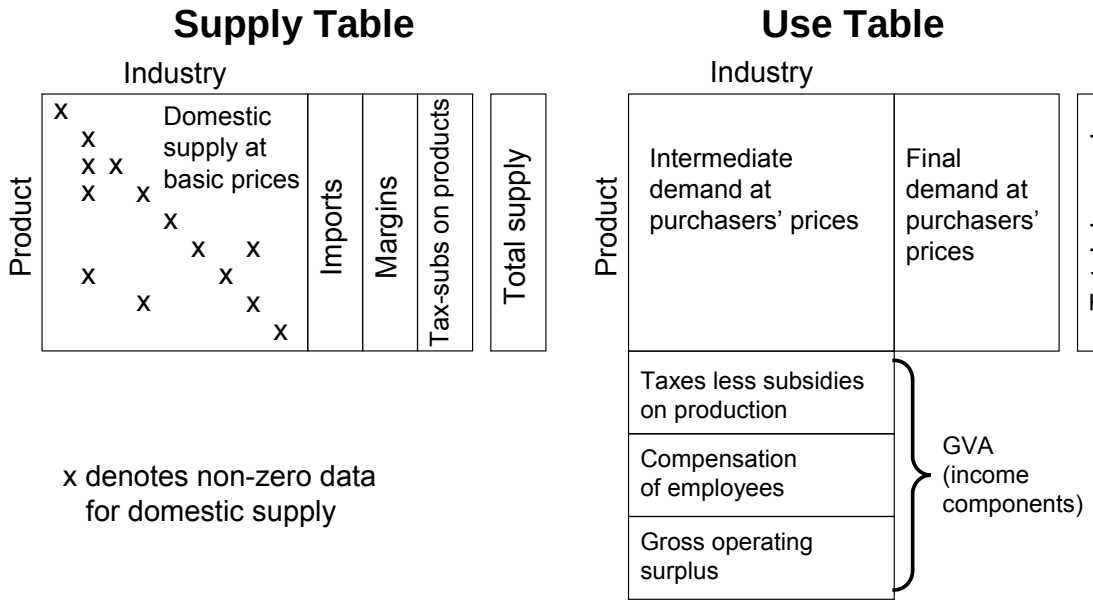
The annual SUTs consist of two matrices, which bring together the *production*, *income* and *expenditure* approaches to measuring GDP (see ONS, 2013a, for the 2010 SUTs). Being balanced, they provide a single estimate of GDP, which integrates the components of value added, output and final demand. Their structures are shown in figure 1, and we note the following:

- The SUTs show the supply and demand for all products. The products are classified into 114 product input-output groups (IOGs) consistent with Eurostat's CPA 2008. Producers are similarly classified into 114 industry IOGs consistent with SIC 2007. Both are consistent with Eurostat's NACE Rev. 2 (*Nomenclature statistique des Activités économiques dans la Communauté Européenne*). The IOG definitions are listed in Annex A
- The supply table shows the output of each product IOG by each industry IOG. As producers are classified according to their principal product, most domestic production lies on the diagonal. However, there are some off-diagonal elements in this table. These represent secondary

production and by-products classified to IOGs other than the principal product of the industry. The supply table is relatively sparse because most producers make a limited range of products. The main body of the supply table (which shows domestic supply at basic prices – the value received by the producers) is not published due to the data being disclosive. The columns on the right of the supply table show imports of products, distributors' trade margins on products and taxes less subsidies on products. Summing across these columns and those in the main body gives the total supply of products at purchasers' prices, the value paid by purchasers excluding any refundable VAT

- The main body of the use table shows, for each industry IOG, the intermediate demand for products. That is, the value of products used-up or altered by the production process. The columns to the left of the main table give the components of final demand for products. Both final demand and intermediate demand are valued at purchasers' prices and cover domestically produced and imported products
- The rows underneath the main body of the use table give the income components of Gross Value Added (GVA) for each industry IOG. These components are labour costs, taxes less subsidies on production, profits, etc
- The construction of the product by product Input-Output table involves matrix operations which require the main bodies of the SUT at basic prices to be square matrices. While previous IOATs have had to transform the Supply and Use tables on which they were based to equalise the number of industry and product groups, since 2011 the UK SUTs have been published as square matrices

Figure 1: Supply and Use tables at purchasers' prices



Once SUTs have been compiled, the first stage in deriving IOATs is to transform the SUTs from purchasers' prices to basic prices. This is achieved by adjusting for imports, margins and taxes less subsidies on products. For the supply table this is a relatively simple task, because the components to be removed are contained in the columns to the right of the main body of the table.

For the use table the task is more complicated; it requires the construction of a use table for each of the components to be adjusted. The sum of these use matrices is called the *transition matrix*; it is subtracted from the use table at purchasers' prices to give the use table at basic prices. With the exception of distributors' trade and transport margins, these components are shown as separate rows in the primary inputs, leaving the industry totals unchanged. This is not needed for margins as they are reallocated across the goods consumed. The transition from purchasers' prices to basic prices is depicted by figure 2; the outcome is shown in figure 3. Tables 1 and 2 show summary 2010 domestic and imports use tables, respectively.

Figure 2: SUT transition from purchasers' prices to basic prices

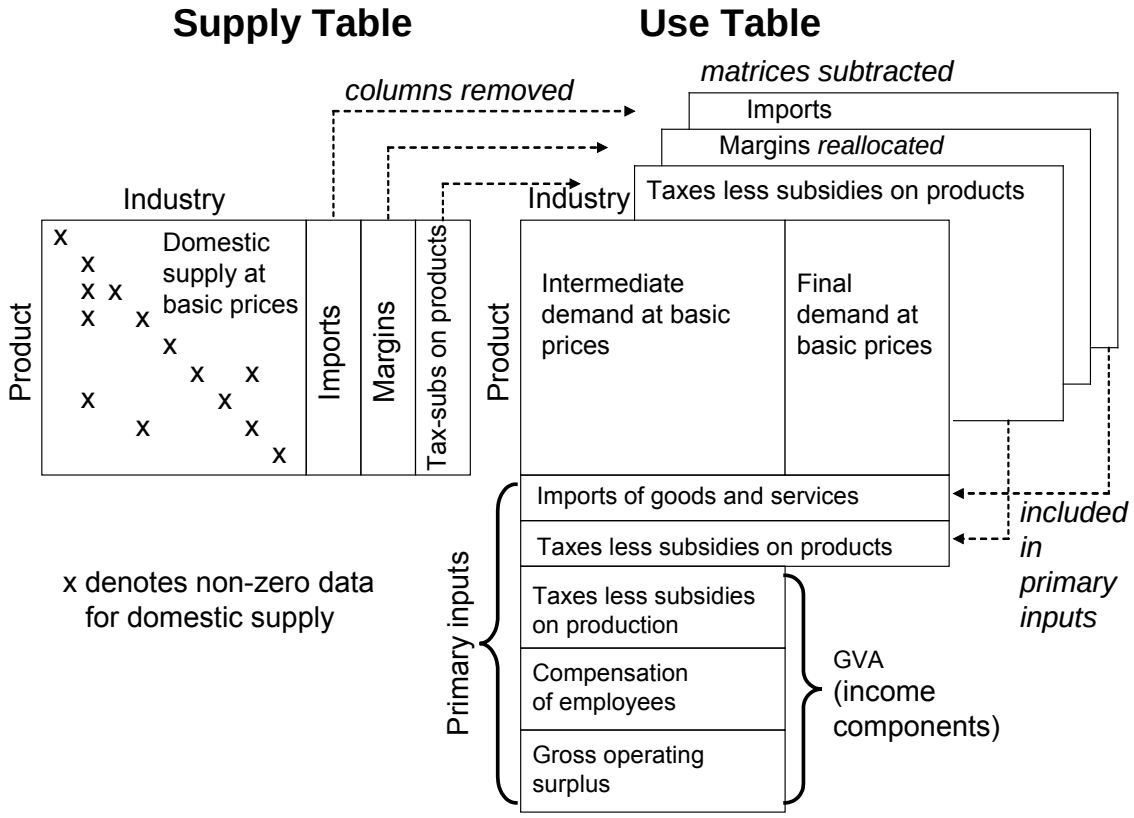


Table 1: Summary Domestic Use table at basic prices (product by industry), 2010

£ million

Product	Intermediate consumption by industries									
	1	2	3	4	5	6	7	8	9	10
	Agriculture [1-3]	Production [5-39]	Construction [41-43]	Distribution, transport, hotels and restaurants [45-56]	Information and communication [58-63]	Financial and insurance [64-66]	Real estate [68.1-2-68.3]	Professional and support activities [69.1-82]	Government, health & education [84-88]	Other services [90-97]
1 Agriculture [1-3]	2,540	8,950	184	1,139	13	0	0	9	23	10
2 Production [5-39]	4,555	165,586	23,228	33,163	6,510	4,192	1,393	5,396	5,404	2,078
3 Construction [41-43]	584	4,707	44,531	18,492	1,660	3,122	16,653	1,652	1,080	417
4 Distribution, transport, hotels and restaurants [45-56]	1,870	36,120	6,525	58,956	5,781	10,643	452	6,720	2,687	1,155
5 Information and communication [58-63]	223	6,319	1,888	15,852	11,761	13,973	1,206	9,390	1,775	2,043
6 Financial and insurance [64-66]	1,185	14,827	3,365	11,369	2,616	21,209	40,973	7,085	1,579	1,306
7 Real estate [68.1-2-68.3]	64	911	1,055	9,154	646	3,317	536	753	509	217
8 Professional and support activities [69.1-82]	417	21,195	15,600	42,481	15,687	28,611	3,865	55,499	4,051	5,727
9 Government, health & education [84-88]	28	1,788	1,014	4,386	1,217	2,170	3,922	7,291	8,250	485
10 Other services [90-97]	23	404	22	1,297	1,634	1,166	21	1,287	867	3,525
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
Total consumption	11,489	260,807	97,412	196,289	47,525	88,403	69,021	95,082	26,225	16,963
Imports of goods and services	3,496	123,185	13,541	41,555	18,507	17,854	3,266	20,090	8,308	4,110
Taxes less subsidies on products	526	5,464	5,380	9,790	566	7,955	2,370	1,319	1,278	1,419
Taxes less subsidies on production	-2,764	4,906	1,169	12,712	1,356	2,064	-863	1,534	510	1,005
Compensation of employees	4,203	115,960	44,144	166,379	49,835	62,507	8,882	95,188	59,813	21,426
Gross operating surplus	7,615	79,347	38,553	62,732	31,870	62,124	120,615	55,269	14,837	14,957
Total output	24,565	589,669	200,199	489,457	149,659	240,907	203,291	268,482	110,971	59,880

Product	Non-market - Industry			NPISHs - Industry				
	11	12	13	14	15	16	17	
	Production (non-market) [5-39]	and communication (non-market) [58-63]	Government, health & education (non-market) [84-88]	Other services (non-market) [90-97]	Professional and support activities (NPISH) [69.1-82]	Government, health & education (NPISH) [84-88]	Other services (NPISH) [90-97]	Total intermediate demand
1 Agriculture [1-3]	0	0	46	0	0	16	2	12,932
2 Production [5-39]	2,371	69	17,375	573	18	928	242	273,081
3 Construction [41-43]	143	14	5,047	111	2	247	59	98,521
4 Distribution, transport, hotels and restaurants [45-56]	141	66	10,724	67	15	965	64	142,951
5 Information and communication [58-63]	52	329	5,073	236	19	495	247	70,881
6 Financial and insurance [64-66]	16	21	4,133	25	18	119	108	109,954
7 Real estate [68.1-2-68.3]	22	18	4,116	67	3	47	9	21,444
8 Professional and support activities [69.1-82]	613	426	18,175	490	120	1,103	524	214,584
9 Government, health & education [84-88]	8	26	36,055	0	18	2,421	94	69,173
10 Other services [90-97]	24	265	2,373	189	1	186	1,006	14,290
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0
Total consumption	3,390	1,234	103,117	1,758	214	6,527	2,355	1,027,811
Imports of goods and services	427	497	40,863	403	184	1,658	510	298,454
Taxes less subsidies on products	666	69	18,253	520	2	1,026	389	56,992
Taxes less subsidies on production	0	0	0	0	0	0	0	21,629
Compensation of employees	1,534	1,116	147,653	1,687	440	16,802	4,227	801,796
Gross operating surplus	152	80	12,889	230	200	1,972	1,056	504,498
Total output	6,169	2,996	322,775	4,598	1,040	27,985	8,537	2,711,180

Product	Final consumption expenditure				Gross capital formation			E xports		
	Households	NPISHs	Central Government	Local Government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total demand at purchasers' prices
1 Agriculture [1-3]	6,455	0	0	0	1,063	0	22	2,387	135	22,994
2 Production [5-39]	78,963	0	0	0	14,464	99	2,301	184,855	2,703	556,466
3 Construction [41-43]	1,053	0	0	0	110,838	0	-1,600	0	1,426	210,238
4 Distribution, transport, hotels and restaurants [45-56]	279,743	0	0	0	6,547	140	0	39,400	34,271	503,052
5 Information and communication [58-63]	29,386	0	0	0	26,394	0	103	3,554	16,206	146,524
6 Financial and insurance [64-66]	53,931	0	0	0	26	3	0	0	58,842	222,756
7 Real estate [68.1-2-68.3]	186,655	0	0	0	8,183	0	0	0	531	216,813
8 Professional and support activities [69.1-82]	11,256	0	0	0	7,124	0	335	39	56,859	290,197
9 Government, health & education [84-88]	34,774	0	0	0	2,544	0	5	0	4,198	110,694
10 Other services [90-97]	38,090	0	0	0	172	-37	79	2,925	1,827	57,346
11 Production (non-market) [5-39]	0	0	0	6,169	0	0	0	0	0	6,169
12 Information and communication (non-market) [58-63]	0	0	2,996	0	0	0	0	0	0	2,996
13 Government, health & education (non-market) [84-88]	0	0	202,144	120,631	0	0	0	0	0	322,775
14 Other services (non-market) [90-97]	0	0	0	4,598	0	0	0	0	0	4,598
15 Professional and support activities (NPISH) [69.1-82]	0	1,040	0	0	0	0	0	0	0	1,040
16 Government, health & education (NPISH) [84-88]	0	27,985	0	0	0	0	0	0	0	27,985
17 Other services (NPISH) [90-97]	0	8,537	0	0	0	0	0	0	0	8,537
Total consumption	720,306	37,562	205,140	131,398	177,355	205	1,245	233,160	176,998	2,711,180
Imports of goods and services	119,811	0	0	0	33,865	12	690	24,515	2,774	480,121
Taxes less subsidies on products	80,917	0	0	0	9,936	34	-9	7,568	2,254	157,692
Total	921,034	37,562	205,140	131,398	221,156	251	1,926	265,243	182,026	3,348,993

Table 2: Summary Imports Use table at basic prices (product by industry), 2010

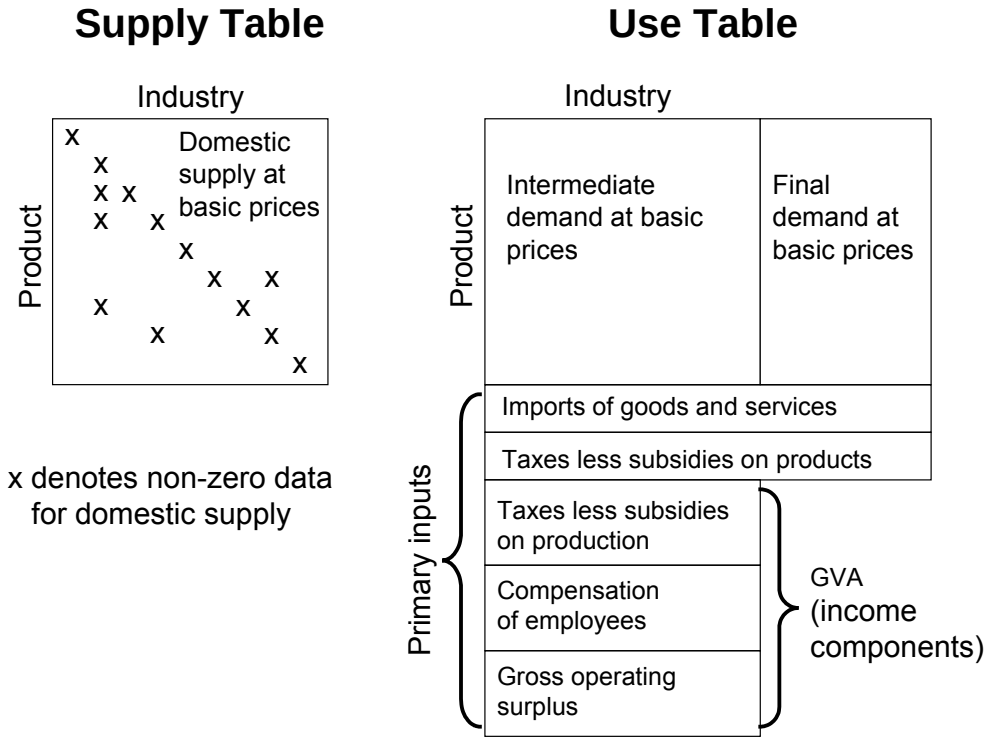
£ million

	Intermediate consumption by industries									
Product	1 Agriculture [1-3]	2 Production [5-39]	3 Construction [41-43]	4 Distribution, transport, hotels and restaurants [45-56]	5 Information and communication [58-63]	6 Financial and insurance [64-66]	7 Real estate [68.1-2-68.3]	8 Professional and support activities [69.1-82]	9 Government, health & education [84-88]	10 Other services [90-97]
1 Agriculture [1-3]	744	2,223	25	195	0	0	0	0	1	0
2 Production [5-39]	2,460	112,171	10,830	24,025	7,023	1,706	295	4,040	3,925	1,254
3 Construction [41-43]	7	134	790	123	11	23	173	65	9	6
4 Distribution, transport, hotels and restaurants [45-56]	148	4,207	399	9,857	932	4,822	115	2,432	3,181	292
5 Information and communication [58-63]	2	526	41	602	5,631	233	18	689	45	377
6 Financial and insurance [64-66]	94	983	209	802	192	7,952	2,375	628	129	81
7 Real estate [68.1-2-68.3]	3	38	49	427	31	182	27	37	48	11
8 Professional and support activities [69.1-82]	38	2,828	1,195	5,351	4,223	2,759	251	11,766	725	1,239
9 Government, health & education [84-88]	0	17	0	29	6	15	1	273	11	3
10 Other services [90-97]	0	58	3	144	458	162	11	160	234	847
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
Total consumption	3,496	123,185	13,541	41,555	18,507	17,854	3,266	20,090	8,308	4,110

	Non-market - Industry				NPISHs - Industry			
Product	11 Production (non-market) [5-39]	12 and communication (non-market) [58-63]	13 Government, health & education (non-market) [84-88]	14 Other services (non-market) [90-97]	15 Professional and support activities (NPISH) [69.1-82]	16 Government, health & education (NPISH) [84-88]	17 Other services (NPISH) [90-97]	Total intermediate demand
1 Agriculture [1-3]	0	0	3	0	0	9	0	3,200
2 Production [5-39]	291	16	32,034	113	8	804	156	201,151
3 Construction [41-43]	1	0	28	0	0	3	0	1,373
4 Distribution, transport, hotels and restaurants [45-56]	2	37	1,912	13	3	252	74	28,678
5 Information and communication [58-63]	4	265	676	35	3	43	16	9,206
6 Financial and insurance [64-66]	2	1	209	1	2	14	7	13,681
7 Real estate [68.1-2-68.3]	1	0	180	3	0	7	0	1,044
8 Professional and support activities [69.1-82]	108	93	3,708	61	166	470	123	35,104
9 Government, health & education [84-88]	0	0	1,202	0	2	2	0	1,561
10 Other services [90-97]	18	85	911	177	0	54	134	3,456
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0
Total consumption	427	497	40,863	403	184	1,658	510	298,454

	Final consumption expenditure				Gross capital formation			E xports		
Product	Households	NPISHs	Central Government	Local Government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total demand at purchasers' prices
1 Agriculture [1-3]	6,313	0	0	0	9	0	8	0	0	9,530
2 Production [5-39]	106,270	0	0	0	32,795	35	638	24,515	113	365,517
3 Construction [41-43]	6	0	0	0	0	0	18	0	0	1,397
4 Distribution, transport, hotels and restaurants [45-56]	473	0	0	0	174	-20	0	0	2,661	31,966
5 Information and communication [58-63]	2,526	0	0	0	743	0	6	0	0	12,481
6 Financial and insurance [64-66]	14	0	0	0	0	-3	0	0	0	13,692
7 Real estate [68.1-2-68.3]	6	0	0	0	0	0	0	0	0	1,050
8 Professional and support activities [69.1-82]	70	0	0	0	0	0	0	0	0	35,174
9 Government, health & education [84-88]	1,013	0	0	0	0	0	0	0	0	2,574
10 Other services [90-97]	3,120	0	0	0	144	0	20	0	0	6,740
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
Total	119,811	0	0	0	33,865	12	690	24,515	2,774	480,121

Figure 3: The outcome of the SUT transition to basic prices



The Input-Output Table

The Input-Output table (IOT) describes how products (and primary inputs) are used to produce further products and to satisfy final demand. To explain how this is achieved, let us first consider the following definitions of the domestic supply and use tables at basic prices:

- The (product by industry) domestic supply table at basic prices shows the outputs of the production process of each industry IOG
- The (product by industry) domestic use table at basic prices shows the inputs of the production process of each industry IOG and each component of final demand

The IOT is constructed by transforming the industry IOGs to product IOGs, so that these definitions become:

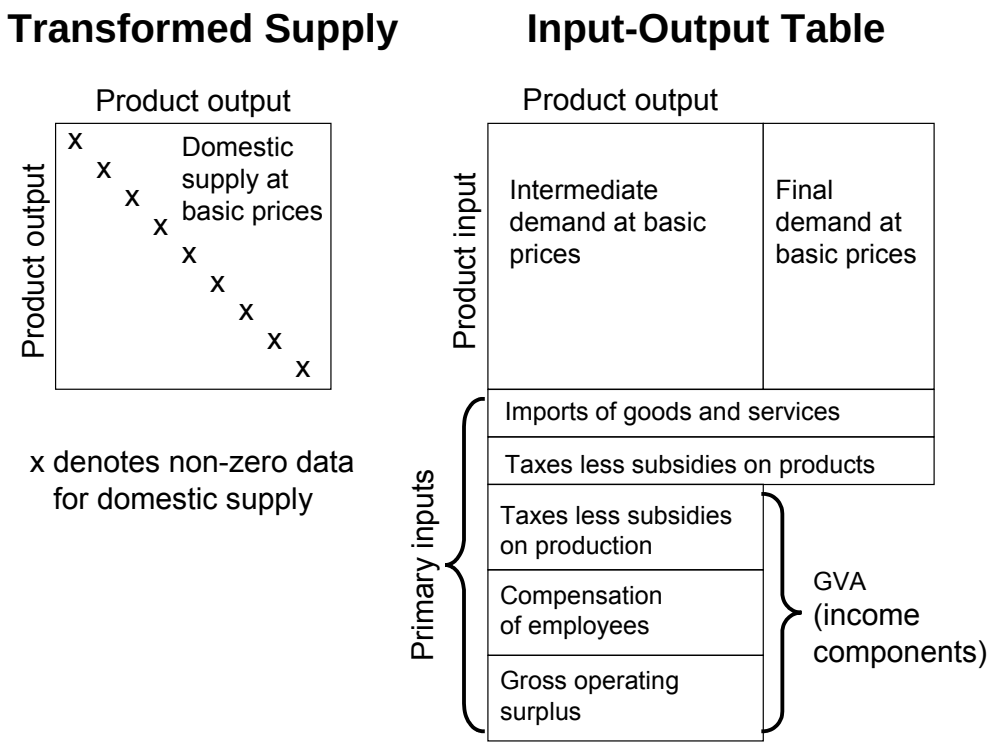
- The transformed (product by product) domestic supply table at basic prices shows the outputs of the production process of each product IOG
- The transformed (product by product) domestic use table at basic prices shows the inputs of the production process of each product IOG and each component of final demand

These transformed tables are depicted in figure 4.

As the output of the production process of a product is simply that product, the transformed domestic supply table is a diagonal matrix. The non-zero elements of this matrix are simply the total domestic supply of each product (at basic prices).

The transformed domestic use table at basic prices is the Input-Output Table. Due to the fact that the production of a product may require many inputs (such as raw materials, energy, office costs, accountancy, legal fees, etc) the IOT is not diagonal.

Figure 4: The result of the transformation to product by product tables



The product by industry domestic supply table describes the relationship between the production processes of industry IOGs and the production process producing product IOGs. The transformation of the supply table involves moving off-diagonal entries onto the industry IOG of which they are the principal product. The total supply of each product IOG remains unaltered; each element of off-diagonal supply is moved to the diagonal element of the same row. For each element of supply thus moved, its corresponding inputs (including primary inputs) need to be moved in the use table. This is complicated by the fact the production of a product may require many inputs. In essence, for each industry IOG we need to separate out the inputs for the various product IOGs produced by that industry. It would be impractical for producers to provide such splits; therefore this is estimated by making a *technology assumption*. Three technology assumptions are described in Eurostat (2008, ch11):

- **Product technology** assumes that all products classified to a product IOG have the same input structure, regardless of the classification of the producer. Unfortunately, it can result in negative entries in the main body of the IOT, even though such negative intermediate use entries are conceptually impossible
- **Industry technology** assumes that all products produced by an industry IOG have the same input structure, regardless of the classification of the product. This is conceptually less satisfactory than the product technology assumption. However, it does not result in negative entries in the main body of the IOT

- It is possible to specify either product technology or industry technology independently for each off-diagonal entry in the supply table. This **hybrid technology** matrix can result in negative entries, but these can be avoided through an appropriate choice of technology for each off-diagonal entry

Each of these technology assumptions generates a set of simultaneous equations which can be solved by matrix algebra. This is not easy to visualise numerically; the interested reader is referred to Eurostat (2008, ch11). The negative entries can have a number of root causes:

- The product technology assumption can be inappropriate for products which can be produced by more than one method (such as electricity produced by hydro and nuclear power stations)
- The set of products classified to a product IOG may be heterogeneous in terms of their inputs (such as beer and cider)
- Different producers of the same product may vary in what production they do in-house. Consider, for example, two bakeries. The first buys flour to make bread. The second buys grain to make flour to make identical bread. Thus the inputs to the first bakery's loaves include flour but no grain. The inputs to the second bakery's loaves include grain but little flour
- There may be quality issues in the data, either in the SUTs or in the transition matrix

For the 2010 IOT, a hybrid technology assumption has been used. The matrix algebra is given in Annex B. Table 3 shows the summary 2010 Input-Output table.

The methodology described above cannot be applied to construct a product by product imports use table. This is because anything produced by consuming an import is not regarded as an import. The inputs (imports) are thus conceptually different from the outputs (domestic production). Furthermore, we have no breakdown of imports by foreign producer, only by product. The 2010 product by product imports use table has therefore been constructed by applying *iterative proportional scaling* (also known as *raking* and *rAs*) to the IOT as follows:

- The intermediate demand parts of the rows are scaled to sum to the intermediate demand totals for the product by industry imports use table
- The intermediate demand parts of the columns are scaled to sum to the value of imports in the primary inputs section of the IOT
- The above scalings are repeated until convergence
- The final demand section of the product by industry imports use table is attached to the right of the resulting table

Table 4 shows the summary 2010 product by product imports use table.

Table 3: Summary Input-Output table (product by product), 2010

£ million

Product	Intermediate consumption by products									
	1	2	3	4	5	6	7	8	9	10
	Agriculture [1-3]	Production [5-39]	Construction [41-43]	Distribution, transport, hotels and restaurants [45-56]	Information and communication [58-63]	Financial and insurance [64-66]	Real estate [68.1-2-68.3]	Professional and support activities [69.1-82]	Government, health & education [84-88]	Other services [90-97]
1 Agriculture [1-3]	2,316	8,671	192	1,505	31	0	21	76	23	33
2 Production [5-39]	4,284	156,229	24,320	37,830	6,471	3,846	1,801	9,361	5,374	1,987
3 Construction [41-43]	549	4,421	44,509	18,325	1,617	2,831	17,184	1,890	1,080	491
4 Distribution, transport, hotels and restaurants [45-56]	1,735	33,839	7,167	59,730	5,595	9,818	1,042	8,157	2,684	1,142
5 Information and communication [58-63]	207	5,928	2,290	15,934	10,748	12,862	1,837	10,679	1,791	2,153
6 Financial and insurance [64-66]	1,119	13,904	3,561	11,740	2,931	19,091	42,700	7,639	1,594	1,234
7 Real estate [68.1-2-68.3]	61	853	1,107	9,088	669	3,029	763	830	509	253
8 Professional and support activities [69.1-82]	391	19,871	16,838	42,451	15,129	26,557	5,408	56,699	4,223	5,565
9 Government, health & education [84-88]	25	1,661	1,332	4,377	1,285	2,013	4,152	7,051	8,201	454
10 Other services [90-97]	20	382	82	1,407	1,664	1,093	64	1,614	866	3,053
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
Total consumption	10,708	245,759	101,399	202,387	46,142	81,143	74,973	103,996	26,346	16,365
Imports of goods and services	3,287	116,704	14,834	45,216	17,935	16,460	4,350	22,780	8,366	3,980
Taxes less subsidies on products	504	5,182	5,458	9,902	827	7,318	2,800	1,563	1,275	1,237
Taxes less subsidies on production	-2,622	4,638	1,334	12,687	1,333	1,903	-815	1,704	508	960
Compensation of employees	3,925	109,515	47,237	168,241	48,735	58,233	11,366	100,547	59,319	21,221
Gross operating surplus	7,193	74,668	39,976	64,619	31,552	57,699	124,140	59,606	14,881	13,583
Total output	22,994	556,466	210,238	503,052	146,524	222,756	216,813	290,197	110,694	57,346

Product	Non-market - Product				NPISH - Product			
	11	12	13	14	15	16	17	
	Production (non-market) [5-39]	Information and communication (non-market) [58-63]	Government, health & education (non-market) [84-88]	Other services (non-market) [90-97]	Professional and support activities (NPISH) [69.1-82]	Government, health & education (NPISH) [84-88]	Other services (NPISH) [90-97]	Total intermediate demand
1 Agriculture [1-3]	0	0	46	0	0	16	2	12,932
2 Production [5-39]	2,371	69	17,375	573	18	928	242	273,081
3 Construction [41-43]	143	14	5,047	111	2	247	59	98,521
4 Distribution, transport, hotels and restaurants [45-56]	141	66	10,724	67	15	965	64	142,951
5 Information and communication [58-63]	52	329	5,073	236	19	495	247	70,881
6 Financial and insurance [64-66]	16	21	4,133	25	18	119	108	109,954
7 Real estate [68.1-2-68.3]	22	18	4,116	67	3	47	9	21,444
8 Professional and support activities [69.1-82]	613	426	18,175	490	120	1,103	524	214,584
9 Government, health & education [84-88]	8	26	36,055	0	18	2,421	94	69,173
10 Other services [90-97]	24	265	2,373	189	1	186	1,006	14,290
11 Production (non-market) [5-39]	0	0	0	0	0	0	0	0
12 Information and communication (non-market) [58-63]	0	0	0	0	0	0	0	0
13 Government, health & education (non-market) [84-88]	0	0	0	0	0	0	0	0
14 Other services (non-market) [90-97]	0	0	0	0	0	0	0	0
15 Professional and support activities (NPISH) [69.1-82]	0	0	0	0	0	0	0	0
16 Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0
17 Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0
Total consumption	3,390	1,234	103,117	1,758	214	6,527	2,355	1,027,811
Imports of goods and services	427	497	40,863	403	184	1,658	510	298,454
Taxes less subsidies on products	666	69	18,253	520	2	1,026	389	56,992
Taxes less subsidies on production	0	0	0	0	0	0	0	21,629
Compensation of employees	1,534	1,116	147,653	1,687	440	16,802	4,227	801,796
Gross operating surplus	152	80	12,889	230	200	1,972	1,056	504,498
Total output	6,169	2,996	322,775	4,598	1,040	27,985	8,537	2,711,180

Product	Final consumption expenditure				Gross capital formation			E xports		
	Households	NPISHs	Central Government	Local Government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total demand
1 Agriculture [1-3]	6,455	0	0	0	1,063	0	22	2,387	135	22,994
2 Production [5-39]	78,963	0	0	0	14,464	99	2,301	184,855	2,703	556,466
3 Construction [41-43]	1,053	0	0	0	110,838	0	-1,600	0	1,426	210,238
4 Distribution, transport, hotels and restaurants [45-56]	279,743	0	0	0	6,547	140	0	39,400	34,271	503,052
5 Information and communication [58-63]	29,386	0	0	0	26,394	0	103	3,554	16,206	146,524
6 Financial and insurance [64-66]	53,931	0	0	0	26	3	0	0	58,842	222,756
7 Real estate [68.1-2-68.3]	186,655	0	0	0	8,183	0	0	0	531	216,813
8 Professional and support activities [69.1-82]	11,256	0	0	0	7,124	0	335	39	56,859	290,197
9 Government, health & education [84-88]	34,774	0	0	0	2,544	0	5	0	4,198	110,694
10 Other services [90-97]	38,090	0	0	0	172	-37	79	2,925	1,827	57,346
11 Production (non-market) [5-39]	0	0	0	6,169	0	0	0	0	0	6,169
12 Information and communication (non-market) [58-63]	0	0	2,996	0	0	0	0	0	0	2,996
13 Government, health & education (non-market) [84-88]	0	0	202,144	120,631	0	0	0	0	0	322,775
14 Other services (non-market) [90-97]	0	0	0	4,598	0	0	0	0	0	4,598
15 Professional and support activities (NPISH) [69.1-82]	0	1,040	0	0	0	0	0	0	0	1,040
16 Government, health & education (NPISH) [84-88]	0	27,985	0	0	0	0	0	0	0	27,985
17 Other services (NPISH) [90-97]	0	8,537	0	0	0	0	0	0	0	8,537
Total consumption	720,306	37,562	205,140	131,398	177,355	205	1,245	233,160	176,998	2,711,180
Imports of goods and services	119,811	0	0	0	33,865	12	690	24,515	2,774	480,121
Taxes less subsidies on products	80,917	0	0	0	9,936	34	-9	7,568	2,254	157,692
Total	921,034	37,562	205,140	131,398	221,156	251	1,926	265,243	182,026	3,348,993

Table 4: Summary Imports Use table (product by product), 2010

£ million

		Intermediate consumption by products									
		1	2	3	4	5	6	7	8	9	10
	Product	Agriculture [1-3]	Production [5-39]	Construction [41-43]	Distribution, transport, hotels and restaurants [45-56]	Information and communication [58-63]	Financial and insurance [64-66]	Real estate [68.1-2-68.3]	Professional and support activities [69.1-82]	Government, health & education [84-88]	Other services [90-97]
1	Agriculture [1-3]	693	2,151	27	283	5	0	7	17	1	4
2	Production [5-39]	2,318	106,274	11,571	27,533	6,957	1,584	489	5,889	3,926	1,189
3	Construction [41-43]	7	130	793	124	12	21	177	62	9	6
4	Distribution, transport, hotels and restaurants [45-5	140	3,946	506	9,849	1,002	4,414	414	2,644	3,176	294
5	Information and communication [58-63]	2	499	296	665	5,116	216	29	891	49	399
6	Financial and insurance [64-66]	89	922	225	828	366	7,335	2,763	710	131	77
7	Real estate [68.1-2-68.3]	3	36	51	424	32	166	40	41	48	13
8	Professional and support activities [69.1-82]	36	2,675	1,354	5,319	4,020	2,560	412	11,992	779	1,229
9	Government, health & education [84-88]	0	16	2	27	3	14	2	276	13	3
10	Other services [90-97]	0	55	10	164	421	151	17	256	234	767
11	Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12	Information and communication (non-market) [58-63	0	0	0	0	0	0	0	0	0	0
13	Government, health & education (non-market) [84-8	0	0	0	0	0	0	0	0	0	0
14	Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15	Professional and support activities (NPISH) [69.1-82	0	0	0	0	0	0	0	0	0	0
16	Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17	Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
	Total imports	3,287	116,704	14,834	45,216	17,935	16,460	4,350	22,780	8,366	3,980

		Non-market - Product				NPISH - Product			
		11	12	13	14	15	16	17	
	Product	Production (non-market) [5-39]	and communication (non-market) [58-63]	Government, health & education (non-market) [84-88]	Other services (non-market) [90-97]	Professional and support activities (NPISH) [69.1-82]	Government, health & education (NPISH) [84-88]	Other services (NPISH) [90-97]	Total intermediate demand
1	Agriculture [1-3]	0	0	3	0	0	9	0	3,200
2	Production [5-39]	291	16	32,034	113	8	804	156	201,151
3	Construction [41-43]	1	0	28	0	0	3	0	1,373
4	Distribution, transport, hotels and restaurants [45-5	2	37	1,912	13	3	252	74	28,678
5	Information and communication [58-63]	4	265	676	35	3	43	16	9,206
6	Financial and insurance [64-66]	2	1	209	1	2	14	7	13,681
7	Real estate [68.1-2-68.3]	1	0	180	3	0	7	0	1,044
8	Professional and support activities [69.1-82]	108	93	3,709	61	166	470	123	35,104
9	Government, health & education [84-88]	0	0	1,201	0	2	2	0	1,561
10	Other services [90-97]	18	85	911	177	0	54	134	3,456
11	Production (non-market) [5-39]	0	0	0	0	0	0	0	0
12	Information and communication (non-market) [58-63	0	0	0	0	0	0	0	0
13	Government, health & education (non-market) [84-8	0	0	0	0	0	0	0	0
14	Other services (non-market) [90-97]	0	0	0	0	0	0	0	0
15	Professional and support activities (NPISH) [69.1-82	0	0	0	0	0	0	0	0
16	Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0
17	Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0
	Total imports	427	497	40,863	403	184	1,658	510	298,454

		Final consumption expenditure				Gross capital formation			Exports		
	Product	Households	NPISHs	Central Government	Local Government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total demand
1	Agriculture [1-3]	6,313	0	0	0	9	0	8	0	0	9,530
2	Production [5-39]	106,270	0	0	0	32,795	35	638	24,515	113	365,517
3	Construction [41-43]	6	0	0	0	0	0	18	0	0	1,397
4	Distribution, transport, hotels and restaurants [45-5	473	0	0	0	174	-20	0	0	2,661	31,966
5	Information and communication [58-63]	2,526	0	0	0	743	0	6	0	0	12,481
6	Financial and insurance [64-66]	14	0	0	0	0	-3	0	0	0	13,692
7	Real estate [68.1-2-68.3]	6	0	0	0	0	0	0	0	0	1,050
8	Professional and support activities [69.1-82]	70	0	0	0	0	0	0	0	0	35,174
9	Government, health & education [84-88]	1,013	0	0	0	0	0	0	0	0	2,574
10	Other services [90-97]	3,120	0	0	0	144	0	20	0	0	6,740
11	Production (non-market) [5-39]	0	0	0	0	0	0	0	0	0	0
12	Information and communication (non-market) [58-63	0	0	0	0	0	0	0	0	0	0
13	Government, health & education (non-market) [84-8	0	0	0	0	0	0	0	0	0	0
14	Other services (non-market) [90-97]	0	0	0	0	0	0	0	0	0	0
15	Professional and support activities (NPISH) [69.1-82	0	0	0	0	0	0	0	0	0	0
16	Government, health & education (NPISH) [84-88]	0	0	0	0	0	0	0	0	0	0
17	Other services (NPISH) [90-97]	0	0	0	0	0	0	0	0	0	0
	Total imports	119,811	0	0	0	33,865	12	690	24,515	2,774	480,121

Matrix of Coefficients

The matrix of coefficients is an important tool in the study of the underlying structure of the economy. It can be calculated by dividing each column of the intermediate demand and primary inputs part of the IOT by the column total. It has the following properties:

- Apart from those primary inputs with negative entries in the IOT, the coefficients take values between 0 and 1 (provided that GVA is positive). They describe the distribution of inputs by monetary value for each product IOG
- It covers intermediate and primary inputs
- Each column sums to 1
- Changes in the coefficients indicate changes in the structure of the economy. They assist, for example, in the study of relative price changes, capital/labour intensity, developments in technology, product substitution and import substitution

Although the matrix of coefficients is conceptually derived from the IOT, applying matrix algebra makes it more convenient to calculate the matrix of coefficients first and then to construct the IOT. The intermediate demand part of the matrix is traditionally denoted by the letter **A**. The 2010 summary matrix of coefficients is shown in table 5.

Table 5: Summary matrix of coefficients (product by product), 2010

	Intermediate consumption by products									
	1	2	3	4	5	6	7	8	9	10
Product	Agriculture [1-3]	Production [5-39]	Construction [41-43]	Distribution, transport, hotels and restaurants [45-56]	Information and communication [58-63]	Financial and insurance [64-66]	Real estate [68.1-2-68.3]	Professional and support activities [69.1-82]	Government, health & education [84-88]	Other services [90-97]
1 Agriculture [1-3]	0.101	0.016	0.001	0.003	0.000	0.000	0.000	0.000	0.000	0.001
2 Production [5-39]	0.186	0.281	0.116	0.075	0.044	0.017	0.008	0.032	0.049	0.035
3 Construction [41-43]	0.024	0.008	0.212	0.036	0.011	0.013	0.079	0.007	0.010	0.009
4 Distribution, transport, hotels and restaurants [45-56]	0.075	0.061	0.034	0.119	0.038	0.044	0.005	0.028	0.024	0.020
5 Information and communication [58-63]	0.009	0.011	0.011	0.032	0.073	0.058	0.008	0.037	0.016	0.038
6 Financial and insurance [64-66]	0.049	0.025	0.017	0.023	0.020	0.086	0.197	0.026	0.014	0.022
7 Real estate [68.1-2-68.3]	0.003	0.002	0.005	0.018	0.005	0.014	0.004	0.003	0.005	0.004
8 Professional and support activities [69.1-82]	0.017	0.036	0.080	0.084	0.103	0.119	0.025	0.195	0.038	0.097
9 Government, health & education [84-88]	0.001	0.003	0.006	0.009	0.009	0.009	0.019	0.024	0.074	0.008
10 Other services [90-97]	0.001	0.001	0.000	0.003	0.011	0.005	0.000	0.006	0.008	0.053
11 Production (non-market) [5-39]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12 Information and communication (non-market) [58-63]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13 Government, health & education (non-market) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14 Other services (non-market) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15 Professional and support activities (NPISH) [69.1-82]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16 Government, health & education (NPISH) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17 Other services (NPISH) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Total consumption	0.466	0.442	0.482	0.402	0.315	0.364	0.346	0.358	0.238	0.285
Imports of goods and services	0.143	0.210	0.071	0.090	0.122	0.074	0.020	0.078	0.076	0.069
Taxes less subsidies on products	0.022	0.009	0.026	0.020	0.006	0.033	0.013	0.005	0.012	0.022
Taxes less subsidies on production	-0.114	0.008	0.006	0.025	0.009	0.009	-0.004	0.006	0.005	0.017
Compensation of employees	0.171	0.197	0.225	0.334	0.333	0.261	0.052	0.346	0.536	0.370
Gross operating surplus	0.313	0.134	0.190	0.128	0.215	0.259	0.573	0.205	0.134	0.237
Total output	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

	Non-market - Product				NPISH - Product		
	11	12	13	14	15	16	17
Product	Production (non-market) [5-39]	Information and communication (non-market) [58-63]	Government, health & education (non-market) [84-88]	Other services (non-market) [90-97]	Professional and support activities (NPISH) [69.1-82]	Government, health & education (NPISH) [84-88]	Other services (NPISH) [90-97]
1 Agriculture [1-3]	0.000	0.000	0.000	0.000	0.000	0.001	0.000
2 Production [5-39]	0.384	0.023	0.054	0.125	0.017	0.033	0.028
3 Construction [41-43]	0.023	0.005	0.016	0.024	0.002	0.009	0.007
4 Distribution, transport, hotels and restaurants [45-56]	0.023	0.022	0.033	0.015	0.014	0.034	0.007
5 Information and communication [58-63]	0.008	0.110	0.016	0.051	0.018	0.018	0.029
6 Financial and insurance [64-66]	0.003	0.007	0.013	0.005	0.017	0.004	0.013
7 Real estate [68.1-2-68.3]	0.004	0.006	0.013	0.015	0.003	0.002	0.001
8 Professional and support activities [69.1-82]	0.099	0.142	0.056	0.107	0.115	0.039	0.061
9 Government, health & education [84-88]	0.001	0.009	0.112	0.000	0.017	0.087	0.011
10 Other services [90-97]	0.004	0.088	0.007	0.041	0.001	0.007	0.118
11 Production (non-market) [5-39]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12 Information and communication (non-market) [58-63]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13 Government, health & education (non-market) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14 Other services (non-market) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15 Professional and support activities (NPISH) [69.1-82]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16 Government, health & education (NPISH) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17 Other services (NPISH) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Total consumption	0.550	0.412	0.319	0.382	0.206	0.233	0.276
Imports of goods and services	0.069	0.166	0.127	0.088	0.177	0.059	0.060
Taxes less subsidies on products	0.108	0.023	0.057	0.113	0.002	0.037	0.046
Taxes less subsidies on production	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Compensation of employees	0.249	0.372	0.457	0.367	0.423	0.600	0.495
Gross operating surplus	0.025	0.027	0.040	0.050	0.192	0.070	0.124
Total output	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Leontief Inverse

Whereas the matrix of coefficients helps to analyse direct relationships within the economy, the Leontief Inverse takes indirect relationships into account. In order to interpret the Leontief Inverse, it is useful to consider its derivation in terms of the identities in the tables. The main identity across the rows of the IOT is:

Total demand ($\mathbf{q}$) = Intermediate demand + Final demand ($\mathbf{f}$) (1)

Intermediate demand is the demand for products which are used-up or changed by producers. **Final demand** consists of final consumption expenditure (of households, NPISHs and general government), gross capital formation (gross fixed capital formation, acquisition less disposal of valuables and changes in inventories) and exports of goods and services.

We can use the intermediate demand part of the matrix of coefficients to rewrite equation (1) as:

$$\mathbf{q} = \mathbf{Aq} + \mathbf{f} \quad (2)$$

After a little matrix algebra (see Annex C), equation (2) can be re-expressed as:

$$\mathbf{q} = (\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{f} \quad (3)$$

where $\mathbf{I}$ is the identity matrix and $(\mathbf{I}-\mathbf{A})^{-1}$ is the Leontief Inverse. The Leontief Inverse thus describes the relationship between total demand and final demand. Table 6 shows a summary version of the Leontief Inverse for 2010. The right hand side of equation (3) may be decomposed as follows:

$$(\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{f} = \mathbf{f} + \mathbf{Af} + \mathbf{A}^2\mathbf{f} + \mathbf{A}^3\mathbf{f} + \mathbf{A}^4\mathbf{f} + \ldots. \quad (4)$$

The first term in this decomposition is final demand. The second term (the first order effects) gives the intermediate demand directly generated from the final demand. This direct intermediate demand generates further intermediate demand represented by the third term (the second order effects). This in turn generates even further intermediate demand represented by the fourth term, and so on. In other words, to produce $\mathbf{f}$ requires the consumption of $\mathbf{Af}$, which requires the further consumption of $\mathbf{A}^2\mathbf{f}$, which in turn requires the further consumption of $\mathbf{A}^3\mathbf{f}$… Equation (4) thus analyses the total demand required to satisfy final demand $\mathbf{f}$. In this way, the Leontief Inverse allows the analyst to study the impact of changes in final demand (for example, due to government programmes) on the economy as a whole. The linear nature of the model described by equation (4) makes it very flexible but unable to take into account factors such as economies of scale. The model is therefore an approximation. The 2010 Leontief inverse matrix is shown in table 6.

Table 6: Summary Leontief Inverse (product by product), 2010

Product	Intermediate consumption by products									
	1	2	3	4	5	6	7	8	9	10
	Agriculture [1-3]	Production [5-39]	Construction [41-43]	Distribution, transport, hotels and restaurants [45-56]	Information and communication [58-63]	Financial and insurance [64-66]	Real estate [68.1-2-68.3]	Professional and support activities [69.1-82]	Government, health & education [84-88]	Other services [90-97]
1 Agriculture [1-3]	1.118	0.025	0.006	0.006	0.002	0.001	0.001	0.002	0.002	0.002
2 Production [5-39]	0.318	1.419	0.226	0.145	0.088	0.054	0.045	0.073	0.087	0.070
3 Construction [41-43]	0.046	0.023	1.278	0.060	0.022	0.027	0.108	0.016	0.018	0.017
4 Distribution, transport, hotels and restaurants [45-56]	0.127	0.108	0.074	1.158	0.062	0.071	0.029	0.052	0.042	0.039
5 Information and communication [58-63]	0.027	0.027	0.029	0.052	1.092	0.081	0.030	0.057	0.026	0.054
6 Financial and insurance [64-66]	0.077	0.048	0.040	0.046	0.035	1.109	0.225	0.044	0.025	0.035
7 Real estate [68.1-2-68.3]	0.007	0.005	0.010	0.023	0.007	0.018	1.008	0.006	0.007	0.007
8 Professional and support activities [69.1-82]	0.071	0.089	0.156	0.150	0.161	0.190	0.086	1.270	0.070	0.150
9 Government, health & education [84-88]	0.007	0.009	0.015	0.017	0.016	0.018	0.027	0.035	1.083	0.015
10 Other services [90-97]	0.003	0.003	0.003	0.005	0.015	0.008	0.003	0.009	0.010	1.058
11 Production (non-market) [5-39]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12 Information and communication (non-market) [58-63]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13 Government, health & education (non-market) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14 Other services (non-market) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15 Professional and support activities (NPISH) [69.1-82]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16 Government, health & education (NPISH) [84-88]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17 Other services (NPISH) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Total	1.801	1.755	1.836	1.662	1.500	1.576	1.562	1.564	1.370	1.447

Product	Non-market - Product				NPISH - Product		
	11	12	13	14	15	16	17
	Production (non-market) [5-39]	Information and communication (non-market) [58-63]	Government, health & education (non-market) [84-88]	Other services (non-market) [90-97]	Professional and support activities (NPISH) [69.1-82]	Government, health & education (NPISH) [84-88]	Other services (NPISH) [90-97]
1 Agriculture [1-3]	0.010	0.001	0.002	0.004	0.001	0.002	0.002
2 Production [5-39]	0.563	0.065	0.102	0.201	0.040	0.067	0.060
3 Construction [41-43]	0.042	0.015	0.028	0.040	0.007	0.017	0.014
4 Distribution, transport, hotels and restaurants [45-56]	0.076	0.047	0.056	0.043	0.028	0.052	0.023
5 Information and communication [58-63]	0.028	0.136	0.029	0.070	0.030	0.028	0.044
6 Financial and insurance [64-66]	0.029	0.025	0.028	0.025	0.028	0.013	0.024
7 Real estate [68.1-2-68.3]	0.007	0.009	0.016	0.017	0.005	0.004	0.003
8 Professional and support activities [69.1-82]	0.170	0.220	0.099	0.169	0.158	0.070	0.108
9 Government, health & education [84-88]	0.009	0.018	0.125	0.007	0.024	0.097	0.017
10 Other services [90-97]	0.006	0.097	0.010	0.046	0.003	0.009	0.126
11 Production (non-market) [5-39]	1.000	0.000	0.000	0.000	0.000	0.000	0.000
12 Information and communication (non-market) [58-63]	0.000	1.000	0.000	0.000	0.000	0.000	0.000
13 Government, health & education (non-market) [84-88]	0.000	0.000	1.000	0.000	0.000	0.000	0.000
14 Other services (non-market) [90-97]	0.000	0.000	0.000	1.000	0.000	0.000	0.000
15 Professional and support activities (NPISH) [69.1-82]	0.000	0.000	0.000	0.000	1.000	0.000	0.000
16 Government, health & education (NPISH) [84-88]	0.000	0.000	0.000	0.000	0.000	1.000	0.000
17 Other services (NPISH) [90-97]	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Total	1.940	1.633	1.494	1.622	1.323	1.358	1.422

Multiplier Analysis

The Leontief Inverse provides the central tool for multiplier analysis, which studies the effect of changes in final demand on output and related aspects of the economy. These effects have three different economic drivers:

- **Direct:** This is the immediate effect caused directly by the change in final demand
- **Indirect:** This is the subsequent effect caused by the consequent changes in intermediate demand
- **Induced:** This is the effect attributable to the ensuing change in compensation of employees and other incomes, which may cause further spending and hence further changes in final demand

Type I multipliers cover direct and indirect effects only, and therefore underestimate the effect on the economy. **Type II multipliers** cover induced effects as well, under the implicit assumption that final consumers do no change their final consumption patterns in response to changes in income. Because this assumption is very unrealistic and because of the lack of consistent employment numbers, only Type I multipliers have been calculated for this publication.

Different multipliers measure the effect on different policy targets:

- **Output multipliers** measure the effect on total economic output caused by a one unit change in the final demand of a specific product. They are calculated as the column totals of the Leontief Inverse
- We have not produced employment multipliers because we do not have employment numbers coherent with the balanced compensation of employees data for 2010. However, we have produced **employment cost multipliers** which measure the relative effect on total compensation of employees resulting from a change in demand for a product that results in a direct increase in compensation of employees of one unit; the multiplier value therefore consists of the direct one unit compensation of employees increase plus the indirect effect. They are calculated as:

  $\mathbf{e}\,(\mathbf{I}-\mathbf{A})^{-1} / \mathbf{e}$ (5)

  where $\mathbf{e}$ is the row of the matrix of coefficients corresponding to compensation of employees, and / denotes to element-wise division
- Alongside those for output and employment cost, the detailed 2010 IOAT data tables include **GVA multipliers**, which measure the effect on total GVA caused by a one unit change in the GVA of a specific product
- All multipliers calculated using (5) are denoted simply as 'multipliers' in the 'Multipliers and effects' table in the IOAT detailed tables file
- Further multipliers by substituting $\mathbf{e}$ in equation (5) with appropriate coefficients (for examples see Scottish Government, 2011). It may be noted that output multipliers can be calculated in this way by substituting a row vector of ones for $\mathbf{e}$
- If element-wise division is excluded from (5), instead of measuring relative multiplier effects, the absolute effects on given components relating to changes in unit changes in final demand can be expressed. They are calculated as

  $\mathbf{e}\,(\mathbf{I}-\mathbf{A})^{-1}$ (6)

  by substituting $\mathbf{e}$ in equation (6) with the row of the matrix of coefficients corresponding to the element in question. The detailed 2010 IOAT tables include absolute employment cost and GVA effects; these are denoted simply as 'effects' in the 'Multipliers and effects' table IOAT detailed tables file

Primary Input Content of Final Demand

The Leontief Inverse can also be used to analyse the content of final demand in terms of primary inputs. These are inputs to the production process which are not the outputs of other domestic producers: imports; taxes less subsidies on products and production; compensation of employees; and gross operating surplus. Indirectly, all final demand is ultimately satisfied by primary inputs. Thus the sum of all primary inputs is equal to the sum of all final demand. This holds at both basic prices and purchasers' prices. They formula for this analysis is:

$$\mathbf{P}\,(\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{F}$$

where $\mathbf{P}$ is that part of the matrix of coefficients corresponding to primary inputs, and
$\mathbf{F}$ is the matrix of final demand at basic prices.

Table 7 shows the decomposition of final demand in terms of primary inputs at basic prices (i.e. excluding taxes less subsidies on products and imports directly consumed by the components of

final demand) for 2010, expressed as percentages of each final demand component. Table 8 shows the same decomposition expressed in monetary values. Rows of totals relating to taxes less subsidies on products and imports directly consumed by the components of final demand have been appended to show totals in purchasers' prices. From this table GDP at market prices (£1,485,615m) can be derived by deducting the direct (£181,667m) and indirect (£298,454m) contributions of imports to final demand from the grand total (£1,965,736m).

The primary input content of final demand can also be used to partition GVA into **direct** and **indirect** effects. The direct effects show how much GVA is directly attributable to the different components of final demand. The formula for their calculation is:

$$\mathbf{i}\,\mathbf{F} \qquad (7)$$

where $\mathbf{i}$ is the row vector of column sums of that part of the matrix of coefficients corresponding to the income measure of GVA (compensation of employees, gross operating surplus, taxes less subsidies on production). The contribution of each product IOG to direct GVA is obtained by diagonalising the vector $\mathbf{i}$ in the formula.

The indirect effects show how much GVA is attributable to all the subsequent demand following the direct effects. They are calculated by subtracting the direct effects from the total effects. The formula for the total effects is:

$$\mathbf{i}\,(\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{F} \qquad (8)$$

The contribution of each product IOG to total GVA is obtained by diagonalising the vector $\mathbf{i}\,(\mathbf{I}-\mathbf{A})^{-1}$ in the formula.

Figure 5 depicts the percentages of direct and indirect GVA by component of final demand.

Table 7: Primary input content of final demand at basic prices (as a percentage), 2010

Percentages

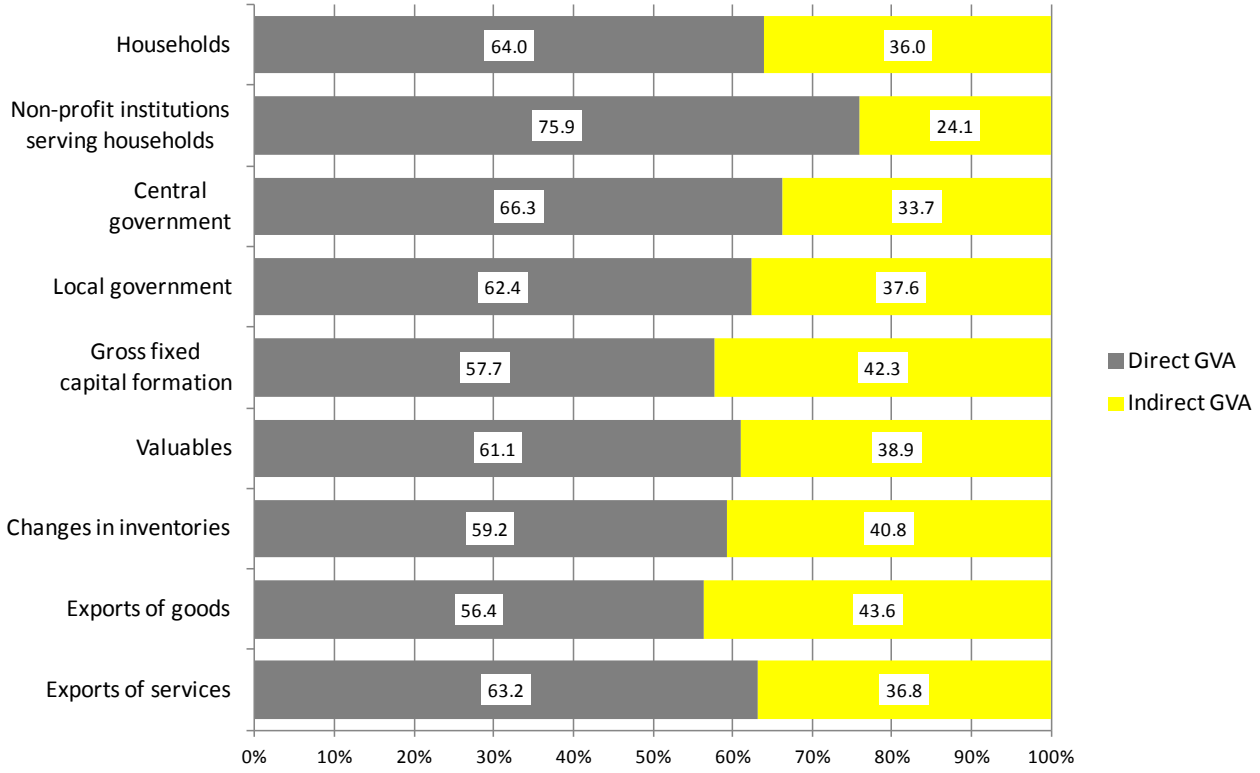
	Final consumption expenditure				Gross capital formation			Exports		
Primary input	Households	NPISHs	Central government	Local government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total final demand
Imports of goods and services	14.7	9.3	20.7	12.7	16.5	16.7	51.4	31.1	15.7	17.7
Taxes less subsidies on products	2.7	4.1	6.3	6.7	2.9	1.9	-2.7	1.7	2.7	3.4
Taxes less subsidies on production	1.8	0.3	0.3	0.4	1.4	3.7	-0.1	1.1	1.2	1.3
Compensation of employees	40.7	71.6	59.4	67.1	47.9	50.8	53.1	40.8	51.3	47.6
Gross operating surplus	40.1	14.6	13.3	13.0	31.3	26.9	-1.7	25.2	29.0	30.0
Total primary input	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Table 8: Primary input content of final demand at purchasers' prices, 2010

£ million

	Final consumption expenditure				Gross capital formation			Exports		
Primary input	Households	NPISHs	Central government	Local government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total final demand
Imports of goods and services	105,581	3,487	42,395	16,683	29,315	34	640	72,467	27,851	298,454
Taxes less subsidies on products	19,731	1,556	12,928	8,856	5,084	4	-34	4,040	4,827	56,992
Taxes less subsidies on production	13,180	115	581	565	2,426	8	-1	2,629	2,128	21,629
Compensation of employees	293,029	26,904	121,858	88,212	85,034	104	662	95,206	90,787	801,796
Gross operating surplus	288,786	5,499	27,379	17,082	55,495	55	-22	58,818	51,405	504,498
Total primary input at basic prices	720,306	37,562	205,140	131,398	177,355	205	1,245	233,160	176,998	1,683,369
Imports in final demand	119,811	0	0	0	33,865	12	690	24,515	2,774	181,667
Taxes less subsidies in final demand	80,917	0	0	0	9,936	34	-9	7,568	2,254	100,700
Total primary input at purchasers' prices	921,034	37,562	205,140	131,398	221,156	251	1,926	265,243	182,026	1,965,736

Figure 5: Composition of final demand in terms of direct and indirect GVA (%), 2010



Industrial Analysis in terms of Primary Inputs

So far, the analysis in this publication has been based on product by product tables. Similar analyses can also be produced based on industry by industry tables. The methodology to produce these can be found in Eurostat (2008, ch11) and Scottish Government (2011). The compilation of industry by industry tables presents certain conceptual issues not encountered in product by product tables. Most notably:

- Assumptions are required relating to the structure of demand of industry outputs. However, demand is by nature product-driven rather than industry-driven
- The input-output model is built on an assumption of homogeneity within IOGs. However, industry IOGs are mostly less homogeneous than their corresponding product IOGs

Eurostat (2008, p301) explains why ESA 1995 (Eurostat, 1996) favours product by product tables for economic analysis.

An alternative approach is to use product by product tables for an industry-based primary input analysis as follows:

- The **Market Share Mix hypothesis** (MSM) is introduced. This uses the assumption that the contribution (market share) of each industry to the supply of a particular product remains a constant proportion of that product's supply

- The product by product Leontief Inverse is used to determine the product output generated by each component of final demand, as was done for the analysis of primary inputs, above
- The MSM is used to obtain the equivalent industry output
- The product by industry domestic use table at basic prices is used to generate a new (product by industry) matrix of coefficients for each industry
- The amount of primary inputs generated by each component of final demand can then be calculated

Tables 9 and 10 show the result of this industrial analysis for 2010 as percentages and monetary values, respectively. The matrix algebra for this analysis is detailed in Annex D.

Table 9: Industrial analysis of primary inputs (as a percentage), 2010

Percentages

	Final consumption expenditure				Gross capital formation			Exports		
Primary input	Households	NPISHs	Central government	Local government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total final demand
Imports of goods and services	35.4	1.2	14.2	5.6	9.8	0.0	0.2	24.2	9.3	100.0
Taxes less subsidies on products	34.6	2.7	22.7	15.5	8.9	0.0	-0.1	7.1	8.5	100.0
Taxes less subsidies on production	60.7	0.5	2.8	2.6	11.1	0.0	0.0	12.2	10.1	100.0
Compensation of employees	36.6	3.4	15.2	11.0	10.6	0.0	0.1	11.9	11.3	100.0
Gross operating surplus	57.2	1.1	5.4	3.4	11.0	0.0	0.0	11.7	10.2	100.0

Table 10: Industrial analysis of primary inputs, 2010

£ million

	Final consumption expenditure				Gross capital formation			Exports		
Primary input	Households	NPISHs	Central government	Local government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	Total final demand
Imports of goods and services	105,777	3,489	42,401	16,689	29,338	36	641	72,315	27,768	298,454
Taxes less subsidies on products	19,725	1,557	12,931	8,858	5,093	4	-34	4,038	4,820	56,992
Taxes less subsidies on production	13,119	116	601	569	2,405	7	0	2,629	2,183	21,629
Compensation of employees	293,322	26,893	121,844	88,199	84,770	104	656	95,280	90,728	801,796
Gross operating surplus	288,537	5,500	27,367	17,061	55,622	55	-21	58,904	51,472	504,498
Total	720,480	37,555	205,145	131,376	177,228	206	1,242	233,166	176,971	1,683,369

Summary of changes in the 2010 Input-Output Analytical Tables

Since the previous publication of IOATs for reference year 2005 (ONS 2011a), a single major change has been introduced, which has been to make the 2010 IOATs consistent with SIC 2007 for industries and CPA 2008 for products, though still on a ESA 95 basis. Future IOATs will remain consistent with these SIC 2007 / CPA 2008, but will be based on the new European System of Accounts 2010 (ESA 10), which is being adopted in the UK National Accounts from September 2014 onwards.

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Annex A: Definition of Input-Output Groups

IOG code (=SIC 2007	Non-market split?	NPISH split	Description	NACE Rev.2 industrial classifications						
				A3	A10	A21	A38	A64	A88	A114
01			Crop and animal production, hunting and related service activities	1	1	A	A	1	1	1
02			Forestry and logging					2	2	2
03			Fishing and aquaculture					3	3	3
05			Mining of coal and lignite	2	2	B	B	4	5	4
06			Extraction of crude petroleum and natural gas						6	5
07			Mining of metal ores						7	6
08			Other mining and quarrying						8	7
09			Mining support service activities						9	8
10.1			Processing and preserving of meat and production of meat products			C	CA	5	10	9
10.2-3			Processing and preserving of fish, crustaceans, molluscs, fruit and ve							10
10.4			Vegetable and animal oils and fats							11
10.5			Dairy products							12
10.6			Grain mill products, starches and starch products							13
10.7			Bakery and farinaceous products							14
10.8			Other food products							15
10.9			Prepared animal feeds							16
11.01-6			Alcoholic beverages						11	17
11.07			Soft drinks; production of mineral waters and other bottled waters							18
12			Tobacco products						12	19
13			Textiles				CB	6	13	20
14			Wearing apparel						14	21
15			Leather and related products						15	22
16			Wood and products of wood and cork, except furniture; Articles of st				CC	7	16	23
17			Paper and paper products					8	17	24
18			Printing and reproduction of recorded media					9	18	25
19			Coke and refined petroleum products				CD	10	19	26
20A			Industrial gases, inorganics and fertilisers (all inorganic chemicals) -				CE	11	20	27
20B			Petrochemicals - 20.14/16/17/60							28
20C			Dyestuffs, agro-chemicals - 20.12/20							29
20.3			Paints, varnishes and similar coatings, printing ink and mastics							30
20.4			Soap and detergents, cleaning and polishing preparations, perfumes		2					31
20.5			Other chemical products							32
21			Basic pharmaceutical products and pharmaceutical preparations				CF	12	21	33
22			Rubber and plastic products				CG	13	22	34
23OTHER			Glass, refractory, clay, porcelain, ceramic, stone products - 23.1-4/7					14	23	35
23.5-6			Cement, lime, plaster and articles of concrete, cement and plaster						23	36
24.1-3			Basic iron and steel				CH	14	24	37
24.4-5			Other basic metals and casting					15		38
25OTHER			Fabricated metal products, excluding weapons and ammunition - 25.					16	25	39
25.4			Weapons and ammunition							40
26			Computer, electronic and optical products				CI	17	26	41
27			Electrical equipment				CJ	18	27	42
28			Machinery and equipment n.e.c.				CK	19	28	43
29			Motor vehicles, trailers and semi-trailers				CL	20	29	44
30.1			Building of ships and boats					21	30	45
30.3			Air and spacecraft and related machinery							46
30OTHER			Other transport equipment - 30.2/4/9							47
31			Furniture				CM	22	31	48
32			Other manufacturing						32	49
33.15			Repair and maintenance of ships and boats					23	33	50
33.16			Repair and maintenance of aircraft and spacecraft							51
33OTHER			Rest of repair; Installation - 33.11-14/17/19/20							52
35.1			Electric power generation, transmission and distribution			D	D	24	35	53
35.2-3			Gas; distribution of gaseous fuels through mains; Steam and air con							54
36			Water collection, treatment and supply			E	E	25	36	55
37			Sewerage					26	37	56
38			Waste collection, treatment and disposal activities; materials recover						38	57
39			Remediation activities and other waste management services						39	58
41			Construction of buildings		3	F	F	27	41	59
42			Civil engineering						42	60
43			Specialised construction activities						43	61

45			Wholesale and retail trade and repair of motor vehicles and motorcy					28	45	62
46			Wholesale trade, except of motor vehicles and motorcycles			G	G	29	46	63
47			Retail trade, except of motor vehicles and motorcycles					30	47	64
49.1-2			Rail transport					31	49	65
49.3-5			Land transport services and transport services via pipelines, excludin							66
50			Water transport		4	H	H	32	50	67
51			Air transport					33	51	68
52			Warehousing and support activities for transportation					34	52	69
53			Postal and courier activities					35	53	70
55			Accommodation			I	I	36	55	71
56			Food and beverage service activities						56	72
58			Publishing activities					37	58	73
59	Yes		Motion picture, video and TV programme production, sound recordin				JA	38	59	74
60	Yes		Programming and broadcasting activities		5	J			60	75
61			Telecommunications				JB	39	61	76
62			Computer programming, consultancy and related activities				JC	40	62	77
63			Information service activities						63	78
64			Financial service activities, except insurance and pension funding					41	64	79
65.1-2			Insurance and reinsurance, except compulsory social security		6	K	K	42	65	80
65.3			Pension funding					42		81
66			Activities auxiliary to financial services and insurance activities					43	66	82
68.1-2			Buying and selling, renting and operating of own or leased real estat							83
68.2IMP			Imputed rental		7	L	L	44	68	84
68.3			Real estate activities on a fee or contract basis							85
69.1			Legal activities					45	69	86
69.2			Accounting, bookkeeping and auditing activities; Tax consultancy				MA			87
70			Activities of head offices; Management consultancy activities	3					70	88
71			Architectural and engineering activities; Technical testing and analys			M		46	71	89
72		Yes	Scientific research and development				MB	47	72	90
73			Advertising and market research					48	73	91
74		Yes	Other professional, scientific and technical activities		8		MC	49	74	92
75			Veterinary activities						75	93
77			Rental and leasing activities					50	77	94
78			Employment activities					51	78	95
79			Travel agency, tour operator and other reservation service and relat			N	N	52	79	96
80			Security and investigation activities						80	97
81			Services to buildings and landscape activities					53	81	98
82		Yes	Office administrative, office support and other business support activ						82	99
84	Yes		Public administration and defence; Compulsory social security			O	O	54	84	100
85	Yes	Yes	Education			P	P	55	85	101
86	Yes	Yes	Human health activities		9	Q	QA	56	86	102
87	Yes	Yes	Residential care activities				QB	57	87	103
88	Yes	Yes	Social work activities without accommodation						88	104
90	Yes	Yes	Creative, arts and entertainment activities						90	105
91	Yes	Yes	Libraries, archives, museums and other cultural activities			R	R	58	91	106
92			Gambling and betting activities						92	107
93	Yes	Yes	Sports activities and amusement and recreation activities					59	93	108
94		Yes	Activities of membership organisations		10			60	94	109
95			Repair of computers and personal and household goods			S	S	61	95	110
96		Yes	Other personal service activities					62	96	111
97			Activities of households as employers of domestic personnel			T	T	63	97	112
98*			Undifferentiated goods- and services-producing activities of private h						98	113
99*			Activities of extra-territorial organisations and bodies			U	U	64	99	114

*Not explicitly measured

Annex B: Matrix Algebra for the Input-Output Table

The following notation is used:

M is the supply table excluding the columns for imports, margins and taxes less subsidies
q is the vector of domestic supply of product IOGs at basic prices
U is the domestic use table at basic prices, including final demand and primary inputs
H is the hybrid technology assumption matrix

Let:

$\mathbf{M}_1 = \mathbf{M} \# \mathbf{H}$ where # denotes element-wise multiplication
$\mathbf{M}_2 = \mathbf{M} - \mathbf{M}_1$
$\mathbf{C}_1 = \mathbf{M}_1 (\text{diag}(\mathbf{1}' \mathbf{M}_1))^{-1}$ where **1** is a column vector of ones
$\mathbf{D}_2 = \mathbf{M}_2' (\text{diag}(\mathbf{q}))^{-1}$
$\mathbf{R} = \mathbf{C}_1^{-1} (\mathbf{I} - \text{diag}(\mathbf{D}_2' \mathbf{1})) + \mathbf{D}_2$
$\mathbf{B} = \mathbf{U}\ \text{diag}(\mathbf{U}\ \mathbf{1})^{-1}$

Now let $\mathbf{B}^*$ be matrix **B** without the columns corresponding to final demand:

$\mathbf{A}^* = \mathbf{B}^* \mathbf{R}$

where $\mathbf{A}^*$ is the matrix of coefficients, including the rows corresponding to primary inputs. Finally,

$\mathbf{S} = \mathbf{A}^* \# (\mathbf{1}\ \mathbf{1}' \mathbf{M}')$

where **S** is the input-output table excluding the columns corresponding to final demand (the components of final demand may be copied from the domestic use table at basic prices).

Annex C: Derivation of the Leontief Inverse

We recall equation (2):

$$\mathbf{q} = \mathbf{Aq} + \mathbf{f}$$

where $\mathbf{q}$ is the vector of total demand,
$\mathbf{f}$ is the vector of final demand (summed over components), and
$\mathbf{A}$ is the main body of the matrix of coefficients (the intermediate demand part only).

Subtracting $\mathbf{Aq}$ from both sides gives:

$$\mathbf{q} - \mathbf{Aq} = \mathbf{f}$$

Which may be rewritten as:

$$(\mathbf{I} - \mathbf{A})\,\mathbf{q} = \mathbf{f}$$

where $\mathbf{I}$ is the identity matrix.
$(\mathbf{I} - \mathbf{A})$ is called the *Leontief matrix*.

Pre-multiplying by the Leontief inverse (the inverse of the Leontief matrix) gives equation (3):

$$\mathbf{q} = (\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{f}$$

Annex D: Matrix Algebra for Industrial Analysis

The following notation is used:

M is the supply table excluding the columns for imports, margins and taxes less subsidies
q is the vector of domestic supply of product IOGs at basic prices
g is the row vector of output of industry IOGs
F is the matrix of final demand at basic prices
P is that part of the matrix of coefficients corresponding to primary inputs
$(\mathbf{I}-\mathbf{A})^{-1}$ is the (product by product) Leontief inverse

Let:

$\mathbf{D}^* = \mathbf{M}' (\text{diag}(\mathbf{q}))^{-1}$
$\mathbf{Q}^* = (\mathbf{I}-\mathbf{A})^{-1}\ \mathbf{F}$
$\mathbf{G}^* = \mathbf{D}^*\ \mathbf{Q}^*$

The industrial analysis of primary inputs is then given by:

$(\mathbf{P} / (\mathbf{1}\ \mathbf{g}))\ \mathbf{G}^*$