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Sensitivity Factors of Aircraft Mass for the Conceptual Design

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1 Introduction

1.1 About main parameter:

Force to ensure the flight $T \Rightarrow L = W = m\,g$ (for cruise flight)

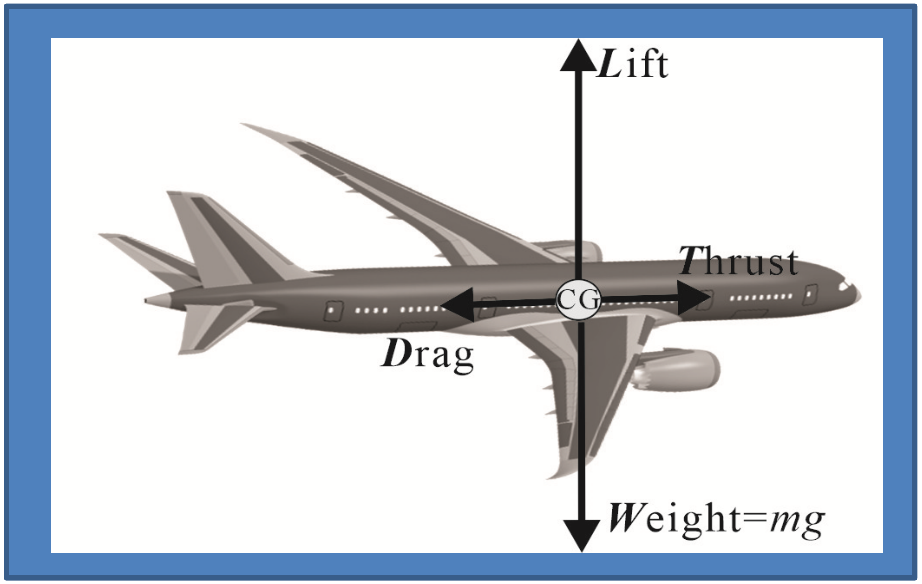


Figure 1. The main forces acting on the aircraft in cruising flight

Weight plays a crucial role in the creation of any aircraft and in the process of its operation. Since the primary is mass, the main parameter used in the work will be mass and not weight.

$L = n\,m\,g$ (for the General case)

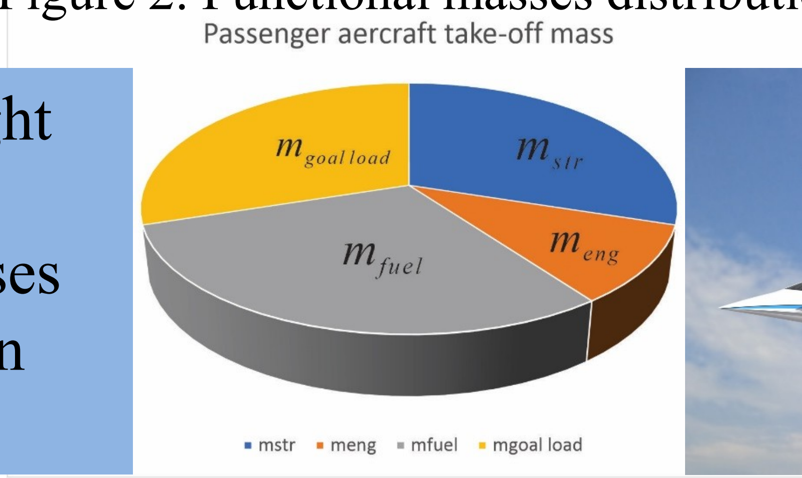
1.2 The takeoff aircraft mass

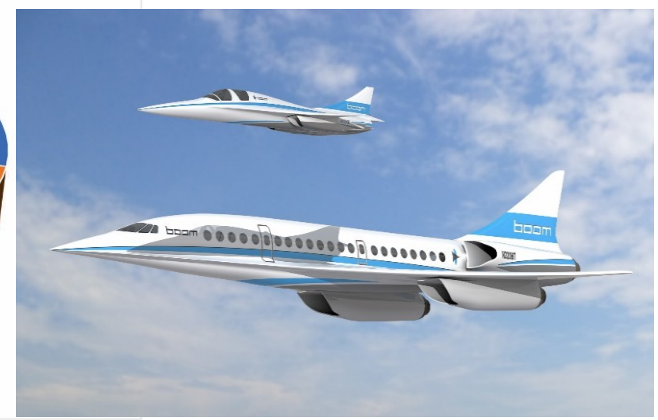
$$m_{TO} = m_{str} + m_{eng} + m_{fuel} + m_{target} \quad (1)$$

where m_i – the components of functional masses: m_{str} is the mass of structures (mass subsystem implements the aerodynamic principle of flight: wing, fuselage, tail, control system, landing gear); m_{eng} – mass of the engine (subsystem, which provides the creation of thrust); m_{fuel} is fuel mass; m_{target} – target mass – it is the mass that is associated with the appointment of aircraft: commercial load (payload) and service load, included the equipment payload, the equipment providing reliable operation of the aircraft, crew; thus $m_{target} = m_{\text{payload}} + m_{\text{service}}$.

Figure 2. Functional masses distribution for passenger aircraft

Semi-empirical weight formulas are usually used to find the masses m_i at the initial design stages [1-3]. But…





2 The basics of the approach. The transition to a new design solution will be reduced to the formation of initial (partial) changes in the mass of the functional parts of the aircraft of the basic project separately for each new technical solution. Any new technical solution for any part of the aircraft begins to be worked out with the "frozen" values of the remaining parts, including the entire mass of the aircraft.

Problem formulation

Changes in the properties of the aircraft and its functions at first with the initial mass of the aircraft will require partial changes in the masses of the functional elements of the system, which will then develop into a General (final) change in the mass of the aircraft.

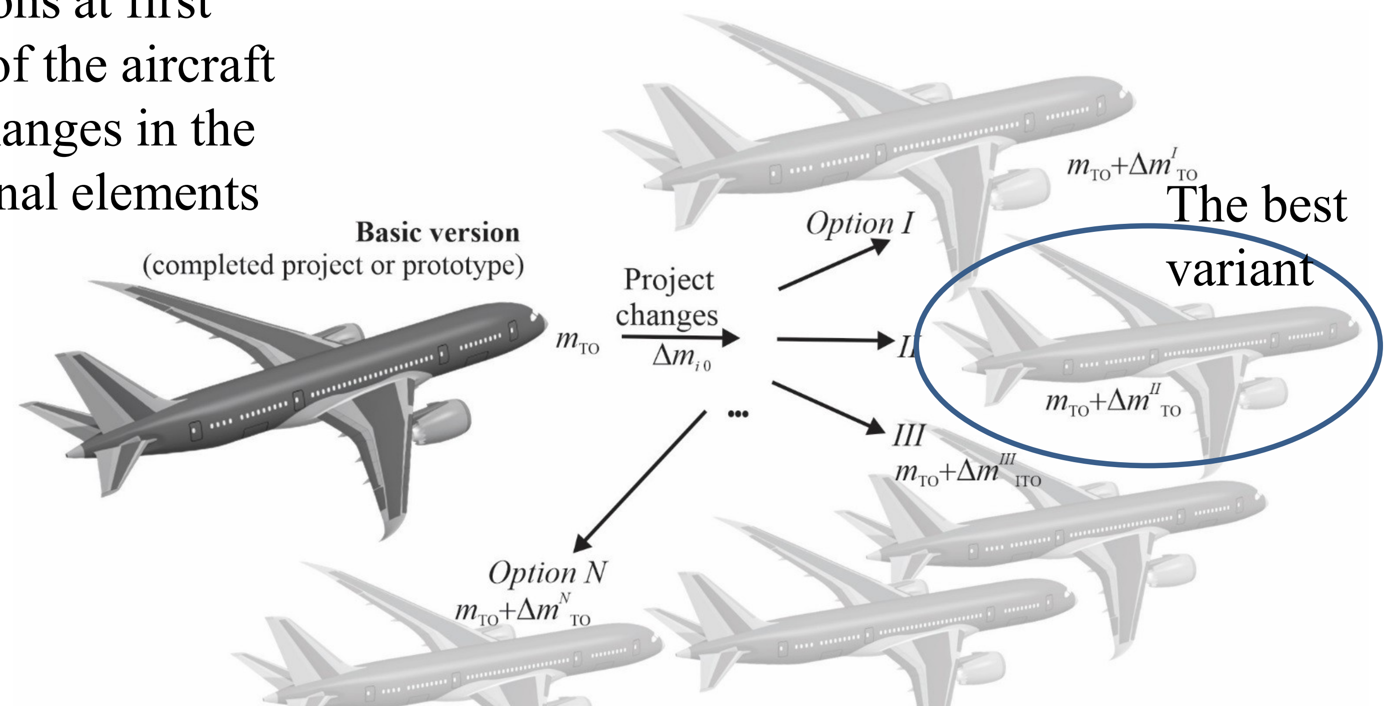


Figure 3. Search for a new project solution based on the basic version

The method is based on the equation of the airplane existence

$$m_{TO} = m_{target} / (1 - \bar{m}_{str} - \bar{m}_{eng} - \bar{m}_{fuel}) \qquad (2)$$

The use of the relative masses $\bar{m}_i = m_i / m_{TO}$ gives a great advantage, since they do not depend much on the mass of the aircraft, but are determined by the purpose of the aircraft and the level of technical condition. The mass m_{target} is determined by the purpose of the aircraft.

This equation can answer the question: what set of properties can be implemented in the project under consideration at a specific level of technology development.

To assess the success of the transition from the original selected aircraft project or from an existing prototype to a new version, it is necessary to select the most important and economically justified criterion. The simplest and most frequently used criterions can be

1. m_{TO} – min

2. m_{fuel} – min in two variants:

or $m_{fuel} / (m_{\text{payload}} R)$ – min; or $m_{fuel} / (nR)$ – min

There are two approaches: Scenario 1 – despite of changes in the initial mass, the payload and flight characteristics must remain unchanged; Scenario 2 – the task is to change the consumer properties of the base project aircraft (payload, flight-take-off and landing characteristics, indicators of technical excellence, component solutions), which will require initial changes in functional parts. Considered method can be as tool to solve both of these problems. In the synthesis of a new project, any changes will be considered as equivalent change in mass.

The main focus in this work is on the consideration of the first direction of application – that is, any changes in the basic project are performed within the Framework of the constant payload mass and the preservation of the aircraft's flight characteristics.

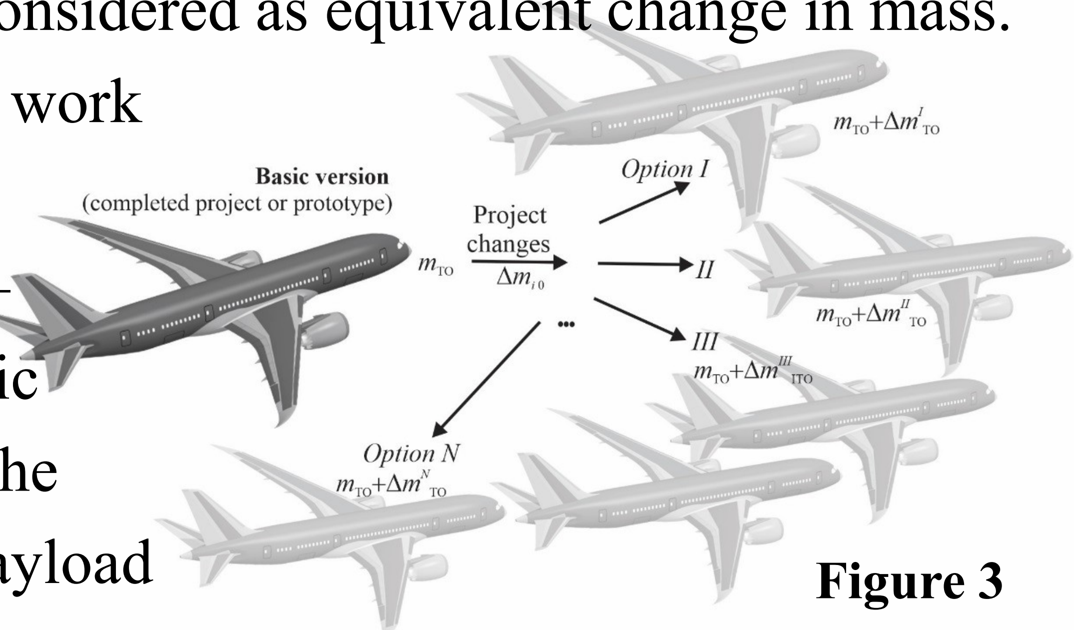


Figure 3

3 Sensitivity factors of mass by the initial mass change

3.1 SFM without taking into account the location of $\Delta m_{i\,0}$

According to the definition of the weight (mass) grow factor is

$$\mu_{m\,i} = dm_{TO}/dm_{i\,0} \approx \Delta m_{TO}/\Delta m_{i\,0} \qquad (3)$$

But it is more correct to call it *Sensitivity Factor of Mass* (SFM) where $\Delta m_{i\,0}$ is initial mass change of the i^{th} component.

$$m_{TO} = m_{str\,new} + m_{eng\,new} + m_{fuel\,new} + m_{target} + \Delta m_{i\,0} \qquad (4)$$

In order to find $\mu_{m\,i}$, first take the derivative of the expression (4) with respect m_{TO}

$$1 = d(m_{str\,new} + m_{eng\,new} + m_{fuel\,new} + m_{target})/d(m_{TO}) + d(m_{i\,0})/d(m_{TO}) \qquad (5)$$

We can accept $dm_{i\,new}/d(m_{TO}) \approx \Delta m_i/\Delta m_{TO} \approx \bar{m}_i$. The result is that

$$d(m_{TO\,new})/d(m_{i\,0}) \approx \Delta m_{TO}/\Delta m_{i\,0} = 1/(1 - \bar{m}_{str} - \bar{m}_{eng} - \bar{m}_{fuel}) \qquad (6)$$

This suggests that

$$\mu_{m\,i} = 1/(1 - \bar{m}_{str} - \bar{m}_{eng} - \bar{m}_{fuel}) = 1/\bar{m}_{target} \qquad (7)$$

Similar formulas were obtained in the works [4-7].

Taking into account the accepted linear dependence

$$\mu_{m\,i} = dm_{TO}/dm_{i\,0} \approx \Delta m_{TO}/\Delta m_{i\,0}$$

we can consider with a sufficient degree of accuracy the variation of the parameter $\Delta m_{i\,0}$ in the range of no more than10-15% - Figure 4.

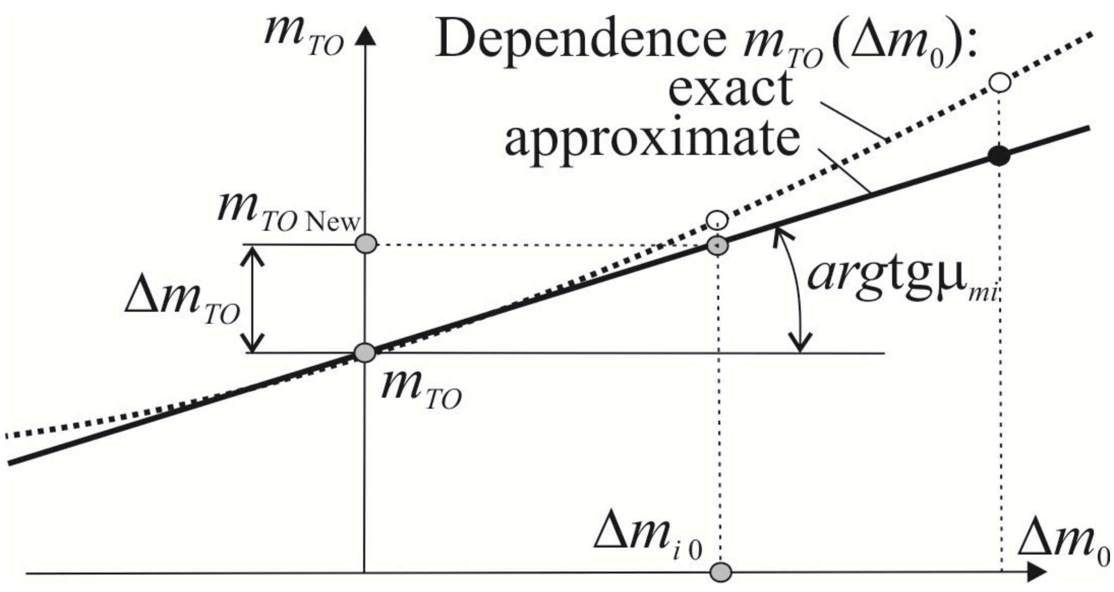


Figure 4. Dependence of the take-off mass on the initial changes $\Delta m_{i\,0}$

Thus, in these studies, the factor $\mu_{m\,i}$ was constant, since it was not taken into account that the relative mass of the components in which the mass change occurred should be corrected.

$$dm_{i\,new}/d(m_{TO})=(\Delta m_i+\Delta m_{i\,0})\Delta m_{TO}=\bar{m}_i+\bar{\Delta}m_{i\,0} \quad (8)$$

$$\mu_{m\,i}=1/(1-\bar{m}_{str}-\bar{m}_{e\,eng}-\bar{m}_{fuel}-\bar{\Delta}m_{i\,0})=1/(\bar{m}_{target}-\bar{\Delta}m_{i\,0}) \quad (9)$$

As we can see in contrast to the traditional form of the writing $\mu_{m\,i}$, in this case there is another term that depends on the value of the initial change $\bar{\Delta}m_{i\,0}$.This feature was first discovered in the works of Prof. Gogolin from the Kazan aviation Institute [8-9]. By the way if we want to keep in the process after initial changes the mass of the payload and the layout of the target load in the previous level (Scenario 1), then the geometry of the fuselage of the passenger aircraft that houses the target load should remain unchanged.

To fulfill this condition, in work [8] it was proposed to present the two components of the mass associated with $m_{e\ eng}$ and m_{fuel} in the form, which in variant of relative masses could be writing so

$$\bar{m}_{eng}+\bar{m}_{fuel}=(\bar{m}_{eng}+\bar{m}_{fuel})C_{D\ fus}/C_D+(\bar{m}_{eng}+\bar{m}_{fuel})(1-C_{D\ fus}/C_D) \quad (10)$$

where $C_{D\ fus}$ and C_D are drag coefficients of the fuselage and the entire aircraft. In this case, the thrust of the power plant is determined by the cruising mode.

The first component of the right part (10) determines the mass cost for the thrust and fuel during transporting of the fuselage of the original (unchanged) dimensions in cruising flight, so it does not change when implementing $\Delta m_{i\ 0}$ and, accordingly, $d((m_{eng}+m_{fuel})C_{D\ fus}/C_D)/d(m_{TO})=0$, in contrast to the second member. And then

$$d((m_{eng}+m_{fuel})/d(m_{TO})=(1-C_{D\ fus}/C_D)(\bar{m}_{eng}+\bar{m}_{fuel}) \quad (11)$$

With this in mind, the expression (9) will take the form

$$\mu_{m\ i}=1/(\bar{m}_{target}-\Delta\bar{m}_{i\ 0}+(\bar{m}_{eng}+\bar{m}_{fuel})C_{D\ fus}/C_D) \quad (12)$$

The general formula for the determining SFM by the changes in the mass of all its functional components is

$$\mu_m = \Delta m_{TO}/(\sum \Delta m_{i\,0}) = 1/(\,\bar{m}_{target} - \bar{\Delta} m_{str\,0} - (\,\bar{\Delta} m_{eng\,0} + \,\bar{\Delta} m_{fuel\,0}) (1 - C_{D\,fus}/C_D) + (\,\bar{m}_{eng} + \,\bar{m}_{fuel}) C_{D\,fus}/C_D) \quad (13)$$

The general changes in the structure of the aircraft mass will be writing

$$\Delta m_{TO} = \mu_m \sum \Delta m_{i\,0}); \quad (14)$$

$$\Delta m_{target} = \Delta m_{target\,0}; \quad (15)$$

$$\Delta m_{str} = \Delta m_{str\,0} + (\,\bar{m}_{str} + \,\bar{\Delta} m_{str\,0})\, \Delta m_{TO}\,; \quad (16)$$

$$\Delta m_{eng} = \Delta m_{eng\,0} + (\,\bar{m}_{eng} + \,\bar{\Delta} m_{eng\,0})\,)(1 - C_{D\,fus}/C_D)\, \Delta m_{TO} \quad (17)$$

$$\Delta m_{fuel} = \Delta m_{fuel\,0} + (\,\bar{m}_{fuel} + \,\bar{\Delta} m_{fuel\,0})\,)(1 - C_{D\,fus}/C_D)\, \Delta m_{TO} \quad (18)$$

Only if the sum of partial changes is relatively small

$$\bar{\Delta} m_{str\,0} + (\,\bar{\Delta} m_{eng\,0} + \,\bar{\Delta} m_{fuel\,0})\,(1 - C_{D\,fus}/C_D) << \bar{m}_{target} + (\,\bar{m}_{eng} + \,\bar{m}_{fuel}) C_{D\,fus}/C_D),$$

then

$$\mu_{target} = \mu_{str} = \mu_{eng} = \mu_{fuel} = \mu_m = 1/(\,\bar{m}_{target} + (\,\bar{m}_{eng} + \,\bar{m}_{fuel}) C_{D\,fus}/C_D) \quad (19)$$

3.2 SFM taking into account the distance from CG

SFM were obtained for the aircraft, which was considered as a material point. But the initial change in mass at a significant distance from the center of mass of the aircraft will cause the appearance of concomitant changes in the mass of parts of the aircraft, which will also have a private initial character.

These changes can be caused by changes in the load, aerodynamics, displacement of the CG and, as a result, changes in the parameters of the tail. This task will be relevant, for example, when assessing the impact of cargo rearrangement on the TOGM – Figure 5.

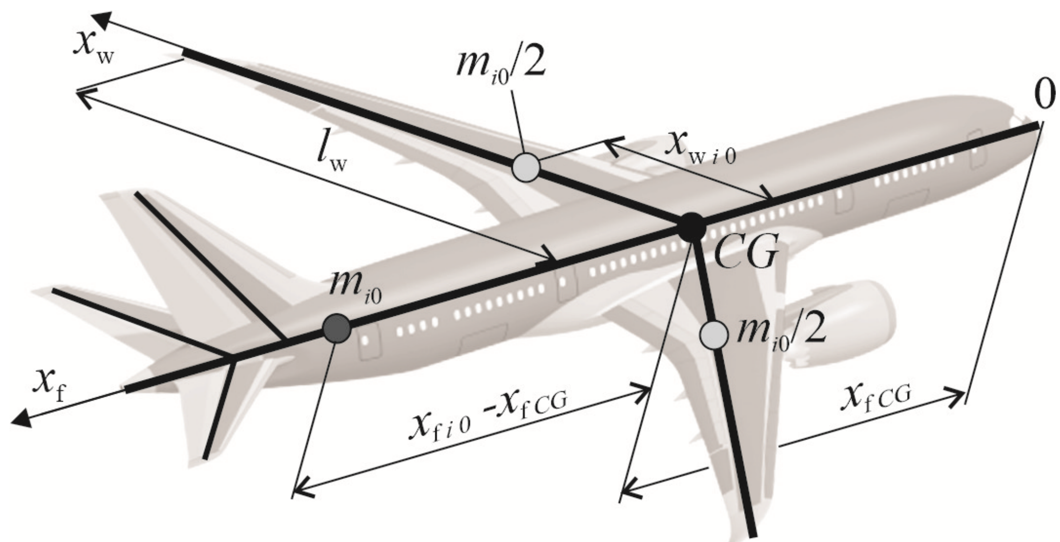


Figure 5. Aircraft – as a system of intersecting beams in the center gravity

In this case, the initial change in mass Δm_{i0} at a distance from the CG will cause the appearance of concomitant initial changes in the mass of parts of the aircraft $\sum \Delta m_{i0\ \mathrm{com}}$ from the condition of preserving the flight characteristics of the aircraft and payload at the same level (the 1st scenario). Because of this, the total initial mass change will already be

$$\Delta m_{i00} = \Delta m_{i0} + \sum \Delta m_{i0\ \mathrm{com}} \tag{20}$$

If we again use the previously obtained parameter SFM then the final change in the aircraft mass can be written as follows

$$\Delta m_{TO} = \mu_{m\ i}\, \Delta m_{i00} = \mu_{m\ i}\, (\Delta m_{i\ 0} + \sum \Delta m_{i0\ \mathrm{com}}) = \mu^*_{m\ i}\, \Delta m_{i0} \tag{21}$$

where $\mu^*_{m\ i}$ is corrected SFM taking into account the location of the place where the initial mass change occurred

$$\mu^*_{m\ i} = \mu_{m\ i}\, (1 + \sum \Delta m_{i0\ \mathrm{com}} / \Delta m_{i\ 0}) = \mu_{m\ i}\, k_{\mu\ mi} \tag{22}$$

We can only estimate very roughly the value of the correction coefficient $k_{\mu\ mi}$ depending on the coordinates of the location of the initial mass change Δm_{i0}. But this assessment will generally answer a fundamental question about such an impact. We will consider the fuselage as a beam directed along the x_{f} axis, while the beginning of this axis is more convenient to place on the nose of the fuselage. And each wing cantilever is considered as a beam directed along the x_{w} axis – Figure 5. At the same time, we assume that these beams intersect at a point close to CG.

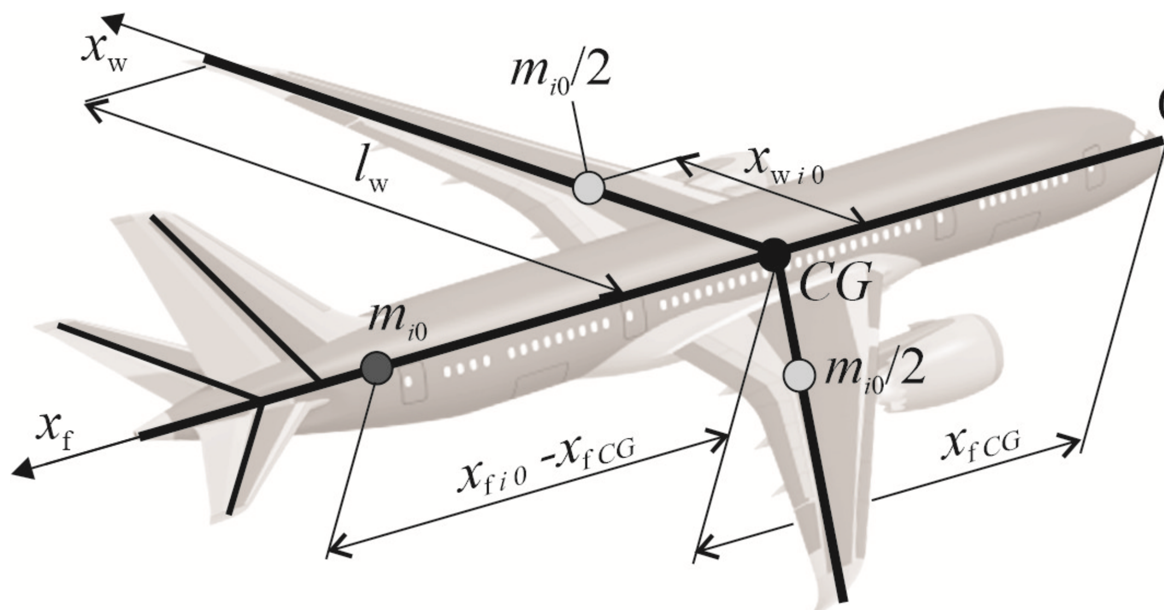


Figure 5.
Aircraft – as a system of intersecting beams in the center gravity

4 Sensitivity factors of mass by the aerodynamic parameter change

The introduced design solutions, along with changes in mass, can be associated with changes in aerodynamics and have a significant impact on the aerodynamic characteristics, in particular on drag. And, as a rule, these effects on mass and aerodynamics are opposite in their effect (for example, the descending nose of a supersonic aircraft). The concept based on SFM is convenient in such a problem of resolving contradictions between the mass and aerodynamic drag of aircraft parts.

4.1 SFM by drag

Assume that the thrust of the power plant is determined by the cruising mode of flight – altitude and speed of flight

$$T_{H,V} = D = C_D q S = mg/(L/D) \quad (23)$$

If we proceed from the constancy of the main flight properties of the aircraft and the specific characteristics of the power plant we can accept. that the initial change in aerodynamically drag ΔD_0 will result in a change in thrust

$$\Delta D_0 = \Delta T_{H,V} = T_{H,V}/(m_{eng} + m_{fuel})\,(\Delta m_{eng\,0} + \Delta m_{fuel\,0}) \qquad (24)$$

$$\Delta D_0 = g/K/(\bar{m}_{eng} + \bar{m}_{fuel})\,(\Delta m_{eng\,0} + \Delta m_{fuel\,0}) \qquad (25)$$

According to definition SFM by drag is

$$\mu_D = dm/dD_0 \qquad (26)$$

and taking into account $\Delta m = \mu_m(\Delta m_{eng\,0} + \Delta m_{fuel\,0})$ we receive

$$\mu_D = dm/dD_0 \approx \Delta m/\Delta D_0 = \mu_m K(\bar{m}_{eng} + \bar{m}_{fuel})/g \qquad (27)$$

4.2 SFM by drag coefficient

As in the previous case we assume a proportional relationship between changes in the drag coefficient and changes in the cost of engine mass and fuel

$$C_D/(m_{eng}+m_{fuel})=\Delta C_D\ /(\Delta m_{eng\ 0}+\Delta m_{fuel\ 0}) \qquad (28)$$

we obtain

$$\mu_D=dm\ /dD_0 \approx \Delta m/\Delta C_D=\mu_m\ (\ \bar{m}_{eng}+\ \bar{m}_{fuel})\ m/\ C_D \qquad (29)$$

4.3 SFM by *L/D* ratio

According to definition SFM by lift to drag ratio $\mu_K=dm/dD_0$

Similarly, given a proportional relationship $(m_{eng}+m_{fuel})\sim T_{H,V}\sim 1/(L/D)$

$$[(m_{eng}+m_{fuel})\ (L/D)]_{\mathrm{Old}}=[(m_{eng}+m_{fuel})\ (L/D) \qquad (30)$$

$$\mu_K=dm\ /dD_0 \approx \Delta m/\Delta(L/D)= -\ \mu_m\ (\ \bar{m}_{eng}+\ \bar{m}_{fuel})\ m/(L/D) \qquad (31)$$

5 Numerical research

Let's analyze the effect of the correction associated with taking into account the dependence of the SFM on the value of the initial mass change. As a basic project, the aircraft is considered with characteristics close to the Boeing 747. The initial data for the Boeing 747-200B is accepted as follows from Table 1 ([10, 11]), and also, we'll assume that C_{Dfus}/C_D=0.3.

Table 1. Calculated component masses for Boeing 747-200B

Masses	m_{TO}	Main functional parts				Elements of structures		
		m_{target}	m_{str}	m_{eng}	m_{fuel}	m_{fus}	m_{wing}	m_{tail}
Absolute masses m_i, t	379	68.18+44.9 = 113.08	97.3	28.4	139	29. 3	43.5	8.5
Relative masses $\bar{m}_i$	1	0.3	0.26	0.07	0.37			

We can analyze the transition from the metal structure of the wing, fuselage and tail, for which mass in the basic project was $m_{fus} + m_{wing} + m_{tail}$ =81.3 t, to the composite one. According to design experience, such a transition can reduce the initial mass of the construction by 20-30%, i.e.

$$\Delta m_{str0} = -81.3 \times 0.3 = -24.4 \text{ t}.$$

Figure 6 shows the values of the coefficient μ_m in the range of the initial change in the mass of the structure Δm_{str0} from –25 t to +25 t (solid line), which was $2.0 < \mu_m < 2.7$. The dotted line corresponds to the constant value $\mu_m = 2.31$, which is obtained by the formula (9) according to [4-7]. The dashed line corresponds to the value $\mu_m = 3.33$, which is obtained by the formula (17).

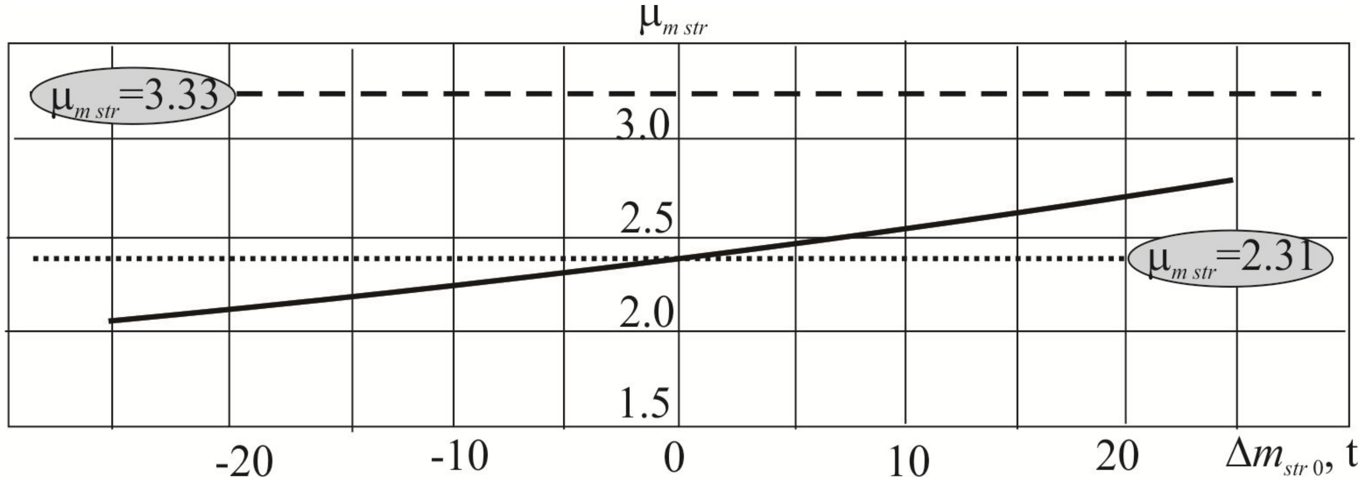


Figure 6. SFM for Boeing 747 200-B obtained on the basis of different approaches

Figure 7 shows the final changes in TOGM and functional masses with th initial change in the mass of the structure

$-25\ t < \Delta m_{str0} <+25\ t$.

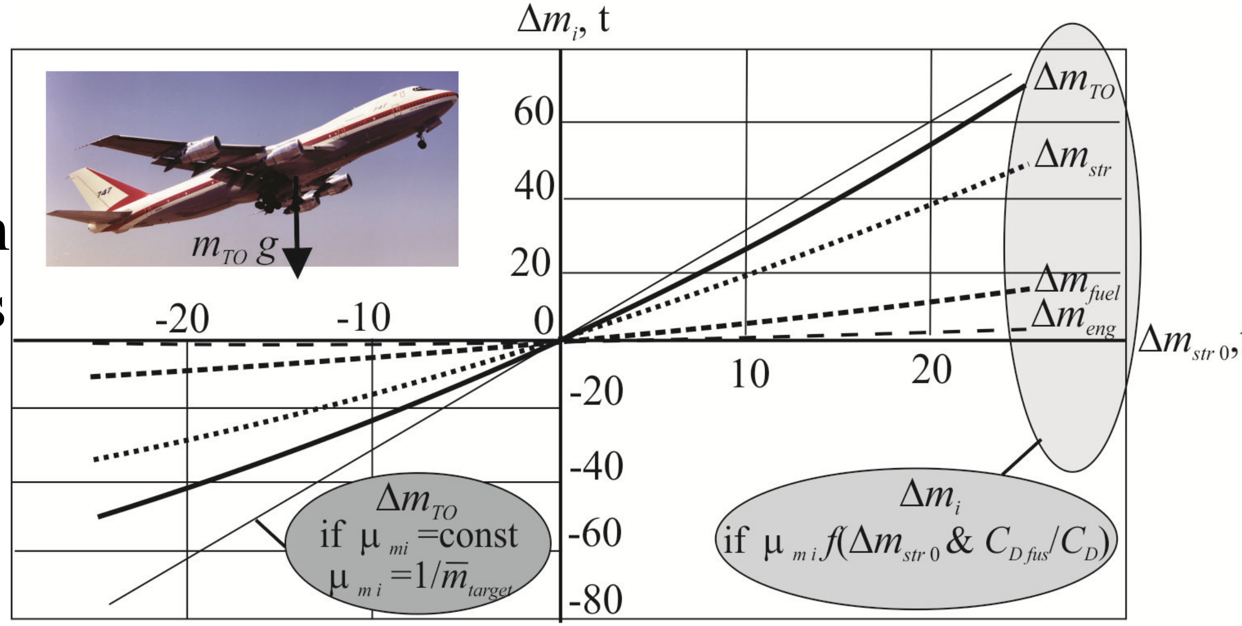


Figure 7. Changes of functional masses components of Boeing 747 200-B as a result of changes in the mass of the structure in the range from -25 tons to +25 tons.

The thin solid line corresponds to the take-off mass calculated using the projection formula (7), which was usually used in the design.

SFM by Drag according to the formula (29), taking K=16

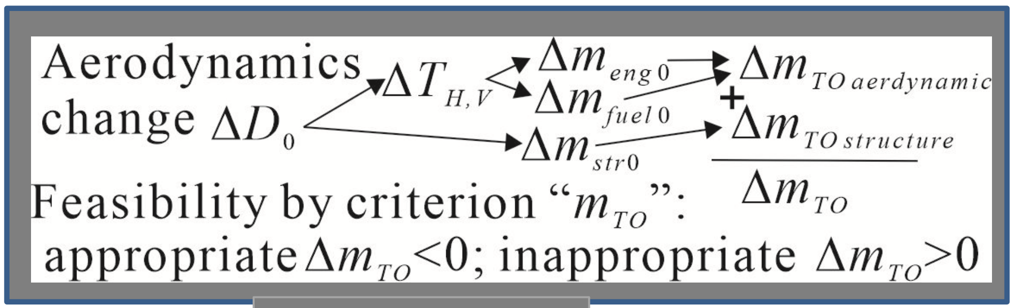
$$\mu_D = 1.66\ \text{kg/N}$$

SFM by L/D ratio according to the formula (31)

$$\mu_{L/D} = -19079\ \text{kg/(per unit } L/D).$$

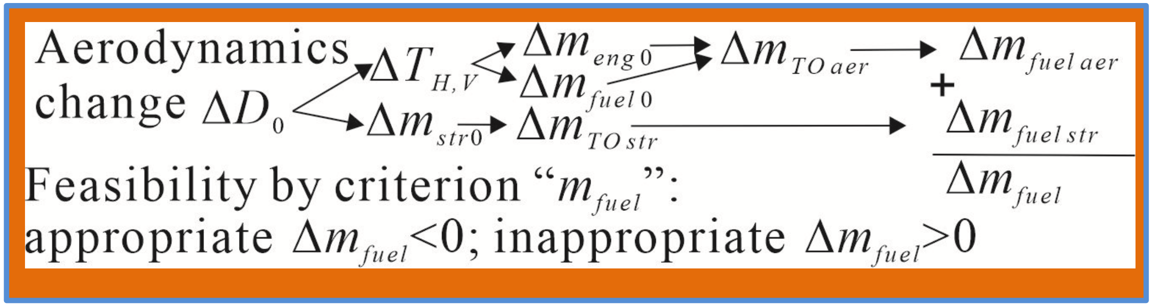
6 Conclusions. The presented method based on SFM allows to make a refined assessment of the take-off mass of the aircraft of new project and its functional parts. Numerical researches have shown that the results of this approach can be adjusted up to 20-25% compared to the traditionally used one.

The approach used allows us to justify the right choice between improving the aerodynamics (for example on value ΔD_0) and increasing the mass of the structure (Δm_{str0}). According to the Δm_{T0} sign, we can conclude that it is advisable to switch to a new version of the design solution according to the criterion “TOGM" (Scheme 1) or "fuel efficiency" (Scheme 2).



Scheme 1

Scheme 2



References

1. Raymer D. P. Aircraft Design: A Conceptual Approach, (6th Edition), AIAA, USA, 2018. 1062p.

2.Torenbeek E. Advanced aircraft design : conceptual design, analysis, and optimization of subsonic civil airplanes. John Wiley and Sons, Ltd. 2013. 410 p.

3.Aircraft design / edited by M. A. Poghosyan. (5th Edition), Moscow: Innovative engineering, 2018. 864 p. (In Russian)

4.Badyagin A. A., Yeger S. M., Mishin V. F. and others. Aircraft design. Moscow: Mashinostroenie, 1972. 516 p. (In Russian)

5.Robinson, A.C., Mass Properties Engineering, Lockheed-California. Course LX3196 notes (unpublished), Sept 1980 - Feb 1981.

6.Hays Anthony P., Zen and the Art of Airplane Sizing, SAE Paper 931255, May 1993.

7.Roskam, Jan, Airplane Design Part V: Component Weight Estimation, Roskam Aviation and Engineering Corp., 1985.

8.Gogolin V. P. Determination of the growth coefficient of take-off weight changes in the implementation of initial changes in the weight of the structure. Works of KAI, Kazan, 1973. vol. 160. pp.11-14 (In Russian)

9.Grebenkov, O. A., Gogolin, V. P., Osokin A. I., etc. The design of the aircraft. Kazan. KSTU publishing house. 1999. 320 p. (In Russian)

10.Jane's All the World's Aircraft, Jane's Annual Publication, various years. See www.janes.com for list of publications.

11. http://en.wikipedia.org/wiki/Boeing_747

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