Euler observers for the perfect fluid without vorticity

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The perfect fluid was already studied for the case where there is vorticity. A new technique was developed in order to locally and covariantly diagonalize the perfect fluid stress-energy tensor. New tetrads were introduced to this purpose. In this manuscript we will analyze the case where there is no vorticity. We will show how to implement for this case the diagonalization algorithm previously built for the case with vorticity. A novel technique will be introduced based only on purely geometrical objects. As an application, a new algorithm will be formulated with the aim of finding Euler observers for this case without vorticity.

I. INTRODUCTION

There are many astrophysical problems that require the use of general relativity. Fundamentally, problems associated to intense gravitational fields. Gravitational collapse, gravitational radiation, stability of different objects, ultrarelativistic flows, etc. Many of these studies[1−8] need the use of relativistic hydrodynamics, in particular ideal perfect fluids. A relevant object to carry out these analysis is the stress-energy tensor,

$$T_{\mu\nu} = (\rho + p)\, u_{\mu}\, u_{\nu} + p\, g_{\mu\nu} \ , \tag{1}$$

where ρ is the energy-density of the fluid, p the isotropic pressure and u^{μ} its four-velocity field, $g_{\mu\nu}$ is the metric tensor. In many schemes or algorithms that confront these complicated relativistic hydrodynamic equations, it is necessary to modify the stress-energy tensor (1) in order to include artificial viscosity Q as[4],

$$T_{\mu\nu} = (\rho + p + Q)\, u_{\mu}\, u_{\nu} + (p + Q)\, g_{\mu\nu} \ . \tag{2}$$

Our goal is to find local and covariant geometrical structures that enable a geometrical understanding of the different problems with simplification in both, the numerical and physical fronts. To this end we carry out the program of finding new tetrads that locally and covariantly diagonalize the stress-energy tensor (1) or the variant (2) in order to develop a covariant algorithm for the construction of Euler observers and Cauchy surfaces[9−10], always in situations with no vorticity. A new technique was already developed in four-dimensional Lorentzian spacetimes where electromagnetic fields are present[11−14]. This technique was already analyzed for the case with vorticity[15] as well. We will introduce the basic elements needed for its development in our case without vorticity in section II. This new technique for the case without vorticity will be novel and a relevant contribution of this paper, since we will make use of purely geometrical objects only, when building the extremal fields. The technique to find Eulerian observers for the Einstein-Maxwell case was introduced in manuscript[16]. We will extend this technique to the perfect fluid case with no vorticity in section III as an application of our new method to build tetrads that locally and covariantly diagonalize the stress-energy tensor (1) or its modification (2).

II. NEW TETRADS FOR THE CASE WITHOUT VORTICITY

The new technique implemented with the goal of finding locally and covariantly new tetrads that diagonalize the perfect fluid stress-energy tensor in the case with vorticity was founded on the antisymmetric nature of the fluid extremal field or the velocity curl extremal field second rank tensor itself[15]. In our present case there is no velocity curl, however we can proceed to introduce other second rank antisymmetric tensors that support the construction of new tetrads that on one hand diagonalize locally and covariantly the perfect fluid stress-energy tensor and on the other hand allow for the construction of Euler vector fields. Let us consider the following objects,

$$A_{\mu\nu} = R_{\mu\nu\rho\lambda}\, g^{\rho\sigma}\, g^{\lambda\tau}\, R_{,\sigma}\;\; u_{\tau} \tag{3}$$

$$B_{\mu\nu} = R_{\mu\nu\rho\lambda;\tau}\, g^{\rho\sigma}\, g^{\lambda\tau}\, R_{,\sigma} \tag{4}$$

$$C_{\mu\nu} = R_{\mu\nu\rho\lambda;\tau}\, g^{\rho\sigma}\, g^{\lambda\tau}\;\; u_{\sigma} \tag{5}$$

$$D_{\mu\nu} = R_{\mu\nu\rho\lambda;[\tau;\sigma]}\, R^{\rho\lambda\tau\sigma}\;\; . \tag{6}$$

Any of these four objects is an antisymmetric tensor field in spacetime. $R_{\mu\nu\rho\lambda}$ is the Riemann tensor, R the Ricci scalar. The symbol ; stands for covariant derivative with respect to the metric tensor $g_{\mu\nu}$. These are just four examples. There could be more, but they suffice in order to proceed with our construction. The next steps are nothing but a replica of the tetrad construction algorithm depicted in detail in manuscripts[11−14]. We introduce an extremal field $\xi_{\mu\nu}$ and a local complexion scalar α exactly as we did for instance in manuscript[15] and follow the necessary analogous steps to obtain the tetrad that locally and covariantly diagonalizes the stress-energy tensor (1). As in manuscript[15] vorticity was present, the extremal field and the complexion found through a local duality transformation, made use of this fact. They were found through a local duality transformation of the velocity curl. In the present paper there is no vorticity and the duality transformation is performed on any of the fields (3-6), instead. As an example, the extremal field could be defined as $\xi_{\mu\nu} = \cos\alpha\;\; B_{\mu\nu} - \sin\alpha\;\; * B_{\mu\nu}$. In this example the complexion would be defined as $\tan(2\alpha) = -\left(B_{\mu\nu}\, g^{\sigma\mu}\, g^{\tau\nu}\; * B_{\sigma\tau}\right) \;/\; \left(B_{\lambda\rho}\, g^{\lambda\alpha}\, g^{\rho\beta}\, B_{\alpha\beta}\right)$. This expression for the complexion stems from the condition imposed on the extremal field $\xi_{\mu\nu}\; * \xi^{\mu\nu} = 0$. It is simple to prove using identities that hold on four-dimensional Lorentzian spacetimes that this extremal field condition is equivalent to $\xi_{\mu\rho} \, * \xi^{\mu\lambda} = 0$, see references[11−14]. The object $*B_{\mu\nu} = \frac{1}{2} \, \epsilon_{\mu\nu\sigma\tau} \, g^{\sigma\rho} \, g^{\tau\lambda} \, B_{\rho\lambda}$ is the dual tensor of $B_{\mu\nu}$. The normalized version of this local tetrad is,

$$U^{\alpha} = u^{\alpha} \tag{7}$$

$$V^{\alpha} = \xi^{\alpha\lambda} \, u_{\lambda} \, / \, (\, \sqrt{u_{\mu} \, \xi^{\mu\sigma} \, \xi_{\nu\sigma} \, u^{\nu}} \,) \tag{8}$$

$$Z^{\alpha} = *\xi^{\alpha\lambda} \, u_{\lambda} \, / \, (\, \sqrt{u_{\mu} * \xi^{\mu\sigma} * \xi_{\nu\sigma} u^{\nu}} \,) \tag{9}$$

$$W^{\alpha} = \left(V^{\alpha}_{(4)} \, (V^{\rho}_{(1)} \, u_{\rho}) - V^{\alpha}_{(1)} \, (V^{\rho}_{(4)} \, u_{\rho}) \right) / \, \sqrt{V^{\beta}_{(5)} \, V_{(5)\beta}} \; , \tag{10}$$

where, $V^{\beta}_{(5)} \, V_{(5)\beta} = (V^{\beta}_{(4)} \, V_{(4)\beta}) \, (V^{\rho}_{(1)} \, u_{\rho})^2 + (V^{\beta}_{(1)} \, V_{(1)\beta}) \, (V^{\rho}_{(4)} \, u_{\rho})^2$. In turn $V^{\alpha}_{(4)} = *\xi^{\alpha\lambda} * \xi_{\rho\lambda} \, u^{\rho}$. We just followed the same method as in papers[11−14], in particular using the same notation as in reference[15]. The only variant so far is the initial antisymmetric field used to find the extremal field and the complexion through a local duality transformation like $\xi_{\mu\nu} = \cos\alpha \; B_{\mu\nu} - \sin\alpha \; * B_{\mu\nu}$. In manuscript[15] it was the velocity curl, in our present manuscript it is any of the antisymmetric tensors (3-6). These vectors (7-10) satisfy,

$$U^{\alpha} \, T_{\alpha}^{\ \ \beta} = -\rho \, U^{\beta} \tag{11}$$

$$V^{\alpha} \, T_{\alpha}^{\ \ \beta} = p \, V^{\beta} \tag{12}$$

$$Z^{\alpha} \, T_{\alpha}^{\ \ \beta} = p \, Z^{\beta} \tag{13}$$

$$W^{\alpha} \, T_{\alpha}^{\ \ \beta} = p \, W^{\beta} \; . \tag{14}$$

The proof can be followed in manuscript[15]. It would be redundant to repeat it in this section. Nonetheless we have the results we need in order to find the Euler vector fields in the next section. These surface forming vector fields along with coordinate observers allow to study spacetime evolution in astrophysical problems of present interest[17−51].

III. APPLICATION: EULER VECTOR FIELDS

We are going to proceed in this section in a very similar way to the analogous section in paper[16]. We introduce the equations satisfied by the hypersurface orthogonal[16,17,18,19] unit vector fields $n_{\mu} \, n^{\mu} = -1$,

$$n_\alpha\, n_{\beta;\gamma} + n_\beta\, n_{\gamma;\alpha} + n_\gamma\, n_{\alpha;\beta} - n_\alpha\, n_{\gamma;\beta} - n_\gamma\, n_{\beta;\alpha} - n_\beta\, n_{\alpha;\gamma} = 0\ . \tag{15}$$

We are going to name $\hat{U}^\mu$ the Euler unit timelike vector field that satisfies equation (15). We are going to name the other three vectors in the new orthonormal tetrad as $\hat{V}^\mu$, $\hat{Z}^\mu$ and $\hat{W}^\mu$. Then, the hypersurface orthogonal vector $\hat{U}^\mu$ must satisfy the equation,

$$\hat{U}_\alpha\, \hat{U}_{\beta;\gamma} + \hat{U}_\beta\, \hat{U}_{\gamma;\alpha} + \hat{U}_\gamma\, \hat{U}_{\alpha;\beta} - \hat{U}_\alpha\, \hat{U}_{\gamma;\beta} - \hat{U}_\gamma\, \hat{U}_{\beta;\alpha} - \hat{U}_\beta\, \hat{U}_{\alpha;\gamma} = 0\ . \tag{16}$$

Next, when we project equation (16) using the four tetrad vectors $(\hat{U}^\alpha, \hat{V}^\alpha, \hat{Z}^\alpha, \hat{W}^\alpha)$ we get only three meaningful equations,

$$\hat{U}_{[\alpha;\beta]}\, \hat{V}^\alpha\, \hat{Z}^\beta = 0 \tag{17}$$

$$\hat{U}_{[\alpha;\beta]}\, \hat{V}^\alpha\, \hat{W}^\beta = 0 \tag{18}$$

$$\hat{U}_{[\alpha;\beta]}\, \hat{Z}^\alpha\, \hat{W}^\beta = 0\ . \tag{19}$$

Equations (17-19) are three conditions on the vector field $\hat{U}^\alpha$. Our intention is to use the tetrad (7-10) that locally and covariantly diagonalizes the perfect fluid stress-energy tensor, and introduce three local scalars that are going to solve the three equations (17-19). To this end, first we perform a rotation on the local plane determined by (V^α, W^α) using the local scalar ϕ,

$$V^\alpha_{(\phi)} = \cos(\phi)\, V^\alpha - \sin(\phi)\, W^\alpha \tag{20}$$

$$W^\alpha_{(\phi)} = \sin(\phi)\, V^\alpha + \cos(\phi)\, W^\alpha\ . \tag{21}$$

Second, we perform another local rotation in the plane $(Z^\alpha, W^\alpha_{(\phi)})$ by the local angle φ,

$$Z^\alpha_{(\varphi)} = \cos(\varphi)\, Z^\alpha - \sin(\varphi)\, W^\alpha_{(\phi)} \tag{22}$$

$$W^\alpha_{(\varphi)} = \sin(\varphi)\, Z^\alpha + \cos(\varphi)\, W^\alpha_{(\phi)}\ . \tag{23}$$

Finally a boost by the local angle ψ in the plane $(U^\alpha, W^\alpha_{(\varphi)})$,

$$\hat{U}^\alpha = \cosh(\psi)\, U^\alpha + \sinh(\psi)\, W^\alpha_{(\varphi)} \tag{24}$$

$$\hat{W}^\alpha = \sinh(\psi)\, U^\alpha + \cosh(\psi)\, W^\alpha_{(\varphi)}\ . \tag{25}$$

Three local scalars (ϕ, φ, ψ) become through these succession of local Lorentz transformations in three local variables that are going to be the solution to the system (17-19). The final orthonormal tetrad that has as a timelike vector field $\hat{U}^\alpha$ the hypersurface orthogonal vector field that will function as an input for our evolution algorithms is given by,

$$\hat{U}^\alpha = \cosh(\psi)\, U^\alpha + \sinh(\psi)\, W^\alpha_{(\varphi)} \tag{26}$$

$$\hat{V}^\alpha = V^\alpha_{(\phi)} \tag{27}$$

$$\hat{Z}^\alpha = Z^\alpha_{(\varphi)} \tag{28}$$

$$\hat{W}^\alpha = \sinh(\psi)\, U^\alpha + \cosh(\psi)\, W^\alpha_{(\varphi)}\ . \tag{29}$$

The algorithm would not work if the vector that involves the three local Lorentz transformations and therefore the three local scalars (ϕ, φ, ψ), were not $\hat{U}^\alpha$. If we would have considered Lorentz transformations only involving the original vectors $(V^\alpha, Z^\alpha, W^\alpha)$ then we would only have produced combinations of the original equations (17-19) and since these can be algebraically decoupled, we would not have introduced any new information. It is through the inclusion of the three local scalars (ϕ, φ, ψ) inside the derivatives of the vector $\hat{U}^\alpha$ that we get equations (17-19) to be meaningful. Next, we contract the tetrad vectors $(\hat{U}^\alpha, \hat{V}^\alpha, \hat{Z}^\alpha, \hat{W}^\alpha)$ with the stress-energy tensor (1),

$$\hat{U}^\alpha\, T_\alpha^{\ \beta} = -\rho\, \cosh(\psi)\, U^\beta + p\, \sinh(\psi)\, W^\beta_{(\varphi)} \tag{30}$$

$$\hat{V}^\alpha\, T_\alpha^{\ \beta} = p\, \hat{V}^\beta \tag{31}$$

$$\hat{Z}^\alpha\, T_\alpha^{\ \beta} = p\, \hat{Z}^\beta \tag{32}$$

$$\hat{W}^\alpha\, T_\alpha^{\ \beta} = -\rho\, \sinh(\psi)\, U^\beta + p\, \cosh(\psi)\, W^\beta_{(\varphi)}\ . \tag{33}$$

Therefore, the only non-zero components of the stress-energy tensor in terms of the new tetrad are,

$$\hat{U}^{\alpha}\, T_{\alpha}^{\ \beta}\, \hat{U}_{\beta} = \rho\, \cosh^{2}(\psi) + p\, \sinh^{2}(\psi) \tag{34}$$

$$\hat{V}^{\alpha}\, T_{\alpha}^{\ \beta}\, \hat{V}_{\beta} = p \tag{35}$$

$$\hat{Z}^{\alpha}\, T_{\alpha}^{\ \beta}\, \hat{Z}_{\beta} = p \tag{36}$$

$$\hat{W}^{\alpha}\, T_{\alpha}^{\ \beta}\, \hat{W}_{\beta} = \rho\, \sinh^{2}(\psi) + p\, \cosh^{2}(\psi) \tag{37}$$

$$\hat{U}^{\alpha}\, T_{\alpha}^{\ \beta}\, \hat{W}_{\beta} = \frac{(\rho + p)}{2}\, \sinh(2\psi)\ . \tag{38}$$

By performing the three local Lorentz transformations in our algorithm we have the following result. First, we ended up with a new local tetrad that adds only one off-diagonal component to the stress-energy tensor, the minimum possible. Second, we found the Euler hypersurface orthogonal congruence. We have found an algorithm that provides both a hypersurface orthogonal congruence and a maximum simplification of the stress-energy tensor given that the tetrad that diagonalized the tensor underwent three Lorentz transformations. When we take the limit $\psi \rightarrow 0$ it can be readily seen from expressions (34-38) that we recover the results for the old tetrad that diagonalizes the stress-energy tensor.

IV. CONCLUSIONS

Many relativistic hydrodynamical problems[1−8] consist of a system of coupled differential equations that possess as a source term a stress-energy tensor of the perfect fluid nature as in equation (1). The resolution of a problem involving artificial viscosity in a relativistic consistent way as in equation (2) would involve similar techniques and we just focus on equation (1). Eulerian observers have proved to be useful in numerous dynamical problems and we set out to find a local and covariant technique to produce them in a geometrical fashion. Using the tensors that play a fundamental role in these schemes. This search is synthesized in the construction of local extremal fields through local duality transformations of second rank antisymmetric tensors as with (3-6). In previous works these second rank antisymmetric tensors have been the electromagnetic field, the velocity curl, etc. Therefore, this new way of building extremal fields by using purely geometrical objects like in equations (3-6) is a decisive contribution of this manuscript. These new tetrads enjoy several useful properties due to its very construction. They diagonalize the stress-energy tensor locally and covariantly. They allow through three local Lorentz transformations to find the Euler hypersurface orthogonal vector fields, minimizing the number of non-zero stress-energy tensor additional components just to one more. They allow in a natural way to find Cauchy surfaces and study the dynamical evolution of a myriad of astrophysical problems. We quote from[4] "With the exception of the vacuum two-body problem (i.e. the coalescence of two black holes), all realistic astrophysical systems and sources of gravitational radiation involve matter. Not surprisingly, the joint integration of the equations of motion for matter and geometry was in the minds of theorists from the very beginning of numerical relativity. Nowadays there is a large body of numerical investigations in the literature dealing with hydrodynamical integrations in static background spacetimes. Most of those are based on Wilson's Eulerian formulation of the hydrodynamic equations and use schemes based on finite differences with some amount of artificial viscosity ". Our new Euler observers built with the tetrads that locally and covariantly diagonalize the stress-energy tensors point into this direction of research.

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