

## Supporting Information

### Master Equation Modelling of Non-equilibrium Chemistry in Stellar Outflows

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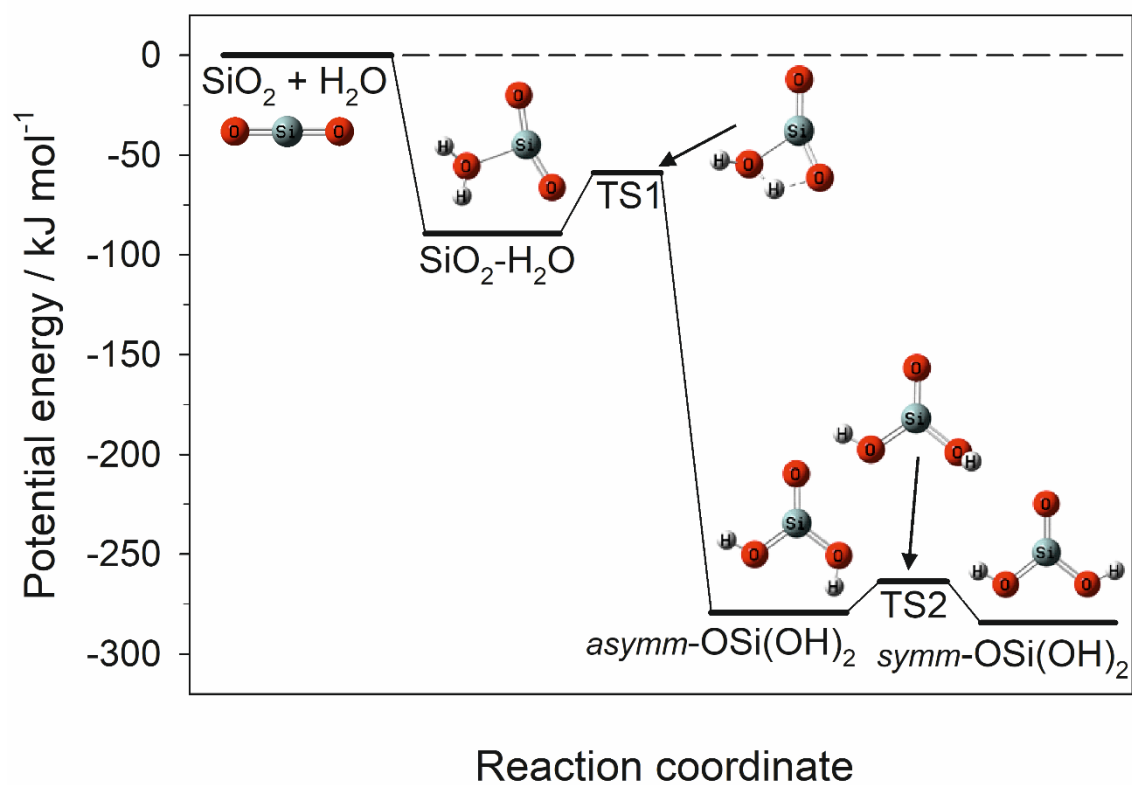
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**Figure S1.** Potential energy surface for the reaction between SiO<sub>2</sub> and H<sub>2</sub>O. Energies are calculated at the cbs-qb3 level of theory [Montgomery *et al.*, 1999].



**Table S1.** Molecular properties and heats of formation (at 0 K) of the stationary points on the SiO<sub>2</sub> + H<sub>2</sub>O potential energy surface.

Molecule (electronic state)	Geometry (Cartesian co-ordinates in Å) <sup>a</sup>	Rotational constants (GHz) <sup>a</sup>	Vibrational frequencies (cm <sup>-1</sup> ) <sup>a</sup>	$\Delta_f H^\circ(0\text{ K})$ (kJ mol <sup>-1</sup> ) <sup>b</sup>
SiO <sub>2</sub>	Si, -0.310, 0.748, 0. O, 1.192, 0.925, 0. O, -1.811, 0.571, 0.	6.9129	297 ( $\times 2$ ), 991, 1437	-277.8
H <sub>2</sub> O	O, 0.001, 0., 0.001 H 0, 0.0123, 0., 0.963 H, 0.933, 0., -0.237	798.21 438.23 282.91	1672, 3802, 3906	-238.9
SiO <sub>2</sub> -H <sub>2</sub> O	Si, -0.249, 0.279, -0.005 O, -1.475, -0.619, 0.067 O, 0.559, 1.568, -0.006 O, 1.151, -1.028, -0.133 H, 2.031, -0.735, 0.149 H, 0.933, -1.916, 0.189	7.8896 6.9337 3.7010	176, 304, 328, 376, 394, 441, 736, 991, 1395, 1569, 3719, 3835	-606.1
TS from SiO <sub>2</sub> -H <sub>2</sub> O to <i>asymm</i> -OSi(OH) <sub>2</sub> (TS1)	Si, 0.285, -0.170, -0.011 O, 1.737, 0.255, 0.0153 O, -0.865, -1.227, 0.043 O, -1.063, 1.052, -0.110 H, -1.556, 0.0415, 0.044 H, -1.185, 1.761, 0.537	10.746 5.9435 3.8530	1227 <i>i</i> , 293, 314, 487, 510, 747, 800, 966, 1276, 1361, 2123, 3791	-575.6
<i>asymm</i> -OSi(OH) <sub>2</sub>	Si, 0.028, 0.035, 0.123 O, 0.032, 1.352, -0.836 H, -0.024, 2.202, -0.385 O, 0.134, -1.240, -0.867 H, 0.182, -1.062, -1.813 O, -0.059, 0.006, 1.635	8.4623 7.2060 3.8919	314, 324, 357, 411, 476, 805, 866, 882, 989, 1295, 3849, 3853	-796.0
TS from <i>asymm</i> - OSi(OH) <sub>2</sub> to <i>symm</i> - OSi(OH) <sub>2</sub> (TS2)	Si, -0.092, -0.103, 0.077 O, 0.556, 1.387, 0.013 H, -0.084, 2.098, -0.112 O, 1.129, -1.118, 0.370 H, 1.674, -1.527, -0.305 O, -1.570, -0.422, -0.042	8.2175 7.3069 3.9011	444 <i>i</i> , 322, 342, 348, 513, 679, 827, 892, 1002, 1279, 3841, 3918	-780.4
<i>symm</i> -OSi(OH) <sub>2</sub>	Si, 0., 0., 0.124 O, 0., 1.275, -0.874 H, 0., 2.137, -0.444 O, 0., -1.275, -0.874 H, 0., -2.137, -0.444 O, 0., 0., 1.642	8.2547 7.3870 3.8984	313, 321, 361, 424, 492, 821, 837, 907, 1019, 1278, 3859, 3861	-801.0

<sup>a</sup> Calculated at the b3lyp/6-311+g(2d,p) level of theory [Frisch *et al.*, 2016].

<sup>b</sup> Calculated at the cbs-qb3 level of theory [Montgomery *et al.*, 1999], using reference values  $\Delta_f H^\circ(\text{SiO}_2) = -277.8$  kJ mol<sup>-1</sup> and  $\Delta_f H^\circ(\text{H}_2\text{O}) = -238.9$  kJ mol<sup>-1</sup> from the *Active Thermochemical Tables* [Ruscic and Bross, 2021].

**Table S2.** Reaction scheme for forming gas-phase Ca, Fe and Mg silicate molecules in the stellar outflow

No.	Reaction	Rate coefficient <sup>a</sup>	$\Delta H_{0K}$ <sup>b</sup> kJ mol <sup>-1</sup>
3	$\text{SiO} + \text{OH} \rightarrow \text{SiO}_2 + \text{H}$	$9.6 \times 10^{-13} 10^{(5.94e-4 T)}$ <sup>c</sup>	-4.1
-3	$\text{SiO}_2 + \text{H} \rightarrow \text{SiO} + \text{OH}$	$1.2 \times 10^{-10} 10^{(4.57e-4 T)}$ <sup>c</sup>	+4.1
1	$\text{SiO}_2 + \text{H}_2\text{O} + \text{M} \rightarrow \text{OSi(OH)}_2 + \text{M}$	$\log_{10}(k / \text{cm}^6 \text{ molecule}^{-2} \text{ s}^{-1}) = -0.0153r^4 + 0.2753r^3 - 1.9485r^2 + 7.2568r - 36.054$ <sup>d</sup>  $\log_{10}(k / \text{cm}^6 \text{ molecule}^{-2} \text{ s}^{-1}) = -0.0077r^4 + 0.1359r^3 - 0.9445r^2 + 3.4265r - 33.278$ <sup>e</sup>	-277
-1	$\text{OSi(OH)}_2 + \text{M} \rightarrow \text{SiO}_2 + \text{H}_2\text{O} + \text{M}$	$\log_{10}(k / \text{cm}^3 \text{ molecule}^{-1} \text{ s}^{-1}) = -0.1425r^5 + 2.6562r^4 - 19.356r^3 + 68.941r^2 - 121.28r + 61.044$ <sup>d</sup>  $\log_{10}(k / \text{cm}^3 \text{ molecule}^{-1} \text{ s}^{-1}) = -1.592r - 10.340$ <sup>e</sup>	+277
2	$\text{Ca} + \text{OSi(OH)}_2 \rightarrow \text{CaSiO}_3 + \text{H}_2$	$2.8 \times 10^{-10} (1000/T)^{0.28}$ <sup>c</sup>	-234
-2	$\text{CaSiO}_3 + \text{H}_2 \rightarrow \text{Ca} + \text{OSi(OH)}_2$	$3.5 \times 10^{-10} \exp(-26590/T)$ <sup>c</sup>	+234
	$\text{Fe} + \text{OSi(OH)}_2 \rightarrow \text{FeSiO}_3 + \text{H}_2$	$7.0 \times 10^{-11} \exp(-5550/T)$ <sup>c</sup>	-81
	$\text{FeSiO}_3 + \text{H}_2 \rightarrow \text{Fe} + \text{OSi(OH)}_2$	$4.8 \times 10^{-11} \exp(-15870/T)$ <sup>c</sup>	+81
	$\text{Mg} + \text{OSi(OH)}_2 \rightarrow \text{MgSiO}_3 + \text{H}_2$	$5.4 \times 10^{-12} \exp(-7560/T)$ <sup>c</sup>	-45
	$\text{MgSiO}_3 + \text{H}_2 \rightarrow \text{Mg} + \text{OSi(OH)}_2$	$1.3 \times 10^{-11} \exp(-12040/T)$ <sup>c</sup>	+45

<sup>a</sup> Units: bimolecular, cm<sup>3</sup> molecule<sup>-1</sup> s<sup>-1</sup>; termolecular, cm<sup>6</sup> molecule<sup>-2</sup> s<sup>-1</sup>; <sup>b</sup> Calculated at the CBS-QB3 level of theory [Montgomery *et al.*, 1999; Frisch *et al.*, 2016]; <sup>c</sup> Plane [2013]. <sup>d</sup> Master Equation calculation including optical transitions, for a distance  $r$  from the star R Dor (see main text). <sup>e</sup> Master Equation calculation *not* including optical transitions, for a distance  $r$  from the star R Dor (see main text).

## References

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