Enrico's Chart of Phase Noise and Two-Sample Variances

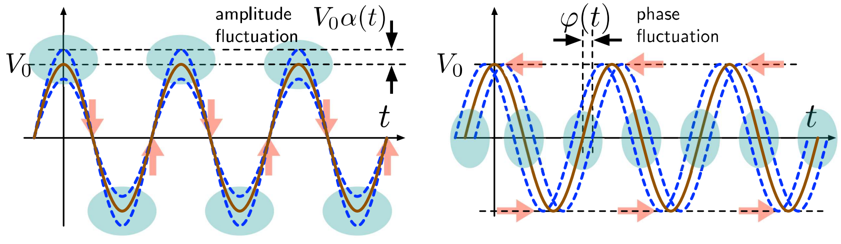
Enrico Rubiola – http://rubiola.org
European Frequency and Time Seminar – http://efts.eu
Oscillator Instability Measurement Platform http://oscillator-imp.com

Download the Companion paper

Download the Enrico's Chart

Thanks to FIRST-TF https://first-tf.com

Clock signal $v(t) = V_0[1+\alpha(t)]\cos[2\pi\nu_0 t + \varphi(t)]$



Boldface notation

total = nominal + fluctuation

$\boldsymbol{\varphi}(t) = 2\pi\nu_0 t + \varphi(t)$ phase

$\boldsymbol{\nu}(t) = \nu_0 + (\Delta\nu)(t)$ frequency

$\mathbf{x}(t) = t + \mathrm{x}(t)$ time

$\mathbf{y}(t) = 1 + \mathrm{y}(t)$ fractional frequency

Phase noise spectrum

Definition

$S_\varphi(f)$ [rad²/Hz] is the one-sided PSD ($f>0$) of $\varphi(t)$

$$S_\varphi(f) = 2\mathcal{F}\{\mathbb{E}\{\varphi(t)\varphi(t+\tau)\}\}, \quad f>0$$

Evaluation

$$S_\varphi(f) = \tfrac{2}{T}\langle \Phi_T(f)\,\Phi_T^*(f)\rangle_m$$

avg on m data, $\Phi_T(f)$ = DFT of $\varphi(t)$ truncated on T

Usage

most often, 'phase noise' refers to $\mathscr{L}(f)$

Only $10\log_{10}[\mathscr{L}(f)]$ is used, given in dBc/Hz

Definition: $\mathscr{L}(f) = \frac{1}{2}S_\varphi(f)$ [the unit c/Hz never used]

Literally, the unit 'c' is a squared angle, $\sqrt{c} = \sqrt{2}$ rad $\simeq 81°$

Two-sample (Allan-like) variances

Definition

$$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\overline{y}_2 - \overline{y}_1\right]^2\right\}$$

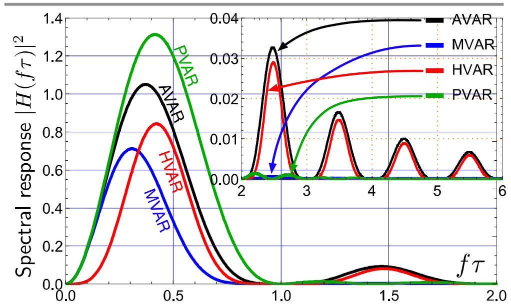
$y(t) \to \overline{y}$ averaged over τ
$\overline{y}_2$ and $\overline{y}_1$ are contiguous

Bare mean $\overline{y}$ → Allan variance AVAR
Weighted averages → MVAR, PVAR, etc.

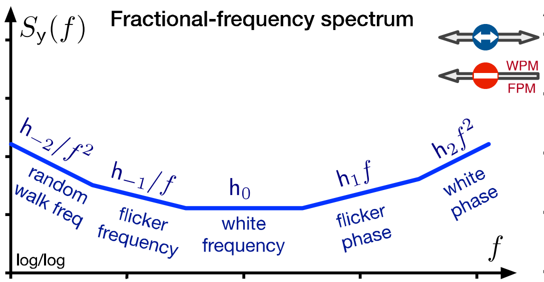
Evaluation

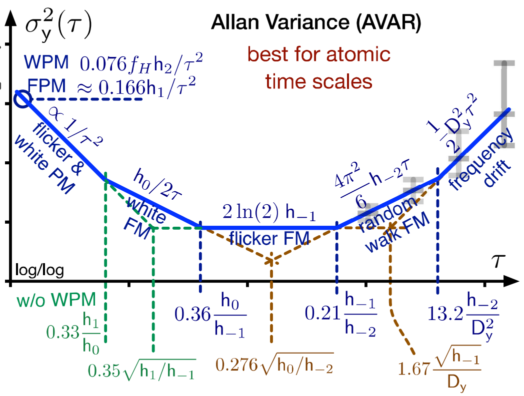
$$\sigma_y^2(\tau) = \frac{1}{2(M-1)}\sum_{k=1}^{M-1}\left[\overline{y}_{k+1} - \overline{y}_k\right]^2$$

M contiguous samples of $\overline{y}$



Frequency fluctuation PSD <—> Allan Variance





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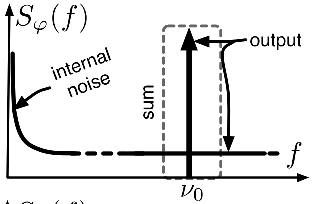
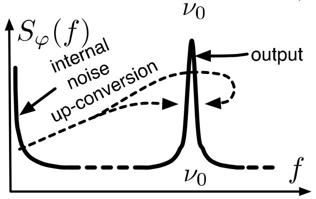
Additive Noise



RF noise added to the carrier

Statistically independent AM & PM

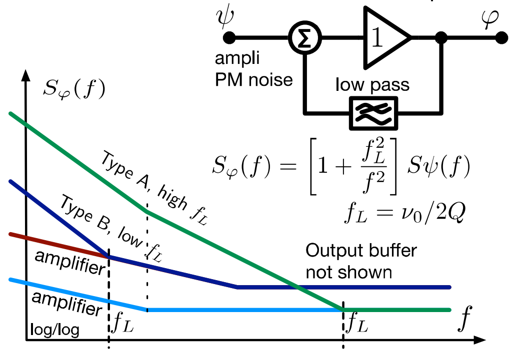
Parametric Noise



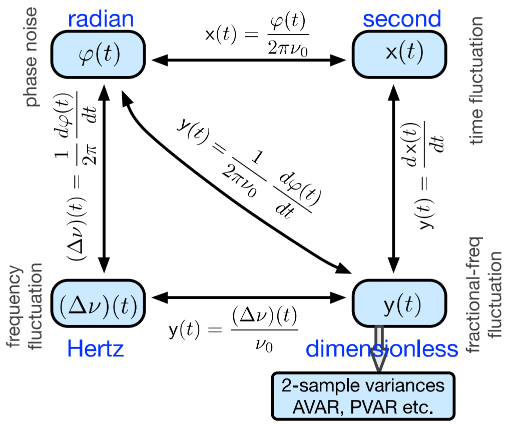
Near-dc noise modulates the carrier

AM & PM related and narrowband

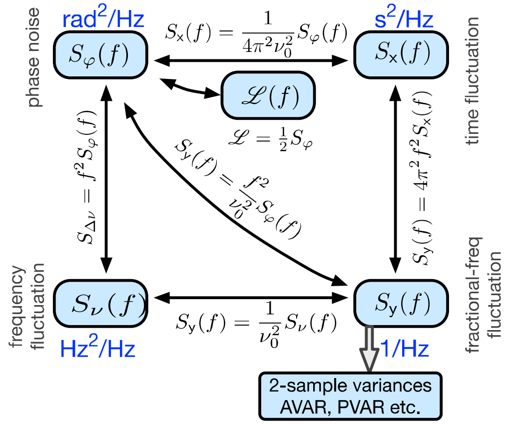
Leeson effect



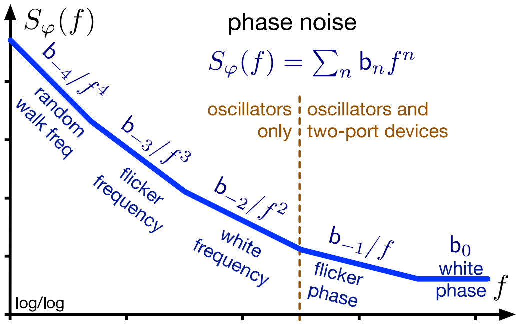
Time Domain

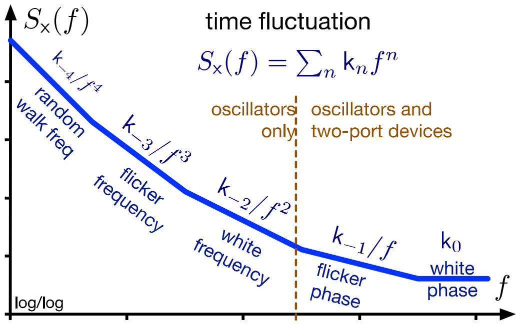


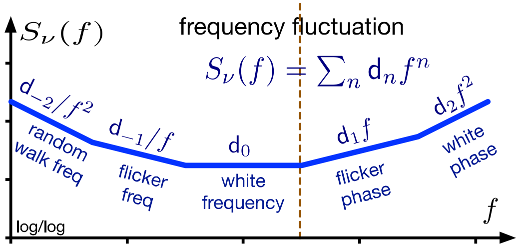
Frequency Domain

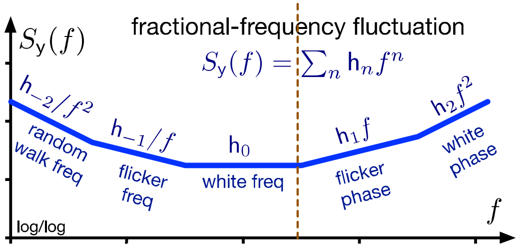


Spectra and Polynomial Law









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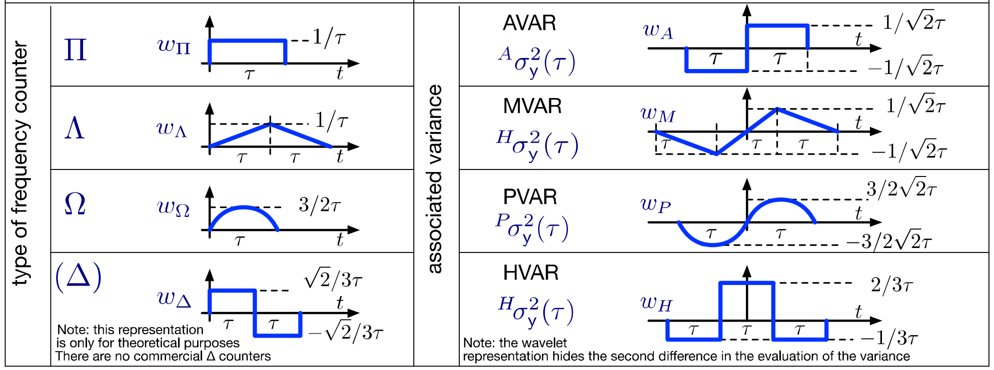
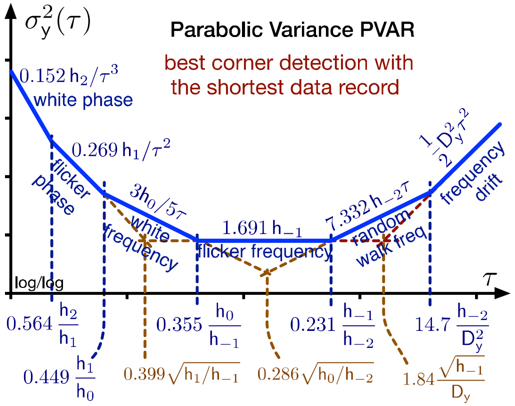
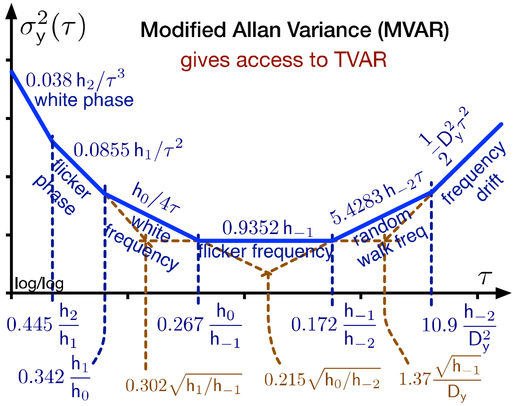
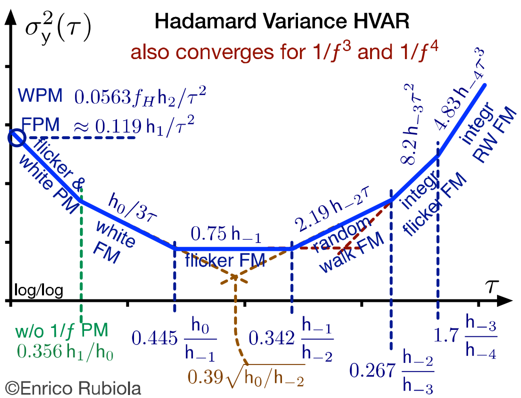
Frequency Counter	Wavelet Variance
$\bar{\mathsf{y}}(\tau) = \int_{\mathbb{R}} \mathsf{y}(t)\, w(t;\tau)\, dt$	$\sigma^2_{\mathsf{y}}(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{\mathsf{y}}_2 - \bar{\mathsf{y}}_1\right]^2\right\} = \mathbb{E}\left\{\int_{\mathbb{R}}\left[\mathsf{y}(t)\, w(t;\tau)\right]^2 dt\right\}$

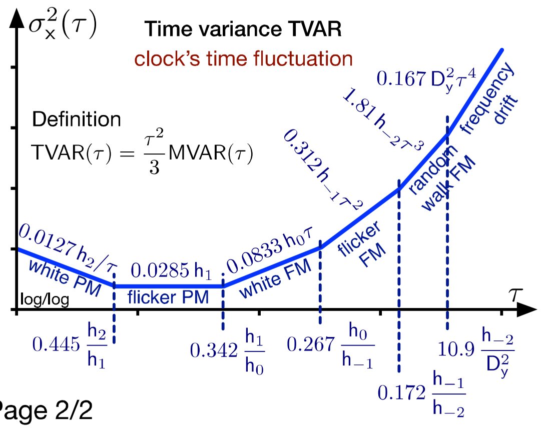


Other Two-Sample Variances









Spectra to Variances Conversion

noise type	$S_{\mathsf{y}}(f)$	AVAR ${}^A\sigma^2_{\mathsf{y}}(\tau)$	MVAR ${}^M\sigma^2_{\mathsf{y}}(\tau)$	HVAR ${}^H\sigma^2_{\mathsf{y}}(\tau)$	PVAR ${}^P\sigma^2_{\mathsf{y}}(\tau)$	TVAR ${}^T\sigma^2_{\mathsf{x}}(\tau)$
white PM	$\mathsf{h}_2 f^2$	$\frac{3f_H}{4\pi^2}\frac{\mathsf{h}_2}{\tau^2}$ $0.0760\, f_H \mathsf{h}_2/\tau^2$	$\frac{3}{8\pi^2}\frac{\mathsf{h}_2}{\tau^3}$ $0.0380\,\mathsf{h}_2/\tau^3$	$\frac{5f_H}{9\pi^2}\frac{\mathsf{h}_2}{\tau^2}$ $0.0563\, f_H \mathsf{h}_2/\tau^2$	$\frac{3}{2\pi^2}\frac{\mathsf{h}_2}{\tau^3}$ $0.1520\,\mathsf{h}_2/\tau^3$	$\frac{1}{8\pi^2}\frac{\mathsf{h}_2}{\tau}$ $0.0127\,\mathsf{h}_2/\tau$
flicker PM	$\mathsf{h}_1 f$	$\frac{3\gamma - \ln 2 + 3\ln(2\pi f_H\tau)}{4\pi^2}\frac{\mathsf{h}_1}{\tau^2}$ $[3\gamma - \ln 2 + 3\ln 2\pi]/4\pi^2 = 0.166$	$\frac{(24\ln 2 - 9\ln 3)}{8\pi^2}\frac{\mathsf{h}_1}{\tau^2}$ $0.0855\,\mathsf{h}_1/\tau^2$	$\simeq \frac{5[\gamma + \ln(\sqrt[10]{48}\,\pi f_H\tau)]}{9\pi^2}\frac{\mathsf{h}_1}{\tau^2}$ $5[\gamma + \ln(\sqrt[10]{48}\,\pi)]/9\pi^2 = 0.1187$	$\frac{3\,[\ln(16) - 1]}{2\pi^2}\frac{\mathsf{h}_1}{\tau^2}$ $0.2694\,\mathsf{h}_1/\tau^2$	$\frac{(8\ln 2 - 3\ln 3)}{8\pi^2}\mathsf{h}_1$ $0.0285\,\mathsf{h}_1$
white FM	h_0	$\frac{1}{2}\frac{\mathsf{h}_0}{\tau}$	$\frac{1}{4}\frac{\mathsf{h}_0}{\tau}$	$\frac{1}{3}\frac{\mathsf{h}_0}{\tau}$	$\frac{3}{5}\frac{\mathsf{h}_0}{\tau}$	$\frac{1}{12}\mathsf{h}_0\tau$
flicker FM	$\mathsf{h}_{-1}f^{-1}$	$2\ln(2)\,\mathsf{h}_{-1}$ $1.386\,\mathsf{h}_{-1}$	$\frac{27\ln 3 - 32\ln 2}{8}\mathsf{h}_{-1}$ $0.935\,\mathsf{h}_{-1}$	$\frac{8\ln 2 - 3\ln 3}{3}\mathsf{h}_{-1}$ $0.750\,\mathsf{h}_{-1}$	$\frac{2\,[7 - \ln(16)]}{5}\mathsf{h}_{-1}$ $1.691\,\mathsf{h}_{-1}$	$\frac{27\ln 3 - 32\ln 2}{24}\mathsf{h}_{-1}\tau^2$ $0.312\,\mathsf{h}_{-1}\tau^2$
random walk FM	$\mathsf{h}_{-2}f^{-2}$	$\frac{2\pi^2}{3}\mathsf{h}_{-2}\tau$ $6.58\,\mathsf{h}_{-2}\tau$	$\frac{11\pi^2}{20}\mathsf{h}_{-2}\tau$ $5.43\,\mathsf{h}_{-2}\tau$	$\frac{2\pi^2}{9}\mathsf{h}_{-2}\tau$ $2.19\,\mathsf{h}_{-2}\tau$	$\frac{26\pi^2}{35}\mathsf{h}_{-2}\tau$ $7.33\,\mathsf{h}_{-2}\tau$	$\frac{11\pi^2}{60}\mathsf{h}_{-2}\tau^3$ $1.81\,\mathsf{h}_{-2}\tau^3$
integrated flicker FM	$\mathsf{h}_{-3}f^{-3}$	not converging	not converging	$\frac{\pi^2[27\ln(3) - 32\ln(2)]}{9}\mathsf{h}_{-3}\tau^2$ $8.205\,\mathsf{h}_{-3}\tau^2$	not converging	not converging
integrated RW FM	$\mathsf{h}_{-4}f^{-4}$	not converging	not converging	$\frac{44\pi^2}{90}\mathsf{h}_{-4}\tau^3$ $4.825\,\mathsf{h}_{-4}\tau^3$	not converging	not converging
linear drift D_{y}		$\frac{1}{2}\mathsf{D}_{\mathsf{y}}^2\tau^2$	$\frac{1}{2}\mathsf{D}_{\mathsf{y}}^2\tau^2$	0	$\frac{1}{2}\mathsf{D}_{\mathsf{y}}^2\tau^2$	$\frac{1}{6}\mathsf{D}_{\mathsf{y}}^2\tau^4$
spectral response $\lvert H(\theta)\rvert^2$, $\theta = \pi f\tau$		$\frac{2\sin^4(\theta)}{\theta^2}$	$\frac{2\sin^6(\theta)}{\theta^4}$	$\frac{16\sin^6(\theta)}{9\theta^2}$	$\frac{9\left[2\sin^2(\theta) - \theta\sin(2\theta)\right]^2}{2\theta^6}$	$\frac{\tau^2}{3}\frac{2\sin^6(\pi f\tau)}{(\pi f\tau)^4}$
The cutoff frequency f_H is explicit in AVAR. In MVAR, PVAR and TVAR, f_H is implicit in sampling frequency $1/\tau_0$.						${}^T\sigma^2_{\mathsf{x}}(\tau) = \frac{\tau^2}{3}\,{}^M\sigma^2_{\mathsf{y}}(\tau)$