IRS Deployment Optimization in Multi-IRS Assisted Two-Way Full-Duplex Communication Systems

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Abstract—**Intelligent reflecting surfaces (IRSs) have emerged as a promising wireless technology for the dynamic configuration of electromagnetic waves. In this context, we study a multi-IRS assisted two-way communication system consisting of two users that employ full-duplex (FD) technology. More specifically, we deal with the joint IRS location and size (i.e., the number of reflecting elements) optimization in order to minimize an upper bound of system outage probability under various constraints. First, the problem is formulated as a discrete optimization problem and, then, a lower bound on the optimum value is computed by solving a linear-programming relaxation (LPR) problem. Subsequently, we design a polynomial-time greedy algorithm based on the LPR solution. The proposed algorithm always computes a feasible solution for which (a posteriori) performance guarantee can be provided. Finally, numerical simulations demonstrate the superiority of the greedy algorithm compared to a baseline scheme and provide useful comparisons between FD and conventional half-duplex (HD) systems.**

Index Terms—**Intelligent reflecting surface (IRS), IRS deployment, full-duplex (FD) communication, discrete optimization, linear-programming relaxation, greedy algorithm.**

I. Introduction

Intelligent reflecting surface (IRS) is a planar surface which is installed on the walls or ceilings of buildings so as to create virtual line-of-sight (LoS) links between the transmitters and receivers, thus overcoming the physical obstacles between them. In particular, IRS consists of passive reflecting elements that can independently induce a controllable phase shift on the incident electromagnetic wave [1], [2]. On the other hand, full-duplex (FD) wireless technology has the potential to double the spectral efficiency, compared to its half-duplex (HD) counterpart, by allowing simultaneous transmission and reception within the same frequency band [3]. Recently, there is a growing interest in combining IRSs with FD systems in order to exploit their benefits and advantages [4]–[6].

In addition, the single-IRS deployment problem, where an access point communicates with multiple users via the IRS, has been investigated in [7]. Specifically, the weighted sum rate maximization problem has been formulated and solved for three different multiple access schemes. Furthermore, the problem of joint IRS deployment, phase-shift design as well as power allocation for maximizing the energy efficiency of a non-orthogonal multiple access (NOMA) network has been recently formulated and solved using machine learning methods [8]. As concerns the coverage of an IRS-assisted network with one base station and one user equipment, [9] has examined the IRS placement problem to maximize the cell coverage by optimizing the IRS orientation and horizontal distance from the base station. Finally, [10], [11] have developed efficient optimization algorithms for the deployment of ground stations in satellite networks with site diversity.

Since IRS locations have a great impact on the overall system performance, their optimization is extremely important and deserves its own study. The major contributions of this paper are the following:

- Extension of the IRS system model introduced in [5] to multi-IRS systems, including not only small-scale fading but also large-scale path loss. In this way, we exploit the *geometric characteristics* of the wireless network.
- Recent works dealing with the IRS deployment often assume a *continuous area for installing an IRS* (see, for example, [7] and [9]). Unlike previous research, in this article we consider a *predetermined and finite set of available IRS locations*, thus taking into account physical constraints for the IRS positions. This is of great practical interest, since IRSs are usually installed on the facades, walls or ceilings of existing buildings.
- Mathematical formulation of a discrete optimization problem in order to minimize an upper bound of system outage probability, which is subject to several constraints, namely, minimum and maximum number of reflecting elements for each IRS, maximum number of installed IRSs, maximum total number of reflecting elements and maximum total IRS installation cost.
- Furthermore, we construct a *linear-programming relaxation (LPR)* so as to lower bound the optimum value. Then, we develop a *greedy algorithm* whose key ingredient is the LPR solution. This algorithm is also guaranteed to find a feasible solution in polynomial time.

The remainder of this paper is organized as follows. Section II describes the system model, Section III formulates the optimization problem, and Section IV develops and analyzes the proposed algorithm. In addition, Section V presents some numerical results, while Section VI concludes the paper.

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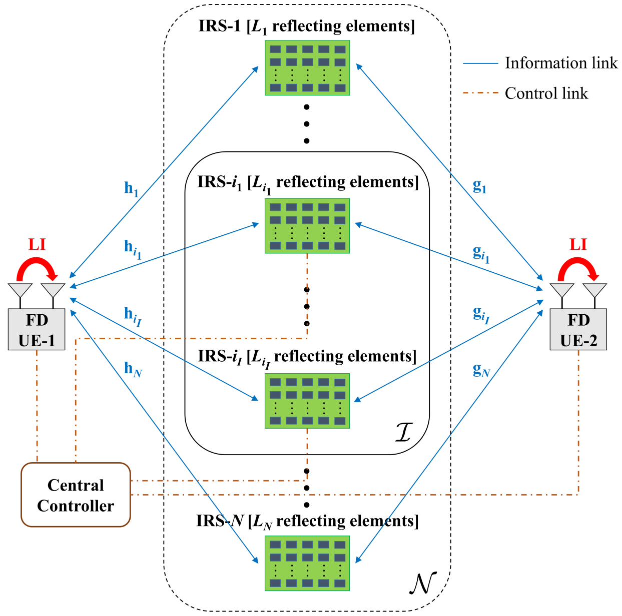

Fig. 1. Multi-IRS system assisting two-way FD communication with reciprocal channels and negligible direct link. The set of available IRS locations, $\mathcal{N}$, and the set of finally installed IRSs, $\mathcal{I}$, are illustrated by the dashed-outline and solid-outline rectangles, respectively. In each time-slot, the central controller activates only one IRS form the set $\mathcal{I}$, while the remaining IRSs are idle.

II. System Model

In this paper, we deal with a multi-IRS system assisting a two-way point-to-point (P2P) communication link, as shown in Fig. 1. In particular, each user equipment (UE) operates in FD mode and therefore is equipped with either a single shared-antenna or a pair of separate antennas for signal transmission and reception, depending on the FD implementation [3]. In addition, $\mathcal{N} = \{1, \ldots, N\}$ represents the set of available locations for installing an IRS, while $\mathcal{I} = \{i_1, \ldots, i_I\} \subseteq \mathcal{N}$ stands for the set of finally installed IRSs. In each time-slot, we assume that exactly one IRS from the set $\mathcal{I}$ is active and the remaining IRSs are idle (i.e., non-reflective). UE-1 transmits its data to UE-2, through the active IRS, and UE-2 transmits its data to UE-1, through the same IRS, simultaneously (i.e., within the same time-slot) and using the same frequency band. The transmit power of each UE is considered *fixed* for all time-slots. Since both UEs suffer from strong loop-interference (LI) due to the FD operation, they employ the same LI-cancellation techniques, resulting in residual LI [3].

Moreover, the total UE-to-IRS and IRS-to-UE transmission time is within a coherence interval of the wireless channel. As a result, the forward and backward channels between a UE and an IRS can be regarded almost identical (*reciprocal channels*) [5]. Also, the direct link between UEs is considered strongly attenuated due to severe blockage by physical obstacles (*no direct link*). We assume *perfect channel state information (CSI)* and *global CSI knowledge*. Furthermore, there is a *central controller* that performs the IRS activation, adjusts the IRS phase-shifts and communicates the necessary CSI knowledge between UEs.

Let L_n be the number of reflecting elements of the n^{th} IRS. The channel coefficient from UE-1 (UE-2) to the ℓ^{th} reflecting element of the n^{th} IRS is denoted by $h_{n,\ell} = |h_{n,\ell}|\, e^{j\vartheta_{n,\ell}}$ (respectively, $g_{n,\ell} = |g_{n,\ell}|\, e^{j\psi_{n,\ell}}$), for every $n \in \mathcal{N}$ and $\ell \in \mathcal{L}_n = \{1, \ldots, L_n\}$. For notational convenience, the channel coefficients corresponding to the n^{th} IRS can be grouped in vector form, i.e., $\mathbf{h}_n = [h_{n,1}, \ldots, h_{n,L_n}]^\top$ and $\mathbf{g}_n = [g_{n,1}, \ldots, g_{n,L_n}]^\top$. All channel coefficients are assumed to be *independent and identically distributed (i.i.d.)* complex normal/Gaussian random variables with zero mean and variance σ^2, therefore, $|h_{n,\ell}|, |g_{n,\ell}| \sim \text{Rayleigh}(\sigma/\sqrt{2})$. Also, the channel coefficients are considered constant within any time-slot. The diagonal $(L_n \times L_n)$ phase-shift matrix of the n^{th} IRS is given by $\mathbf{\Phi}_n = \text{diag}(e^{j\phi_{n,1}}, \ldots, e^{j\phi_{n,L_n}})$.

Under the above assumptions and following a similar approach with [5], the received signals at UE-1 and UE-2 in time-slot t (after the LI mitigation), when only the n^{th} IRS is active, are expressed as follows

$$\begin{aligned} y_1(t) =& \sqrt{P_2}\sqrt{\delta_{n,2}\delta_{n,1}}\mathbf{g}_n^\top \mathbf{\Phi}_n \mathbf{h}_n s_2(t) \\ &+ \sqrt{P_1}\delta_{n,1}\mathbf{h}_n^\top \mathbf{\Phi}_n \mathbf{h}_n s_1(t) + \xi_1(t) + w_1(t), \end{aligned} \tag{1}$$

$$\begin{aligned} y_2(t) =& \sqrt{P_1}\sqrt{\delta_{n,1}\delta_{n,2}}\mathbf{h}_n^\top \mathbf{\Phi}_n \mathbf{g}_n s_1(t) \\ &+ \sqrt{P_2}\delta_{n,2}\mathbf{g}_n^\top \mathbf{\Phi}_n \mathbf{g}_n s_2(t) + \xi_2(t) + w_2(t), \end{aligned} \tag{2}$$

where P_k, $s_k(t)$, $\xi_k(t)$ and $w_k(t)$ are the transmit power, information symbol, residual LI and additive white Gaussian noise (AWGN) of UE-k, respectively, for $k \in \{1, 2\}$. In addition, $\delta_{n,k} = A_0 d_{n,k}^{-\alpha}$ accounts for the large-scale path loss between the n^{th} IRS and UE-k, where A_0 is a positive constant that depends on the carrier frequency, $d_{n,k}$ is their Euclidean distance, and α is the path-loss exponent which depends on the wireless propagation environment. Note that, in the above equations, the first term represents the desired signal, while the second term is the self-interference (SI) induced by the IRS reflection of users' own transmitted symbols. Given that UE-k has knowledge of P_k, $s_k(t)$, $\delta_{n,k}$, $\mathbf{h}_n$ (required for $k = 1$), $\mathbf{g}_n$ (needed for $k = 2$), and $\mathbf{\Phi}_n$, it can completely remove the SI. Moreover, the residual LI $\xi_k(t)$ and AWGN $w_k(t)$ are modeled as independent zero-mean complex Gaussian random variables with variances $\sigma^2_{\text{LI}_k}$ and $\sigma^2_{w_k}$, respectively.

For the sake of simplicity, we assume the following: $P_1 = P_2 = P$, $\mathbb{E}(|s_1(t)|^2) = \mathbb{E}(|s_2(t)|^2) = 1$, $\sigma^2_{w_1} = \sigma^2_{w_2} = \sigma^2_w$ and $\sigma^2_{\text{LI}_1} = \sigma^2_{\text{LI}_2} = \sigma^2_{\text{LI}}$. Consequently, the *instantaneous* signal-to-interference-plus-noise ratio (SINR) at both UEs, after the SI elimination, when communicating via the n^{th} IRS is given by

$$\gamma_n = \rho_n \left| \sum_{\ell \in \mathcal{L}_n} |h_{n,\ell}|\, |g_{n,\ell}|\, e^{j(\phi_{n,\ell} + \vartheta_{n,\ell} + \psi_{n,\ell})} \right|^2, \tag{3}$$

where

$$\rho_n = \frac{P\delta_n}{\sigma^2_{\text{LI}} + \sigma^2_w} \tag{4}$$

with $\delta_n = \delta_{n,1}\delta_{n,2}$ being the overall path-loss between the two UEs through the n^{th} IRS. This SINR formula is quite similar to that in [5], except for the total path-loss term δ_n.

Furthermore, the IRS phase-shifts are optimally designed in order to maximize the instantaneous SINR, i.e.,

$$\phi_{n,\ell}^{\star} = -\vartheta_{n,\ell} - \psi_{n,\ell}, \ \forall \ell \in \mathcal{L}_n = \{1, \ldots, L_n\}. \quad (5)$$

Note that the IRS phase-shifts are adjusted by the central controller after obtaining the necessary CSI knowledge (channel coefficients' phases) from the UE that performs the channel estimation. Therefore, the maximum SINR at both UEs (when communicating via the n^{th} IRS) is written as follows

$$\gamma_n^{\star} = \rho_n \left(\sum_{\ell \in \mathcal{L}_n} |h_{n,\ell}| \, |g_{n,\ell}| \right)^2 = \rho_n \zeta_n^2, \quad (6)$$

where $\zeta_n = \sum_{\ell \in \mathcal{L}_n} \zeta_{n,\ell}$, with $\zeta_{n,\ell} = |h_{n,\ell}| \, |g_{n,\ell}| \geq 0$, $\forall \ell \in \mathcal{L}_n$. Observe that the Rayleigh-product random variables $\{\zeta_{n,\ell}\}_{\ell \in \mathcal{L}_n}$ are i.i.d. and, according to [5], the cumulative distribution function (CDF) of each one can be expressed as

$$F(u) \triangleq \Pr(\zeta_{n,\ell} \leq u) = 1 - \frac{2u}{\sigma^2} K_1 \left(\frac{2u}{\sigma^2} \right), \ \forall u \geq 0, \quad (7)$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν.

In addition, given an SINR threshold γ_{th}, the outage probability of each UE, defined as $P_{\text{out},n}(L_n) \triangleq \Pr(\gamma_n^{\star} \leq \gamma_{\text{th}}) = \Pr(\zeta_n \leq \sqrt{\gamma_{\text{th}}/\rho_n})$, can be approximated by the CDF of the Gamma distribution [5]. Despite being very useful for performance analysis, this approximation is not flexible enough for optimization purposes. Nevertheless, in order to achieve mathematical tractability, we can construct an upper bound of outage probability as follows

$$\begin{aligned} P_{\text{out},n}(L_n) &= \Pr\left(\sum_{\ell \in \mathcal{L}_n} \zeta_{n,\ell} \leq \sqrt{\gamma_{\text{th}}/\rho_n} \right) \\ &\overset{\text{(a)}}{\leq} \Pr\left(\max_{\ell \in \mathcal{L}_n} \{\zeta_{n,\ell}\} \leq \sqrt{\gamma_{\text{th}}/\rho_n} \right) \\ &= \Pr\left(\bigcap_{\ell \in \mathcal{L}_n} \{\zeta_{n,\ell} \leq \sqrt{\gamma_{\text{th}}/\rho_n}\} \right) \\ &\overset{\text{(b)}}{=} \prod_{\ell \in \mathcal{L}_n} \Pr(\zeta_{n,\ell} \leq \sqrt{\gamma_{\text{th}}/\rho_n}) \\ &\overset{\text{(c)}}{=} \left[F(\sqrt{\gamma_{\text{th}}/\rho_n}) \right]^{L_n} \triangleq \overline{P}_{\text{out},n}(L_n), \end{aligned} \quad (8)$$

where inequality (a) is due to the fact that $\sum_{\ell \in \mathcal{L}_n} \zeta_{n,\ell} \geq \max_{\ell \in \mathcal{L}_n} \{\zeta_{n,\ell}\}$, while equalities (b) and (c) follows from the independence of $\{\zeta_{n,\ell}\}_{\ell \in \mathcal{L}_n}$ and (7), respectively.

A. IRS Activation Policy

As we mentioned earlier, exactly one IRS is activated in each time-slot by the central controller, while the remaining IRSs are inactive. In particular, the central controller activates the IRS that achieves the maximum (instantaneous) SINR at UEs among the installed IRSs, that is,

$$i^{\star} \in \arg\max_{i \in \mathcal{I}} \{\gamma_i^{\star}\} \ \Leftrightarrow \ \gamma_{i^{\star}}^{\star} = \max_{i \in \mathcal{I}} \{\gamma_i^{\star}\}, \quad (9)$$

where $\gamma_i^{\star}$ is given by (6).

B. Upper Bound of System Outage Probability

Based on the aforementioned IRS activation strategy, the *system outage probability* can be computed as follows

$$\begin{aligned} P_{\text{out}}(\mathcal{I}, \mathbf{L}) &\triangleq \Pr(\gamma_{i^{\star}}^{\star} \leq \gamma_{\text{th}}) \overset{\text{(d)}}{=} \Pr\left(\max_{i \in \mathcal{I}} \{\gamma_i^{\star}\} \leq \gamma_{\text{th}} \right) \\ &= \Pr\left(\bigcap_{i \in \mathcal{I}} \{\gamma_i^{\star} \leq \gamma_{\text{th}}\} \right) \overset{\text{(e)}}{=} \prod_{i \in \mathcal{I}} \Pr(\gamma_i^{\star} \leq \gamma_{\text{th}}) \\ &\overset{\text{(f)}}{=} \prod_{i \in \mathcal{I}} P_{\text{out},i}(L_i), \end{aligned} \quad (10)$$

where $\mathbf{L} = [L_1, \ldots, L_N]^\top$ and γ_{th} is the SINR threshold. Equalities (d), (e) and (f) follow from (9), the independence of $\{\gamma_i^{\star}\}_{i \in \mathcal{I}}$ (due to the independence of $\{\zeta_i\}_{i \in \mathcal{I}}$) and the definition of $P_{\text{out},i}(L_i) \triangleq \Pr(\gamma_i^{\star} \leq \gamma_{\text{th}})$, respectively.

Afterwards, by combining (10) with (8), we obtain the following upper bound of system outage probability

$$P_{\text{out}}(\mathcal{I}, \mathbf{L}) \leq \prod_{i \in \mathcal{I}} \left[F(\sqrt{\gamma_{\text{th}}/\rho_i}) \right]^{L_i} \triangleq \overline{P}_{\text{out}}(\mathcal{I}, \mathbf{L}). \quad (11)$$

C. IRS Installation Cost Model

We model the installation cost of IRS $n \in \mathcal{N}$ as an *affine function* of the number of reflecting elements, that is,

$$C_n(L_n) = c_n + \lambda_n L_n, \quad (12)$$

where $c_n \geq 0$ is the fixed deployment cost and $\lambda_n \geq 0$ is the cost rate (measured in cost-units per element) of the corresponding IRS. In addition, the total installation cost is defined as the sum of the costs of all IRSs in the set $\mathcal{I}$, i.e.,

$$C_{\text{tot}}(\mathcal{I}, \mathbf{L}) = \sum_{i \in \mathcal{I}} C_i(L_i) = \sum_{i \in \mathcal{I}} (c_i + \lambda_i L_i). \quad (13)$$

III. Optimization Problem Formulation

In this section, we study the minimization of system outage probability under various constraints. More precisely, we minimize the upper bound $\overline{P}_{\text{out}}(\mathcal{I}, \mathbf{L})$ given by (11) instead of $P_{\text{out}}(\mathcal{I}, \mathbf{L})$ in (10); this is done for *mathematical tractability when constructing a relaxation problem*. In addition, the IRS deployment problem consists of two components, namely, the selection of locations for installing IRSs and the determination of IRS sizes. Herein, we consider a *predetermined* and *finite* set of available IRS locations.

In this context, the *joint IRS location and size optimization* problem is formulated as follows

$$\begin{aligned} \min_{\mathcal{I}, \mathbf{L}} \quad & \overline{P}_{\text{out}}(\mathcal{I}, \mathbf{L}) \triangleq \prod_{i \in \mathcal{I}} \left[F(\sqrt{\gamma_{\text{th}}/\rho_i}) \right]^{L_i} && \text{(14a)} \\ \text{s.t.} \quad & \mathcal{I} \subseteq \mathcal{N} && \text{(14b)} \\ & L_n \in \{L_n^{\min}, \ldots, L_n^{\max}\}, \ \forall n \in \mathcal{N} && \text{(14c)} \\ & |\mathcal{I}| \leq M && \text{(14d)} \\ & L_{\text{tot}}(\mathcal{I}, \mathbf{L}) \triangleq \sum_{i \in \mathcal{I}} L_i \leq L_{\text{tot}}^{\max} && \text{(14e)} \\ & C_{\text{tot}}(\mathcal{I}, \mathbf{L}) \triangleq \sum_{i \in \mathcal{I}} (c_i + \lambda_i L_i) \leq C_{\text{tot}}^{\max}, && \text{(14f)} \end{aligned}$$

where $L_n^{\min}, L_n^{\max} \geq 0$ are the minimum and maximum number of reflecting elements of the n^{th} IRS, respectively (with $L_n^{\min} \leq L_n^{\max}$). For example, in a specific location/building there are some limitations on the area (dimensions) that an IRS can occupy. Also, $M \in \{0, 1, \ldots, N\}$ is the maximum number of installed IRSs, resulting in an *IRS-cardinality constraint*. Finally, $L_{\text{tot}}^{\max}, C_{\text{tot}}^{\max} \geq 0$ denote the maximum total number of reflecting elements and the maximum total IRS installation cost, respectively. Note that constraint (14e) implicitly imposes an upper bound on the *overall signaling overhead*, which is required for IRS activation and phase adjustments.

Now, let us introduce a vector of binary (0/1) variables $\mathbf{x} = [x_1, \ldots, x_N]^\top$ such that, for all $n \in \mathcal{N}$, $x_n = 1$ if and only if (iff) $n \in \mathcal{I}$. Subsequently, the set $\mathcal{I}$ is replaced by the vector $\mathbf{x}$ in all functions that contained it with a slight abuse of notation. With these in mind, we can make the following observations:[1] 1) $\overline{P}_{\text{out}}(\mathcal{I}, \mathbf{L}) = \prod_{n \in \mathcal{N}} \left[F(\sqrt{\gamma_{\text{th}}/\rho_n})\right]^{x_n L_n}$, 2) $|\mathcal{I}| = \sum_{n \in \mathcal{N}} x_n$, 3) $L_{\text{tot}}(\mathcal{I}, \mathbf{L}) = \sum_{n \in \mathcal{N}} x_n L_n$ and 4) $C_{\text{tot}}(\mathcal{I}, \mathbf{L}) = \sum_{n \in \mathcal{N}} (c_n + \lambda_n L_n) x_n$. Therefore, problem (14) can be written as follows

$$\min_{\mathbf{x}, \mathbf{L}} \quad \overline{P}_{\text{out}}(\mathbf{x}, \mathbf{L}) \triangleq \prod_{n \in \mathcal{N}} \left[F(\sqrt{\gamma_{\text{th}}/\rho_n})\right]^{x_n L_n} \tag{15a}$$
$$\text{s.t.} \quad x_n \in \{0, 1\}, \ \ \forall n \in \mathcal{N} \tag{15b}$$
$$L_n \in \{L_n^{\min}, \ldots, L_n^{\max}\}, \ \ \forall n \in \mathcal{N} \tag{15c}$$
$$\sum_{n \in \mathcal{N}} x_n \leq M \tag{15d}$$
$$L_{\text{tot}}(\mathbf{x}, \mathbf{L}) \triangleq \sum_{n \in \mathcal{N}} x_n L_n \leq L_{\text{tot}}^{\max} \tag{15e}$$
$$C_{\text{tot}}(\mathbf{x}, \mathbf{L}) \triangleq \sum_{n \in \mathcal{N}} (c_n + \lambda_n L_n) x_n \leq C_{\text{tot}}^{\max}, \tag{15f}$$

where $\mathbf{x}$, $\mathbf{L}$ are the decision/optimization variables. Since $\log(\cdot)$ is a monotonically increasing function, we can replace $\overline{P}_{\text{out}}(\mathbf{x}, \mathbf{L})$ with its logarithm, without altering the set of optimal solutions. Hence, we obtain the equivalent *discrete optimization problem*

$$\min_{\mathbf{x}, \mathbf{L}} \quad G(\mathbf{x}, \mathbf{L}) \triangleq \log\left(\overline{P}_{\text{out}}(\mathbf{x}, \mathbf{L})\right) = \sum_{n \in \mathcal{N}} \beta_n (x_n L_n) \tag{16a}$$
$$\text{s.t.} \quad x_n \in \{0, 1\}, \ \ \forall n \in \mathcal{N} \tag{16b}$$
$$L_n \in \{L_n^{\min}, \ldots, L_n^{\max}\}, \ \ \forall n \in \mathcal{N} \tag{16c}$$
$$\sum_{n \in \mathcal{N}} x_n \leq M \tag{16d}$$
$$\sum_{n \in \mathcal{N}} x_n L_n \leq L_{\text{tot}}^{\max} \tag{16e}$$
$$\sum_{n \in \mathcal{N}} c_n x_n + \sum_{n \in \mathcal{N}} \lambda_n (x_n L_n) \leq C_{\text{tot}}^{\max}, \tag{16f}$$

where $\beta_n = \log\left(F(\sqrt{\gamma_{\text{th}}/\rho_n})\right) \leq 0$ for all $n \in \mathcal{N}$. Let $G^\star$ be the global minimum of (16). Due to its discrete (and, thus, nonconvex) structure, this problem is rather unlikely to be globally solved in polynomial time. Note that the brute-force exhaustive search is impractical because of its extremely high (exponential) complexity.

Remark 1 (Feasibility): Problem (16) is *always* feasible, since the solution $(\mathbf{x}, \mathbf{L}) = (\mathbf{0}_N, \mathbf{L}^{\min})$, where $\mathbf{0}_N$ is the N-dimensional zero vector and $\mathbf{L}^{\min} = [L_1^{\min}, \ldots, L_N^{\min}]^\top$, satisfies all constraints.

IV. LPR-Based Greedy Algorithm

Despite the difficulty of computing the global minimum of (16), we will show how to efficiently compute a lower bound of $G^\star$. Firstly, by using auxiliary decision variables $\mathbf{z} = [z_1, \ldots, z_N]^\top$, problem (16) can be equivalently written in the following form

$$\min_{\mathbf{x}, \mathbf{L}, \mathbf{z}} \quad \sum_{n \in \mathcal{N}} \beta_n z_n \tag{17a}$$
$$\text{s.t.} \quad x_n \in \{0, 1\}, \ \ \forall n \in \mathcal{N} \tag{17b}$$
$$L_n \in \{L_n^{\min}, \ldots, L_n^{\max}\}, \ \ \forall n \in \mathcal{N} \tag{17c}$$
$$z_n = x_n L_n, \ \ \forall n \in \mathcal{N} \tag{17d}$$
$$\sum_{n \in \mathcal{N}} x_n \leq M \tag{17e}$$
$$\sum_{n \in \mathcal{N}} z_n \leq L_{\text{tot}}^{\max} \tag{17f}$$
$$\sum_{n \in \mathcal{N}} c_n x_n + \sum_{n \in \mathcal{N}} \lambda_n z_n \leq C_{\text{tot}}^{\max}. \tag{17g}$$

Secondly, by relaxing the integer/discrete constraints, $x_n \in \{0, 1\}$ and $L_n \in \{L_n^{\min}, \ldots, L_n^{\max}\}$, we have

$$\min_{\mathbf{x}, \mathbf{L}, \mathbf{z}} \quad \sum_{n \in \mathcal{N}} \beta_n z_n \tag{18a}$$
$$\text{s.t.} \quad 0 \leq x_n \leq 1, \ \ \forall n \in \mathcal{N} \tag{18b}$$
$$L_n^{\min} \leq L_n \leq L_n^{\max}, \ \ \forall n \in \mathcal{N} \tag{18c}$$
$$z_n = x_n L_n, \ \ \forall n \in \mathcal{N} \tag{18d}$$
$$\sum_{n \in \mathcal{N}} x_n \leq M \tag{18e}$$
$$\sum_{n \in \mathcal{N}} z_n \leq L_{\text{tot}}^{\max} \tag{18f}$$
$$\sum_{n \in \mathcal{N}} c_n x_n + \sum_{n \in \mathcal{N}} \lambda_n z_n \leq C_{\text{tot}}^{\max}. \tag{18g}$$

Notice that this problem is nonlinear due to the equality constraints $z_n = x_n L_n$. In order to obtain a linear problem, we apply further relaxation by replacing the set of constraints $L_n^{\min} \leq L_n \leq L_n^{\max}$ and $z_n = x_n L_n$ with the linear constraints $L_n^{\min} x_n \leq z_n \leq L_n^{\max} x_n$. In this way, we can

[1] In order to avoid the undefined quantity 0^0, we assume that $P_{\text{out},n}(L_n) > 0$, which implies $\left[F(\sqrt{\gamma_{\text{th}}/\rho_n})\right]^{L_n} > 0$, for all $n \in \mathcal{N}$.

remove the decision variable $\mathbf{L}$ and formulate the following *linear-programming relaxation (LPR)* problem

$$\min_{\mathbf{x},\mathbf{z}} \quad \sum_{n\in\mathcal{N}} \beta_n z_n \tag{19a}$$
$$\text{s.t.} \quad 0 \le x_n \le 1, \ \ \forall n \in \mathcal{N} \tag{19b}$$
$$L_n^{\min} x_n \le z_n \le L_n^{\max} x_n, \ \ \forall n \in \mathcal{N} \tag{19c}$$
$$\sum_{n\in\mathcal{N}} x_n \le M \tag{19d}$$
$$\sum_{n\in\mathcal{N}} z_n \le L_{\text{tot}}^{\max} \tag{19e}$$
$$\sum_{n\in\mathcal{N}} c_n x_n + \sum_{n\in\mathcal{N}} \lambda_n z_n \le C_{\text{tot}}^{\max}. \tag{19f}$$

Note that the guaranteed feasibility of (16) (see Remark 1) implies the feasibility of (17), (18) and (19). In what follows, $(\mathbf{x}^\dagger, \mathbf{z}^\dagger)$ and $G^\dagger = \sum_{n\in\mathcal{N}} \beta_n z_n^\dagger$ denote an optimal solution and the global minimum of (19), respectively. Obviously, $G^\dagger \le G^\star$, that is, $G^\dagger$ is a lower bound of $G^\star$.

Finally, given that (19) has $V = 2N = \Theta(N)$ decision variables and $U = 4N+3 = \Theta(N)$ constraints, a globally optimal solution can be computed in $O((U+V)^{1.5}V^2) = O(N^{3.5})$ time using an interior-point method [12].

Algorithm 1 LPR-based Greedy Algorithm (LPR-GA)

Input: N, $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_N]^\top$, $\mathbf{L}^{\min} = [L_1^{\min}, \ldots, L_N^{\min}]^\top$, $\mathbf{L}^{\max} = [L_1^{\max}, \ldots, L_N^{\max}]^\top$, M, $L_{\text{tot}}^{\max}$, $\mathbf{c} = [c_1, \ldots, c_N]^\top$, $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_N]^\top$, $C_{\text{tot}}^{\max}$

Output: A feasible solution $(\mathbf{x}', \mathbf{L}')$ of discrete problem (16)

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1:  Solve the LPR problem (19) to obtain an optimal solution (x†, z†).
2:  ▷ Computation of L' = [L'_1, ..., L'_N]^⊤
3:  for all n ∈ N do
4:      if x†_n ≠ 0 then
5:          L'_n := round(z†_n / x†_n)
6:      else
7:          L'_n := round((1/2)(L_n^min + L_n^max))
8:      end if
9:  end for
10: ▷ Computation of x' = [x'_1, ..., x'_N]^⊤
11: Sort the entries of x† in descending order. Let (σ_1, ..., σ_N) ∈ Σ_N be their order after sorting, where Σ_N is the set of all permutations of N, therefore x†_{σ_1} ≥ ··· ≥ x†_{σ_N}.
12: m := 1, L_tot := 0, C_tot := 0, x' := 0_N
13: while (m ≤ M) ∧ (L_tot ≤ L_tot^max) ∧ (C_tot ≤ C_tot^max) do
14:     i := σ_m, x'_i := 1
15:     L_tot := L_tot + L'_i, C_tot := C_tot + c_i + λ_i L'_i
16:     m := m + 1
17: end while
18: if (L_tot > L_tot^max) ∨ (C_tot > C_tot^max) then
19:     x'_i := 0
20: end if
21: return (x', L')
```

Now, we are ready to develop a greedy algorithm of polynomial complexity to obtain a feasible solution for problem (16). This procedure is given in Algorithm 1 and is called *LPR-based greedy algorithm (LPR-GA)*. First, the proposed algorithm applies *deterministic rounding*, using the solution of LPR, to compute the decision variable $\mathbf{L}'$ (lines 3–9), i.e.,

$$L_n' = \begin{cases} \text{round}\left(z_n^\dagger / x_n^\dagger\right), \ \text{if } x_n^\dagger \ne 0 \\ \text{round}\left(\frac{1}{2}(L_n^{\min} + L_n^{\max})\right), \ \text{otherwise} \end{cases}, \ \forall n \in \mathcal{N}, \tag{20}$$

where $\text{round}(y) = \lfloor y + 0.5 \rfloor$. Observe that, if $x_n^\dagger \ne 0$, then $L_n^{\min} \le z_n^\dagger / x_n^\dagger \le L_n^{\max}$ (due to the feasibility of LPR problem) and therefore $\text{round}\left(z_n^\dagger / x_n^\dagger\right) \in \{L_n^{\min}, \ldots, L_n^{\max}\}$. Also, the same holds for $\text{round}\left(\frac{1}{2}(L_n^{\min} + L_n^{\max})\right)$ in case of $x_n^\dagger = 0$. In other words, the above deterministic rounding guarantees that $L_n' \in \{L_n^{\min}, \ldots, L_n^{\max}\}$ for all $n \in \mathcal{N}$.

Concerning the computation of $\mathbf{x}'$, the proposed algorithm sorts the entries of $\mathbf{x}^\dagger \in [0,1]^N$ in descending order (line 11). Then, by starting from the zero vector, it successively selects IRS locations (based on their sorting) until the violation of at least one of the constraints: $\sum_{n\in\mathcal{N}} x_n \le M$, $L_{\text{tot}}(\mathbf{x}, \mathbf{L}) \le L_{\text{tot}}^{\max}$ and $C_{\text{tot}}(\mathbf{x}, \mathbf{L}) \le C_{\text{tot}}^{\max}$ (lines 12–17). Finally, it removes the last selected IRS location if $L_{\text{tot}} > L_{\text{tot}}^{\max}$ or $C_{\text{tot}} > C_{\text{tot}}^{\max}$ (lines 18–20); note that the IRS-cardinality constraint is automatically satisfied due to the construction of the while-loop, so there is no need to check it in line 18.

Obviously, the solution $(\mathbf{x}', \mathbf{L}')$ returned by Algorithm 1 is guaranteed to be feasible for problem (16), thus $G^\star \le G' \triangleq G(\mathbf{x}', \mathbf{L}')$, i.e., G' is an upper bound of $G^\star$.

Remark 2 (A posteriori performance guarantee): It is possible to provide an approximation guarantee after the termination of Algorithm 1, using the already obtained solution of LPR, as follows: $0 \le G' - G^\star \le G' - G^\dagger$.

Regarding the complexity of Algorithm 1, the LPR problem can be solved in $O(N^{3.5})$ time, the computation of $\mathbf{L}'$ requires $\Theta(N)$ time, while the computation of $\mathbf{x}'$ requires $O(N\log N + N) = O(N \log N)$ arithmetic operations in total. Hence, the overall complexity of LPR-GA is $O(N^{3.5} + N + N\log N) = O(N^{3.5})$. In other words, it has the same asymptotic complexity (up to a constant) as the LPR problem.

V. Numerical Results

In this section, we generate random system configurations, where UE-1 and UE-2 are constantly located at $(0,0)$ and $(100,0)$, respectively, while each IRS location is uniformly distributed either inside the rectangle $[30,70] \times [20,40]$ or $[30,70] \times [-40,-20]$, with probability $1/2$ of being in each rectangle. The remaining system parameters are the following: $P = 25$ dBm, $\sigma^2 = 1$, $\sigma_w^2 = -80$ dBm, path-loss model with $\{A_0 = 1, \alpha = 2.7\}$, $N = 25$, $M = 7$, $L_{\text{tot}}^{\max} = 250$, $L_n^{\min} = L^{\min} = 5$, $L_n^{\max} = L^{\max} = 40$, $c_n \sim \text{Uniform}[1,5]$, $\lambda_n \sim \text{Uniform}[0.1, 0.5]$ for all $n \in \mathcal{N}$. Note that, for a given problem, the IRS locations and $\{c_n, \lambda_n\}_{n\in\mathcal{N}}$ are all fixed (the randomization is only used for problem generation). In addition, all figures present average values obtained from 10^3 independent simulation scenarios.

In order to evaluate the performance of LPR-GA, we consider (besides LPR) a baseline scheme, namely, *Average-element greedy algorithm (AEGA)*, which is described as follows. First, L_n is set equal to $L_n^{\text{avg}} \triangleq \left\lceil \frac{1}{2}(L_n^{\min} + L_n^{\max}) \right\rceil \in$

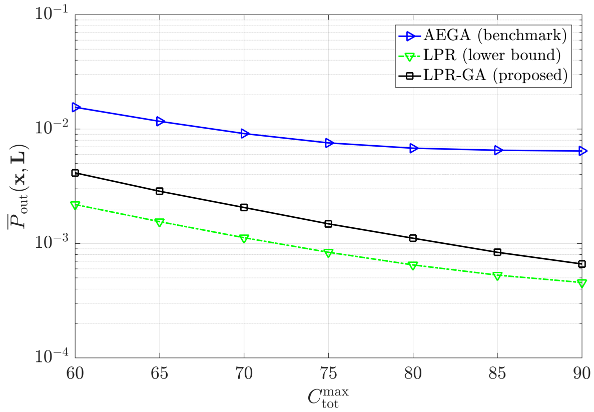

Fig. 2. Upper bound of system outage probability versus the maximum total IRS installation cost, for $\gamma_{\text{th}} = 8$ dB and $\sigma_{\text{LI}}^2 = -70$ dBm.

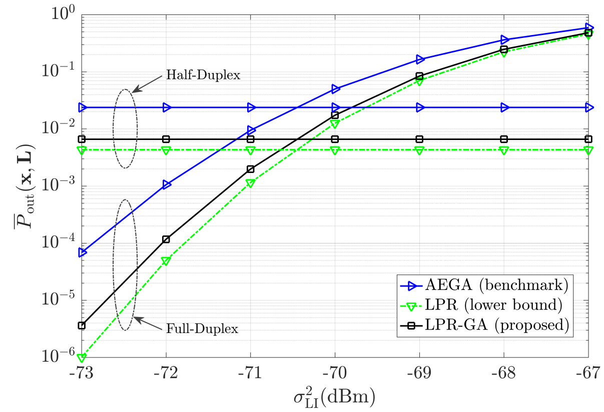

Fig. 3. Upper bound of system outage probability versus the residual-LI power at each UE, for $\gamma_{\text{th}} = 9$ dB and $C_{\text{tot}}^{\max} = 75$.

$\{L_n^{\min}, \ldots, L_n^{\max}\}$ for all $n \in \mathcal{N}$. Then, we sort the IRS locations in ascending order in terms of the product $\beta_n L_n^{\text{avg}}$ (≤ 0). Let $(\sigma_1, \ldots, \sigma_N) \in \Sigma_{\mathcal{N}}$ be the order of IRS locations after sorting, where $\Sigma_{\mathcal{N}}$ is the set of all permutations of $\mathcal{N}$, therefore $\beta_{\sigma_1} L_{\sigma_1}^{\text{avg}} \leq \cdots \leq \beta_{\sigma_N} L_{\sigma_N}^{\text{avg}}$. Finally, AEGA follows the steps 12–20 of Algorithm 1 to compute the binary vector $\mathbf{x}$. This method finds a feasible solution with computational complexity $O(N \log N)$ because of the sorting procedure.

Fig. 2 shows the upper bound of system outage probability, against the maximum total IRS installation cost, achieved by AEGA, LPR and LPR-GA. As expected, the outage probability is a nonincreasing function of $C_{\text{tot}}^{\max}$ for all algorithms, since larger $C_{\text{tot}}^{\max}$ translates to a less restricted feasible set. Furthermore, the proposed algorithm achieves much higher performance than the benchmark. It is interesting to observe that the LPR-GA remains very close to the lower bound (and, therefore, to the global minimum) for all values of $C_{\text{tot}}^{\max}$.

As concerns the comparison between FD and HD wireless technologies, we study how the residual-LI power affects the system outage probability.[2] Based on Fig. 3, it is obvious that LPR-GA significantly outperforms the baseline scheme, not only in FD but also in HD scenario. Moreover, for the FD scheme, the upper bound of system outage probability increases rapidly with the residual-LI power. Finally, FD performs better than HD system when σ_{LI}^2 is approximately less than -70.4 dBm, whereas HD is preferable when σ_{LI}^2 is greater than -70.4 dBm (regardless of the comparison algorithm). In other words, FD is more beneficial than HD technology, provided that the LI is sufficiently suppressed.

VI. Conclusion

In this paper, we have dealt with the minimization of outage probability in a FD system assisted by multiple IRSs. In particular, we have transformed the joint IRS location and size optimization problem into a discrete problem and developed an efficient greedy algorithm to find a near-optimal solution. Moreover, we have observed that FD outperforms HD scheme, provided that the LI at each UE is adequately mitigated.

[2]For the HD scheme, where each UE is equipped with a single antenna, we should replace ρ_n in (4) with $\rho_n^{\text{HD}} = P\delta_n/\sigma_w^2$ (since $\sigma_{\text{LI}}^2 = 0$) and also γ_{th} with $\gamma_{\text{th}}^{\text{HD}} = (1+\gamma_{\text{th}})^2 - 1$, which is obtained by equating the spectral efficiencies of the two schemes, i.e., $\log(1+\gamma_{\text{th}}) = \frac{1}{2}\log(1+\gamma_{\text{th}}^{\text{HD}})$. The latter replacement is made for fair comparison between FD and HD scenarios in terms of outage probability.

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