

Developed a Relation between True and Engineering Stress – Strain for Compressibility

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ABSTRACT

Every material shows compressible characteristics under any types of loading and this characteristic expresses actual behavior of materials in reality. Strains and stresses are gradually changed in each loading step during tests because of variations of volumes. Volumetric true strains are non – zero in case of compressible conditions. In this study, poisson's ratio is considered to be constant in three perpendicular directions and stiffness is not changed during the application of tensile forces. In any stage of loading, true stress is the ratio between incremental load and area but engineering stress is the ratio between total load and initial area. Similarly, true strain is the ratio between incremental deflection and length in any stage of loading. On the other hand, engineering strain is the ratio between final deflection and initial length. Final deflection means additives of all stage deflections. True strain is the function of engineering strain which relates with the incremental lengths. Similarly, true stress is the function of engineering stress which relates with the incremental diameters. Proposed equations of true stress and strain have expressed compressible behaviors of materials.

Keywords:-Compressibility, engineering, stress-strain, true.

INTRODUCTION

Internal resisting force per unit area defines stress and shortening or lengthening per unit length defines strain [1]. However, every material in the universe shows stress – strain behavior hence stress is the function of strain. There were a good correlation found by the digital image method and finite element method for metal [4]. In engineering point of view, stress is the ratio between the final load and area for multistage loading so, it can't describe actual behavior. Maximum true stress was 15% high as maximum engineering stress and maximum true strain was 1.5% smaller than engineering strain [2]. In stress for cold formed section, corner region showed better performance than other region [6]. Conventional methods of calculation of true stress and strain are developed by considering incompressible condition. But

in real life stress – strain behavior is fully compressible. In incompressible condition, cyclic true stress variations of various materials were nearly close with each other [4]. Stainless steel represented larger amount of strain which was higher than others materials [3]. The rate of compressibility is very low but it has no option to neglect it because of accuracy. In each step of tensile loading, material length increases and at the same time diameter either size of material decreases. Tensile stress capacity of rectangular section was higher than triangular section of material of steel [7]. In compressive loading, length of material decreases and at the same time diameter of material increases. Loading steps are independent each other for compressive and tensile loadings. In laboratory test, loading is gradually increased at a certain level then reloading, it decreases gradually.

However, loading, re-loading and reverse loading takes very shorter time during seismic shaking. Elastic modulus has not changed within elastic limit. After crosses elastic limit, elastic modulus changes that means to form non-linearity. In reality, all cases of loading show non-linear behavior after crosses elastic limit.

To avoid complexity, elastic modulus is taken constant which means that stress – strain behaviors are stand within elastic limit. Incremental lengths are used to express actual stress – strain behaviors and

$$l_{n(T)} = l_i \left[1 + \sum_{r=1}^n \epsilon_{n-(n-r)} \right] \quad (1)$$

Where, $l_{n(T)}$ is the incremental load under staged tensile loading, l_i is the initial length, r is expressed as the number of loading step, n is the total number of steps

$$l_{n(C)} = l_i \left[1 - \sum_{r=1}^n \epsilon_{n-(n-r)} \right] \quad (2)$$

Where, $l_{n(C)}$ is the incremental load under staged compressive loading. Engineering stress is represented by the Eq. (3) under tensile and compressive loadings.

$$\sigma_{ES(T/C)} = \frac{\sum_{r=1}^n P_{r(T/C)}}{\frac{\pi}{4} d_i^2} \quad (3)$$

Where, $P_{r(T/C)}$ is the staged tensile either compressive loading, $\sigma_{ES(T/C)}$ is

$$\epsilon_{ES(T/C)} = \left[\sum_{r=1}^n \epsilon_{n-(n-r)} \right] \quad (4)$$

Where, $\epsilon_{ES(T/C)}$ is the engineering strain under tensile either compressive loadings. True stress is the function of engineering stress in compressible condition. Final true

also variations of area for each steps of loading. In compressible conditions, variable volumes are expressed actual behavior of stresses and strains.

STRESS – STRAIN FORMULAE FOR COMPRESSIBLE CONDITION

Length increases gradually during tensile loading and it decreases under compressive loading. Incremental length for tensile loading is expressed by the Eq. (1). Also, incremental load for compressive loading is expressed by the Eq. (2).

and $\epsilon_{n-(n-r)}$ is the incremental longitudinal strain.

Engineering strain is expressed by the Eq. (4) under tensile and compressive loadings.

engineering stress and d_i is the initial diameter.

stress depends on final loading and area. Final loading is the summation of all loading.

Final area depends on the incremental loading step. Final true stress for tensile and compressive loadings in compressible

$$\sigma_{FTS(T)} = \frac{\sum_{r=1}^n P_{r(T)}}{\frac{\pi}{4} [d_i(1 - \mu \sum_{r=1}^n \epsilon_{n-(n-r)})]^2} \quad (5)$$

Where, $\sigma_{FTS(T)}$ is the final true stress under tensile loading, $P_{r(T)}$ is the staged tensile

condition is expressed by the Eq. (5) and Eq. (6).

$$\sigma_{FTS(C)} = \frac{\sum_{r=1}^n P_{r(C)}}{\frac{\pi}{4} [d_i(1 + \mu \sum_{r=1}^n \epsilon_{n-(n-r)})]^2} \quad (6)$$

Where, $\sigma_{FTS(C)}$ is the final true stress under compressive loading, $P_{r(C)}$ is the staged compressive loading. When $r = 0$, it expresses no loading condition so, in this case initial strain is equal o zero. $\epsilon_{n-(n-r)}$ is the incremental strain which comes from strain gage. Final true strain is calculating

from tensile and compressive loadings. Calculation procedure of true strain is similar for tensile and compressive loadings. Final true strain is expressed by the Eq. (7) and Eq. (8) under tensile and compressive loadings.

$$\epsilon_{FTS(T)} = \frac{[\sum_{r=1}^n \epsilon_{\{n-(n-r)\}}] - [\sum_{r=0}^{(n-1)} \epsilon_{\{(n-1)-\{(n-1)-r\}}]}]{1 + \sum_{r=0}^{(n-1)} \epsilon_{\{(n-1)-\{(n-1)-r\}}]}} \quad (7)$$

Where, $\epsilon_{\{(n-1)-\{(n-1)-r\}}}$ is the incremental strain and $\epsilon_{FTS(T)}$ is the final true strain for tensile loading.

$$\epsilon_{FTS(C)} = \frac{[\sum_{r=1}^n \epsilon_{\{n-(n-r)\}}] - [\sum_{r=0}^{(n-1)} \epsilon_{\{(n-1)-\{(n-1)-r\}}]}]{1 - \sum_{r=0}^{(n-1)} \epsilon_{\{(n-1)-\{(n-1)-r\}}]}} \quad (8)$$

Where, $\epsilon_{FTS(C)}$ is the final true strain for compressive loading. There have a relation of final true stress and engineering stress under tensile and compressive loadings. These relations are expressed by the Eq. (9) and Eq. (10). Eq. (9) represents the

relation between true and engineering stress under tensile loading and Eq. (10) represents the relation between true and engineering stress under compressive loading.

$$\sigma_{FTS(T)} = \frac{\sigma_{ES(T)}}{[1 - \mu \sum_{r=1}^n \epsilon_{n-(n-r)}]^2} \quad (9)$$

$$\sigma_{FTS(C)} = \frac{\sigma_{ES(C)}}{[1 + \mu \sum_{r=1}^n \epsilon_{n-(n-r)}]^2} \quad (10)$$

Where, $\sigma_{ES(T)}$ is engineering stress under tensile loading and $\sigma_{ES(C)}$ is the engineering

stress under compressive loading. True strain is the function of engineering strain.

Relation between true strain and engineering strain under tensile loading is expressed by the Eq. (11). Similarly,

$$\varepsilon_{\text{FTS(T)}} = \frac{\varepsilon_{\text{ES(T)}} - [\sum_{r=0}^{(n-1)} \varepsilon_{[(n-1)-\{(n-1)-r\}]}]}{[1 + \sum_{r=0}^{(n-1)} \varepsilon_{[(n-1)-\{(n-1)-r\}]}]} \quad (11)$$

$$\varepsilon_{\text{FTS(C)}} = \frac{\varepsilon_{\text{ES(C)}} - [\sum_{r=0}^{(n-1)} \varepsilon_{[(n-1)-\{(n-1)-r\}]}]}{[1 - \sum_{r=0}^{(n-1)} \varepsilon_{[(n-1)-\{(n-1)-r\}]}]} \quad (12)$$

Compressibility is the complex phenomena for every types of materials specially, stresses and strains. Therefore, these above developed formulae are expressed this complex situation.

CONCLUSION

Stresses and strains are different during every steps of loading in compressible condition. Change of area is inversely proportional to the change of length during every steps of loading.

For each loading steps, volumes are variable. Final true stress during tensile and compressive loading is inversely proportional to variable strains.

Length of any steps loading is the function of variable strains under tensile and compressive loading. Final true stresses and strains are proportional to the engineering stresses and strains during tensile and compressive loading.

Engineering strains are equal to variable strains under tensile and compressive loading. Area decreases and length increases under tensile loading.

During compressive loading, area increases and length decreases. Newly developed formulae are fully expressed stress – strain behavior under tensile and compressive loading in compressible condition.

relation between true strain and engineering strain under compressive loading is expressed by the Eq. (12).

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APPENDIX A: Details Functional Mechanism of Some Formulae

$$l_1 = l_i(1+\varepsilon_1)$$

$$l_2 = l_i(1+\varepsilon_1)(1+\varepsilon_2)$$

$$l_3 = l_i(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)$$

$$= l_i(1+\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_1\varepsilon_2+\varepsilon_2\varepsilon_3+\varepsilon_3\varepsilon_1+\varepsilon_1\varepsilon_2\varepsilon_3)$$

For small strain, $\varepsilon_1\varepsilon_2 = 0$, $\varepsilon_2\varepsilon_3 = 0$, $\varepsilon_3\varepsilon_1 = 0$ and $\varepsilon_1\varepsilon_2\varepsilon_3 = 0$

$$\varepsilon_1 = \frac{l_i - l_1}{l_i} \text{ (for compressive loading)}$$

$$\varepsilon_{ES(C)} = \frac{l_i - l_n}{l_i} \text{ (for compressive loading)}$$

$$\varepsilon_{FTS(C)} = \frac{l_{(n-1)} - l_n}{l_{(n-1)}} \text{ (for compressive loading)}$$

$$\varepsilon_1 = \frac{l_1 - l_i}{l_i} \text{ (for tensile loading)}$$

$$\varepsilon_{ES(T)} = \frac{l_n - l_i}{l_i} \text{ (for tensile loading)}$$

$$\varepsilon_{FTS(T)} = \frac{l_n - l_{(n-1)}}{l_{(n-1)}} \text{ (for tensile loading)}$$

$$d_0 = 0, l_0 = 0, \varepsilon_0 = 0$$