

# DATA BASE OF THE CLASSIFICATION OF MATROIDS ON EIGHT ELEMENTS WITH RESPECT TO THE HALF-PLANE PROPERTY AND SOS-RAYLEIGH

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Using the classification given in [4], we provide the list of all simple matroids on 8 elements of rank 3 and 4 that

- have the half-plane property (HPP),
- don't have the half-plane property
- it is unknown whether they are SOS-Rayleigh (they have HPP),

and the list of some matroids on 8 elements that are not SOS-Rayleigh (in particular, they are not weakly determinantal).

The matroids are listed in an encoded format first given by Matsumoto *et. al.* in [5]. In this encoding, each line in the file keeps the fingerprint of the collection of the bases of a matroid  $M$  in the following way: for a fixed  $r$ , all size  $r$  subsets of  $E = [8]$  are ordered increasingly in the reverse lexicographic order, and each subset  $S$  is represented with the character “\*” if it appears in the collection of bases of  $M$  and with the character “0” if it doesn't appear in the collection. For example,

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**0*****0
```

represents the matroid on 8 elements of rank 2 that has all size two subsets of  $E = [8]$  as bases except the subsets  $\{2, 3\}$  and  $\{7, 8\}$  represented on the 3rd and 28th positions respectively. The formal definition of the reverse lexicographic order of sets is as follows.

**Definition 0.1.** Two distinct  $r$  element sets  $S_1, S_2$  are called to have the relation  $S_1 \prec S_2$  with respect to the reverse lexicographic order if  $\max(S_1) < \max(S_2)$  or  $\max(S_1) = \max(S_2) = a$  and  $S_1 \setminus \{a\} \prec S_2 \setminus \{a\}$ .

All matroids of rank  $r \leq 2$  have the half-plane property, and taking the dual and direct sums preserve the half-plane property. The first examples of matroids that have the half-plane property that are not SOS-Rayleigh appear on rank 4 on 8 elements. Moreover, the half-plane property and SOS-Rayleigh are minor closed properties (see [4] for more information). Therefore, providing the list of simple matroids of rank 3 and 4 with or/and without the respective properties provide a complete database information to determine whether a given matroid on 8 elements has the desired property.

Among the files provided, one can find Macaulay2 code for encoding and decoding matroids, and for sorting them with respect to the properties.

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## REFERENCES

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