

Mermin Device: a rather mundane explanation

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Abstract

Mermin Device and conundrum presented by it is discussed. An alternate explanation for the conundrum is discussed which is rather mundane.

Keywords and phrases: EPR paradox; Bell's inequality; Mermin Device.

1 Introduction

In 1981 N. D. Mermin published his now famous paper titled, “Bringing home the atomic world: Quantum mysteries for anybody” (1). Feynman called this “One of the most beautiful papers in physics that I know”. In this paper Mermin proposed a device containing some black boxes, which is not very difficult to build with technology available at our disposal today. The device shown in his Figure 2 below, illustrates in a simple way for anyone with basic knowledge of theory of probability, to understand the EPR paradox also known as Einstein-Podolsky-Rosen conundrum.

Two particles are emitted from the Source C towards the two detectors A and B controlled by Alice and Bob respectively. Experimentalists may choose any one of three settings (1, 2, or 3) for their detector on each run of the experiment. This decision can be made at any time before the particle reaches the detector. There are two possible outcomes for any given setting, red (R) or green (G). The two possible Mermin device outcomes R and G represent two possible spin measurement outcomes “up” or “down” respectively, and the three possible Mermin device settings represent three different orientations of the SG magnets along a preferred direction (e.g. vertical) in the plane normal to electron's motion. The experiment is performed thousands of times with Alice and Bob changing their detector settings randomly before the particle hits the detectors. They note down their results as below

1. When the settings are the same for both detectors (11, 22, or 33) they always flash the same color: RR and GG with equal frequency; RG and GR never occur.

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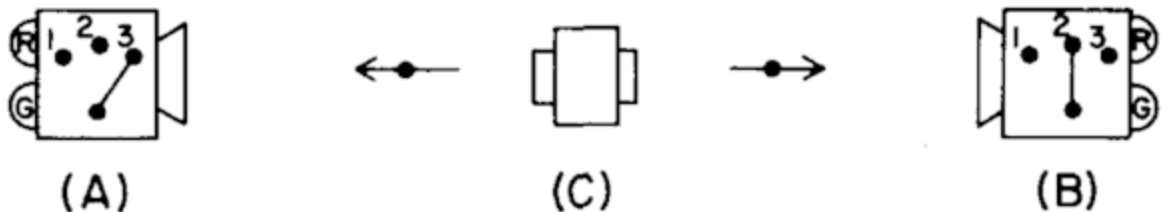


Fig. 2. Complete device. *A* and *B* are the two detectors. *C* is the box from which the two particles emerge.

Table 1: Mermin's calculations with all possible combinations

Setting	RGG	RGR	GRG	GRR	RRG	GGR	RRR	GGG
12	RG	RG	GR	GR	RR	GG	RR	GG
13	RG	RR	GG	GR	RG	GR	RR	GG

Table 2: Mermin's calculations excluding GGG and RRR

Setting	RGG	RGR	GRG	GRR	RRG	GGR
12	RG	RG	GR	GR	RR	GG
13	RG	RR	GG	GR	RG	GR

2. When the settings are different for both detectors (12, 13, 21, 23, 31 or 32) they flash the same color (RR and GG occurring with equal frequency) only a quarter of times; the other three quarters of the time the detectors flash different colors (RG and GR occurring with equal frequency)

Result 1 means that both particles emitted by box C are having an instruction set which is same for both. For example instruction GGR means that both detectors will flash green for setting 1, green for setting 2 and Red for setting 3. If we use this fact and there are no other constraints we get 8 possible instruction sets (i.e. GGG, RRR, GGR, GRR, GRG, RRG, RGG and RGR). This means there is a probability of 1/2 of getting similar results (GG or RR) when settings are different on the two detectors assuming all instruction sets occur with same frequency. This contradicts with result 2 which states that we get same color only a quarter of times. See table 1 for settings 12 and 13. Similar results are obtained for other pair of settings 21, 23 and 31, 32 as well.

Part of this discrepancy can be explained by assuming that all 3 settings cannot have same instructions at once. This means there are only 6 instruction sets (GGR, GRR, GRG, RRG, RGG and RGR). This reduces the probability of getting same color for a setting to 1/3 (see table 2) but still does not match with result 2 which states that same color is flashed only a quarter of the times.

Proposed solutions to this discrepancy are:

1. Superposition of states with entanglement, where pair of particles does not have a definite spin at the time of emission but collapses to one as soon as one of them is measured. This involves spooky action at a distance.
2. Retro causality, where setting used for a particular run of the experiment is known to particles beforehand and based on that the instruction is encoded in the particle pair
3. Super determinism, where everything is predestined and there is no free will. Nature not only knows what is going to happen in future but also controls actions of Alice and Bob as well.

I will show in next section that this discrepancy can be explained using basic theory of probability itself without resorting to any such fancy explanations. I will do this by discussing a thought experiment involving game of cards.

2 Game of 3 cards

Mermin device has 3 settings on each detector and result 1 shows that instruction set for particle pair reaching both detectors must be exactly same. This means that if both Alice and Bob use different settings then it's equivalent to measuring 2 different properties for the same particle. As total number of possible settings is 3, it is akin to a game of 3 cards where each card can have either green or red color and the player picks any 2 cards randomly out of the 3. What is the chance of getting same color for both the cards? Let's play this game in two different ways to find out the answer

1. We put no condition about the cards, i.e. assume all cards have equal probability of having either green (G) or red (R) color. In this case first card we pick can either be G or R with equal probability. Let's

Table 3: Chances of first card being R (or G)

Card Combination	Probability of Combination	Probability for G	Probability for R
2Red 1Green	$1/2$	$1/3$ of $1/2 = 1/6$	$2/3$ of $1/2 = 1/3$
2Green 1Red	$1/2$	$2/3$ of $1/2 = 1/3$	$1/3$ of $1/2 = 1/6$
	1	$1/2$	$1/2$

Table 4: Chances of second card being R if first card is G

Second card is	Probability	Color of card	Probability of color	Total Probability
Opposite of first	$1/2$	R	1	$1/2$ of 1 = $1/2$
Independent of first	$1/2$	R	$1/2$	$1/2$ of $1/2 = 1/4$
				$3/4$
Independent of first	$1/2$	G	$1/2$	$1/2$ of $1/2 = 1/4$
				$1/4$

say it's G. Now we have two choices for the second card. But since both cards have equal probability of having R (or G), no matter which card we pick change of getting a different (or same) color is 50%. This is double of what result 2 says above.

2. Now let's play the game again but this time let's put a condition that not all 3 cards can be of same color (i.e. at least R and at least one G). It is not very difficult to imagine that in this case also the first card we pick can either be G or R with equal probability (see table 3). Let's say it's G. What is the probability now that the second card we pick is R? We know that out of the two remaining cards one must be R. As there is 50% chance of player picking that card there is 50% chance of both cards being different in color. There is also 50% chance of player picking the other card but as that card also has 50% chance of being R that means there is additional 25% chance of both cards being different in color. That makes it total 75% chance of two cards being of different color. Exactly same as result 2. See table 4 for all scenarios.

Using the same approach, let us now re-calculate the probability of getting different colors flashed on two detectors of the Mermin device (when the settings selected are different). Let's say Alice has selected setting 2 and her detector flashes G. What is the probability that Bob's detector will flash R? If instruction sets GGG and RRR are not allowed and all other instruction sets have equal probability then we know for sure that for at least one of settings (1 and 3) Bob's detector will flash R, as instruction set are exactly same for Alice and Bob for a given run. Let's say that setting is 1. Since Bob has 50% chance of selecting setting 1 there is 50% chance of his detector flashing R. For the setting 3 also there is equal chance of detector flashing R or G and since there is also 50% chance of Bob selecting that setting, there is additional 25% (50% of 50%) chance of detector flashing R color. This makes it total 75% probability his detector flashing R, which is exactly the result 2 from Mermin device.

3 Reviewing the Mermin logic

After reading the section above, most readers will likely go back and read the Mermin paper again to figure out what is wrong with his logic, where he calculates the probability of getting same color on both detectors as one third instead of a quarter. It is not an obvious mistake but some aspects which should have been considered more rigorously are listed below.

1. The fact that only two of the three colors in an instruction set can be same (i.e. each instruction set has at least one R and one G) has big implications on the calculation as we saw in the game of cards.
2. Even though actions of Alice and Bob seem completely independent of each other, it is not really the case. There is a statistical relation between their actions. For example when Alice selects setting 2

Table 5: Instruction sets built using logical constraints

Setting 1	Setting 2	Setting 3	Instruction set
R	G	G	RGG
R	G	R	RGR
G	R	G	GRG
G	R	R	GRR

Table 6: Results using the instruction sets built in table 5

Setting	RGG	RGR	GRG	GRR
12	RG	RG	GR	GR
13	RG	RR	GG	GR

then chances of Bob selecting setting 1 or 3 is 50-50. (for cases where settings selected by two are different).

- Numbers mentioned on detectors are arbitrary and have no bearing on the results. So instead of 1, 2, and 3 we could as well label them as α, β and γ and the results will still be same. We can even use a different label each time. As long as we are able to identify the settings uniquely and consistently between Alice and Bob it makes no difference. Using the same numbers (written on the detectors) for every run is just a convention. For example an alternate convention would be to always label the setting chosen by Alice as 1. Using this convention means setting selected by Bob (when settings selected by two are different) is either 2 or 3 (irrespective of what is written on the detectors). If we follow this convention, then we have to do all our calculations only for 2 pair of settings (12 and 13) instead of 6. And this is not just a matter of convention. Setting pair 12 is exactly same as pairs 23 and 31 in reality as well, since all these correspond to measuring spin of an electron along two different directions separated by 120 degrees clockwise. Similarly setting pair 13 is exactly same as pairs 21 and 32.
- As number of unique pair of settings depends on the convention used, so doe the possible instruction sets. Let's say we use a convention mentioned above, where the setting selected by Alice is always labeled as 1 (irrespective of what is written on her detector) and the other two settings are assumed to be 2 and 3. Since one of these two settings is bound to give result opposite of what Alice gets, we can call that setting as 2 and the other as 3 (irrespective of what is written on the detector). If Alice choses setting labeled 2 on detector, then we will call it setting 1 and may call setting 1 as 2 and setting 3 as 3 (or 1 as 3 and 3 as 2). Following this convention the instruction sets we get has only four permutations, viz. RGR, RGG, GRR and GRG (or RRG, RGG, GRR and GGR) instead of 6. Please see table 5 for the instruction sets. Table 6 shows using these instruction sets that chance of getting same color is 1/4 when Alice and Bob's settings are different. As discussed we need to do this calculation for setting pairs 12 and 13 only.

For those who may still have some doubts, let's consider the card game once again. If we start writing numbers 1, 2 and 3 on the back of the cards based on some convention (e.g. left to right), will it make any difference to the chances of 2 cards picked by the player having same color? Does it make any difference whether player picked cards numbered 12, 13, 21, 23, 31 or 32? Do we need to consider numbers written on the back of cards or the 6 possible ways in which the cards can be numbered and arranged from left to right (GGR, GRG, GRR, RRG, RGR and RGG) for calculating probabilities? The answer is clearly no for all these questions because whether or what we write on back of cards, is immaterial to color of cards and which two cards user picks. In fact using these unnecessary constructs may lead to lot of confusion and thereby wrong calculations, which is exactly what happened in Mermin's paper. If at all one has to use a numbering scheme for ease of doing calculations it must be logical in nature like the one discussed above.

4 Discussion

Thought experiment proposed by Mermin has certain aspects which are unnecessary and cause confusion. Biggest source of confusion is the instruction sets which are assumed to be superset of all possible ways nature can create electrons, while in reality these are just different ways in which one can arrange a set of 3 binary results, two of which must have opposite value. Just like numbering and arranging a set of 3 green or red colored cards in any way does not influence in any way the outcome of randomly drawing two cards out of the set, instruction sets created based on numbers written on a detector do not influence the outcome of the experiment.

I showed in this paper that the chances of the detectors (of Mermin device) flashing same colors when settings on the two are different, is in fact quarter of times. This matches with the observations of SG experiment done with a pair of coherent electrons.

References

- [1] N. D. Mermin, “Bringing home the atomic world: Quantum mysteries for anybody,” *Am. J. Phys.* **49** no. 10, (Jun, 1998) 940. <https://aapt.scitation.org/doi/abs/10.1119/1.12594>.