Zep Tepi Mathematics 101

How Giza was probably designed

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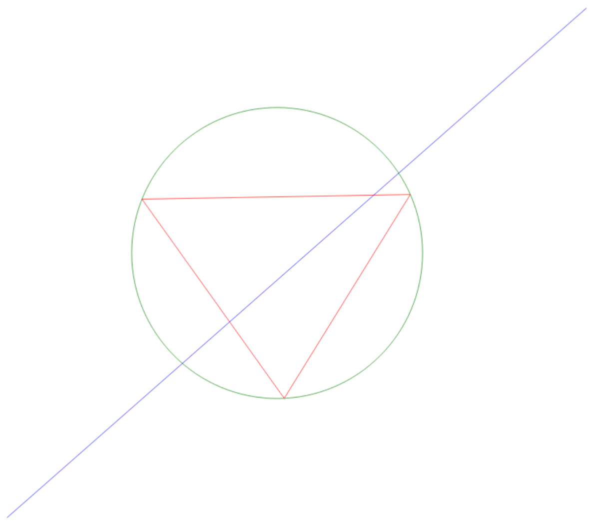
Abstract

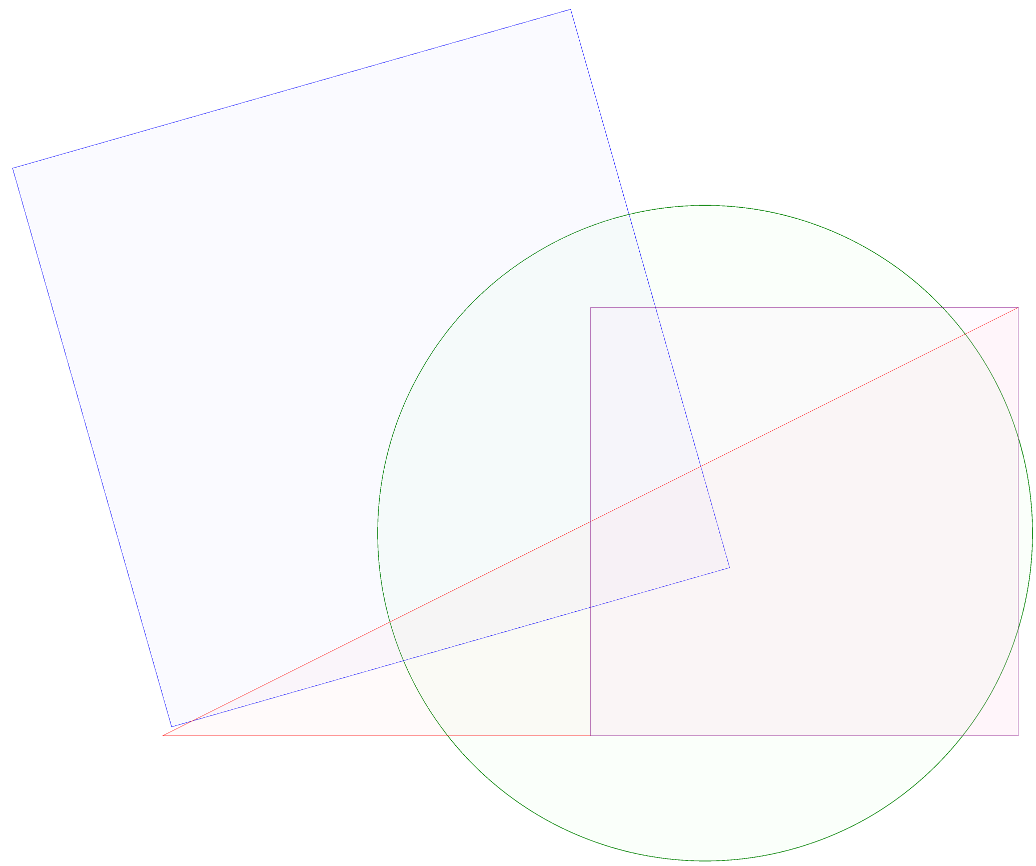
A mathematics course from the Zep Tepi era, where we plan and analyse a large building site, showing how the design mirrors the stars.

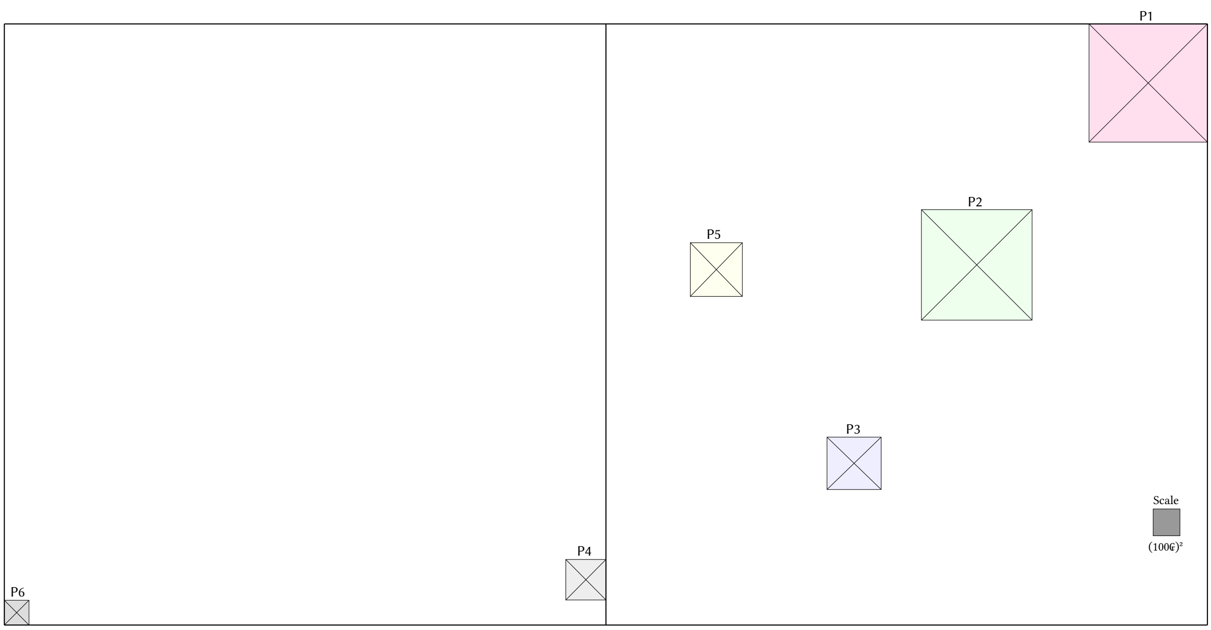
A simple and elegant explanation of how Giza, with six main pyramids, was laid out, using $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π and φ. The design incorporates the necessary elements for squaring the circle, area-wise. The design matches the heavens around 55.5k BCE. This could force a rethink of at least the history of mathematics, if not the broader human timeline, and effectively solves the puzzle of how Giza was laid out.

Keywords: Egyptology, Giza, pyramids, alignment, geometry, archaeogeometry, archaeoastronomy, history of mathematics, π, pi, φ, golden ratio, squaring the circle.

Best viewed and printed in colour.






P1
P2
P5
P3
Scale
(100₵)²
P4
P6

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Updates:
1.1.0 Re-arranged discussion of P1 base size, expanded discussion of 2143, expanded discussion about the big triangle and put it in its own section. Added "Map of Khufu?" section. Assorted other bug-fixes and embellishments.
1.1.1 Added overview diagram, and accuracies to symbol table. Expanded discussion re Khafre to include π and φ.
1.1.2 Added equation for e-1. Fix 198 -> 195 in φ √π diagram
1.2.0 Fixed some typos. Added new π and e diagrams. Added formulas for relation between P1, P2 and P3 base sizes. Removed "A map of Khufu?" section, it is wrong and I have a better idea. Added alternate P4 P5 layout. Updated right side master plan. Integrated results from the Douglas Triangle. Added more π ratios in site plan. Added figures index. Added tables index.

1. Introduction

"*The language of Giza is mathematics.*"
Robert Bauval

"*You will believe.*"
The architects of Giza

This is my fifth and a half attempt at writing this since early 2020, each time starting at a different point and following a different path. As with the rest of my explorations of Giza, it has been a cyclical process, one thing leading to another, forcing revisions and refinements. It feels like I have been guided along like a child learning science, where what they teach you in junior school is "corrected" in high school, and then again at university level.

So what started off innocently as a quest to see if Giza was hiding any circles divided in the golden ratio has now progressed to a good understanding of exactly how the site was laid out. The design presented here largely supersedes the methods presented in my earlier paper. The design is elegant and demonstrates undeniable knowledge of π, φ, and square roots. The final surprise is that the design includes squaring the circle area-wise, not perimeter-wise as in Khufu. There are also numerous mathematical “tricks” or “jokes” along the way. As such, Giza rewrites the history of mathematics, and we need to stop giving the Greeks credit for things that originated in Egypt.

This paper is a continuation of my previous efforts, Diskerfery and the Alignment of the Four Main Giza Pyramids (Douglas 2019 [1]) (henceforth *Diskerfery*) and 55,550 BCE and the 23 Stars of Giza (Douglas 2019 [2]) (henceforth *55K*). The first attempted to determine the location of the fourth pyramid, as documented by Norden [3], while the second showed a stellar alignment around 55.5k BCE.

This document introduces the fifth and sixth pyramids, long demolished, for which we currently have no conventional evidence. However, the stellar alignment strongly suggests their presence. Once we add them to the layout map, multiple fascinating mathematical relationships surface, which can not be by chance. The relationships show advanced (compared to what we think they knew) mathematical knowledge. Thus, I am convinced they were there.

When we analyse the layout at Giza, we have to overcome these obstacles:

1. We are dealing with “as is”, which is not the same as “as renovated” (multiple times? [4]), which is not the same as “as built”, which is not the same as “as designed.”

2. As discussed in a previous paper (55.5k), I have come to the conclusion that Giza was built around 55.5k BCE. This is a problematic date, but it’s when the star map behind the design aligns.

Figure 1 shows how the three main pyramids, plus P4 and P5, align with the stars around 55.5k BCE. Giza elements are in green, while stars and labels are red, blue and black. The “+” sign is the celestial north pole.

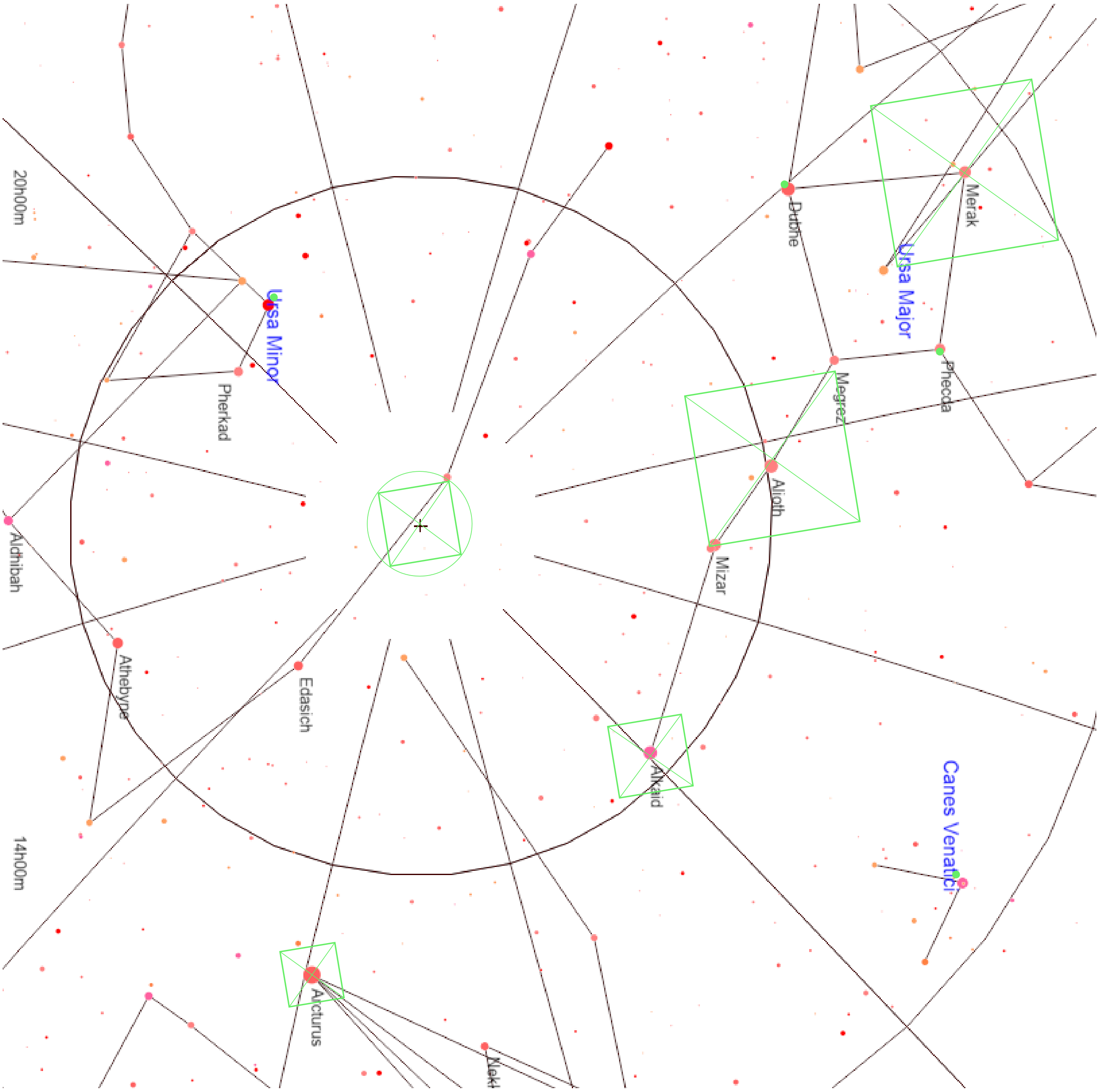


Figure 1: How the right hand side of Giza aligns with the stars, 55.5k BCE

This shows five pyramids, and Thuban's orbit around P5. Thuban was the pole star at the time. Apart from the five pyramids, I will also show how Cor Caroli, Kochab, Dubhe, and Phecda positions are mathematically related to the pyramids. They are shown as green dots. I don't know if anything was built there. Kochab in Ursa Minor is actually off the Giza plateau at the moment, but it may still have been plateau when Giza was built. Kochab is close to the current entrance to the Cave of Birds.

Showing the alignment for six pyramids is difficult, because the stars involved span across more than half the sky, which creates problems trying to map the curved sky to a flat screen or page. The constellations distort differently, depending on which projection you use. Here are two attempted alignments using the same sky and date, just projected differently. This is the Hammer-Aitoff projection, with a 183.5° field of view, at Cairo on 21 March, 55.5k BCE.

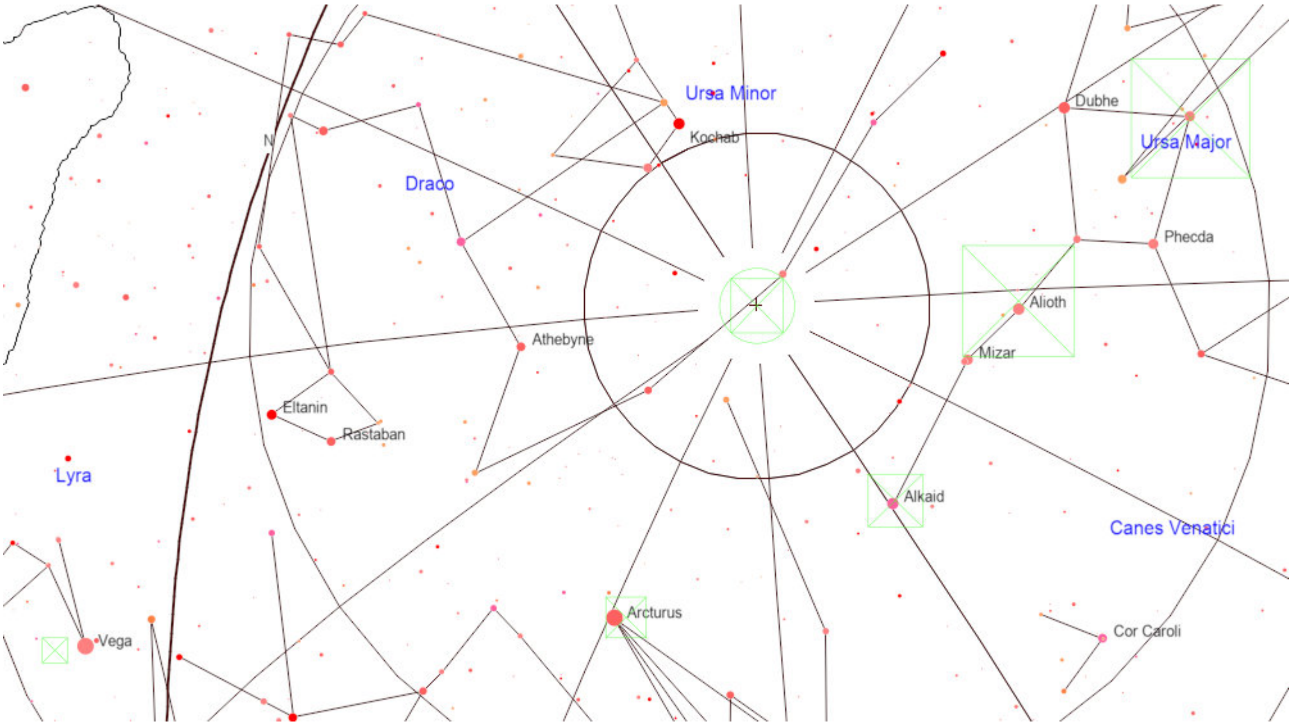


Figure 2: Giza stellar alignment, HAI projection

Here is the ARC Zenithal equidistant projection version, with the same field of view.

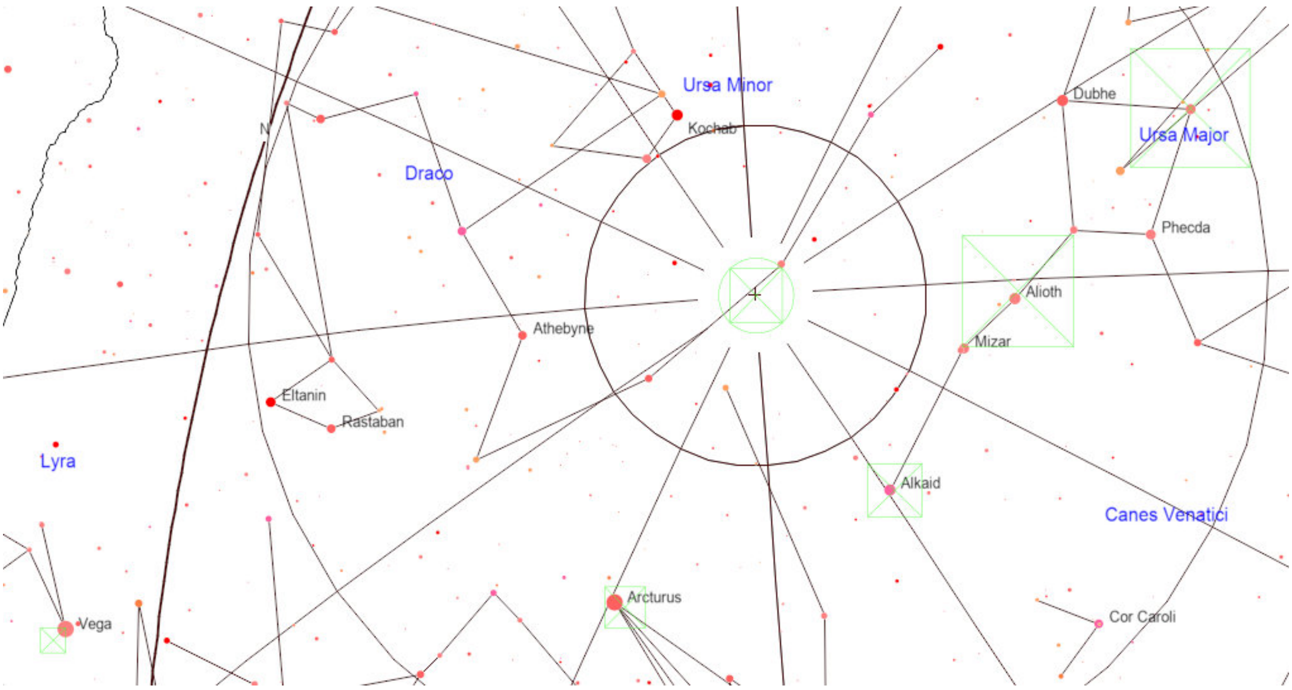


Figure 3: Giza stellar alignment, ARC projection

3. In the years between being built, and the 4th dynasty, I would imagine that the pyramids suffered a lot of damage. I interpret "Khufu's Horizon" as the project initiated by Khufu, and completed by Khafre and Menkaure, to "renovate and restore" what was left. In the process they cleared the area west of P2, creating the "horizon." I speculate that they took what was left of the 5th and 6th pyramids, and possibly Djedefre's, and used that to rebuild the other four pyramids, shipping in Tura limestone for the final layer. Khufu may have had to rebuild the top section of his pyramid, from the area above the granite slabs. Starting from there could explain the graffiti, a twenty year time span, and the odd changes in course thickness.

4. Judging by the mathematics, Menkaure's rebuilding resulted in the footprint moving slightly south and west, and possibly re-centring the entire pyramid. Legon's triangle [5] actually ends inside the pyramid, so I think Menkaure was differently sized to now.

5. Africa has rotated slightly since 57.5k years ago. This twist, along with Menkaure's problematic renovations, has complicated any analysis of the site.

6. If my speculation in (3) is correct, then having pyramids 4, 5 and 6 missing, has also complicated any analysis of the site. At least we have Norden's documentation [3] of pyramid 4 as a starting point.

7. Pyramid 4, 5 and 6 were strongly suggested by the stellar alignment. Adding them to the design, and what comes out of that as documented here, convinces me that they did indeed exist. We can even get a good idea of their base sizes from the mathematics of the site, and potentially their heights.

So I have gone back the drawing board. Based on my own analysis, as well as that by John Legon [5], it became clear mathematics was at the heart of the design. So, what happens if we take that as our starting point, and design Giza so that it meets assorted mathematical relationships, which are now "almost exact".

The problem with things being "almost exact" is that researchers are never sure if the error is their miscalculation, the renovators changed things, the builders didn't build exactly according to plan, or if the designers erred. We also don't know what level of precision the designers and builders thought was acceptable.

My starting assumption is that Giza was built according to mathematics, while being modelled after the stars to record the date. I assume it was originally "perfectly" (i.e. "as best possible") aligned with North. This means we don't need to worry about skewness interfering with the calculations. It also appears that they used only whole-cubit dimensions for the base sizes, heights, and inter-pyramid spaces, which has a knock-on effect on some accuracies.

A note on style

I don't like the usual phrases "The current author" or "The present author." I will refer to myself in the first person, or frequently as "we," not because I am schizophrenic but I've been using that term since childhood, and it's even more relevant now. While investigating Giza, I have had constant help from sources unknown, and they deserve due credit. Tesla experienced the same phenomenon, and could not explain the source either.

> “*My brain is only a receiver. In the universe there is a core from which we obtain knowledge, strength, inspiration. I have not penetrated into the secrets of this core, but I know that it exists.*”
>
> Attributed to Nikola Tesla

The guides are my shepherd;
I shall not wonder.
They make me ponder plans,
and lead me above still waters.
They restore my hope.
They lead down the paths of mathematics
to admire them.
Even though I walk through the pyramids
among the shadow of death,
I will have no doubts:
for they are with me;
their π and their φ
they comfort me.
And I shall dwell
in the house of Thoth
Forever.

2. Notation, accuracy and methodology

2.1 Notation

The main pyramids at Giza are usually designated as G1 for Khufu, G2 for Khafre, or G3 for Menkaure, alternatively as P1 to P3. I have used the P1 to P3 notation to maintain consistency with my previous papers.

The corners and centre are abbreviated respectively as NW, NE, SW, SE and C, for North West, North East, South West, South East, and Centre, following the cardinal directions. We can then refer to Pyramid 1, North West corner as P1 NW without confusion.

I have also used the traditional names associated with P1, P2 and P3 as a convenience, although I don’t think those people had anything to do with the original construction, only maintenance or appropriation.

I take the royal cubit as $\pi/6$ metres, to 4 decimal places. (*The Beautiful Cubit System*, Douglas 2019 [6]). Over the last year it became apparent they thought 3 or 4 decimals were accurate enough, so I have used that, and things “just work.”

Symbols used in this and other papers:

Symbol	*Name*	*Approximate / practical value*	*% Accuracy to true*
π	Archimedes' constant	3.1416	99.9998
π́	π - 1	2.1416	99.9997
τ	Circle constant	6.2832 = 2π	99.9998
e	Euler's number	2.7183	99.9993
é	e - 1	1.7183	99.9989
φ	Golden ratio	1.618 φ + 1 = φ² = 2.618	99.9979
♳	Silver ratio	1+√2 = 2.4142	99.9994
ρ	Plastic number / ratio	1.3247 ρ+1 = ρ³ = 2.3247	99.9986
α	Fine structure constant	0.007297... ≈ 1/137	
ꝺ	π/3 (pioth?)	1.0472 (i.e. τ/6, cf. π/6)	99.9998
f	Foot, Imperial	0.3048m or 0.3047 (from Ɠ/é)	
Ͼ	Short cubit	0.4488m (π/7)	
Ɠ	Royal cubit aka cubit	0.5236m (π/6)	
Y	"Megalithic yard"	0.8283m = 1Ɠ + 1f	
M	Grand metre	1.5236m = m + Ɠ ("5 feet")	
S	"Six" feet	1.8283m = m + Ɠ + f	

Table 1: Symbols, names and values

I use "cubit" for Royal cubit Ɠ, any references to the short cubit will be "short cubit" Ͼ.

We can approximate the value of M well using famous mathematical constants:

$$M = 1 + \text{Ɠ} \approx \frac{1+\pi}{e} \approx \frac{\varphi^2}{é} \left(= \frac{\varphi+1}{e-1} \right) \approx \pi - \varphi \approx 1.5236\, m$$

I invented names and symbols for Ɠ, Ͼ, M, S, é, π́ and ꝺ since they pop up so often.

If you use a practical number of decimals (3 or 4) for the irrationals, then the string of approximations above becomes closer to equalities.

That gives us the following close relationships, which is discussed in more detail in *The foot, cubit, metre, and φ, π and e* (Douglas, 2020 [7])

$$\frac{\text{Ɠ}}{\text{f}} \approx \frac{\text{e}\,\text{Ɠ}}{\text{Y}} \approx \frac{\text{é}}{\text{m}} \approx \frac{\varphi^2}{\text{M}} \approx \frac{\pi}{\text{S}} \approx \text{é} \approx 1.7183$$

$$é = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n!}$$

2.2 Accuracy

How accurate must things be? We have no idea what tools or technologies the builders had, what they considered "accurate" or "good enough," nor exactly how earthquakes or tectonic shifts have affected the relative positions over time. We can not assume that their standards were the same as ours. There is no such thing as perfect accuracy in building construction, despite which, we can demonstrate in excess of 99% accuracy.

Note that "close" in context of this discussion refers to practical measurements on a large-scale building project using unknown instruments, not something on the scale of modern micro-electronics.

I am indebted to the late Glen Dash and the Giza Plateau Mapping Project (GPMP) for their work on providing accurate measurements for the pyramids at Giza. Note that their co-ordinates for Menkaure and Khafre are not as accurate as for Khufu. Co-ordinates are given accurate to the nearest tenth of a metre.

This document contains many ratios approximating well-known irrationals like π or $\sqrt{2}$. By definition, these can not be expressed as a/b, but we can approximate them with varying degrees of accuracy, like 22/7 or 355/113 for π. Approximations are frequently integer-based, which in itself immediately limits the possible accuracy. The accuracy for the skeleton blueprint decreased after I switched to the grid as the starting point for the pyramid locations. It appears that the builders stuck to whole-cubit dimensions for the pyramid bases and inter-pyramid spaces. In truth, ⅓ of a cubit is 17.45 cm or just over 6 inches, which doesn't make much difference on a line a hundred metres long. However, it reduces accuracy from 100% down to 99.83%.

If the accuracy of the approximation is 99%, that equates to a measuring error of 1 metre in 100 metres, as measured on the ground. At 99.9%, we can say the error is less than 10cm in 100 metres. At 99.99%, the error would be less than 1cm in 100 metres. The reader can imagine the difficulties getting this sort of accuracy when measuring long distances over uneven ground.

Despite that, most ratios presented here are over 99%, some over 99.9%. In truth, when I was searching for the location of P5, higher accuracies for various relationships were not unusual, for example here's some accuracy "debug statements" from one of the programs, before I implemented whole-cubit restrictions.

Pi Triangle: 99.97911886%
Triangle area 99.99999671%
Phi/Pi Cross: 99.99709498%
Long phi: 100%
Short phi: 100%
$\sqrt{2}$ 99.73851304%

Giza as analysed consists of 6 pyramids, each with a 4 sides, 4 corners and a centre. Each point or length is involved in multiple relationships, both linear and areal. It is impossible to achieve 100% accuracy in all relationships under such circumstances, even more so with whole-cubit restrictions. The designers had to compromise some ratios so that the average was still good, and the intent still clear.

Instead of quoting accuracy in terms of percentage, I will instead quote it in cm/100m, as that puts it in an easier to visualise form. Most grass-based sports fields are 90 or 100m long, and the ruler on your desk is typically 30cm, so our minds can do the comparison.

3. The pyramids and their locations

The plan has six main pyramids, which are sited according to mathematical rules, inspired by a stellar arrangement of the stars in and around Ursa Major (the big dipper).

Giza / stellar name	*P number*	*Base X ₢*	*Base Y ₢*	*Height ₢*	*Design*
Khufu	P1	440	440	280	√φ
Khafre	P2	411	411	274	4/3
Menkaure	P3	201	195	126	√φ
Arcturus	P4	149	151	100	4/3
North Pole / Thuban	P5	193	200	125	√φ
Vega	P6	92	92	65	√2

Table 2: Summary of the six pyramids

I previously believed that all pyramids are square, but the way the mathematics works has convinced me otherwise. So P1, P2 and P6 remain square, while P3, P4 and P5 are squarish rectangles. The Pyramid of Djoser and Pyramid of Khui were also rectangular. I think they were of necessity non-square, to make the mathematics work. In *Diskerfery* I concluded that P4 was 162₢ square, but that size, being 100φ, always bothered me as being "too pat." Following the mathematics of the grid, I have resized it to 149×151, with a 100₢ height.

Menkaure is usually specified as a 202 square base and 125 height, although online discussions show various researchers questioning those values. Keith Hamilton (The Pyramid of Menkaure, at Giza. [8]) provides the following summary:

> So what can we say about the size of the pyramid? It is a complex puzzle, with practically every publication publishing a different finding; for example in Lehner's "Complete Pyramids, page 134" he strangely gives the base as 335 x 343 feet: this is amended in his latest "Giza and the Pyramids, page 244" with Zahi Hawass, who give 346 feet/201 cubits, and an angle which relates to a tangent of 5/4. Other authors see the angle of the pyramid as that of the Great Pyramid tangent 14/11, by using the finished limestone fragments as reflecting the intended angle. Others suggest that the structure was simply intended to be a base of 200 cubits; the list of permutations can be quite lengthy. John Legon who has done several articles on pyramid geometry, including a Giza site plan, which brought forward interesting geometric relationships, suggested that the pyramid base was 201.5 cubits [5].

My copy of Lehner gives Menkaure as 102.2 × 104.6 m or 335 × 343 feet as quoted from Hamilton.

The diagram shows the view from the east, so I assume the dimensions are thus north-south first rather than east-west first (i.e. the x-axis) as I do.

I'm using what I think the original dimensions were before it was vandalised, then restored by Menkaure. Various calculations work better with a rectangular base, and the curious internal passage structure suggests it was originally smaller.

Legon's triangle (see next section) is a $\sqrt{2} : \sqrt{3} : \sqrt{5}$ triangle. If you draw such a triangle on the current map of Giza, starting at P1 NE corner, the "Legon point" ends up inside P3, close to P3 SW. Exactly where inside depends on whether you allow for the skewness or not, and if you round the numbers. Here is the south-west corner of Menkaure, zoomed in, showing where the Legon point calculates to, for different criteria.

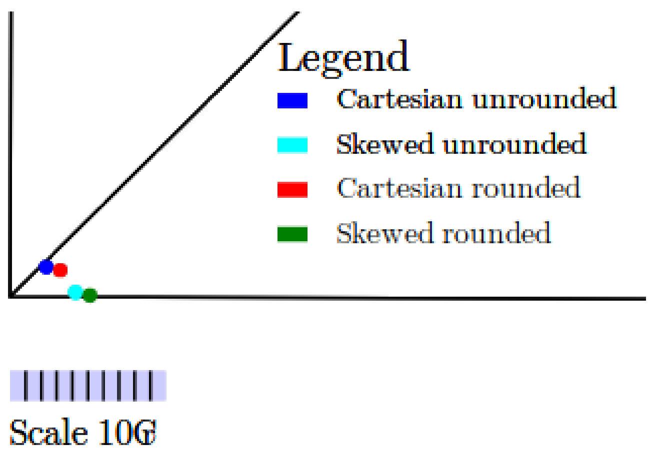


Figure 4: South west corner of Menkaure, showing Legon point for various calculations.

The spacing ratios hint directly at a base of 201×195, and it also gave the best results for P5 position. So I have used that.

Comparing the bases visually.

P1	P2	P3	P4	P5	P6

Table 3: Pyramid colour key

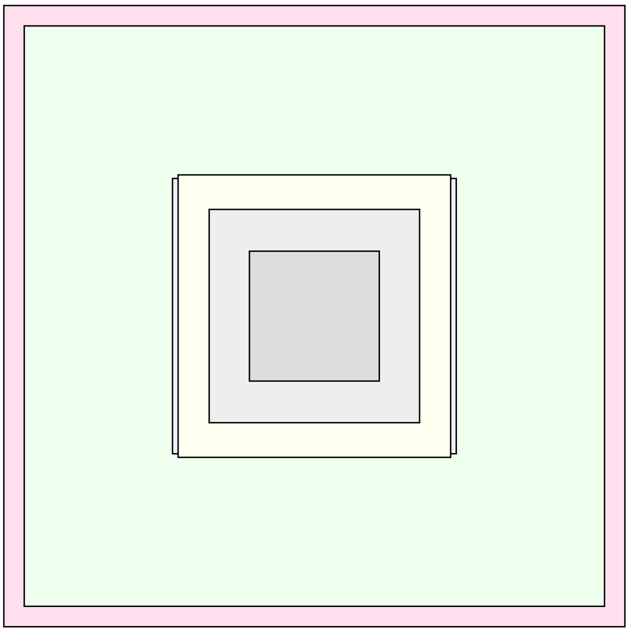


Figure 5: Comparing the pyramid size and shape from above.

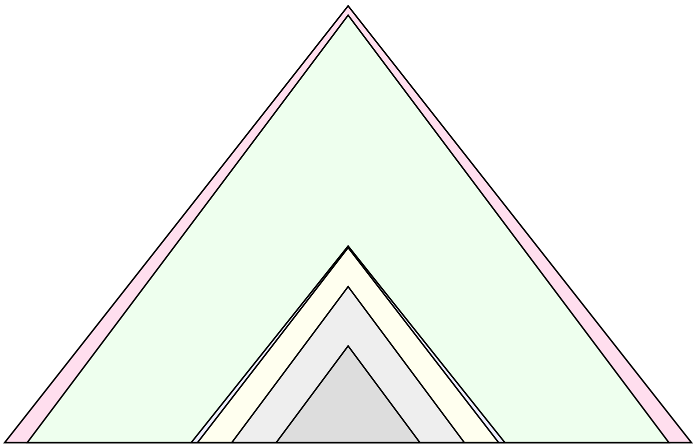


Figure 6: Comparing the pyramids, looking north.

The actual order in which I stumbled across (or it was revealed to me) what follows is
a) Skeleton blueprint

b) Squared circle one
c) Vega and the Thoth grid
d) The large squared circle
e) The large triangle

However, the whole process is continuously cyclical, meaning frequent revisions as things become clearer and new insights surface. Even now, as of this fifth and ~~a half~~ three-quarter attempt at writing this, I am not entirely sure I have everything correct. It is, however, good enough to proceed with.

The issues revolve around the exact size and locations of P4 and P5. Every little change has knock-on effects, improving some measurements and worsening others. At the moment, the locations and sizes are optimised for the first squared circle, meaning ratios in the blueprint are not as good as can be achieved.

We start then with the Thoth grid.

4. Module 1: The Thoth Grid

> *"**Contradictions do not exist.** Whenever you think you are facing a contradiction, check your premises. You will find that one of them is wrong."*
>
> Ayn Rand

That quote from Ayn Rand turned out to be key to getting things to work nicely. The incorrect assumptions were that all pyramids are square, and that P4 would sit inside a rectangle like P3. It actually sits outside a rectangle and square..

The designers used a few favourite numbers repeatedly. These include φ, π, √2, √3, 3, √5, and multiples of 137. They also used π-1 and e-1. As mentioned above, the irrationals were rounded to 3 or 4 decimal places for practical reasons.

Using 137 is "curious," now we know the Fine Structure Constant and its inverse approximation of 137. So there is some speculation that this is a nod to α. However, it is probably a reference to φ^2:

$$100\,\varphi = 100\times 2.618 = 261.8$$

$$261.8\times\frac{3.1416}{6} = 137.07848 \approx 137$$

The Thoth grid consists of rectangles and squares, with the main sides multiples of square roots. Dimensions are in whole ₵. We start with P1, which has sides of 440₵. We will discuss the 440 size presently. P1 centre is at latitude 29.9791667°N. If the centre had been a mere 25m further north, it would have been 29.98… °N. I have been unable to find any reason for the longitude yet.

Location	29.9791667° N
Speed of light	29.9792458 $\times 10^7$m/s
Difference	00.0000791

Table 4: Comparing P1 location and the speed of light.

The difference equates to 0° 0' 0.28", which is about 8.6 metres, comparable to modern consumer-grade GPS equipment like smart-phones.
So either we accept that a semi-primitive people randomly decided to build what was the biggest building for millennia if not tens of millennia at that precise latitude, or it was not random.

If it was not random, then we are forced to conclude:

1. They had the metre and second, with values identical to or very close to what we use. This is the part that bothers me.
2. They were able to measure the speed of light.
3. They divided the circle into 360°.
4. They knew the earth was round, at least is a north-south direction, and probably in an east-west direction as well, based on logic or exploration.
5. They were able to measure latitude accurately.

As far as I know, there is no evidence of the 4th dynasty having these capabilities, which means they didn't build Giza.

Using the P1 north east corner as a starting point, draw two squares, of side 1000√5 ₢. If we had not rounded to whole-cubit dimensions, this would have been 10,000,000₢².

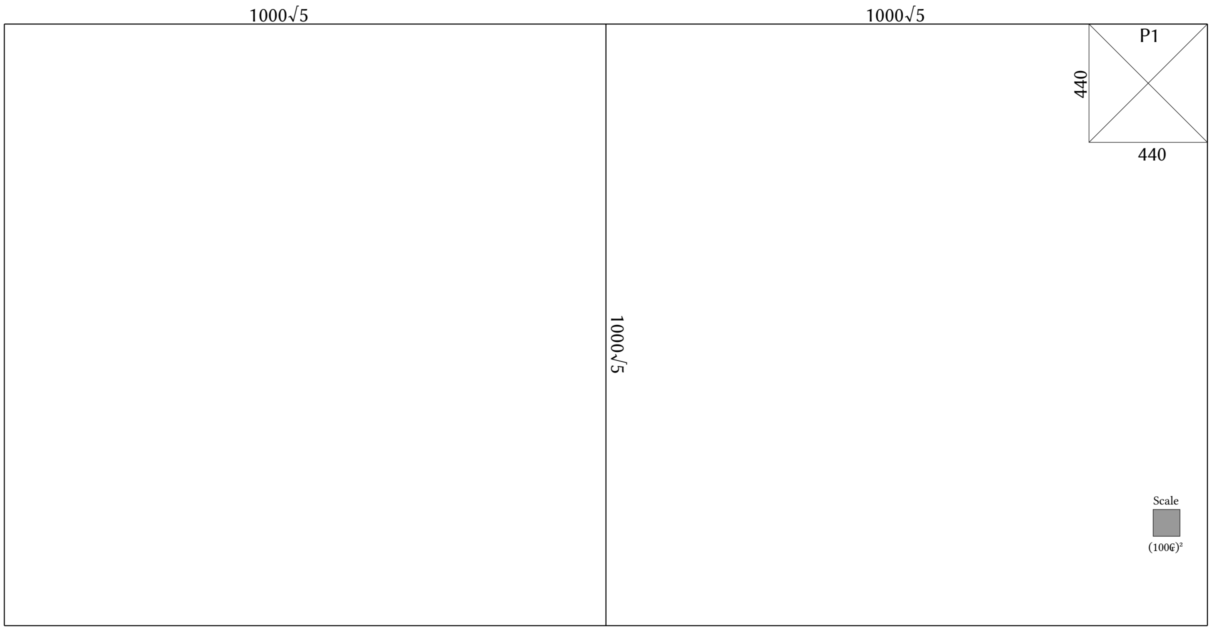


Figure 7: The Giza plan outer bounding rectangle

The perimeter is 6 × 2236 = 13416, an anagram of 3.1416. Giza maths humour.

$$1000\sqrt{5} = 2236\,(rounded)$$

$$\frac{2236}{\pi\varphi} = \frac{2236}{3.1416 \times 1.618} = 440\,(rounded)$$

Right away, we find π and φ hiding. I have seen πφ numerous times while investigating Giza, and was surprised to learn that it's the area of a Golden Ellipse.

We then mark off 1000√1, 1000√2, 1000√3, and 1000√4.

The 1000√2 × 1000√3 rectangle is from Legon [5].

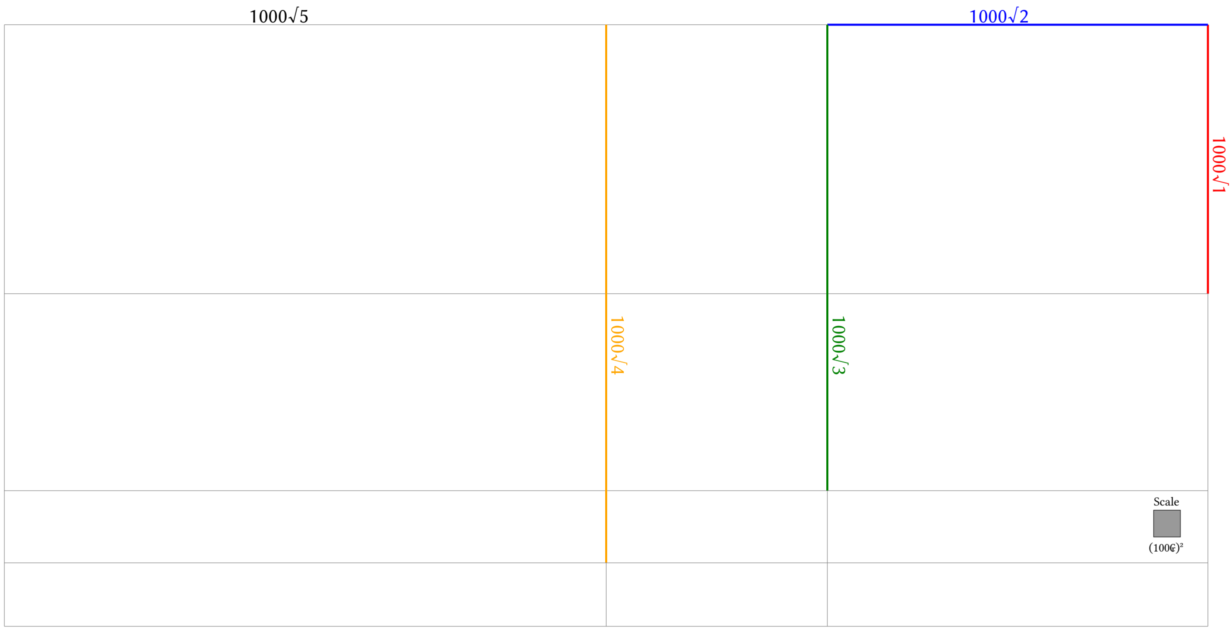


Figure 8: Step 2, marking off required distances and adding lines.

1000√5 - 1000√2 = 822.
822 ≈ 100πφ² (rounded). It's also 6 × 137.
411 ≈ 50πφ² (rounded), and is the P2 base size.

We draw two 411₢ squares, imitating the larger √5 squares.

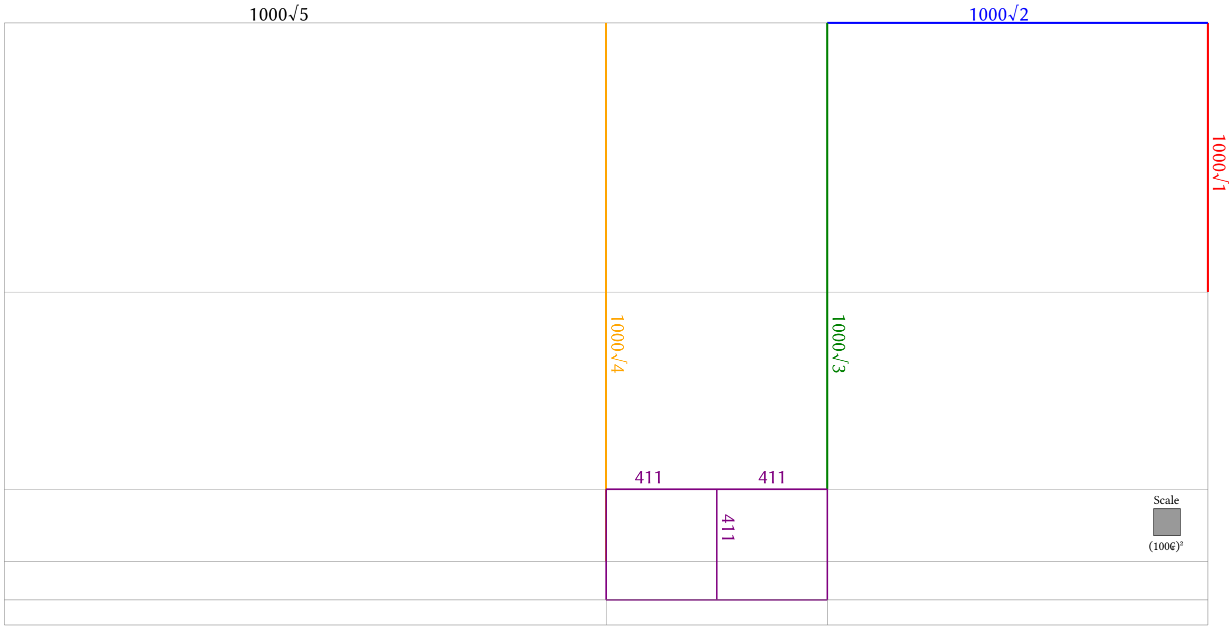


Figure 9: Step 3, adding the double 411 squares.

Curiously, if we round e to 2.72, then

$$822\times(e-1) = 822\times1.72 = 1413.84 \approx 1414 = 1000\sqrt{2}\ (rounded)$$

$$822\times(e) = 822\times2.72 = 2235.84 \approx 2236 = 1000\sqrt{5}\ (rounded)$$

The distance from the top of the grid to the bottom of the 411 squares is 1732 + 411 = 2143. That number caused me a lot of trouble until I stumbled onto the double 2236^2 grid.

The famous Indian mathematician Ramanujan received this formula for π in a dream:

$\sqrt[4]{\frac{2143}{22}} = 3.1415926525\ldots$, which is 99.99999996794...% accurate for π.

I tried to find 22 in the plan, but it is not there directly. After asking my guides, the only answer I got is that "it is there, hidden in plain sight." Eventually it surfaced ... another little joke. For those "in the know," 2143 is all you need. For example:

1. The first four natural numbers, each pair switched.
2. Four digits, so take the 4^{th} root
3. The difference between 21 and 43 is ... 22. (or 12 and 34, for that matter.)
4. Or, 2143, **divided** by **2** pairs of **2**.

So it's a π formula hidden in the design. Others will surface in due course.

This completes the Thoth grid.

5. Module 2: Thoth's Law, or "The square on the hypotenuse ..."

The design uses the roots of the first three primes. The only rational reason for irrational square root dimensions is to demonstrate that you understand that in right-angled triangles, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Naming it after Pythagoras is misguided.

We can add the hypotenuses.

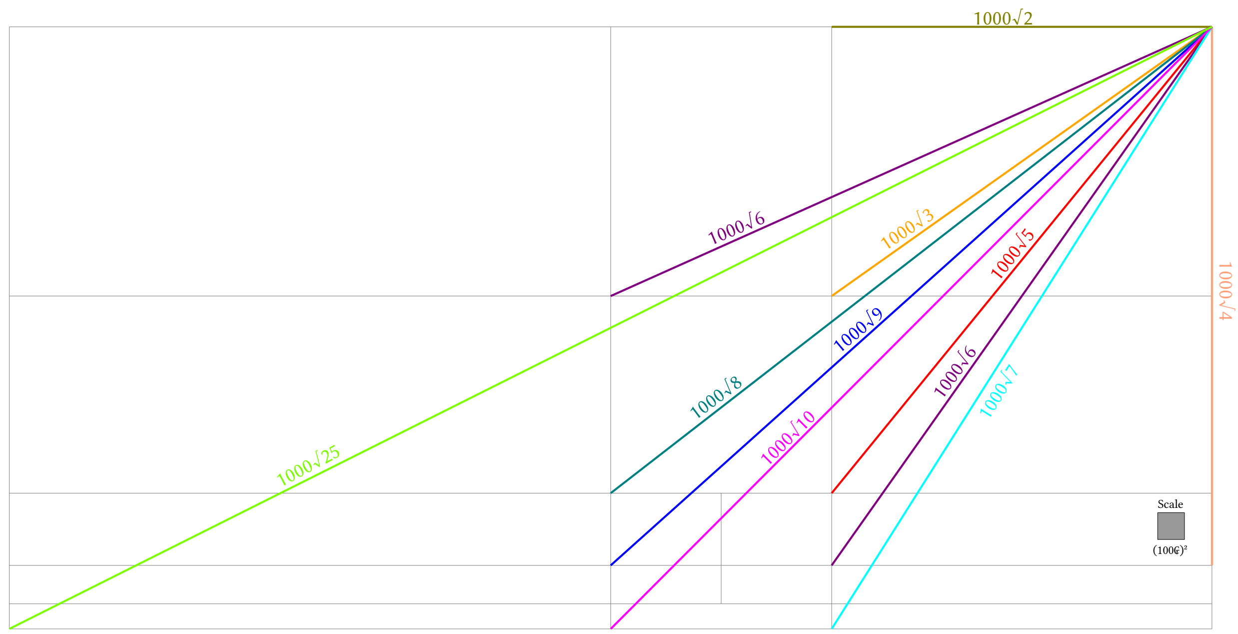


Figure 10: Thoth's law in action.

Figure 10 has 1000 times √2, √3, √4, √5, √6, √7, √8, √9, √10 and √25. It is also possible to draw √21, √23, and √24, ending down the west edge.

History will credit the Greeks for this.

Roots are radical, but what about π and φ? First add P1 back, and mark off P3.

Figure 11: Thoth gets irrational.

$$\sqrt{(1414-440)^2+(1732-440)^2} = 1618$$

$$\sqrt{(1414-201)^2+(1732-440)^2} = 1772$$

$$\sqrt{(2236+2236)^2+2236^2} = 5000\,\text{₲} = 2618\,m$$

$$\sqrt{2236^2+2000^2} = 3000\,\text{₲} = 1570.8\,m$$

Values are rounded to nearest ₲ or metre as per usual. Stumbling across that 1000φ line was one of those “You must be kidding me!” moments that left me stunned at the ingenious design. Adding √π was the cherry on top. There is a line to the other corner that is almost 1000é (1720 ₲) but I’m not including that.

This is the clearest use of φ that I have seen in the Giza layout. It provides a fifth reason why P1 has to be 440 ₲ square. The other reasons are related to the speed of light, balancing integer approximations of π and φ, and “squaring the circle,” perimeter- and area-wise. It would have been easier to build a smaller pyramid with the same ratios, but it had to have a 440₲ base.

Item	*Formula*
Approximate π	2 × base / height
Approximate φ	(height / half-base)²
Approximate speed of light c	base × π × (√2 - 1) --> convert to metres
Square the circle, perimeter : circumference	(4 × side) / (2 × height × π)
Square the ellipse, area	(π × height × base/2) / base²

Table 5: Formulas for approximating targets using P1 dimensions.

The formula for the speed of light is the short version of "difference between circumscribed and inscribed circles."

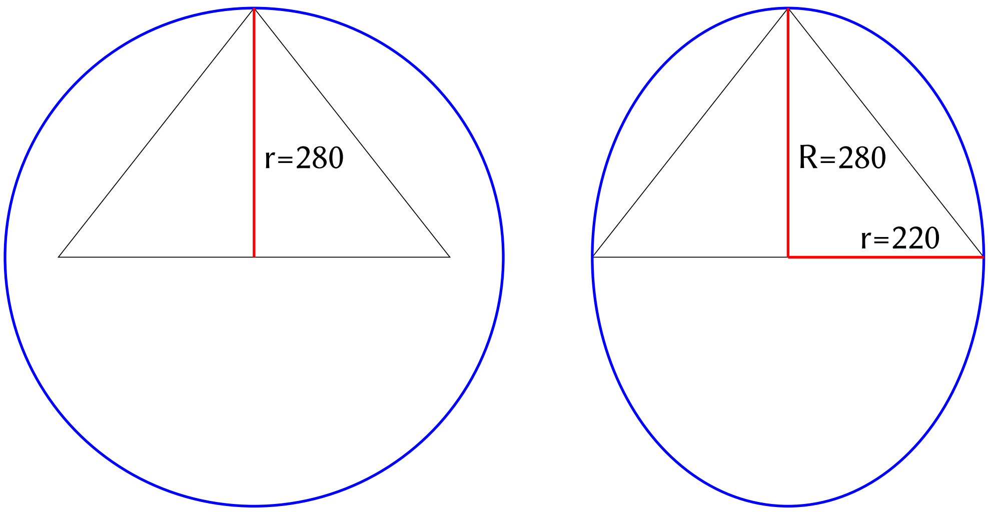


Figure 12: Circle for perimeter, ellipse for area.

A comparison of some combinations of different base and height options, and the resulting approximations as per the above formulas. **Best**, *worst.*

Side ₢	*Height ₢*	*π*	*φ*	*c (Mm/s)*	*P / C*	*Area*	*Ave Accuracy %*
440	280	3.1429	1.6198	**299.7959599**	1.0004	0.9996	**99.9535**
440.25	280	*3.1446*	**1.6180**	299.9660984	*1.0010*	**1.0001**	99.9474
439.8	280	**3.1414**	*1.6213*	299.6594891	**0.9999**	*0.9990*	99.9298
220	140	3.1429	1.6198	*149.8978799*	1.0004	0.9996	*89.9538*
443	282	3.1418	1.6209	301.8398218	1.0001	0.9999	99.8235

Table 6: Comparing alternate design possibilities for P1

We will see more of both π and φ presently. History will credit the Greeks for π and φ ....

There is also the Douglas Triangle [9] linking the base sizes of Khufu and Khafre, of which one side scales to 440₢.

6. Module 3: Dividing space in a given ratio

The Thoth grid indicates the locations of pyramids P1, P3, P4 and P6. For now, we will work with only the right hand side of the plan. We add P4 to the left of the centre line, with its south east corner at the south west corner of the double 411 squares. Extend the inside edges of P1 and P3. P6 will go in the south-west corner of the left hand side later.

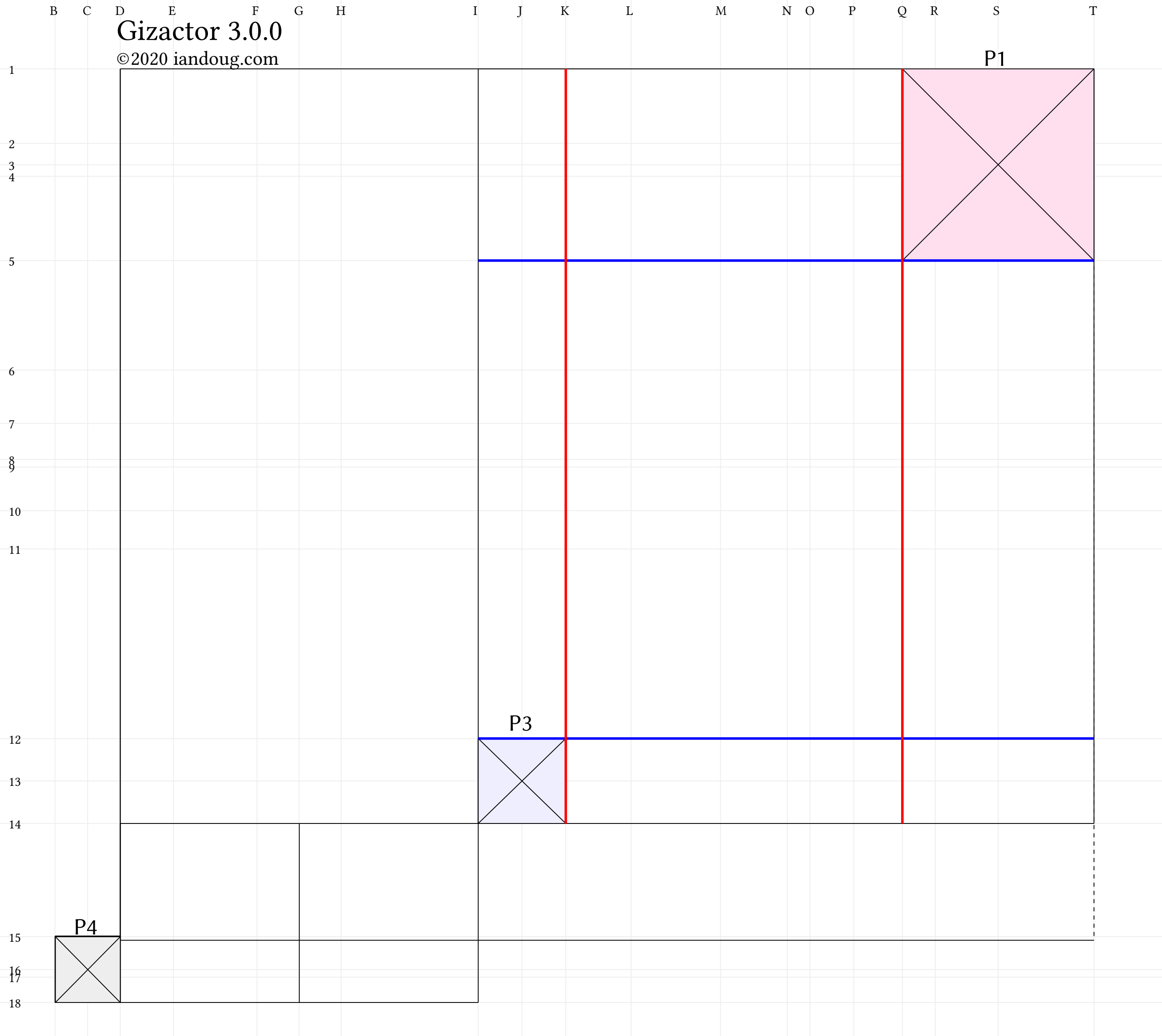


Figure 13: The preparation to calculate P2 position.

P2 has a base side of 411Cr.

$$1000\sqrt{5} = 2236(\mathit{rounded})$$

$$\frac{2236}{2e} = \frac{2236}{2 \times 2.7183} = 411(\mathit{rounded})$$

$$\frac{440}{\left(\frac{\pi-1}{2}\right)} = \frac{880}{2.1416} = 411\,(\textit{rounded})$$

$$\frac{1000\sqrt{5} - 1000\sqrt{2}}{2} = \frac{2236 - 1414}{2} = \frac{822}{2} = 411$$

$$\frac{440}{411} \approx \frac{\sqrt{(3)}}{\varphi}$$

The Douglas Triangle [9] , which has sides in the ratio 1 : φ^2 : $\sqrt{3}\varphi$, links the base sizes of Khufu and Khafre, when scaled by 50π (rounded). That gives us 157 : 411 : 440.

Khafre has a design paradigm based on the famous 3 : 4 : 5 triangle, actually using multiples of 137 in a 1½ : 2 : 2½ ratio. The base side of 411 ₢ (3 × 137) is "curious," but there is method in their madness, apart from the Douglas Triangle. The secret lies is treating ₢ as if it were metres, and converting up to ₢ by dividing by π/6. Then we find reasonable approximations for π and φ hiding, within the limits of whole-cubit dimensions. Consider:

$$137 \div \frac{\pi}{6} \approx 261.65 \approx 100\,\varphi^2\ (\textit{rounded})$$

$$(411\times4)\div\frac{\pi}{6} \approx 3139.8 \approx 1000\,\pi\ (\textit{rounded})$$

$$\left(\frac{411^2}{274}\right) = 616.5 \approx \frac{1000}{\varphi}$$

$$(411\times2) - 274 = 548$$

$$548 \div \frac{\pi}{6}\times3 \approx 3139.8 \approx 1000\,\pi\ (\textit{rounded})$$

$$(411^2)\div\left(\frac{\pi}{6}\right)^2 \approx 616147 \approx \frac{1000000}{\varphi}$$

$$\text{volume} = \frac{\textit{base}\times\textit{height}}{3} = \frac{411^2\times274}{3} = 15428118\,₢^3 = 2214684.497\,m^3$$

$$\text{diameter of sphere with that volume} = 2\times\sqrt[3]{\frac{\textit{volume}\times3}{4\times\pi}} = 161.7226\,m \approx 100\,\varphi$$

$$\log(411\sqrt{\varphi}) \approx \log(411\sqrt{1.618}) = 2.71833 \approx e$$

$$\frac{\ln\left(411\frac{\varphi^2}{2}\right)}{2} \approx \frac{\ln\left(\frac{411\times2.618}{2}\right)}{2} \approx 3.1439 \approx \pi$$

We now place P2 between P1 and P3, so that it divides the east-west space between P1 and P3 in the ratio √2 : 1, and the north-south space between P1 and P3 in the ratio 1 : √3.

Doing the east-west spacing first, the distance from P3 left to P1 right is 1000√2 = 1414 ₲.

We subtract the three pyramid widths to get the total empty space that will be left.
1414 - 440 - 411 - 201 = 362.
To split this space into √2 : 1, we divide by the silver ratio ♁, which is 1+√2.

362 / 2.4142 = 150 ₲ (rounded). That means the other portion is 362 - 150 = 212 ₲.

For the north-south division, we have a total length of 1000√3 = 1732 ₲.
So the empty space is 1732 - 440 - 411 - 195 = 686₲.

To get the division, we divide 686 by (1+√3), so
686 / 2.732 = 251. That means the other portion is 686 - 251 = 435₲.

We measure these distances and place P2.

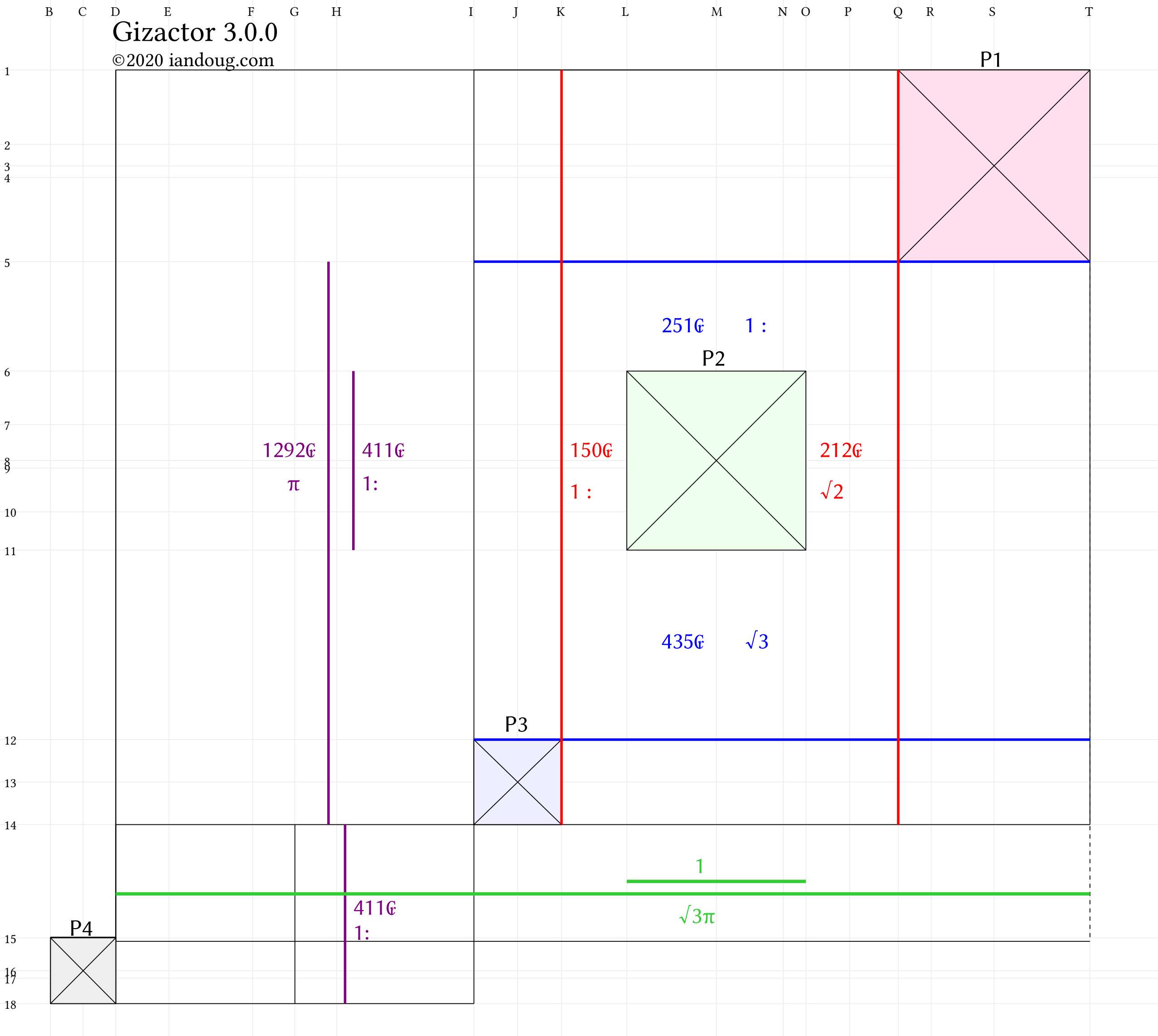


Figure 14: P2 position.

This also pops out three π ratios. 1292/411 = 3.1436, limited by whole-cubit dimensions.
$411 \times 3.1416 = 1291.1976$.
$411 \times \sqrt{3} \times 3.1416 = 2236.354$.

In truth, I did this partially backwards. Initially, I wrote programs to search for the centre of P2, down to sub-millimetre accuracy, but no matter what I did, I could not get all aspects of the blueprint to work together. When I realised that the designers used whole-cubit dimensions, I calculated the spaces between P1 and P2 using the GPMP values, then calculated what the spaces between P2 and P3 should be.

	Eastings	*Average*
P1 NW	499884.6	P1: 499884.75
P1 SW	499884.9	
P2 NE	499773.5	P2: 499773.7
P2 SE	499773.9	
Difference m		111.050
Difference ₢		212.089

Table 7: Calculating east-west space between P1 and P2

	Northings	*Average*
P1 SW	99884.7	P1: 99884.8
P1 SE	99884.9	
P2 NW	99753.1	P2: 99753.25
P2 NE	99753.4	
Difference m		131.55
Difference ₢		251.24

Table 8: Calculating north-south space between P1 and P2

At that point my suspicions regarding the size of P3 were confirmed, and I adjusted the size from 202₢ square to 201×195₢ instead. Then other things suddenly started working as well, and I accepted the size as correct.

Finding how to site P5 took considerable effort. I had an idea of where it should be, but could not find any simple mathematical relationship. Nothing worked. I was getting frustrated, so I asked my guides, "Are you sure the ratios are there?" The answer came back, "Yes, and they're beautiful."

So I tried again, and "thunk different.' Lo and behold, they surfaced... and yes, they were beautiful and I had to laugh. For P5, we use (drum roll...) π and φ. Of course.

We place P5 so that P2 divides the space between P1 and P5 in the ratio 1 : π. So multiply the east-west distance of 212 between P1 and P2 by 3.1416.

212 × 3.1416 = 666 ₲.

For φ, we divide the north-south distance between P1 and P4 so that P5 will split the space in the ratio 1 : $φ^2$.

The vertical distance is 1732 + 411 = 2143 ₲.

So the available space is 2143 - 440 - 151 - 200 = 1352. We divide that by $1+φ^2$ = 3.618 so

1352 / 3.618 = 373.687, which rounds to 374, leaving the other space as 978.

Measure, and place P5.

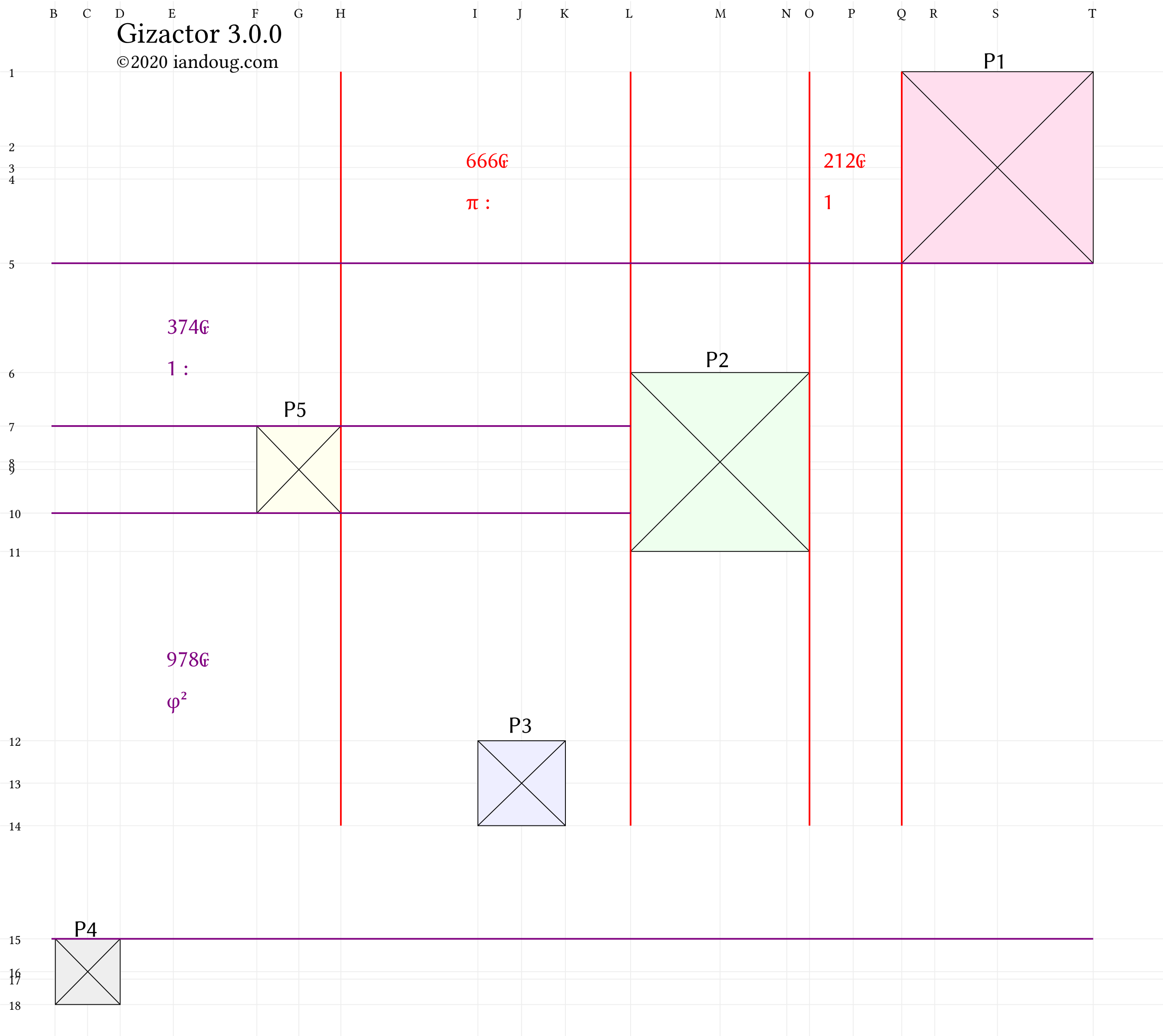


Figure 15: P5 position.

Some interesting relationships now pop up.

If we round φ to 1.62, then 411 (i.e. P2) × 1.62 = 666 (rounded).

The east-west space between P4 and P5 = 2236 - 440 - 212 - 411 - 666 - 193 = 314 = 100π.

374₢ = 314₢ + 60₢ = 100π₢ + 10πm.

The perimeter of P4 is 149 + 151 + 149 + 151 = 600₢ = 100πm.

There are only four good simple integer fraction approximations for π: $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$ and $\frac{377}{120}$.

We can compare their accuracy, and 3.1416, to π:

Numerator	*Denominator*	*Value*	*Value / π*	*Accuracy %*
22	7	3.14285714	100.04024994	99.95975006
333	106	3.14150943	99.99735104	99.99735104
355	113	3.14159292	100.00000849	99.99999151
377	120	3.14166667	100.00235591	99.99764409
		3.1416	100.00023384	99.99976616

Table 9: Comparing π approximations.

22/7 is in Khufu… twice base / height = $\frac{2 \times 440}{280} = \frac{880}{280} = \frac{22}{7}$.

The site plan includes $\frac{333}{106}$ by doubling it to $\frac{666}{212}$. My guides insisted that $\frac{355}{113}$ was also there. I could not find it, despite several attempts, so I thought I was imagining things… and they kept insisting… So after many months, I tried again… and instead found (to my surprise) $\frac{377}{120}$ as $\frac{754}{240}$. And still they insisted about $\frac{355}{113}$ …. so I looked some more… and there it was.

I laughed and apologised…

So that puts all the low integer approximations for π, plus 2143, in the Giza plan. 2143 surfaces again later. The $\frac{754}{240}$ and $\frac{355}{113}$ formula locations involve subtraction, as shown in Figure 16.

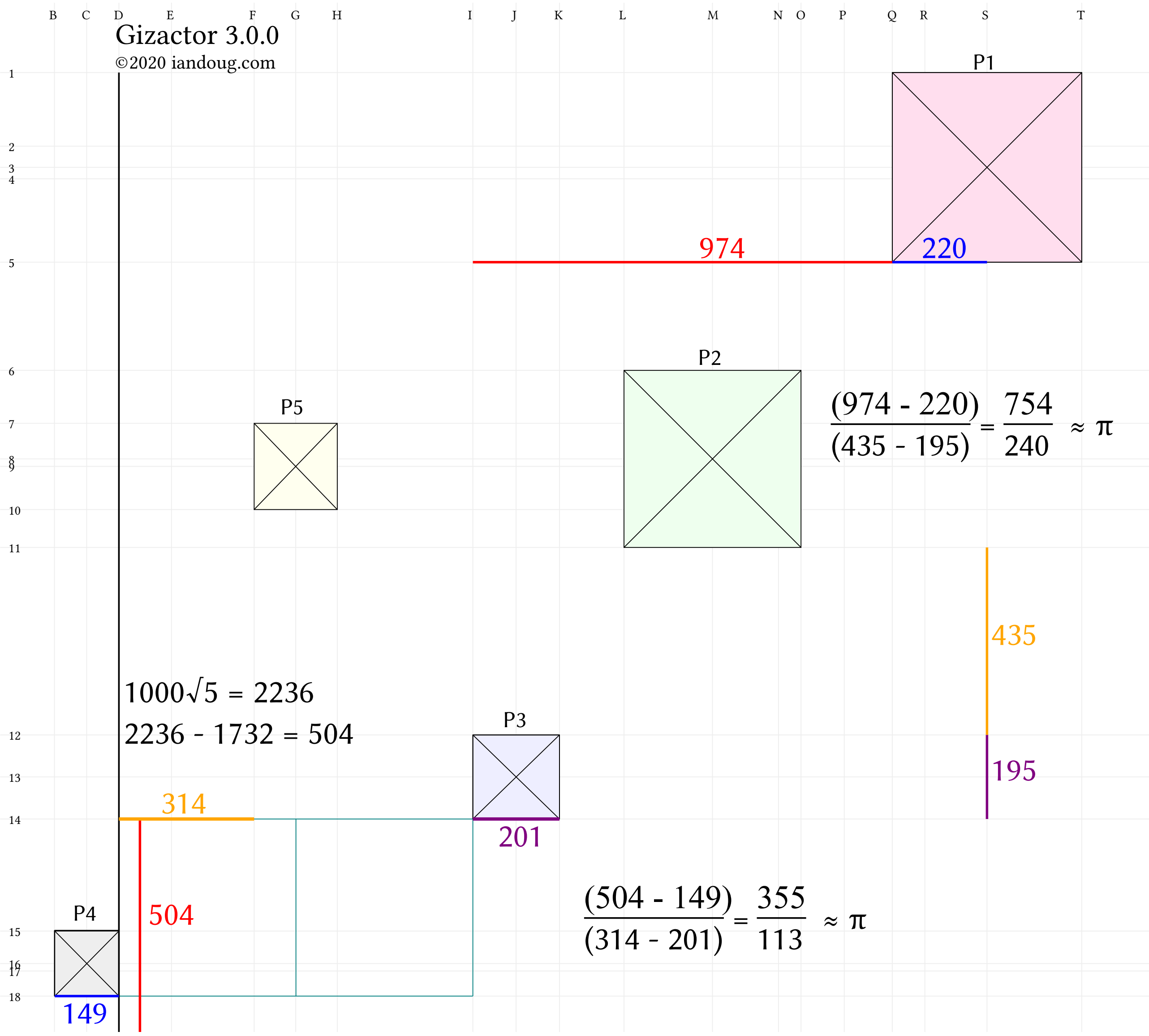


Figure 16: 754/240 and 355/113 formula locations.

These only work if P3 is 201 × 195, which for me further confirms the sizing.

Their rounded value of 3.1416 is very good.

I was still dealing with the shock of finding $\frac{355}{113}$ as had been promised, when they asked, "And what about e?"

"Seriously? You're just messing with me."

"No."

So I found some integer approximations for e on the web: $\frac{19}{7}$ and $\frac{878}{323}$. I started doubling $\frac{19}{7}$ and got to $\frac{212}{78}$, which looked promising because there is a 212 in the plan, but I could not find 78. Then I noticed that 666 + 212 = 878 ... and a bit of searching found 323.

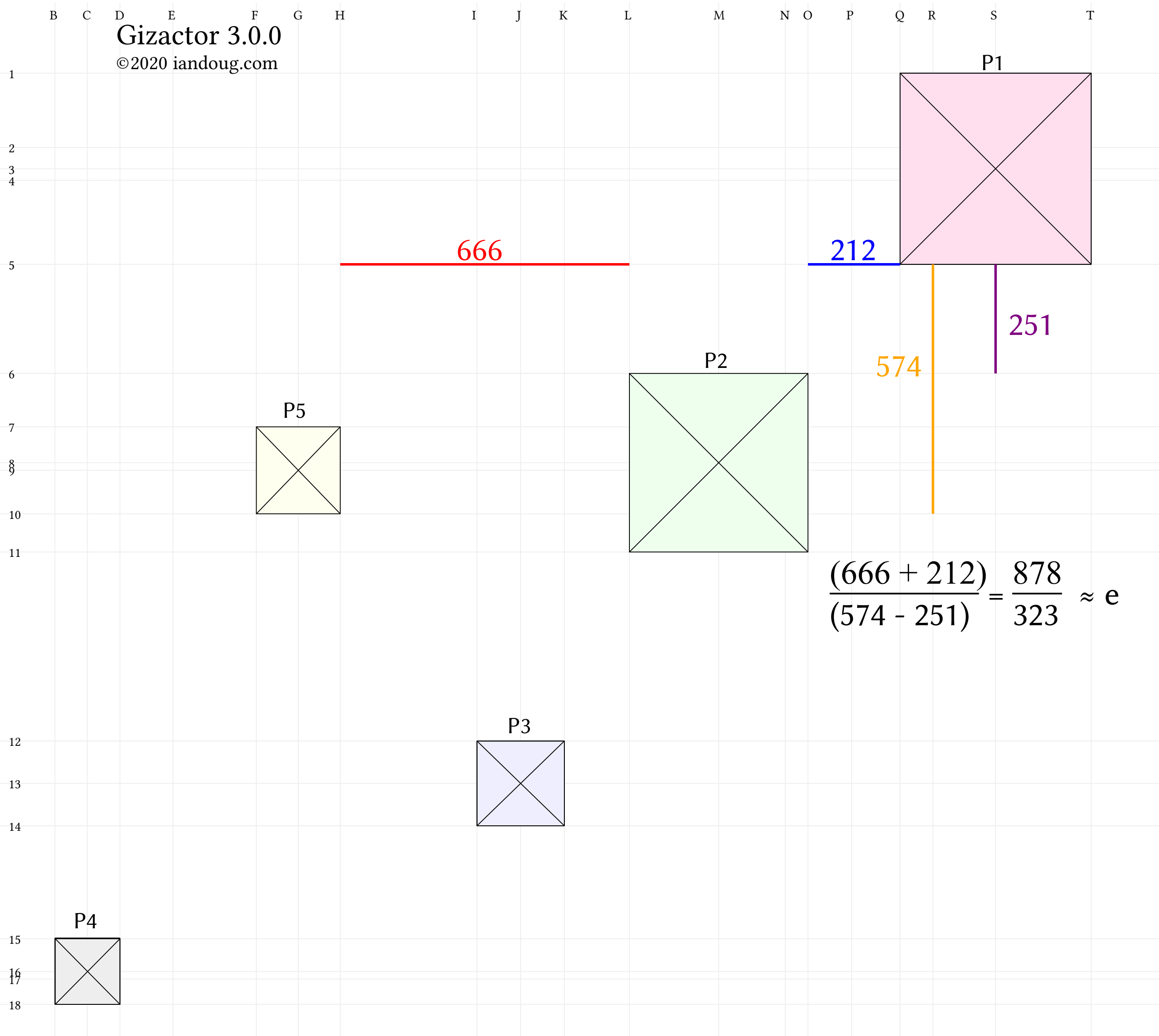


Figure 17: Formula for approximating e

878/323 = 2.718266254...; e = 2.718281828....; difference is 0.000015574...

When I was doing The Douglas Triangle [9] in 2021, I was trying to find its dimensions in the site plan. The sides are in the ratio 1 : φ^2 : $\sqrt{3}\varphi$, and we have just seen 1 : φ^2. I was unable to find the matching $\sqrt{3}\varphi$, but did find something else.

Firstly, a nice 1 : $\pi/2$ ratio between the centre of P4 and the west edge of P2. Then if we move P5 ½ a cubit south, making the distance from P1 to be 374.5Ꞡ, and adjust the size of 200 × 193 to 196 × 193, and increase P4 from 149 ×151 to 149 × 152, then we would have excellent π, φ^2 and Ꞡ ratios, as shown:

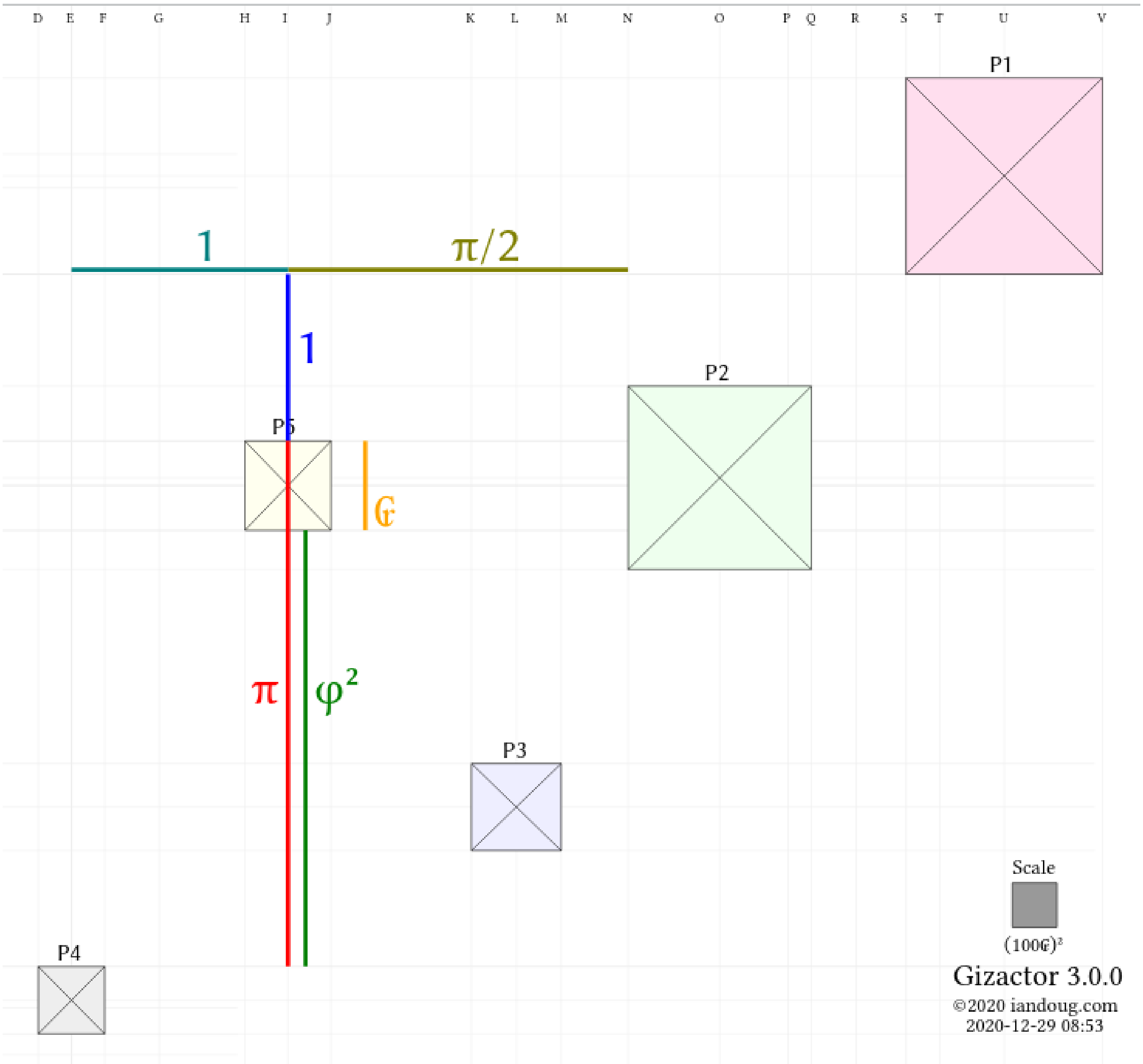


Figure 18: Alternative compelling arrangement for P4 and P5

The ratios are:

Item	*Ratios*	*Value*
π	1176.5 / 374.5	3.141522
φ^2	980.5 / 374.5	2.618158
₲	196 / 374.5	0.523364
$\pi/2$	644.5 / 410	1.57195

Table 10: Analysis of alternative arrangement.

However, making these changes produces worse values for most of the other ratios shown earlier, and below, especially the $\sqrt{\pi}$ value crucial for squaring the circle. I did try playing with the P4 width but was unable to get it all to work together nicely. It is possible that P5 had a complex structure, for example a raised base or boundary wall, that enabled both ideas simultaneously.

Putting it all together gives us the site plan.

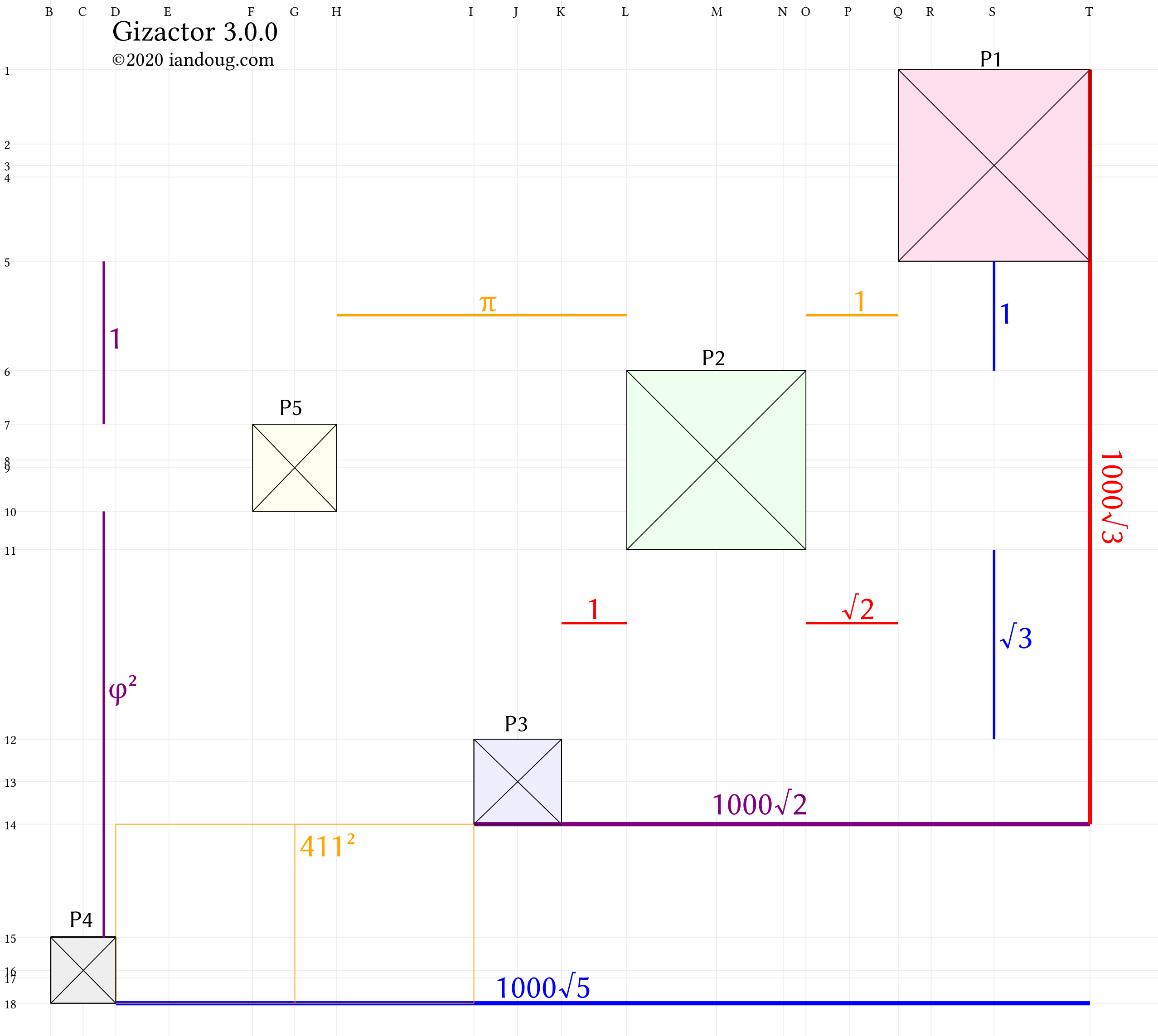


Figure 19: Master plan for laying out Giza right side, with additional π ratios.

This simple yet elegant design has puzzled many people over the years, but like all the best magic tricks, the secret is simple and "obvious" once revealed.

The design displays absolute knowledge of π, φ, square roots, and ~~Pythagoras~~ Thoth.

Here is a summary of the additional π ratios that come with this design.

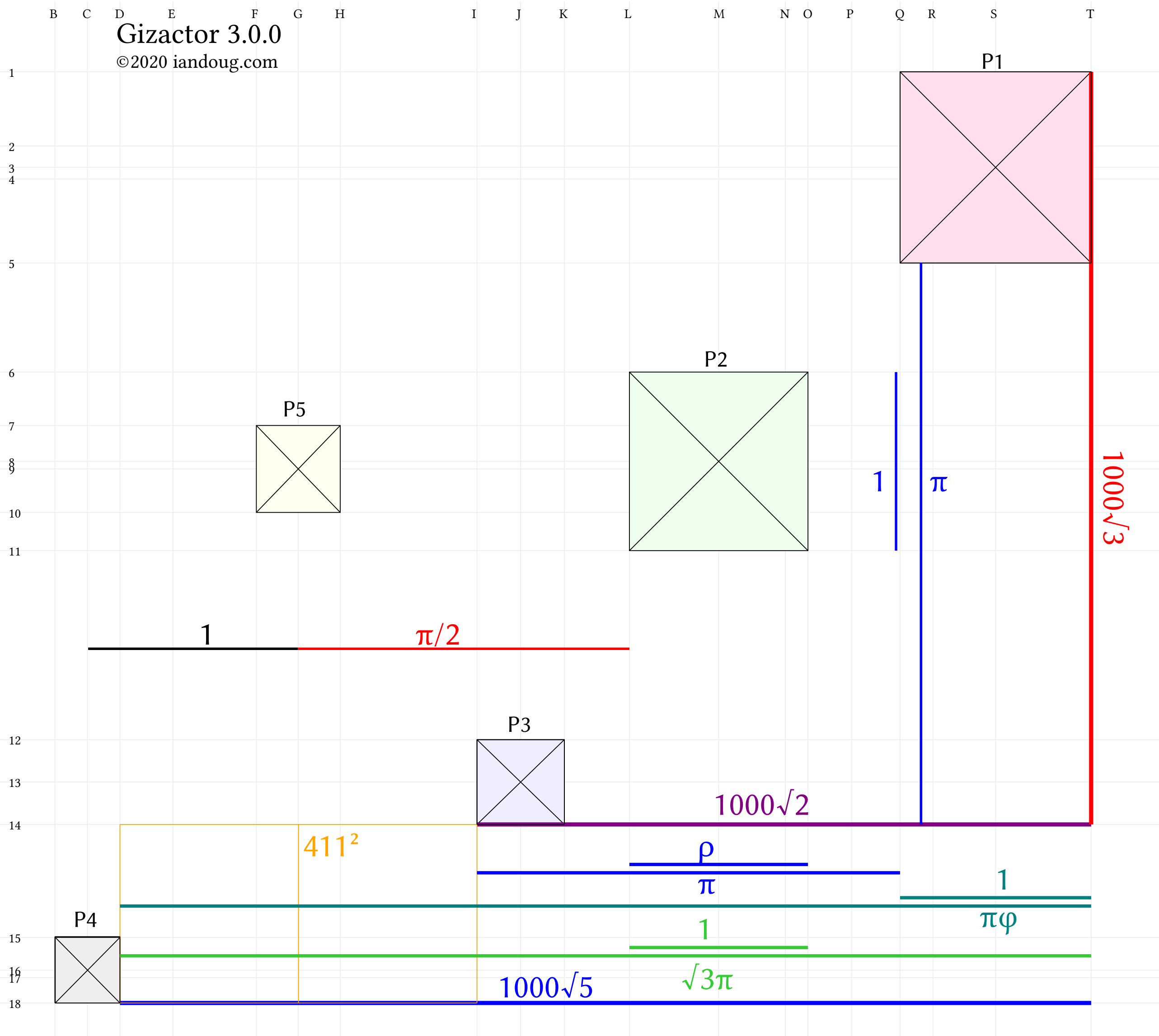


Figure 20: Additional π approximations inherent in the design

This design uses three missing pyramids, a slightly resized P3, and is strictly North-aligned. We can compare this version to Giza, which is slightly twisted with respect to north. Giza is plotted in a thicker red line using the co-ordinates from Glen Dash and the GPMP, while the above design is overlaid using a thin black line.

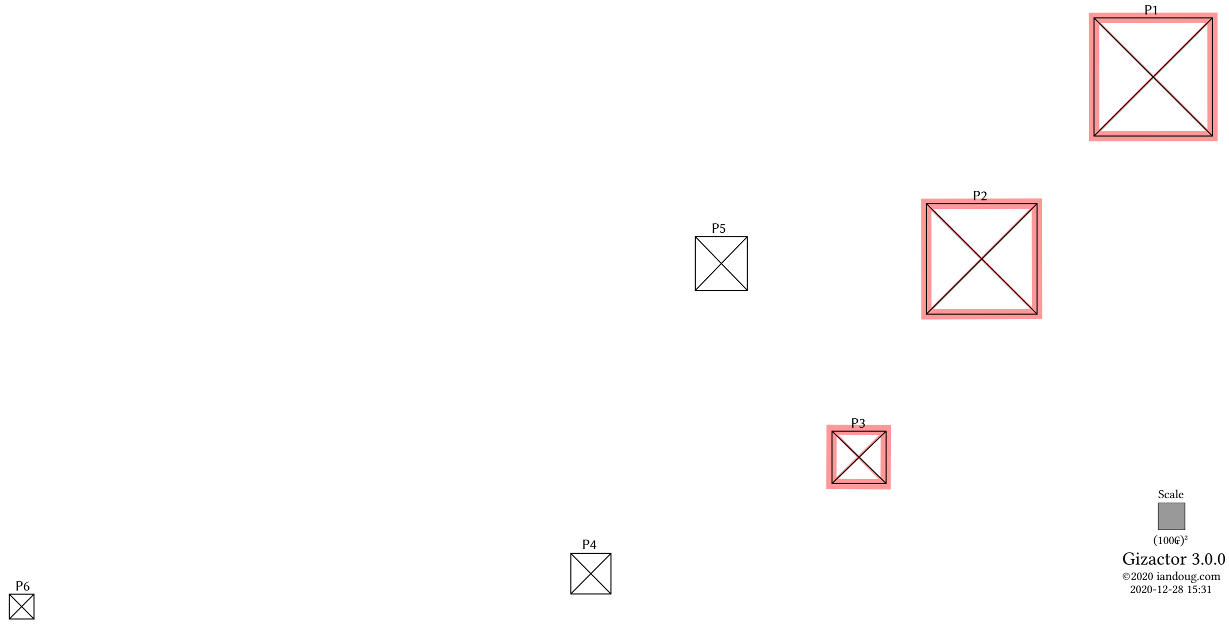


Figure 21: New locations overlaid on three extant pyramids.

A comparison of the pyramids as per GPMP, the location for P4 in *Diskerfery*, and as determined in this paper. These are the SVG co-ordinates at a scale of 1 pixel : 1 metre, which map directly to the GPMP grid. P1 centre on GPMP is (500,000; 100,000) while the SVG point is (2300; 200).

Pyramid	*GPMP / Diskerfery centre*	*Current centre*	*GPMP / Diskerfery base*	*Current base*
P1	2300; 200	2300; 200	440 × 440	440 × 440
P2	1966; 554.4	1966.205; 554.215	411 × 411	411 × 411
P3	1726.5; 940.1	1727.443; 940.632	202 × 202	201 × 195
P4	1204.2; 1165.7	1205.414; 1167.351	162 ×162	149 × 151
P5		1459.36; 563.378		193 × 200
P6		97.738; 1231.492		92 × 92

Table 11: Locations and sizes comparison.

The differences are very small, considering Giza is not aligned with the Cartesian grid, while my current values are. Also, both P3 and P4 have different sizes.

We can calculate the revised GPMP co-ordinates. Perhaps someone with a ground-penetrating radar can take a look at the P4, P5 and P6 locations.

Pyramid	*Eastings*	*Northings*
P1	500,000.000	100,000.000
P2	499,666.205	99,645.785

Pyramid	*Eastings*	*Northings*
P3	499,427.443	99,259.368
P4	498,905.414	99,032.649
P5	499,159.360	99,636.622
P6	497,797.738	98,968.508

Table 12: GPMP co-ordinates for all six pyramids.

My mantra with Giza is that "Everything works together" ... and it's not just the mathematics agreeing with the stars. Here is an updated version of a diagram in *Diskerfery,* using the latest pyramid positions. It's quite amazing how this pops out of the design, and hints at some relationship between the square roots, and π and e. As a reminder, we have ₵ as $\pi/6 = 0.5236$m, and the foot as ₵/é = 0.3047m. The grand Metre is 1 metre plus one cubit.

The correlation is very good, considering that sizes and spaces are limited to whole cubits.

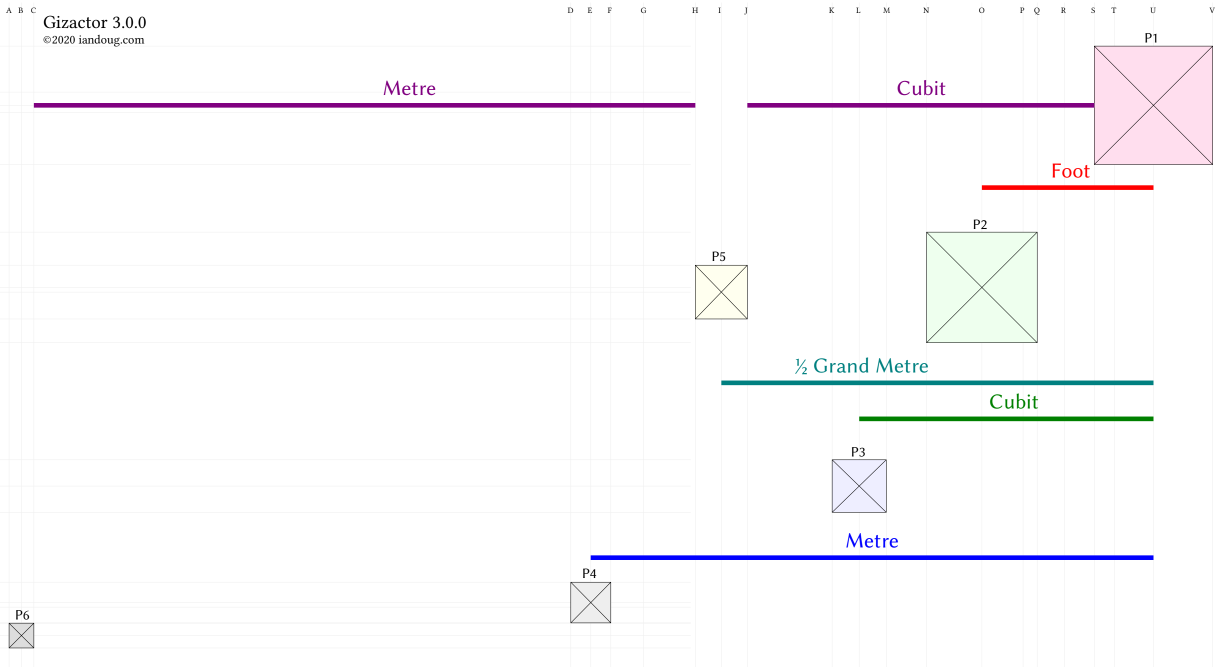


Figure 22: The spacing closely matches the length units.

Label	Line	Distance m	Ratio	Maps to	Value => m	Accuracy %
A	P1C - P2C	333.8	A/C	Foot	0.3049	99.95
B	P1C - P3C	572.56	B/C	₢	0.5230	99.88
C	P1C - P4C	1095.85	C/C	Metre	1.000	100.00
D	P1C - P5C	840.64	D/C	½ Grand Metre	0.7678	99.21
E	P1L - P5R	674.92				
F	P5L - P6R	1287.01	E/F	Cubit	0.5244	99.85

Table 13: Accuracy calculations for length units.

7. Module 4: The skeleton blueprint

The above site plan uses simple distances, and space ratios, to lay out the site. However, the designers embedded other mathematical curiosities in the design. I call this "The skeleton blueprint."

In truth, I stumbled across this before the ratios above, or adding Vega to the plan. For now, we will just use the right hand side, and include Vega later.

The blueprint is all about distance ratios between the centres of the pyramids. Once I had made progress analysing these, my guides innocently suggested (as is their wont) that I apply the same techniques to the spaces between the pyramids, and *voila*!, I was flabbergasted when those ratios popped out. That led to the Thoth grid discussed above.

One night I was staring at the stellar alignment trying to figure out how to calculate the celestial north pole from the other four pyramids. I noticed that a line from the north pole to P3 appeared to divide a line from P1 to P4 in the golden ratio. That sent me down the rabbit hole that resulted in the skeleton blueprint.

Before we get to the blueprint itself, we need two diagrams that are not in the geometry textbooks. They are not in the textbooks because the Greeks didn't know about them, and the Greeks didn't know about them because the dynastic Egyptians, who taught the Greeks mathematics, didn't know about them. The dynastic Egyptians didn't know about them because they didn't build Giza. If they had, they would have. Thus, our ignorance proves that Giza pre-dates the dynastic Egyptians. Q.E.D.

The first diagram is the πφ cross.

Take two lines that are in the ratio 1:π, for example 200 and 628.32.

Now arrange them so that each line cuts the other in the golden ratio.

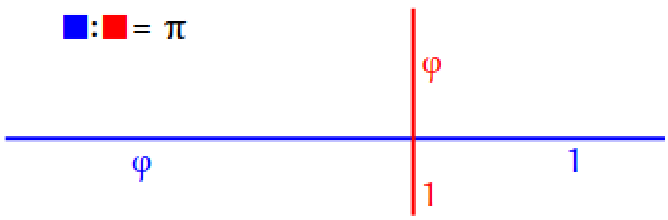


Figure 23: The $\pi\varphi$ cross.

The lines do not need to cross at right angles, I suppose we can call the general case a Golden Cross or "πφ cross," and the right-angled version a "right πφ cross" or "proper πφ cross."

Note that both the long arms and short arms are also in π ratio to each other.

The second diagram is a π triangle, which is a triangle where the ratio of one side to the perimeter is 1:π. This could be scalene, isosceles, or equilateral.

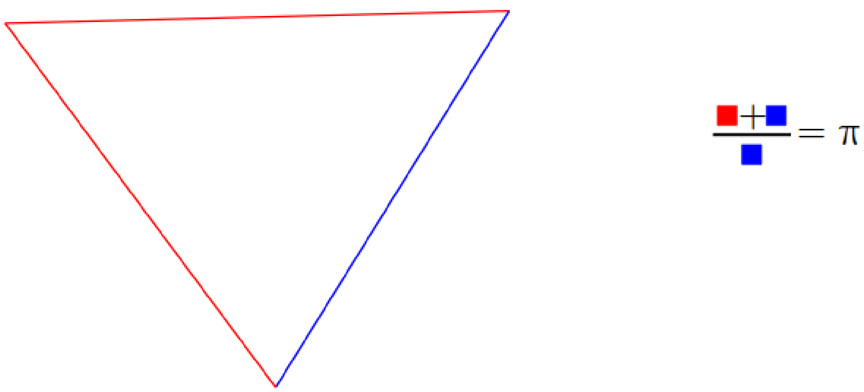


Figure 24: The π triangle.

A special case of this triangle is the Golden Pi triangle, where the two red sides above are in the golden ratio. The sides of such a triangle are in the ratio

$$\frac{1}{\varphi} : \frac{1}{\varphi^2} : \frac{1}{\pi - 1} \quad .$$

We might as well also define the Ultimate Triangle, which has sides in the ratio φ : e : π, and comes out rather close to a 30°:60°:90° triangle.

We now combine the πφ cross and π triangle into a masterpiece. This masterpiece encapsulates the very basics of geometry: line; circle, and triangle, as well as π, φ, √π, √2, √3, √5, and others, in a beautiful, elegant and minimalist design.

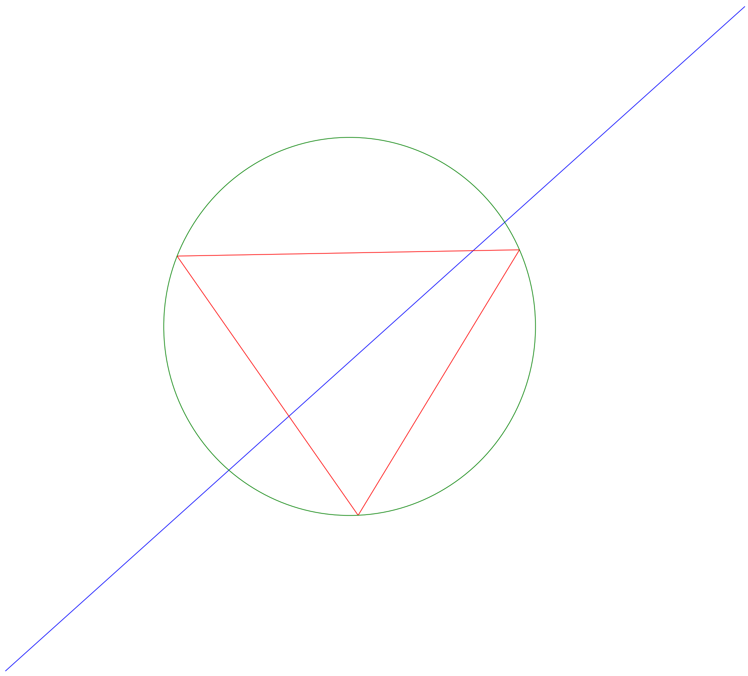


Figure 25: The skeleton blueprint.

Adding the pyramids back shows how it all works together:

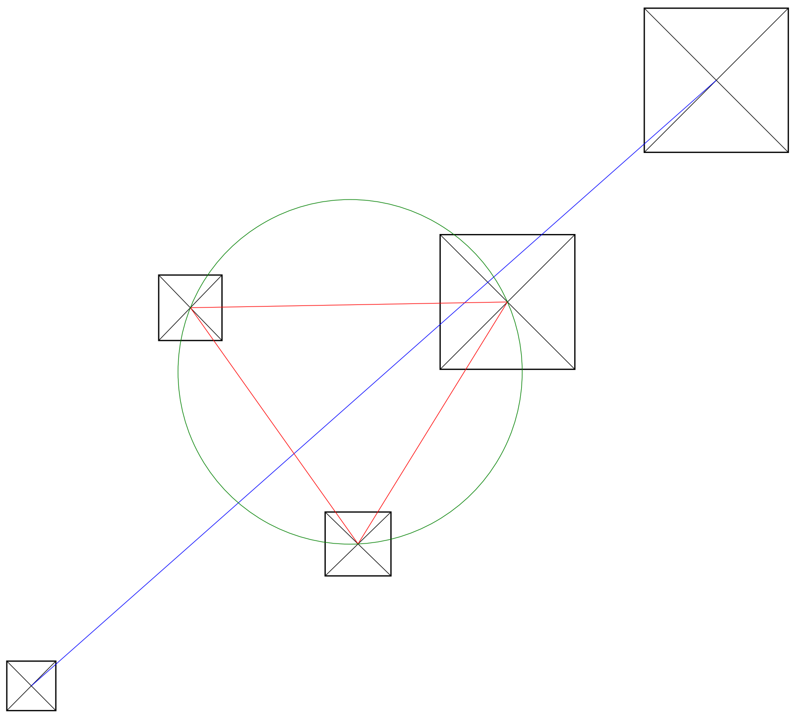


Figure 26: Skeleton blueprint with pyramids.

Let us explore the magic in this design.

The accuracy of the various relationships is heavily dependent on where exactly P5 is located. Varying this location within a circle of about 5m diameter dramatically affects the accuracy, but in the grand scheme of things on a site the size of Giza, 5m is not very much.

So again it becomes a question of "how accurate is enough" and how close must it be to demonstrate intent of design?

As mentioned previously, if P5 is in one spot, you get accuracies like these, when I was researching this, and siting the pyramids with millimetre-level precision.

Pi Triangle: 99.97911886%
Triangle area 99.99999671%
πφ Cross: 99.99709498%
Long φ: 100%
Short φ: 100%
$\sqrt{2}$: 99.73851304%

After limiting pyramid sizes and inter-pyramid spaces to whole cubits, and focusing on getting the best result for squaring the circle, those ratios now come out as:

Pi Triangle: 99.7865%
Triangle area 99.4074%
πφ Cross: 99.5301%
Long φ: 99.7726%
Short φ: 99.9885%
$\sqrt{2}$: 98.8397%

On the flip side, other ratios come out well, for example:

$\sqrt{\pi}$	99.9999%
$\sqrt{\tau}$	99.9996%
$\sqrt{8}$	99.9988%
$\sqrt{6}$	99.9942%
φ^2	99.9890%
$\sqrt[3]{3}$	99.9886%
φ	99.9885%
e	99.9853%
π	99.9799%

We start with a Golden Cross, which neatly combines π and φ, and is in some ways the major axes of the design. In the diagrams, I've joined various points with grey lines for what follows.

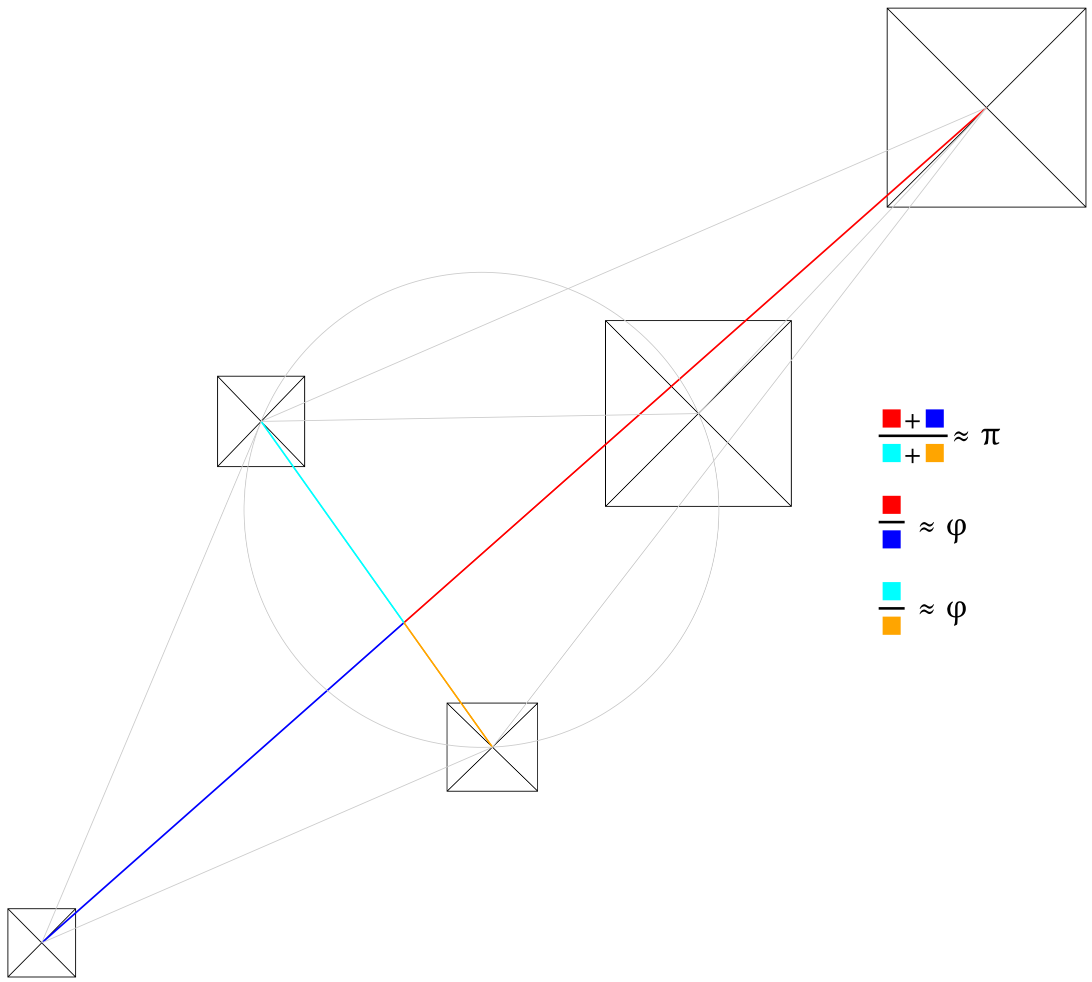


Figure 27: π,φ and φ

The reader is invited to compare Figures 1, 19, 22, and 27, and contemplate how the designers got them all to work at the same time.

The long blue-red diagonal runs from the centre of P1 to centre of P4. The length in metres is 1460.9786m, which is 4 × 365.2447 ... remarkably close to the number of days in a year, or the total number of days in a 4-year cycle.

Discrepancies for the above were better before I catered for the squaring of the circle. However, the intent is still clear. They are now:

π : 47cm/100m
long φ: 23cm/100m
short φ: 1cm/100m

That was one π and two φ. We also have one φ and two π:

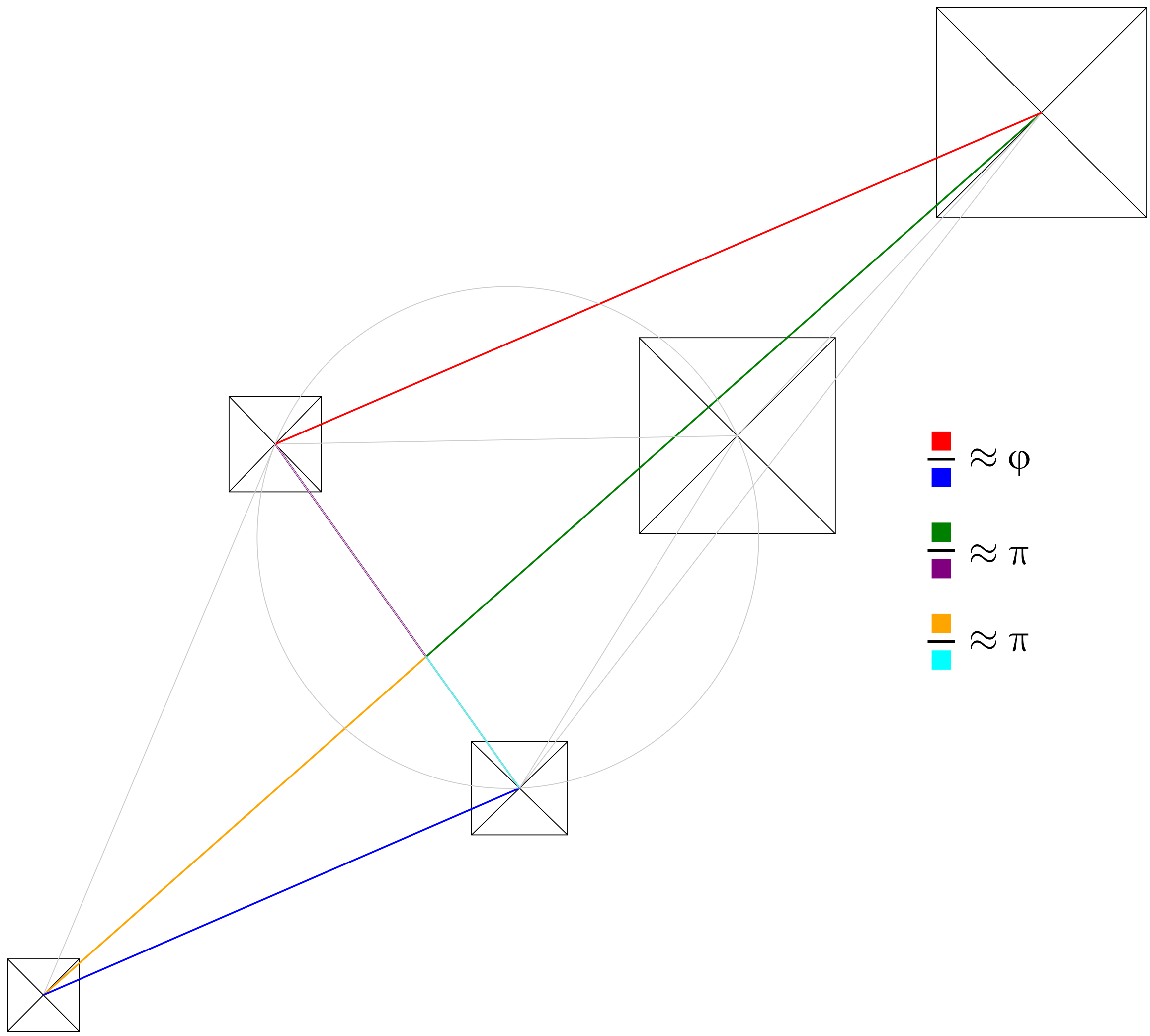


Figure 28: φ,π and π

Discrepancies for above:
φ: 55cm/100m
Long π: 25cm/100m
Short π: 81cm/100m

The square roots of the first three primes:

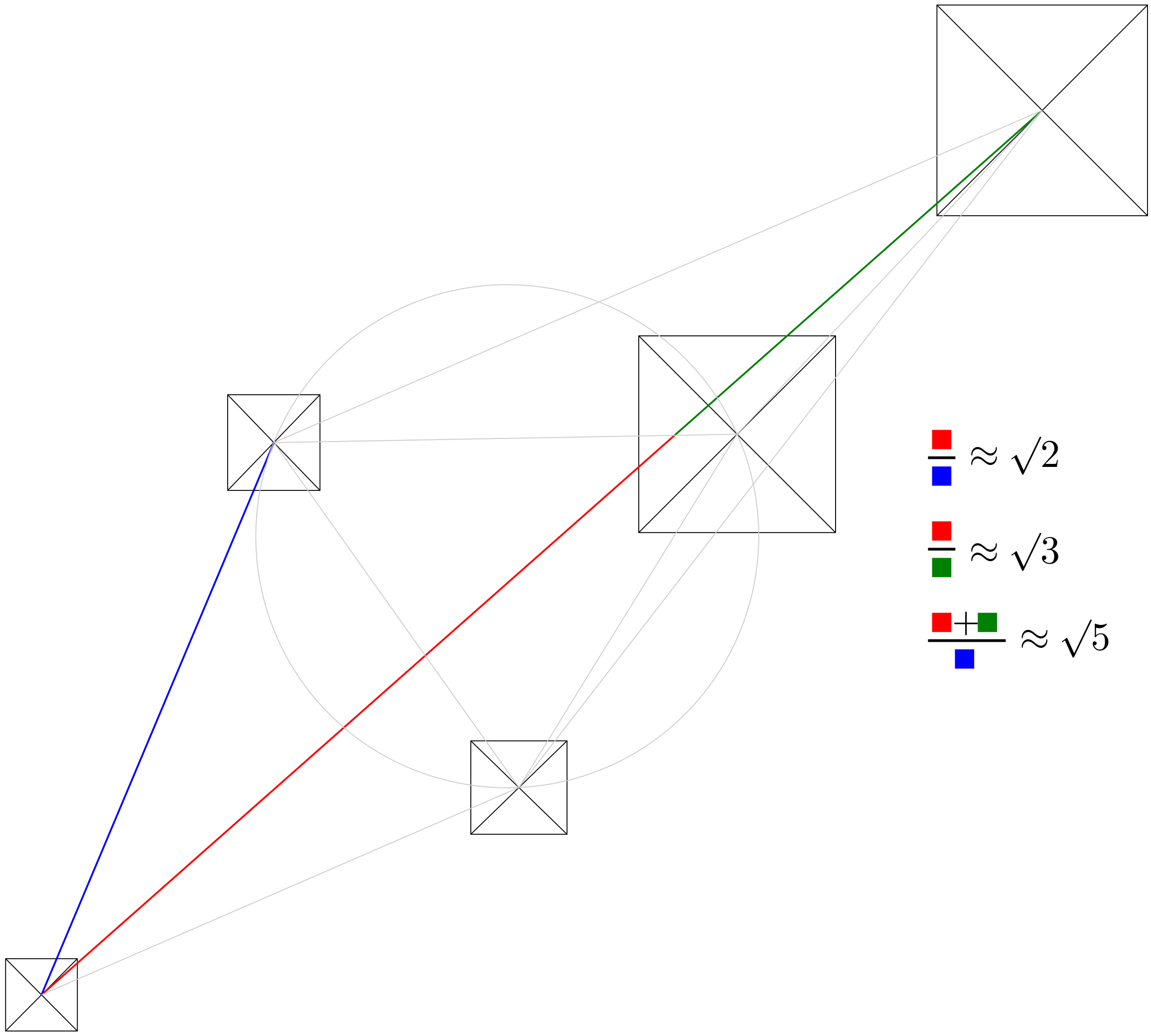


Figure 29: Square roots of the first three primes.

This is one possible grouping. It's not the best but the "neatest."

Discrepancies for above:
$\sqrt{2}$: 28cm/100m
$\sqrt{3}$: 61cm/100m
$\sqrt{5}$: 29cm/100m.

The pair P4P5 / P5P3 has an error for $\sqrt{2}$ of 11cm/100m.

The cube roots of the first three primes:

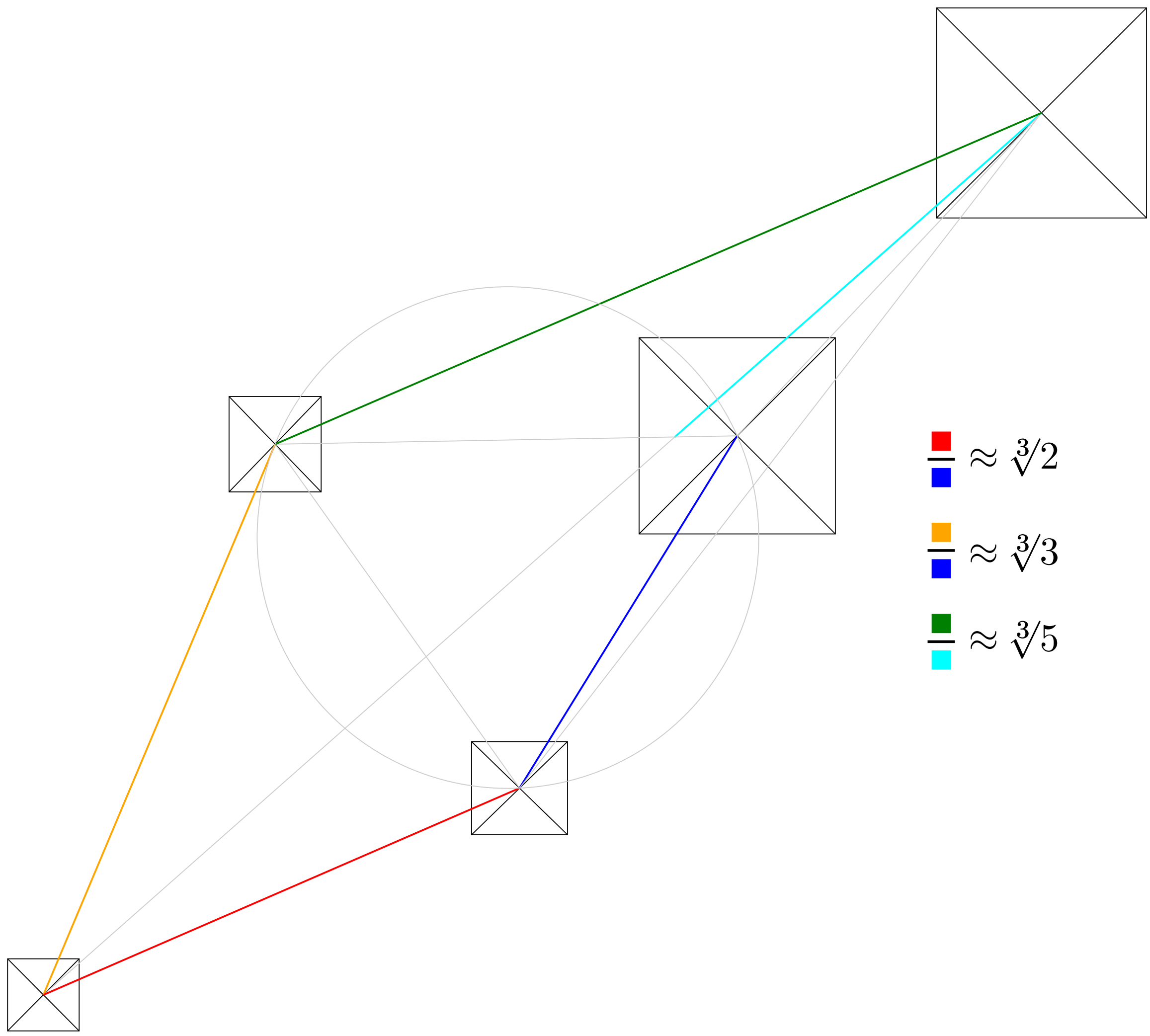


Figure 30: Cube roots of the first three primes.

The discrepancies above are:
$\sqrt[3]{2}$: 55cm/100m
$\sqrt[3]{3}$: 1cm:100m
$\sqrt[3]{5}$: 22cm:100m

Roots of the famous irrationals:

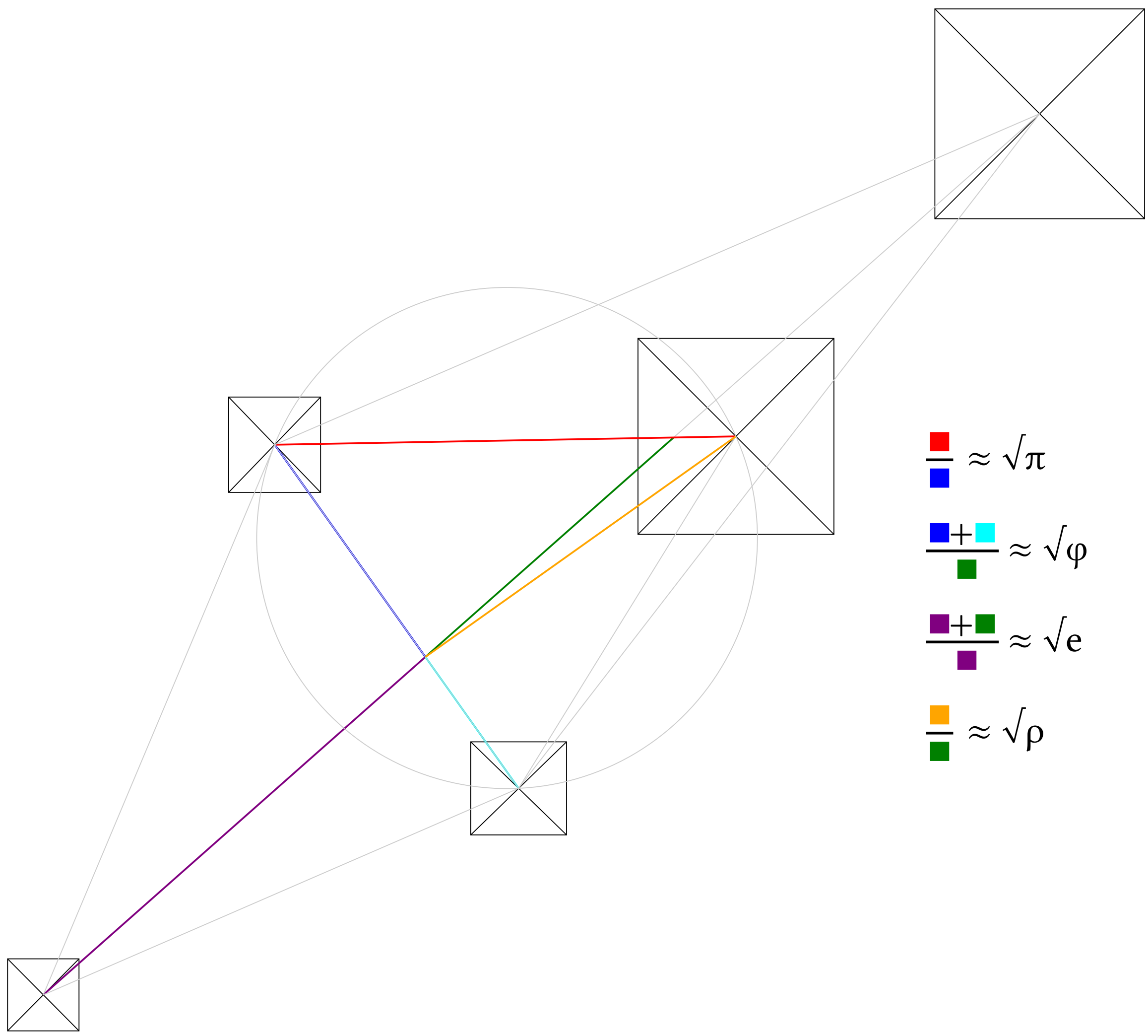


Figure 31: Square roots of π, φ, e and ρ

The discrepancies above are:
$\sqrt{\pi}$: 0cm/100m
$\sqrt{\varphi}$: 5cm:100m
$\sqrt{e}$: 8cm:100m
$\sqrt{\rho}$: 12cm/100m

There are many others but I think you get the idea.
We shall use $\sqrt{\pi}$ again shortly.

We turn our attention to the triangle.
The area of the triangle, in square metres on the ground, based on the proposed blueprint, is 96832.7866m², which is 99.4074% accurate compared to $1000\pi^4$.

$1000\pi^4$ is 97410, which is 9.74Ha or about 13.6 soccer fields. The difference between 96832.7866 and 97410 equates to a square of side about 24m over that 9.74Ha. As mentioned previously in the example accuracies, depending on where exactly P5 is, the accuracy could be 99.99999%.

So we need to ask ourselves, why would they design a triangle that size? Pure chance?

The plot thickens when we consider that it is a π triangle:

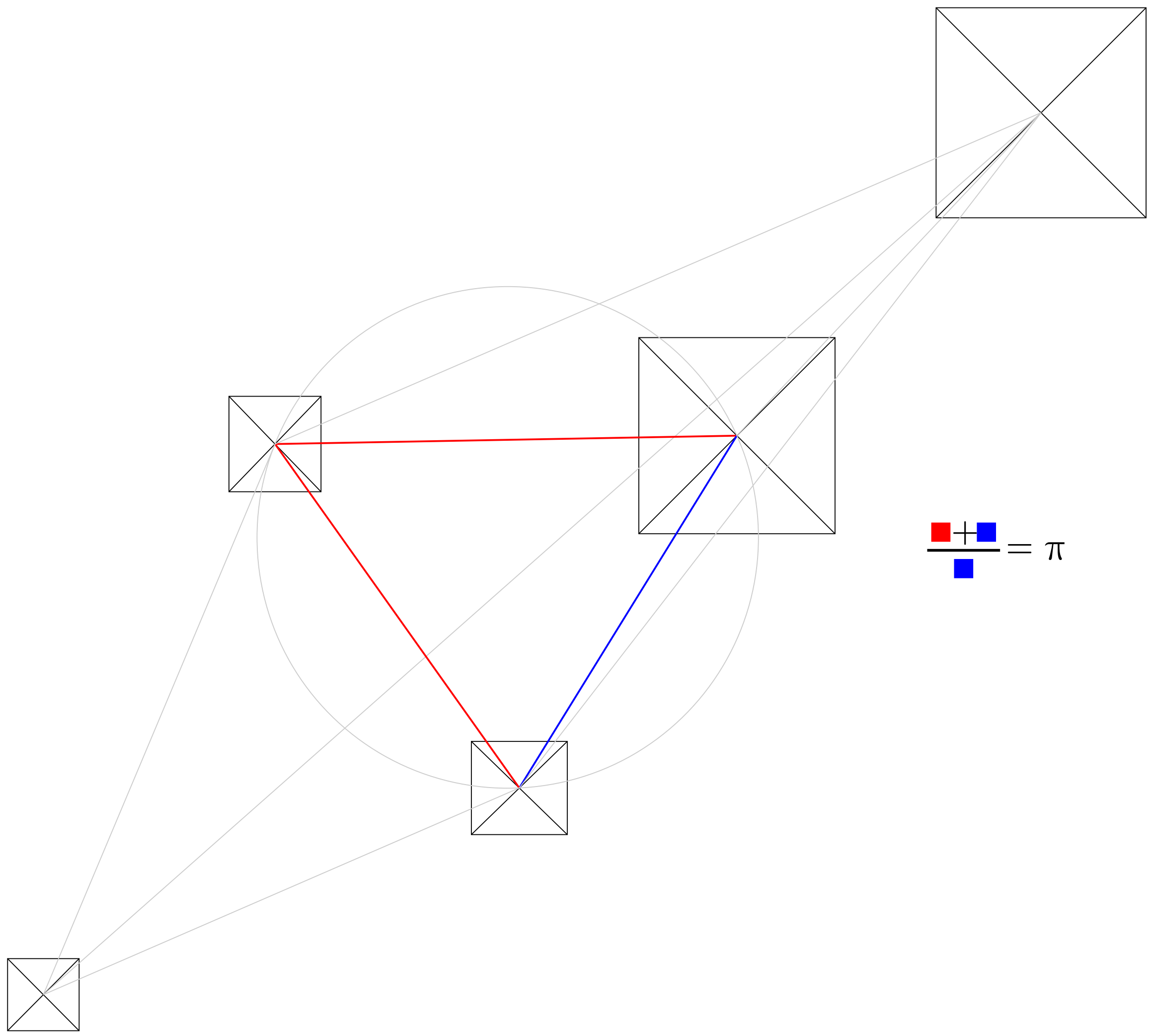


Figure 32: The π triangle between P5, P3 and P2.

The discrepancy is 21cm/100m.
Despite π getting all this attention, they did not forget φ. The slope of the line from P3 to P2, is φ (99.974%).

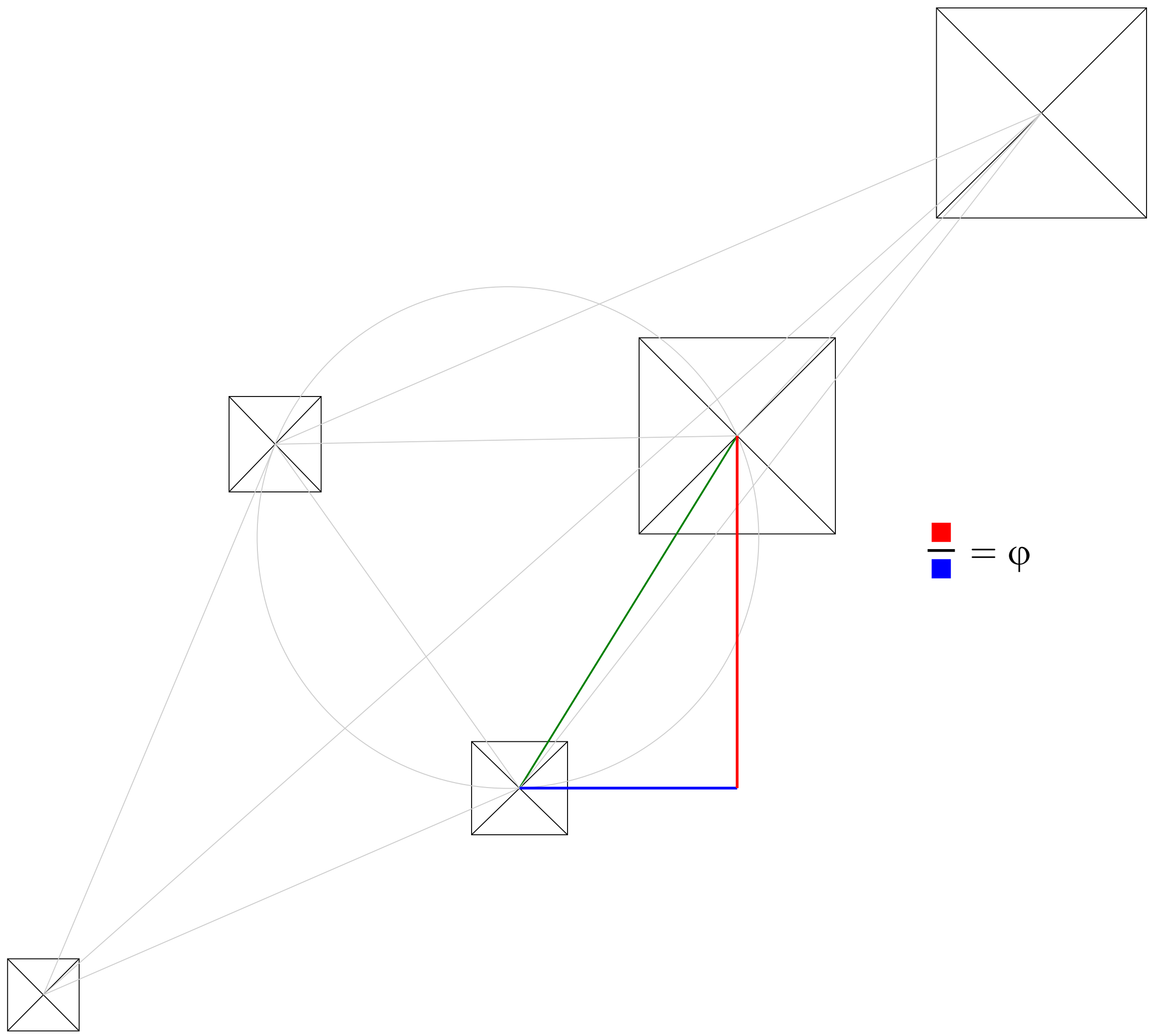


Figure 33: Golden ratio slope between P3 and P2.

The slope from P4 to P3 is 23.465°, while the slope from P5 to P1 is 23.377°. So those two lines are effectively parallel, and in the golden ratio to each other. See Figure 28 above.

We can now turn our attention to the circle, which circumscribes the triangle. We look at how the main P1 - P4 diagonal cuts the diameter, in a manner similar to the πφ cross.

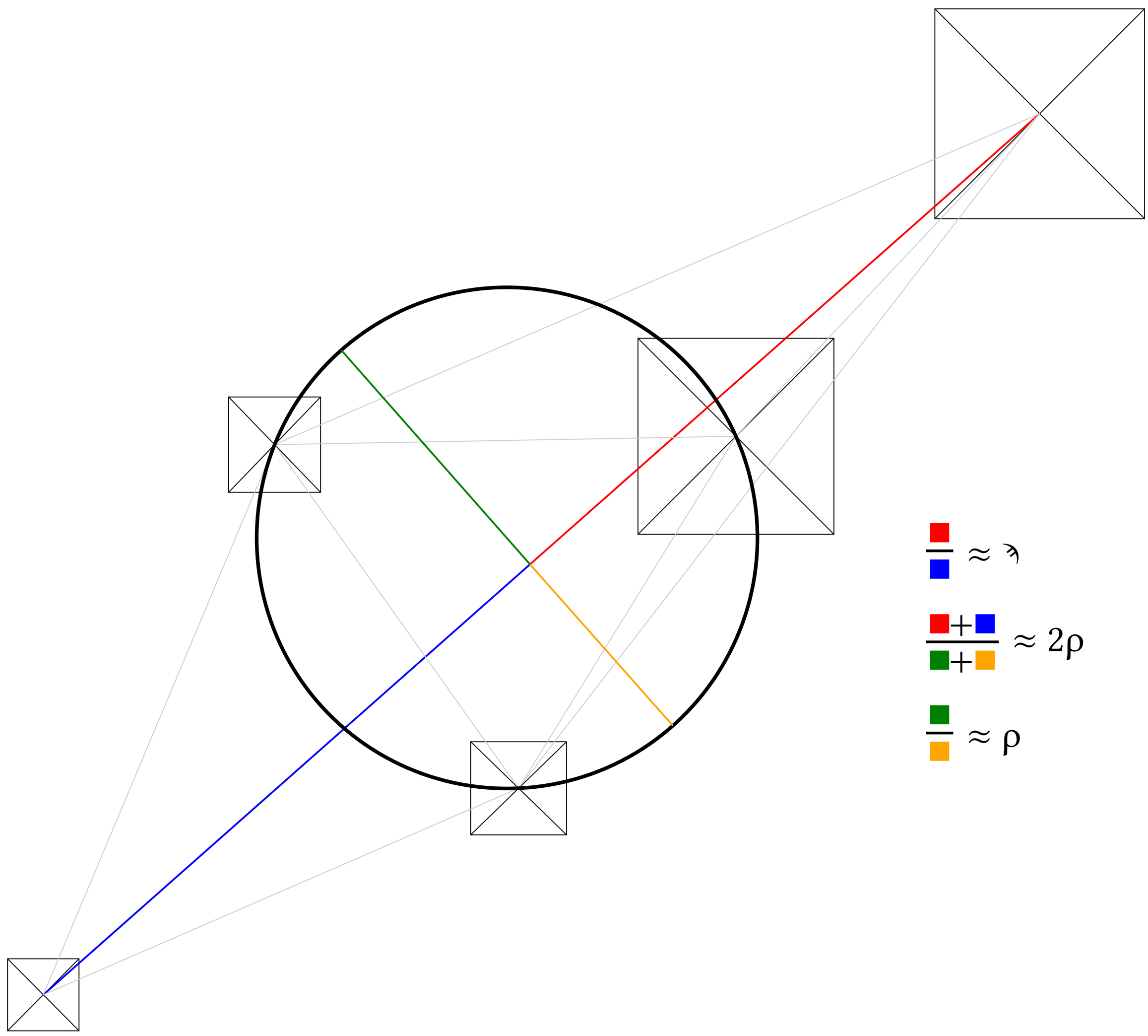


Figure 34: How the main diagonal cuts the diameter.

The discrepancies above are:
ↄ: 6cm/100m
2ρ: 20cm:100m
ρ: 9cm:100m
The angle from the cut to the circle centre is 89.68°.

The ratio of the diagonal from P1 to P4, to the circle diameter, is 2ρ.

The perpendicular from the circle centre to the line, cuts the line so that the parts are in the ratio ↄ ($\pi/3$).

The diagonal cuts the diameter so that the parts are in the ratio ρ. Compare these ratios and splits to the similar construction between P1P4 and P5P3.

Line 1	*Line 2*	*Ratio*	*Cut 1 ratio*	*Cut 2 ratio*
P1 P4	P5 P3	π	φ	φ
P1 P4	Perpendicular diameter	2ρ	ↄ (π/3)	ρ

Table 14: Comparing how the long diagonal cuts two lines.

I don't know why ↄ ... their mathematics seems focused on different things to ours.
π/3 is the ratio of 2𝕲 : 1m or 𝕲 : 0.5m. Perhaps I should look at it in those terms.

Comparing cutting a line in golden and plastic ratios:

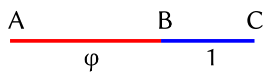
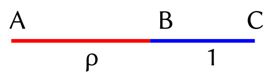
Ratio	*Golden ratio φ*	*Plastic ratio ρ*
		
AB / BC	φ	ρ
AC / AB	φ	$ρ^2$
AC / BC	$φ^2$	$ρ^3$
AB + BC	$φ^2$	$ρ^3$

Table 15: Comparing dividing a line by φ and ρ

If we compare the area of the circle to that of the triangle, then we get $(π/2)^2$.

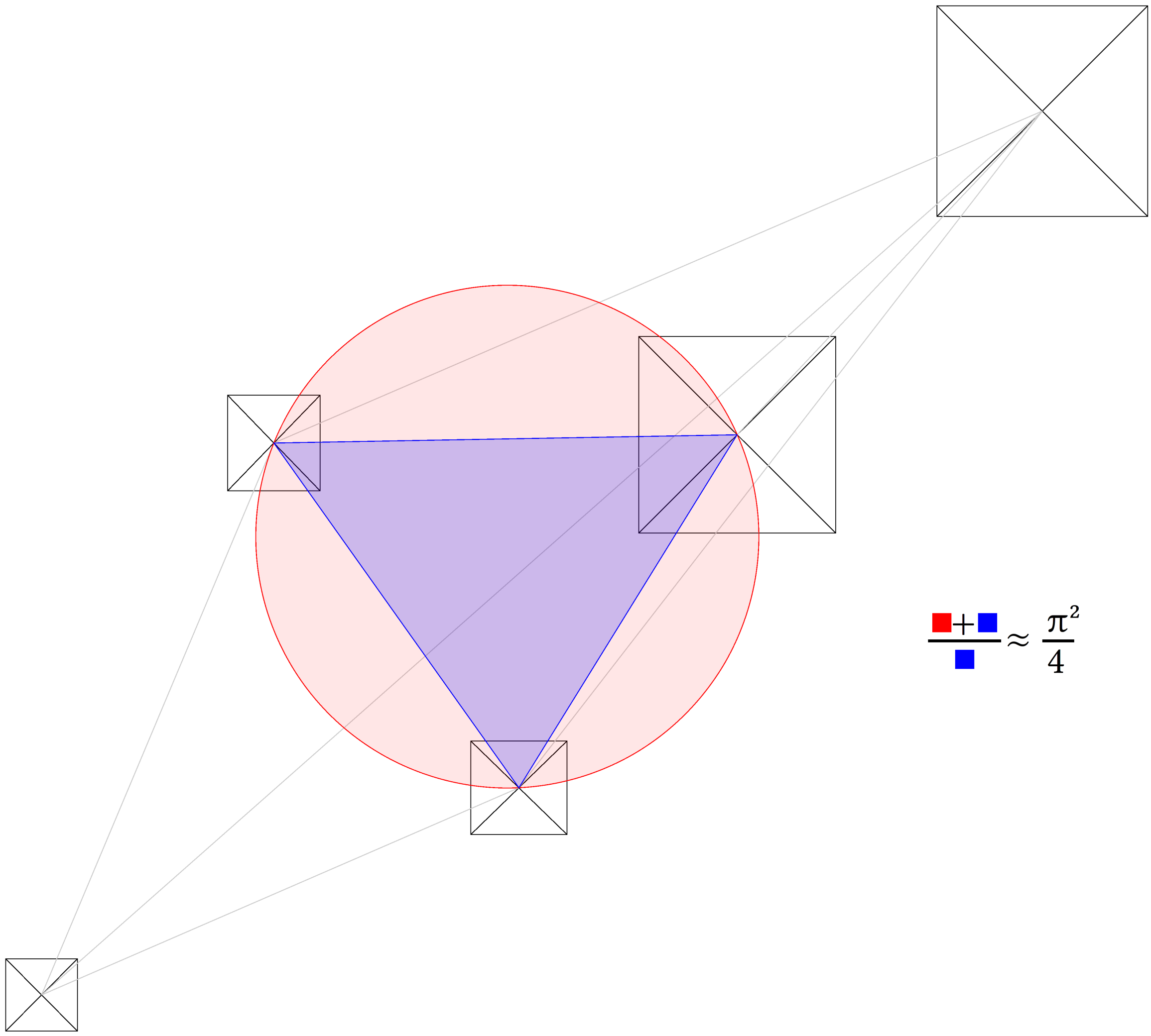


Figure 35: Comparing circle and triangle areas.

The discrepancy is 47cm/100m.

Adding the other four stars

The first star chart (Figure 1) has four green dots, closely or exactly aligned with four stars. We now show how these positions were calculated, using the same style as in the rest of the blueprint.

They are connected to the skeleton blueprint using $\sqrt{2}$, $\sqrt{3}$, φ, π́, ₠, M, and a right angle. These ratios and angle are accurate (i.e. 100.0000%) by design, and were found programmatically by a cascading brute-force search, after measurements on paper showed the likely relationships. First is Cor Caroli.

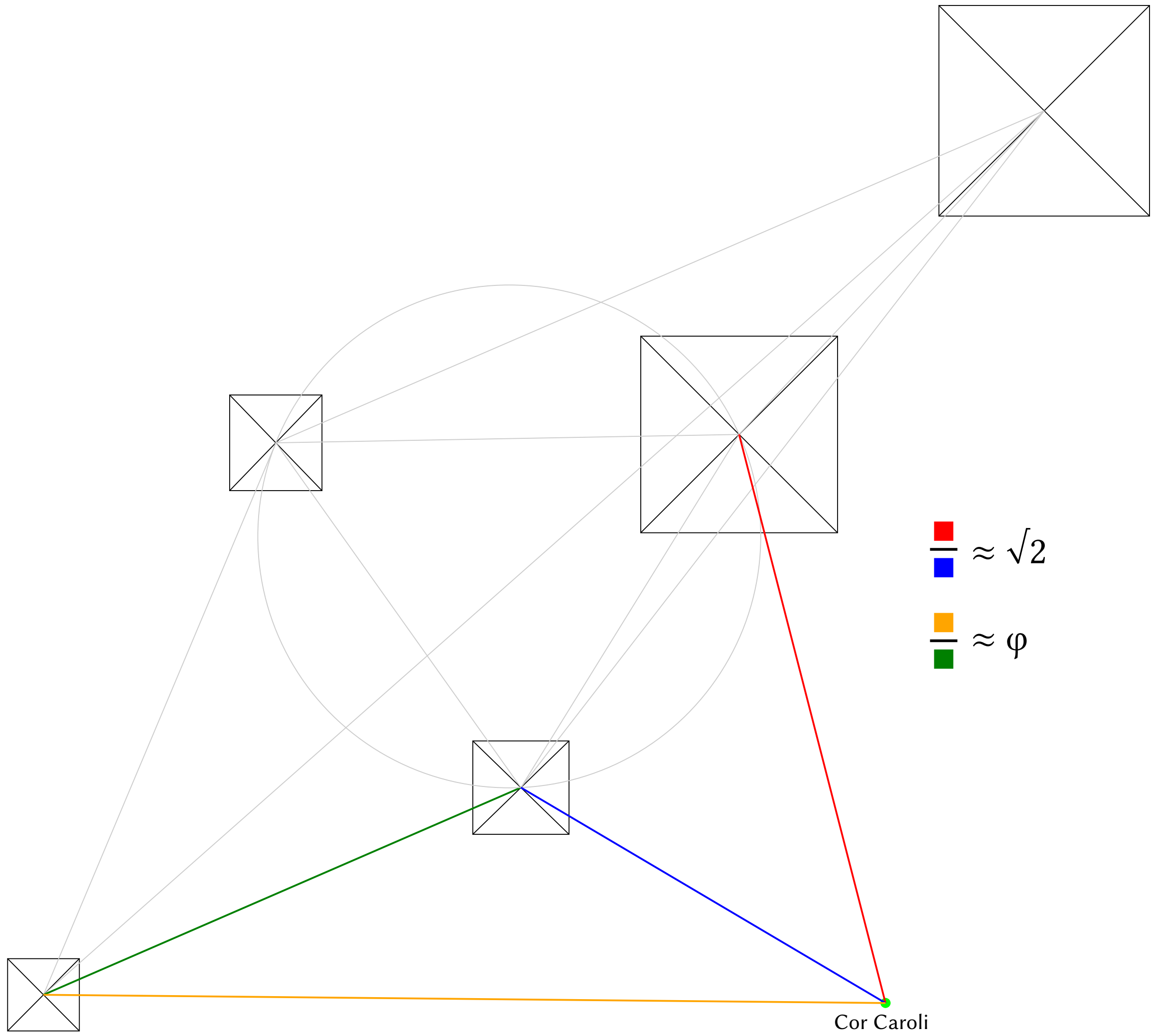


Figure 36: Calculating the position for Cor Caroli.

Kochab features a right angle alignment. I don't see anything interesting about the hypotenuse.

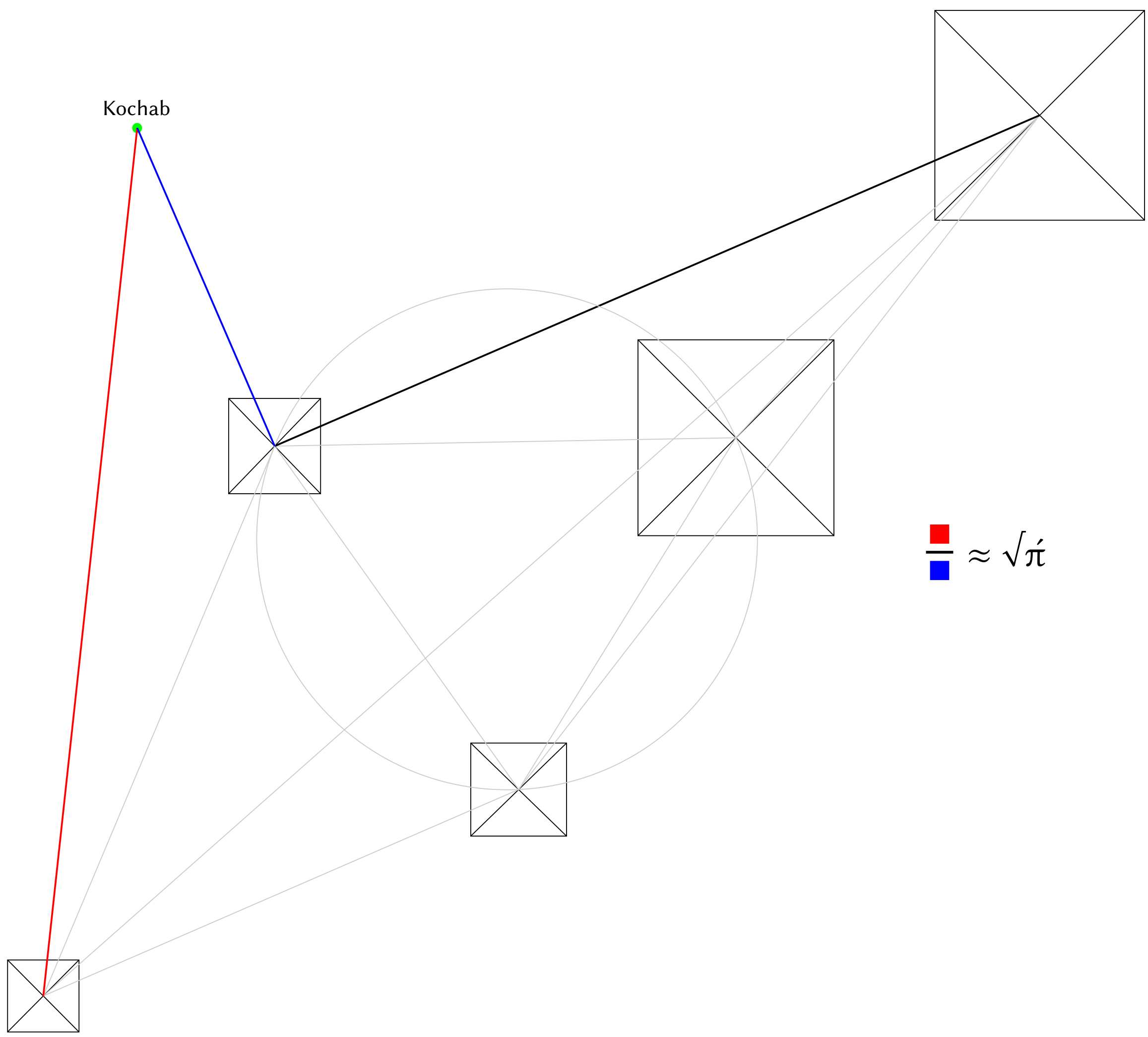


Figure 37: Calculating the position for Kochab.

Both Dubhe and Phecda incorporate $\mathfrak{G}$. I find Dubhe particularly interesting.

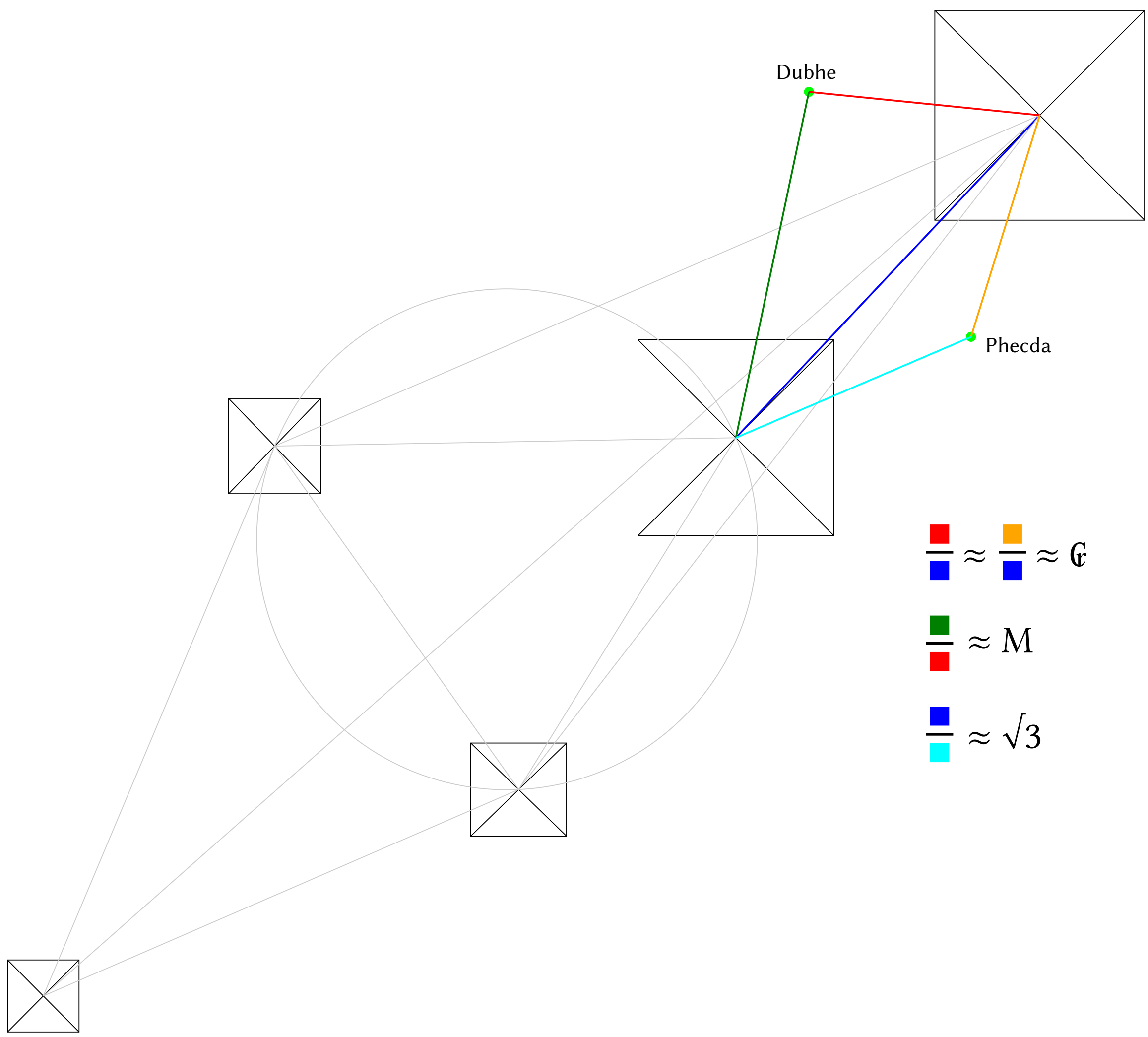


Figure 38: Calculating the positions for Dubhe and Phecda.

The reader is referred to the star map in Figure 1 to see how well these spots align.

Expanding the blueprint to full width

We can now add the left hand site of the plan. Adding more lines produces many more possibilities, I'll just show a few of the better ones.

We start with the roots of the first three primes:

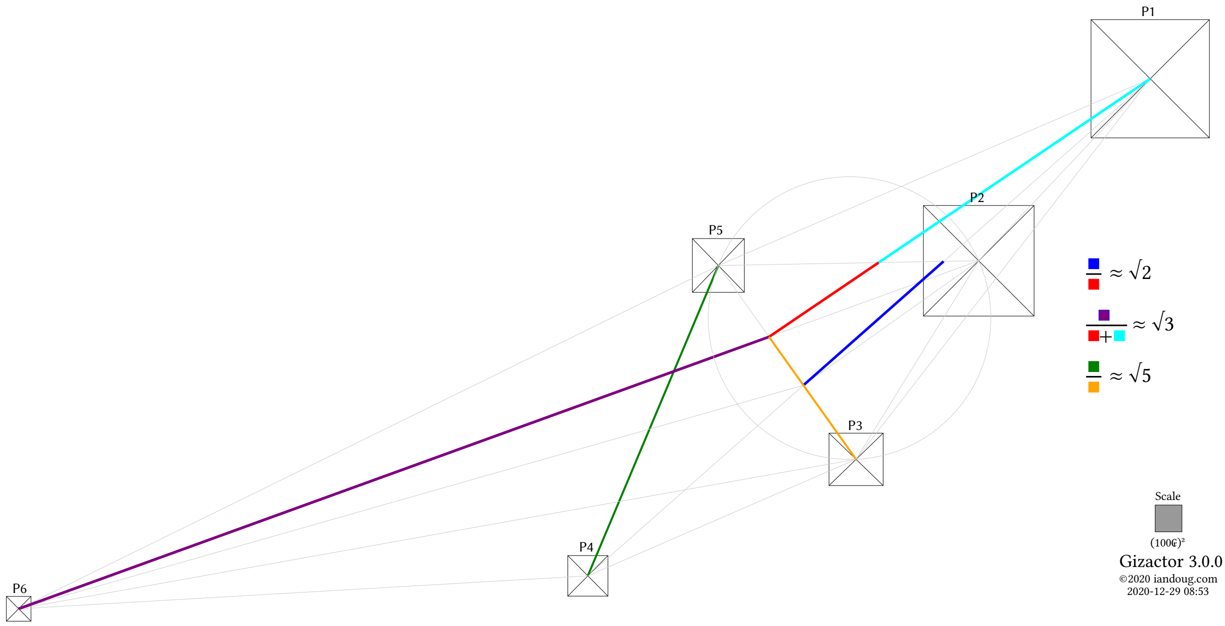


Figure 39: Square roots of the first three primes.

The discrepancies above are:
$\sqrt{2}$: 12cm/100m
$\sqrt{3}$: 8.5cm:100m
$\sqrt{5}$: 16.5cm:100m

The famous irrationals:

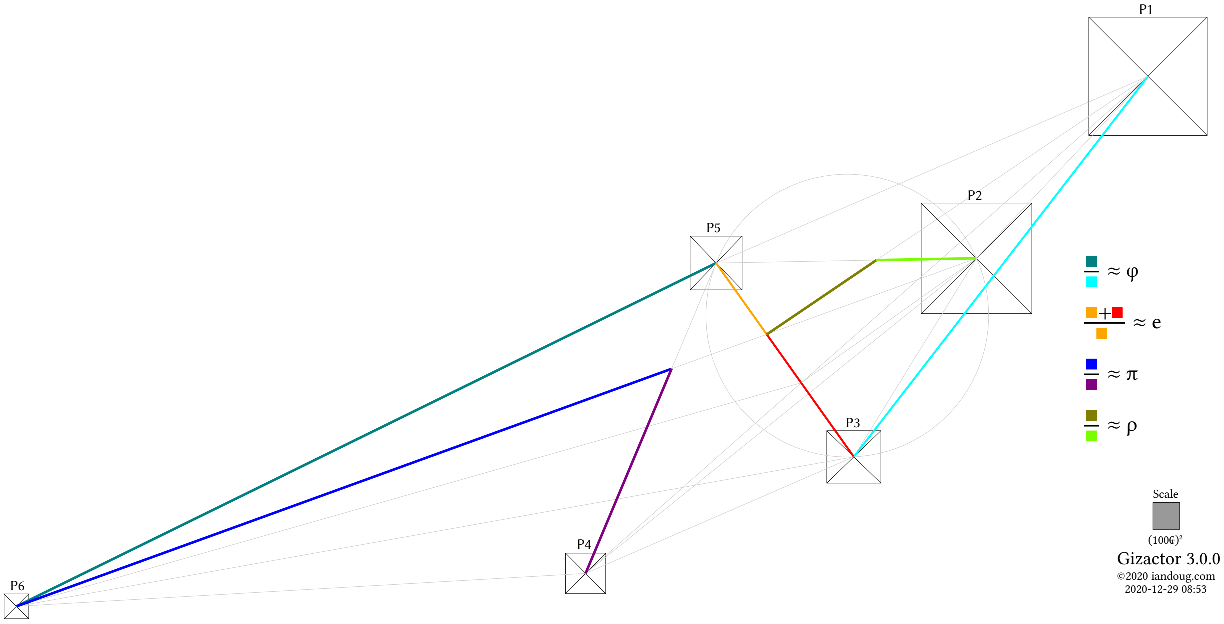


Figure 40: π, φ, e and ρ.

The discrepancies above are:
φ: 13.4cm/100m
e: 1.5cm:100m
π: 2cm:100m
ρ: 14cm:100m

If we compare this to Figure 28, then we see two pairs of “outer” lines in φ ratio with each other: P6P5/P3P1, and P5P1/P4P3.

Irrational roots:

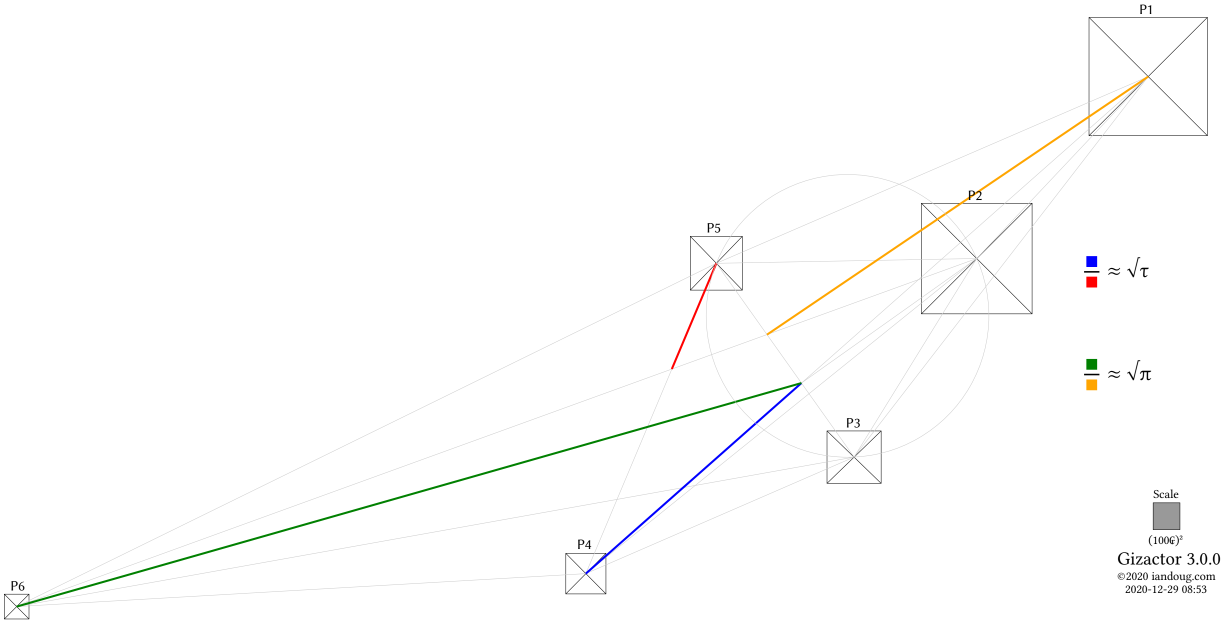


Figure 41: $\sqrt{\pi}$ and $\sqrt{\tau}$

The discrepancies above are:
$\sqrt{\pi}$: 0.33cm/100m
$\sqrt{\tau}$: 0.00cm:100m

We shall use both of these pairs again shortly.

8. Module 5: The big triangle

We consider the diagonal from P1 NE to P6 SW ... i.e. the diagonal of the outer bounding $2000\sqrt{5} \times 1000\sqrt{5}$ rectangle. Firstly, it goes right through the centre of P5, which stunned me when I checked (by calculation) the angle between P1 NE, P5 C and P6 SW ... 180°.

Figure 15 shown earlier shows how the position of P5 was calculated, using π and φ. There is nothing in that procedure that would end up on the diagonal. That was "curiosity 1."

My guides were also insisting that the location on the line was not random... It took a while with the calculator but eventually the secret surfaced ... $\sqrt{6}$. That was " curiosity 2." I take these curiosities as proof that I (with considerable help) have somehow managed to reverse-engineer Giza, which is the closest thing to a miracle I've experienced lately.

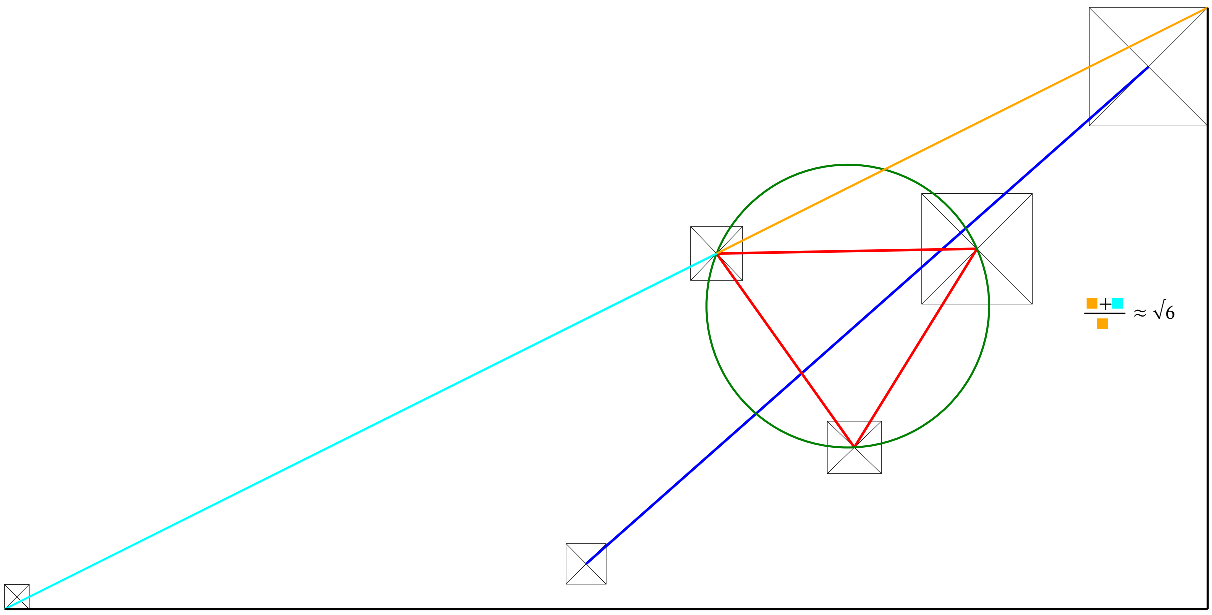


Figure 42: How P5 divides the diagonal.

The discrepancy is 1.6cm:100m.

$$\frac{5000}{\sqrt{6}} = 2041.241\, \text{₠} = 1068.794\, m$$

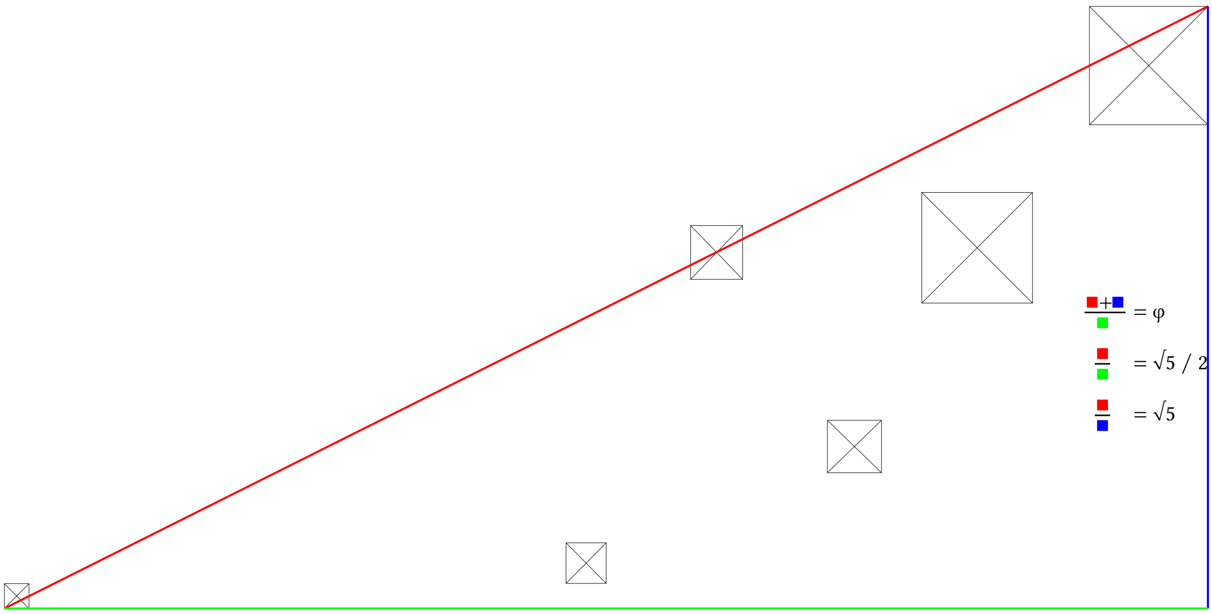
The distance from P5C to P1C is 1068.945m, an error of 15.1cm on a line 2618m long.

I don't know why √6 ... I have not seen it much at Giza. I felt like it is a hint to something else, but didn't know what. Indeed, my guides were suggesting that I draw vertical and horizontal lines at that point, but in truth I was so baffled by their achievement, that I didn't.

Surely anything more would be a bridge too far?

Oh ye of little faith...

The triangle is a $2 : 1 : \sqrt{5}$ triangle, which automatically gives us



π is also there, as will be shown shortly.

Drawing in the horizontal and vertical lines from P5 gives us:

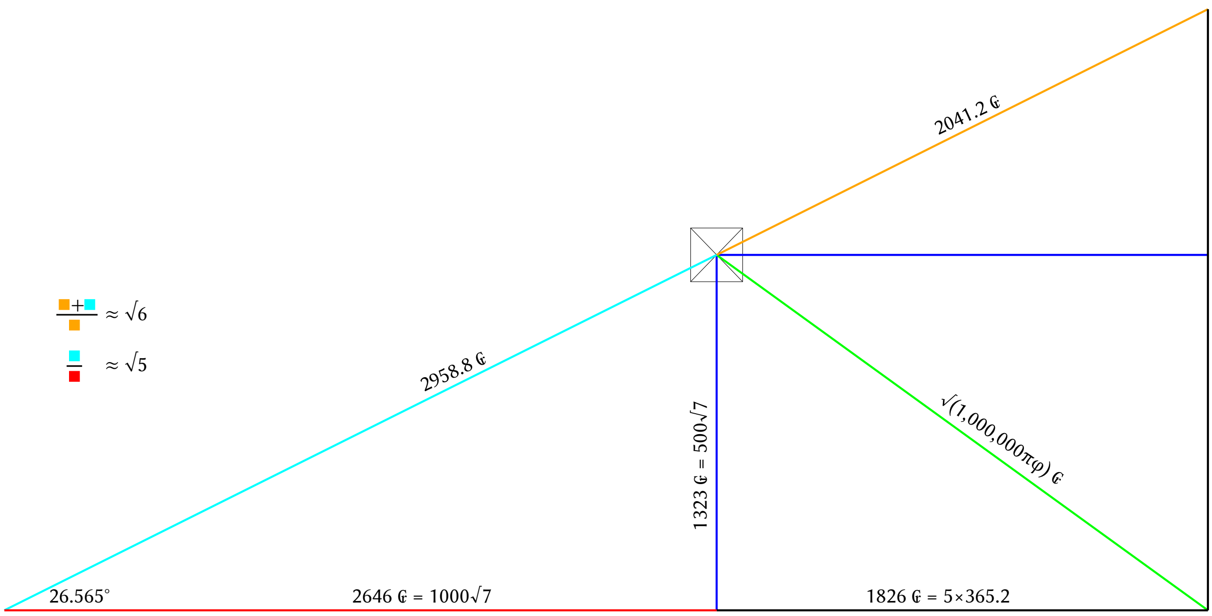


Figure 43: From √6 to √7, √5 and a year.

$$\frac{5000}{\sqrt{6}} = 2041.2$$

$$5000 - 2041.2 = 2958.8$$

$$\arctan\left(\frac{1000\sqrt{5}}{2000\sqrt{5}}\right) = \arctan\left(\frac{1}{2}\right) = 26.565°$$

$$2958.8 \times \cos(26.565°) = 2646(\mathit{rounded}) = 1000\sqrt{7}$$

$$(2 \times 2236) - 2646 = 1826$$

$$1826 = \frac{365.2 \times 10}{2}$$

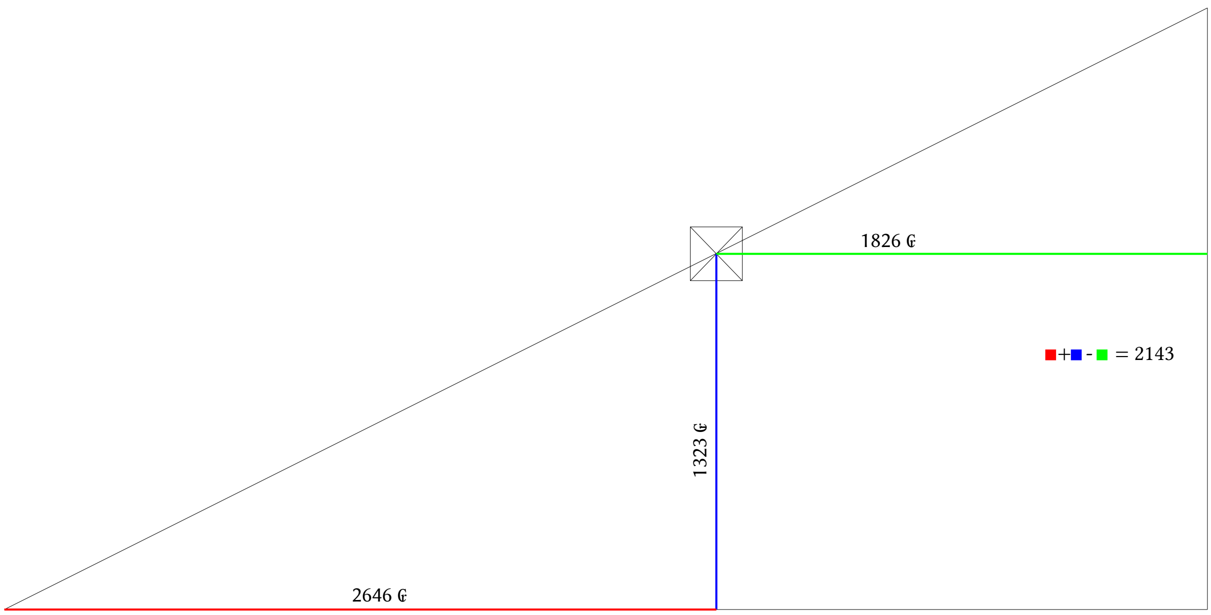
$$2958.8 \times \sin(26.565°) = 1323(\mathit{rounded}) = 500\sqrt{7}$$

$$1826^2 + 1323^2 = 5084605 \approx 1{,}000{,}000\,\pi\varphi$$

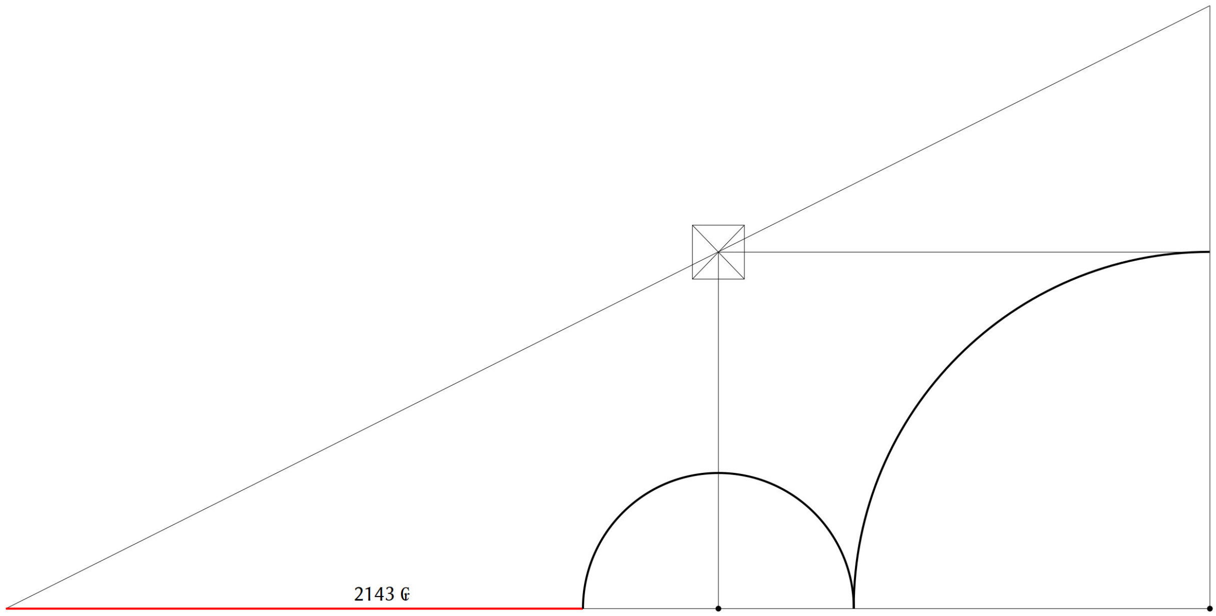
$$\frac{2958.8}{2646} = 1.118$$

$$1.118 \times 2 = 2.236 \approx \sqrt{5}$$

In Module 1 I showed that the number 2143 is a self-contained extremely accurate approximation for π, as $\sqrt[4]{\frac{2143}{22}}$. The triangle has that number, firstly algebraically:



Or effectively the same thing, geometrically:



The same size circles lead to the silver ratio (1+√2), and φ^{φ}.

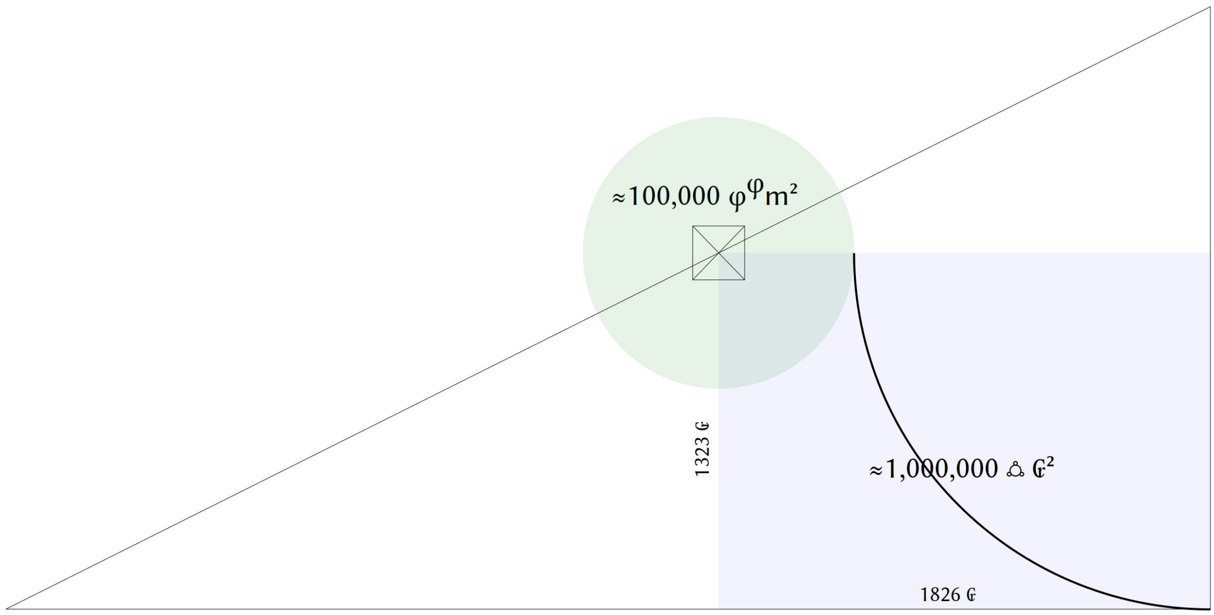


Figure 44: Silver ratio and φ^{φ}

The silver ratio:

$$1826 \times 1323 = 2415798$$

$$\frac{2415798}{1000000} = 2.415798$$

$$2.415798 - 1 = 1.415798$$

$$\frac{1.4142}{1.415798} \times \frac{100}{1} = 99.887\,\%$$

φ^{φ}:

$$1826 - 1323 = 503$$

$$503^2 \times 3.1416 = 794853$$

$$794853 \times 0.5236^2 = 217914.5 (convert \rightarrow m^2)$$

$$\frac{217914.5}{100000} = 2.179145$$

$$\varphi^{\varphi} = 1.618^{1.618} = 2.1783 \ (cf. e = 2.7183)$$

$$\frac{2.1783}{2.179145} \times \frac{100}{1} = 99.963\%$$

In truth, I stumbled onto these before finding φ and π. My guides were insisting "there was more" but I couldn't see it. Eventually φ dawned on me, and naturally I immediately wondered about π, but how can you hide π in a simple triangle? I could not believe it was there, but my guides insisted. I even had a rerun of the "Are you sure?" "Yes, and it's beautiful" conversation. So I tried to find 355/113 or similar, then thought of 2143, and realised that 2646 - 2143 = 503 ... which had just surfaced in the drawing above. Cue laughter and amazement ....

9. Module 6: Squaring the circle

"You can never know both the diameter and the circumference exactly, or the radius and area."
The Thoth Uncircularity Principle

The ancient Greeks put a lot of effort into the problem of "squaring the circle" ... how to construct a circle and a square with the same area, using only a compass and a straight-edge.

We now know that the construction is impossible because π is transcendental, but it is possible to get quite close... especially if you take a practical value for π with a fixed number of decimals, like 3.1416.

One such construction takes advantage of the fact that $6\varphi^2/5$ is very close to π, a fact that is also used for approximations of the ℭ, as $\pi/6 \approx \varphi^2/5$.

Robert Dixon published such a construction in 1991.

Here is the Wikipedia illustration [10] of the technique, which also shows the method in the text.

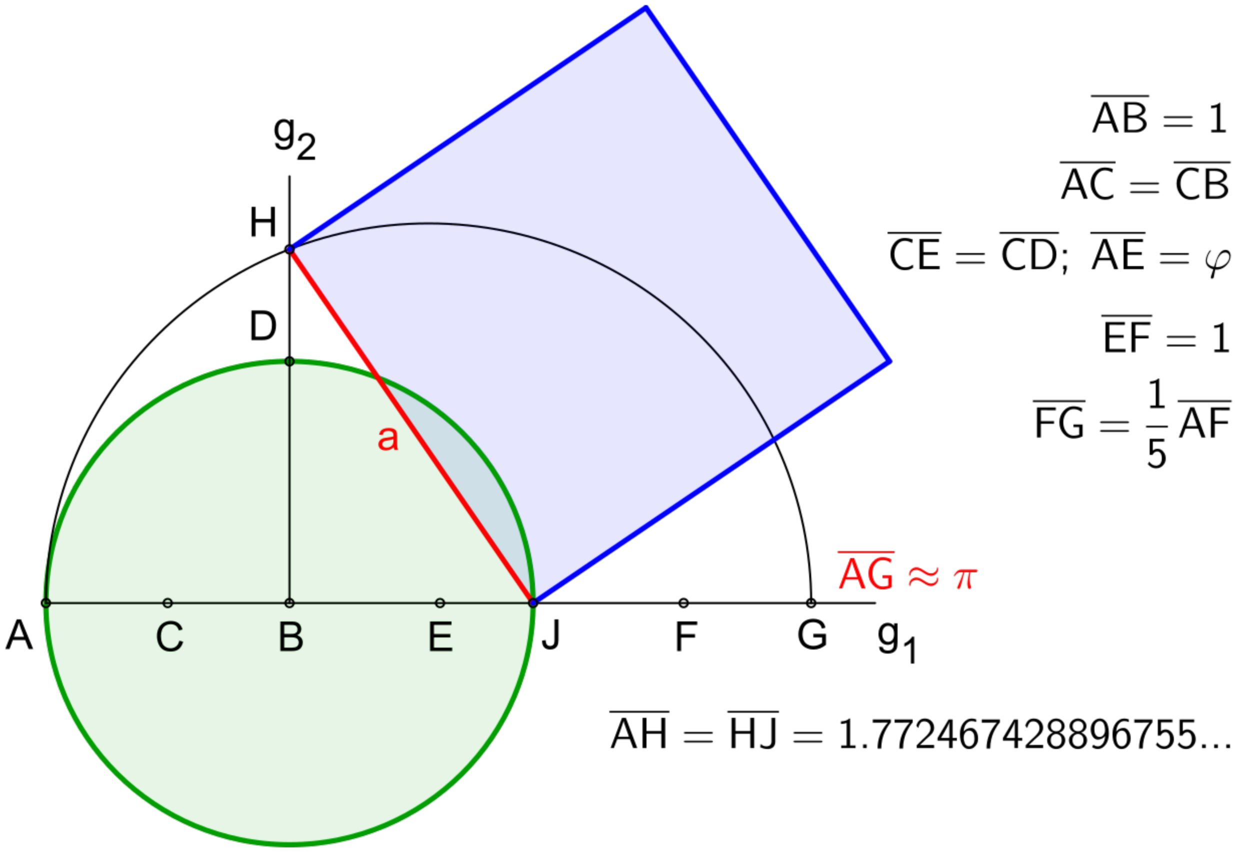


Figure 45: Squaring the circle, Robert Dixon "Golden Ratio" method.

Point B divides the line AE in the golden ratio.

Now it just so happens that the skeleton blueprint features two lines divided in the golden ratio. We can map the above construction to the Giza blueprint, as follows. I've only included the necessary letter markers. Point G was calculated by scaling the φ distance to π, rather than by the method shown above. The centre of the outer circle is then AG/2, which is just left of point E.

The green dot is the centre of the green circle, and likewise for the orange.

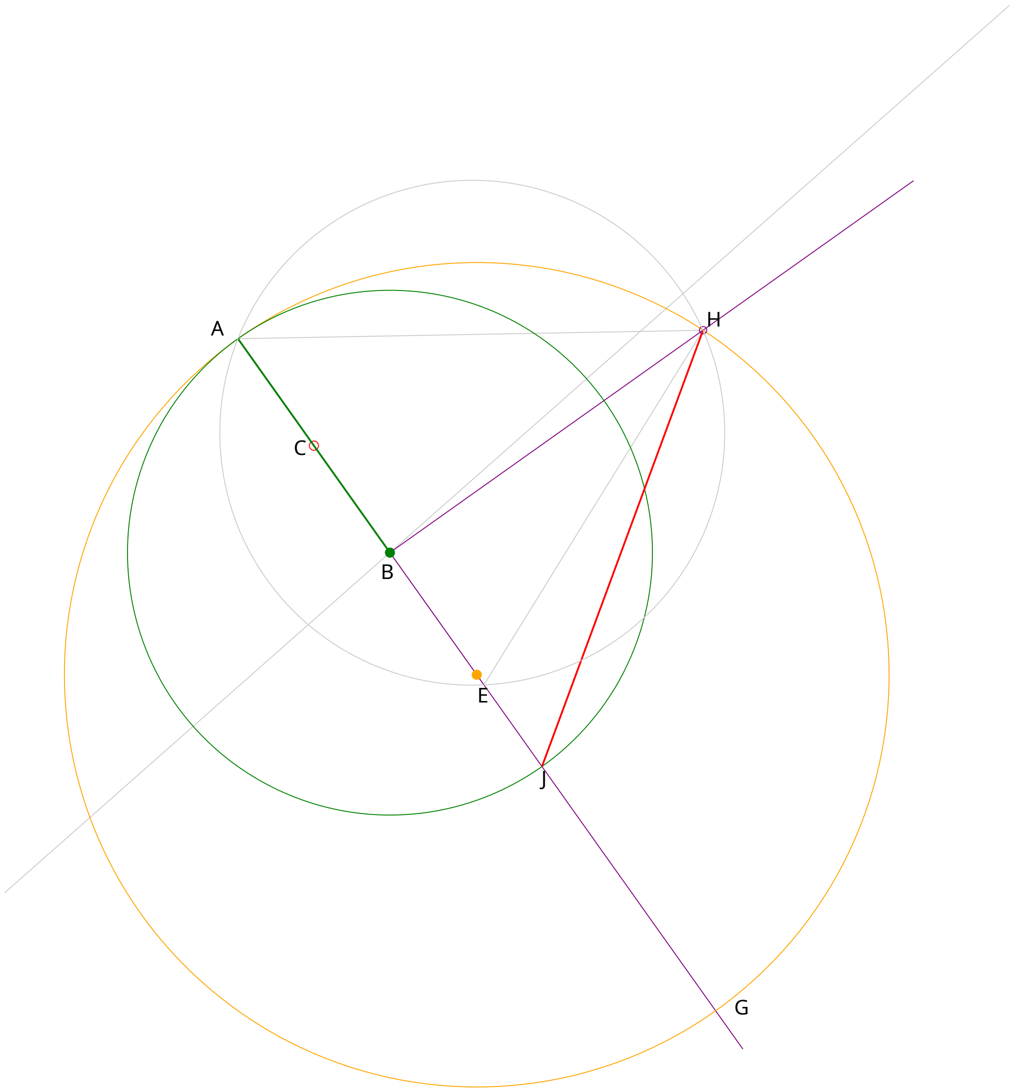


Figure 46: Dixon's method transposed to the skeleton blueprint.

The line BH is drawn vertically from point B, which cuts AE at the golden ratio point.

Triangle AHJ is isosceles, with AH and HJ equal, so we can draw in the square on AH instead of HJ. I've dropped the unnecessary constructions and added the pyramids back.

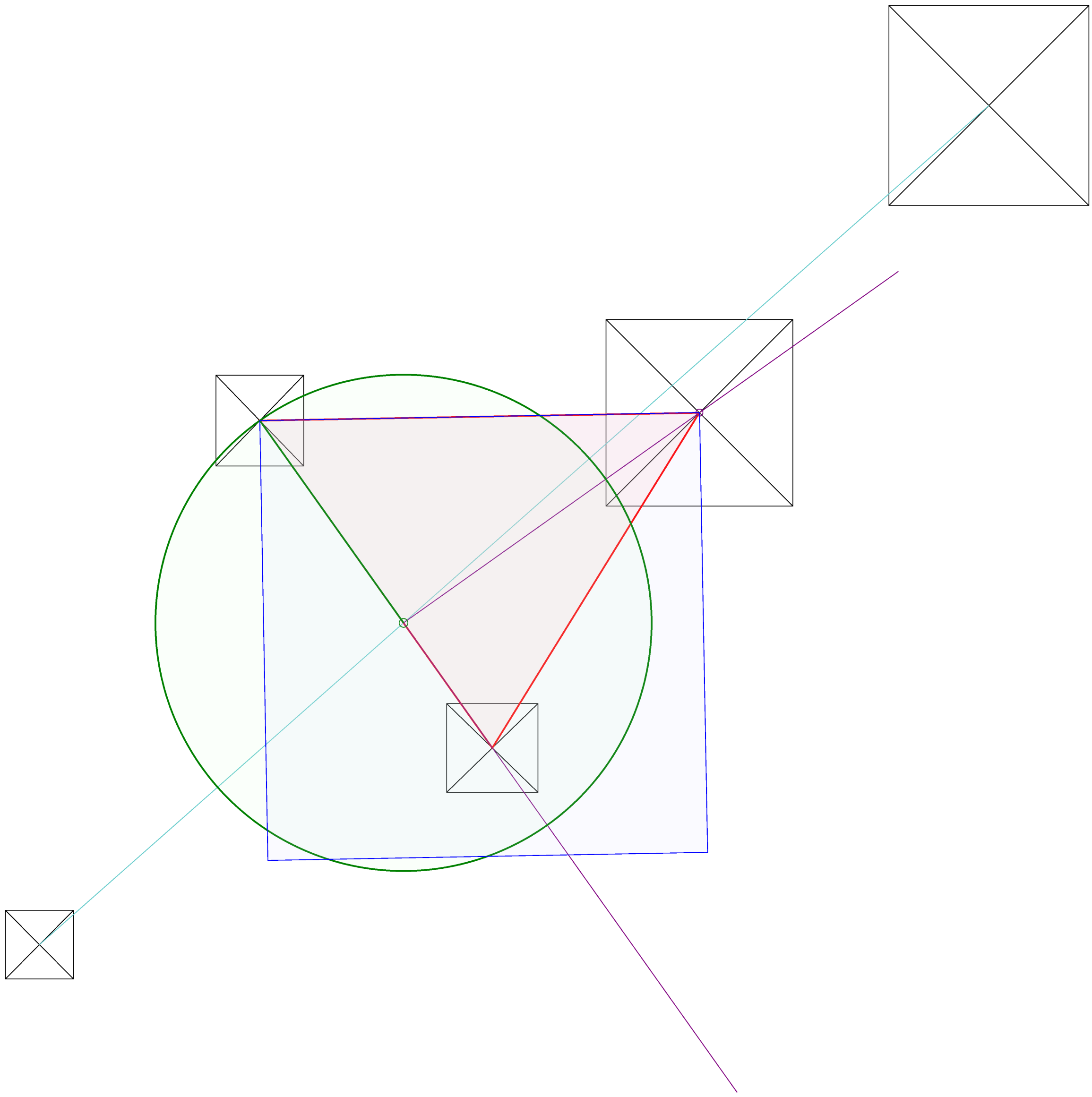


Figure 47: Squaring the circle, Giza style.

Squaring the circle, Giza style.

A thing of beauty and a joy forever, and you can't make it up.

Area of circle: 257034.2997m²
Area of square: 257034.3088m²
Calculated $\sqrt{\pi}$ 1.77245595 versus $\sqrt{3.1416} = 1.77245592$.

The difference between point H and P2 centre is 0.15767m.

The difference between point B and the split between P5 and P3 is 0.03290m.

So the average error for these two points is 0.0953m, or less than four inches. Quite remarkable.

As per usual, the designers suspected people may think it was random luck. The design has a few other line pairs that produce close values for $\sqrt{\pi}$. The most interesting is the pair involving the two long diagonals, where the radius and square side are lines to where the opposite diagonal cuts the line from P5 to P3. See the black lines in Figure 48.

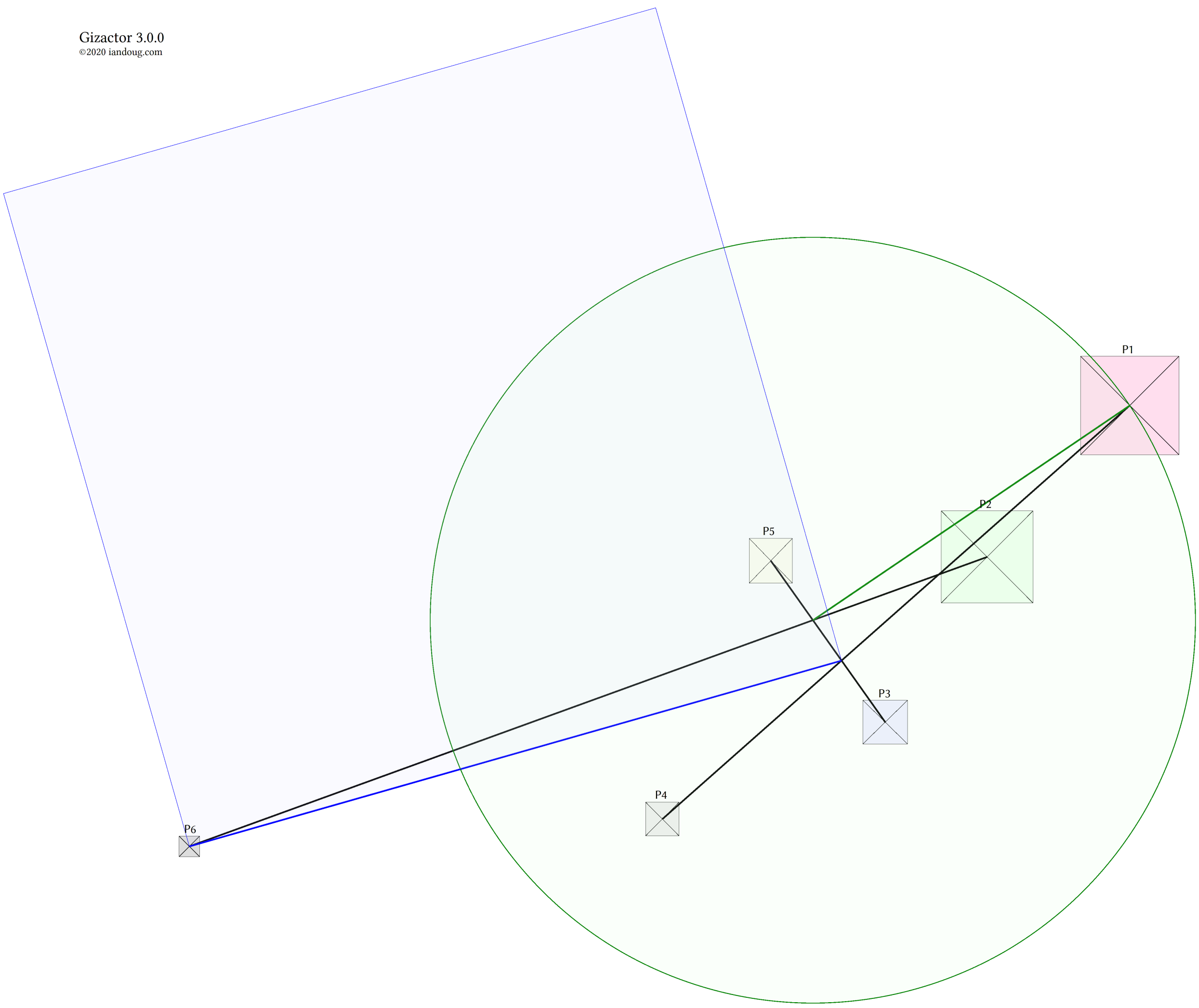


Figure 48: Squaring the circle, Texas style, where everything is bigger.

The accuracy for $\sqrt{\pi}$ is 99.99774556%, but given the large areas involved, that small error gets multiplied.

The area of the circle calculates out at 2,521,938.218m², while the square calculates out at 2,521,824.508m², giving an accuracy of 99.99549%.

The side of the square is 1,588m. This is a large area. Curiously, 1000 × 3.1416 / 2 = 1,570.8.

That brings us to this diagram, in which the small square and triangle are exactly equal, and the large square and circle as above are almost equal.

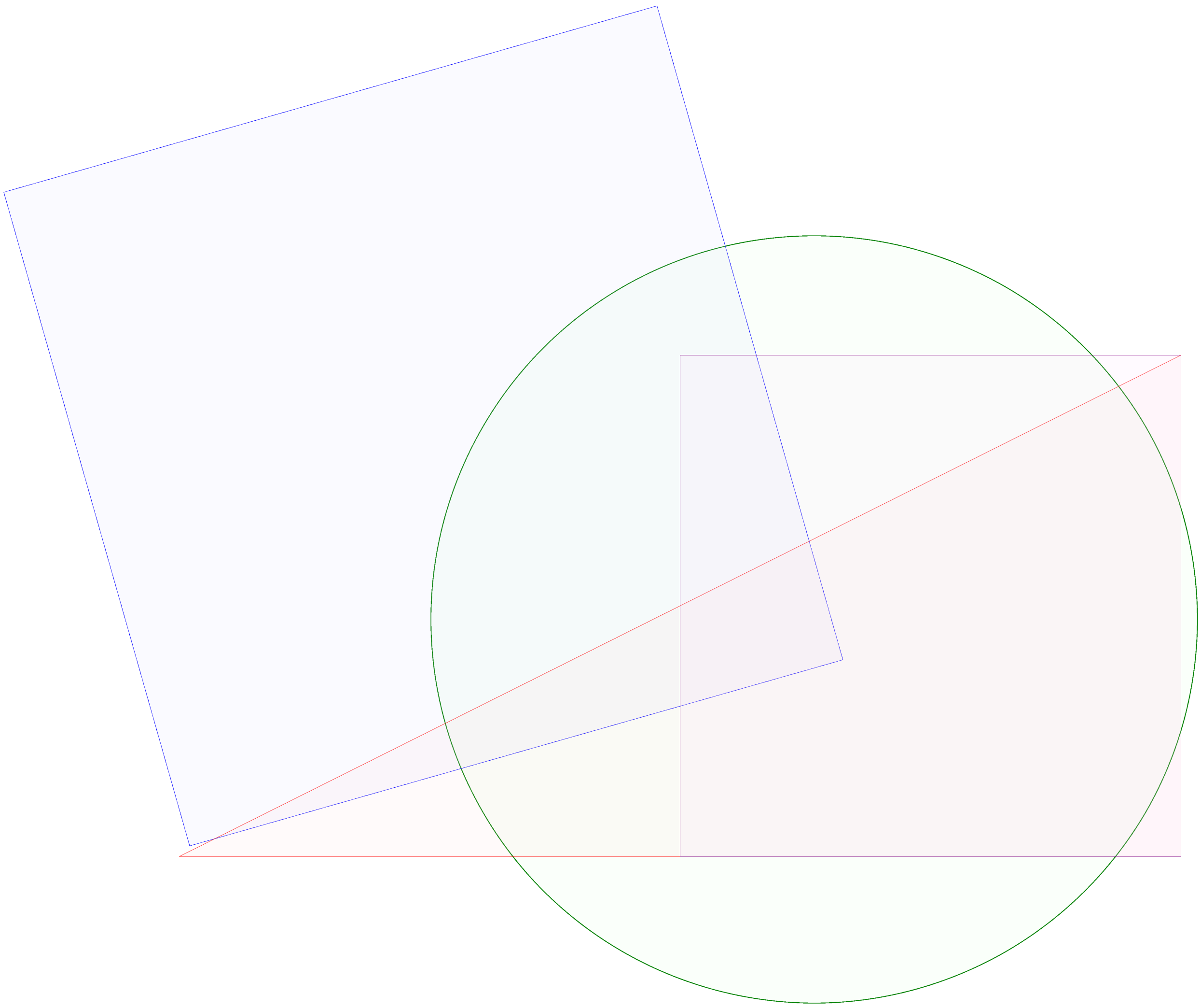


Figure 49: The search for equality.

The ratio of this large square (Figure 49) to that from the first square-circle diagram (Figure 47), is

$$\frac{2521824.508}{257034.3088} = 9.8112$$

That is annoyingly close to the value for g, defined as 9.80665 m/s^{2}. It's also annoying because I have been looking for that ever since becoming aware of the references to c at Giza. I wasn't expecting it to suddenly pop up here as I was wrapping this up, after doing a random comparison because my guides insisted. Instead of laughter somewhere off in the distance, I just got a sense of *Voila!,* like a magician pulling a rabbit out of the hat.

Figure 41 showed a very accurate $\sqrt{\tau}$ ratio, which allows us to construct a square double the size of a circle.

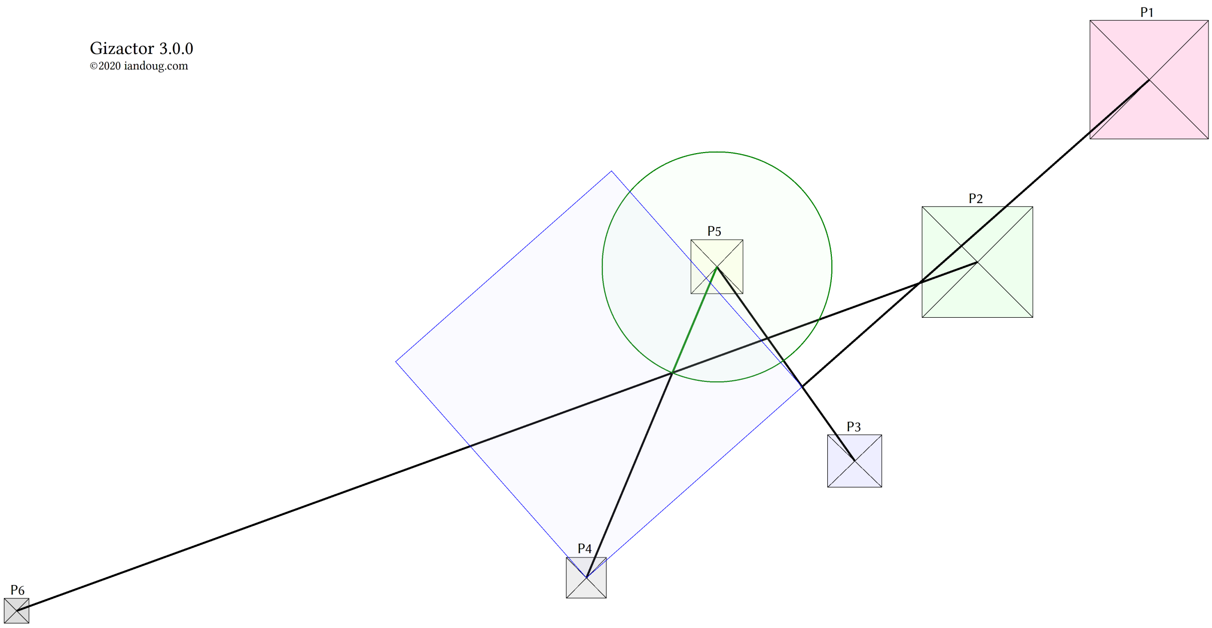


Figure 50: Doubling the area, circle to square.

Area of circle: 156,800.2497m²
Area of square: 313598.1611m²

Ratio of square to circle is 1.999985 : 1.
Doubling accuracy: 99.99925%

Compare the points used for the radius and square side to those for the large square and circle in Figure 48. Both constructions are based on intersections on the two long diagonals.

10. Conclusion

After my previous efforts (Diskerfery and 55.5k) I though I was done with Giza, but Giza was not done with me.

The whole process started off something like this, with my guides challenging me... “You’ve solved the square and triangles, now what about the circles?”

I had no idea what they were talking about, but I tried various things, getting nowhere fast. So I asked for clarification, and they replied, “The circles of heaven.”. Clear as mud.

After more “going around in circles” I started to focus on the area of the north pole, and slowly things began to fall in place. So we had Thuban’s orbit as one circle, and the circle around the triangle as the second. I tried the inscribed circle but that didn’t lead anywhere.

Then I stumbled across the squaring of the circle. I began to suspect that perhaps the phrase “circle of heaven” may be how they referred to the circle with the same area as a given square... a somewhat Chinese turn of phrase. Indeed, the paper by Rachel Fletcher on this topic is entitled, “Squaring the Circle: Marriage of Heaven and Earth.”[11]

Eventually I saw how Dixon’s method was already “designed in” at Giza. The circles of heaven are thus resolved.

Giza was clearly laid out mathematically, using a general design modelled on the stars in and around the Big Dipper (Ursa Major), around 55,500 BCE.

I realise this is a problematic date, but perhaps our history timeline needs an even bigger rethink than what recent discoveries in Turkey imply.

Who built it, and why, I don’t know. I get the sense that the final design evolved as a product of their mathematics, rather than as a designer sitting down one day to draw up a plan. It may be that they noticed interesting patterns in the stars, and one thing led to another.

They must have had a very good reason for embarking on such a massive and expensive project, using it to record both mathematical knowledge and a date.

I understand that my views will remain fringe until such time as tangible physical evidence surfaces, at which time I will be vindicated. If the people working at Giza can check out the locations of P4, P5 and P6, then we can perhaps prove or disprove the design. From my point of view, there is too much correlation between the stars and the mathematics for it to be random.

I certainly am not talented enough to have created the design, with its interlocking relationships, out of thin air. I just followed the mathematics and the logic wherever it led.

My argument is:

1. Giza’s design shows mathematical knowledge that, as far as we know, the 4th dynasty and later did not have.
2. That means the 4th dynasty did not build Giza.

3. We have no evidence for other contemporary peoples having this knowledge either.
4. That means Giza was built before… in the First Times (Zep Tepi), as the Egyptians said.
5. Since we have no evidence of anyone on this side of the Younger Dryas being capable, we need to look on the other side of the Younger Dryas.
6. It then becomes a question of how far back to look… and the only clue we have for that is the stellar alignment, which is 55.5k BCE. Perhaps there is a closer or better one with a more believable date, but I was guided to 55.5k BCE …
7. These people would have lived in cities, not caves. Their mathematical, stone-working, wealth and organisational skills imply a mature culture, not hunter-gatherers or subsistence farmers. Doing that level of mathematics presupposes written language, which presupposes writing materials, which presupposes a certain level of development.

We just need to find the evidence…

11. Acknowledgements

Thanks as always to my “guides,” whoever or whatever they are, for their constant prompting and odd ideas, which frequently lead to shocking discoveries and amazed laughter. I wish I knew them.

Thanks also to Patrick Chevalley, author of Skychart [12] and the team behind the Libertinus fonts [13].

12. Index of numbered figures

13. Index of Tables

14. Bibliography

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15. Gizactor

The current version of the Gizactor is now available separately: https://doi.org/10.5281/zenodo.5856237