Double Domination Number of Some Families of Graph

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Abstract: In a graph G = (V, E) each vertex is said to dominate every vertex in its closed neighborhood. In a graph G, if S is a subset of V then S is a double dominating set of G if every vertex in V is dominated by at least two vertices in S. The smallest cardinality of a double dominating set is called the double domination number γ_{x2} (G). [4]. In this paper, we computed some relations between double domination number, domination number, number of vertices (n) and maximum degree (Δ) of Helm graph, Friendship graph, Ladder graph, Circular Ladder graph, Barbell graph, Gear graph and Firecracker graph.

Keywords: dominating set, domination number, double dominating set, double domination number. We denote n, Δ respectively by number of vertices, maximum degree of a graph G.

I. INTRODUCTION

Frank Harary and T. W. Haynes defined and studied the concept of double domination in graphs. Domination and double domination numbers are defined only for graphs with non- isolated vertices. Every graph with no-isolated vertex has a double dominating set. In a graph G= (V, E), a subset S ⊆ V is a dominating set of G if every vertex v of V−S has a neighbor in S.The domination number γ (G) is the minimum size of a dominating set of vertices in G. In a graph G with vertex set V and edge set E, a subset S of V is a double dominating set of G if every vertex v ∈ V, is in V−S and has at least two neighbors in S. The smallest cardinality of a double dominating set of G is known as the double domination number γ_{x2}. [6] [4] [8].

II. DEFINITIONS

2.1. Dominating set: A non-empty set D⊆ V of a graph G is a "dominating set" of G if every vertex in V-D is adjacent to some vertex in D. [1]

2.2. Domination number: The "domination number" γ(G) is the number of vertices in a smallest dominating set for G. [7]

2.3. Double dominating set: A subset S ⊆ V (G) is a "double dominating set" for G if S dominates every vertex of G at least twice. [4]

2.4. Double domination number: The "double domination number" γ_{x2} (G) is the smallest cardinality of a double dominating set of G. [4]
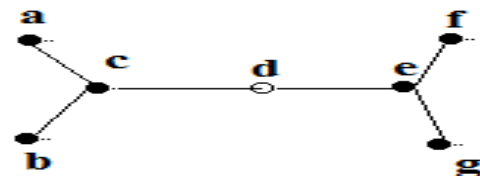
Example:

Fig.1

Now consider the graph Fig.1. The set S = {a, b, c, e, f, g} forms a double dominating set of G and double domination number γ_{x2} = 6.

III. DEFINITIONS OF SOME FAMILES OF GRAPHS

3.1. "Helm Graph": It is designed from a wheel graph with n vertices by adjoining a pendant edge at each node of the cycle and denoted by H_n . [8]

3.2. "Friendship Graph": It is constructed by joining n copies of the cycle graph C_3 with a common vertex and represented by F_n. It is a planar undirected graph with 2n+1 vertices and 3n edges. [7]

3.3. Ladder Graph: The "Ladder graph" L_n can be obtained as the Cartesian product of two path graphs P_2 and P_n ,where P_n is a path graph. The Ladder graph L_n is a planar undirected graph with 2n vertices and 3n−2 edges. [7]

3.4. "Circular Ladder graph" CL_n: It is a connected planar and Hamiltonian graph which is constructible by connecting the four 2-degree vertices in a straight way or by the Cartesian product of a cycle of length n ≥ 3 and an edge. In symbols, CL_n=C_n x P_2 . It has 2n nodes and 3n edges. They are the polyhedral graphs of prisms, so they are more commonly called prism graphs. [7]

3.5. "n-Barbell graph" B_n: It is obtained by connecting two copies of a complete graph K_n by a cut edge. It has 2n vertices and 2n+1 edges. [8]

3.6. "Gear graph" G_n: It is a wheel graph W_n with a vertex added between every pair of adjacency vertices of the n-cycle. It has 2n+1 vertices and 3n edges. [3]

3.7. "Firecracker Graph" F(n,k) : It is framed by the concatenation of nk-stars by linking one leaf from each. It has order nk and size nk−1. [2]

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IV RESULTS ON DOUBLE DOMINATION OF SOME FAMILES OF GRAPHS

Theorem 4.1:

For any Helm graph H_n $(n \geq 3)$, double domination number

$$\gamma_{\times 2}(H_n) = \begin{cases} \Delta , & \text{if } n = 3 \\ \Delta + 1 , & \text{if } n \geq 4 \end{cases}$$

Proof:

Case (i) for n=3,

The Helm graph H_n has a single universal vertex v connecting all the vertices of the cycle with vertex set $\{x_1, x_2, x_3\}$ and each node of the cycle adjoining a pendant edge. Let y_1, y_2, y_3 be the pendant vertices. Here vertex v and pendant vertices are enough to dominate all the vertices of graph H_n twice. Here, $\Delta = n+1$. Let us consider the double dominating set of $H_n = \{v, y_1, y_2, y_3\}$. Thus double domination number $\gamma_{\times 2}(H_n)$ is $\{v, y_1, y_2, y_3\}$. Hence $\gamma_{\times 2}(H_n) = \Delta$.

Case (ii) for $n \geq 4$,

The Helm graph H_n has a universal vertex v connecting the vertices of the cycle having vertex set $\{x_1, x_2, x_3, \ldots\ldots, x_n\}$ and each node of the cycle adjoining a pendant edge. Let $\{y_1, y_2, y_3, \ldots\ldots, y_n\}$ be the pendant vertices. Here vertex v and pendant vertices will dominate the remaining vertices of graph H_n twice. Here $\Delta = n$. Let us consider the double dominating set of $H_n = \{v, y_1, y_2, y_3, \ldots\ldots, y_n\}$. Thus double domination number is $\gamma_{\times 2}(H_n) = \{v, y_1, y_2, y_3, \ldots\ldots, y_n\}$.

Hence $\gamma_{\times 2}(H_n) = \Delta + 1$.

Theorem 4.2:

For any Friendship graph F_n $(n \geq 2)$, double domination number $\gamma_{\times 2}(F_n) = 1 + \frac{\Delta}{2}$.

Proof: The Friendship graph F_n has $2n+1$ vertices and 3n edges. Let the vertex set be $\{v, v_1, v_2, v_3, \ldots\ldots, v_{2n}\}$ and $\Delta = 2n \Longrightarrow n = \frac{\Delta}{2}$. Here vertex v and n vertices dominates all the vertices of the graph F_n twice. Let us consider the double dominating set of $F_n = \{v, v_1, v_3, v_5, \ldots\ldots, v_n\}$. Thus double domination number is

$\gamma_{\times 2}(F_n) = \{v, v_1, v_3, v_5, \ldots\ldots, v_n\}$. Hence $\gamma_{\times 2}(F_n) = 1 + \frac{\Delta}{2}$.

Theorem 4.3:

For any Ladder graph L_n $(n \geq 2)$, double domination number $\gamma_{\times 2}(L_n) = n$.

Proof: The Ladder graph L_n has 2n nodes and $(3n-2)$ edges. Let the vertex set be $\{x_1, x_2, x_3, \ldots\ldots, x_n, y_1, y_2, y_3, \ldots\ldots, y_n\}$ and the edge set be $\{x_i y_i, x_i x_{i+1}, y_i y_{i+1}\}$ for $1 \leq i \leq n$. Let us consider the double dominating set of $L_n = \{x_1, x_3, x_5, \ldots\ldots, x_{n-1}, y_2, y_4, y_6, \ldots\ldots, y_n\}$. Thus double domination number is

$\gamma_{\times 2}(L_n) = \{x_1, x_3, x_5, \ldots\ldots x_{n-1}, y_2, y_4, y_6, \ldots\ldots, y_n\}$.

Hence $\gamma_{\times 2}(L_n) = \frac{n}{2} + \frac{n}{2} = n$.

Theorem 4.4:

For any Circular Ladder graph CL_n $(n \geq 3)$, double domination number $\gamma_{\times 2}(CL_n) = n$.

Proof: Circular Ladder graph CL_n has 2n nodes and 3n edges with vertex set $\{v_1, v_2, v_3, \ldots\ldots, v_{2n}\}$. Here n number of nodes is enough to dominate all the vertices of the graph CL_n twice. Let us consider the double dominating set of $CL_n = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$. Thus double domination number is $\gamma_{x2}(CL_n) = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$. Hence $\gamma_{\times 2}(CL_n) = n$.

Theorem 4.5:

For any Barbell graph B_n $(n \geq 3)$, double domination number $\gamma_{\times 2}(B_n) = 4$.

Proof : The Barbell graph B_n has 2n vertices and $2n+1$ edges with vertex set $\{x_1, x_2, x_3, \ldots\ldots, x_n, y_1, y_2, y_3, \ldots\ldots, y_n\}$. Since B_n is obtained by connecting two copies of complete graph. So, any two vertices say x_1 and x_2 dominates all the vertices of single copy of complete graph twice and since it has two copies. The double dominating set of $B_n = \{x_1, x_2, y_1, y_2\}$. Thus double domination number is $\gamma_{\times 2}(B_n) = \{x_1, x_2, y_1, y_2\}$.

Hence $\gamma_{\times 2}(B_n) = 4$.

Theorem 4.6:

For any Gear graph G_n $(n \geq 3)$, double domination number $\gamma_{\times 2}(G_n) = \Delta$.

Proof : The Gear graph has $2n+1$ vertices with vertex set $\{v, v_1, v_2, v_3, \ldots\ldots, v_n, u_1, u_2, u_3, \ldots\ldots, u_n\}$ and 3n edges with $\Delta = n$. As G_n is constructed from the wheel graph W_n by adding a vertex between every pair of adjacent vertices of the n-cycle. Here the vertices of the cycle $\{v_1, v_2, v_3, \ldots\ldots, v_n\}$ dominates all the vertices of G_n twice. Let us consider the double dominating set of $G_n = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$. Thus double domination number is $\gamma_{\times 2}(G_n) = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$. Hence $\gamma_{\times 2}(G_n) = \Delta$.

Theorem 4.7:

For any Firecracker graph $F(n, k)$ $(n = 2, k \geq 3)$, double domination number $\gamma_{\times 2}[F(n,k)] = n\Delta$.

Proof: The Firecracker graph $F(n, k)$ $(n = 2, k \geq 3)$, has nk vertices and $nk-1$ edges. The graph $F(n, k)$ consists of $(k-2)$ pendant vertices in each star. Let the vertex set of $F(n, k)$ be $\{v_1, v_2, v_3, \ldots\ldots, v_k, u_1, u_2, u_3, \ldots\ldots, u_k\}$ and $\Delta = (k-1)$. Here all pendant vertices and one of the remaining vertex dominates all the vertices of the graph $F(n, k)$ twice, for single star. Since the graph $F(n, k)$ has two stars, let us consider the double dominating set of $F(n, k) = \{v_1, v_2, v_3, \ldots\ldots, v_{k-2}, v_{k-1}, u_1, u_2, u_3, \ldots\ldots, u_{k-2}, u_{k-1}\}$. Thus double domination number is

$\gamma_{\times 2}[F(n, k)] = \{v_1, v_2, v_3, \ldots\ldots, v_{k-2}, v_{k-1}, u_1, u_2, u_3, \ldots\ldots, u_{k-2}, u_{k-1}\}$.

Hence $\gamma_{\times 2}[F(n, k)] = 2(k-1) = n\Delta$.

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Theorem 4.8:

For any Firecracker graph F(n,k) with odd values of n (n ≥ 3, k ≥ 4), double domination number

$$\gamma_{x2}\,[F(n,k)] = \left\lceil nk - \left(\tfrac{3n-1}{2}\right)\right\rceil.$$

Proof: The Firecracker graph F(n,k) (n ≥ 3, k ≥ 4), has nk vertices and nk−1 edges. The graph F(n,k) consists of (k−2) pendant vertices in each star. Let the vertex set of F(n,k) be $\{u_1,u_2,u_3,\ldots,u_k, \quad v_1,v_2,v_3,\ldots,v_k, \quad w_1,w_2,w_3,\ldots, w_k,x_1,x_2,x_3,\ldots,x_k,y_1,y_2,y_3,\ldots,y_k,\ldots,k_1,k_2,k_3,\ldots \ldots,k_{nk}\}$. Here, all pendant vertices and some of the other remaining vertices will dominate all the vertices of the graph F(n,k) twice. Let us consider double dominating set of F(n,k) as $\{u_1, u_2, u_3,\ldots,u_{k-2}, u_{k-1}, v_1, v_2, v_3,\ldots,v_{k-2}, w_1, w_2, w_3,\ldots,w_{k-2}, w_{k-1}, x_1, x_2, x_3,\ldots,x_{k-2},y_1,y_2,y_3,\ldots,y_{k-2},y_{k-1},\ldots,k_1,k_2,k_3,\ldots, k_{nk-2}, k_{nk-1}\}$. The double domination number is $\gamma_{x2}[F(n,k)] = \{u_1,u_2,u_3,\ldots, u_{k-2}, u_{k-1}, v_1, v_2, v_3,\ldots,v_{k-2},w_1,w_2,w_3,\ldots,w_{k-2},w_{k-1},x_1,x_2,x_3,\ldots,x_{k-2},y_1,y_2,y_3,\ldots \ldots,y_{k-2}, y_{k-1},\ldots,k_1, k_2, k_3,\ldots,k_{nk-2}, k_{nk-1}\}$.Thus we have

$$\gamma_{x2}[F(n,k)] = \{nk-4,\,nk-7,\,nk-10,\,nk-13,\,\ldots\ldots\,\}.$$

Hence, $\gamma_{x2}[F (n,k)] = \left\lceil nk - \left(\tfrac{3n-1}{2}\right)\right\rceil$.

Theorem 4.9:

For any Firecracker graph F(n,k) with even values of n (n ≥ 4,k ≥ 4), double domination number

$$\gamma_{x2}\,[F(n,k)] = \left\lceil nk - \left(\tfrac{3n-2}{2}\right)\right\rceil$$

Proof: The Firecracker graph F(n,k) (n ≥ 4,k ≥ 4), has nk vertices and nk−1 edges. The graph F(n,k) consists of (k−2) pendant vertices in each star.

Let the vertex set of F(n,k) be$\{u_1,u_2,u_3,\ldots,u_k, v_1,v_2,v_3,\ldots,v_k,w_1,w_2,w_3,\ldots,w_k,x_1,x_2,x_3,\ldots,x_k,y_1,y_2,y_3,\ldots,y_k,\ldots,k_1,k_2,k_3,\ldots,k_{nk}\}$. Here all pendant vertices along with some of the rest of vertices will dominate all the vertices of the graph F(n,k) twice.

Let us consider double dominating set of F(n,k) as $\{u_1,u_2,u_3,\ldots,u_{k-1},v_1,v_2,v_3,\ldots,v_{k-2},w_1,w_2,w_3,\ldots,w_{k-1},x_1,x_2,x_3,\ldots,x_{k-2},y_1,y_2,y_3,\ldots,y_{k-1},\ldots k_1,k_2,k_3,\ldots,k_{nk-1}\}$.

The double domination number is

$\gamma_{x2}[F(n,k)] = \{u_1,u_2,u_3,\ldots,u_{k-1},v_1,v_2,v_3,\ldots,v_{k-2}, w_1,w_2,w_3,\ldots,w_{k-1},x_1,x_2,x_3,\ldots,x_{k-2},y_1,y_2,y_3,\ldots,y_{k-1},\ldots, k_1,k_2,k_3,\ldots,k_{nk-1}\}$. Thus we have

$$\gamma_{x2}\,[F(n,k)] = \{nk-5,\,nk-8,\,nk-11,\,nk-14,\,\ldots\ldots\,\}.$$

Hence, $\gamma_{x2}[F (n,k)] = \left\lceil nk - \left(\tfrac{3n-2}{2}\right)\right\rceil$.

V. RELATION BETWEEN DOUBLE DOMINATION AND DOMINATION NUMBER.

Theorem 5.1:

For any Helm graph H_n (n ≥ 3), double domination number $\gamma_{\times2}(H_n) = \gamma(H_n) + 1$.

Proof : The Helm graph H_n has 2n+1 vertices and 3n edges which consists of single universal vertex v connecting all the vertices of the cycle and each node of the cycle adjoining a pendant edge with vertex set $\{v, x_1, x_2, x_3, x_4,\ldots, x_{2n}\}$ Here, vertex v and pendant vertices dominate all the other vertices of graph H_n twice. Let us consider the double dominating set of $H_n = \{v, x_1, x_2, x_3, x_4,\ldots, x_n\}$. Thus double domination number is $\gamma_{\times2}(H_n) = \{v, x_1, x_2, x_3, x_4,\ldots, x_n\}$. Hence $\gamma_{\times2}(H_n) = n+1$------equation (1). Whereas, in a dominating set of H_n all the vertices of the cycle are enough to dominate all the vertices of the graph. Let us consider the dominating set of $H_n=\{x_1, x_2, x_3, x_4,\ldots, x_n\}$. Thus domination number is $\gamma(H_n) =\{x_1, x_2, x_3, x_4,\ldots, x_n\}$.

Hence $\gamma(H_n) = n$------ equation (2). From equations (1) and (2), $\gamma_{\times2}(H_n) = \gamma(H_n) + 1$.

Theorem 5.2:

For any Friendship graph F_n (n ≥ 2), double domination number $\gamma_{\times2}(F_n) = \gamma(F_n) + n$.

Proof: The Friendship graph F_n has 2n+1 vertices and 3n edges. Let the vertex set be $\{v, v_1, v_2, v_3,\ldots,v_{2n}\}$. Here, vertex v and n vertices dominates all the vertices of the graph F_n twice. Let us consider the double dominating set of $F_n = \{v, v_1, v_3, v_5,\ldots,v_n\}$. The double domination number is $\gamma_{\times2}(F_n) = \{v, v_1, v_3, v_5,\ldots,v_n\}$. Hence $\gamma_{\times2}(F_n) = n + 1$------- equation (1). Whereas, in a dominating set of F_n common vertex will dominate all the vertices of the graph. Let us consider the domination set of $F_n = \{v\}$. Thus domination number is $\gamma(F_n) = \{v\}$.

Hence $\gamma(F_n) = 1$------- equation (2). From equations (1) and (2) $\gamma_{\times2}(F_n) = \gamma(F_n) + n$.

Theorem 5.3:

For any Barbell graph B_n (n ≥ 3), double domination number $\gamma_{\times2}(B_n) = 2\,\gamma(B_n)$.

Proof : The Barbell graph B_n has 2n vertices and 2n+1 edges with vertex set $\{x_1, x_2, x_3, x_4,\ldots, x_{2n}\}$. Since B_n is obtained by connecting two copies of complete graph. In dominating set of B_n only two vertices dominate the remaining vertices of the graph. The dominating set of $B_n = \{x_1, x_2\}$.Thus domination number is $\gamma(B_n) = \{x_1, x_2\}$. Hence $\gamma(B_n) = 2$------- equation (1). From theorem 4.5 $\gamma_{\times2}(B_n) = 4$-------- equation (2). From equations (1) and (2) $\gamma_{\times2}(B_n) = 2\,\gamma(B_n)$.

Theorem 5.4 :

For any Firecracker graph F(n , k) (n = 2, k ≥ 3), double domination number $\gamma_{x2}[F(n,k)] = \gamma[F(n,k)]\,\Delta$.

Proof : F(n , k) (n = 2,k ≥ 3), has nk vertices and nk−1 edges. It consists of (k−2) pendant vertices in each star. Let the vertex set of F (n,k) be $\{v_1,v_2,v_3,\ldots,v_k, u_1,u_2,u_3,\ldots,u_k\}$ and $\Delta = (k-1)$. Here two vertices (i.e., one from each star) are enough to dominate all the vertices of the graph.

The dominating set of F(n,k) = {v_1 , u_1}. Thus domination number is γ [F (n,k)] = {v_1, u_1}. Hence γ [F (n,k)] = 2 = n- ---------equation (1). From theorem 4.7

γ_{x2} [F (n,k)] = n Δ------equation (2). From equations (1) and (2) γ_{x2} [F (n,k)] = γ [F (n,k)] Δ.

Theorem 5.5:

For any Firecracker graph F(n , k) where n is odd (n ≥ 3, k ≥ 4), the double domination number is

$$\gamma_{x2}[F(n,k)] = \left[k\gamma(F) - \left(\frac{3n-1}{2}\right)\right].$$

Proof : The Firecracker graph F(n,k) (n ≥ 3, k ≥ 4), has nk vertices and nk−1 edges. The graph F(n , k) consists of (k−2) pendant vertices in each star. Let the vertex set of F(n,k) be {u_1 ,u_2 ,u_3 ,….,u_k ,v_1 ,v_2 ,v_3 ,…, v_k ,w_1 ,w_2 ,w_3 ,…,w_k ,…….,k_1 ,k_2 ,k_3 ,……, k $_{nk}$}. Here n vertices (i.e., central vertex of each star) are enough to dominate all the vertices of the graph. The dominating set of F(n,k) is {u_1,v_1,w_1,…..,k_1}. Thus the domination number is γ ={u_1, v_1,w_1,….k_1}. Hence γ [F (n,k)] = n ----------equation (1). From theorem 4.8

$\gamma_{x2}[F(n,k)] = \left[nk - \left(\frac{3n-1}{2}\right)\right]$ ------- -equation (2). From equations (1) and (2)

$$\gamma_{x2}[F(n,k)] = \left[k\,\gamma(F) - \left(\frac{3n-1}{2}\right)\right].$$

Theorem 5.6 :

Every Firecracker graph F(n,k) ($n \geq 4$, $k \geq 4$), where n is even, the double domination number is

$$\gamma_{x2}[F(n,k)] = \left[k\gamma[F] - \left(\frac{3n-2}{2}\right)\right].$$

Proof : The Firecracker graph F (n,k) (n ≥ 4, k ≥ 4), has nk vertices and nk-1 edges. The graph F(n , k) consists of (k-2) pendant vertices in each star. Let the vertex set of F(n,k) be {u_1 ,u_2 ,u_3 ,….u_k , v_1 ,v_2 ,v_3 ,……v_k ,w_1 w_2 ,w_3 ,…w_k ,…….k_1 ,k_2 ,k_3 ,……k $_{nk}$}. Here n vertices (i.e., central vertex of each star) are enough to dominate all the vertices of the graph. The dominating set of F(n,k) is {u_1,v_1,w_1,…..k_1}. Thus the domination number is γ = {u_1, v_1,w_1,….k_1}. Hence γ (F(n,k) = n ---------- equation (1). From theorem 4.9

$\gamma_{x2}[F(n,k)] = \left[nk - \left(\frac{3n-2}{2}\right)\right]$ -------- equation (2)

From equations (1) and (2)

$$\gamma_{x2}[F(n,k)] = \left[k\gamma[F] - \left(\frac{3n-2}{2}\right)\right].$$

VI. CONCLUSION

We have computed relation between double domination number, domination number, number of vertices and maximum degree of some graphs.

REFERENCE:

1. K. Ameenai Bibi and R.Selvakumar, "The Inverse Split and Non-split Domination in Graphs", International Journal of Computer Applications (0975-8887) , Volume 8-No 7.October 2010.
2. Ashaq Ali , Hifza Iqbal , Waqas Nazeer , Shin Min Kang, "On Topological Indices for the line graph of firecracker graph".
International Journal of Pure and Applied Mathematics Volume 116 No. 4 2017, 1035-1042.
3. Ersin Aslan and Alpay Kirlangic, "Computing The Scattering Number and The Toughness for Gear Graphs", Bulletin of Society of Mathematicans Banja Luka, ISSN 0354-5792, ISSN 1986-521X(o) Vol. 18(2011), 5-15.
4. F. Harary and T.W.Haynes, "Double domination in graphs", Ars Combis. 55 April(2000) 201-213.
5. T.W.Haynes, S.Teresa. Hedetniemi and P.J.Slater, "Domination in Graphs": Advanced Topics (Marcel Dekker, New york, (1998).
6. Mustapha. Chellali, Abdelkader Khelladi and Frederic Maffray," EXACT DOUBLE DOMINATION IN GRAPHS", Discussiones Mathematicae 291 Graph Theory 25 (2005) , Volume :25, Issue:3 , page 291–302, ISSN: 2083-5892 .
7. From Wikipedia, the free encyclopedia https://en.wikipedia.org/wiki/.
8. Wolfram Math World , https://mathworld.wolfram.com/.

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