```
function fe2dx_d_fast ( alpha, beta, gamma, delta, T, delt, u0f, v0f, guf, gvf )
%*****************************************************************************80
%
%% FE2DX_D_FAST applies Scheme 1 with Kinetics 1 to predator prey in a region.
%
%  Discussion:
%
%    FE2DX_D_FAST is a "fast" version of FE2DX_D.
%
%    FE2DX_D is a finite element Matlab code for Scheme 1 applied
%    to the predator-prey system with Kinetics 1 solved over a region
%    which has been triangulated.  The geometry and grid are read
%    from user-supplied files 't_triang.dat' and 'p_coord.dat' respectively.
%
%    Dirichlet boundary conditions are applied.
%
%    This function has 10 input parameters.  All, some, or none of them may
%    be supplied as command line arguments or as functional parameters.
%    Parameters not supplied through the argument list will be prompted for.
%
%    The parameters ALPHA, BETA, GAMMA and DELTA appear in the predator-prey
%    equations as follows:
%
%      dUdT =         nabla U +      U*V/(U+ALPHA) + U*(1-U)
%      dVdT = delta * nabla V + BETA*U*V/(U+ALPHA) - GAMMA * V
%
%  Licensing:
%
%    Copyright (C) 2014 Marcus R. Garvie.
%    See 'mycopyright.txt' for details.
%
%  Modified:
%
%    29 April 2014
%
%  Author:
%
%    Marcus R. Garvie and John Burkardt.
%
%  Reference:
%
%    Marcus R Garvie, John Burkardt, Jeff Morgan,
%    Simple Finite Element Methods for Approximating Predator-Prey Dynamics
%    in Two Dimensions using MATLAB,
%    Submitted to Bulletin of Mathematical Biology, 2014.
%
%  Parameters:
%
%    Input, real ALPHA, a parameter in the predator prey equations.
%    0 < ALPHA.
%
%    Input, real BETA, a parameter in the predator prey equations.
%    0 < BETA.
%
%    Input, real GAMMA, a parameter in the predator prey equations.
```

```
%    0 < GAMMA.
%
%    Input, real DELTA, a parameter in the predator prey equations.
%    0 < DELTA.
%
%    Input, real T, the maximum time.
%    0 < T.
%
%    Input, real DELT, the time step to use in integrating from 0 to T.
%    0 < DELT.
%
%    Input, string U0F or function pointer @U0F, a function for the initial
%    condition of U(X,Y).
%
%    Input, string V0F or function pointer @V0F, a function for the initial
%    condition of V(X,Y).
%
%    Input, string GUF or function pointer @GUF, a function for the Dirichlet
%    boundary condition of U(X,Y,T).
%
%    Input, string GVF or function pointer @GVF, a function for the Dirichlet
%    boundary condition of V(X,Y,T).
%
%*****************************************************************************80
%  Enter data for mesh geometry.
%*****************************************************************************80
%
%  Read in 'p(2,n)', the 'n' coordinates of the nodes.
  load p_coord.dat -ascii
  p = ( p_coord )';
%
%  Read in 't(3,no_elems)', the list of nodes for 'no_elems' elements,
%  and force the entries to be integers.
%
  load t_triang.dat -ascii
  t = ( round ( t_triang ) )';
%
%  Construct the connectivity for the nodes on Gamma.
%
  edges = boundedges ( p', t' );
%
%  BN = boundary nodes on Gamma.
%
  bn = unique ( edges(:) );
%
%  ISN = number of boundary nodes.
%
  [ ~, isn ] = size ( bn );
%
%  N = degrees of freedom per variable.
%
  [ ~, n ] = size ( p );
%
%  NO_ELEMS = number of elements.
%
  [ ~, no_elems ] = size ( t );
%
```

```
%  Extract vector of 'x' and 'y' values.
%
  x = p(1,:);
  y = p(2,:);
%*****************************************************************************80
%  Enter data for model.
%*****************************************************************************80
  if ( nargin < 1 )
    alpha = input ( 'Enter parameter alpha:  ' );
  elseif ( ischar ( alpha ) )
    alpha = str2num ( alpha );
  end
  if ( nargin < 2 )
    beta = input ( 'Enter parameter beta:  ' );
  elseif ( ischar ( beta ) )
    beta = str2num ( beta );
  end
  if ( nargin < 3 )
    gamma = input ( 'Enter parameter gamma:  ' );
  elseif ( ischar ( gamma ) )
    gamma = str2num ( gamma );
  end
  if ( nargin < 4 )
    delta = input ( 'Enter parameter delta:  ' );
  elseif ( ischar ( delta ) )
    delta = str2num ( delta );
  end
  if ( nargin < 5 )
    T = input ( 'Enter maximum time T:  ' );
  elseif ( ischar ( T ) )
    T = str2num ( T );
  end
  if ( nargin < 6 )
    delt = input ( 'Enter time-step delt:  ' );
  elseif ( ischar ( delt ) )
    delt = str2num ( delt );
  end
  fprintf ( 1, '  Using ALPHA = %g\n', alpha );
  fprintf ( 1, '  Using BETA = %g\n', beta );
  fprintf ( 1, '  Using GAMMA = %g\n', gamma );
  fprintf ( 1, '  Using DELTA = %g\n', delta );
  fprintf ( 1, '  Using T = %g\n', T );
  fprintf ( 1, '  Using DELT = %g\n', delt );
%
%  Initial conditions.
%
  if ( nargin < 7 )
    u0_str = input ( 'Enter initial data function u0(x,y):  ', 's' );
    u0f = @(x,y) eval ( u0_str );
  elseif ( ischar ( u0f ) )
    u0_str = u0f;
    u0f = @(x,y) eval ( u0_str );
  end
  u = ( arrayfun ( u0f, x, y ) )';
  if ( nargin < 8 )
    v0_str = input ( 'Enter initial data function v0(x,y):  ', 's' );
    v0f = @(x,y) eval ( v0_str );
```

```
  elseif ( ischar ( v0f ) )
    v0_str = v0f;
    v0f = @(x,y) eval ( v0_str );
  end
  v = ( arrayfun ( v0f, x, y ) )';
%
%  Boundary conditions.
%
  if ( nargin < 9 )
    gu_str = input('Enter the Dirichlet b.c. gu(x,y,t) for u  ','s');
    guf = @(x,y,t)eval(gu_str);
  elseif ( ischar ( guf ) )
    gu_str = guf;
    guf = @(x,y,t)eval(gu_str);
  end
  if ( nargin < 10 )
    gv_str = input('Enter the Dirichlet b.c. gv(x,y,t) for v  ','s');
    gvf = @(x,y,t)eval(gv_str);
  elseif ( ischar ( gvf ) )
    gv_str = gvf;
    gvf = @(x,y,t)eval(gv_str);
  end
%
%  N = number of time steps.
%
  N = round ( T / delt );
  fprintf ( 1, '  Taking N = %d time steps\n', N );
%**********************************************************************80
%  Assembly.
%**********************************************************************80
  m_hat = zeros ( n, 1 );
  K = sparse ( n, n );
  for elem = 1 : no_elems
%
%  Identify nodes ni, nj and nk in element 'elem'.
%
    ni = t(1,elem);
    nj = t(2,elem);
    nk = t(3,elem);
%
%  Identify coordinates of nodes ni, nj and nk.
%
    xi = p(1,ni);
    xj = p(1,nj);
    xk = p(1,nk);
    yi = p(2,ni);
    yj = p(2,nj);
    yk = p(2,nk);
%
%  Calculate the area of element 'elem'.
%
    triangle_area = abs(xj*yk-xk*yj-xi*yk+xk*yi+xi*yj-xj*yi)/2;
%
%  Calculate some quantities needed to construct elements in K.
%
    h1 = (xi-xj)*(yk-yj)-(xk-xj)*(yi-yj);
    h2 = (xj-xk)*(yi-yk)-(xi-xk)*(yj-yk);
```

```
    h3 = (xk-xi)*(yj-yi)-(xj-xi)*(yk-yi);
    s1 = (yj-yi)*(yk-yj)+(xi-xj)*(xj-xk);
    s2 = (yj-yi)*(yi-yk)+(xi-xj)*(xk-xi);
    s3 = (yk-yj)*(yi-yk)+(xj-xk)*(xk-xi);
    t1 = (yj-yi)^2+(xi-xj)^2;
    t2 = (yk-yj)^2+(xj-xk)^2;
    t3 = (yi-yk)^2+(xk-xi)^2;
%
%  Calculate local contributions to m_hat.
%
    m_hat_i = triangle_area/3;
    m_hat_j = m_hat_i;
    m_hat_k = m_hat_i;
%
%  Calculate local contributions to K.
%
    K_ki = triangle_area*s1/(h3*h1);
    K_ik = K_ki;
    K_kj = triangle_area*s2/(h3*h2);
    K_jk = K_kj;
    K_kk = triangle_area*t1/(h3^2);
    K_ij = triangle_area*s3/(h1*h2);
    K_ji = K_ij;
    K_ii = triangle_area*t2/(h1^2);
    K_jj = triangle_area*t3/(h2^2);
%
%  Add contributions to vector m_hat.
%
    m_hat(nk) = m_hat(nk)+m_hat_k;
    m_hat(nj) = m_hat(nj)+m_hat_j;
    m_hat(ni) = m_hat(ni)+m_hat_i;
%
%  Add contributions to K.
%
    K=K+sparse(nk,ni,K_ki,n,n);
    K=K+sparse(ni,nk,K_ik,n,n);
    K=K+sparse(nk,nj,K_kj,n,n);
    K=K+sparse(nj,nk,K_jk,n,n);
    K=K+sparse(nk,nk,K_kk,n,n);
    K=K+sparse(ni,nj,K_ij,n,n);
    K=K+sparse(nj,ni,K_ji,n,n);
    K=K+sparse(ni,ni,K_ii,n,n);
    K=K+sparse(nj,nj,K_jj,n,n);
  end
%
%  Construct matrix L.
%
  ivec = 1 : n;
  IM_hat = sparse(ivec,ivec,1./m_hat,n,n);
  L = delt * IM_hat * K;
%
%  Construct fixed parts of matrices A_{n-1} and C_{n-1} .
%
  A0 = L + sparse(1:n,1:n,1-delt,n,n);
  C0 = delta * L + sparse(1:n,1:n,1+delt*gamma,n,n);
%
%  Adjust A0 and C0 for Dirichlet boundary conditions on Gamma
```

```
%
  for i = 1:isn
    node = bn(i);
    C(node,:) = 0;
    C(node,node) = 1;
    A(node,:) = 0;
    A(node,node) = 1;
  end
  fprintf ( 1, '\n' );
  fprintf ( 1, '  Matrix size N = %d\n', n );
  fprintf ( 1, '  A0 nonzeros = %d\n', nnz ( A0 ) );
  fprintf ( 1, '  C0 nonzeros = %d\n', nnz ( C0 ) );
%*****************************************************************************80
%  Time-stepping.
%*****************************************************************************80
  for nt = 1 : N
    tn = nt * delt;
%
%  Update coefficient matrices of linear system
%
    diag = abs ( u );
    diag_entries = u ./ ( alpha + abs ( u ) );
%
%  Impose Dirichlet boundary conditions on Gamma, and zero out
%  entries of DIAG and DIAG_ENTRIES that would interfere.
%
    for i = 1 : isn
      node = bn(i);
      xx = p(1,node);
      yy = p(2,node);
      v(node) = gvf ( xx, yy, tn );
      u(node) = guf ( xx, yy, tn );
      diag(node) = 0.0;
      diag_entries(node) = 0.0;
    end
    A = A0 +        delt * sparse ( 1:n, 1:n, diag, n, n );
    B =             delt * sparse ( 1:n, 1:n, diag_entries, n, n );
    C = C0 - beta * delt * sparse ( 1:n, 1:n, diag_entries, n, n );
%
%  Do the incomplete LU factorisation of A and C.
%
    [ LC, UC ] = ilu ( C, struct('type','ilutp','droptol',1e-5) );
    [ LA, UA ] = ilu ( A, struct('type','ilutp','droptol',1e-5) );
%
%  Solve for v using GMRES.
%
    [v,flagv,relresv,iterv] = gmres ( C, v,[],1e-6,[],LC,UC,v );
    if flagv ~= 0
      flagv
      relresv
      iterv
      error('GMRES did not converge')
    end
    r = u - B * v;
%
%  Solve for u using GMRES
%
```

```
    [u,flagu,relresu,iteru] = gmres ( A, r,[],1e-6,[],LA,UA,u );
    if flagu ~= 0
      flagu
      relresu
      iteru
      error('GMRES did not converge')
    end

  end
%*****************************************************************************80
%  Plot the solutions.
%*****************************************************************************80
%
%  Plot U;
%
  figure;
  set(gcf,'Renderer','zbuffer');
  trisurf(t',x,y,u,'FaceColor','interp','EdgeColor','interp');
  colorbar;
  axis off;
  title('u');
  view ( 2 );
  axis equal on tight;
  filename = 'fe2dx_d_fast_u.png';
  print ( '-dpng', filename );
  fprintf ( 1, '  Saved graphics file "%s"\n', filename );
%
%  Plot V.
%
  figure;
  set(gcf,'Renderer','zbuffer');
  trisurf(t',x,y,v,'FaceColor','interp','EdgeColor','interp');
  colorbar;
  axis off;
  title('v');
  view ( 2 );
  axis equal on tight;
  filename = 'fe2dx_d_fast_v.png';
  fprintf ( 1, '  Saved graphics file "%s"\n', filename );
  print ( '-dpng', filename );
  return
end
```