```
function fe2dx_nr_alt_fast ( alpha, beta, gamma, delta, T, delt, u0f, v0f, ...
  k1, k2, g2uf, g2vf )
%*****************************************************************************80
%
%% FE2DX_NR_ALT_FAST: version of FE2DX_NR_FAST with implicit Robin condition.
%
%  Discussion:
%
%    FE2DX_NR_ALT_FAST is a "fast" version of FE2DX_NR_ALT.
%
%    FE2DX_NR_ALT is similar to FE2DX_NR, but uses an implicit approximation
%    of the Robin boundary condition.  It is a finite element Matlab code
%    for Scheme 1 applied to the predator-prey system with Kinetics 1
%    solved over a region which has been triangulated.  The geometry and grid
%    are read from user-supplied files 't_triang.dat' and 'p_coord.dat'
%    respectively, as are the list of nodes on which Robin and Neumann boundary
%    conditions are to be imposed (from 'bn1_nodes.dat' and 'bn2_nodes.dat'
%    respectively).
%
%    This function has 12 input parameters.  All, some, or none of them may
%    be supplied as command line arguments or as functional parameters.
%    Parameters not supplied through the argument list will be prompted for.
%
%    The parameters ALPHA, BETA, GAMMA and DELTA appear in the predator-prey
%    equations as follows:
%
%      dUdT =         nabla U +      U*V/(U+ALPHA) + U*(1-U)
%      dVdT = delta * nabla V + BETA*U*V/(U+ALPHA) - GAMMA * V
%
%  Licensing:
%
%    Copyright (C) 2014 Marcus R. Garvie.
%    See 'mycopyright.txt' for details.
%
%  Modified:
%
%    29 April 2014
%
%  Author:
%
%    Marcus R. Garvie and John Burkardt.
%
%  Reference:
%
%    Marcus R Garvie, John Burkardt, Jeff Morgan,
%    Simple Finite Element Methods for Approximating Predator-Prey Dynamics
%    in Two Dimensions using MATLAB,
%    Submitted to Bulletin of Mathematical Biology, 2014.
%
%  Parameters:
%
%    Input, real ALPHA, a parameter in the predator prey equations.
%    0 < ALPHA.
%
%    Input, real BETA, a parameter in the predator prey equations.
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```
%    0 < BETA.
%
%    Input, real GAMMA, a parameter in the predator prey equations.
%    0 < GAMMA.
%
%    Input, real DELTA, a parameter in the predator prey equations.
%    0 < DELTA.
%
%    Input, real T, the maximum time.
%    0 < T.
%
%    Input, real DELT, the time step to use in integrating from 0 to T.
%    0 < DELT.
%
%    Input, string U0F or function pointer @U0F, a function for the initial
%    condition of U(X,Y).
%
%    Input, string V0F or function pointer @V0F, a function for the initial
%    condition of V(X,Y).
%
%    Input, real K1, the coefficient for the Robin boundary condition
%    to be applied to U: dU/dn = k1 * U.
%
%    Input, real K2, the coefficient for the Robin boundary condition
%    to be applied to V: dV/dn = k2 * V.
%
%    Input, string G2UF or function pointer @G2UF, a function for the Neumann
%    boundary condition of U(X,Y,T).
%
%    Input, string G2VF or function pointer @G2VF, a function for the Neumann
%    boundary condition of V(X,Y,T).
%
%*****************************************************************************80
%  Enter data for mesh geometry.
%*****************************************************************************80
%
%  Read in 'p(2,n)', the 'n' coordinates of the nodes.
%
  load p_coord.dat -ascii
  p = ( p_coord )';
%
%  Read in 't(3,no_elems)', the list of nodes for 'no_elems' elements.
%
  load t_triang.dat -ascii
  t = ( round ( t_triang ) )';
%
%  Read in 'bn1(1,isn1)', the nodes on Gamma1.
%
  load bn1_nodes.dat -ascii
  bn1 = ( round ( bn1_nodes ) )';
%
%  Read in 'bn2(1,isn2)', the nodes on Gamma2.
%
  load bn2_nodes.dat -ascii
  bn2 = ( round ( bn2_nodes ) )';
%
%  Construct the connectivity for the nodes on Gamma1.
```

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%
  cpp1 = subsetconnectivity ( t', p', bn1' );
%
%  Construct the connectivity for the nodes on Gamma2.
%
  cpp2 = subsetconnectivity ( t', p', bn2' );
%
%  E1 = number of edges on Gamma1.
%
  [ e1, ~ ] = size ( cpp1 );
%
%  E2 = number of edges on Gamma2.
%
  [ e2, ~ ] = size ( cpp2 );
%
%  N = degrees of freedom per variable.
%
  [ ~, n ] = size ( p );
%
%  NO_ELEMS = number of elements.
%
  [ ~, no_elems ] = size ( t );
%
%  Extract vector of 'x' and 'y' values.
%
  x = p(1,:);
  y = p(2,:);
%*****************************************************************************80
%  Enter data for model.
%*****************************************************************************80
  if ( nargin < 1 )
    alpha = input ( 'Enter parameter alpha:  ' );
  elseif ( ischar ( alpha ) )
    alpha = str2num ( alpha );
  end
  if ( nargin < 2 )
    beta = input ( 'Enter parameter beta:  ' );
  elseif ( ischar ( beta ) )
    beta = str2num ( beta );
  end
  if ( nargin < 3 )
    gamma = input ( 'Enter parameter gamma:  ' );
  elseif ( ischar ( gamma ) )
    gamma = str2num ( gamma );
  end
  if ( nargin < 4 )
    delta = input ( 'Enter parameter delta:  ' );
  elseif ( ischar ( delta ) )
    delta = str2num ( delta );
  end
  if ( nargin < 5 )
    T = input ( 'Enter maximum time T:  ' );
  elseif ( ischar ( T ) )
    T = str2num ( T );
  end
  if ( nargin < 6 )
    delt = input ( 'Enter time-step delt:  ' );
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  elseif ( ischar ( delt ) )
    delt = str2num ( delt );
  end
  fprintf ( 1, '  Using ALPHA = %g\n', alpha );
  fprintf ( 1, '  Using BETA = %g\n', beta );
  fprintf ( 1, '  Using GAMMA = %g\n', gamma );
  fprintf ( 1, '  Using DELTA = %g\n', delta );
  fprintf ( 1, '  Using T = %g\n', T );
  fprintf ( 1, '  Using DELT = %g\n', delt );
%
%  Initial conditions.
%
  if ( nargin < 7 )
    u0_str = input ( 'Enter initial data function u0(x,y):  ', 's' );
    u0f = @(x,y) eval ( u0_str );
  elseif ( ischar ( u0f ) )
    u0_str = u0f;
    u0f = @(x,y) eval ( u0_str );
  end
  u = ( arrayfun ( u0f, x, y ) )';
  if ( nargin < 8 )
    v0_str = input ( 'Enter initial data function v0(x,y):  ', 's' );
    v0f = @(x,y) eval ( v0_str );
  elseif ( ischar ( v0f ) )
    v0_str = v0f;
    v0f = @(x,y) eval ( v0_str );
  end
  v = ( arrayfun ( v0f, x, y ) )';
%
%  Boundary conditions.
%
  if ( nargin < 9 )
    k1 = input('Enter the parameter k1 in the Robin b.c. for u  ');
  elseif ( ischar ( k1 ) )
    k1 = str2num ( k1 );
  end
  if ( nargin < 10 )
    k2 = input('Enter the parameter k2 in the Robin b.c. for v  ');
  elseif ( ischar ( k2 ) )
    k2 = str2num ( k2 );
  end
  if ( nargin < 11 )
    g2u_str = input('Enter the Neumann b.c. g2u(x,y,t) for u  ','s');
    g2uf = @(x,y,t)eval(g2u_str);
  elseif ( ischar ( g2uf ) )
    g2u_str = g2uf;
    g2uf = @(x,y,t)eval(g2u_str);
  end
  if ( nargin < 12 )
    g2v_str = input('Enter the Neumann b.c. g2v(x,y,t) for v  ','s');
    g2vf = @(x,y,t)eval(g2v_str);
  elseif ( ischar ( g2vf ) )
    g2v_str = g2vf;
    g2vf = @(x,y,t)eval(g2v_str);
  end
%
%  N = number of time steps.
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```
%
  N = round ( T / delt );
  fprintf ( 1, '  Taking N = %d time steps\n', N );
%*****************************************************************************80
%  Assembly.
%*****************************************************************************80
  m_hat = zeros ( n, 1 );
  K = sparse ( n, n );
  for elem = 1 : no_elems
%
%  Identify nodes ni, nj and nk in element 'elem'.
%
    ni = t(1,elem);
    nj = t(2,elem);
    nk = t(3,elem);
%
%  Identify coordinates of nodes ni, nj and nk.
%
    xi = p(1,ni);
    xj = p(1,nj);
    xk = p(1,nk);
    yi = p(2,ni);
    yj = p(2,nj);
    yk = p(2,nk);
%
%  Calculate the area of element 'elem'.
%
    triangle_area = abs(xj*yk-xk*yj-xi*yk+xk*yi+xi*yj-xj*yi)/2;
%
%  Calculate some quantities needed to construct elements in K.
%
    h1 = (xi-xj)*(yk-yj)-(xk-xj)*(yi-yj);
    h2 = (xj-xk)*(yi-yk)-(xi-xk)*(yj-yk);
    h3 = (xk-xi)*(yj-yi)-(xj-xi)*(yk-yi);
    s1 = (yj-yi)*(yk-yj)+(xi-xj)*(xj-xk);
    s2 = (yj-yi)*(yi-yk)+(xi-xj)*(xk-xi);
    s3 = (yk-yj)*(yi-yk)+(xj-xk)*(xk-xi);
    t1 = (yj-yi)^2+(xi-xj)^2;
    t2 = (yk-yj)^2+(xj-xk)^2;
    t3 = (yi-yk)^2+(xk-xi)^2;
%
%  Calculate local contributions to m_hat.
%
    m_hat_i = triangle_area/3;
    m_hat_j = m_hat_i;
    m_hat_k = m_hat_i;
%
%  Calculate local contributions to K.
%
    K_ki = triangle_area*s1/(h3*h1);
    K_ik = K_ki;
    K_kj = triangle_area*s2/(h3*h2);
    K_jk = K_kj;
    K_kk = triangle_area*t1/(h3^2);
    K_ij = triangle_area*s3/(h1*h2);
    K_ji = K_ij;
    K_ii = triangle_area*t2/(h1^2);
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    K_jj = triangle_area*t3/(h2^2);
%
%  Add contributions to vector m_hat.
%
    m_hat(nk)=m_hat(nk)+m_hat_k;
    m_hat(nj)=m_hat(nj)+m_hat_j;
    m_hat(ni)=m_hat(ni)+m_hat_i;
%
%  Add contributions to K.
%
    K=K+sparse(nk,ni,K_ki,n,n);
    K=K+sparse(ni,nk,K_ik,n,n);
    K=K+sparse(nk,nj,K_kj,n,n);
    K=K+sparse(nj,nk,K_jk,n,n);
    K=K+sparse(nk,nk,K_kk,n,n);
    K=K+sparse(ni,nj,K_ij,n,n);
    K=K+sparse(nj,ni,K_ji,n,n);
    K=K+sparse(ni,ni,K_ii,n,n);
    K=K+sparse(nj,nj,K_jj,n,n);
  end
%
%  Construct matrix L.
%
  ivec = 1 : n;
  IM_hat = sparse(ivec,ivec,1./m_hat,n,n);
  L = delt * IM_hat * K;
%
%  Construct fixed parts of matrices A_{n-1} and C_{n-1}.
%
  A0 = L + sparse(1:n,1:n,1-delt,n,n);
  C0 = delta * L + sparse(1:n,1:n,1+delt*gamma,n,n);
%
%  Set up A1 and C1 matrices that impose Robin boundary condition on Gamma1.
%
  A1 = sparse ( n, n );
  C1 = sparse ( n, n );
  for i = 1 : e1
    node1 = cpp1(i,1);
    node2 = cpp1(i,2);
    x1 = p(1,node1);
    y1 = p(2,node1);
    x2 = p(1,node2);
    y2 = p(2,node2);
    im_hat1 = 1/m_hat(node1);
    im_hat2 = 1/m_hat(node2);
    gamma12 = sqrt((x1-x2)^2 + (y1-y2)^2);
    A1(node1,node1) = A1(node1,node1) - delt * k1 * im_hat1 * gamma12 / 2;
    A1(node2,node2) = A1(node2,node2) - delt * k1 * im_hat2 * gamma12 / 2;
    C1(node1,node1) = C1(node1,node1) - delt * k2 * im_hat1 * gamma12 / 2;
    C1(node2,node2) = C1(node2,node2) - delt * k2 * im_hat2 * gamma12 / 2;
  end
%***************************************************************************80
%  Time-stepping.
%***************************************************************************80
  for nt = 1 : N
    tn = nt * delt;
%
```

```
% Initialize right-hand-side functions.
%
  rhs_u = u;
  rhs_v = v;
%
% Update coefficient matrices of linear system.
%
  diag = abs ( u );
  diag_entries = u ./ ( alpha + abs ( u ) );
  A = A0 +        delt * sparse(1:n,1:n,diag,n,n);
  B =             delt * sparse(1:n,1:n,diag_entries,n,n);
  C = C0 - beta * delt * sparse(1:n,1:n,diag_entries,n,n);
%
% Impose implicit Robin boundary condition on Gamma1.
%
  A = A + A1;
  C = C + C1;
%
% Do the incomplete LU factorisation of C and A.
%
  [ LC, UC ] = ilu ( C, struct('type','ilutp','droptol',1e-5) );
  [ LA, UA ] = ilu ( A, struct('type','ilutp','droptol',1e-5) );
%
% Impose Neumann boundary condition on Gamma2.
%
  for i = 1:e2
    node1 = cpp2(i,1);
    node2 = cpp2(i,2);
    x1 = p(1,node1);
    y1 = p(2,node1);
    x2 = p(1,node2);
    y2 = p(2,node2);
    im_hat1 = 1/m_hat(node1);
    im_hat2 = 1/m_hat(node2);
    gamma12 = sqrt((x1-x2)^2 + (y1-y2)^2);
    rhs_u(node1) = rhs_u(node1) + delt * g2uf (x1,y1,tn) * im_hat1*gamma12/2;
    rhs_u(node2) = rhs_u(node2) + delt * g2uf (x2,y2,tn) * im_hat2*gamma12/2;
    rhs_v(node1) = rhs_v(node1) + delt * g2vf (x1,y1,tn) * im_hat1*gamma12/2;
    rhs_v(node2) = rhs_v(node2) + delt * g2vf (x2,y2,tn) * im_hat2*gamma12/2;
  end
%
% Solve for v using GMRES.
%
  [v,flagv,relresv,iterv] = gmres ( C,rhs_v,[],1e-6,[],LC,UC,v );
  if flagv ~= 0
    flagv
    relresv
    iterv
    error('GMRES did not converge')
  end
  r = rhs_u - B * v;
%
% Solve for u using GMRES.
%
  [u,flagu,relresu,iteru] = gmres ( A,r,[],1e-6,[],LA,UA,u );
  if flagu ~= 0
    flagu
```

```
      relresu
      iteru
      error('GMRES did not converge')
    end
  end
%*****************************************************************************80
%  Plot the solutions.
%*****************************************************************************80
%
%  Plot U;
%
  figure;
  set(gcf,'Renderer','zbuffer');
  trisurf(t',x,y,u,'FaceColor','interp','EdgeColor','interp');
  colorbar;
  axis off;
  title('u');
  view ( 2 );
  axis equal on tight;
  filename = 'fe2dx_nr_alt_fast_u.png';
  print ( '-dpng', filename );
  fprintf ( 1, '  Saved graphics file "%s"\n', filename );
%
%  Plot V.
%
  figure;
  set(gcf,'Renderer','zbuffer');
  trisurf(t',x,y,v,'FaceColor','interp','EdgeColor','interp');
  colorbar;
  axis off;
  title('v');
  view ( 2 );
  axis equal on tight;
  filename = 'fe2dx_nr_alt_fast_v.png';
  fprintf ( 1, '  Saved graphics file "%s"\n', filename );
  print ( '-dpng', filename );
  return
end
```

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