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1 function fe2d_d
2 % 'fe2d_d.m' 2D finite element Matlab code for Scheme 2 applied
3 % to the predator-prey system with Kinetics 1. The nodes and elements
4 % of the unstructured grid are loaded from external files 't_triang.dat'
5 % and 'p_coord.dat' respectively.
6 %
7 % Boundary conditions:
8 %   Gamma: Dirichlet
9 %
10 % (C) 2009 Marcus R. Garvie. See 'mycopyright.txt' for details.
11 %
12 % Modified April 7, 2014
13 %
14 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
15 %                               Enter data for mesh geometry
16 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
17 % Read in 'p(2,n)', the 'n' coordinates of the nodes
18 load p_coord.dat -ascii
19 p = (p_coord)';
20 % Read in 't(3,no_elems)', the list of nodes for 'no_elems' elements
21 load t_triang.dat -ascii
22 t = (round(t_triang))';
23 % Construct the connectivity for the nodes on Gamma
24 edges = boundedges (p',t');
25 % Identify the boundary nodes on Gamma
26 bn = unique(edges(:));
27 % Number of nodes (isn) on Gamma
28 [junk,isn]=size(bn);
29 % Degrees of freedom per variable (n)
30 [junk,n]=size(p);
31 % Number of elements (no_elems)
32 [junk,no_elems]=size(t);
33 % Extract vector of 'x' and 'y' values
34 x = p(1,:); y = p(2,:);
35 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
36 %                               Enter data for model
37 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
38 % User inputs of parameters
39 alpha = input('Enter parameter alpha ');
40 beta = input('Enter parameter beta ');
41 gamma = input('Enter parameter gamma ');
42 delta = input('Enter parameter delta ');
43 T = input('Enter maximum time T ');
44 delt = input('Enter time-step Delta t ');
45 % User inputs of initial data
46 u0_str = input('Enter initial data function u0(x,y) ','s');
47 u0_anon = @(x,y)eval(u0_str); % create anonymous function
48 u = arrayfun(u0_anon,x,y)';
49 v0_str = input('Enter initial data function v0(x,y) ','s');
50 v0_anon = @(x,y)eval(v0_str); % create anonymous function
51 v = arrayfun(v0_anon,x,y)';
52 % Enter the boundary conditions
53 gu_str = input('Enter the Dirichlet b.c. gu(x,y,t) for u ','s');
54 gu = @(x,y,t)eval(gu_str); % create anonymous function
55 gv_str = input('Enter the Dirichlet b.c. gv(x,y,t) for v ','s');

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56 gv = @(x,y,t)eval(gv_str);    % create anonymous function
57 % Calculate and assign some constants
58 N=round(T/delt);
59 % Degrees of freedom per variable (n)
60 [junk,n]=size(p);
61 % Number of elements (no_elems)
62 [junk,no_elems]=size(t);
63 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
64 %                                     Assembly
65 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
66 m_hat=zeros(n,1);
67 K=sparse(n,n);
68 for elem = 1:no_elems
69     % Identify nodes ni, nj and nk in element 'elem'
70     ni = t(1,elem);
71     nj = t(2,elem);
72     nk = t(3,elem);
73     % Identify coordinates of nodes ni, nj and nk
74     xi = p(1,ni);
75     xj = p(1,nj);
76     xk = p(1,nk);
77     yi = p(2,ni);
78     yj = p(2,nj);
79     yk = p(2,nk);
80     % Calculate the area of element 'elem'
81     triangle_area = abs(xj*yk-xk*yj-xi*yk+xk*yi+xi*yj-xj*yi)/2;
82     % Calculate some quantities needed to construct elements in K
83     h1 = (xi-xj)*(yk-yj)-(xk-xj)*(yi-yj);
84     h2 = (xj-xk)*(yi-yk)-(xi-xk)*(yj-yk);
85     h3 = (xk-xi)*(yj-yi)-(xj-xi)*(yk-yi);
86     s1 = (yj-yi)*(yk-yj)+(xi-xj)*(xj-xk);
87     s2 = (yj-yi)*(yi-yk)+(xi-xj)*(xk-xi);
88     s3 = (yk-yj)*(yi-yk)+(xj-xk)*(xk-xi);
89     t1 = (yj-yi)^2+(xi-xj)^2;    % g* changed to t*
90     t2 = (yk-yj)^2+(xj-xk)^2;
91     t3 = (yi-yk)^2+(xk-xi)^2;
92     % Calculate local contributions to m_hat
93     m_hat_i = triangle_area/3;
94     m_hat_j = m_hat_i;
95     m_hat_k = m_hat_i;
96     % calculate local contributions to K
97     K_ki = triangle_area*s1/(h3*h1);
98     K_ik = K_ki;
99     K_kj = triangle_area*s2/(h3*h2);
100    K_jk = K_kj;
101    K_kk = triangle_area*t1/(h3^2);
102    K_ij = triangle_area*s3/(h1*h2);
103    K_ji = K_ij;
104    K_ii = triangle_area*t2/(h1^2);
105    K_jj = triangle_area*t3/(h2^2);
106    % Add contributions to vector m_hat
107    m_hat(nk)=m_hat(nk)+m_hat_k;
108    m_hat(nj)=m_hat(nj)+m_hat_j;
109    m_hat(ni)=m_hat(ni)+m_hat_i;
110    % Add contributions to K
111    K=K+sparse(nk,ni,K_ki,n,n);
112    K=K+sparse(ni,nk,K_ik,n,n);

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113 K=K+sparse(nk,nj,K_kj,n,n);
114 K=K+sparse(nj,nk,K_jk,n,n);
115 K=K+sparse(nk,nk,K_kk,n,n);
116 K=K+sparse(ni,nj,K_ij,n,n);
117 K=K+sparse(nj,ni,K_ji,n,n);
118 K=K+sparse(ni,ni,K_ii,n,n);
119 K=K+sparse(nj,nj,K_jj,n,n);
120 end
121 % Construct matrix L
122 ivec=1:n;
123 IM_hat=sparse(ivec,ivec,1./m_hat,n,n);
124 L=delt*IM_hat*K;
125 % Construct matrices B1 & B2
126 B1=sparse(1:n,1:n,1,n,n)+L;
127 B2=sparse(1:n,1:n,1,n,n)+delta*L;
128 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
129 %                               Time-stepping procedure
130 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
131 for nt=1:N
132     tn = nt*delt;
133     % Evaluate modified functional response
134     hhat = u./(alpha + abs(u));
135     % Update right-hand-side of linear system
136     F = u - u.*abs(u) - v.*hhat;
137     G = beta*v.*hhat - gamma*v;
138     rhs_u = u + delt*F;
139     rhs_v = v + delt*G;
140     % Impose dirichlet boundary conditions on Gamma
141     for i = 1:isn
142         node = bn(i);
143         xx = p(1,node);
144         yy = p(2,node);
145         B1(node,:)=0;
146         B1(node,node)=1;
147         rhs_u(node)=gu(xx,yy,tn);
148         B2(node,:)=0;
149         B2(node,node)=1;
150         rhs_v(node)=gv(xx,yy,tn);
151     end
152     % Do the incomplete LU factorisation of B1 and B2
153     [LB1,UB1] = ilu(B1,struct('type','ilutp','droptol',1e-5));
154     [LB2,UB2] = ilu(B2,struct('type','ilutp','droptol',1e-5));
155     % Solve for u and v using GMRES
156     [u,flagu,relresu,iteru]=gmres(B1,rhs_u,[],1e-6,[],LB1,UB1,u);
157     if flagu~=0 flagu,relresu,iteru,error('GMRES did not converge'),end
158     [v,flagv,relresv,iterv]=gmres(B2,rhs_v,[],1e-6,[],LB2,UB2,v);
159     if flagv~=0 flagv,relresv,iterv,error('GMRES did not converge'),end
160 end
161 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
162 %                               Plot solutions
163 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
164 % Plot solution for u
165 figure;
166 set(gcf,'Renderer','zbuffer');
167 trisurf(t',x,y,u,'FaceColor','interp','EdgeColor','interp');
168 colorbar;axis off;title('u');
169 view ( 2 );

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170 axis equal on tight;
171 % Plot solution for v
172 figure;
173 set(gcf,'Renderer','zbuffer');
174 trisurf(t',x,y,v,'FaceColor','interp','EdgeColor','interp');
175 colorbar;axis off;title('v');
176 view ( 2 );
177 axis equal on tight;
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