

On two approaches to radar band fusion

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Abstract—Due to the frequency constraints imposed by the necessary coexistence of radar and communications, the increasing range resolution requirements of modern radar systems can only be achieved by fusing multiple frequency bands. There are a variety of published approaches to solve this task. In this paper we will present two algorithms, the first one based on a high resolution spectral estimation method and the second one based on a compressive sensing algorithm.

Keywords—High resolution radar, radar band fusion, Iterative adaptive approach, IAA, Iterative Shrinkage-Thresholding Algorithm, ISTA

I. INTRODUCTION

Improvement of range resolution in radars systems has been a topic of interest for many years. Since the range resolution is inversely proportional to radar bandwidth, a wide-band radar allows for a good range resolution. However, wide frequency bands necessary for close-spaced target separation and high resolution imaging are not easily available due to the congestion of the spectrum by radars and communication systems [1], [2].

While one approach is the dual use of spectrum for radar and communication, it is also possible to split the spectrum for radar into small dedicated sub-bands. However, the latter approach necessitates the fusion of sub-bands into one. Therefore, the goal is to use the existing narrow-band radars and estimate the gapped-band data from the available sub-bands. A good estimate of the missing band would result in the desired wide-frequency band, which could be used to detect targets at a higher resolution.

The problem of band fusion has been investigated under the topic of ‘spectral estimation’ in the past. Fusion of data from disjoint sources has also been an active research topic in the field of sensor networks [3]. The three important factors of an ideal band fusion method are low computational complexity, robustness at low signal-to-interference-plus-noise ratio (SINR), and a near-ideal estimate of the missing band. In practice, a trade-off must be achieved between these factors.

Traditional spectral estimation methods largely relied on different minimization techniques and exploited the notion of similarity between missing and known bands. However, this resulted in inaccurate estimations and a large computational load. Researchers have tried to counter these disadvantages by moving to feature-based detection methods [4], [5]. Recent work has heavily focused on using the natural sparsity of targets to reduce the problem size.

This paper presents two band fusion methods. The first is based on a high resolution spectrum estimation, the second on a compressive sensing (CS) technique.

Section II introduces the problem with a basic signal model. Section III presents an ‘Iterative Adaptive Approach (IAA)’ and shows simulation results. Section IV presents a compressive sensing fusion algorithm based on ‘Iterative Shrinkage-Thresholding Algorithm (ISTA)’ and also includes simulation results. The paper ends with the summary in section V.

II. SIGNAL MODEL

A. Motivation

In some radar situations, the scene is illuminated in two or more separate frequency bands, e.g., because of disallowed bands or with the goal of fusing two or more different phase-coherent radar units to achieve finer range resolution. In either case, the Tx signal(s) in the frequency domain may be represented with gaps in the spectrum. The signals received in the time domain can be converted to the frequency domain via a DFT without loss of information. Conventional matched filtering results in an unacceptable side lobe level.

Without additional information about the scattering distribution, it is not possible to retrieve in the missing frequency data. However, assuming that the distribution is sparse in the range (equivalently, fast time), CS methods can be applied to obtain an excellent estimate of this. As we will see in the IV section, this is accompanied by an estimation (‘interpolation’) of the missing frequency data.

B. Continuous formulation

The basic model can be formulated as follows:

We consider an (unknown) complex-valued time signal consisting of a finite set of delta functions at the time points $t = t_1, \dots, t_K$

$$x(t) = \sum_{k=1}^K a_k \delta(t - t_k). \quad (1)$$

In the frequency domain the signal is

$$z(f) = \int x(t) \exp\{-j2\pi ft\} dt = \sum_{k=1}^K a_k \exp\{-j2\pi ft_k\}. \quad (2)$$

Suppose that the frequency signal is available only on a subset $\Omega \in \mathbb{R}$ and corrupted by noise $n(f)$:

$$y(f) = z(f) + n(f) \text{ for } f \in \Omega. \quad (3)$$

Ω may consist in the finite union of different frequency intervals (frequency bands). The basic problem is to estimate x - or equivalently $K, a_1, \dots, a_K, t_1, \dots, t_K$ - from y .

C. Discrete formulation

We consider a time grid $t_n = n\Delta t, n = 1, \dots, N$ and a grid $f_m, m = 1, \dots, M$ in the frequency domain with a Nyquist spacing Δf with respect to the time interval $M\Delta t$. Then, $z_m = \sum_{n=1}^N x_n \exp\{-j2\pi f_m t_n\}$ is the discrete Fourier transform of the unknown time signal x_1, \dots, x_N . Again, the frequency data can only be observed on a subset $\Omega \in \{1, \dots, M\}$ and are corrupted by noise:

$$y_m = \mathbf{z}_m + \mathbf{n}_m \text{ for } m \in \Omega. \quad (4)$$

In vector notation this can be written as

$\mathbf{z} = \mathbf{F}\mathbf{x}, \mathbf{y} = \mathbf{z}|_{\Omega} + \mathbf{n}$, where \mathbf{F} is the (generalized) Fourier matrix with the coefficients $\exp\{-j2\pi f_m t_n\}$ and $\mathbf{z}|_{\Omega}$ denotes the restriction of \mathbf{z} to indices contained in Ω .

Of course, the task of estimating \mathbf{x} from \mathbf{z} is in general only possible if further assumptions are made about the signal \mathbf{x} . In the context of this paper, we assume that \mathbf{x} is S -sparse, i.e. it has not more than S nonzero coefficients.

D. Consideration of the radar Tx signal

In the radar application, a signal \mathbf{s} is transmitted and reflected in the scene, which is characterized by the (unknown) vector \mathbf{x} of reflection amplitudes.

The deterministic part of the received signal is given by $\mathbf{z} = \mathbf{s} \star \mathbf{x}$, where ' \star ' stands for the convolution operator. To achieve equal lengths of \mathbf{s} and \mathbf{x} , the missing coefficients are filled with zeros.

If \mathbf{F} denotes the ordinary Fourier matrix, then

$$\mathcal{Z} = \mathcal{S} \odot \mathcal{X} + \mathcal{N} \quad (5)$$

where $\mathcal{Z} = \mathbf{F}\mathbf{z}$, $\mathcal{S} = \mathbf{F}\mathbf{s}$ and $\mathcal{X} = \mathbf{F}\mathbf{x}$ are the Fourier transforms of the corresponding signals. This is exactly the case if the convolution of the two signals is identical with the cyclic convolution, i.e., if \mathbf{x} is equal to zero in the initial and final sections.

We now consider the case where \mathcal{S} is concentrated over a set Ω of indices with non-vanishing coefficients, representing the frequency bands used. Then only the constraints on Ω are meaningful: $\mathcal{Z}|_{\Omega} = \mathcal{S}|_{\Omega} \odot \mathcal{X}|_{\Omega}$.

Let $\mathbf{D} = \text{diag}(\mathcal{S}|_{\Omega})$ and Θ be the restriction operator, i.e., the identity matrix whose rows are cancelled with indices outside Ω : $\mathcal{Z}|_{\Omega} = \mathbf{D}\Theta\mathcal{X}$.

Since the diagonal elements of \mathbf{D} are non-zero, this matrix is invertible, such that

$$\mathbf{y} = \mathbf{D}^{-1}\mathcal{Z}|_{\Omega} + \mathbf{n} \quad (6)$$

$$= \mathbf{A}\mathbf{x} + \mathbf{n} \quad (7)$$

with $\mathbf{A} = \Theta\mathbf{F}$.

\mathbf{y} represents the measurements after preprocessing by Fourier transform and inverse filter. The task is to estimate \mathbf{x} on the basis of \mathbf{y} . Of course if we have an estimator $\hat{\mathbf{x}}$ it is possible to complete the missing frequency data by $\hat{\mathcal{Z}} = \mathbf{F}\hat{\mathbf{x}}$.

The model 6 is applicable not only to the case of a radar with a transmitted signal whose spectrum is distributed over two or more subbands, but also to multiple RF units that are phase stable with each other. In this case, the baseband signals of these units can be combined into a single signal after compensating for the difference in LO frequencies in the processor.

III. IAA

We consider that a part of the transmitted signal used for scene detection is missing. The received echo after the detection may be sampled in the time domain or frequency domain. In order to estimate the missing samples in either domain, compressed sensing techniques are used under the assumption that the scene reflectivity is sparse.

The iterative adaptive approach (IAA) [6] is one such non-parametric high resolution spectral estimation method that uses an iterative weighted least square minimization based on the reflectivity of the scattering centers in the region of interest.

The MIAA [7] is a variation of IAA used to estimate missing data. It removes the requirement of having overlapping data segments in the available band, resulting in higher resolution and lower side-lobes. In [7], MIAA first uses IAA to obtain a spectral estimate from the given samples. This spectral estimate is then used to recover the missing time samples. Since the direct implementation of MIAA is computationally expensive, faster approaches have been studied in [8].

Our work addresses the frequency band fusion problem. Therefore, we consider that a part of the frequency band is missing and apply MIAA in order to estimate this missing band. We assume sparsity in the time domain and first obtain the time samples using IAA. Then these time samples are used to recover the missing frequency band.

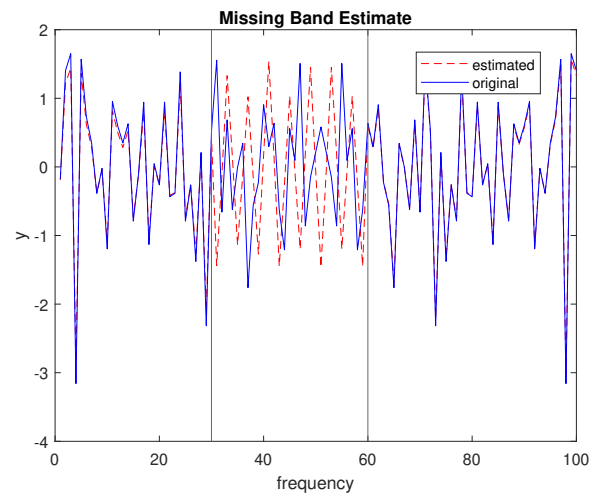


Fig. 1. Missing Frequency Band Estimate

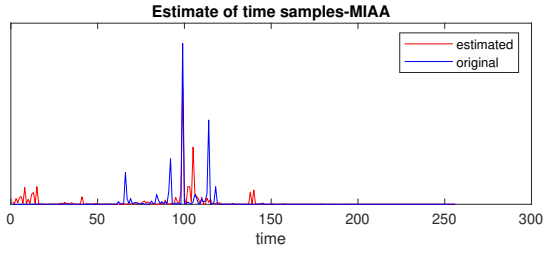


Fig. 2. Time Sample Estimate

Fig. 1 shows the frequency band estimated using MIAA. Fig. 2 shows the sparsity-based time sample estimation as a part of MIAA.

IV. AN APPROACH TO FUSION USING COMPRESSIVE SENSING

In this section, we will discuss an approach to solving the fusion problem using a compressive sensing algorithm called "Approximated Observation". In [9] this idea was presented for a SAR stripmap mode with undersampled data, but we will show that it also works with our band fusion problem. It is based on the "Iterative Shrinkage-Thresholding Algorithm (ISTA)" [10], which can be applied to the general CS problem for estimating \mathbf{x} from the underdetermined system of equations $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$ and consists of a relatively simple iteration:
Initialization: $it = 0$, initial estimate $\mathbf{x}^{(0)}$, given numbers μ and η
Iteration: repeat

$$\begin{aligned} \mathbf{x}^{(it+1)} &:= T_{\eta} \left(\mathbf{x}^{(it)} + \mu \mathbf{A}^H (\mathbf{y} - \mathbf{A}\mathbf{x}^{(it)}) \right) \\ it &:= it + 1 \end{aligned} \quad (8)$$

until a stopping criterion is met.

$$t_{\eta}(z) = \begin{cases} \frac{z}{|z|} (|z| - \eta) & \text{if } |z| > \eta \\ 0 & \text{else} \end{cases} \quad (9)$$

Here, t_{η} is the shrinkage-thresholding operator and $T_{\eta}(\mathbf{z})$ the vector with t_{η} applied to each coefficient of \mathbf{z} .

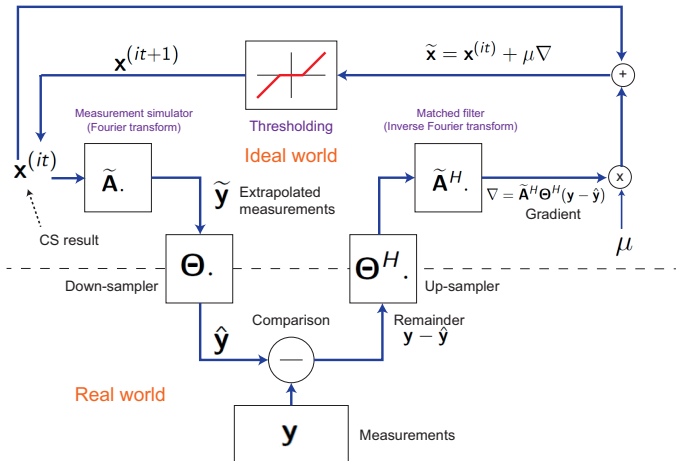


Fig. 3. The principal processing structure

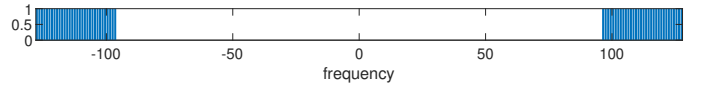


Fig. 4. The measured frequency bands

The advantage of this algorithm is that only matrix multiplications with \mathbf{A} and \mathbf{A}^H are performed. In the SAR environment, the application of \mathbf{A}^H is a traditional matched-filter SAR processor and that of \mathbf{A} is the inverse process, referred to in [9] as the "measurement simulator" since $\mathbf{A}\mathbf{x}$ produces the expected measurements for a given reflectance distribution \mathbf{x} . In our case, \mathbf{A} is the thinned Fourier matrix and \mathbf{A}^H corresponds to the inverse Fourier transform. This allows for fast processing compared to other algorithms since the FFT can be used.

The numbers η and μ influence the sparsity of the result and the convergence velocity. It is possible to adjust these parameters in an optimum way, see [11] for η and [12] for μ .

The proposed algorithm, illustrated in Fig. 3, can be vividly described as follows: We distinguish a part, which we will call 'real world', with missing data, and a part 'ideal world', where the data are assumed to be completely available. In the ideal world there exists a complete sensing matrix $\tilde{\mathbf{A}}$, which is related to the real matrix \mathbf{A} by $\mathbf{A} = \Theta \tilde{\mathbf{A}}$. The application of Θ corresponds to removing rows from $\tilde{\mathbf{A}}$ and is called a 'down-sampler'.

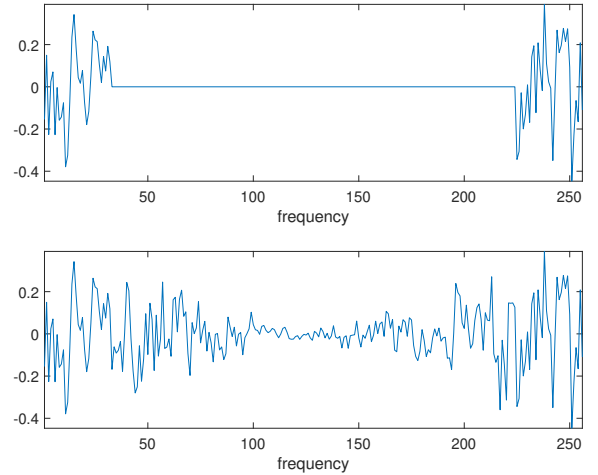


Fig. 5. Top: Real part of measured frequency data. Bottom: Real part of filled frequency data

Suppose that in the iteration step it , the estimate $\mathbf{x}^{(it)}$ exists. The application of $\tilde{\mathbf{A}}$ serves as 'measurement simulator' without missing measurements. The downsampler restricts $\tilde{\mathbf{y}}$ to the real usable part of the spectrum with the result $\hat{\mathbf{y}}$. This is compared with the actual measurements \mathbf{y} . The difference is sent through the 'up-sampler' Θ^H . The unavailable parts of the spectrum are filled with zeros. This is followed by an application of $\tilde{\mathbf{A}}^H$, which corresponds to a matched filter. The result is the gradient of the data fidelity terms $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2$. Multiplied by μ and added to $\tilde{\mathbf{x}}$, the renewal estimate is obtained. This is passed through the thresholder and yields the new estimator $\mathbf{x}^{(it+1)}$.

Incidentally, the filled spectrum \tilde{y} can be used as a starting point for a Fourier analysis to provide an estimate for x in a traditional way.

We have simulated the algorithms for a case of a large gap in the center of the whole frequency band, see Fig. 4. Only 64 of the 256 frequency points are used. The sparsity of the simulated signal was $S = 10$. Fig. 5 shows the measured (y) and the filled spectrum (\tilde{y}), in Fig. 6 the sparse signal (x), the matched filter response ($A^H y$), the result of the algorithm (\hat{x}), and the application of the matched filter to the filled spectrum ($\tilde{A}^H \tilde{y}$) are depicted. Fig. 7 shows the results of a Monte Carlo simulation for varying S and four different filling ratios. The quality measure was the magnitude of the correlation coefficient averaged over 5000 trials.

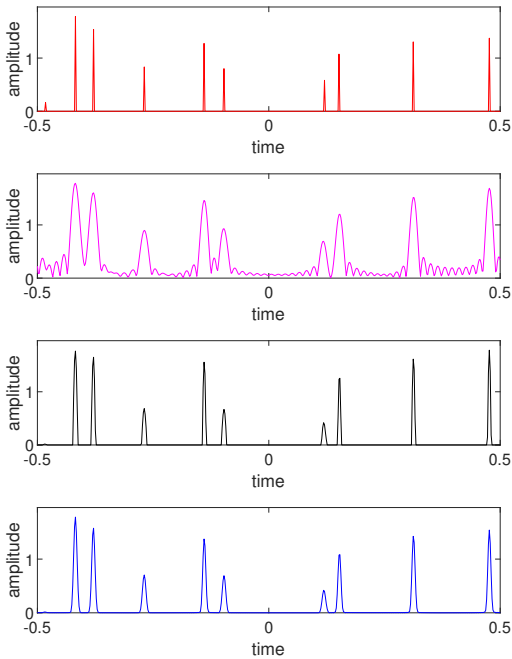


Fig. 6. From top to bottom: The sparse signal. Matched filter response with missing center frequencies. Sparse estimation. Matched filter response to filled frequencies

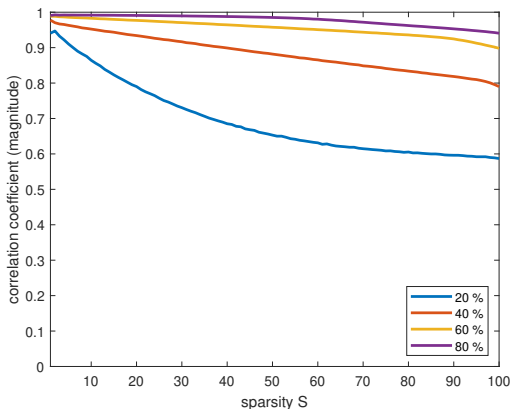


Fig. 7. Averaged magnitude of the correlation coefficient between the full Fourier transform of x and the filled spectrum \tilde{y} as function of the sparsity for four different sizes of the missing data. The percentages give the fraction of measured data to the full size M . $N = 512$, $M = 256$

V. CONCLUSION

In this paper, two algorithms for radar band fusion were presented. Simulation examples showed the principal feasibility of these approaches. A performance analysis for varying levels of sparsity and different sizes of missing data was also performed.

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REFERENCES

- [1] H. Griffiths, S. Blunt, L. Cohen, and L. Savy, "Challenge problems in spectrum engineering and waveform diversity," in *2013 IEEE Radar Conference (RadarCon13)*, 2013, pp. 1–5.
- [2] H. Griffiths, L. Cohen, S. Watts, E. Mokole, C. Baker, M. Wicks, and S. Blunt, "Radar spectrum engineering and management: Technical and regulatory issues," *Proceedings of the IEEE*, vol. 103, no. 14, pp. 85–102, 2015.
- [3] R. Aceska, J.-L. Bouchot, and S. Li, "Fusion frames and distributed sparsity," *Contemporary mathematics*, pp. 47–61, 2018.
- [4] H. Liu, B. Jiu, F. Li, and Y. Wang, "Attributed scattering center extraction algorithm based on sparse representation with dictionary refinement," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 5, pp. 2604–2614, 2017.
- [5] L. C. Potter, Da-Ming Chiang, R. Carriere, and M. J. Gerry, "A gtd-based parametric model for radar scattering," *IEEE Transactions on Antennas and Propagation*, vol. 43, no. 10, pp. 1058–1067, 1995.
- [6] T. Yardibi, J. Li, P. Stoica, M. Xue, and A. B. Baggeroer, "Source localization and sensing: A nonparametric iterative adaptive approach based on weighted least squares," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 1, pp. 425–443, 2010.
- [7] P. Stoica, J. Li, J. Ling, and Y. Cheng, "Missing data recovery via a nonparametric iterative adaptive approach," in *2009 IEEE International Conference on Acoustics, Speech and Signal Processing*, no. 12, 2009, pp. 3369–3372.
- [8] J. Karlsson, W. Rowe, L. Xu, G. Glentis, and J. Li, "Fast missing-data iaa with application to notched spectrum sar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 13, pp. 959–971, 2014.
- [9] J. Fang, Z. Xu, B. Zhang, W. Hong, and Y. Wu, "Fast compressed sensing sar imaging based on approximated observation," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 7, no. 1, pp. 352–363, Jan 2014.
- [10] I. Daubechies, M. Defrise, and C. De Mol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Communications on Pure and Applied Mathematics*, vol. 57, no. 11, pp. 1413–1457, 2004.
- [11] Z. Xu, X. Chang, F. Xu, and H. Zhang, " $l_{1/2}$ regularization: A thresholding representation theory and a fast solver," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 7, pp. 1013–1027, July 2012.
- [12] T. Blumensath and M. E. Davies, "Normalized iterative hard thresholding: Guaranteed stability and performance," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 298–309, April 2010.