Millimeter-Wave Channel Estimation using Alternating Direction Method of Multipliers

Aarab Mohamed Nassim, Chakkor Otman

Abstract: ***With the explosive growth in demand for mobile data traffic, the contradiction between capacity requirements and spectrum scarcity becomes more and more prominent. The bandwidth is becoming a key issue in 5G mobile networks. However, with the huge bandwidth from 30 GHz to 300 GHz, mmWave communications considered an important part of the 5G mobile network providing multi communication services, where channel state information considers a challenging task for millimeter wave MIMO systems due to the huge number of antennas. Therefore, this paper discusses the channel and signal models of the mmWave, with a novel formulation for mmWave channel estimation inclusive low rank features, that we improved using a developed theory of matrix completion with Alternating Direction Method.***

Keywords: millimeter wave; channel estimation; low rank approximations; matrix completion; ADMM.

I. INTRODUCTION

The propagation condition in millimeter wave bands is not very stable thanks to their great path loss, thus, channel estimation process is a very important task to ameliorates the estimation of the location of User equipment's, where the procedure of the channel estimation keeps being a challenging task thanks to the wide dynamic SNR ranges. Note that, even though the massive number of antennas at the base station will compensate the large and changing path loss, the channel must be estimated before this compensation is operative [1]. In other hand, the channel model is thought-about sparse in millimeter wave, where there are just a few paths due to the narrow beams produced by the high frequency carrier [2]. where there are several efficient channel estimation techniques for mm-Wave massive multiple antenna systems assuming the sparsity nature of the channel, like compressive sensing [3] and subspace methods [4]. Matrix completion base method is much attractive thanks to the low complexity methods and they require a reduced number of training sequences, as compared to other methods. Our approach combines compressive sensing and matrix completion using the alternating direction method of multipliers (ADMM) that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle. However, we tend to present a novel formulation to estimate mmWave for massive MIMO system inclusive low rank features, that we improve a developed theory of matrix completion by using an algorithm that based on the Alternating Direction Method for efficient recovery of large MIMO channel matrices.

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The main contributions of this work are as follows, first we explain the system and channel models, second a combined optimization for the channel estimation problem which we based on the alternating direction method of multipliers algorithm where it's formulated with the help of side information in matrix completion theory, then we propose an algorithm of the ADMM method, finally the simulation and results that explain the efficiency of the proposed algorithm, along with orthogonal matching pursuit (OMP), vector approximate message passing (VAMP) and the alternating direction method of multipliers (ADMM) in terms of mean squared error (MSE), number of paths and achievable spectral efficiency (ASE).

THE NOTATIONS OF THIS PAPER:

a, a and A	Vector, scaler and matrix.
A^H , A^T A^*	Conjugale transpose, matrix transpose and conjugale.
$\|(.)\|_*$, $\|(.)\|_l$ and $\|(.)\|_F$	Nuclear norm, $l_1 - norm$ and frobenius norm
Operands $\otimes$ and $\bigcirc$	Kronecker products, Matrix Hadamard
$\mathrm{vec}(.)$	Vectorisation of $(.)$.
$\mathrm{unvec}(.)$	Inverse operation of $\mathrm{vec}(.)$.
$\mathrm{E}\{.\}$	Expected value of {.}.
$\mathrm{diag}(.)$	Diagonal of (.).
I_N	$\mathrm{N} \times \mathrm{N}$ identity matrix.

II. SYSTEM AND CHANNEL MODELS

This system model equipped with Massive MIMO for N_T transmitters at the base station along with a combination of two consecutive joint segments: an analog RF precoder F_{RF} and digital MIMO baseband F_{BB} in other side at the receiver we have N_R antennas in the mobile user where the signal is processed by the consecutive joint segments of the RF combiner W_{RF} and baseband combiner W_{BB} , to get the closest estimation, at the receiver we employs N_R^{Beam} pilot beam patterns, Meanwhile, at the transmitter employs N_T^{Beam} pilot beam patterns, for empower communication, we use an analog Hybrid Beamforming in which by steering the beam to the receiver it opens a path for the transmitter to apply a baseband precoder.

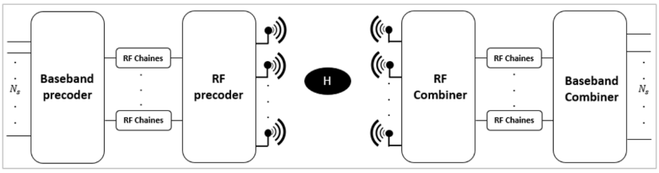

Fig. 1. Block structure of mmWave MIMO system.

As we assume the following small-scale fading model for the millimeter wave channel and consider the downlink mmWave transmission system [6]:

$$A = \frac{1}{\sqrt{L}} \sum_{k=1}^{K} \sum_{l=1}^{L} g_{kl} a_r(\emptyset_{kl}^r, \theta_{kl}^r) a_t^H(\emptyset_{kl}^t, \theta_{kl}^t) \quad (1)$$

Where:

- L is the number of rays within each cluster.
- K is the number of clusters.

The small-scale complex gain fading g_{kl} on the $(l-th)$ ray of the $(k-th)$ cluster follows a complex Gaussian distribution, which lead us to an alternative representation for the channel matrix A is based on the beam space model [7] that is defined as:

$$A = D_R Z D_T^H \quad (2)$$

Where:

- D_R and D_T are the unitary matrices.

III. PROBLEM FORMULATION

There are few dominant spatial paths in the millimeter wave channel, channel, where the channel can be built again by using the information of those paths due to the low scattering nature at millimeter waves. Training resources are required for obtaining the information paths, and thus we can reduce the training overhead. As such, we could solve the channel estimation problem by finding the angle of departure and the angle of arrival, and the path gains in the channel, along with the mean square error (MSE) which is related to the signal noise ratio (SNR) and symbol error rate (SER). Our propose is to formulate the CSI problem including estimating subset of the entries channel A, and recovering the full channel matrix by exploiting the low-rank nature using the alternating direction method of multipliers.

Define a sampling operator $P_\Psi(.)$ as

$$[P_\Psi(A)]_{i,j} = \begin{cases} [A]_{i,j}, & (i,j) \in \Psi \\ 0 & otherwise \end{cases} \quad (3)$$

Where:

- $[A]_{i,j}$denotes the (i,j)-*th* entry of **A.**
- Ψ represents the sampling domain.
- $N = pN_t N_r$ is the number of sampled entries of **A** in the operator $P_\Psi(.)$,
- *P is* sampling density.

To retrieval the elements of the channel **A**, where it's expanded by combining with the structure of the matrix **A** [8]. where the beamspace exemplification of **A** as its side-information; As we suppose that the unknown channel is defined as $A = D_R C D_T^H$.

So, for the joint recovery of the matrix A, we formulate it the following constrained Optimization Problem via the obscure sparse channel gain matrix C:

$$\underset{A,\,C}{minmize}\ \tau_A \|A\|_* + \tau_C \|C\|_1 \quad (4)$$

Where:

- A is the nuclear norm in the objective imposes its low rank properties.

As $\|\Psi\|_0 = M$ the placements of it's elements in a steady way from the set $\Psi \triangleq \{1,2,\dots,N_R N_T\}$.

Where the matrix A_Ψ represents the subsampled estimated channel matrix and contains M nonzero entries following the same pattern with Ψ. These entries are derived prior to the solution of (3), based on the training procedure which is described in the next subsection.

Clearly, A is the estimation error from (3) depends on the value of M and the estimation reliability of A_Ψ is elements. In other hand (2) introduce additional errors thanks to the angle discretization impact.

Estimation Error from (3) count on the value of M and the estimation efficiency of A_Ψ is elements, where (2) introduce additional errors thanks to the angle discretization effect.

IV. DETAILS OF THE PROPOSED ALGORITHM ADMM

Numerous algorithms can be employed to solve the MC problems. In this paper will review alternating direction method of multipliers (ADMM), which is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle [9].

This proposed method for the CSI of a MmWave MIMO system is described below, where the matrix Ψ has the non-zero uniformly distributed entries at their respective $ij-th$ position in such a way that $\Psi = \{1,2,3,\dots N_R N_T\}$. Thus, it could be saying the matrix Ψ has M ones and $(N_R N_T - M)$ zeros.

The error caused during the estimation of A lean on the estimation reliability of A_Ψ's elements and the M non-zero values of A_Ψ, where the Matrix Ψ is followed by a subsampled matrix A_Ψ. Therefore, the entries of A_Ψ is also followed by the entries of Ψ, so the positions of non-zero entries in A_Ψ are also similar to the positions on non-zero entries in Ψ. The threshold point, wherever the training symbols length are similar to the status of the non-zero entries in A i.e., T = M and $M \ll N_R N_T$, is considered as a stopping criterion for the proposed alternating direction scheme.

In other hand the alternating direction method of multipliers is the best efficient AOT's for resolving the stringent problems. thus, to get the solutions for Eq (4) we need to reformulate it and introduced two auxiliary matrices, $B \in C^{N_R \times N_T}$ and $D \triangleq B - D_R C D_T^H$. Hence, the new targeted problem expressed as:

$$\underset{A,B,C,D}{minmize}\ \tau_A \|A\|_* + \tau_C \|C\|_1 + \frac{1}{2}\|D\|_F^2 + \frac{1}{2}\|\Psi \circ B - A_\Psi\|_F^2 \quad (5)$$

$$Subject\ to\ A = B\ and\ D \triangleq B - D_R C D_T^H$$

Thence, we can write Eq (5) as the augmented Lagrangian function where:

$$L(A,B,C,D,Z_1,Z_2) \triangleq \tau_A\|A\|_* + \tau_C\|C\|_1 + \frac{1}{2}\|D\|_F^2 + \frac{1}{2}\|\Psi \circ B - A_\Psi\|_F^2 + tr\left(Z_1^H(A-B)\right) + \frac{\rho}{2}\|A-B\|_F^2 + tr\left(Z_2^H(B - D_R C D_T^H - D)\right) + \frac{\rho}{2}\|B - D_R C D_T^H - D\|_F^2 \quad (6)$$

In the Algorithm below we describe the channel estimation method, where we updated first the essential variable then at the last, we updated the dual variables, wherever it utilized to settle the function described in Eq (6) and it create its order as follows:

$$A^{(I+1)} = \underset{A}{argmin}\ L\,(A,,B^{(1)},C^{(1)},D^{(1)},Z_1^{(1)},Z_2^{(1)}) \quad (7)$$

$$B^{(I+1)} = \underset{B}{argmin}\ L\,(A^{(I+1)},B,C^{(1)},D^{(1)},Z_1^{(1)},Z_2^{(1)}) \quad (8)$$

$$C^{(I+1)} = \underset{C}{argmin}\ L\,(A^{(I+1)},B^{(I+1)},C,D^{(1)},Z_1^{(1)},Z_2^{(1)}) \quad (9)$$

$$D^{(I+1)} = \underset{C}{argmin}\ L\,(A^{(I+1)},B^{(I+1)},C^{(I+1)},D,Z_1^{(1)},Z_2^{(1)}) \quad (10)$$

$$Z_1^{(I+1)} = Z_1^{(1)} + \rho\alpha(A^{(I+1)} - B^{(I+1)}) \quad (11)$$

$$Z_2^{(I+1)} = Z_2^{(1)} + \rho\alpha(B^{(I+1)} - D_R C^{(I+1)} D_T^H - D^{(I+1)}) \quad (12)$$

Algorithm Proposed:

Input	$A_\Psi, \Psi, D_R, D_T, \rho, \alpha, \tau_A, \tau_C$ and I_{max}
Output	Estimating the output for the channel matrix $\hat{A} = A^{(I_{max})}$ **Introduction:** $A^{(0)} = B^{(0)} = C^{(0)} = D^{(0)} = Z_1^{(0)} = Z_2^{(0)} =$
Phase 1	**for** $1 = 0,1,2 \ldots \ldots I_{max} - 1$
Phase 2	Upgrade $A^{(I+1)}$
Phase 3	Upgrade $Z_1^{(I+\frac{1}{2})}$ and $Z_2^{(I+\frac{1}{2})}$
Phase 4	Upgrade $B^{(I+1)}$
Phase 5	Upgrade $C^{(I+1)}$
Phase 6	Upgrade $D^{(I+1)}$
Phase 7	Upgrade $Z_1^{(I+1)}$ and $Z_2^{(I+1)}$
Phase 8	end for

V. SIMULATION AND RESULTS

In this simulation we take the mean square error (MSE) as unite of measure, which the variants of the MSE are related to other important performance measures (symbol error rate (SER) and signal to interference plus noise ratio (SNR)) [10], in which we take $N_{T,R} = 60$ equipped with large-scale antenna arrays on both transmitter and receiver sides operating through 90 GHz mmWave channel, where the angles of departure and arrival have been created of Laplace distribution. As benchmark channel state information techniques, we have considered first the Orthogonal Matching Pursuit (OMP) to exploit only the sparsity of the channel matrix, second Our method the Alternating Direction of Multipliers (ADMM) to capitalizes the low rank property, and last the Vector Approximate Message Passing (VAMP), with adding the perfect channel estimation to compare to.

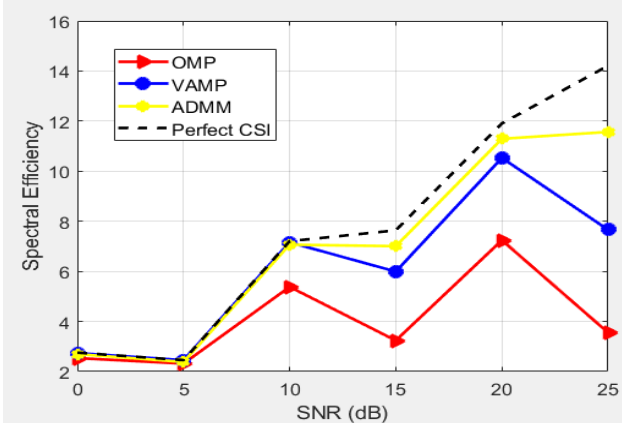

Fig.2. Achievable Spectral Efficiency transmit SNR for a 60 × 60 MIMO channel

As we discuss the theorized channel estimation techniques both in terms of Achievable Spectral Efficiency (ASE) and Normalized performance (MSE), for the Alternating Direction of Multipliers (ADMN) techniques that are based on MC; The performance of the Orthogonal Matching Pursuit and Vector Approximate Message Passing gets better at mid-to-high SNR points as the signal to noise ratio is increased, and it's moderate in terms of ASE at low-to-mid SNR points, As is clear from the Figure, but, as the SNR range increased from mid to high, the Alternating Direction of Multipliers (ADMM) outperformed OMP, VAMP, in which was very close to achieving the perfect CSI.

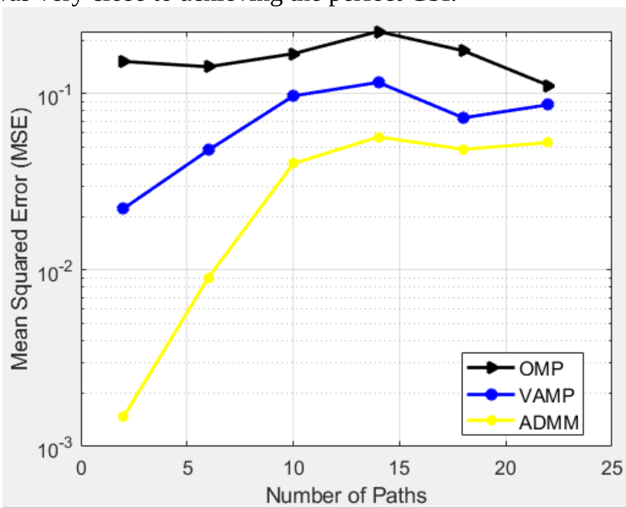

Fig. 3. Mean Squared Error for a 60 × 60 MIMO channel

In Fig. 3 we describe the impact of the number of mmWave channel propagation paths N_p with Normalized performance for $N_{T,R} = 60$ antennas of considered estimation techniques; As expected, we see in the figure the CSI estimation performance gets worse as number of paths improves, yet, the ADMM technique outperforms all others providing good MSE for quite large N_p values, which shown through the results of simulation in terms of ADMM algorithm offers faster convergence and an improved performance in terms of Mean Squared Error (MSE) for channel estimation with short training length, when compared with other techniques.

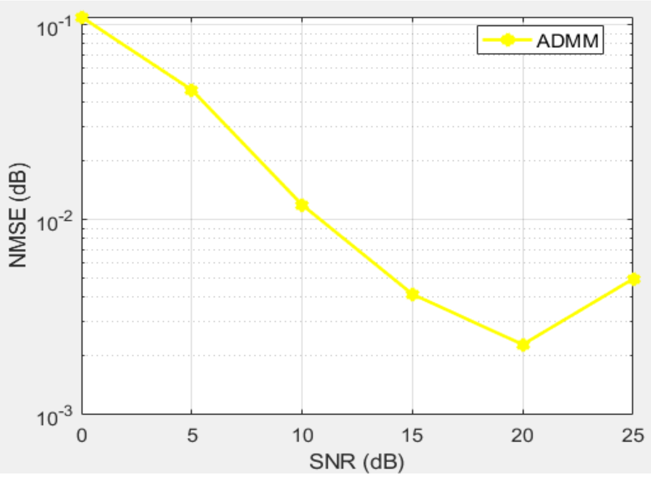

Fig. 4. NMSE with SNR for 60 × 60 MIMO channel

Finally, for this part of simulation, we present the derivation of post-processing Signal to Noise Radio (SNR) for Minimum Mean Squared Error (MMSE) receivers with imperfect channel estimates which shows an accurate indicator of the error rate performance of MIMO systems in the presence of channel estimation error; The simulation results shows that the noise effects are much bigger than the error due A_{Ψ} estimation where it will be more noisy and becomes less impactful and less severe as transmit SNR increases.

VI. CONCLUSION

This paper reviewed briefly the background and requirements of mmWave communication and discussed the extended version of ADMM that proposed for mmWave channel estimation due to the large path loss and poor penetration, with utilizing the low rank properties and the sparsity of the channel matrix, with all that, the estimated matrix completion leveraging jointly their low rank properties and sparsity and outperformed other algorithms in terms of ASE, MSE.

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