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Graceful Labelling for Complete Bipartite Fuzzy Graphs

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The concept of fuzzy graceful labelling is introduced. A graph which admits a fuzzy graceful labelling is called a fuzzy graceful graph. Fuzzy graceful labelled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications. In this paper the concept of fuzzy graceful labelling is applied to complete bipartite graphs. Also we discussed the edge and vertex gracefulness of some complete bipartite graphs.

Keywords: Fuzzy labelling; graceful labelling; fuzzy graceful labelling; fuzzy edge- vertex graceful labelling; fuzzy bipartite graph.

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1 Introduction

A mathematical framework to describe the phenomena of uncertainty in real life situation is first suggested by L. A. Zadeh [1] in 1965. Rosenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness [2]. There are many problems, which can be solved with the help of the fuzzy graphs.

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A fuzzy graph is the generalisation of the crisp graph. Therefore it is natural that many properties are similar to crisp graph and also it deviates at many places [3]. A fuzzy graph has ability to solve uncertain problems in a range of fields that's why fuzzy graph theory has been growing rapidly and consider it in numerous applications of various fields like mathematical sciences and technology [4] and [5].

Alfred L. Guiffrida, Rakesh Nagi [6] discussed the applications of fuzzy set theory in production management. Fuzzy set theory is now applied to problems in engineering, business, medical and health sciences and the natural sciences.

Fuzzy labelling has been studied by A. Nagoor Gani and D. Rajalaxmi (a) Subahashini [3] and [7] who discussed fuzzy labelling in magic graph. A research on fuzzy labelling has been witnessing an exponential growth, both within mathematics and in its applications in science and technology- especially in the fields information theory, neural networks, expert systems, cluster analysis, medical diagnosis, control theory, etc. [8] and [9]. Also fuzzy graceful labelling was discussed by R. Jebesty Shajila and S. Vimala [10] on wheel and fan graphs.

The concept of a graceful labelling has been introduced by Rosa [11] and many variations of graceful labelling have been introduced in recent years by researchers [12] and [13]. Graceful labelling for complete bipartite graphs has been discussed by V. J. Kaneria, H. M. Makadia, M. M. Jariya and Meera Meghapara [14].

This note is a further contribution on fuzzy labelling. Fuzzy labelling for some fuzzy complete bipartite graphs which satisfy fuzzy graceful labelling based on few conditions.

2 Preliminaries

Definition: 2.1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of UxV.

Definition: 2.2

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : VxV \rightarrow [0,1]$, where for all $u,v \in V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition: 2.3

A labelling of a graph is an assignment of values to the vertices and edges of a graph.

Definition: 2.4

A graceful labelling of a graph G with q edges is an injection $f: V(G) \rightarrow \{0, 1, 2, \ldots, q\}$ such that when each edge $xy \in E(G)$ is assigned the label $|f(x) - f(y)|$, all of the edge labels are distinct.

Definition: 2.5

A graph $G = (\sigma, \mu)$ is said to be a fuzzy graceful labelling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : VxV \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$.

Definition: 2.6

A graph $G = (V(G),E(G))$ is said to be **Bipartite** [15] if and only if there exists a partition $V(G) = V_1 \cup V_2$ and $V_1 \cap V_2 = \phi$. Hence all edges share a vertex from both set V_1 and V_2 and there are no edges formed between two vertices in the set V_1 and there are no edges formed between the two vertices in V_2.

Definition: 2.7

A graph G = (V(G), E(G)) is said to be **Complete Bipartite** if and only if there exists a partition V(G) = $V_1 \cup V_2$ and $V_1 \cap V_2 = \phi$ so that all edges share a vertex from both set V_1 and V_2 and all possible edges that join vertices from set V_1 to set V_2 are drawn.

We denote a complete bipartite graph as $K_{m,n}$ where m refers to the number of vertices in subset V_1 and n refers to the number of vertices in subset V_2.

Definition: 2.8

For any graph G, we define if all the points of G have the same degree r that is $\delta(G) = \Delta(G) = r$ then in this case G is called a regular graph of degree r.

3 Main Results

Definition: 3.1

A bipartite graph with fuzzy labelling is called a fuzzy bipartite graph.

Definition: 3.2

In a fuzzy complete bipartite graph if all vertices (edges) are distinct then it is called fuzzy vertex (edges) graceful labelling bipartite graph.

Theorem 3.3

Every Fuzzy Complete Bipartite graph $K_{m,n}$ is a fuzzy graceful labelling graph.

Proof:

Let Km,n be a complete bipartite graph with vertex group V = $\{V_1,V_2\}$ such that $V_1=\{u_1,u_2,u_3,...u_m\}$ and $V_2 = \{v_1,v_2,v_3,...v_n\}$, where $m \geq 2$ and $n \geq 2$.

Let $V\rightarrow(0,1]$ such that if the starting point of the vertex may be assigned (nm)/100, where $2 \leq m,n \leq 24$ and (nm)/1000, where $m,n \geq 2$ then the fuzzy complete bipartite graph we get would be a Fuzzy Graceful Labelling Complete Bipartite graph.

Case (i): m=n, where $2 \leq m,n \leq 7$

Let us take $\sigma(u_1) = (mn)/100$

If we assign the value of the edge

$\mu(u_1,v_n) = |\sigma(u_1) - \sigma(v_n)| = 0.01$,

then the fuzzy labelling will continue in the following order such as

$\mu(u_1, v_n) = 0.01$

$\mu(u_1, v_{n-1}) = 0.02$, etc.,

According to this order of fuzzy labelling, we get

$\mu(u_1,v_j) = [\, i^2 + (n\text{-}j) \,]/100$, where i=1 , j=1,2,...n , $2 \le n \le 7$

$\mu(u_2,v_j) = [\, i^2 + (j+(n\text{-}4)) \,]/100$, where i=2 , j=1,2,...n, $2 \le n \le 7$

$\mu(u_3,v_j) = [\, i^2 + (j+(2n\text{-}9)) \,]/100$, where i=3 , j=1,2,...n , $3 \le n \le 7$

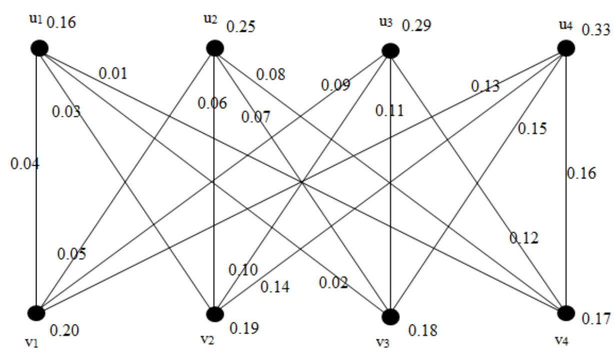
$\mu(u_4,v_j) = [\, i^2 + (j+(3n\text{-}16)) \,]/100$, where i=4 , j=1,2,...n , $4 \le n \le 7$, etc.,

$\mu(u_{n-1},v_j) = [\, i^2 + (j\text{-}1) \,]/100$, where i=n-1 , j=1,2,...n , $2 \le n \le 7$

$\mu(u_n,v_j) = [\, i^2 + (j\text{-}n) \,]/100$, where i=n , j=1,2,...n , $2 \le n \le 7$

Example: 3.4

When m=n, ie, m=4 and n=4



Fuzzy graceful labelling graph $K_{4,4}$

Case (ii): m< n , 2≤ m ≤ 6 and 3≤ n ≤ 24

Let us take $\sigma(u_1) = (mn)/100$

If we assign the value of the edge

$\mu(u_1,v_n) = |\sigma(u_1) - \sigma(vn)| = 0.01$,

then the fuzzy labelling will continue in the following order such as

$\mu(u_1,v_n) = 0.01$

$\mu(u_1,v_{n-1}) = 0.02$, etc.,

According to this order of fuzzy labelling, we get Fuzzy Graceful Labelling Complete Bipartite graph based on the following sub cases:

Sub case (i): m=2, 3≤ n ≤ 24

$\mu(u_1,v_j) = [\ i^2 + (n-j)\ ]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\ i^2 + (j+(n-2m))\ ]/100$, where i=2 , j=1,2,...n

Sub case (ii): m=3, 4≤ n ≤ 16

$\mu(u_1,v_j) = [\ i^2 + (n-j)\ ]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\ i^2 +(j+ (n-(m+1)))\ ]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\ i^2 +(j+(2n-3m))\ ]/100$, where i=3 , j=1,2,...n

Sub case (iii): m=4, 5≤ n ≤ 12

$\mu(u_1,v_j) = [\ i^2 + (n-j)\ ]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\ i^2 +(j+ (n-m)\ ]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\ i^2 +(j+(2n-(2m+1)))\ ]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\ i^2 +(j+(3n-4m))\ ]/100$, where i=4 , j=1,2,...n

Sub case (iv): m=5, 6≤ n ≤ 9

$\mu(u_1,v_j) = [\ i^2 + (n-j)\ ]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\ i^2 +(j+ (n-(m-1)))\ ]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\ i^2 +(j+(2n-(2m-1)))\ ]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\ i^2 +(j+(3n-(3m+1)))\ ]/100$, where i=4 , j=1,2,...n

$\mu(u_5,v_j) = [\ i^2 +(j+(4n-5m))\ ]/100$, where i=5 , j=1,2,...n

Sub case (v): m=6, 7≤ n ≤ 8

$\mu(u_1,v_j) = [\ i^2 + (n-j)\ ]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\ i^2 +(j+ (n-(m-2)))\ ]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\ i^2 +(j+(2n-(2m-3)))\ ]/100$, where i=3 , j=1,2,...n

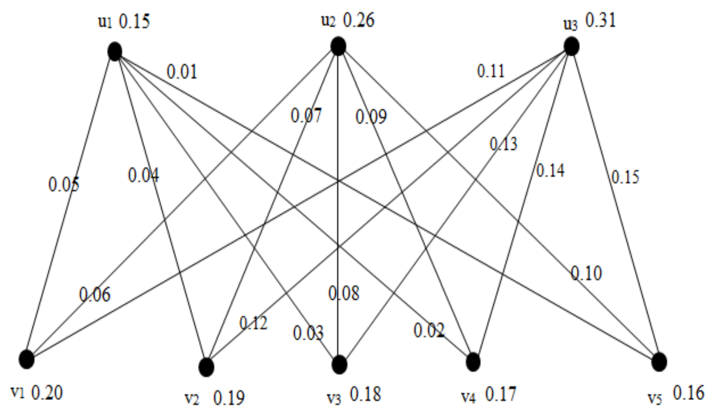
$\mu(u_4,v_j) = [\ i^2 +(j+(3n-(3m-2)))\ ]/100$, where i=4 , j=1,2,...n

$\mu(u_5,v_j) = [\ i^2 +(j+(4n-(4m+1)))\ ]/100$, where i=5 , j=1,2,...n

$\mu(u_6,v_j) = [\ i^2 +(j+(5n-6m))\ ]/100$, where i=6 , j=1,2,...n

Example: 3.5

When m<n ie, m=3 and n=5



Fuzzy graceful labelling graph $K_{3,5}$

Case (iii): m > n, 3≤ m ≤ 24 and 2≤ n ≤ 6

Let us take $\sigma(u_1) = (mn)/100$

If we assign the value of the edge

$\mu(u_1,v_n) = |\sigma(u_1) - \sigma(vn)| = 0.01$,

then the fuzzy labelling will continue in the following order such as

$\mu(u_1,v_n) = 0.01$

$\mu(u_1,v_{n-1}) = 0.02$, etc.,

According to this order of fuzzy labelling, we get Fuzzy Graceful Labelling Complete Bipartite graph based on the following sub cases:

Sub case (i): m=3, n = 2

$\mu(u_1,v_j) = [\, i^2 + (n-j)\,]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\, i^2 + (j-2)\,]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\, i^2 + (j-5)\,]/100$, where i=3 , j=1,2,...n

Sub case (ii): m = 4, n= 2,3

$\mu(u_1,v_j) = [\, i^2 + (n-j)\,]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\, i^2 + (j-(m-n))\,]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\, i^2 + (j-(2m-(2n-1)))\,]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\, i^2 + (j-(3m-(3n-4)))\,]/100$, where i=4 , j=1,2,...n

Sub case (iii): m= 5, 2≤ n ≤ 4

$\mu(u_1,v_j) = [\, i^2 + (n-j) \,]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\, i^2 +(j- (m-(n+1))) \,]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\, i^2 +(j-(2m-(2n+1))) \,]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\, i^2 +(j-(3m-(3n-1))) \,]/100$, where i=4 , j=1,2,...n

$\mu(u_5,v_j) = [\, i^2 +(j-(4m-(4n-5))) \,]/100$, where i=5 , j=1,2,...n

Sub case (iv): m=6, 2≤ n ≤ 5

$\mu(u_1,v_j) = [\, i^2 + (n-j) \,]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\, i^2 +(j- (m-(n+2))) \,]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\, i^2 +(j-(2m-(2n+3))) \,]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\, i^2 +(j-(3m-(3n+2))) \,]/100$, where i=4 , j=1,2,...n

$\mu(u_5,v_j) = [\, i^2 +(j-(4m-(4n-1))) \,]/100$, where i=5 , j=1,2,...n

$\mu(u_6,v_j) = [\, i^2 +(j-(5m-(5n-6))) \,]/100$, where i=6 , j=1,2,...n

Sub case (v): m=7, 2≤ n ≤ 6

$\mu(u_1,v_j) = [\, i^2 + (n-j) \,]/100$, where i=1 , j=1,2,...n

$\mu(u_2,v_j) = [\, i^2 +(j- (m-(n+3))) \,]/100$, where i=2 , j=1,2,...n

$\mu(u_3,v_j) = [\, i^2 +(j-(2m-(2n+5))) \,]/100$, where i=3 , j=1,2,...n

$\mu(u_4,v_j) = [\, i^2 +(j-(3m-(3n+5))) \,]/100$, where i=4 , j=1,2,...n

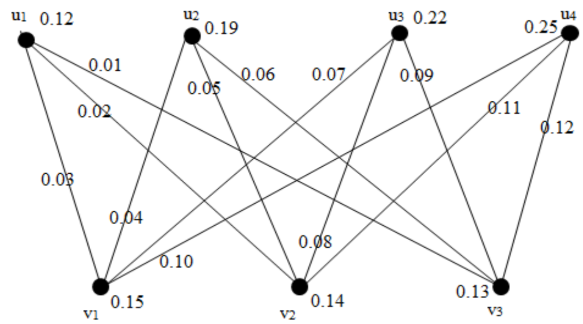
$\mu(u_5,v_j) = [\, i^2 +(j-(4m-(4n+3))) \,]/100$, where i=5 , j=1,2,...n

$\mu(u_6,v_j) = [\, i^2 +(j-(5m-(5n-1)) \,]/100$, where i=6 , j=1,2,...n

$\mu(u_7,v_j) = [\, i^2 +(j-(6m-(6n-7)) \,]/100$, where i=7 , j=1,2,...n

Example: 3.6

When m>n, ie, m=4 and n=3



Fuzzy graceful labelling graph $K_{4,3}$

In all the above cases Fuzzy Complete Bipartite graph $K_{m,n}$ is satisfied the conditions of fuzzy graceful labelling graph.

4 Remarks

The fuzzy complete bipartite graph will be a Fuzzy Graceful Labelling Complete Bipartite graph for the following conditions also.

(i) When m=8 and $2 \leq n \leq 6$
(ii) When m=9 and $2 \leq n \leq 5$
(iii) When m=10 and $2 \leq n \leq 4$
(iv) When m=11 and $2 \leq n \leq 4$
(v) When m=12 and $2 \leq n \leq 4$
(vi) When m=13 and n = 2,3
(vii) When m=14 and n = 2,3
(viii) When m=15 and n = 2,3
(ix) When m=16 and n = 2,3
(x) When m=17 and n = 2
(xi) When m=18 and n = 2
(xii) When m=19 and n = 2
(xiii) When m=20 and n = 2
(xiv) When m= 21 and n = 2
(xv) When m=22 and n = 2
(xvi) When m=23 and n = 2
(xvii) When m=24 and n = 2

5 Results

1. Every Fuzzy Complete Bipartite graph $K_{m,n}$ is a fuzzy regular graph when m=n.
2. Every Fuzzy Complete Bipartite graph $K_{m,n}$ is not a fuzzy regular graph when m>n and m<n

6 Conclusion

In this paper the concept of fuzzy edge(vertex) graceful labelling for complete bipartite graph has been introduced. We plan to extend our research work to fuzzy graceful labelling for even vertex and odd edges of complete bipartite graph.

Competing Interests

Authors have declared that no competing interests exist.

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