The paper presents an analytical solution to the problem of optimal dynamic balancing of the six-link converting mechanism of the sucker-rod pumping unit. This problem is solved numerically using a computer model of dynamics, namely by selecting the value of the correction factor k. Here we will consider an analytical method for solving this problem, that is, we find the location of the counterweight on the third link of the six-link converting mechanism for balancing. To solve the problem, we use the principle of possible displacement and write an equation where we express the torque through the unknown parameter of the counterweight. Further, such a value of the unknown parameter is found, at which the minimum of the root-mean-square value of torque M is reached. From the condition of the minimum of the function, we obtain an equation for determining the location of the counterweight. Thus, we obtain an analytical solution to the problem of optimal dynamic balancing of the six-link converting mechanism of the sucker-rod pumping drive in various settings.

According to the results, it was found that with the combined balancing method, the value of the maximum torque M and the value of the maximum power are reduced by 20 % than when the counterweight is placed on the third link of the converting mechanism, as well as when the value of the maximum torque is determined through the correction factor k.

In practice, balancing is carried out empirically by comparing two peaks of torque M on the crank shaft per cycle of the mechanism movement. Solving the analytical problem, we determine the exact location of the counterweight

Keywords: sucker-rod pump drive, converting mechanism, balancer, optimal balancing, dynamic synthesis

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ANALYTICAL SOLUTION OF THE PROBLEM OF DYNAMIC SYNTHESIS OF A SIX-LINK STRAIGHT-LINE CONVERTING MECHANISM OF THE SUCKER-ROD PUMPING DRIVE

Ayaulym Rakhmatulina
PhD*, Associate Professor**

Nurbibi Imanbayeva
Corresponding author
PhD, Associate Professor*
E-mail: imanbaevan@mail.ru

Sayat Ibrayev
Doctor of Technical Sciences, Professor*

Assemgul Uderbayeva
PhD, Associate Professor**

Aiman Nurmaganbetova
PhD, Associate Professor
Department of General Construction
Kazakh Leading Academy of Architecture and Civil Engineering
Ryskulbekova str., 28, Almaty, Republic of Kazakhstan, 050043

*Department of Mechanical Engineering and Robotics
Institute of Mechanics and Engineering Science
named after academician U. A. Dzholdasbekov KN MES RK
Shevchenko str., 28, Almaty, Republic of Kazakhstan, 050000

**Department of Automation and Robotics
Almaty Technological University
Tole bi str., 100, Almaty, Republic of Kazakhstan, 050012

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1. Introduction

Technological machines for various purposes are widely used in technology. In these machines, the transfer of energy from the engine to the working body is carried out through the mechanisms of levers. Modern technology increases the power of such machines, which leads to an increase in the speed of movement of the working bodies of the machines. In addition, inertial loads increase sharply in mechanisms, and the problem of equilibrium is of particular importance. The level of vibration of the machine depends on the quality of balancing, as well as its performance, reliability and accuracy of work, and the quality of technological processes.

There is a static and dynamic balance of the mechanism; their elimination in the designed mechanism will correspond to its static and dynamic balancing. Depending on the degree of balance, an exact or approximate solution (balancing) can be obtained.

Traditionally, the criterion for accurate static balancing of the mechanism is the condition that the main vector of inertia of its links is equal to zero

$$\bar{F}^{I}=0, \tag{1}$$

which corresponds to the immobility of the general mass center of the mechanism. With precise dynamic balancing, simultaneously with the abovementioned condition, it is also required to zero the main moment of the inertial forces of the links, i. e.

$$\bar{F}^{I}=0,\ M_{0}^{I}=0. \tag{2}$$

If it was possible for the mechanism in some way to satisfy the conditions of exact balancing, then these conditions will persist for any law of motion of the input link and, therefore, the balance of the mechanism (both static and dynamic) becomes an intrinsic quality of the mechanism [1].

Approximate balancing of the mechanism can be considered as an approximation to the precise one, if some secondary conditions can be neglected when solving a specific problem.

The tasks associated with reducing the swinging moment, the load on the gearbox and the efforts on the crank pins, which cause shocks in the drive mechanism of the rocker machine, do not have an analytical solution.

This allows us to assert that it is expedient to analytically solve the problem of optimal dynamic balancing of the converting mechanism of sucker-rod pumping units using the analytical method to reduce the required engine power.

The goal of balancing is to cancel or reduce variable dynamic responses. When solving the problem of balancing by adding counterweights, the method of selecting the value of the correction factor was used. Therefore, an analytical solution to the problem in this direction is relevant.

In practice, according to the results of the analytical solution of the problem of optimal balancing of the converting mechanism of sucker-rod pumping units, the location and the weight of the counterweight are determined, which provides the minimum swing moment to the foundation.

2. Literature review and problem statement

In [2], the general foundations of the structural analysis of mechanisms, features of kinematic analysis using Lagrange variables and dynamic analysis based on the energy model of mechanics with the fulfillment of the law of conservation of energy on any elements and for the mechanism as a whole for any time interval are given. But there were still unresolved issues related to balancing.

The paper [3] presents an optimization method for finding the link shapes for a dynamically balanced flat four-link mechanism. The force of shock and the moment of shock, arising in the mechanism due to inertia, are minimized due to the optimal distribution of the masses of the links. It is shown that using cubic B-spline curves, the link shapes are found and an optimization problem is formulated to minimize the percentage error in the resulting link inertia values, in which the control points of the B-spline curve are taken as design variables. Since in the paper the dynamic balancing of the mechanism is achieved due to the optimal distribution of the masses of the links, an analytical solution to the problem is not given.

The paper [4] considers the mechanism of balancing the upper roll significantly affecting the accuracy of rolling and the operating conditions of the pressure device. Balancing mechanism calculations are usually limited to the case of static balancing for some middle position of the mechanism. The dependence of the rebalancing coefficient on the position of the mechanism has been investigated; a refined calculation of the balancing device was carried out taking into account the friction forces and the dynamic operating conditions of the mechanism.

However, the mechanism operates in a dynamic mode, which means that large masses are accelerated and decelerated in short periods of time, and, in addition, friction forces in kinematic pairs have a significant effect on the mechanism operation. But the analytical approach to solving balancing problems is not considered.

The paper [5] presents the research results of the developed method of graphic synthesis, which allows determining the initial values of free parameters of the considered crank-slider mechanisms of the 3rd class, realizing the required cyclogram with an approximate height, the output link of which moves along the guide. Analytical expressions are obtained for calculating the parameters of the kinematic diagram of the mechanism. Multi-link crank-link mechanisms are used in crank presses. But there were still unresolved issues related to balancing the mechanisms of the 3rd class.

The paper [6] is devoted to the study of the peculiarities of using the analytical method of kinematic analysis of lever mechanisms. It is shown how to use a number of analytical methods based on obtaining formal mathematical expressions describing the position functions in the form of functions of the angles of rotation of the movable links or in the form of functions of movement of the characteristic points of the mechanism for the mathematical modeling of multi-link lever mechanisms. However, questions related to kinetostatic analysis and balancing of multi-link linkage mechanisms remained unresolved.

In [7], a dynamic system simulation is presented, compiled in Visual Basic V6.0, and the correctness and practicality of the simulation are verified by field measurements. To study the complex forces acting in a system, static and dynamic models of the injection pump, hermetic piston, and production pump are set up from the bottom up using a mechanical method, and the solution is found using the difference method. The drive mechanism is not considered here, as the research is carried out by a mechanical method.

The paper [8] shows that the inherently balanced communication architecture, based on four rods, is only part of a complete or grandiose four-rod based inherently balanced communication architecture that is presented here. It is shown that this architecture includes all related theories and they all depend on the principal vectors. Various new balanced relationships are also displayed. Here the problem of mechanism balancing is solved due to the balanced communication architecture and the analytical method for solving the problem is not considered.

The work [9] shows the modes of the frequency-elastic drive for the pumping system to reduce the peak loads on the polished rod and the total energy consumption. It is shown that the variable-frequency drive mode is a software solution for variable-speed drive systems that can be applied in the controller and does not require any hardware settings. The new drive mode adjusts the reference frequency transmitted by the controller to the frequency converter based on the actual power requirements. But the power consumption of the sucker-rod pumping drive mechanism is not given. The reason for this may be objective difficulties associated with the analytical solution of the problem of dynamic analysis and synthesis.

Based on the analysis of the kinematics, dynamics and running characteristics of the beam pumping unit, in [10] a full-fledged mathematical model of the engine, pumping unit, sucker rod and oil pump was created. The system of differential equations for pumping out the pumping unit used the method of cyclic iterations to solve the problem of strong adhesion between the engine, pumping unit, sucker rod and pump. The model is confirmed by experimental data on production pumping wells.

But there are still unresolved issues related to the reduction of the swinging moment, the load on the gearbox and the efforts on the crank fingers, which cause shocks in the drive mechanism of the rocking machine.

All this allows us to assert that it is expedient to conduct a study on the optimal dynamic balancing of the converting mechanism of sucker-rod pumping units by the analytical method to reduce the required engine power and its uniform load per cycle of movement and to determine the optimal values of the counterweight weight.

3. The aim and objectives of the study

The aim of the study is to solve the problem of optimal dynamic balancing of the six-link converting mechanism of sucker-rod pumping units by the analytical method to reduce the required engine power and uniform its load during the movement cycle and to determine the optimal values $l_{\Pi}=OL$ of the counterweight weight G_{Π} and the distance from the crank axis at which the minimum peak value of the balancing moment on the crank shaft is ensured. This will allow optimal placement of the counterweight to counterbalance the conversion mechanism of the sucker-rod pumping unit.

To achieve the aim, the following objectives were set:

– by selecting the value of the correction factor *k*, to solve the problem of balancing using a computer model of dynamics;

– to analytically solve the balancing problem, that is, find the location of the counterweight on the third link of the six-link converting mechanism and for which the principle of possible displacement is used;

– to analytically solve the problem of combined balancing, that is, placing the counterweight on the third link and on the crank of the six-link converting mechanism.

4. Research materials and methods

In [1], the movement of a six-link hinge-lever mechanism was investigated and the following problems were solved:

– the problem of kinematic analysis of a six-link rectilinear guiding converting mechanism; defining the functions of the mechanism positions;

– the problem of the power analysis of a six-link rectilinear guiding converting mechanism;

– determination of the reaction force in kinematic pairs. Also, a computer model for studying the kinematics and kinetostatics of the six-link rectilinear guiding mechanism of the SRP drive has been developed.

Based on these studies, we will solve analytically the problem of optimal dynamic balancing of the six-link converting mechanism of the sucker-rod pump drive.

The converting mechanism of the pumping unit, shown in Fig. 1, is a class II mechanism, which consists of a crank – 1, a double-drive group (2, 3) FCO, also attached to it double-drive group (4, 5) ABC. The working point is the suspension point *D* of the rod string.

The crank is affected at the point S_1 in the center of mass of the crank by G_1 – the weight of the crank and M_D – engine torque, the connecting rod is affected in the center of mass of the connecting rod by G_2 – the weight of the connecting rod. The third link is affected at the point S_3 by the force G_3 – the weight of the third link. The connecting rod 4 is affected in the center of mass of the connecting rod by G_4 – the weight of the connecting rod. G_5 – the weight of the fifth link operates on the 5th link at the center of mass, as well as load *P* (the weight of the rod string and pumped liquids) in the suspension point of the rod string *D*.

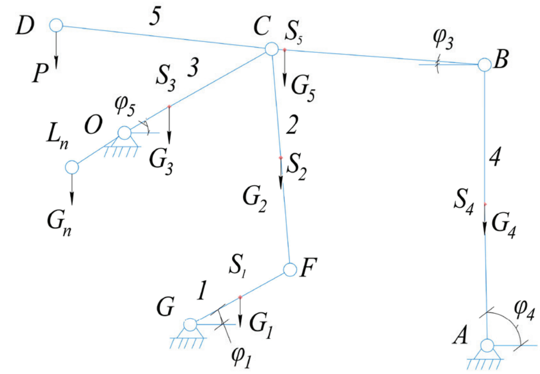

Fig. 1. Converting mechanism of the rod pump

The purpose of the balancing task is to minimize input torque *M* on the crank shaft. To do this, one needs to properly pick up the mass of the counterbalance and the distance of the center of the counterweight from the axis of rotation. In the case of rotary balancing, the counterweight is set on the crank. And in the case of combined balancing, a second counterweight is added, which is installed on the balancer.

The distance of the center of the counterweight mass from the axis of crank rotation is defined in the first approximation as:

$$OL=\frac{k\cdot H_E\left(P_{up}+P_{down}\right)}{4\cdot G_{\Pi}}, \tag{3}$$

where H_E – length of the rod string, P_{up}, P_{down} – loads at the point of rod suspension at the motion up and down, G – total weight of counterweights;

k – correcting coefficient that is manually entered by the user until the two peak values of torque M on the crank shaft will be equal.

Let's use the well-known principle of possible movements

$$\sum \delta A_i=0 \text{ or } \sum N_i=0. \tag{4}$$

According to the principle of possible movements, the power of these forces should be zero. Let's write this down for our problem:

$$\begin{gathered}\overline{G}_1 V_{s_{1i}}+\overline{G}_2 V_{s_{2i}}+\overline{G}_3 V_{s_{3i}}+\overline{G}_4 V_{s_{4i}}+\\+\overline{G}_5 V_{s_{5i}}+\overline{G}_P V_{s_{Di}}+\overline{G}_{\Pi} V_{L_i}+M_i\omega_i=0.\end{gathered} \tag{5}$$

Here, V_i are the velocities of the corresponding points of gravity forces application;

ω_i – angular speed of the crank;

M – torque on the crank shaft.

In this section, we consider an analytical method for solving this problem. The starting point is the principle of possible displacement, which is written in the form of expression (5).

Let us introduce the notations: $x_1=G_{\Pi}l_{\Pi}$, where $l_1=OL$.

Then

$$\overline{G}_{\Pi}\overline{V}_L=-G_{\Pi}V_L^y=-G_{\Pi}\cdot l_{\Pi}\cdot\cos\varphi_{3i}\cdot\omega_{3i}. \tag{6}$$

Next, we rewrite our expression (5), taking into account the introduced designation for each i-th position of the mechanism, i=1, ..., N

$$\overline{G}_1 V_{s_{1i}} + \overline{G}_2 V_{s_{2i}} + \overline{G}_3 V_{s_{3i}} + \overline{G}_4 V_{s_{4i}} + \overline{G}_5 V_{s_{5i}} + \\ + \overline{G}_P V_{D_i} - x_1 \omega_{3i} \cos\varphi_{3i} + M_i \omega_i = 0. \quad (7)$$

To find the unknown variable x_1 at which the minimum value of the balancing moment is achieved at full rotation of the crank, two methods are proposed.

2) From the obtained expression, we find M_i, expressing it in terms of the remaining values of the powers:

$$M_i = \frac{1}{\omega_1}[-\overline{G_1 V}_{s_{1i}} - \overline{G_2 V}_{s_{2i}} - \overline{G_3 V}_{s_{3i}} - \overline{G_4 V}_{s_{4i}} - \\ - \overline{G_5 V}_{s_{5i}} - \overline{G_P V}_{s_{Di}}] + x_1 \frac{\omega_{3i}}{\omega_1} \cos\varphi_{3i} = 0. \quad (8)$$

Let us introduce the following notations:

$$d_i = \begin{bmatrix} -\overline{G_1 V}_{s_{1i}} - \overline{G_2 V}_{s_{2i}} - \overline{G_3 V}_{s_{3i}} - \\ -\overline{G_4 V}_{s_{4i}} - \overline{G_5 V}_{s_{5i}} - \overline{G_P V}_{s_{Di}} \end{bmatrix}, \quad (9)$$

$$c_i = \frac{\omega_{3i}}{\omega_1} \cos\varphi_{3i}, \quad (10)$$

and rewrite the expression (8)

$$M_i = b_i + x_1 c_i. \quad (11)$$

Then the problem is reduced to finding the minimum of the S function depending on the x_1 variable

$$S(x_1) = \frac{1}{N}\sum_{i=1}^{N} M_i^2 \Rightarrow \min_{x_1}. \quad (12)$$

Thus, such an x_1 value is sought, at which the minimum of the root-mean-square value of torque M is reached.

As you know, in order to reach the minimum of a function, it is necessary that its first derivative be equal to zero, that is

$$\frac{dS}{dx_1} = 0, \quad (13)$$

$$\sum_{i=1}^{N} \frac{2\left(d_i c_i + c_i^2 x_1\right)}{N} = 0. \quad (14)$$

2) The essence of the second method is as follows.

Consider the problem of balancing, when the counterweight on the third link is displaced by the α_Π angle (Fig. 2).

Fig. 2 shows the third link, intermediate links are shown in dotted lines. Then, for the corresponding terms in the expression (15) we have

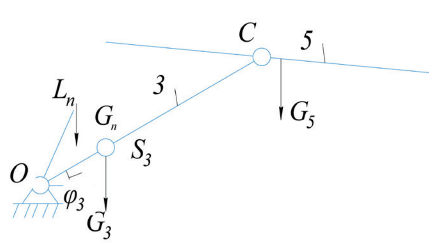

Fig. 2. Placement of additional weight on the third link

$$\overline{G_\Pi V_\Pi} = -\omega_{OCi} G_\Pi l_\Pi \cos\left(\varphi_{3i} + \alpha_\Pi\right) = \\ = -\omega_{OCi} G_\Pi l_\Pi \cos\alpha_\Pi \cos\varphi_{3i} + \omega_{OCi} G_\Pi l_\Pi \sin\alpha_\Pi \sin\varphi_{3i}. \quad (15)$$

Let us introduce variables:

$$x_1 = G_\Pi l_\Pi \cos\alpha_\Pi, \quad x_2 = G_\Pi l_\Pi \sin\alpha_\Pi \quad (16)$$

and notations:

$$c_i = \frac{\omega_{OCi}}{\omega_{GF}} \cos\alpha_{3i}, \quad s_i = \frac{\omega_{OCi}}{\omega_{GF}} \sin\alpha_{3i},$$

$$d_i = -\frac{1}{\omega_{GF}} \cdot \sum \overline{F}_i \overline{V}_i. \quad (17)$$

We rewrite the expression (11), with variables and notations introduced for each i-th position of the mechanism, i=1, ..., N.

$$M_i = d_i + x_1 \cdot c_i - x_2 \cdot s_i. \quad (18)$$

We find such an x_1 value, at which the minimum of the root-mean-square value of torque M is reached.

$$S(x_1, x_2) = \frac{1}{N}\sum_{i=1}^{N} M_i^2 \Rightarrow \min_{x_1, x_2}. \quad (19)$$

From the necessary condition for the minimum of the S function, we obtain two equations with two unknowns

$$\begin{cases} \sum_{i=1}^{N} c_i^2 \cdot x_1 - \sum_{i=1}^{N} c_i \cdot s_i \cdot x_2 = -\sum_{i=1}^{N} c_i \cdot d_i, \\ \sum_{i=1}^{N} s_i \cdot c_i \cdot x_1 - \sum_{i=1}^{N} s_i^2 \cdot x_2 = -\sum_{i=1}^{N} s_i \cdot d_i. \end{cases} \quad (20)$$

3) Now we are solving the problem of combined balancing, when the counterweight is placed not only on the 3rd link, but also on the crank. Let's try to solve analytically the problem of optimal dynamic balancing in general form. In addition, we assume that the counterweight on the crank is displaced by the third link angle and the counterweight on the third link is displaced by the α_Π angle (Fig. 3).

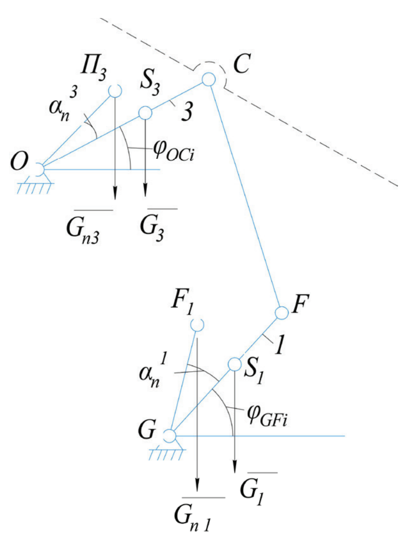

Fig. 3. The third link and the crank of the six-link mechanism are shown, the intermediate links are shown in dotted lines

With combined balancing, the principle of possible displacements is written as follows:

$$\overline{G}_1 V_{s_{1i}} + \overline{G}_2 V_{s_{2i}} + \overline{G}_3 V_{s_{3i}} + \overline{G}_4 V_{s_{4i}} + \overline{G}_5 V_{s_{5i}} + \\ + \overline{G}_P S_{D_i} + \overline{G_\Pi^3} V_{\Pi_i}^3 + \overline{G_\Pi^1} V_{\Pi_i}^1 + M_i \omega_{GF} = 0. \quad (21)$$

For the corresponding terms in the expression for the principle of possible displacements (21), we have

$$\overline{G_\Pi^1} V_{\Pi_i}^1 = -G_\Pi^1 \cdot l_\Pi^1 \cos\left(\varphi_{GF_i} + \alpha_\Pi^1\right) = \\ = -\omega_{GF} G_\Pi^1 \cdot l_\Pi^1 \cos\alpha_\Pi^1 \cos\varphi_{1i} + \\ + \omega_{GF} G_\Pi^1 \cdot l_\Pi^1 \cdot \sin\alpha_\Pi^1 \cdot \sin\varphi_{1i}, \quad (22)$$

$$\overline{G_\Pi^3} \overline{V}_{\Pi_i}^3 = -G_\Pi^3 \cdot l_\Pi^3 \cos\left(\varphi_{OC_i} + \alpha_\Pi^3\right) = \\ = -\omega_{OCi} G_\Pi^3 l_\Pi^3 \cos\alpha_\Pi^3 \cos\varphi_{3i} + \omega_{OCi} G_\Pi^3 l_\Pi^3 \sin\alpha_\Pi^3 \sin\varphi_{3i}. \quad (23)$$

Let us introduce variables

$$x_1 = G_\Pi^1 l_\Pi^1 \cos\alpha_\Pi^1, \quad x_2 = G_\Pi^1 l_\Pi^1 \sin\alpha_\Pi^1,$$

$$x_3 = G_\Pi^3 l_\Pi^3 \cos\alpha_\Pi^3, \quad x_4 = G_\Pi^3 l_\Pi^3 \sin\alpha_\Pi^3. \quad (24)$$

Then (16) takes the form:

$$M_i = \frac{1}{\omega_{GF}} \begin{bmatrix} -\overline{G}_1 V_{s_{1i}} - \overline{G}_2 V_{s_{2i}} - \overline{G}_3 V_{s_{3i}} - \\ -\overline{G}_4 V_{s_{4i}} - \overline{G}_5 V_{s_{5i}} - \overline{G}_P S_{D_i} \end{bmatrix} + \\ + x_1 \cos\varphi_{1_i} - x_2 \sin\varphi_{1_i} + \\ + x_3 \frac{\omega_{OCi}}{\omega_{GF}} \cos\varphi_{3_i} - x_4 \cdot \frac{\omega_{OCi}}{\omega_{GF}} \sin\varphi_{3_i} = 0. \quad (25)$$

Let us introduce notations

$$s_i = \sin\varphi_{1_i} \;\; c_i = \cos\varphi_{1_i}, \;\; a_i = \frac{\omega_{OCi}}{\omega_{GF}} \cos\varphi_{3_i},$$

$$b_i = \frac{\omega_{OCi}}{\omega_{GF}} \sin\varphi_{3_i}, \;\; d_i = -\frac{1}{\omega_{GF}} \sum \overline{G_i V_i}. \quad (26)$$

We rewrite the expression (26), with variables and notations introduced for each i-th position of the mechanism, i=1, ..., N, and derive

$$M_i = d_i + x_1 c_i - x_2 s_i + x_3 a_i - x_1 b_i. \quad (27)$$

We search for x_1–x_5 values, at which the minimum of the root-mean-square value of torque M is reached.

$$S(x_1, x_2, x_3, x_4) = \frac{1}{N} = \sum M_i^2 \to \min x_1, x_2, x_3, x_4. \quad (28)$$

From the necessary condition for the minimum of the S function

$$\begin{cases} \dfrac{\partial S}{\partial x_1} = 0 \\ \dfrac{\partial S}{\partial x_2} = 0 \\ \dfrac{\partial S}{\partial x_3} = 0 \\ \dfrac{\partial S}{\partial x_4} = 0 \end{cases},$$

we get four equations with four unknowns

$$\begin{cases} \sum\limits_{i=1}^{N} c_i^2 \cdot x_1 - \sum c_i \cdot s_i x_2 + \\ + \sum c_i \cdot a_i x_3 - \sum c_i \cdot b_i x_4 = \sum c_i d_i, \\ \sum\limits_{i=1}^{N} s_i \cdot c_i x_1 - \sum s_i^2 x_2 + \\ + \sum s_i \cdot a_i x_3 - \sum s_i \cdot b_i x_4 = \sum s_i d_i, \\ \sum\limits_{i=1}^{N} a_i \cdot c_i x_1 - \sum a_i \cdot s_i x_2 + \\ + \sum a_i^2 x_3 - \sum a_i \cdot b_i x_4 = \sum a_i d_i, \\ \sum\limits_{i=1}^{N} b_i \cdot c_i x_1 - \sum b_i \cdot s_i x_2 + \\ + \sum b_i \cdot a_i x_3 - \sum b_i^2 x_4 = \sum b_i d_i. \end{cases} \quad (29)$$

4) Assuming that M_i=M*=const, we introduce a new variable x_5=M^* and rewrite (28) as follows:

$$\frac{1}{\omega_{GF}} \sum_{i=1}^{N} \begin{pmatrix} d_i + x_1 \cdot c_i - x_2 \cdot s_i + \\ + x_3 \cdot a_i - x_4 \cdot b_i + x_5 \end{pmatrix} = 0. \quad (30)$$

The left part of (30) we designate as Δ_i and then, according to the synthesis condition, we find x_1–x_5 values, at which Δ_i approaches zero. For what, we will minimize the mean square value of the S function.

$$S(x_1, x_2, x_3, x_4, x_5) = \\ = \frac{1}{N} \sum \Delta_i^2 \to \min x_1, x_2, x_3, x_4, x_5. \quad (31)$$

From the minimum condition, we get five equations with five unknowns.

$$\begin{cases} \sum\limits_{i=1}^{N} c_i^2 \cdot x_1 - \sum c_i \cdot s_i x_2 + \sum c_i \cdot a_i x_3 - \\ - \sum c_i \cdot b_i x_4 + \sum c_i x_5 = -\sum\limits_{i=1}^{a} c_i d_i, \\ \sum\limits_{i=1}^{N} s_i \cdot c_i x_1 - \sum s_i^2 x_2 + \sum s_i \cdot a_i x_3 - \\ - \sum s_i \cdot b_i x_4 + \sum s_i x_5 = -\sum s_i d_i, \\ \sum\limits_{i=1}^{N} a_i \cdot c_i x_1 - \sum a_i \cdot s_i x_2 + \sum a_i^2 x_3 - \\ - \sum a_i \cdot b_i x_4 + \sum a_i x_5 = -\sum a_i d_i, \\ \sum\limits_{i=1}^{N} b_i \cdot c_i x_1 - \sum b_i \cdot s_i x_2 + \sum b_i \cdot a_i x_3 - \\ - \sum b_i^2 x_4 + \sum b_i x_5 = -\sum b_i d_i, \\ \sum\limits_{i=1}^{N} c_i x_1 - \sum\limits_{i=1}^{N} s_i x_2 + \sum\limits_{i=1}^{N} a_i x_2 - \\ - \sum\limits_{i=1}^{N} b_i x_2 + N \cdot x_5 = -\sum\limits_{i=1}^{N} d_i. \end{cases} \quad (32)$$

Thus, the unknown parameters x_1–x_4 and torque value x_4 are determined, where M_i=M*=const is considered as a constant parameter.

5. Results of the study of the balancing mechanism of rod pumping units by the analytical method

5. 1. Results of solving the balancing problem using a computer model of dynamics

By choosing the value of the correction factor k, the mass of the counterweight and the distance of the center of the counterweight from the axis of rotation were selected. The distance of the center of mass of the counterweight from the axis of rotation of the crank is determined in the first approximation by (1). In the 2nd column of Table 1, the balancing results are obtained through the correction factor, which adjusts the distance of the center of mass of the counterweight from the axis of rotation and is entered manually until the two peak values of shaft torque M become equal. Moore values change – is the torque shown in Fig. 4, *a*.

Also, changes in the values of torque M after determining the place of the counterweight are shown in Fig. 4, *b*, *c*.

5. 2. Results of analytical solution of the balancing problem

Unknown parameter of the counterweight. Further, the value of the unknown parameter is determined, at which the minimum of the root-mean-square value of torque M is achieved. From the condition of the minimum of the function, an equation is obtained to determine the location of the counterweight.

Solving the problem of optimal balancing, an expression was obtained that allows one to determine the mass and location of the counterweight along the 3rd link of the six-link converting mechanism of the rocking machine.

$$x_1 = \frac{-\sum_{i=1}^{N} d_i c_i}{\sum_{i=1}^{N} c_i^2} =$$

$$= \frac{\left[-\overline{G_1 V_{s_{1i}}} - \overline{G_2 V_{s_{2i}}} - \overline{G_3 V_{s_{3i}}} - \overline{G_4 V_{s_{4i}}} - \overline{G_5 V_{s_{5i}}} - \overline{G_P V_{s_{Di}}}\right] \cos \varphi_{3i}}{\omega_i \cos^2 \varphi_i}. \quad (33)$$

Table 1

Results of the analytical solution of the optimal balancing problem

ω_1=0.7, P_{up}/P_{down}=30/10 kN			
Results	SK_R2_02	1-method	2-method
M_{max}, kNm	8.260	8.412	8.365
N_{max} kNm	56.99	58.04	57.71
M_{min}, kNm	3.823	3.892	3.992
m_3, kg	478	479	478
l_3, m	0.525	0.653	0.753
α_3, degree	0	0.000	10.5414

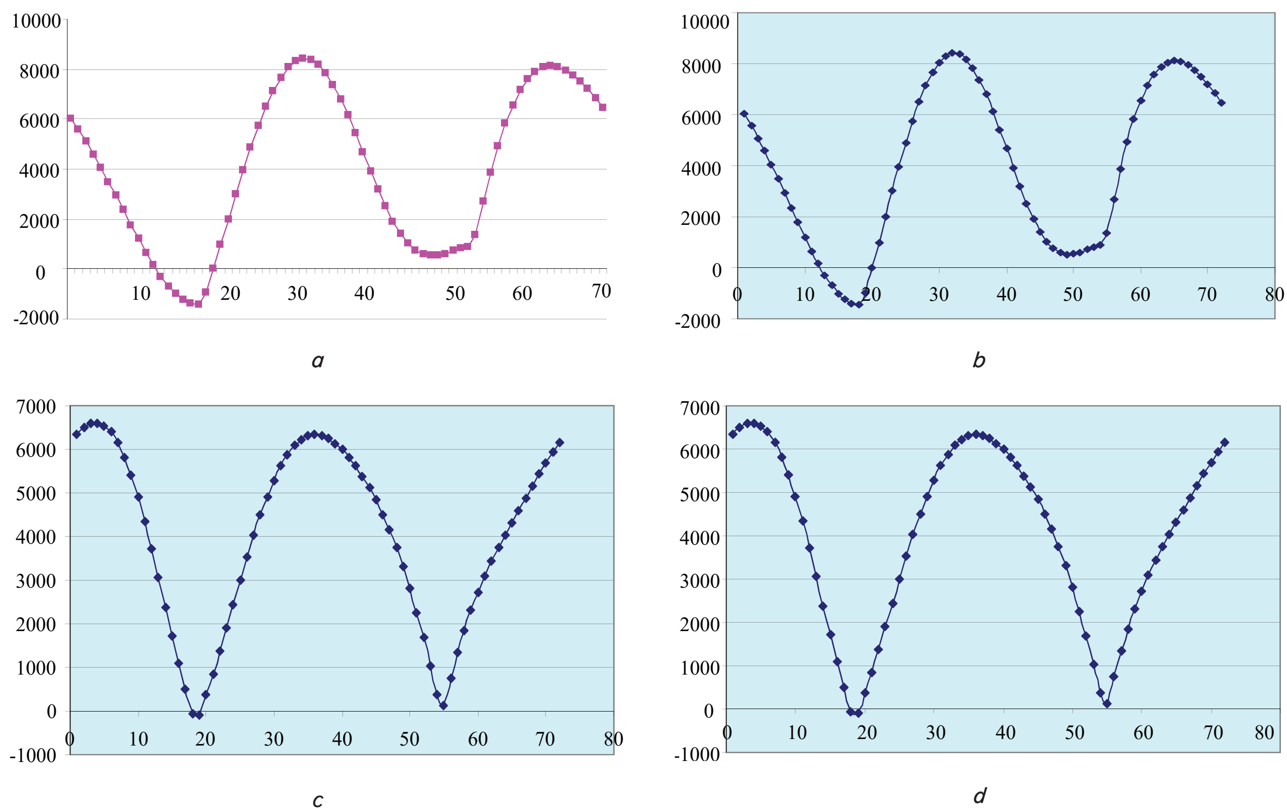

Fig. 4. Graphs of torque changes at optimal balancing: *a* — solving the balancing problem by choosing the value of the correction factor; *b* — solving by an analytical method; *c* — solving the combined balancing problem; *d* — solving the combined balancing problem when the counterweight on the third link is displaced by an angle and on the crank is displaced by an angle

The balancing problem is also analytically solved, when the counterweight on the third link is displaced by an angle and the unknown variables are

$$x_1 = G_\Pi l_\Pi \cos\alpha_\Pi, \quad x_2 = G_\Pi l_\Pi \sin\alpha_\Pi. \tag{34}$$

The analytical solution of the problem for $\det A \neq 0$ is obtained as follows

$$\overline{x} = A^{-1} \cdot \overline{d}, \tag{35}$$

where

$$A = \begin{bmatrix} \sum_{i=1}^{N} c_i^2 & -\sum_{i=1}^{N} c_i \cdot s_i \\ \sum_{i=1}^{N} s_i \cdot c_i & -\sum_{i=1}^{N} s_i^2 \end{bmatrix}, \quad \overline{d} = \begin{bmatrix} \sum_{i=1}^{N} c_i \\ \sum_{i=1}^{N} s_i \end{bmatrix}, \quad \overline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{36}$$

The 2nd column of Table 2 gives the result of the analytical method where the value of $G_\Pi l_\Pi$ is found, at which the minimum of the torque M value is reached.

The 3rd column of Table 2 gives the result of the analytical method for solving the balancing problem, when the counterweight on the third link is displaced by an angle.

Table 2

Results of the analytical solution of the optimal balancing problem

ω_1=0.7, P_{up}/P_{down}=30/10 kN		
Results	3-method	4-method
$M_{\max}$, kNm	6.605	6.605
$N_{\max}$ kNm	45.57	45.57
$M_{\min}$, kNm	3.992	3.992
m_3, kg	478	478
l_3, m	0.844	0.844
α_3, degree	8.5324	8.5233
m_1, kg	400	400
l_1, m	0.334	0.334
α_1, degree	–18.2414	–17.3421

5. 3. Results of the analytical solution of the combined balancing problem

An analytical solution to the problem of optimal dynamic balancing is obtained in general form, where the counterweight on the crank is displaced by the α_Π^1 angle, and the counterweight on the third link is displaced by the α_Π angle (Fig. 4).

From the condition for the minimum of the function (28), a system of equations was obtained to determine the location of the counterweight on the third link and on the crank (29).

An analytical solution to the problem of optimal dynamic balancing is obtained in general form, where the counterweight on the crank is displaced by the angle α_Π^1, and the counterweight on the third link is offset by the angle α_Π^3. Then the unknown variables are

$$x_1 = G_\Pi^1 l_\Pi^1 \cos\alpha_\Pi^1, \quad x_2 = G_\Pi^1 l_\Pi^1 \sin\alpha_\Pi^1,$$

$$x_3 = G_\Pi^3 l_\Pi^3 \cos\alpha_\Pi^3, \quad x_4 = G_\Pi^3 l_\Pi^3 \sin\alpha_\Pi^3. \tag{37}$$

The analytical solution of the problem for $\det A \neq 0$ is as follows

$$\overline{X} = A^{-1} \cdot \overline{d}, \tag{38}$$

where

$$A = \begin{bmatrix} \sum_{i=1}^{N} c_i^2 & -\sum c_i \cdot s_i & \sum c_i \cdot a_i & -\sum c_i \cdot b_i \\ \sum_{i=1}^{N} s_i \cdot c_i & -\sum s_i^2 & \sum s_i \cdot a_i & -\sum s_i \cdot b_i \\ \sum_{i=1}^{N} a_i \cdot c_i & -\sum a_i \cdot s_i & \sum a_i^2 & -\sum a_i \cdot b_i \\ \sum_{i=1}^{N} b_i \cdot c_i & -\sum b_i \cdot s_i & \sum b_i \cdot a_i & -\sum b_i^2 \end{bmatrix}, \tag{39}$$

$$\overline{d} = \begin{bmatrix} -\sum_{i=1}^{N} c_i \\ -\sum_{i=1}^{N} s_i \\ \sum_{i=1}^{N} a_i \\ \sum_{i=1}^{N} b_i \end{bmatrix}, \quad \overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}. \tag{40}$$

Also, assuming that the torque M_i=M*=const, we introduce a new variable $x_5=M^*$.

Similarly, the analytical solution of the problem for $\det A \neq 0$ is as follows

$$\overline{X} = A^{-1} \cdot \overline{d}, \tag{41}$$

where

$$A = \\ = \begin{bmatrix} \sum_{i=1}^{N} c_i^2 & -\sum c_i \cdot s_i & \sum c_i \cdot a_i & -\sum c_i \cdot b_i & \sum c_i \\ \sum_{i=1}^{N} s_i \cdot c_i & -\sum s_i^2 & \sum s_i \cdot a_i & -\sum s_i \cdot b_i & \sum s_i \\ \sum_{i=1}^{N} a_i \cdot c_i & -\sum a_i \cdot s_i & \sum a_i^2 & -\sum a_i \cdot b_i & \sum a_i \\ \sum_{i=1}^{N} b_i \cdot c_i & -\sum b_i \cdot s_i & \sum b_i \cdot a_i & -\sum b_i^2 & \sum b_i \\ \sum c_i & -\sum s_i & \sum a_i & -\sum b_i & N \end{bmatrix},$$

$$\overline{d} = \begin{bmatrix} -\sum_{i=1}^{N} c_i d_i \\ -\sum_{i=1}^{N} s_i d_i \\ -\sum_{i=1}^{N} a_i d_i \\ -\sum_{i=1}^{N} b_i d_i \\ -\sum_{i=1}^{N} d_i \end{bmatrix}, \quad \overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}. \tag{42}$$

Changes in the values of torque M after determining the locations of the counterweights are shown in Fig. 4, *d*.

6. Discussion of the results of solving the problem of balancing the six-link drive mechanism of the sucker-rod pumping unit

The results confirm that by the analytical solution of the problem, it is possible to more accurately determine the location of the counterweight when balancing the converting mechanism of the sucker-rod pumping unit.

As can be seen from Table 1, the result obtained when solving the balancing problem by selecting the value of the correction factor *k*, comparing two peak values of torque *M* on the crank shaft per cycle of movement of the mechanism, was also obtained when solving the balancing problem by the analytical method.

This proves the correctness of the analytical expressions obtained for the parameters of the counterweight of the transmission mechanism of sucker-rod pumping units.

Also, with the combined balancing method, the value of the maximum torque *M* and the value of the maximum power are reduced by 20 % than when the counterweight is placed on the third link of the converting mechanism, as well as when the value of the maximum torque is determined through the correction factor *k*. This shows the optimality of the combined balancing of the six-link converting mechanism of the SRP drive, which allows reducing the swinging moment, load on the gearbox and efforts on the crank pins, which cause impacts.

As you can see, the combined balancing significantly reduces the swinging moment on the foundation of the converting mechanism of sucker-rod pumping units, thereby ensuring optimal balancing of the mechanism. Therefore, the obtained analytical solutions of the balancing problem can be widely applied in practice.

The disadvantage of the combined balancing is the increased crank bearing response due to the weight of the counterweight.

In the future, a multicriteria synthesis of the six-link converting mechanism of the SRP drive will be carried out to reduce the upper strut and improve the angle of motion transmission. It is also required to reduce the values of the reactions of hinges and supports.

7. Conclusions

1. The mass of the counterweight and the distance of the center of the counterweight from the axis of rotation when solving the problem, by choosing the value of the correction factor k are m_3=478 kg and l_3=0.525 m. Maximum torque value $M_{\max}$=8.260 kNm.

2. An analytical solution to the problem of optimal dynamic balancing of the six-link converting mechanism of the sucker-rod pumping drive is obtained in various formulations. In the case when the counterweight is placed along the 3rd link, the following values are obtained m_3=479 kg and l_3=0.653 m, the value of the maximum torque $M_{\max}$=8.412 kNm. In the case when the counterweight on the third link is displaced by the angle, m_3=479 kg and l_3=0.653, the value of the maximum torque $M_{\max}$=8.365 kNm. Comparison of the results obtained shows that the value of the maximum torque and the location of the counterweight have insignificant differences when solving by choosing the value of the correction factor and when solving by an analytical method.

3. A system of equations is obtained for solving the problem of combined balancing, that is, when the counterweight is placed on the third link, displaced by an angle and on the crank, displaced by an angle. The following values are obtained m_3=478 kg and l_3=0.844 m, m_1=400 kg, l_1=0.334, the value of the maximum torque $M_{\max}$=6.605 kNm. The results show that with the combined balancing method, the value of the maximum torque *M* and the value of the maximum power are reduced by 20 % than when the counterweight is placed on the third link of the sucker-rod pumping drive converter mechanism.

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