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HARVESTING OPTIMISATION AND POST-HOC ANALISYS*

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Summary: *A formulated model of linear programming, which minimizes the total harvest and transport time for the observed farm company, is necessary for achieving the maximum profit from combine harvester rentals. Interdependent field operations have been mutually connected. The calculated optimal operating time showed that more human labour and machinery were used than necessary. This was later used for the post-hoc analysis which determined the profit from the combine harvester rentals for the observed period.*

Key words: *optimisation, maximum profit, minimum operating time, machinery pool.*

INTRODUCTION

Linear programming is one of the methods that enable the fulfilment of criteria related to planning the capacities and structure of machinery pool, as well as the exploitation of the machinery. Fokkens and Puylaert (1981) developed a mathematical model as a tool for organisation of harvest operation at a large scale grain farm. They created three types of variables in order to determine combine and transport capacity, transfers of combine harvesters and number of unloading pits for each crop. In the paper of Camarena et al. (2004), a programme for making decisions on investing in the machinery utilised in a multifarm system was presented based on the mixed integer programming. Ouhimmou et al. (2009) presented a comparative study of traditional decision making versus optimal decision making.

If there are showers, the greatest losses occur due to grain dispersal and if it rains slightly and repeatedly for a longer period of time the losses occur due to physiological processes in the grain. In Serbia, oilseed rape, wheat and winter barley are harvested during June and July when the probability of rainy days was in some years even 70% in the observed region (Đurić et al., 2010). From the aspect of machinery exploitation, they are most often used in September and October during sunflower and corn harvesting. However, on small farms, corn is harvested by corn pickers and corncobs are stored in the barns so there is no special need for renting a combine harvester. Certainly, this is not the case with small grains which is harvested by combine harvesters only, and which harvesting period is short because of the climatic conditions and variety characteristics. Therefore, main goal of this study is to minimize

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the operating time of the combine harvesters and tractors in June-July in order to perform harvesting on "Pobeda" farm as fast as possible. Moreover, this will lead to a significant profit from combine harvester rentals. This research specifies which type of combine harvester would operate on a specific parcel and which type of a trailer would be used for unloading the combine harvester bunker. It is assumed that, after being loaded, combine harvester bunkers are emptied into two trailers with load bearing capacity of 7 or 8 tons and then towed by a tractor. Moreover, the determined number of tractors is sufficient to provide one free tractor with two trailers which will carry the harvested crops as soon as the combine harvester bunkers are loaded. It was further assumed that there was no possibility to change the machinery pool and that available mechanization and human resources should be used to minimize the total operating time. Post-hoc analysis was additionally conducted in order to determine the profit from renting combine harvesters to small and medium farms in the observed period. In comparison to the cited papers, integer variables were not used in our mathematical model which reduced the model complexity. Also, interdependent field operations were additionally connected in the constraints.

MATERIAL AND METHODS

Notation and characteristics of machinery pool and operations necessary for optimisation

Notation			
t_{vpki}	total operating time per combine harvester or tractor type *v* with trailer type *p* at parcel *k* in operation *i* (h)	sl_{vk}	speed of combine harvester or tractor type *v* on the parcel *k* (km h^{-1})
s_{vpki}	operation or transport productivity per combine harvester or tractor type *v* with trailer type *p* at parcel *k* in operation *i* (ha h^{-1}), or (t h^{-1})	s_p	tractor speed with loaded trailer type *p*, (km h^{-1})
		w_v	working width of combine harvester type *v* (m)
a_k	surface area of parcel *k* (ha)	s	tractor speed on asphalt with empty trailers, (km h^{-1})
d_k	distance from parcel *k* to silos (km)	D	number of working days in the observed period
y_k	crop yields of parcel *k* (t ha^{-1})	W	number of operators on the farm company
tb_{vk}	time of bunker loading for combine harvester type *v* at parcel *k* (h)	tu_p	unload time of trailer type *p* (h)
n_v	number of combine harvesters or tractors type *v*	*Subscripts*	
m_p	number of trailers type *p*	v	combine harvester or tractor
wh	working hours per day (h)	p	trailer
ct_p	capacity of trailer type *p* (t)	k	parcel
c_v	bunker capacity of combine harvester type *v* (t)	i	operation (*i*=1 harvesting, *i*=2 crop transport)

Operating characteristics of combine harvesters JD (*v*=1), C (*v*=2) and Z (*v*=3) are given in Table 1.

Table 1: Speed sl_{vk} and working width w_v of combine type *v* on the parcel *k*, *v*□1,2,3,*k*□1,...,7 and transport speed s_p with loaded trailers type *p*, (*p*=1and *p*=2 for two trailers of 8 *t* and 7 *t*, respectivily)

Type of combine	JD	C	Z
Oilseed rape - field speed sl_{vk} , km h^{-1}	sl_{11}=5.5	sl_{21}=5.2	sl_{31}=4.2
Wheat - field speed sl_{vk} , km h^{-1}	sl_{12}=6	sl_{22}=5.7	sl_{32}=4.5
Winter barley - field speed sl_{vk} , km h^{-1}	sl_{13}=6.5	sl_{23}=6.2	sl_{33}=4.8
Working width w_v, m	w_1=6	w_2=6	w_3=5
Tractor speed s_p, km h^{-1}	s_1=14 and s_2=16		

The following will be valid in the sequel: when subscript *v* takes values 1, 2 or 3, then the only operation is harvest (*i*□1); when *v*□4, it will refer to tractor M performing the transport of the harvested crops (*i*□2).

Condition for the number of tractors used for harvested crop transportation. Unloading the grain from combine bunker to the tractor trailers is performed simultaneously with the harvesting (drive-by procedure) until the trailers are full. In order to avoid the situations when there is no free two trailer tractor for crop transfer after the bunker is loaded, it is necessary to fulfil the condition (1) which implies that the time necessary to load a bunker operating on the parcel k should be longer than the time necessary for a tractor with any type of trailer to leave the parcel, empty the load and return to the parcel k. Combine type v will always have the tractor with trailer type p available if the following is valid for every parcel k:

$$\min(tb_{vk}, v = 1,2,3) > \max\left(\frac{d_k}{s_p} + \frac{d_k}{s} + tu_p, p = 1,2\right) \qquad k = 1,2,...7. \tag{1}$$

Here, s_p is the speed of the tractor with two loaded trailers type p (Table 1), tb_{vk} is the loading time of a bunker for combine type v on the parcel k (Table 2); tu_p is the unloading time and s is the tractor speed on asphalt with empty trailers (Table 3). Table 2 shows the time tb_{vk} of bunker loading for combine type v on the parcel k which is obtained by

$$tb_{vk} = \frac{c_v \cdot 10}{w_v \cdot sl_{vk} \cdot y_k}, v = 1,2,3, k = 1,2,..7\,. \tag{2}$$

Here, c_v is the bunker capacity for combine type v (Table 2), sl_{vk} and w_v are speed and working width of combine type v on the parcel k (Table 2), and y_k stands for crop yields of the parcel k.

Table 2: Capacity c_v and loading time tb_{vk} of bunker for combine type v on the parcel k, p□1,2,k□1,2,..7

	c_v, t	Oilseed rape, h	Wheat, h	Winter barley, h
		k=1,2	k=3,4,5	k=6,7
JD	7	tb_{1k}=0.65	tb_{1k}=0.65	tb_{1k}=0.65
C	7	tb_{2k}=0.71	tb_{2k}=0.71	tb_{2k}=0.71
Z	5	tb_{3k}=0.83	tb_{3k}=0.83	tb_{3k}=0.83

Table 3: Two trailers of capacity ct_p, unloading time tu_p and tractor speed s on asphalt

capacity ct_1□2•8 t	capacity ct_2□2•7 t
unloading time,□h	
tu_1□□□ h	tu_2□□□ h
tractor speed on asphalt s□20 kmh^{-1}	

If the inequation (1) is not valid, which usually happens for more distant parcels, parcels with lower yield or combine harvesters with high harvesting speed, it is necessary to use more tractors and one driver per tractor in order to ensure that there will always be an available tractor with two trailers. Therefore, the required number of tractors nt_{vpk} with two trailers type p, p□1,2, that should be additionally employed and which would be used on every parcel k, k□1,2,...7, for the combine type v (if the combine operates on the parcel k), is:

$$\left\lceil \left(\frac{d_k}{s_p} + \frac{d_k}{s} + tu_p\right) \cdot c_v \Big/ (tb_{vk} \cdot ct_p) \right\rceil = nt_{vpk} \tag{3}$$

for every v, v□1,2,3 and p□1,2. Here, $\lceil x \rceil$ represents the smallest integer which is higher than x, while ct_p represents the capacity of trailer type p (Table 3), and c_v is the bunker capacity for combine type v (Table 2). One additional tractors are required on the parcel k=1,3,6, and 7 for all combine and both trailer type. On the 5-th parcel, there is no need for additional tractor in the case of combine Z with 7 t trailers, only.

Finally, Table 4 shows the productivity values s_{vpk1} of combine and s_{4pk2} of tractor needed for model (5-11).

Table 4: Combine productivity s_{vpk1} and tractor productivity s_{4pk2}, v=1,2,3, p□1,2,k□1,2,…7

Combine or tractor type		Harvesting productivity on the parcels					
		Oilseed rape k□1,2		Wheat k□□ ,4,5		Winter barley k□□ ,7	
JD	s_{1pk2}, ha h^{-1}	s_{1pk1}□3.3		s_{1pk1}□3.6		s_{1pk1}□3.9	
KJ	s_{2pk1}, ha h^{-1}	s_{2pk1}□□ .12		s_{2pk1}□□ .42		s_{2pk1}□□ .72	
Z	s_{3pk1}, ha h^{-1}	s_{3pk1}□2.1		s_{3pk1}□2.25		s_{3pk1}□2.4	
		Transport productivity from the parcels to silos					
		p□1	p□2	p□1	p□2	p□1	p□2

M	s_{4pk2}, t h^{-1}	s_{1pk1}□3.9	s_{1pk1}□3.9	s_{1pk1}□3.9	s_{1pk1}□3.9	s_{1pk1}□3.9	s_{1pk1}□3.9
		s_{2pk1}□□ .72	s_{2pk1}□□ .72	s_{2pk1}□□ .72	s_{2pk1}□□ .72	s_{2pk1}□□ .72	s_{2pk1}□□ .72
		s_{3pk1}□2.4	s_{3pk1}□2.4	s_{3pk1}□2.4	s_{3pk1}□2.4	s_{3pk1}□2.4	s_{3pk1}□2.4

Combine operating productivity s_{vpki} is calculated based on the equation (4) for combine type v and for tractor with trailers type p on the parcel k.

$$s_{vpk1} = \frac{w_v \cdot sl_{vk}}{10}, v = 1,2,3; k = 1,2,...7 \qquad \text{and} \qquad s_{4pk2} = \frac{ct_p \cdot s_p}{d_k},\ p = 1,2; k = 1,2,...7 \qquad (4)$$

Formulation of linear programme. The problem of linear programming was solved by using the software Mathematica (Wolfram, 2011). This software is applied on problems related to agriculture (Matić-Kekić et al., 2011) and optimisation (Savin et al., 2014). The advantage of this software is that one can easily handle the problems while programming if the problems include matrices, lists, numerical fitting, integral calculus, symbolic calculus, statistics, etc. The suggested model: 1) minimizes total harvesting time for a real situation in the farm company; 2) determines the type of combine and period during which it will be used on each parcel; 3) determines the type of trailer that will be used during the transport of crops from the parcel to silos.

Objective function. The objective function which minimizes the total operating time needed for harvest completion and transport operations during the harvesting agrotechnical period (D=20 days) is given in (5).

$$F = \min\left(\sum_{v=1}^{3}\sum_{p=1}^{2}\sum_{k=1}^{7} t_{vpk1} + \sum_{p=1}^{2}\sum_{k=1}^{7} t_{4pk2} \right) \qquad (5)$$

Here, t_{vpk1} represents the time that one combine type v, served by trailer type p on the parcel k, spends during the harvesting process. Time spent during the crop transport from parcel k to silos, using tractors with trailers type p, is denoted as t_{4pk2}. The objective function is subject to four types of constraints.

Constraint type Ia: Harvesting must be completed on all parcels. This type of constraint implies that all crops on the parcel k have to be harvested (operation i□1). Constraint for each parcel is needed

$$\sum_{v=1}^{3}\sum_{p=1}^{2} s_{vpk1} \cdot t_{vpk1} = a_k \qquad \text{for } k□1,2,...7 \qquad (6)$$

where s_{vpk1} (see equation (4) and Table 4) is operating productivity per combine type v, served by trailer type p on the parcel k during the harvest (i□1), and a_k is the surface area of parcel k.

Constraint type Ib: Harvested crops must be transported to silos. This type of constraint implies that all harvested crops have to be transported by tractors (v□4). The number of loaded pairs of trailers with the capacity of 8t (p=1) or 7t (p=2) must be equal to the number of trailers with the same capacities transported to the silos:

$$\sum_{v=1}^{3} y_k \cdot s_{vpk1} \cdot t_{vpk1} = s_{4pk2} \cdot t_{4pk2} \qquad \text{for } p□1,2 \text{ and } k□1,2, ...7, \qquad (7)$$

where y_k represents the crop yields of the parcel k and s_{4pk2} (Table 4) is the tractor productivity with loaded trailer type p on the parcel k during the transportation (i□2).

Constraint type IIa: Working hours of available combines and tractors must not be exceeded. Number of working days D in the observed period, multiplied by the number of working hours per day wh, gives maximum working hours for each combine or tractor. These constraints ensure that the total working hours of combines type v (first constraint in (8)) and tractors (second constraint in (8)) never exceed the limit. Restriction for each combine type v and for each tractor is represented by

$$\sum_{p=1}^{2}\sum_{k=1}^{7} t_{vpk1} \le n_v \cdot D \cdot wh \qquad \text{for } v□1,2,3 \text{ and } \qquad \sum_{p=1}^{2}\sum_{k=1}^{7}\sum_{v=1}^{3} \left(1 + nt_{vpk}\right) \cdot t_{vpk1} \le n_4 \cdot D \cdot wh \qquad (8)$$

where n_v is the number of combines type v; n_4 is the number of tractors; D is the number of working days in the observed period; nt_{vpk} is the number of tractors that should be additionally employed for the combines type v with trailer p on the parcel k, and wh is the number of working hours per day. Values of n_1, n_2, n_3 and n_4 are 1, 1, 2 and 15, respectively. The second inequation (8) shows the operating time of one tractor expressed as total harvest time t_{vpk1}, since tractors are involved in either transport and reloading during the harvest, or wait on the parcel until their trailers are loaded. The total operating time of a tractor should also include the operation times of additionally employed tractors which number is denoted by nt_{vpk}.

Constraint type IIb: Working hours of available trailers must not be exceeded. This constraint refers to the total working hours of trailers type p, on different parcels, serving the combines type v:

$$\sum_{v=1}^{3}\sum_{k=1}^{7}\left(1+nt_{vpk}\right)\cdot t_{vpk1} \le m_p \cdot D \cdot wh \qquad \text{for } p=1,2 \tag{9}$$

where m_p is the number of pairs of trailers type p (m_1=m_2=18).

Constraint type III: Total working hours of manpower must not be exceeded. This constraint implies that the total working hours per combine and tractor operator should never be exceeded. Then, the following constraint must be met:

$$\sum_{v=1}^{3}\sum_{p=1}^{2}\sum_{k=1}^{7} t_{vpk1}\cdot\left(2+nt_{vpk}\right) \le W \cdot D \cdot wh \tag{10}$$

where W is the number of needed combine and tractor operators. Since every combine needs to operate with at least one tractor for grain transport, in the brackets in inequation (10), is the sum of two operators (one combine and one tractor operator) with the number of additionally hired tractor operators (nt_{vpk}).

Constraint type IV: Unknown variables should be non-negative. This constraint is quite logical because the total time per combine type v, served by trailer type p, on the parcel k during the operation i cannot be negative:

$$t_{vpki} \ge 0 \qquad \text{for } v☐1,2,3,4, p=1,2,\ k=1,2,...,7 \text{ and } i=1,2. \tag{11}$$

RESULTS AND DISCUSSION

Objective function (5) with constraints (6-11) reaches minimum F_{min} = 252.5 h = 217.5 h (harvesting time) + 35 h (transport time from parcels to silos), with the involvement of all available combines at the farm (Table 5). Operation time for combines JD and C is 60 h, while two combines Z work together 97.5 h. For example, harvesting time for combine JD at parcel k=3 is 16.8 h. This combine is served by a tractor with two trailers capacity 8 tons which spends 3.4 h transporting grain from parcels to silos. Values of operating times t_{4pk2} for tractors M (Table 5), represent time spent during crops transport from parcel to silos, only. Tractors M, with two trailers of capacity 8 tons (7 tons), during transportation, spend 18.8 h (16.2 h).

Table 5. Total operating time required for harvesting and transport using all available resources at farm "Pobeda"

		Oilseed rape				Wheat						Winter barley				F_{min}
Parcel		k=1		k=2		k=3		k=4		k=5		k=6		k=7		(h) **252.5**
Trailers		16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	Total (h)
JD	t_{1pk1}	0.0	0.0	0.0	0.0	16.8	0.0	0.0	2.6	37.3	3.3	0.0	0.0	0.0	0.0	**60.0**
C	t_{2pk1}	0.0	0.0	0.0	0.0	0.0	0.0	12.1	0.0	0.0	0.0	0.0	7.8	0.0	40.1	**60.0**
Z	t_{3pk1}	50.5	0.0	9.5	0.0	0.0	37.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**97.5**
M	t_{41k2}	8.0	-	0.3	-	3.4	-	1.1	-	6.0	-	0.0	-	0.0	-	**18.8**
	t_{42k2}	-	0.0	-	0.0	-	4.7	-	0.3	-	0.5	-	2.3	-	8.4	**16.2**

Since working time is 10 hours per day and harvest on the farm is done for 60 working hours (the harvest organisation is given in Table 5), it follows that theoretically 6 working days is needed for harvest. However, it is not realistic because of the possibility of bad weather and malfunctions. It is important to consider the fact that not every working day can be used during the specified agrotechnical period. The number of working days is a function of climate conditions, soil type, terrain inclination and performed operation (ASAE D497.6 JUN2009).

Probability of working days spent during an agrotechnical period, represents a ratio between the numbers of working days during which the harvesting is performed and total number of available days in the specified agrotechnical period for a given region. The influence of climatic factors on the number of available working days in different regions was investigated by a considerable number of authors, such as Dyer (1980), Rotz et al. (1983), while in the Republic of Serbia important research was conducted by Nikolić (1983), Savin (2004) and Mileusnić et al. (2010). The real number of working days also depends on the *operational reliability* of combine harvester. Operational reliability is defined as the statistical probability that machine will function under specified conditions at any given time (ASAE D497.6 JUN2009). Probability of real working days spent during an agrotechnical period for the harvesting operation in the observed region was calculated to be 0.81. The operational reliability is high and it is 0.94 for considered type of combines. Harvest of oilseed rape, wheat and barley is the first one to be performed, since the soybean, sunflower and corn harvests are conducted later in autumn. Operational reliability is expected to be high since prior to the harvest combines are technically prepared. According to this, the product of probability of working days and operational reliability (here: *meteo-technical coefficient*) is 0.81·0.94□0.76. It increases theoretical number of working days (D^{opt}) to the real number of working days (D^{real}) needed for harvesting to, i.e. $D^{real} \approx D^{opt}/0.76$. Under the considered conditions (meteorological and technical), maximum real number of working days spent on combine harvesting, during an agrotechnical period, is $D \cdot 0.76 = 20 \cdot 0.76 \approx 15$ days.

Post-hoc analysis. Post-hoc analysis should give an answer to the question which organisation of the harvest and renting provides the highest profit. Organisation of the harvest and renting includes the following rules: 1. if the combine harvests in the considered farm, it cannot be rented until the harvest of the farm is completed; 2. if the combine is rented at the beginning of harvest, then it remains rented till the end of the harvest. In the example l=3 (Table 6), $n_2^u = 0$ means that combine C will be rented from the beginning of harvest for $D_2^{rent} = 15$ working days, while $n_1^u = 1$ and $n_3^u = 2$ mean that combine JD and two combines Z will harvest on parcels "Pobeda" for D^{opt}=8 working days (11 real days), firstly, and then will be rented for 7 working days ($D_1^{rent} = 7, D_3^{rent} = 2 \cdot 7 = 14$), while maximum theoretically number of days for renting are $D_1^{rent,\max} = 12$, $D_2^{rent,\max} = 20$ and $D_3^{rent} = 2 \cdot 12 = 28$. Suggested model (5-11) gave output values D^{opt} (Table 6) for all eleven values of input parameters n_1^u, n_2^u, n_3^u.

If no malfunctions occur in the period of 20 days, and if every day is suitable for the field work, then the maximum number of renting days, marked as $D_v^{rent,\max}$, is calculated for every combine type v. Maximum profit gained in this way is marked as $T_l^{rent,\max}$. The real profit from combine renting is marked as T_l^{rent}.

Table 6. Inputs: n_1^u, n_2^u, n_3^u are the number of combines type JD, C and Z operating in the parcels, respectively; Outputs: D^{opt} and D^{real} are the minimum number and real number of working days needed in order to complete field and transport operations on the farm, respectively; $D_v^{rent,\max}$ and D_v^{rent} are the maximum and real number of days for combine type v renting, respectively; $T_l^{rent,\max}$ and T_l^{rent} are the maximum and real total profit from combines renting for l-th example. Subscripts 1, 2 and 3 correspond to combines type JD, C and Z, respectively.

	l-th example					$D_v^{rent,\max}$			D_v^{rent}			$T_l^{rent,\max}$		T_l^{rent}
l	n_1^u	n_2^u	n_3^u	D^{opt}	D^{real}	v=1	v=2	v=3	v=1	v=2	v=3	dinars[a]	€	€
1	1	1	2	6	8	14	14	28	9	9	18	4,298,842	41,736	26,830
2	0	0	2	15	20	20	20	10	15	15	0	4,727,062	45,894	30,988
3	1	0	2	8	11	12	20	24	7	15	14	4,506,075	43,748	28,842
4	0	1	2	9	12	20	11	22	15	6	12	4,368,696	42,415	27,509
5	1	1	1	7	9	13	13	33	8	8	23	4,321,748	41,959	27,053
6	1	1	0	10	13	10	10	40	5	5	30	4,013,362	38,965	24,059
7	1	0	1	11	14	9	20	29	4	15	19	4,411,420	42,829	27,923
8	0	1	1	12	16	20	8	28	15	3	18	4,343,516	42,170	27,264
9	1	0	0	18	24	Harvest not completed in the agrotechnical period								
10	0	1	0	19	25	Harvest not completed in the agrotechnical period								
11	0	0	1	29	38	Harvest not completed in the agrotechnical period								

[a]One euro was equal to 103 dinars during the period June – July 2011.

Renting profit. In order to calculate the profit gained from renting, total expenses e_v^{rent} and renting price p_v^{rent} per combine type v per hour (dinars h^{-1}) must be taken into account. Therefore, the following factors were taken: renting price of 6000 dinars per hectare for one combine, agrotechnical period of 20 days for harvesting (from 1st July to 20th July), number of working days when harvesting is performed on the parcel (in the example l□1, there are 6 working days, Table 6). Other data needed for the calculation are: number of working hours per day (10), fuel consumption per combine (JD - 54 l h^{-1}, C - 49.5 l h^{-1} and Z - 32 l h^{-1}), considering that one litre of euro-diesel fuel costs 130 dinars, while one litre of diesel fuel costs 117 dinars. Sum of total (variable and fixed) costs per combine type v, which are the costs of fuel and lubricants, personal income of combine operator, amortization costs, costs for the maintenance of combine technical validity, insurance and loan costs are all included in e_v^{rent}, Savin (2004). Total expenses e_v^{rent} and renting price p_v^{rent} per combine type v per hour (dinars h^{-1}) are equal 10,588.51 and 21,600.00, 10,253.08 and 20,520, 8786.20 and 13,500.00 per combine JD, C and Z, respectively. The real profit from combine renting is calculated by $T_l^{rent} = \sum_{v=1}^{3} (p_v^{rent} - e_v^{rent}) \cdot D_v^{rent} \cdot wh$, for *l*-th example in Table 6. The least profit from combine renting is in the example *l*□6 (Table 6), when harvest can be completed in 13 real days (100 working hours) with combines JD and C, while the renting is performed during the remaining 7 real days (50 working hours), in comparison to the two combines Z which can be rented for 20 real days (2·15·10=300 working hours).

The highest profit can be achieved with the types of combines that have the greatest engine power and which are used for renting only, like in the example *l*□2. Inthat case, harvesting period on the farm company is prolonged 2.5 times. This example can hardly be considered as a global optimal solution due to the increased risk of bad weather in the period of 20 days during which harvest is performed.

Global optimal solution. Two conflicting requests: (**a**) to harvest crops on the "Pobeda" Farm Company as fast as possible and (**b**) to achieve the maximum profit from combine harvester rentals, can be met by three organisations of the harvest (basic parameters are given in Table 7) of all possible organisations of the harvest (Table 6). If it requires only (**a**), the optimal solution is given in Table 5. In the case that requires only (**b**), the optimal solution is given in Table 8. If both requests (**a**,**b**) need to be fulfilled, then global optimal solution is given in Table 9.

Table 7. Number of engaged combines JD^e, C^e, Z^e, tractors M^e, number of trailers, number of hired operators W^e, working hours wh^a per day and minimum days D^{opt} needed for the harvest completion at considered farm household

	JD^e	C^e	Z^e	M^e	8 t	7 t	*wh* (h)	W^e	D^{opt}
	n_1^u	n_2^u	n_3^u	n_4^u	m_1^u	m_2^u			
Request (**a**)	1	1	2	7	8	6	10	11	**6**
Request (**b**)	0	0	2	4	8	0	10	6	**15**
Requests (**a**,**b**)	1	0	2	6	6	6	10	9	**8**

[a]working shift is from 7am to 8pm

Optimal solution for the observed problem (5-11), with request (**a**), is given in Table 7. This solution suggests engagement of all available combines so that the harvest can be completed in the shortest period possible. However, this engagement still does not provide the maximum profit. Maximum profit (Table 6, *l*=2) can be achieved by the organisation of the harvest given in Table 8, but with harvesting period prolonged from 6 to 15 days.

Table 8. Total operating time required for harvesting and transport for the input parameters given in Table 7 at farm "Pobeda", request (b)

	Oilseed rape				Wheat						Winter barley				F_{min} (h)
Parcel	*k*□1		*k*□□		*k*□□		*k*□□		*k*□□		*k*□□		*k*□□		**321.1**
Trailers	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	Total (h)
JD t_{1pk1}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**
C t_{2pk1}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**
Z t_{3pk1}	50.5	0.0	9.5	0.0	64.4	0.0	22.7	0.0	64.9	0.0	12.1	0.0	62.1	0.0	**286.2**
M t_{41k2}	8.0	-	0.3	-	8.1	-	1.4	-	6.5	-	2.3	-	8.4	-	**35.0**
M t_{42k2}	-	0.0	-	0.0	-	0.0	-	0.0	-	0.0	-	0.0	-	0.0	**0.0**

Example from Table 9 could be considered as the most favourable organisation of the harvest, considering the two opposite goals (**a**,**b**): the fastest harvest on the parcel and the highest profit from renting.

Table 9. Total operating time required for harvesting and transport for the input parameters given in Table 7 at farm "Pobeda", requests (a,b)

	Oilseed rape				Wheat						Winter barley				Fmin (h)
Parcel	*k*=1		*k*=2		*k*=3		*k*=4		*k*=5		*k*=6		*k*=7		**272.1**
Trailers	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	16 t	14 t	Total (h)
JD t_{1pk1}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	34.4	0.0	7.4	0.0	38.2	0.0	**80.0**
C t_{2pk}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**

	1															
Z	t_{3pk} 1	0.0	50.5	9.3	0.2	0.0	64.4	22.7	0.0	0.0	9.9	0.0	0.0	0.0	0.0	**157.0**
M	t_{41k} 2	0.0	-	0.3	-	0.0	-	1.4	-	5.5	-	2.3	-	8.4	-	**17.9**
	t_{42k} 2	-	8.0	-	0.1	-	8.1	-	0.0	-	1.0	-	0.0	-	0.0	**17.2**

CONCLUSION

Linear programme for minimising total harvesting time was applied to “Pobeda” Farm Company from Vojvodina, Serbia, located in North Bačka district. Out of 1380 ha (soil type: clay loam), “Pobeda” Farm Company has: wheat on 342 ha (25%), winter barley on 178 ha (13%) and oilseed rape on 126 ha (9%). The developed programme is flexible since it can be easily modified for the examinations which include more parcels, combines, tractors, implements and operators.

One of the problem solutions shows that renting only the combines with good characteristics provides the highest profit, but then the number of days for harvesting increases 2.5 times in comparison to the minimum time needed for harvesting. For optimal solution can be taken the case when one JD and two Z combines harvest crops on the observed farm for 11 real days (80 working hours) and after that 9 real days (70 working hours) are rented. Similiar problem was solved by Savin et al. (2014) where the objective function included the risk of yield reduction due to bad weather conditions and the deliberate extension of the harvest on the primary farm. They created a general LP model and a profit maximization algorithm for harvesting during an agrotechnical period.

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OPTIMIZACIJA ŽETVE I POST-HOC ANALIZA

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Izvod: Formiran je model koji minimizira ukupno vreme žetve i vreme transporta useva do silosa u cilju dobijanja maksimalnog profita od iznajmljivanja kombajna i prodaje useva. Nezavisne operacije u polju su međusobno povezane. Dobijeno optimalno vreme rada potvrđuje da se koristi više ljudske snage i više poljoprivrednih mašina nego što je potrebno. Ovo je kasnije korišćeno u post-hoc analizi koja određuje profit od iznajmljivanja kombajna.

Ključne reči: optimizacija, maksimalan profit, optimalno vreme rada , mašinski park.

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