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Existence and Uniqueness of Positive Almost Periodic Solutions for a Class of Impulsive Lotka-Volterra Cooperation Models with Delays

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

This paper discusses an almost periodic Lotka-Volterra cooperation system with time delays and impulsive effects. By constructing a suitable Lyapunov functional, a sufficient condition which guarantees the existence, uniqueness and uniformly asymptotically stable of almost periodic solution of this system is obtained. A new result has been provided. A suitable example indicates the feasibility of the criterion.

Keywords: Lotka-Volterra cooperation model; impulse; delay; almost periodic solution.

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1 Introduction

It is well known that the Lotka-Volterra competitive systems with time delays and impulses have been studied extensively. Considerable work on the permanence, the extinction, the global asymptotic stability, the existence of periodic solutions and almost periodic solutions of autonomous, non-autonomous or nonlinear Lotka-Volterra competitive systems has been reported. See, for example, [1]-[21] and the references cited therein. Meanwhile, the books by Gopalsamy [22] and Kuang [23] are very good sources for dynamical behavior of Lotka-Volterra systems. For almost periodic models in impulsive ecological systems, Stamov *et al* have discussed the existence of almost periodic solutions [24]-[27]. In comparison with the Lotka-Volterra competitive system, fewer results of the Lotka-Volterra cooperation systems were considered in the literature [28, 29]. In [29], Wei and Wang have investigated the asymptotic behavior of the periodic solutions of the following Lotka-Volterra cooperation system with finite delays:

$$x_i^{'}(t) = r_i(t)x_i(t)[1 - \frac{x_i(t-\tau_{ii})}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_j(t)x_j(t-\tau_{jj})} - c_i(t)x_i(t-\tau_{ii})] \tag{1.1}$$

In order to investigate the role of impulses in control of asymptotic behavior of this cooperation system, Stamova has extended system (1.1) to an impulsive functional differential system as follows [28]:

$$\begin{cases} x_i^{'}(t) = x_i(t)[r_i(t) - \frac{x_i(t-\tau_{ii}(t))}{a_i(t)+\sum_{j=1,j\neq i}^{n} b_j(t)x_j(t-\tau_{jj}(t))} - c_i(t)x_i(t)], t \neq t_k; \\ x_i(t_k^+) = x_i(t_k) + I_{ik}(x_i(t_k)), t = t_k, i = 1,2,...,n, k = 1,2,... \end{cases} \tag{1.2}$$

where $x_i(t_k)(= x_i(t_k^-))$ and $x_i(t_k^+)$ are, respectively, the population densities of species i before and after impulse perturbation at the moment t_k; and I_{ik} are functions which characterize the magnitude of the impulse effect on the species i at the moments t_k. By means of piecewise continuous Lyapunov functions and Razumikhin technique, sufficient conditions for uniform asymptotic stability of a nonzero solution are obtained.

In this paper, by employing the piecewise continuous Lyapunov functional technique we shall investigate the existence and uniqueness of positive almost periodic solutions for the following impulsive Lotka-Volterra cooperation model:

$$\begin{cases} x_i^{'}(t) = x_i(t)[r_i(t) - \frac{d_i(t)x_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^{n} b_j(t)x_j(t-\tau_{jj})} - c_i(t)x_i(t-\tau_{ii})], t \neq t_k; \\ x_i(t_k^+) = x_i(t_k) + I_{ik}(x_i(t_k)), t = t_k, i = 1,2,...,n, k \in Z. \end{cases} \tag{1.3}$$

Let $\tau = \max\{\tau_{ij}\}$. We denote by $x(t) = x(t, t_0, \varphi)$ the solution of system (1.3), satisfying the initial conditions

$$x_i(s; t_0, \varphi) = \varphi_i(s) \geq 0, s \in [-\tau, 0], x_i(0^+; t_0, \varphi) = \varphi_i(0) > 0, i = 1,2,...,n, \tag{1.4}$$

where $\varphi = [\varphi_1, \varphi_2, ..., \varphi_n]^T$.

2 Preliminaries

Definition 2.1 ([30]) The set of sequences $\{t_k^l\} = \{t_{k+l} - t_k\}, k, l \in Z$ is said to be uniformly almost periodic if for any $\varepsilon > 0$, there exists a relatively dense set in R, i.e., for any $\varepsilon > 0, k \in Z$, there exists $q \in Z$ such that $|\tau_{k+q} - \tau_k| < \varepsilon$.

By $B = \{t_k | t_k \in R, t_k < t_{k+1}, k \in Z, \lim_{k\to\pm\infty} \tau_k = \pm\infty\}$, we denote the set of all sequences unbounded and strictly increasing with distance $\rho(t_k^1, t_k^2)$, the set $PC(R, R) = \{\varphi | R \to R$ is continuously differentiable everywhere except at the points $t_k, t_k \in B$ at which $\varphi(t_k^-)$ and $\varphi(t_k^+)$ exist, and $\varphi(t_k^-) = \varphi(t_k^+)\}$.

Definition 2.2 ([30]) The function $\varphi \in PC(R, R)$ is said to be almost periodic, if the following conditions hold:

(C1) The set of sequences $\{t_k^l\}, k, l \in Z$ is uniformly almost periodic;
(C2) For any $\varepsilon > 0$, there exists a real number $\delta > 0$, such that if the points t_1 and t_2 belong to same interval of continuity of $\varphi(t)$ and satisfy the inequality $|\varphi(t_1) - \varphi(t_2)| < \varepsilon$ if $|t_1 - t_2| < \delta$.
(C3) For any $\varepsilon > 0$, there exists a relatively dense set M such that if $r \in M$ then $|\varphi(t+r) - \varphi(t)| < \varepsilon$ for all $t \in R$ satisfying the condition $|t - t_k| > \varepsilon, k \in Z$. The elements of M are called ε-almost periods.

For system (1.3), we introduce the following conditions:

(H1) The functions $r_i(t), a_i(t), b_j(t), c_i(t), d_i(t) (i, j = 1, 2, ..., n)$ are all bounded continuous positive almost periodic functions in the sense of Bohr;
(H2) The set of sequence $\{t_k^l\} = \{t_{k+l} - t_k\}, k, l \in Z$ is uniformly almost periodic and there exists $l > 0$ such that $\inf\limits_{k \in Z} t_k^l = \theta > 0$.
(H3) $I_{ik} \in PC(R, R)$ and I_{ik} is uniformly almost periodic satisfying $|I_{ik}(u) - I_{ik}(v)| < L|u - v|$.

For convenience, let $\phi^+ = \sup\limits_{t \in R} \phi(t), \phi^- = \inf\limits_{t \in R} \phi(t)$ for a given bounded continuous function $\phi(t)$ defined on R. According to the condition (H1), it will be assumed that $r_i^-, a_i^-, b_j^-, c_i^-, d_i^- > 0$ $(i, j = 1, 2, ..., n)$. Corresponding to equation (1.3), first we consider the following almost periodic differential equation with delays and impulses:

$$\begin{cases} x^{'}(t) = x(t)f(t, x(t), x_t), t \neq t_k, k \in Z; \\ x(t_k^+) = x(t_k) + I_k(x(t_k)), t = t_k; \\ x(t) = \varphi(t), t \in [-\tau, 0]. \end{cases} \tag{2.1}$$

where $x_t = x(t+\theta), \theta \in [-\tau, 0], f : R \times \Lambda \times \Lambda \to R^n, \Lambda = \{x \in R^n : |x| \leq \upsilon, \upsilon > 0\}, \varphi(t) \in PC(R, R)$. The product system of (2.1) is the follows:

$$\begin{cases} x^{'}(t) = x(t)f(t, x(t), x_t), t \neq t_k, k \in Z; \\ x(t_k^+) = x(t_k) + I_k(x(t_k)), t = t_k; \\ x(t) = \varphi(t), t \in [-\tau, 0]; \\ y^{'}(t) = y(t)f(t, y(t), y_t), t \neq t_k, k \in Z; \\ y(t_k^+) = y(t_k) + I_k(y(t_k)), t = t_k; \\ y(t) = \psi(t), t \in [-\tau, 0]. \end{cases} \tag{2.2}$$

Define the Lyapunov functional $V(t, x, y)$ and calculate the upper right derivative of $V(t, x, y)$ along solutions of system (2.2):

$$V'_{(2.2)}(t, x, y) = \overline{\lim_{h \to 0^+}} \frac{1}{h}\{V(t + h, x_{t+h}(t, \varphi), y_{t+h}(t, \psi)) - V(t, x, y)\}. \tag{2.3}$$

Using Lyapunov functional, we first have

Theorem 2.1 Suppose that the Lyapunov functional $V(t, x, y)$ exists, and the following conditions hold

(i) $a(|x(t) - y(t)|) \leq V(t, x(t), y(t)) \leq b(|x(t) - y(t)|)$;
(ii) $D^+_{(2.2)}V(t, x, y) \leq -cV(t, x, y), t \neq t_k$, and $V(t_k^+, x(t_k) + I_kx(t_k), y(t_k) + I_ky(t_k)) \leq V(t, x(t_k), y(t_k)), t = t_k$.

where c is a positive constant, $a(v), b(v)$ are continuous and increasing functions, satisfying that $b(0) = 0$. In addition, assume that there exists a bounded solution $\xi(t)$ of system (2.1), such that

$\mid \xi(t) \mid \leq \upsilon$, then there exists a unique asymptotically stable almost periodic solution of system (2.1).

Proof: The proof is similar to Theorem 3.1 [24], we omit the details.

3 Main Results

Theorem 3.1 If conditions (H1)-(H3) hold. And furthermore assume that

(H4) For any $i, k, -1 < I_{ik}(x_i(t_k)) < 0$ and $x_i(t_k) + I_{ik}(x_i(t_k)) > 0$.

(H5) $c_i^- + \dfrac{d_i^-}{a_i^+ + \sum_{j=1,j\neq i}^n b_j^+ M_j} > \sum\limits_{i=1,j\neq i}^{n} \dfrac{d_i^+ b_j^+ M_i}{(a_i^- + \sum\limits_{k=1,k\neq i}^{n} b_k^- m_k)^2}$,

where $M_i = \frac{r_i^+ \exp(r_i^+ \tau)}{c_i^-}, m_i = \frac{a_i^- r_i^-}{d_i^+ + a_i^- c_i^+}, (i = 1, 2, ..., n)$.

Then there exists a unique asymptotically stable almost periodic solution of system (1.3).

Proof: Firstly, we shall prove that there exists a bounded positive solution of system (1.3) and (1.4). From the point of view of biology, in the sequel, we assume that $x(t_0) > 0, t_0 \geq 0$. Then it is easy to see that, for given $x(t_0) > 0$, the systems (1.3) and (1.4) have a positive solution $x(t)$ passing through (t_0, φ) for $t \in R^+$. This is due to that

$$\begin{aligned} x_i(t) = x_i(t_0) \exp\{\textstyle\int_{t_0}^{t} [r_i(s) - \frac{d_i(s) x_i(s-\tau_{ii})}{a_i(s) + \sum_{j=1,j\neq i}^n b_j(s) x_j(s-\tau_{jj})} - c_i(s) x_i(s-\tau_{ii})] ds\} \\ > 0, t \neq t_k, i = 1, 2, ..., n. \end{aligned} \tag{3.1}$$

On the one hand, from

$$x_i^{'}(t) \leq r_i(t) x_i(t), t \neq t_k, i = 1, 2, ..., n. \tag{3.2}$$

Both sides integrating from $t-\tau_{ii}$ to t of (3.2) we get $x_i(t) \leq x_i(t-\tau_{ii}) \exp(r_i^+ \tau_{ii})$ or $x_i(t) \exp(-r_i^+ \tau_{ii}) \leq x_i(t-\tau_{ii}), t \neq t_k, i = 1, 2, ..., n$. So from system (1.3) we have

$$x_i^{'}(t) \leq x_i(t)[r_i^+ - c_i^- \exp(-r_i^+ \tau_{ii}) x_i(t)], t \neq t_k, i = 1, 2, ..., n. \tag{3.3}$$

Eq.(3.3) is a Bernoulli's equation, it is easily to get

$$x_i(t) \leq \frac{x_i(0) r_i^+}{r_i^+ \exp(-r_i^+ t) + x_i(0) c_i^- \exp(-r_i^+ \tau_{ii})(1 - \exp(-r_i^+ t))}, t \neq t_k, i = 1, 2, ..., n. \tag{3.4}$$

Thus, we obtain

$$\limsup_{t\to\infty} x_i(t) \leq \frac{r_i^+ \exp(r_i^+ \tau_{ii})}{c_i^-} = M_i, t \neq t_k, i = 1, 2, ..., n. \tag{3.5}$$

On the other hand, let $x_i(t) = \frac{1}{u_i(t)}$, we have

$$u_i^{'}(t) = u_i(t)[-r_i(t) + \frac{d_i(t)}{[a_i(t) + \sum_{j=1,j\neq i}^n \frac{b_j(t)}{u_j(t-\tau_{jj})}] u_i(t-\tau_{ii})} + \frac{c_i(t)}{u_i(t-\tau_{ii})}], t \neq t_k, i = 1, 2, ..., n. \tag{3.6}$$

From (3.6) we get

$$u_i^{'}(t) \leq u_i(t)[-r_i^- + \frac{d_i^+}{a_i^- u_i(t-\tau_{ii})} + \frac{c_i^+}{u_i(t-\tau_{ii})}], t \neq t_k, i = 1, 2, ..., n. \tag{3.7}$$

Noting that as t is sufficiently large, $u_i(t) \sim u_i(t-\tau_{ii})$. Thus we have

$$\liminf_{t\to\infty} u_i(t) \le \frac{d_i^+ + a_i^- c_i^+}{a_i^- r_i^-}, t \neq t_k, i = 1,2,...,n. \tag{3.8}$$

or

$$\liminf_{t\to\infty} x_i(t) \ge \frac{a_i^- r_i^-}{d_i^+ + a_i^- c_i^+} = m_i, t \neq t_k, i = 1,2,...,n. \tag{3.9}$$

This indicates that there exists a bounded positive solution of system (1.3) and (1.4).
Noting that the product system of (1.3) is the following:

$$\begin{cases} x_i^{'}(t) = x_i(t)[r_i(t) - \frac{d_i(t)x_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{jj})} - c_i(t)x_i(t-\tau_{ii})], t \neq t_k; \\ x_i(t_k^+) = x_i(t_k) + I_{ik}(x_i(t_k)), t = t_k, i = 1,2,...,n, k \in Z; \\ y_i^{'}(t) = y_i(t)[r_i(t) - \frac{d_i(t)y_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)y_j(t-\tau_{jj})} - c_i(t)y_i(t-\tau_{ii})], t \neq t_k; \\ y_i(t_k^+) = y_i(t_k) + I_{ik}(y_i(t_k)), t = t_k, i = 1,2,...,n, k \in Z. \end{cases} \tag{3.10}$$

Corresponding to system (3.10), we construct a Lyaponov functional as follows:

$$V(t) = \sum_{i=1}^n |\ln x_i(t) - \ln y_i(t)| \tag{3.11}$$

As $t \neq t_k$, calculating the upper right derivative we get

$$\begin{aligned} & D^+_{(3.10)}V(t) \\ = & \sum_{i=1}^n (\frac{x_i^{'}(t)}{x_i(t)} - \frac{y_i^{'}(t)}{y_i(t)}) sign(x_i(t) - y_i(t)) \\ = & \sum_{i=1}^n \{ -\frac{d_i(t)x_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{ij})} - c_i(t)x_i(t-\tau_{ii}) \\ & + \frac{d_i(t)y_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)y_j(t-\tau_{ij})} + c_i(t)y_i(t-\tau_{ii})\} sign(x_i(t-\tau_{ii}) - y_i(t-\tau_{ii})) \\ = & \sum_{i=1}^n \{ -\frac{d_i(t)x_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{ij})} + \frac{d_i(t)y_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{ij})} \\ & - \frac{d_i(t)y_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{ij})} - c_i(t)x_i(t-\tau_{ii}) \\ & + \frac{d_i(t)y_i(t-\tau_{ii})}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)y_j(t-\tau_{ij})} + c_i(t)y_i(t-\tau_{ii})\} sign(x_i(t-\tau_{ii}) - y_i(t-\tau_{ii})) \\ \le & \sum_{i=1}^n \{ -\frac{d_i(t)}{a_i(t)+\sum_{j=1,j\neq i}^n b_j(t)x_j(t-\tau_{ij})} - c_i(t)\} |x_i(t-\tau_{ii}) - y_i(t-\tau_{ii})| \\ & + \sum_{i=1}^n \sum_{j=1,j\neq i}^n \frac{d_i(t)b_j(t)y_i(t-\tau_{ii})|x_i(t-\tau_{ii}) - y_i(t-\tau_{ii})|}{[a_i(t)+\sum_{k=1,k\neq i}^n b_k(t)x_k(t-\tau_{ij})][a_i(t)+\sum_{k=1,k\neq i}^n b_k(t)y_k(t-\tau_{ij})]} \\ \le & \sum_{i=1}^n \{ -\frac{d_i^-}{a_i^+ + \sum_{j=1,j\neq i}^n b_j^+ M_j} - c_i^- + \sum_{j=1,j\neq i}^n \frac{d_i^+ b_j^+ M_i}{[a_i^- + \sum_{k=1,k\neq i}^n b_k^- m_k]^2} \} \sup_{s\in[t-\tau,t]} |x_i(s) \\ & - y_i(s)| \end{aligned} \tag{3.12}$$

Noting that $|\ln x_i(t) - \ln y_i(t)| = \frac{1}{\eta_i(t)}|x_i(t) - y_i(t)|$, where $\eta_i(t)$ lies between $x_i(t)$ and $y_i(t)$. From condition (H5), there exists $c > 0$ such that

$$D^+_{(3.10)}V(t) \le -cV(t), t \neq t_k. \tag{3.13}$$

When $t = t_k$, From condition (H4), we have

$$\begin{aligned} V(t_k^+) &= \sum_{i=1}^{n} |\ln x_i(t_k^+) - \ln y_i(t_k^+)| \\ &= \sum_{i=1}^{n} |\ln[x_i(t_k) + I_{ik}(x_i(t_k))] - \ln[y_i(t_k) + I_{ik}(y_i(t_k))]| \\ &\leq \sum_{i=1}^{n} |\ln x_i(t_k) - \ln y_i(t_k)| = V(t_k) \end{aligned} \tag{3.14}$$

Hence based on Theorem 2.1, there exists a unique asymptotically stable almost periodic solution of system (1.3) and (1.4).

An example: Now consider the following impulsive almost periodic Lotka-Volterra cooperation system in the form

$$\begin{cases} x'(t) = x(t)[12 - \sin\sqrt{2}t - \frac{(2+\cos t)x(t-0.1)}{7-\sin t+(0.4-0.1\cos\sqrt{3}t)y(t-0.12)} - (8-\cos\sqrt{2}t)x(t-0.1)], t \neq t_k, \\ y'(t) = y(t)[10 - \cos\sqrt{2}t - \frac{(2-\sin t)y(t-0.2)}{5+\cos t+(0.3+0.1\cos\sqrt{2}t)x(t-0.1)} - (6+\cos\sqrt{3}t)y(t-0.12)], t \neq t_k, \\ x(t_k^+) = x(t_k) - \frac{1}{48}(x(t_k) + \frac{1}{8}), \\ y(t_k^+) = y(t_k) - \frac{1}{60}(y(t_k) + \frac{1}{10}). \end{cases} \tag{3.15}$$

where $r_1^+ = 13, r_1^- = 11, r_2^+ = 11, r_2^- = 9; a_1^+ = 8, a_2^+ = 6, a_1^- = 6, a_2^- = 4; b_1^+ = 0.5, b_1^- = 0.3, b_2^+ = 0.4, b_2^- = 0.2; c_1^+ = 9, c_1^- = 7, c_2^+ = 7, c_2^- = 5; d_1^+ = d_2^+ = 3, d_1^- = d_2^- = 1, \tau_1 = 0.1, \tau_2 = 0.12$. Therefore, $m_1 = 0.667, m_2 = 0.529, M_1 = 6.814, M_2 = 7.304$. It is easy to see that the condition (H4) is satisfied. Noting that
$c_1^- + \dfrac{d_1^-}{a_1^+ + b_2^+ M_2} = 7 + \dfrac{1}{8 + 0.4 \times 7.304} = 7.092$,
$\dfrac{d_1^+ b_2^+ M_1}{(a_1^- + b_2^- m_2)^2} + \dfrac{d_2^+ b_1^+ M_2}{(a_2^- + b_1^- m_1)^2} = \dfrac{3 \times 0.4 \times 6.814}{(6 + 0.2 \times 0.529)^2} + \dfrac{3 \times 0.5 \times 7.304}{(4 + 0.3 \times 0.667)^2} = \dfrac{8.177}{49.815} + \dfrac{10.956}{17.64} = 0.164 + 0.622 = 0.786 < 7.092$.
$c_2^- + \dfrac{d_2^-}{a_2^+ + b_1^+ M_1} = 5 + \dfrac{1}{6 + 0.5 \times 6.814} = 5.107$,
$\dfrac{d_2^+ b_1^+ M_2}{(a_2^- + b_1^- m_1)^2} = \dfrac{3 \times 0.5 \times 7.304}{(4 + 0.3 \times 0.667)^2} = \dfrac{10.956}{17.724} = 0.618 < 5.107$. Thus the condition (H5) is also satisfied. Based on Theorem 3.1, there exists a unique asymptotically stable almost periodic solution of system (3.15).

4 Conclusion

This paper investigates the existence and stability of almost periodic solutions for a class of impulsive Lotka-Volterra cooperation models with delays. To the best of our knowledge, this result is new. The example indicates that the restrictive conditions are suitable.

Competing Interests

Author has declared that no competing interests exist.

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