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Single Valued Neutrosophic VIKOR and Its Application to Wastewater Treatment Selection

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Abstract: The fuzzy VIsekriterijumska optimizacija i KOmpromisno Resenje (VIKOR) has been used in solving various multi-criteria decision-making problems where triangular fuzzy numbers are utilized in defining decision-makers' linguistic judgements. Most of the fuzzy VIKOR are built from linguistic variables based on fuzzy sets and its generalization such as intuitionistic fuzzy sets. Recent literature suggests that single-valued neutrosophic sets (SVNSs) can offer a better alternative particularly when fuzzy sets have some extent of limitations in handling indeterminacy and uncertainty. This paper proposes the SVNS-VIKOR where single-valued neutrosophic numbers are utilized in defining linguistic variables of VIKOR. Differently from the typical fuzzy VIKOR which directly utilizes fuzzy numbers with a single membership, the proposed method introduces three independent memberships of truth, indeterminacy and false to enhance judgments in the group decision-making environment. The obtained solutions would help policy makers in identifying the best solution that could enhance the efficiency of treating wastewater.

Keywords: multi-criteria decision-making; single-valued neutrosophic; VIKOR; linguistic variables; wastewater treatment; compromise solution

1. Introduction

The multi-criteria decision-making (MCDM) methods are got significant consideration in decision sciences discipline. In recent years, the necessity of concurrent consideration to the criteria and alternatives in decision problems is more vital especially in presence of uncertain data sets. So, decision makers use subjective evaluation methods to deal with this obstacle. Zadeh [1] introduced Fuzzy Sets (FSs) theory to overcome on uncertain and imprecise data sets. Besides, generalized FSs such as interval-valued fuzzy set [2], fuzzy soft set [3], interval type-2 fuzzy set [4], hesitant fuzzy set [5], intuitionistic fuzzy set [6], [7], intuitionistic fuzzy soft set [8] and interval-valued intuitionistic fuzzy set [9] were developed for the same purpose of FS. Although FS theory has been developed and extended, it still cannot deal with all possible uncertainties. For instance, when a decision-maker is asked on the possibility of the true answer, he/she may think the possibility is equal to 0.5, the possibility of the false answer is 0.8 and the degree of uncertainty is 0.3. This problem is beyond the scopes of FS and intuitionistic fuzzy set (IFS). Therefore, Smarandache [10] proposed the neutrosophic logic and neutrosophic set to generalized the concepts of the classic set, FS, IFS, etc. In neutrosophic set (NS), the truth-membership, indeterminacy membership, and false-membership are completely independent and lie in the nonstandard unit interval $]0^{-}, 1^{+}[$. From scientific point of view, neutrosophic set and set-theoretic view, it is difficult to apply in the real situation since operators need to be specified. Hence, Wang et al [11] defined a single-valued neutrosophic set (SVNS) and proposed the set theoretic operations and some properties of SVNSs.

There are several researches about using the SVNSs with MCDM methods in the literature. For example, Biswas et al [12] extended the grey relational analysis method to the neutrosophic environment and applied it to the selection of the investment sector. In this study, neutrosophic grey relational coefficient is calculated by using Hamming distance between each alternative to ideal neutrosophic estimates reliability solution and the ideal neutrosophic estimates un-reliability solution. Then, Then, for ranking/ordering all alternatives, the neutrosophic relational degree is determined [12]. Gomes and Lima [13] developed TODIM (An acronym in Portuguese of interactive and decision-making method named Tomada de decisao interativa e multicrite´vio) method to consider the risk preferences of decision-makers. Xu et al. [14] continued the TODIM method to the MCDM with the single-valued neutrosophic numbers (SVNNs). The authors proposed the extended classical TODIM method to solve the selection of emerging technology enterprise with the SVNNs. The complete evaluation of the decision-makers' bounded rationality, which is genuine action in the decision-making process, is a significant element of this method [14]. Biswas, et al. [15] introduced a new approach for multi-attribute group decision- making problems by extending the technique for order preference by similarity to ideal solution (TOPSIS) method to single-valued neutrosophic environment. They used SVNS to rank alternatives based on the characteristic, which expresses the opinion of the decision-makers based on the information provided [15]. The applicability and effectiveness of the proposed approach are shown with the tablet selection case study. Stanujkij [16] applied SVNs with Multi-Objective Optimization by a Ratio Analysis (MULTIMOORA) method for selection of communication circuit designs case study. The proposed method has the potential to be more efficient in handling a large number of complicated decision issues involving imprecise and insufficient data sets [16]. Sahin and Yigider [17] used single valued neutrosophic information with the TOPSIS method for supplier selection decision problems. The author agreed about the usage of SVNS beside MCDM methods in vast knowledge domains of the real-life as business, management, environmental sustainability, financial, scientific, and engineering. They addressed wastewater treatment (WWT) decision problems from sustainability engineering processes which get considerable attention in the usage of SVNS with MCDM methods [17].

Various methods of MCDM in selecting WWT technologies have been extensively discussed in many literatures [18]–[22]. Kalbar [23] developed the multiple attribute decision-making methodology TOPSIS and applied it to the WWT selection. The four most commonly used WWT technologies for treatment of municipal wastewater in India are ranked in many scenarios. A commonly used compensatory method, TOPSIS has been most preferred as the best to rank the WWT alternatives [23]. Abdullah [24] selected the most suitable WWT technology with the participation of three decision makers for providing WWT information and evaluating criteria using Fuzzy simple additive weighting (SAW) method . Ilangkumaran [25] recommended the best WWT technology using Fuzzy analytic hierarchy process (AHP) to assign the weights to the criteria and applying grey relation analysis technique to rank alternatives. Molinos-Senante [26] determined weights to the attributes and selected the most sustainable WWT technology using the AHP method. Abdullah and Rahman [27] implemented the analytic network process (ANP) to rank the alternatives and select the best WWT for the related case study. Zhou et al. [28] introduced a group decision-making model for WWT selection utilizing the intuitionistic fuzzy set to deal with uncertainties associated with the decision problem.

Apart from TOPSIS, AHP, ANP and SAW methods, VIKOR is another MCDM method which helps decision makers to solve MCDM problems in presence of conflicting and incommensurable criteria. VIKOR stands for VlseKriterijumska Optimizaciji I Kompromisno Resenje was translated in English as Multi-criteria Optimization and Compromise Solution. It was first developed in 1998 by Opricovic [29] that solves discrete decision problems with conflicting criteria. VIKOR rates alternatives by compromising solution from a set of conflicting criteria and comparing the proximity to the ideal solution [30]. The main advantage of the VIKOR method is its practicality in real case problems and the final results can be accomplished owing to the initial characteristics and capabilities. Moreover, regarding VIKOR ability is solving MCDM problems with discrete data sets and, the obtained compromise solution gives a maximum group utility for the ''majority'' and a minimum individual regret for the ''opponent''. The multi-criteria ranking index is ranked based on the particular measure of ''closeness'' to the ideal solution [30].

Recently, some approaches are introduced to generalize the crisp VIKOR method into fuzzy environment to cover uncertain information [31]–[33], however, there is a lack of a proper approach for solving the multi-criteria group decision-making with IFS. So, an extension of fuzzy VIKOR method, namely intuitionistic fuzzy VIKOR is proposed for handling fuzzy MCDM problems based on the IFSs [34]–[36], where the characteristics of the alternatives and attributes are represented by the IFS. The generalizing into interval-valued intuitionistic fuzzy VIKOR models was done [37]–[39] because the interval-valued intuitionistic fuzzy set (IVIFS) is more suitable for dealing with imprecise and uncertain information than FS or IFS. Although there is success in solving decision-making problems, still fuzzy-based VIKOR method cannot handle the problems with neutrosophic information. Therefore, it is essential to describe the generalization of the MCDM approaches under the NSs environment [40]–[44].

Since its introduction, the VIKOR method has received much attention from a number of scholars to solve MCDM problems. For instance, Ghorabaee [45] introduced VIKOR with interval type-2 fuzzy numbers to handle the robot selection decision problem. Liu et al [46] proposed an interval 2-tuple linguistic VIKOR method for solving the material selection under uncertain and incomplete information considering the subjective and objective weights of criteria simultaneously. The proposed method has exact characteristics and could avoid information distortion and loss in linguistic information processing. Devi [34] introduced an intuitionistic fuzzy VIKOR to solve robot selection problems and expressed the performance rating values as well as the weights of criteria with linguistic terms using triangular IFSs. Peng et al. [35] presented an efficient VIKOR method which optimizes multi-response problems in intuitionistic fuzzy environments. They evaluated the importance weights of various responses in terms of IFSs and applied the proposed approach in two case studies which are plasma-enhanced chemical vapor deposition and a double-sided surface mount technology electronic assembly operation. Other than that, VIKOR method has been widely applied in many other decision-making problems such as supplier selection [47]–[49], e-government website evaluation [50], doctor selection [51], robot selection [34], [45], renewable energy selection [52], [53], WWT technology [19], [54], insurance company selection [55], [56] and material selection [57]–[59].

To deal with imprecise and inconsistent information, we extend the crisp VIKOR method applying the SVNS environment, namely single-valued neutrosophic based VIKOR (SVN-VIKOR). The main advantage of the proposed method is dealing with imprecise and inconsistent information. Nowadays, the way of rating alternatives and selecting the best one based on uncertain conditions and given requirement based on SVNSs is an interesting and significant research topic that is motivated us to do this research. This paper has twofold purposes. Firstly, we would like to present the SVNSs into the VIKOR method as a new approach that is the extension of traditional FSs. Comparing results of crisp, fuzzy, IFS, and IVIFS with the results of SVNS provides significantly greater flexibility, which can be helpful to solve decision-making problems associated with ambiguity and uncertain. Secondly, the proposed approach contributes to ease the interpretation of a complex decision-making problems. For testing the proposed method, a case study of WWT selection problem is used to rate and select the best WWT technology. The paper is organized as follows. In second section, we review some basic concepts of NS and SVNS. The third section explains the proposed SVN-VIKOR. Subsequently, we illustrate a case study to show the decision-making steps and the applicability of the proposed method to WWT decision-making case study. The last section refers to the conclusion and future works.

2. Preliminaries

In this section, some definitions with regard to neutrosophic set, single-valued neutrosophic set and entropy are reviewed. These definitions can be retrieved from the references [51, 10].

2.1. Neutrosophic set [10]

Let X be a nonempty set. A neutrosophic set A of X defined as $\boldsymbol{A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X, T(x), I_A(x), F_A(x) \in \,]^-0, 1^+[\,\}}$, where $\boldsymbol{T_A(x)}$, $\boldsymbol{I_A(x)}$ and $\boldsymbol{F_A(x)}$ are truth membership function, indeterminacy-membership function, and falsity-membership function respectively.

2.2. Single-valued neutrosophic set [11]

Let X be a nonempty set. Single-valued neutrosophic set A of X is defined as $\boldsymbol{A = \{\langle x, T(x), I_A(x), F_A(x)\rangle | x \in X\}}$ where $\boldsymbol{T_A(x)}$, $\boldsymbol{I_A(x)}$ and $\boldsymbol{F_A(x) \in [0, 1]}$ for each $\boldsymbol{x \in X}$ and $\boldsymbol{0 \le T_A(x) + I_A(x) + F_A(x) \le 3}$.

2.2.1. Set theoretic operations and relations [11]

The notion of union, intersection, inclusion and equality have been defined on single-valued neutrosophic sets as follows.
Given that two SVNS sets $\boldsymbol{H_1}$ and $\boldsymbol{H_2}$ in U:

1. Union of two sets formed $\boldsymbol{H_3}$ written as:
$$\boldsymbol{H_3 = H_1 \cup H_2}$$
$$\boldsymbol{T_{H_3}(u) = \max\{T_{H_1}(u), T_{H_2}(u)\}},$$
$$\boldsymbol{I_{H_3}(u) = \min\ \{I_{H_1}(u), I_{H_2}(u)\}},$$
$$\boldsymbol{F_{H_3}(u) = \min\ \{F_{H_1}(u), F_{H_2}(u)\}}$$

2. Intersection of two sets denoted by $\boldsymbol{H_4}$ defined as:
$$\boldsymbol{H_4 = H_1 \cap H_2}$$
$$\boldsymbol{T_{H_4}(u) = \min\ \{T_{H_1}(u), T_{H_2}(u)\}},$$
$$\boldsymbol{I_{H_4}(u) = \max\ \{I_{H_1}(u), I_{H_2}(u)\}},$$
$$\boldsymbol{F_{H_4}(u) = \max\ \{F_{H_1}(u), F_{H_2}(u)\}}$$

3. Inclusion of two sets denoted by $\boldsymbol{H_1 \subseteq H_2}$ defined as:
$\boldsymbol{T_{H_1}(u) \le T_{H_2}(u), I_{H_1}(u) \ge I_{H_2}(u), F_{H_1}(u) \ge F_{H_2}(u)}$ for all $\boldsymbol{u \in U}$.

4. Equality of two sets denoted by $\boldsymbol{H_1 = H_2}$ defined as:
$\boldsymbol{T_{H_1}(u) = T_{H_2}(u), I_{H_1}(u) = I_{H_2}(u), F_{H_1}(u) = F_{H_2}(u)}$ for all $\boldsymbol{u \in U}$.

2.2.2. Axiomatic of Entropy

Let $N(X)$ be all SVNSs on x and $A \in N(x)$. An entropy on SVNSs is a function $E_N: N(X) \to [0,1]$ which satisfies the following axioms:
The entropy of SVNS set A is:

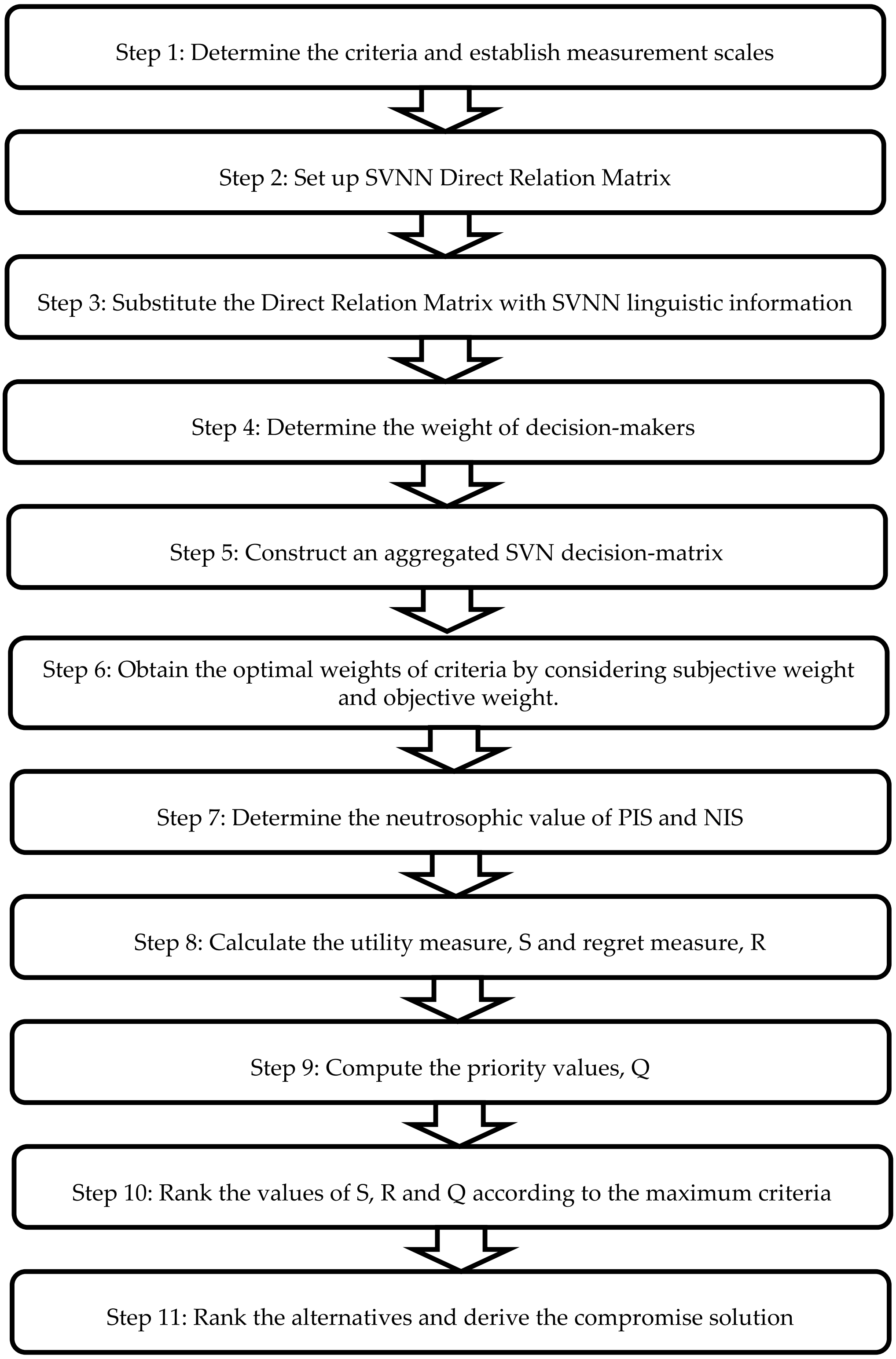
$$E_N(A) = \frac{1}{n}\sum_{t=1}^{n}\left(1 - \frac{1}{b-a}\int_a^b |T_A(x_i) - F_A(x_i)|\left|I_A(x_i) - I_{A^c}(x_i)\right|dx\right) \text{ for all } x \in X.$$

1. $E_N(A) = 0$ if A is crisp set.
2. $E_N(A) = 1$ if $\left(T_A(x), I_A(x), F_A(x)\right) = (0.5, 0.5, 0.5)$ for all $x \in X$.
3. $E_N(A) \geq E_N(B)$ if $A \subset B$.
4. $E_N(A) = E_N(A^C)$ for all $A \in N(X)$.

3. Proposed SVN-VIKOR method

MCDM evaluation is a complex, imprecise and time-consuming process. Moreover, it is a very significant process for choosing the best alternative. In this section, we extend the VIKOR method to solve MCDM problems in which all preference information provided by decision-makers are expressed as single-valued neutrosophic values, and the interaction phenomena among the preference of individual decision-makers and conflicting criteria are taken into account. To deal with vague and inconsistent information, we apply the SVN-VIKOR approach since it can compromise the multiple, conflict criteria and dealing efficiently with vague and inconsistent information. The SVN-VIKOR method is developed to provide a rational, systematic decision-making process by which one discovers the best solution and a compromise solution that can be used to resolve a MCDM problem in neutrosophic environment. The extended VIKOR decision procedure of MCDM based on SVNS is summarized as in Figure 1.

Figure 1 A general procedure of the proposed SVN-VIKOR method



Based on the Figure 1, the proposed method consists of eleven steps. Differently with other VIKOR based methods, the proposed SVN-VIKOR method has two types of optimal weights of criteria which are subjective weight and objective weight.

Let $D^{(k)}$ be a committee of decision-makers where $k = 1, 2, \ldots, p$. A_i represents alternatives, where $i = 1, 2, \ldots, m$ and C_j represents criteria, where $j = 1, 2, \ldots, n$. The criteria can be classified as cost criteria and benefit criteria.

Step 1: Define criteria and establish measurement scales

Step 2: Set up single valued neutrosophic number (SVNN) Direct-Relation Matrix [42]

We will get a single-valued neutrosophic decision-matrix $X_k, (k = 1, 2, \ldots, p)$ for k th decision-maker as shown as follows:

$$X_k = \begin{matrix} & C_1 & \cdots & C_j & \cdots & C_n \\ A_1 & \langle t_{11}^k, i_{11}^k, f_{11}^k \rangle & \cdots & \langle t_{1j}^k, i_{1j}^k, f_{1j}^k \rangle & \cdots & \langle t_{1n}^k, i_{1n}^k, f_{1n}^k \rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_i & \langle t_{i1}^k, i_{i1}^k, f_{i1}^k \rangle & \cdots & \langle t_{ij}^k, i_{ij}^k, f_{ij}^k \rangle & \cdots & \langle t_{in}^k, i_{in}^k, f_{in}^k \rangle \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_m & \langle t_{m1}^k, i_{m1}^k, f_{m1}^k \rangle & \cdots & \langle t_{mj}^k, i_{mj}^k, f_{mj}^k \rangle & \cdots & \langle t_{mn}^k, i_{mn}^k, f_{mn}^k \rangle \end{matrix}$$

Step 3: Substitute the Direct Relation Matrix with SVNN linguistic information [12]

Step 4: Determine the weight of decision-makers

The weight of the k th decision-maker can be obtained through the following formula [17]:

$$\psi_k = \frac{T_k + I_k\left(\frac{T_k}{T_k + F_k}\right)}{\sum_{k=1}^{p}\left(T_k + I_k\left(\frac{T_k}{T_k + F_k}\right)\right)}, where\ \psi_k \geq 0, \sum_{k=1}^{p} \psi_k = 1. \quad (1)$$

and

T_k represents the truth-membership function of k th decision-maker;
I_k represents the indeterminacy-membership function of k th decision-maker;
F_k represents the falsity-membership function of k th decision-maker.

Step 5: Construct an aggregated single-valued neutrosophic decision-matrix

Let $D^{(k)} = (D_{ij}^k)_{mxn}$ be a single-valued decision-matrix of the k th decision-maker. The simplified neutrosophic weighted averaging (SNWA) operator [60] is used to aggregate all individual decision-matrices of $D^{(k)} = (D_{ij}^k)_{mxn}$ where k = 1, 2, …, p, i = 1, 2, …, m and j = 1, 2, …, n into a collective decision-matrix $D = (d_{ij})_{mxn}$.

$$d_{ij} = SNWA\left(d_{ij}^{(1)}, d_{ij}^{(2)}, \ldots, d_{ij}^{(p)}\right)$$

$$= \left(1 - \prod_{k=1}^{p}\left(1 - T_{ij}^{(k)}\right)^{\frac{1}{p}}, \prod_{k=1}^{p}\left(1 - I_{ij}^{(k)}\right)^{\frac{1}{p}}, \prod_{k=1}^{p}\left(1 - F_{ij}^{(k)}\right)^{\frac{1}{p}}\right) \quad (2)$$

The aggregated decision-matrix D is defined as follows:

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}, \text{ where } d_{ij} = (T_{ij}, I_{ij}, F_{ij}).$$

Step 6: Obtain the optimal weights of criterion:

In this section, there are two types of weight of criteria that need to be considered which are subjective weight and objective weight.

1. Subjective weight

The rating of alternatives with regards to each criterion is collecting through decision-makers' opinion. The weight of importance of the criteria correspond to alternatives are identified through linguistic rating scale as follows:

Table 1 Five-point linguistic rating scale and its linguistic terms.

Linguistic Terms	Influence Score	SVNNs
Very Low	1	⟨0.1, 0.8, 0.9⟩
Low	2	⟨0.35, 0.6 ,0.7⟩
Medium	3	⟨0.5, 0.4, 0.45⟩
High	4	⟨0.8, 0.2, 0.15⟩
Very High	5	⟨0.9, 0.1, 0.1⟩

Assume that the weight of the criterion is obtained using eq (3):

$$w_j = SVNSWA(w_j^1, w_j^2, \dots, w_j^l)$$
$$= \left(1 - \prod_{k=1}^{l}\left(1 - T_{ij}^{(k)}\right)^{\psi_k}, \prod_{k=1}^{l}\left(I_{ij}^{(k)}\right)^{\psi_k}, \prod_{k=1}^{l}\left(F_{ij}^{(k)}\right)^{\psi_k}\right) \tag{3}$$

where $j = 1,2, \dots, n$ and $w_j = (T_j, I_j, F_j)$ is the importance weight of the j th criterion. Normalized subjective weight of each criterion can be obtained using eq (4). Assume that our decision group has k decision-makers and $A_j = (T_j, I_j, F_j)$ is an SVNN expresses j th decision-maker.

$$w_j^s = T_j + I_j\left(\frac{T_j}{T_j+F_j}\right)\left[-\left(\frac{1}{\ln m}\right)\sum_{i=1}^{m} T_j + I_j\left(\frac{T_j}{T_j+F_j}\right)\right]^{-1} \tag{4}$$

where $j = 1,2, \dots, n,$ and $\sum_{j=1}^{n} w_j^s = 1$.

2. Objective weight [46]

The evaluation criterion should be normalized by using eq (5):

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{5}$$

where P_{ij} is the projected outcome of criterion j.

Next, entropy E_j of the set of projected outcomes of criterion j should be calculated using eq (6):

$$E_j = -\left(\frac{1}{\ln m}\right)\sum_{i=1}^{m} P_{ij} \ln P_{ij} \tag{6}$$

where m is the number of criteria and $0 \le E_j \le 1$.

After that, the divergence, div_j which is the divergence degree of the intrinsic information of criterion *j*, should be defined in order to obtain the objective weights of the criteria.

$$div_j = 1 - E_j \tag{7}$$

The greater the divergence degree of the criterion, the more important the criterion in the decision-making process. Finally, objective weights can be obtained using eq (8).

$$w_j^o = \frac{div_j}{\sum_{j=1}^{n} div_j} \tag{8}$$

Step 7: Determine the neutrosophic value of positive ideal solution (PIS) $f_j^+ = \langle T_j^+, I_j^+, F_j^+ \rangle$ and the neutrosophic value of negative ideal solution (NIS) $f_j^- = \langle T_j^-, I_j^-, F_j^- \rangle$ for all criteria rating, $j = 1,2,\ldots,n$.

$$f_j^- = \begin{Bmatrix} \min_i r_{ij}, \text{for Benefit Criteria} \\ \max_i r_{ij}, \text{for Cost Criteria} \end{Bmatrix} \tag{9}$$

$$f_j^+ = \begin{Bmatrix} \max_i r_{ij}, \text{for Benefit Criteria} \\ \min_i r_{ij}, \text{for Cost Criteria} \end{Bmatrix} \tag{10}$$

where $j = 1,2,\ldots,n$.

Step 8: Calculate the utility measure, (S_i), and the regret measure, (R_i) for the alternative as follow:

$$S_i = \sum_{j=1}^{n} \frac{w_j ||\tilde{f}_j^+ - \tilde{f}_{1j}||}{||\tilde{f}_j^+ - \tilde{f}_j^-||}, i = 1,2,\ldots,m \tag{11}$$

$$R_i = \max_j \left\{ \frac{w_j ||\tilde{f}_j^+ - \tilde{f}_{1j}||}{||\tilde{f}_j^+ - \tilde{f}_j^-||} \right\}, i = 1,2,\ldots,m \tag{12}$$

where w_j indicates the combination weight for each criterion.

$$w_j = v w_j^s + (1 - v) w_j^o \tag{13}$$

where v denotes relative importance between subjective weights and objective weights. It can be taken in any value from 0 to 1 but usually it is set as 0.5.

Step 9: Compute the priority values, $Q_i, i = 1,2,\ldots,m$ by using the formula as follow:

$$Q_i = v \frac{(S_i - S^+)}{(S^- - S^+)} + (1 - v) \frac{(R_i - R^+)}{(R^- - R^+)} \tag{14}$$

where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$, $R^- = \max_i R_i$,

v indicates the weight of the strategy of the majority of criteria, usually it is assumed as 0.5.

Step 10: Rank the value of S_i, R_i, and Q_i according to maximum criteria. Rank the alternatives in decreasing order results.

Step 11: Rank the alternatives and derive the compromise solution.

The alternative $A^{(1)}$ (top alternative) that ranks the best in minimum value of Q fulfills the following two conditions:

Condition 1: Acceptable advantages

$$Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{m-1} \quad (15)$$

where $A^{(1)}$ and $A^{(2)}$ are the top two alternatives in Q_i.

Condition 2: Acceptable Stability

The top alternatives should be the best ranked by S_i and R_i.

If one of the above conditions cannot be satisfied, a set of compromise solutions has been proposed:

1. Alternatives $A^{(1)}$ and $A^{(2)}$ are accepted if only stability condition is not satisfied;
2. Alternatives $A^{(1)}$, $A^{(2)}$, …, $A^{(u)}$ are accepted if advantage condition is not satisfied. $A^{(u)}$ is determined by the relation $Q(A^{(u)}) - Q(A^{(1)}) \geq \frac{1}{m-1}$ for maximum u (the positions of these alternatives are in closeness).

4. Implementation

In order to test the efficiency and effectiveness of proposed method for the WWT selection, we present a case study which are includes scenario, implementation methods and data analysis. The WWT selection is a complex process, where alternatives and criteria are inherited some extent of imprecise information. In the midst of this complexity, it is a very vital process for finding the best alternative to solve the problem. Our case study includes the evaluation process of various WWT which contains five alternatives as follows: Activated Sludge (A1), Aerated Lagoons (A2), Rotating Biological Contactors (A3), Oxidation Ditch (A4) and Trickling Filter (A5). Three interview sessions of three decision-makers were conducted to evaluate the importance of alternatives with respect to criteria of WWT.

The decision-making for the best WWT alternatives is done by considering the basis of nine core factors or criteria. The amount of pollutants removed (C1), lifetime (C2), operation and maintenance cost (C3), reliability (C4), capital cost (C5), environmental impacts (C6), sustainability (C7), land area requirement (C8) and safety risk to worker (C9) are nine criteria of WWT. The procedure of decision-making is presented as follows:

Step 1: Define criteria and establish measurement scales

Structured interviews were used to choose three distinct decision-makers (DM1, DM2, DM3) as expert groups in order to gather their perspectives. The DMs were carefully chosen to ensure that they are well recognised and specialists in WWT technology. Tables 2 and 3 describe decision-makers' perspectives on the weight of importance of criteria and the evaluation of WWT alternatives against the criteria.

Table 2 Influence score of three decision-makers on importance of criteria.

	DM1	**DM2**	**DM3**
C1	5	4	5
C2	3	5	5
C3	5	5	4
C4	3	4	5

C5	5	3	5
C6	5	3	3
C7	4	3	5
C8	4	4	3
C9	5	2	3

Table 3 Rating on evaluation of WWT alternatives with respected to criteria.

		C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	DM1	7	7	7	7	7	5	7	5	6
	DM2	7	7	7	7	5	7	7	7	7
	DM3	7	7	8	8	6	5	4	4	6
A2	DM1	6	6	5	6	4	4	5	1	6
	DM2	7	7	7	7	5	7	7	3	7
	DM3	6	6	4	8	3	4	6	7	6
A3	DM1	7	7	5	7	6	8	7	5	6
	DM2	7	5	7	7	7	7	7	7	7
	DM3	7	7	4	7	6	8	7	5	6
A4	DM1	6	6	5	6	4	4	5	1	6
	DM2	7	7	7	5	5	7	7	5	7
	DM3	7	7	5	6	4	4	5	3	6
A5	DM1	7	6	4	7	4	8	7	6	5
	DM2	7	5	3	7	5	7	7	7	7
	DM3	7	6	5	5	4	6	7	3	5

Step 2 & Step 3: Construct SVNN Direct Relation Matrix

SVNNs Linguistic Phrases are substituted into table 3 in order to construct SVNN Direct Relation Matrix (see Table 4).

Table 4 SVNN Direct Relation Matrix.

		C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	DM1	0.8	0.8	0.8	0.8	0.8	0.5	0.8	0.5	0.65
		0.2	0.2	0.2	0.2	0.2	0.5	0.2	0.5	0.35
		0.15	0.15	0.15	0.15	0.15	0.45	0.15	0.45	0.3
	DM2	0.8	0.8	0.8	0.8	0.5	0.8	0.8	0.8	0.8
		0.2	0.2	0.2	0.2	0.5	0.2	0.2	0.2	0.2
		0.15	0.15	0.15	0.15	0.45	0.15	0.15	0.15	0.15
	DM3	0.8	0.8	0.9	0.9	0.65	0.5	0.35	0.35	0.65
		0.2	0.2	0.1	0.1	0.35	0.5	0.65	0.65	0.35
		0.15	0.15	0.05	0.05	0.3	0.45	0.6	0.6	0.3
A2	DM1	0.65	0.65	0.5	0.65	0.35	0.35	0.5	0.05	0.65
		0.35	0.35	0.5	0.35	0.65	0.65	0.5	0.9	0.35

		0.3	0.3	0.45	0.3	0.6	0.6	0.45	0.95	0.3
		0.8	0.8	0.8	0.8	0.5	0.8	0.8	0.2	0.8
	DM2	0.2	0.2	0.2	0.2	0.5	0.2	0.2	0.75	0.2
		0.15	0.15	0.15	0.15	0.45	0.15	0.15	0.8	0.15
		0.65	0.65	0.35	0.9	0.2	0.35	0.65	0.8	0.65
	DM3	0.35	0.35	0.65	0.1	0.75	0.65	0.35	0.2	0.35
		0.3	0.3	0.6	0.05	0.8	0.6	0.3	0.15	0.3
		0.8	0.8	0.5	0.8	0.65	0.9	0.8	0.5	0.65
	DM1	0.2	0.2	0.5	0.2	0.35	0.1	0.2	0.5	0.35
		0.15	0.15	0.45	0.15	0.3	0.05	0.15	0.45	0.3
		0.8	0.5	0.8	0.8	0.8	0.8	0.8	0.8	0.8
A3	DM2	0.2	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2
		0.15	0.45	0.15	0.15	0.15	0.15	0.15	0.15	0.15
		0.8	0.8	0.35	0.8	0.65	0.9	0.8	0.5	0.65
	DM3	0.2	0.2	0.65	0.2	0.35	0.1	0.2	0.5	0.35
		0.15	0.15	0.6	0.15	0.3	0.05	0.15	0.45	0.3
		0.65	0.65	0.5	0.65	0.35	0.35	0.5	0.05	0.65
	DM1	0.35	0.35	0.5	0.35	0.65	0.65	0.5	0.9	0.35
		0.3	0.3	0.45	0.3	0.6	0.6	0.45	0.95	0.3
		0.8	0.8	0.8	0.5	0.5	0.8	0.8	0.5	0.8
A4	DM2	0.2	0.2	0.2	0.5	0.5	0.2	0.2	0.5	0.2
		0.15	0.15	0.15	0.45	0.45	0.15	0.15	0.45	0.15
		0.8	0.8	0.5	0.65	0.35	0.35	0.5	0.2	0.65
	DM3	0.2	0.2	0.5	0.35	0.65	0.65	0.5	0.75	0.35
		0.15	0.15	0.45	0.3	0.6	0.6	0.45	0.8	0.3
		0.8	0.65	0.35	0.8	0.35	0.9	0.8	0.65	0.5
	DM1	0.2	0.35	0.65	0.2	0.65	0.1	0.2	0.35	0.5
		0.15	0.3	0.6	0.15	0.6	0.05	0.15	0.3	0.45
		0.8	0.5	0.2	0.8	0.5	0.8	0.8	0.8	0.8
A5	DM2	0.2	0.5	0.75	0.2	0.5	0.2	0.2	0.2	0.2
		0.15	0.45	0.8	0.15	0.45	0.15	0.15	0.15	0.15
		0.8	0.65	0.5	0.5	0.35	0.65	0.8	0.2	0.5
	DM3	0.2	0.35	0.5	0.5	0.65	0.35	0.2	0.75	0.5
		0.15	0.3	0.45	0.45	0.6	0.3	0.15	0.8	0.45

Step 4: Determine the weights of decision-makers

The weights of decision-makers are obtained as in eq (1).

$$T_k + I_k\left(\frac{T_k}{T_k + F_k}\right)$$

I: $0.8 + 0.2\left(\frac{0.8}{0.8+0.15}\right) = 0.9684$

I: $0.8 + 0.2\left(\frac{0.8}{0.8+0.15}\right) = 0.9684$

M: $0.5 + 0.4\left(\frac{0.5}{0.5+0.45}\right) = 0.7105$

$$\sum_{k=1}^{p} \mu_k + \pi_k\left(\frac{\mu_k}{\mu_k + v_k}\right) = 0.9684 + 0.9684 + 0.7105 = 2.6474$$

ψ_k, where $k = 1, 2, 3$

DM1 = $\frac{0.9684}{2.6474} = 0.3658$, DM2 = $\frac{0.9684}{2.6474} = 0.3658$, DM3 = $\frac{0.7105}{0.6474} = 0.2684$

Step 5: Construct an aggregated SVN decision-matrix

The importance weight of decision-makers is shown in Table 5.

Table 5 Importance weight of decision-makers.

DM	Linguistic Variable	IFNs	Weights
1	Important	$\langle 0.8, 0.2, 0.15\rangle$	0.3658
2	Important	$\langle 0.8, 0.2, 0.15\rangle$	0.3658
3	Medium	$\langle 0.5, 0.4, 0.45\rangle$	0.2684

To fuse the weight of all decision-makers into one, SNWA operator is applied by using eq (2).
$T_{1,1} = 1 - \left((1-0.8)^{0.3658} \times (1-0.8)^{0.3658} \times (1-0.8)^{0.2684}\right) = 0.8$
$I_{1,1} = 0.2^{0.3658} \times 0.2^{0.3658} \times 0.2^{0.2684} = 0.2$
$F_{1,1} = 0.15^{0.3658} \times 0.15^{0.3658} \times 0.15^{0.2684} = 0.15$
The rest of calculations are calculated in similarly. The detailed calculation of aggregated SVNS matrix is shown in the Table 6.

Table 6 Aggregated SVNS matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	0.8	0.8	0.8339	0.8340	0.6750	0.6424	0.7256	0.6163	0.7148
A1	0.2	0.2	0.1660	0.1660	0.3250	0.3576	0.2744	0.3837	0.2852
	0.15	0.15	0.1116	0.1117	0.2700	0.3011	0.2176	0.3252	0.2328
	0.7148	0.7148	0.6163	0.79623	0.3756	0.5777	0.6750	0.4128	0.7148
A2	0.2852	0.2852	0.3837	0.20377	0.6136	0.4223	0.3250	0.5623	0.2852
	0.2328	0.2328	0.3252	0.14393	0.5834	0.3613	0.2700	0.5436	0.2328
	0.8	0.7204	0.6163	0.8	0.7148	0.8711	0.8	0.6424	0.7148
A3	0.2	0.2796	0.3837	0.2	0.2852	0.1289	0.2	0.3576	0.2852
	0.15	0.2242	0.3252	0.15	0.2328	0.0747	0.15	0.3011	0.2328
A4	0.7546	0.7546	0.6424	0.6012	0.4095	0.5777	0.6424	0.2827	0.7148

	0.2454	0.2454	0.3576	0.3988	0.5905	0.4223	0.3576	0.6912	0.2852
	0.1932	0.1932	0.3011	0.3479	0.5401	0.3613	0.3011	0.6902	0.2328
	0.8	0.6012	0.3464	0.7442	0.4095	0.8196	0.8	0.6439	0.6424
A5	0.2	0.3988	0.6384	0.2558	0.5905	0.1804	0.2	0.3499	0.3577
	0.15	0.3480	0.6171	0.2014	0.5401	0.1209	0.15	0.3029	0.3011

Step 6: Obtain the optimal weight of criterion

The linguistic variables are substituted into Table 2. Aggregated subjective weight of criterion is calculated by using eq (3).

$T_1 = 1 - \left((1-0.9)^{0.3658} \times (1-0.8)^{0.3658} \times (1-0.9)^{0.2684}\right) = 0.8711$

$I_1 = 0.1^{0.3658} \times 0.2^{0.3658} \times 0.1^{0.2684} = 0.1289$

$F_1 = 0.1^{0.3658} \times 0.15^{0.3658} \times 0.1^{0.2684} = 0.116$

The result of calculation of aggregated subjective weight is shown in Table 7.

Table 7 Aggregated subjective weight.

	T	***I***	***F***
C1	0.8711399	0.12886	0.115989
C2	0.8198282	0.166049	0.17336
C3	0.8795537	0.120446	0.111496
C4	0.7678305	0.213971	0.201078
C5	0.8198282	0.166049	0.17336
C6	0.7224871	0.240893	0.259576
C7	0.7678305	0.213971	0.201078
C8	0.7442398	0.240893	0.20144
C9	0.694533	0.279408	0.30511

The subjective weight of criterion is calculated using eq (4) and presented as follows:

$$T_j + I_j\left(\frac{T_j}{T_j + F_j}\right)$$

$$0.8711+0.129\left(\frac{0.8711}{0.8711+0.116}\right)=0.985$$
$$0.8198+0.166\left(\frac{0.8198}{0.8198+0.173}\right)=0.957$$
$$0.8796+0.12\left(\frac{0.8796}{0.8796+0.111}\right)=0.986$$
$$0.7678+0.214\left(\frac{0.7678}{0.7678+0.201}\right)=0.937$$
$$0.8198+0.166\left(\frac{0.8198}{0.8198+0.173}\right)=0.957$$
$$0.7225+0.241\left(\frac{0.7225}{0.7225+0.26}\right)=0.9$$
$$0.7678+0.214\left(\frac{0.7678}{0.7678+0.201}\right)=0.937$$
$$0.7442+0.241\left(\frac{0.7442}{0.7442+0.201}\right)=0.934$$
$$0.6945+0.279\left(\frac{0.6945}{0.6945+0.305}\right)=0.889$$

$$\sum_{j=1}^{n} T_j+I_j\left(\frac{T_j}{T_j+F_j}\right)=0.985+0.957+0.986+0.937+0.957+0.9+0.937+0.934+0.889$$

$$=8.482$$

$$w_1^s=\frac{0.985}{8.482}=0.116$$

$$w_2^s=\frac{0.957}{8.482}=0.113$$

$$w_3^s=\frac{0.986}{8.482}=0.116$$

The rest answers are given as follow:

$w_4^s=0.111,\quad w_5^s=0.113,\quad w_6^s=0.106,\quad w_7^s=0.111,\quad w_8^s=0.11,\quad w_9^s=0.105$

Crisp Value are calculated using the following equation:

$$s(x_{ij})=\frac{2+T_{ij}-I_{ij}-F_{ij}}{3}$$

$$s(x_{11})=\frac{2+0.8-0.2-0.15}{3}=0.8167$$

$$s(x_{21})=\frac{2+0.7148-0.2852-0.2328}{3}=0.7323$$

The aggregated crisp matrix is given in Table 8.

Table 8 Aggregated crisp matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
A1	0.8167	0.8167	0.8521	0.8521	0.6934	0.6612	0.7445	0.6358	0.7323
A2	0.7323	0.7323	0.6358	0.8162	0.3929	0.598	0.6934	0.4356	0.7323
A3	0.8167	0.7388	0.6358	0.8167	0.7323	0.8892	0.8167	0.6612	0.7323

A4	0.7719	0.7719	0.6612	0.6182	0.4263	0.598	0.6612	0.3004	0.7323
A5	0.8167	0.6182	0.3637	0.7623	0.4263	0.8395	0.8167	0.6637	0.6612

After that, the evaluation of criterion is normalized by using eq (5).

$$P_{11} = \frac{0.8167}{0.8167 + 0.7323 + 0.8167 + 0.7719 + 0.8167} = 0.2065$$
$$P_{21} = \frac{0.7323}{0.8167 + 0.7323 + 0.8167 + 0.7719 + 0.8167} = 0.1852$$
$$P_{31} = \frac{0.8167}{0.8167 + 0.7323 + 0.8167 + 0.7719 + 0.8167} = 0.2065$$
$$P_{41} = \frac{0.7719}{0.8167 + 0.7323 + 0.8167 + 0.7719 + 0.8167} = 0.1952$$
$$P_{51} = \frac{0.8167}{0.8167 + 0.7323 + 0.8167 + 0.7719 + 0.8167} = 0.2065$$

Next, entropy E_j is calculated by using eq (6).

$\ln P_{ij}$:

$\ln P_{11} = \ln 0.2065 = -1.5773$
$\ln P_{21} = \ln 0.1852 = -1.6864$
$\ln P_{31} = \ln 0.2065 = -1.5773$
$\ln P_{41} = \ln 0.1952 = -1.6336$
$\ln P_{51} = \ln 0.2065 = -1.5773$

$$\sum_{i=1}^{m} P_{ij} \ln P_{ij} = (0.2065 \times -1.5773) + (0.1852 \times -1.6864) + (0.20644 \times -1.5773)$$
$$+(0.1952 \times -1.6336) + (0.2065 \times -1.5773)$$
$$= -1.6085$$

$$\therefore E_1 = -\left(\frac{1}{\ln 5}\right)(-1.6085) = 0.9994$$

After that, the divergence is calculated as formula in eq (7).

$$div_1 = 1 - 0.9994 = 0.0006$$

$$\sum_{j=1}^{n} div_j = 0.0005 + 0.0026 + 0.0202 + 0.0037 + 0.0229 + 0.0091 + 0.0022 + 0.0249 + 0.0005$$
$$= 0.0867$$

Objective weights are calculated through the formula given in eq (8).

$$w_1^o = \frac{0.0005821}{0.08671} = 0.0067$$

The rest of objective weights are calculated in similar manner. The result of calculated objective weight and subjective weight are shown in the Table 9.

Table 9 Subjective weight and objective weight of criteria.

	Subjective Weight	Objective Weight
C1	0.1161	0.0067

C2	0.1128	0.0295
C3	0.1163	0.2334
C4	0.1105	0.0427
C5	0.1128	0.2644
C6	0.1061	0.1045
C7	0.1105	0.0257
C8	0.1101	0.2874
C9	0.1048	0.0057

The result of subjective weight for each criterion implies that operational and maintenance cost (C3) is the most important criteria and safety risk to worker (C9) is the least important based on decision-makers.

For analysis of data for objective weight of criteria, we found that capital cost of WWT (C5) is the most important criteria while safety risk to worker (C9) is the least important.

Step 7: Determine the neutrosophic value of Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

Benefit Criteria and Cost Criteria are categorized. Benefit Criteria includes the amount of pollutant removed (C1), Lifetime (C2), Reliability (C4), Sustainability (C7), and Safety Risk to Worker (C9). On the other hand, the cost criteria are Operational and Maintenance Cost (C3), Capital Cost (C5), Environmental Impacts (C6), and Safety Risk to Worker (C8).

The PIS and NIS from Table 8 is obtained using eq (9) and eq (10) respectively. The PIS and NIS of all criteria are given in Table 10.

Table 10 The PIS and NIS of all criteria.

	PIS	**NIS**
C1	0.8167	0.7279
C2	0.8167	0.6182
C3	0.1114	0.2741
C4	0.8521	0.6182
C5	0.1114	0.2741
C6	0.1114	0.2741
C7	0.8167	0.6612
C8	0.1114	0.2741
C9	0.7326	0.6565

Step 8: The utility measure (S_i), and the regret measure, (R_i) for the alternative are calculated using eq (11) and eq (12) respectively. The combination weight for each criterion is given in eq (13).

Let $v = 0.5$, $w_{C1} = (0.5)0.1161 + (1 - 0.5)0.0067 = 0.0614$,

Then,

$$\frac{w_{C1}||\tilde{f}_1^+ - \tilde{f}_{11}||}{||\tilde{f}_1^+ - \tilde{f}_1^-||} = \frac{0.0614121(0.8167 - 0.8167)}{0.8167 - 0.7323} = 0$$

So,

$S_{A1} = 0 + 0 + 0.1748 + 0 + 0.167 + 0.0229 + 0.0316 + 0.1835 + 0 = 0.5798$
$S_{A2} = 0.0614 + 0.0303 + 0.0974 + 0.0118 + 0 + 0 + 0.0541 + 0.074 + 0 = 0.3289$
$S_{A3} = 0 + 0.0279 + 0.0974 + 0.0116 + 0.1886 + 0.1053 + 0 + 0.1974 + 0 = 0.62818$
$S_{A4} = 0.0325 + 0.016 + 0.1065 + 0.0166 + 0.0185 + 0 + 0.068126 + 0 + 0 = 0.3184$
$S_{A5} = 0 + 0.0711 + 0 + 0.0294 + 0.0186 + 0.0873 + 0 + 0.1987 + 0.0552 = 0.4604$

and

$R_1 = 0.1098$
$R_2 = R_3 = R_4 = R_5 = 0.1123.$

The rest of S_i and R_i are calculated in similar manner.

Step 9: Compute priority value, Q_i as in eq (14).

From above,
$S^+ = \min_i S_i = 0.3184$
$S^- = \max_i S_i = 0.6282$
$R^+ = \min_i R_i = 0.09741$
$R^- = \max_i R_i = 0.19874$

$$Q_{A1} = (0.5)\frac{(0.4003 - 0.4003)}{(0.4866 - 0.4003)} + (1 - 0.5)\frac{(0.1098 - 0.1098)}{(0.1123 - 0.1098)} = 0$$

Other calculations are obtained in the similar manner.
Step 10: S_i, R_i, Q_i are ranked according to the maximum criteria.

Table 11 The result of S_i, R_i, Q_i and their ranking according to the maximum criteria

	S	Ranking	R	Ranking	Q	Ranking
A1	0.57978575	4	0.18347188	3	0.84656246	4
A2	0.31841157	1	0.09741361	1	0.0168618	1
A3	0.62817945	5	0.19739361	4	0.99337543	5
A4	0.32885806	2	0.10652321	2	0.04495352	2
A5	0.46039547	3	0.19873605	5	0.72917789	3

Step 11: Ranking the alternatives
According to the results of the analysis, the ranking of alternatives based on S_i and R_i values are obtained as $A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$. The ranking order according to Q value is obtained as $A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$. Since $0.045 - 0.0169 \leq \frac{1}{4}$ (condition 1), the acceptable advantage does not fulfill.

To fulfill the condition 2 which is acceptable stability, the top alternatives should be the best ranked by S_i and R_i. From the Table 11, alternative A_2 presents the best ranked S_i and R_i. Because of one of the conditions does not fulfill, thus alternative A_2 is not the best ranking. Therefore, a set of compromise solutions is proposed. From the result, only the acceptable advantage condition is not satisfied. Which mean, alternative A_u and the top two alternatives represent a group of compromise solutions. The closeness of these options is determined.

Based on the closeness formula, $0.72917789 - 0.0168618 \geq \frac{1}{5-1}$, it shows that the third rank of the alternatives is alternative A_u. As a result, the compromise solutions found using the SVN-VIKOR approach are Aerated Lagoons (A2), Oxidation Ditch (A4), and Trickling Filter (A5), indicating that these three alternatives are in close competition for the best position. Dursun [19], Wongburi and Park [61] and Maurya et al. [62]'s results are comparable. Dursun [19] revealed that aerated lagoons is the best WWT alternative. According to the findings of Wongburi and Park [61]'s research, aerated lagoon is also the best option, followed by oxidation ditch. However, Maurya et al. [62] discovered that Trickling Filter is the best WWT option. They came to the conclusion that the best WWT alternatives are 'Aerated Lagoon,' 'Oxidation Ditch,' and 'Trickling Filter'. This comparable finding demonstrates the efficacy of the SVN-VIKOR method as well.

5. Conclusions

In this study, we presented the definitions related to SVNS, entropy weight, set-theoretic operations and relations of SVNs, and described the steps of the VIKOR method for MCDM problems. Next, the ratings of each alternative and the weights of each criterion were interpreted in linguistic terms expressed by single-valued neutrosophic numbers regarding the importance of different elements in the decision-making procedure, namely the weight of decision-makers, the weight of criteria and the impact of alternatives on criteria with respect to decision-makers. Other than that, SNWA operator is used to aggregating all individual decision-makers' opinions in SVN assessments. Then, we determine the weight of criteria by considering the subjective weight and objective weight. The PIS and NIS of all criteria were also determined. Next, the utility measure, regret measure and priority values were calculated ranked according to the maximum criteria. Finally, the alternatives are ranked according to the previous value and the compromise solution was derived if one of the conditions cannot be fulfilled. To show the applicability of the proposed method, a case study of WWT alternatives selection was used. The obtained results show that Aerated Lagoons (A2), Oxidation Ditch (A4) and Trickling Filter (A5) are the three alternatives that have a close position as the best alternatives for the WWT. Our findings appear to be correlated when compared to those of other studies on WWT selection [19], [61], [62], demonstrating the feasibility and efficacy of the proposed approach.

The benefit of the SVN-VIKOR method is that it is more beneficial for addressing MCDM problems since it takes into account the significance of decision-makers and may be used to identify the best solution in a conflicting criteria environment. Moreover, considering that IFSs are sometimes unable to deal with ambiguity and uncertainty, the SVN-VIKOR method was proposed in this work to handle MCDM issues. WWT selection was investigated using the suggested SNV VIKOR technique. The findings would be extremely useful to policymakers in determining the best technology for WWT. Our proposed method deals efficiently with imprecise, inconsistent and inadequate information by considering all aspects of the decision-making process. Therefore, SVN-VIKOR can be preferable for dealing with incomplete and unpredictable information in MCDM problems such as supplier selection, landfill sites selection and many other decision-making problems. There are some recommendations suggested for future research. In the future, a sensitivity analysis with alteration of some parameters will be presented to analyze changes in the results. Furthermore, we will consider more decision-makers with specific years in the related field. The linguistic variable used may consider decision-makers' opinions to increase the accuracy of the calculation in decision-making.

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References

[1] L. A. Zadeh, "Fuzzy Sets and Systems," *Proc. Symp. Syst. Theory*, pp. 29–37, 1965.

[2] M. B. Gorzalzany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," *Fuzzy Sets Syst.*, vol. 21, pp. 1–17, 1987.

[3] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.

[4] L. Abdullah and Z. Nurnadiah, "A new aggregating phase for interval type-2 fuzzy TOPSIS using the ELECTRE I method," in *Lecture Notes in Electrical Engineering*, vol. 315, 2015.

[5] V. Torra, "Hesitant Fuzzy Set," *Int. J. Intell. Syst.*, vol. 25, pp. 529–539, 2010.

[6] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, pp. 87–96, 1986.

[7] S. Khaleie and M. Fasanghari, "An intuitionistic fuzzy group decision making method using entropy and association coefficient," *Soft Comput.*, vol. 16, no. 7, pp. 1197–1211, 2012.

[8] P. K. Maji, A. R. Roy, and R. Biswas, "Intuitionistic fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 677–692, 2001.

[9] K. T. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 3, no. 31, pp. 343–349, 1989.

[10] F. Smarandache, "Neutrosophic set – A generalization of the intuitionistic fuzzy set," *Neutrosophic Probab. Set, Logic. Amer. Res.*, vol. 1, no. 1, pp. 1–15, 1995.

[11] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multisp. Multistruct*, vol. 4, pp. 410–413, 2010.

[12] P. Biswas, S. Pramanik, and B. C. Giri, "Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments," *Neutrosophic Sets Syst.*, vol. 2, pp. 105–113, 2014.

[13] L. Gomes and M. Lima, "TODIM: Basics and application to multicriteria ranking of projects with environmental impacts," *Found Comput Decis. Sci.*, vol. 16, no. 4, pp. 113–127, 1992.

[14] D.-S. Xu, C. Wei, and G.-W. Wei, "TODIM method for single-valued neutrosophic multiple attribute decision making," *Information*, vol. 8, no. 4, p. 125, 2017.

[15] P. Biswas, S. Pramanik, and B. C. Giri, "TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment," *Neural Comput. Appl.*, vol. 27, pp. 727–737, 2016.

[16] D. Stanujkij, E. K. Zavadskas, F. Smarandache, W. K. M. Brauers, and D. Karabasevic, "A neutrosophic extension of the MULTIMOORA method," *Informatica*, vol. 28, no. 1, pp. 181–192, 2017.

[17] R. Şahin and M. Yiğider, "A multi-criteria neutrosophic group decision making metod based TOPSIS for supplier selection," *Appl. Math. Inf. Sci.*, vol. 10, no. 5, pp. 1843–1852, 2016.

[18] M. Bottero, E. Comino, and V. Riggio, "Application of the analytic hierarchy process and the analytic network process for the assessment of different wastewater treatment systems," *Environ. Model. Softw.*, vol. 26, no. 10, pp. 1211–1224, 2011.

[19] M. Dursun, "Evaluation of wastewater treatment alternatives using fuzzy VIKOR method," *J. Adv. Manag. Sci.*, vol. 4, no. 4, pp. 333–336, 2015.

[20] A. R. Karimi, N. Mehrdadi, S. J. Hashemian, G. R. Nabi Bidhendi, and R. Tavakkoli Moghaddam, "Selection of Wastewater Treatment Process Based on Analytical Hierarchy Process," *Int. J. Civ. Eng. Technol.*, vol. 5, no. 9, pp. 27–33, 2014.

[21] G. Zeng, R. Jiang, G. Huang, M. Xu, and J. Li, "Optimization of wastewater treatment alternative selection by hierarchy grey relational analysis," *J. Environ. Manage.*, vol. 82, no. 2, pp. 250–259, 2007.

[22] M. Dursun, "An integrated approach for the evaluation of wastewater treatment alternatives," *Proc. World Congr. Eng. Comput. Sci. 2015*, vol. 2, 2015.

[23] P. P. Kalbar, S. Karmakar, and S. R. Asolekar, "Selection of wastewater treatment alternative: Significance of choosing MADM method," *Environ. Eng. Manag. J.*, vol. 14, no. 5, pp. 1011–1020, 2015.

[24] L. Abdullah, "Developing decision on suitable wastewater treatment technology using fuzzy simple additive weighting," *Int. J. Eng. Technol.*, vol. 7, no. 2, pp. 405–413, 2015.

[25] M. Ilangkumaran *et al.*, "Optimization of wastewater treatment technology selection using hybrid MCDM," *Manag. Environ. Qual. An Int. J.*, vol. 24, no. 5, pp. 619–641, 2013.

[26] M. Molinos-Senante, T. Gómez, M. Garrido-Baserba, R. Caballero, and R. Sala-Garrido, "Assessing the sustainability of small wastewater treatment systems: A composite indicator approach," *Sci. Total Environ.*, vol. 497–498, pp. 607–617, 2014.

[27] L. Abdullah and N. A. A. Rahman, "Analytic network process for developing relative weight of wastewater treatment technology selection," *Mod. Appl. Sci.*, vol. 11, no. 5, p. 64, 2017.

[28] Z. Zhou, Y. Dou, X. Zhang, D. Zhao, and Y. Tan, "A group decision-making model for wastewater treatment plans selection based on intuitionistic fuzzy sets," *J. Environ. Eng. Landsc. Manag.*, vol. 26, no. 4, pp. 251–260, 2018.

[29] S. Opricovic, "Multicriteria optimization of civil engineering systems," *Multicriteria Optim. Civ. Eng. Syst. Fac. Civ. Eng. Serbian)*, 1998.

[30] S. Opricovic and T. H. Tzeng, "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS," *Eur J Oper Res*, vol. 2, no. 156, pp. 445–455, 2004.

[31] A. Shemshadi, H. Shirazi, M. Toreihi, and M. J. Tarokh, "A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting," *Expert Syst. Appl.*, vol. 10, no. 38, pp. 12160– 12167, 2011.

[32] M. K. Saydi, M. Heydari, and K. Shahanaghi, "Extension of VIKOR method for decision making problem with interval numbers," *Appl. Math. Model.*, no. 33, pp. 2257–2262, 2009.

[33] J. P. Park, H. J. Cho, and Y. C. Kwun, "Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy information," *Fuzzy Optim. Decis. Mak.*, no. 10, pp. 233–253, 2011.

[34] K. Devi, "Extension of VIKOR method in intuitionistic fuzzy environment for robot selection," *Expert Syst. Appl.*, vol. 38, no. 11, pp. 14163–14168, 2011.

[35] J. P. Peng, W. C. Yeh, T. C. Lai, and C. Bin Hsu, "The incorporation of the Taguchi and the VIKOR methods to optimize multi-response problems in intuitionistic fuzzy environments," *J. Chinese Inst. Eng. Trans. Chinese Inst. Eng. A/Chung-kuo K. Ch'eng Hsuch K'an*, vol. 38, no. 7, pp. 897–907, 2015.

[36] S. M. Mousavi, B. Vahdani, and S. Sadigh Behzadi, "Designing a model of intuitionistic fuzzy VIKOR in multi-attribute group decision-making problems," *Iran. J. Fuzzy Syst.*, vol. 13, no. 1, pp. 45–65, 2016.

[37] L. Chaojie and J. Hongli, "Extension of VIKOR method with interval-valued intuitionistic fuzzy sets," *Int. Conf. Manag. Serv. Sci. MASS 2011*, pp. 0–3, 2011.

[38] G. Büyüközkan and F. Göçer, "Smart medical device selection based on interval valued intuitionistic fuzzy VIKOR," *Adv. Fuzzy Log. Technol.*, pp. 306–317, 2018.

[39] C. Tan and X. Chen, "Interval-valued intuitionistic fuzzy multicriteria group decision making based on VIKOR and Choquet integral," *J. Appl. Math.*, vol. 2013, pp. 1–16, 2013.

[40] A. Awang, A. T. A. Ghani, L. Abdullah, and M. F. Ahmad, "A DEMATEL method with single valued neutrosophic set (SVNS) in identifying the key contribution factors of Setiu Wetland's coastal erosion," *AIP Conf. Proc.*, vol. 1974, 2018.

[41] A. Awang, N. Aizam, and L. Abdullah, "An integrated decision-making method based on neutrosophic numbers for investigating factors of coastal erosion," *Symmetry (Basel).*, vol. 11, no. 3, p. 328, 2019.

[42] R. Bausys and E. K. Zavadskas, "Multicriteria decision making approach by VIKOR under interval neutrosophic set environment," *Econ. Comput. Econ. Cybern. Stud. Res.*, vol. 49, no. 4, pp. 33–48, 2015.

[43] F. Karaaslan, "Neutrosophic soft sets with applications in decision making," *Int. J. Inf. Sci. Intell. Syst.*, vol. 4, no. 2, pp. 1–20, 2015.

[44] P. Chi and P. Liu, "An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set," *Neutrosophic Sets Syst.*, vol. 1, pp. 1–8, 2013.

[45] M. K. Ghorabaee, "Developing an MCDM method for robot selection with interval type-2 fuzzy sets," *Robot. Comput. Integr. Manuf.*, vol. 37, pp. 221–232, 2016.

[46] H. C. Liu, L. Liu, and J. Wu, "Material selection using an interval 2-tuple linguistic VIKOR method considering subjective and objective weights," *Mater. Des.*, vol. 52, pp. 158–167, 2013.

[47] J. Ahmad, J. Xu, M. Nazam, and M. Kashif, "A fuzzy linguistic VIKOR multiple criteria group decision making method for supplier selection," *Int. J. Sci. Basic Appl. Res.*, vol. 4531, pp. 1–16, 2015.

[48] S. Kumar and A. G. Barman, "Fuzzy TOPSIS and fuzzy VIKOR in selecting green suppliers for sponge iron and steel manufacturing," *Soft Comput.*, vol. 25, no. 8, pp. 6505–6525, 2021.

[49] J. Zhang, D. Yang, Q. Li, B. Lev, and Y. Ma, "Research on sustainable supplier selection based on the rough DEMATEL and FVIKOR methods," *Sustain.*, vol. 13, no. 88, pp. 1–20, 2021, doi: 10.3390/su13010088.

[50] M. Abdel-Basset, Y. Zhou, M. Mohamed, and V. Chang, "A group decision making framework based on neutrosophic VIKOR approach for e-government website evaluation," *J. Intell. Fuzzy Syst.*, vol. 34, no. 6, pp. 4213–4224, 2018.

[51] J. Hu, L. Pan, and X. Chen, "An interval neutrosophic projection-based VIKOR method for selecting doctors," *Cognit. Comput.*, vol. 9, no. 6, pp. 801–816, 2017.

[52] A. Mardani, E. K. Zavadskas, K. Govindan, A. A. Senin, and A. Jusoh, "VIKOR technique: A systematic review of the state of the art literature on methodologies and applications," *Sustain.*, vol. 8, no. 1, pp. 1–38, 2016.

[53] J. M. Shariff, D. Mohamad, S. Amin, and N. F. Baha, "A preference analysis of renewable energy alternatives in Malaysia using fuzzy VIKOR method," *J. Qual. Meas. Anal.*, vol. 16, no. 2, pp. 283–292, 2020.

[54] T. Gichamo, H. Gökçeku¸, D. U. Ozsahin, G. Gelete, and B. Uzun, *Ranking of Natural Wastewater Treatment Techniques by Multi-criteria Decision Making (MCDM)Methods*, no. May. 2021.

[55] G. N. Yücenur and N. E. Demirel, "Group decision making process for insurance company selection problem with extended VIKOR method under fuzzy environment," *Expert Syst. Appl.*, vol. 39, no. 3, pp.

3702–3707, 2012.

[56] M. Ercan and E. Onder, "Ranking insurance companies in Turkey based on their financial performance indicators using VIKOR method," *Int. J. Acad. Res. Accounting, Financ. Manag. Sci.*, vol. 6, no. 2, pp. 104–113, 2016.

[57] J. Ahmad, J. Xu, and M. Nazam, "Multi-criteria group decision making for pipe material selection: Comparative analysis of HF-VIKOR and HF-ELECTRE II," *Int. J. Dev. Res.*, vol. 5, no. 6, pp. 4826–4841, 2015.

[58] S. Dev, A. Aherwar, and A. Patnaik, "Material selection for automotive piston component using entropy-VIKOR method," *Silicon*, vol. 12, pp. 155–169, 2020.

[59] I. Emovon and O. S. Oghenenyerovwho, "Application of MCDM method in material selection for optimal design: A review," *Results Mater.*, vol. 7, no. 100115, pp. 1–21, 2020.

[60] J. Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 26, no. 5, pp. 2459–2466, 2014.

[61] P. Wongburi and J. K. Park, "Decision making tools for selecting sustainable wastewater treatment technologies in Thailand," *IOP Conf. Ser. Earth Environ. Sci.*, vol. 150, no. 1, 2018.

[62] S. P. Maurya, A. Ohri, P. K. Singh, and A. Debnath, "Development of a decision making tool for selection of waste water treatment technology based on urban water reuse," *Int. J. Civ. Eng. Technol.*, vol. 9, no. 2, pp. 497–506, 2018.

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