

Ice-shelf vibrations modeled by a full 3-D elastic model based on the depth-integrated momentum equations: field equations

Y.V. Konovalov^{1,2}

¹Financial University under the Government of the Russian Federation, Leningradsky Prospekt 49, Moscow, Russian Federation, 125993.

²MIREA - Russian Technological University, Vernadsky avenue 78, Moscow, Russian Federation, 119454.

Correspondence to: Y.V. Konovalov (yu-v-k@yandex.ru)

Abstract

The propagation of high-frequency elastic-flexural waves through an ice shelf was modeled by a full 3-D elastic model. This model based on the momentum equations that were written as the integro-differential equations. The integro-differential form implies the vertical integration of the momentum equations from the current vertical coordinate z to the ice surface like, for instance, in the Blatter-Pattyn ice flow model. The sea water flow under the ice shelf is described by the wave equation. The numerical solutions were obtained by a finite-difference method. Numerical experiments were undertaken for a crevasse-ridden ice shelf with different spatial periodicities of the crevasses.

1. Model description and field equations

1.1 Basic equations

The movement of the elastic continuum substance (in a gravitational field) is described by the well-known momentum equations, e.g. [1], [2]. Here, these equations are applied for the description of an ice shelf vibrations. These equations are

$$\left\{ \begin{array}{l} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 U}{\partial t^2}; \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial^2 V}{\partial t^2}; \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 W}{\partial t^2} + \rho g; \\ 0 < x < L; y_1(x) < y < y_2(x); h_b(x, y) < z < h_s(x, y); \end{array} \right. \quad (1)$$

where (XYZ) is a rectangular coordinate system with X axis along the central line, and Z axis is pointing vertically upward; U, V and W are two horizontal and one vertical ice displacements, respectively; σ is the stress tensor; and ρ is ice density. The ice shelf is of length L along the central line. The geometry of the ice shelf is assumed to be given by lateral boundary functions $y_{1,2}(x)$ at sides labeled 1 and 2 and functions for the surface and base elevation, $h_{s,b}(x, y)$, denoted by subscripts s and b, respectively. Thus, the domain on which Eqs. (1) are considered is $\Omega = \{0 < x < L, y_1(x) < y < y_2(x), h_b(x, y) < z < h_s(x, y)\}$.

The depth-integrated model implies the vertical integration of the momentum equations (1) from the current vertical coordinate z to the ice surface like, for instance, in the Blatter-Pattyn ice flow model, e.g.,[3],[4]. The integration yields the following equations

$$\begin{cases} \frac{\partial}{\partial x} \int_z^{h_s} \sigma_{xx} dz + \frac{\partial}{\partial y} \int_z^{h_s} \sigma_{xy} dz - \sigma_{xz} = \rho \int_z^{h_s} \frac{\partial^2 U}{\partial t^2} dz; \\ \frac{\partial}{\partial x} \int_z^{h_s} \sigma_{yx} dz + \frac{\partial}{\partial y} \int_z^{h_s} \sigma_{yy} dz - \sigma_{yz} = \rho \int_z^{h_s} \frac{\partial^2 V}{\partial t^2} dz; \\ \frac{\partial}{\partial x} \int_z^{h_s} \sigma_{zx} dz + \frac{\partial}{\partial y} \int_z^{h_s} \sigma_{zy} dz - \sigma_{zz} = \rho g(h_s - z) + \rho \int_z^{h_s} \frac{\partial^2 W}{\partial t^2} dz; \\ 0 < x < L; y_1(x) < y < y_2(x); h_b(x, y) < z < h_s(x, y). \end{cases} \quad (2)$$

Sub-ice water is assumed to be an incompressible inviscid fluid of uniform density. Another assumption is that water depth in the cavity below the ice shelf changes gradually in the horizontal directions. Thus, the ice-front and other such features are not considered here. Moreover, the ice is considered to be a continuous solid elastic plate. Under these three assumptions, sub-ice water flow is independent of z in a vertical column. Manipulating the governing equations of the shallow sub-ice water layer yields the wave equation [5]:

$$\frac{\partial^2 W_b}{\partial t^2} = \frac{1}{\rho_w} \frac{\partial}{\partial x} \left(d_0 \frac{\partial P'}{\partial x} \right) + \frac{1}{\rho_w} \frac{\partial}{\partial y} \left(d_0 \frac{\partial P'}{\partial y} \right), \quad (3)$$

where ρ_w is sea water density; $d_0(x, y)$ is the depth of the sub-ice water layer; $W_b(x, y, t)$ is the vertical deflection of the ice-shelf base, and $W_b(x, y, t) = W(x, y, h_b(x, y), t)$; and $P'(x, y, t)$ is the deviation of the sub-ice water pressure from the hydrostatic value.

1.2 Boundary conditions

The boundary conditions are: (i) a stress free ice surface (**Appendix A**); (ii) the normal stress exerted by seawater at the ice-shelf free edges (**Appendix A** and **Appendix B**) and at the ice-shelf base (**Appendix B**); and (iii) rigidly fixed edges at the grounding line of the ice-shelf.

In this model, the boundary conditions at the ice-shelf free edges are considered in the form of the linear combination [6]

$$\alpha_1 F_i(U, V, W) + \alpha_2 \Phi_i(U, V, W) = 0, \quad i = 1, 2, 3, \quad (4)$$

where:

- (i) $F_i(U, V, W) = 0$ is the typical and well-known form of the boundary conditions where, for example, the condition on the ice-shelf surface is expressed as $\sigma_{ik} n_k = 0$ (\vec{n} is the unit vector normal to the surface);
- (ii) $\Phi_i(U, V, W) = 0$ is the approximation based on the integration of the typical boundary conditions into the momentum equations (1); and
- (iii) the coefficients α_1 and α_2 satisfy the condition $\alpha_1 + \alpha_2 = 1$.

Thus, these boundary conditions (4) are the superposition of the typical boundary conditions (see **Appendix A**) and those based on the integration of the basic/typical boundary conditions into the momentum equations (see **Appendix C**). Thus, the boundary conditions formulated here are notable because they are "mixed".

The boundary conditions to the sea water layer correspond to the frontal incident wave. They are

- (i) at $x = 0$: $\frac{\partial P'}{\partial x} = 0$;
- (ii) at $y = y_1, y = y_2$: $\frac{\partial P'}{\partial y} = 0$;
- (iii) at $x = L$: $P' = A_0 \rho_w g e^{i\omega t}$, where A_0 is the amplitude of the incident wave.

1.3 Discretization of the model

The numerical solutions were obtained by a finite-difference method, which is based on the standard coordinate transformation $x, y, z \rightarrow x, \eta = \frac{y-y_1}{y_2-y_1}, \xi = (h_s - z)/H$, where H is the ice thickness ($H = h_s - h_b$). The coordinate transformation maps the ice domain Ω into the rectangular parallelepiped $\Pi = \{0 \leq x \leq L; 0 \leq \eta \leq 1; 0 \leq \xi \leq 1\}$, which presents simplification to the numerical discretization.

In x, η, ξ variables Eq. (2) is rewritten as

$$\left\{ \begin{array}{l} \frac{\partial}{\partial x} \int_0^\xi \sigma_{xx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{xx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{xy} H d\xi + \xi'_x \sigma_{xx} H + \xi'_y \sigma_{xy} H - \sigma_{xz} = \\ \quad = \rho \int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi; \\ \frac{\partial}{\partial x} \int_0^\xi \sigma_{yx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{yx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{yy} H d\xi + \xi'_x \sigma_{yx} H + \xi'_y \sigma_{yy} H - \sigma_{yz} = \\ \quad = \rho \int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi; \\ \frac{\partial}{\partial x} \int_0^\xi \sigma_{zx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zy} H d\xi + \xi'_x \sigma_{zx} H + \xi'_y \sigma_{zy} H - \sigma_{zz} = \\ \quad = \rho g H \xi + \rho \int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi; \\ 0 < x < L; 0 < \eta < 1; 0 < \xi < 1. \end{array} \right. \quad (5)$$

1.4 Equations for ice-shelf displacements

Constitutive relationships between stress tensor components and displacements correspond to Hooke's law, e.g., [2], [7]:

$$\sigma_{ij} = \frac{E}{1+\nu} \left(u_{ij} + \frac{\nu}{1-2\nu} u_{ll} \delta_{ij} \right) \quad , \quad (6)$$

where u_{ij} are the strain components, E - Young's modulus, ν - Poisson's ratio.

1.5 Ice-shelf harmonic vibrations. The eigenvalue problem.

(The content of this item is the same like in the description of the previous model/versions)

It is assumed that for harmonic vibrations all variables are periodic in time, with the periodicity of the incident wave (of the forcing) given by the frequency ω , i.e.,

$$\tilde{\zeta}(x, y, z, t) = \zeta(x, y, z) e^{i\omega t}, \quad (7)$$

where $\tilde{\zeta} = \{U, V, W, \sigma_{ij}\}$,

where we are interested in the real part of the variables expressed in complex form.

This assumption also implies that the full solution of the linear partial differential Eqs. (2), (5) is a sum of the solution for the steady-state flexure of the ice shelf and solution (7) for the time-dependent problem. In other words, solution (7) implies that the deformation due to the gravitational forcing can be separated from the vibration problem, i.e. the term ρg as well as the appropriate terms in the boundary conditions (4) are absent from the final equations formulated for the vibration problem, because a time-independent solution accounting for them applies and is not of interest in this study.

The separation of variables in Eq. (7) and its substitution into Eqs. (2), (5) yields the same equations, but with the operator $\frac{\partial^2}{\partial t^2}$ replaced with the constant $-\omega^2$, i.e. we obtain equation for $\zeta(x, y, z)$:

$$\mathcal{L} \zeta = -\omega^2 \zeta, \quad (8)$$

where \mathcal{L} is a linear partial differential operator.

The numerical solution of Eq. (8) at different values of ω yields the dependence of ζ on the frequency of the forcing ω . When the frequency of the forcing converges to the eigenfrequency of the system, we observe the typical rapid increase of deformation/stresses in the spectra in the form of the resonant peaks.

Note that here, the term “eigenvalue” refers to the eigenfrequency (ω_n) of the ice/water system or corresponding periodicity ($T_n = \frac{2\pi}{\omega_n}$). As mentioned previously, the term “eigenvalue” is employed in the same meaning like in a Sturm-Liouville Eigenvalue

Problem, e.g. [8]. Eigenvalues (where resonant peaks would be observed) are denoted by the letters ω_n or T_n with the subscript n (or other), which is integer, because the array of the eigenvalues is a countable set.

Letters ω or T without the subscript denote the non-resonant values of frequency or periodicity of the ice/water system. They are defined by the frequency of the incident wave (of the forcing).

The eigenvalues can be derived from the equation $D(\omega) = 0$, where D is the determinant of the matrix, which results from the discretization of Eq. (8) and of the corresponding boundary conditions. However, the probability of the appearance of the forcing at any specific frequency is practically zero. This can be seen when we consider only events within the frequency range $(\omega_i - \Delta\omega, \omega_i + \Delta\omega)$. The probability of a forcing that is within the frequency range, is non-zero:

$$p\{\omega \in (\omega_i - \Delta\omega, \omega_i + \Delta\omega)\} = \frac{2\Delta\omega}{\Omega}, \quad (9)$$

where Ω is the width of the range in omega space, which includes all possible frequencies of the forcing. Eq. (9) also assumes that the events have equal probabilities in different parts of Ω .

Thus, the probability of the resonant-like motion is higher when the value $\Delta\omega$, which is defined by the width of the resonant peak, is higher too. Therefore, the width of the resonant peaks is an important parameter, from a practical standpoint, because it defines the probability of the suitable resonant-like motion.

Computation of the spectra, such as provided below, thus provides important information about the width of resonant peaks within the likely range of forcing frequencies found in nature. By assessing the widths of such peaks, a better understanding of the probability that any one specific forcing event, at a specific ω can be assessed.

2. The output results

In this version the crevasse-ridden ice shelf [9] is considered. The parameters of the crevasses are listed in the [line 27-31](#) of the program code. They are

- a) the spatial periodicity of the crevasses T_{cr}
- b) the crevasses depth D_{cr} ;
- c) the crevasses width W_{cr} .

The shape of the crevasses was assumed as rectangular ([lines 160-180](#) in the code) or as triangular ([lines 183-210](#) in the code)

The [lines 25512-25606](#) in the code contain the algorithm, which allows to derive the wavenumber for the mode obtained for a given frequency of the forcing. The algorithm is based on the counting of the maxima and the minima in the deflection profile along the central line. The algorithm is the same as in the previous model/version. The case of the frontal forcing is considered in the program code.

Some of the modes generated by the model based on the depth-integrated momentum equations are listed below (Fig.1-20).

The output results of the code are the dispersion spectra (wave number versus periodicity/frequency of the forcing): [lines 25637-25670](#)

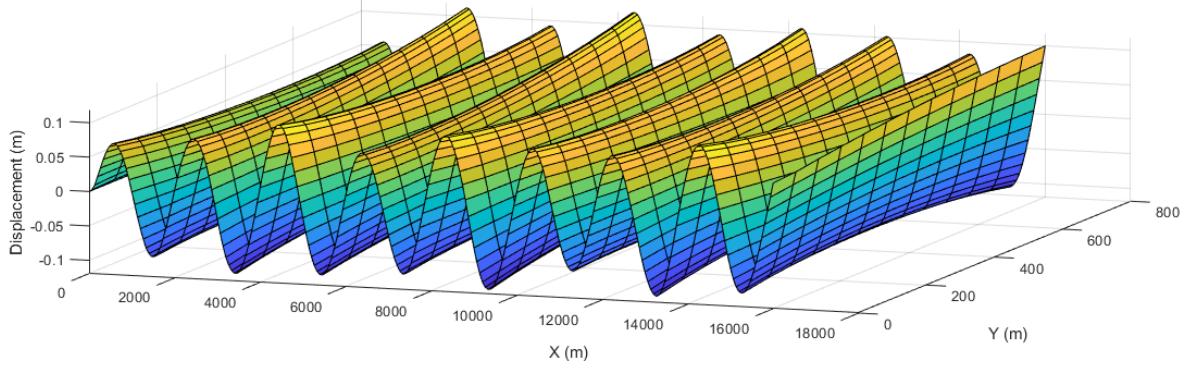


Figure 1. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 20\text{s}$.

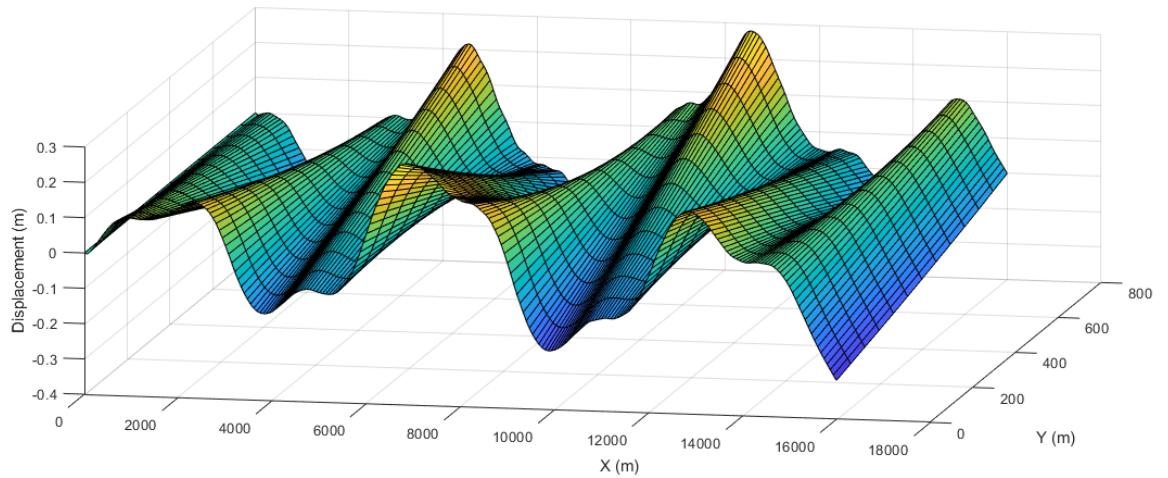


Figure 2. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 20\text{s}$.

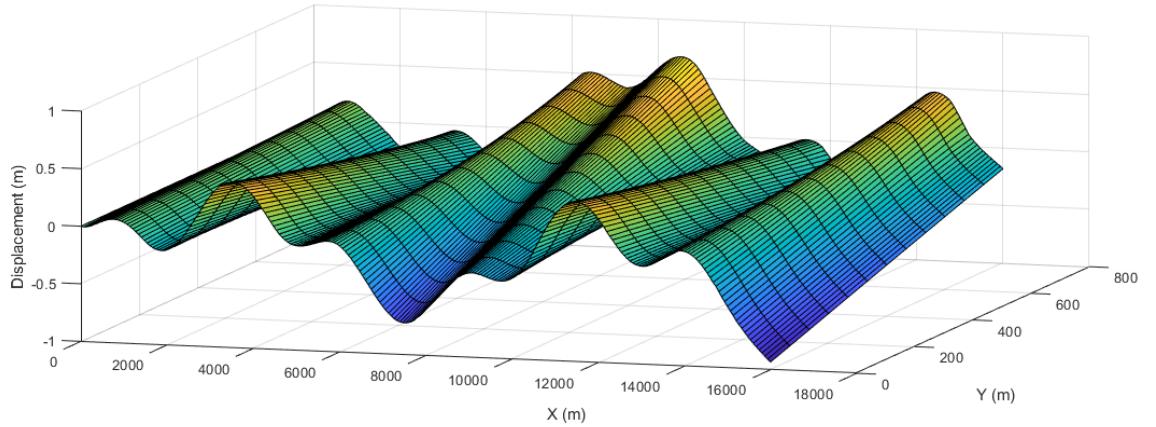


Figure 3. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 1.5\text{km}, D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 50\text{s}$.

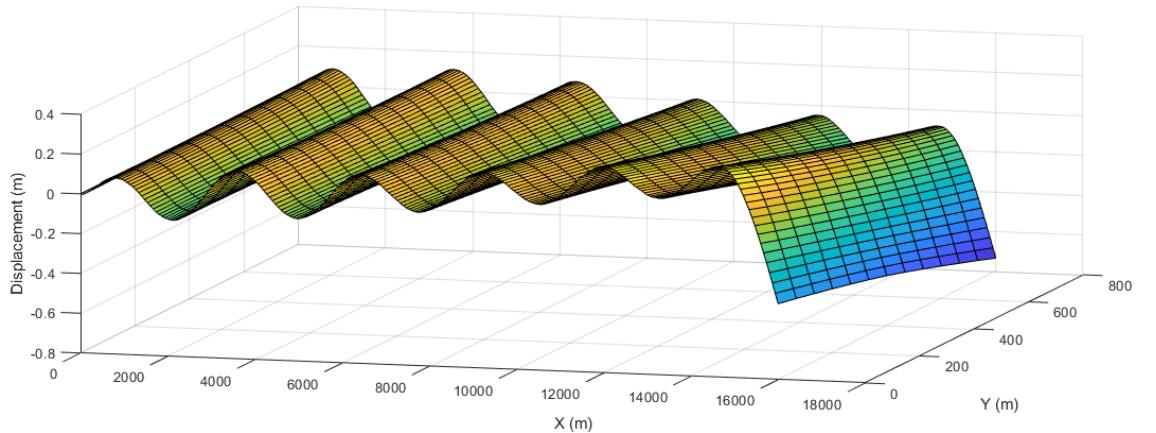


Figure 4. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 1.5\text{km}, D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 50\text{s}$.

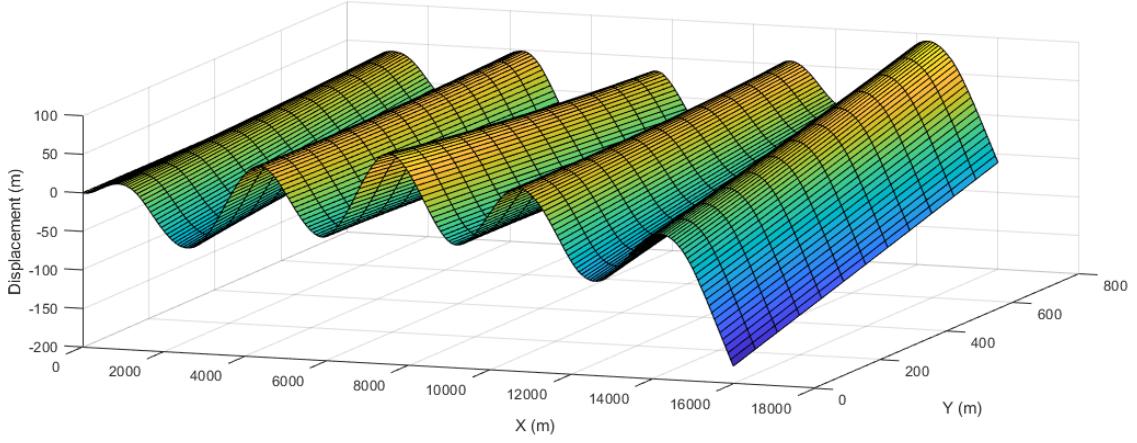


Figure 5. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 100\text{s}$.

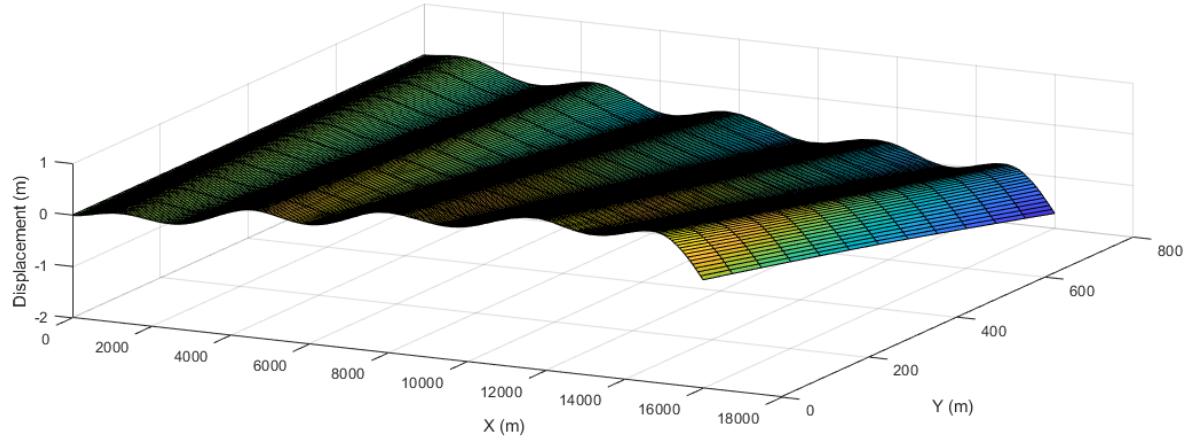


Figure 6. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 100\text{s}$.

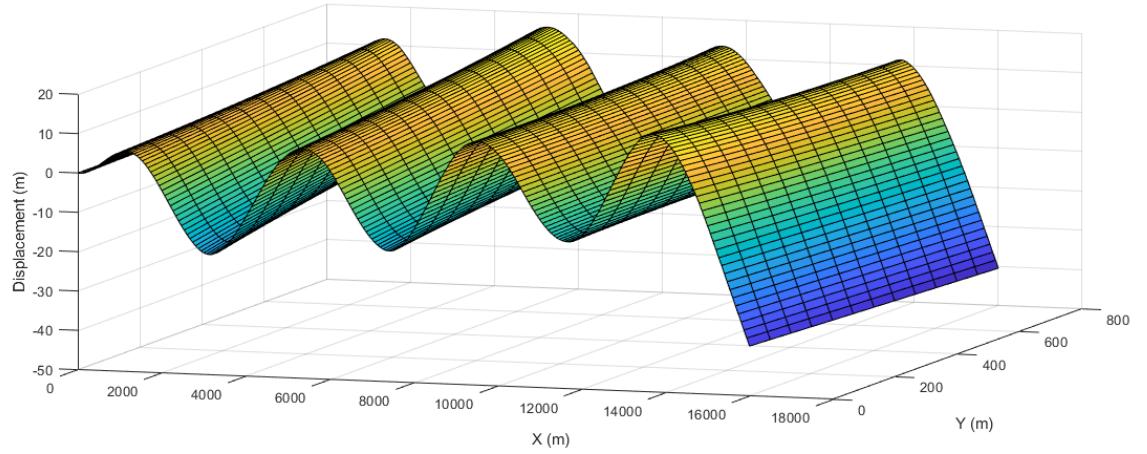


Figure 7. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 200\text{s}$.

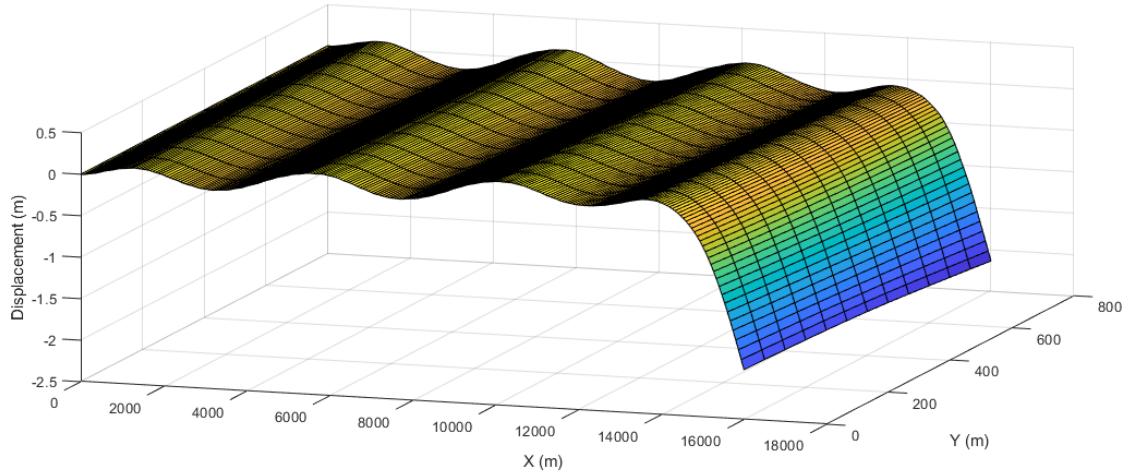


Figure 8. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 1.5\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 200\text{s}$.

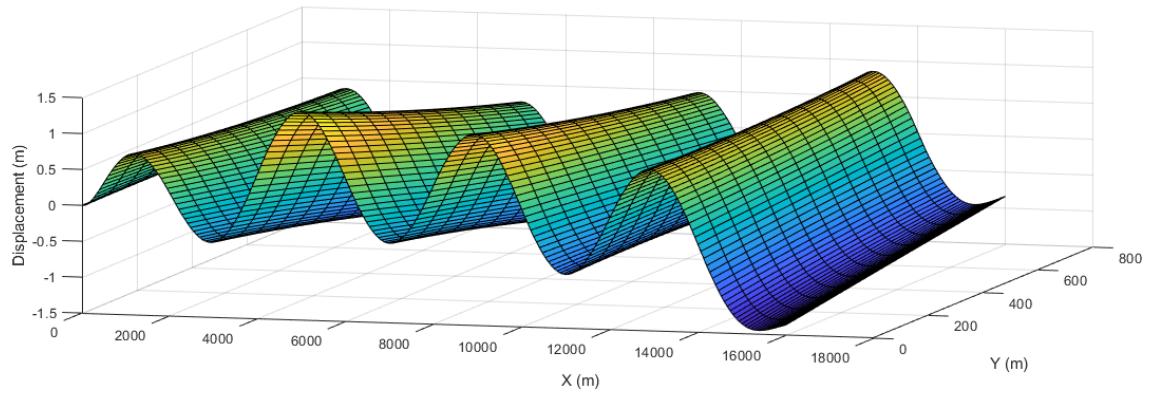


Figure 9. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 2km, D_{cr} = 10m$; The periodicity of the forcing $T = 150s$.

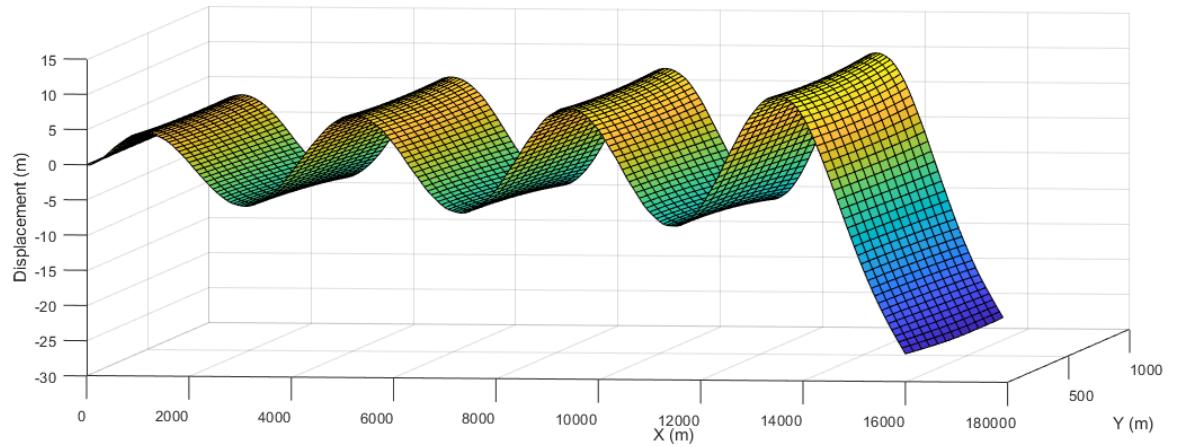


Figure 10. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 2km, D_{cr} = 10m$; The periodicity of the forcing $T = 150s$.

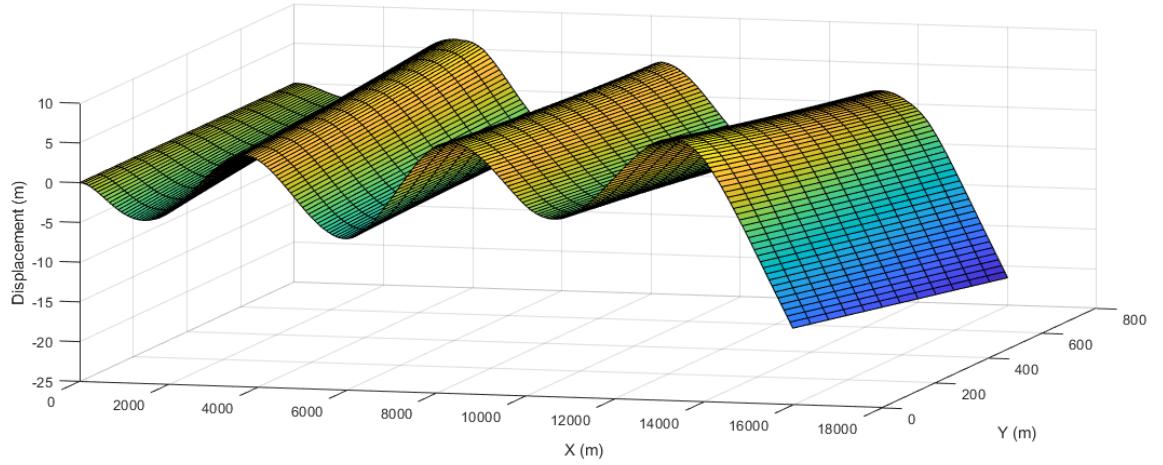


Figure 11. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0, T_{cr} = 2\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 300\text{s}$.

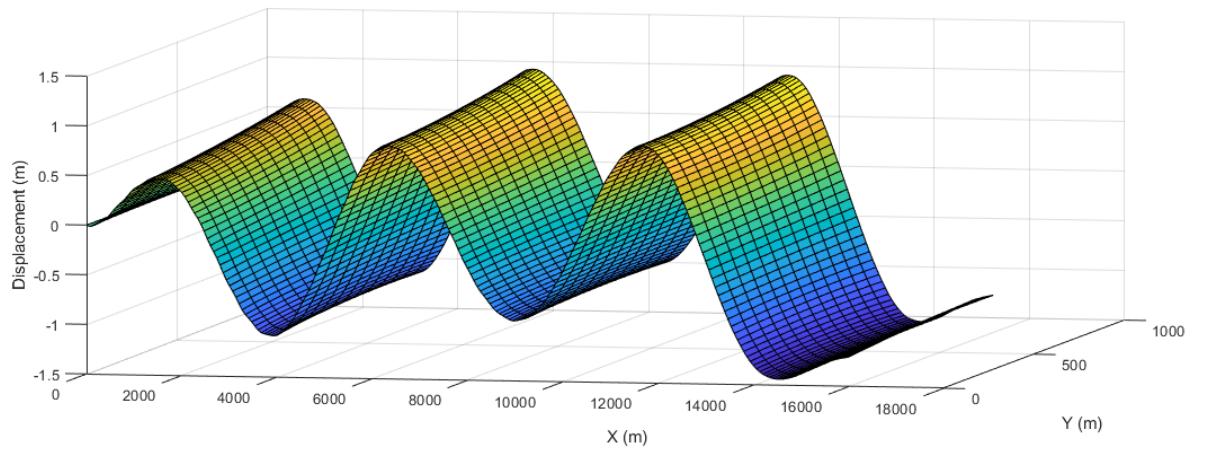


Figure 12. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8, T_{cr} = 2\text{km}; D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 300\text{s}$.

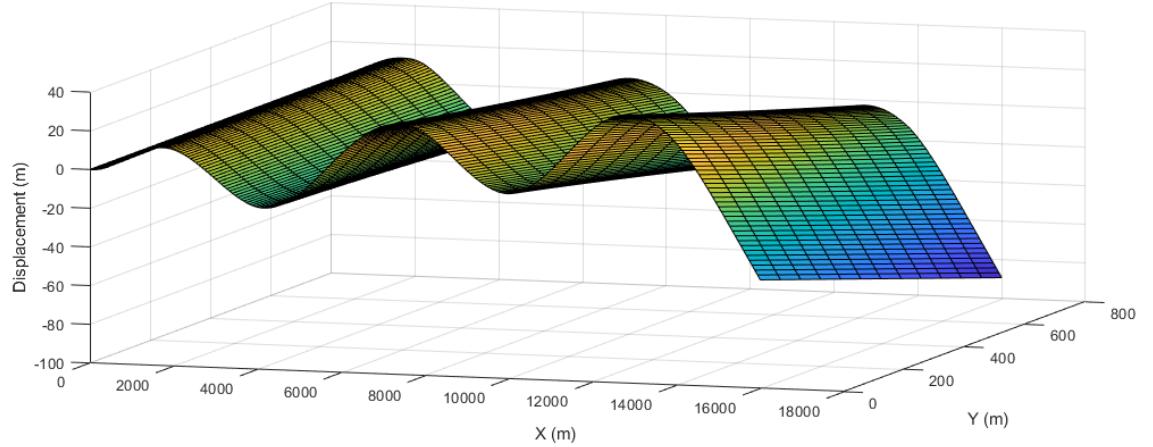


Figure 13. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0$. $T_{cr} = 2\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 500\text{s}$.

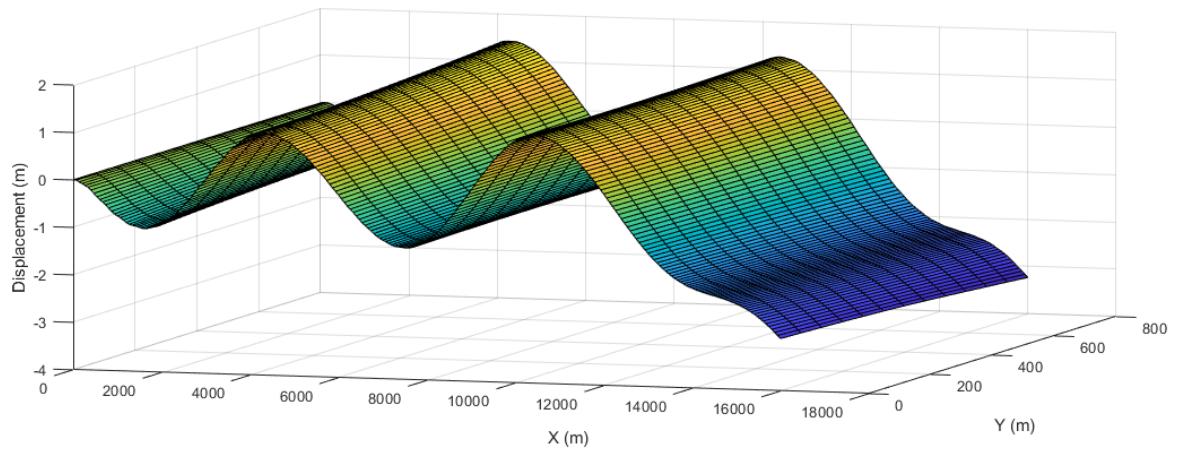


Figure 14. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8$. $T_{cr} = 2\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 500\text{s}$.

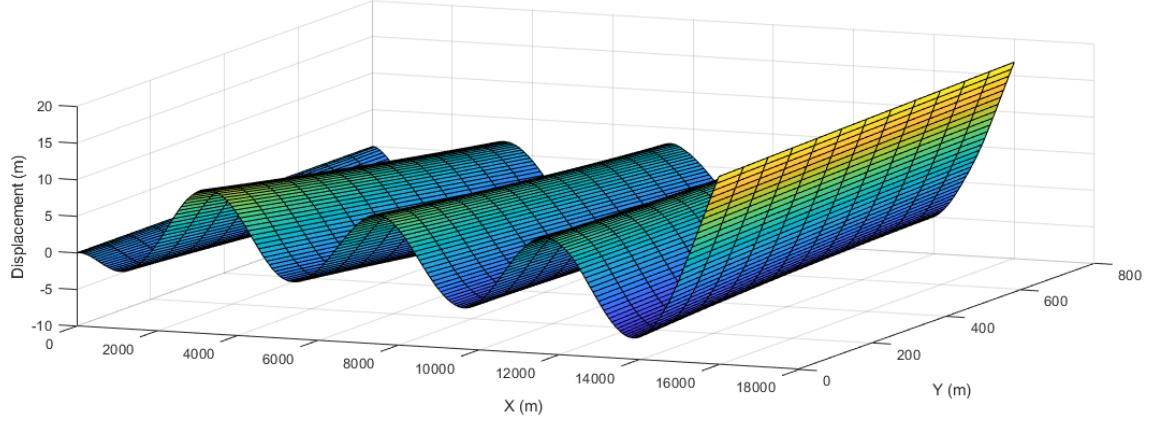


Figure 15. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 200\text{s}$.

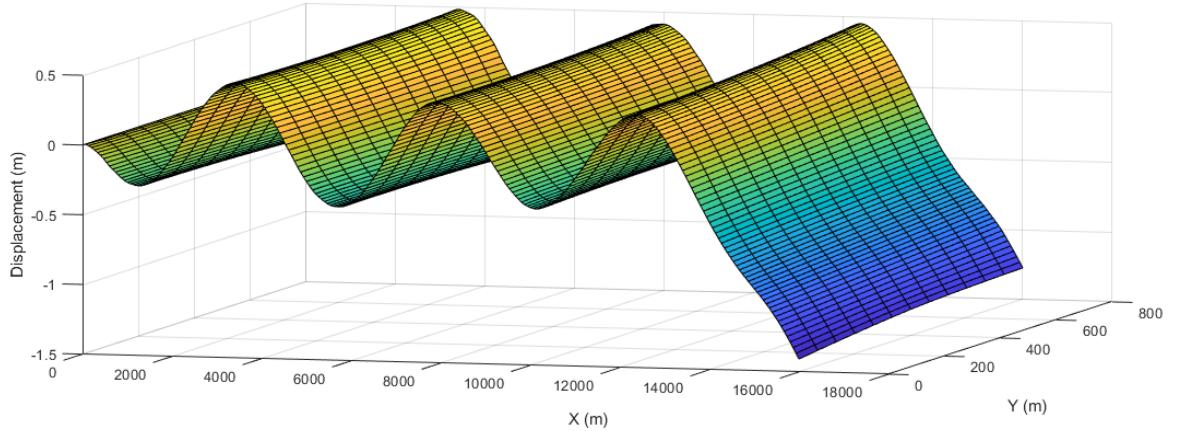


Figure 16. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 200\text{s}$.

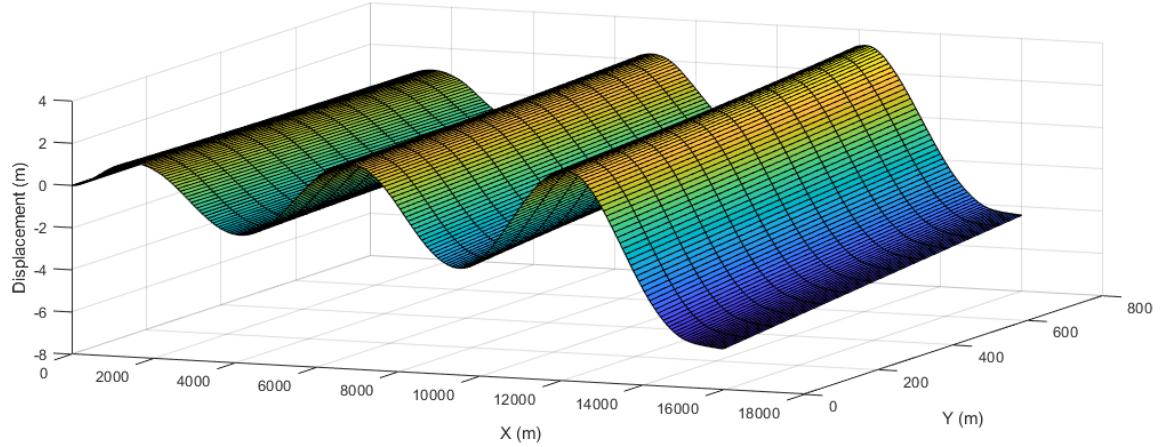


Figure 17. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 400\text{s}$.

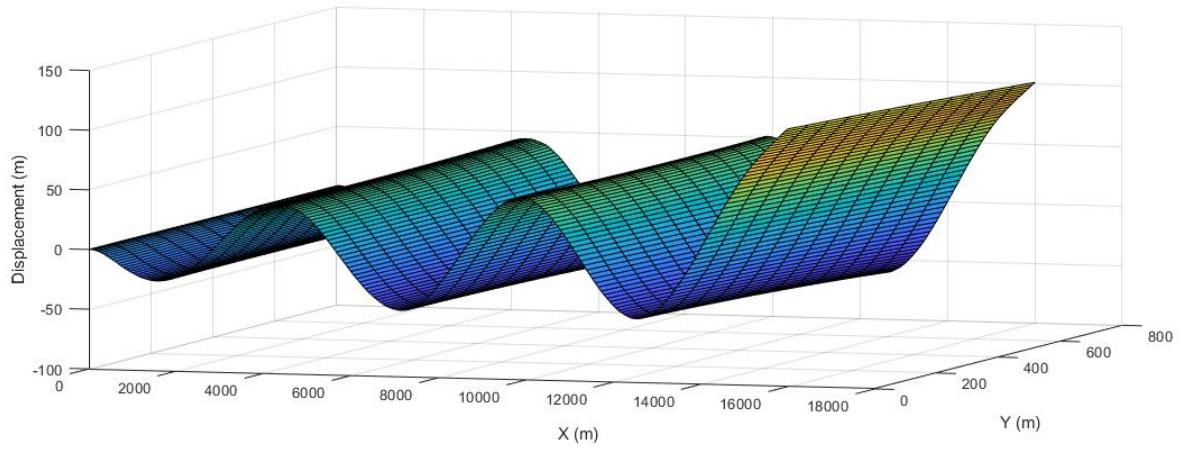


Figure 18. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 400\text{s}$.

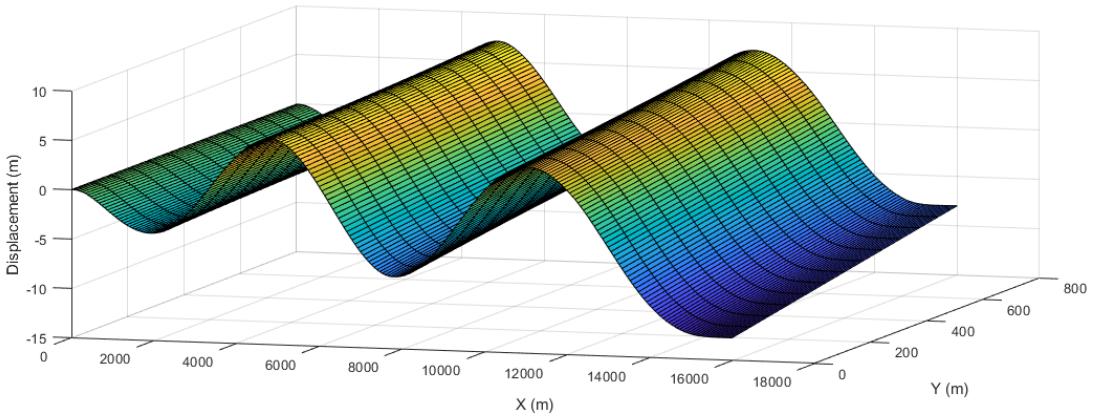


Figure 19. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 1, \alpha_2 = 0$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 700\text{s}$.

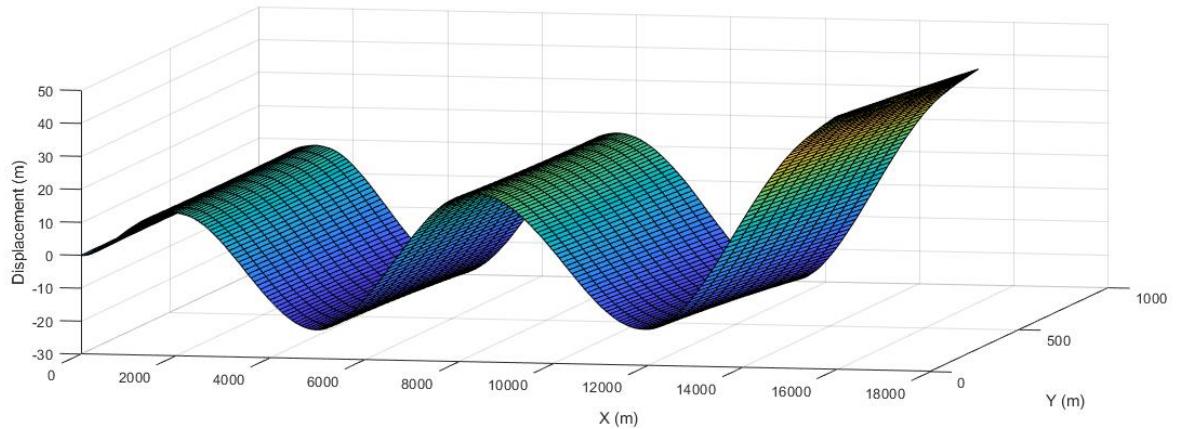


Figure 20. Ice shelf vertical deflections result from the impact of the frontal incident wave. The parameters of the model $\alpha_1 = 0.2, \alpha_2 = 0.8$. $T_{cr} = 2.5\text{km}$; $D_{cr} = 10\text{m}$; The periodicity of the forcing $T = 700\text{s}$.

Appendix A: Basic boundary conditions on the ice-shelf surface and on the ice-shelf edges in x, η, ξ variables

(The approximations considered in this item are the same like in the description of the previous model/versions)

- 1) The **boundary conditions on the ice-shelf surface** (stress-free surface) are expressed as

$$\begin{cases} -\sigma_{xx} \frac{\partial h_s}{\partial x} - \sigma_{xy} \frac{\partial h_s}{\partial y} + \sigma_{xz} = 0; \\ -\sigma_{yx} \frac{\partial h_s}{\partial x} - \sigma_{yy} \frac{\partial h_s}{\partial y} + \sigma_{yz} = 0; \\ -\sigma_{zx} \frac{\partial h_s}{\partial x} - \sigma_{zy} \frac{\partial h_s}{\partial y} + \sigma_{zz} = 0. \end{cases} \quad (10)$$

Respectively, using Eq. (6), we obtain the follow boundary conditions on the ice-shelf surface for the displacements in x, y, z variables

$$\begin{cases} -\frac{2}{1-2\nu} \left\{ (1-\nu) \frac{\partial U}{\partial x} + \nu \left(\frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) \right\} \cdot \frac{\partial h_s}{\partial x} - \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \cdot \frac{\partial h_s}{\partial y} + \left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right) = 0; \\ -\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \frac{\partial h_s}{\partial x} - \frac{2}{1-2\nu} \left\{ (1-\nu) \frac{\partial V}{\partial y} + \nu \left(\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} \right) \right\} \frac{\partial h_s}{\partial y} + \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) = 0; \\ -\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right) \cdot \frac{\partial h_s}{\partial x} - \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) \cdot \frac{\partial h_s}{\partial y} + \frac{2}{1-2\nu} \left\{ (1-\nu) \frac{\partial W}{\partial z} + \nu \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) \right\} = 0. \end{cases} \quad (11)$$

Essentially, Eq. (5) at $\xi = 0$ and Eq. (10) are the same equations.

Therefore, in x, η, ξ variables we can write the equations that express the stress-free conditions on the ice surface like Eq. (5) at $\xi = 0$:

- a) **the first equation (lines 19194-19469 in the program code)** is

$$\begin{aligned} & \frac{2(1-\nu)}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_x H)_1^{i,j} + \frac{2\nu}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_x H)_1^{i,j} + \\ & \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_x H)_1^{i,j} + \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_y H)_1^{i,j} - \\ & \left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_1^{i,j} = 0; \end{aligned} \quad (12)$$

b) the second equation ([lines 19473-19730 in the program code](#)) is

$$\begin{aligned} & \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_x H)_1^{i,j} + \frac{2(1-\nu)}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_1^{i,j} \cdot \\ & (\xi'_y H)_1^{i,j} + \frac{2\nu}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_y H)_1^{i,j} + \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_y H)_1^{i,j} - \\ & \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_1^{i,j} = 0; \end{aligned} \quad (13)$$

c) the third equation ([lines 19734-19963 in the program code](#)) is

$$\begin{aligned} & \left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_x H)_1^{i,j} + \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_1^{i,j} \cdot (\xi'_y H)_1^{i,j} - \\ & \frac{2(1-\nu)}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_1^{i,j} - \frac{2\nu}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_1^{i,j} - \frac{2\nu}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_1^{i,j} = 0; \end{aligned} \quad (14)$$

where the bottom index "1" corresponds to the grid layer located on the ice shelf surface.

2) The **boundary conditions on the ice-shelf front ($x = L$)** are expressed as

$$\begin{cases} \sigma_{xx} = f(\xi); \\ \sigma_{yx} = 0; \\ \sigma_{zx} = 0; \end{cases} \quad (15)$$

where

$$f(\xi) = \begin{cases} 0, \xi < \frac{h_s}{H}; \\ \rho_w g(h_s - \xi H), \xi \geq \frac{h_s}{H}. \end{cases} \quad (16)$$

In x, η, ξ variables equations (15) in terms of the displacements are expressed as

a) the first equation ([lines 981-1066 in the program code](#)) is

$$\frac{2(1-\nu)}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{N_{x,j}} + \frac{2\nu}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{N_{x,j}} + \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{N_{x,j}} = \frac{2(1+\nu)}{E}. \\ f(\xi); \quad (17)$$

b) **the second equation (lines 1071-1137 in the program code)** is

$$\left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{N_{x,j}} = 0; \quad (18)$$

c) **the third equation (lines 1141-1198 in the program code)** is

$$\left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{N_{x,j}} = 0. \quad (19)$$

where the first superscript " N_x " corresponds to the grid layer located at the ice shelf terminus.

3) The **boundary conditions on the ice-shelf lateral edge $y = y_1(x)$** are expressed as

$$\begin{cases} \sigma_{xx} \frac{dy_1}{dx} - \sigma_{xy} = f_x^1(\xi); \\ \sigma_{yx} \frac{dy_1}{dx} - \sigma_{yy} = f_y^1(\xi); \\ \sigma_{zx} \frac{dy_1}{dx} - \sigma_{zy} = 0; \end{cases} \quad (20)$$

where

$$f_x^1(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \rho_w g (h_s - \xi H) \frac{dy_1}{dx}, & \xi \geq \frac{h_s}{H}; \end{cases} \quad (21)$$

and

$$f_y^1(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\rho_w g (h_s - \xi H), & \xi \geq \frac{h_s}{H}. \end{cases} \quad (22)$$

Respectively, in x, η, ξ variables equations (20) in terms of the displacements are expressed as

a) **the first equation (lines 3123-3292 in the program code)** is

$$\begin{aligned} & \frac{2(1-\nu)}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx} \right)^i + \frac{2\nu}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx} \right)^i + \\ & \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx} \right)^i - \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,1} = \frac{2(1+\nu)}{E} \cdot f_x^1(\xi); \end{aligned} \quad (23)$$

b) **the second equation (lines 3296-3462 in the program code)** is

$$\begin{aligned} & \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx} \right)^i - \frac{2(1-\nu)}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,1} - \\ & \frac{2\nu}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,1} - \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} = \frac{2(1+\nu)}{E} \cdot f_y^1(\xi); \end{aligned} \quad (24)$$

c) **the third equation (lines 3466-3578 in the program code)** is

$$\left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx} \right)^i - \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_k^{i,1} = 0. \quad (25)$$

where the second superscript "1" corresponds to the grid layer located at the ice shelf first lateral edge $y = y_1(x)$.

4) The **boundary conditions on the ice-shelf lateral edge $y = y_2(x)$** look like Eq. (20) and are expressed as

$$\begin{cases} -\sigma_{xx} \frac{dy_2}{dx} + \sigma_{xy} = f_x^2(\xi); \\ -\sigma_{yx} \frac{dy_2}{dx} + \sigma_{yy} = f_y^2(\xi); \\ -\sigma_{zx} \frac{dy_2}{dx} + \sigma_{zy} = 0; \end{cases} \quad (26)$$

where

$$f_x^2(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\rho_w g (h_s - \xi H) \frac{dy_2}{dx}, & \xi \geq \frac{h_s}{H}; \end{cases} \quad (27)$$

and

$$f_y^2(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \rho_w g(h_s - \xi H), & \xi \geq \frac{h_s}{H}. \end{cases} \quad (28)$$

Respectively, in x, η, ξ variables equations (26) in terms of the displacements look like Eqs (23)-(25) and are expressed as

a) **the first equation (lines 11172-11336 in the program code)** is

$$\begin{aligned} & -\frac{2(1-\nu)}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,N_\eta} \cdot \left(\frac{dy_2}{dx} \right)^i - \frac{2\nu}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} \cdot \left(\frac{dy_2}{dx} \right)^i - \\ & \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} \cdot \left(\frac{dy_2}{dx} \right)^i + \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} = \frac{2(1+\nu)}{E} \cdot f_x^2(\xi); \end{aligned} \quad (29)$$

b) **the second equation (lines 11340-11506 in the program code)** is

$$\begin{aligned} & - \left\{ \eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} \cdot \left(\frac{dy_2}{dx} \right)^i + \frac{2(1-\nu)}{1-2\nu} \left\{ \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} + \\ & \frac{2\nu}{1-2\nu} \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} = \frac{2(1+\nu)}{E} \cdot f_y^2(\xi); \end{aligned} \quad (30)$$

c) **the third equation (lines 11510-11621 in the program code)** is

$$-\left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} \cdot \left(\frac{dy_2}{dx} \right)^i + \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} = 0. \quad (31)$$

where the second superscript " N_η " corresponds to the grid layer located at the ice shelf second lateral edge $y = y_2(x)$.

Appendix B: The boundary conditions on the ice-shelf base in x, η, ξ variables

The **boundary conditions on the ice-shelf base** (the surface under the water pressure forcing) are expressed as

$$\begin{cases} \sigma_{xx} \frac{\partial h_b}{\partial x} + \sigma_{xy} \frac{\partial h_b}{\partial y} - \sigma_{xz} = -P \frac{\partial h_b}{\partial x}; \\ \sigma_{yx} \frac{\partial h_b}{\partial x} + \sigma_{yy} \frac{\partial h_b}{\partial y} - \sigma_{yz} = -P \frac{\partial h_b}{\partial y}; \\ \sigma_{zx} \frac{\partial h_b}{\partial x} + \sigma_{zy} \frac{\partial h_b}{\partial y} - \sigma_{zz} = P; \end{cases} \quad (32)$$

where P is the sum of the hydrostatic pressure and the pressure perturbations result from ocean swell:

$$P = \rho g H + P'. \quad (33)$$

The vertical integration of the momentum equations (1) from the ice base h_b to the ice surface h_s , changing the order between the integration and the differentiation and accounting for the boundary conditions (10) and (32) yields the following equations

$$\begin{cases} \frac{\partial}{\partial x} \int_{h_b}^{h_s} \sigma_{xx} dz + \frac{\partial}{\partial y} \int_{h_b}^{h_s} \sigma_{xy} dz - P \frac{\partial h_b}{\partial x} = \rho \int_{h_b}^{h_s} \frac{\partial^2 U}{\partial t^2} dz; \\ \frac{\partial}{\partial x} \int_{h_b}^{h_s} \sigma_{yx} dz + \frac{\partial}{\partial y} \int_{h_b}^{h_s} \sigma_{yy} dz - P \frac{\partial h_b}{\partial y} = \rho \int_{h_b}^{h_s} \frac{\partial^2 V}{\partial t^2} dz; \\ \frac{\partial}{\partial x} \int_{h_b}^{h_s} \sigma_{zx} dz + \frac{\partial}{\partial y} \int_{h_b}^{h_s} \sigma_{zy} dz + P = \rho g H + \rho \int_z^{h_s} \frac{\partial^2 W}{\partial t^2} dz; \end{cases} \quad (34)$$

where P is defined from Eq. (33).

Essentially, Eq. (34) are considered as boundary conditions on the ice shelf base. In x, η, ξ Eq. (34) are expressed as

$$\left\{ \begin{array}{l} \frac{\partial}{\partial x} \int_0^1 \sigma_{xx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^1 \sigma_{xx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^1 \sigma_{xy} H d\xi - P' \frac{\partial h_b}{\partial x} = \\ \quad = \rho g H \frac{\partial h_b}{\partial x} + \rho \int_0^1 \frac{\partial^2 U}{\partial t^2} H d\xi; \\ \frac{\partial}{\partial x} \int_0^1 \sigma_{yx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^1 \sigma_{yx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^1 \sigma_{yy} H d\xi - P' \frac{\partial h_b}{\partial y} = \\ \quad = \rho g H \frac{\partial h_b}{\partial y} + \rho \int_0^1 \frac{\partial^2 V}{\partial t^2} H d\xi; \\ \frac{\partial}{\partial x} \int_0^\xi \sigma_{zx} H d\xi + \eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zx} H d\xi + \eta'_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zy} H d\xi + P' = \\ \quad = \rho \int_0^1 \frac{\partial^2 W}{\partial t^2} H d\xi. \end{array} \right. \quad (35)$$

Respectively, the boundary conditions on the ice base in terms of the displacements are expressed as:

a) **the first equation (lines 19975-20869 in the program code)** is

$$\begin{aligned} & \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^1 \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^1 \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\ & \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^1 \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\ & \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\ & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\ & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\ & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \\ & \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \\ & \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} - P' \cdot \frac{\partial h_b}{\partial x} \cdot \frac{2(1+\nu)}{E} = \rho g H \frac{\partial h_b}{\partial x} \cdot \\ & \frac{2(1+\nu)}{E} + \rho \left(\int_0^1 \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{i,j} \cdot \frac{2(1+\nu)}{E}; \end{aligned} \quad (36)$$

b) the second equation (lines 20873-21743 in the program code) is

$$\begin{aligned}
& \left(\frac{\partial}{\partial x} \left(H \int_0^1 \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^1 \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^1 \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} - P' \frac{\partial h_b}{\partial y} \cdot \frac{2(1+\nu)}{E} = \rho g H \frac{\partial h_b}{\partial y} \cdot \frac{2(1+\nu)}{E} + \rho \left(\int_0^1 \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{i,N_\eta} \cdot \\
& \frac{2(1+\nu)}{E}; \tag{37}
\end{aligned}$$

c) the third equation (lines 21747-22289 in the program code) is

$$\begin{aligned}
& \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^1 \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^1 \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^1 \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_z \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \eta'_y \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^1 \xi'_y \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + P' \cdot \frac{2(1+\nu)}{E} = \\
& \rho \left(\int_0^1 \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,j} \cdot \frac{2(1+\nu)}{E}; \tag{38}
\end{aligned}$$

Appendix C: The approximation of the boundary conditions on the edges based on the integration of the basic boundary conditions into the momentum equations (2)

1) The boundary conditions on the ice-shelf front ($x = L$)

Accounting for the basic boundary conditions (15)-(16) and for the momentum equations (5) we can obtain the following equations at the ice-shelf front ($x = L$):

$$\left\{ \begin{array}{l} \frac{1}{2 \Delta x} \left(\int_0^\xi \sigma_{xx} H d\xi \right)_k^{N_x-2,j} - \frac{4}{2 \Delta x} \left(\int_0^\xi \sigma_{xx} H d\xi \right)_k^{N_x-1,j} \approx -\frac{3}{2 \Delta x} \left(\int_0^\xi f(\tilde{\xi}) H d\tilde{\xi} \right)_k^{N_x,j} - \\ \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi f(\tilde{\xi}) H d\tilde{\xi} \right)_k^{N_x,j} - (\xi'_x f(\xi) H)_k^{N_x,j} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{N_x,j}; \\ \\ \frac{1}{2 \Delta x} \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{N_x-2,j} - \frac{4}{2 \Delta x} \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{N_x-1,j} + \left(\eta'{}_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{yy} H d\xi \right)_k^{N_x,j} + \\ \left(\xi'{}_y \sigma_{yy} H \right)_k^{N_x,j} - (\sigma_{yz})_k^{N_x,j} \approx \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{N_x,j}; \\ \\ \frac{1}{2 \Delta x} \left(\int_0^\xi \sigma_{zx} H d\xi \right)_k^{N_x-2,j} - \frac{4}{2 \Delta x} \left(\int_0^\xi \sigma_{zx} H d\xi \right)_k^{N_x-1,j} + \left(\eta'{}_y \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zy} H d\xi \right)_k^{N_x,j} + \\ \left(\xi'{}_y \sigma_{zy} H \right)_k^{N_x,j} - (\sigma_{zz})_k^{N_x,j} \approx (\rho g H \xi)_k^{N_x,j} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{N_x,j}; \end{array} \right. \quad (39)$$

In terms of the displacements Eqs (39) are expressed as

- a) **the first equation on the ice-shelf front (lines 1212-1648 in the program code)** is

$$\begin{aligned} & \frac{1}{2 \Delta x} \cdot \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \\ & \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \end{aligned}$$

$$\begin{aligned}
& \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{N_{x-2,j}} + \frac{1}{2\Delta x} \cdot \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{N_{x-2,j}} - \frac{4}{2\Delta x} \cdot \\
& \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \\
& \frac{2(1-\nu)}{1-2\nu} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \\
& \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \frac{2\nu}{1-2\nu} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{N_{x-1,j}} \approx -\frac{3}{2\Delta x} \left(\int_0^\xi \tilde{f}(\tilde{\xi}) H d\tilde{\xi} \right)_k^{N_{x,j}} - \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \tilde{f}(\tilde{\xi}) H d\tilde{\xi} \right)_k^{N_{x,j}} - \left(\xi'_x \check{f}(\xi) H \right)_k^{N_{x,j}} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{N_{x,j}} \cdot \frac{2(1+\nu)}{E}; \tag{40}
\end{aligned}$$

where

$$\tilde{f}(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \rho_w g(h_s - \xi H) \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \tag{41}$$

$$\int_0^\xi \tilde{f}(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\frac{1}{2} \rho_w g (h_s - \xi H)^2 \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \tag{42}$$

$$\frac{\partial}{\partial \eta} \int_0^\xi \tilde{f}(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\rho_w g (h_s - \xi H) \left\{ \frac{\partial h_s}{\partial \eta} - \xi \frac{\partial H}{\partial \eta} \right\} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}. \end{cases} \tag{43}$$

b) **the second equation on the ice-shelf front (lines 1652-2524 in the program code)** is

$$\begin{aligned}
& \frac{1}{2\Delta x} \cdot \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{N_{x-2,j}} + \frac{1}{2\Delta x} \cdot \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_{x-2,j}} + \frac{1}{2\Delta x} \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{N_{x-2,j}} + \frac{1}{2\Delta x} \cdot \\
& \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{N_{x-2,j}} + \frac{1}{2\Delta x} \cdot \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{N_{x-2,j}} - \frac{4}{2\Delta x} \cdot \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \\
& \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{N_{x-1,j}} - \frac{4}{2\Delta x} \cdot
\end{aligned}$$

$$\begin{aligned}
& \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{N_x-1,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{N_x,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{N_x,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{N_x,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{N_x,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{N_x,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{N_x,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_y \eta'_y \frac{\partial V}{\partial \eta} + H (\xi'_y)^2 \frac{\partial V}{\partial \xi} \right\}_k^{N_x,j} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_y \frac{\partial U}{\partial x} + \right. \\
& \left. H \xi'_y \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_y \xi'_x \frac{\partial U}{\partial \xi} + H \xi'_y \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{N_x,j} - \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_k^{N_x,j} \approx \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{N_x,j} \cdot \\
& \frac{2(1+\nu)}{E}. \tag{44}
\end{aligned}$$

c) the third equation on the ice-shelf front ([lines 2528-3104 in the program code](#)) is

$$\begin{aligned}
& \frac{1}{2 \Delta x} \cdot \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right)_k^{N_x-2,j} + \frac{1}{2 \Delta x} \cdot \\
& \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right)_k^{N_x-2,j} - \frac{4}{2 \Delta x} \cdot \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right)_k^{N_x-1,j} - \frac{4}{2 \Delta x} \cdot \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right)_k^{N_x-1,j} - \frac{4}{2 \Delta x} \cdot \\
& \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right)_k^{N_x-1,j} - \frac{4}{2 \Delta x} \cdot \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right)_k^{N_x-1,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{N_x,j} + \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{N_x,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{N_x,j} + \left\{ H \xi'_y \xi'_z \frac{\partial V}{\partial \xi} + H \xi'_y \eta'_y \frac{\partial W}{\partial \eta} + \right. \\
& \left. H (\xi'_y)^2 \frac{\partial W}{\partial \xi} \right\}_k^{N_x,j} - \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{N_x,j} - \frac{2\nu}{1-2\nu} \cdot \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} + \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{N_x,j} \approx \\
& (\rho g H \xi)_k^{N_x,j} \cdot \frac{2(1+\nu)}{E} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{N_x,j} \cdot \frac{2(1+\nu)}{E}. \tag{45}
\end{aligned}$$

2) The boundary conditions on the ice-shelf lateral edge $y = y_1(x)$

Accounting for the basic boundary conditions (20)-(22) and for the momentum equations

(5) we can write the following **equations on the ice-shelf lateral edge $y = y_1(x)$:**

$$\left\{ \begin{array}{l} \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{xx} H d\xi \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{xx} H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{xx} \frac{dy_1}{dx} H d\xi \right)_k^{i,1} + \\ + \frac{4}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{xy} H d\xi \right)_k^{i,2} - \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{xy} H d\xi \right)_k^{i,3} + \left(\xi'_x \sigma_{xx} H \right)_k^{i,1} + \\ \left(\xi'_y \sigma_{xx} \frac{dy_1}{dx} H \right)_k^{i,1} - \left(\sigma_{xz} \right)_k^{i,1} \approx - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi f_x^1(\xi) H d\xi \right)_k^{i,1} + \\ \left(\xi'_y f_x^1(\xi) H \right)_k^{i,1} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{i,1}; \\ \\ \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{xx} \frac{dy_1}{dx} H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_x \right)^i \left(\int_0^\xi \sigma_{xx} \frac{dy_1}{dx} H d\xi \right)_k^{i,1} + \frac{4}{2\Delta y} \left(\eta'_x \right)^i \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{i,2} - \\ - \frac{1}{2\Delta y} \left(\eta'_x \right)^i \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{i,3} - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yx} \frac{dy_1}{dx} H d\xi \right)_k^{i,1} + \frac{4}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yy} H d\xi \right)_k^{i,2} - \\ - \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yy} H d\xi \right)_k^{i,3} + \left(\xi'_x \sigma_{xx} \frac{dy_1}{dx} H \right)_k^{i,1} + \left(\xi'_y \sigma_{yx} \frac{dy_1}{dx} H \right)_k^{i,1} - \left(\sigma_{zx} \frac{dy_1}{dx} \right)_k^{i,1} \approx \\ \approx \left(\frac{\partial}{\partial x} \int_0^\xi f_x^1(\xi) H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_x \right)^i \left(\int_0^\xi f_x^1(\xi) H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi f_y^1(\xi) H d\xi \right)_k^{i,1} + \\ \left(\xi'_x f_x^1(\xi) H \right)_k^{i,1} + \left(\xi'_y f_y^1(\xi) H \right)_k^{i,1} + \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{i,1}; \\ \\ \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{zx} H d\xi \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zx} H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zx} \frac{dy_1}{dx} H d\xi \right)_k^{i,1} + \\ + \frac{4}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zy} H d\xi \right)_k^{i,2} - \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zy} H d\xi \right)_k^{i,3} + \left(\xi'_x \sigma_{zx} H \right)_k^{i,1} + \\ \left(\xi'_y \sigma_{zx} \frac{dy_1}{dx} H \right)_k^{i,1} - \left(\sigma_{zz} \right)_k^{i,1} \approx (\rho g H \xi)_k^{i,1} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,1}; \end{array} \right. \quad (46)$$

where the second superscripts “1-3” in Eq (46) denote respectively the numbers of the grid layers starting from the lateral edge $y = y_1(x)$, moving in the horizontal transverse direction in the glacier.

In terms of the displacements Eqs (46) are expressed as

a) **the first equation on the ice-shelf lateral edge $y = y_1(x)$ (lines 3591-6461**

in the program code) is

$$\begin{aligned}
& \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,1} + \\
& \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \left(\eta'_{yy} \right)^i \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \frac{\partial U}{\partial x} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{yy} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{yy} \right)^i \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2\nu}{1-2\nu} \left(\eta'_{yy} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{yy} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,1} + \frac{4}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,2} + \\
& \frac{4}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \\
& \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,2} - \frac{1}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot \\
& \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot \left(\eta'_{yy} \right)^i \cdot \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,3} + \\
& \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_x \frac{\partial U}{\partial x} + H \xi'_x \eta'_x \frac{\partial U}{\partial \eta} + H (\xi'_x)^2 \frac{\partial U}{\partial \xi} \right\}_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_x \eta'_y \frac{\partial V}{\partial \eta} + H \xi'_x \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_x \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_y \frac{dy_1}{dx} \frac{\partial U}{\partial x} + H \xi'_y \frac{dy_1}{dx} \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_y \frac{dy_1}{dx} \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_y \frac{dy_1}{dx} \eta'_y \frac{\partial V}{\partial \eta} + \right. \\
& \left. H \xi'_y \frac{dy_1}{dx} \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_y \frac{dy_1}{dx} \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} - \left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,1} \approx \\
& - \frac{3}{2\Delta y} \left(\eta'_{,y} \right)^i \left(\int_0^\xi \tilde{f}_x^1(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,1} + \left(\xi'_{,y} \tilde{f}_x^1(\xi) H \right)_k^{i,1} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\tilde{\xi} \right)_k^{i,1} \cdot \frac{2(1+\nu)}{E}; \quad (47)
\end{aligned}$$

where

$$\tilde{f}_x^1(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \rho_w g (h_s - \xi H) \cdot \frac{dy_1}{dx} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \quad (48)$$

$$\int_0^\xi \tilde{f}_x^1(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\frac{1}{2} \rho_w g (h_s - \xi H)^2 \cdot \frac{dy_1}{dx} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}. \end{cases} \quad (49)$$

b) **the second equation on the ice-shelf lateral edge $y = y_1(x)$ (lines 6466-9236 in the program code)** is

$$\begin{aligned}
& \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \frac{\partial U}{\partial x} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2\nu}{1-2\nu} \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2\nu}{1-2\nu} \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \frac{2\nu}{1-2\nu} \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,1} + \frac{4}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \\
& \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot (\eta'_{,x})^{i,1} \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot
\end{aligned}$$

$$\begin{aligned}
& \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,2} - \frac{1}{2 \Delta y} \cdot \left(\eta'_{x} \right)^{i,1} \cdot \\
& \left(H \int_0^{\xi} \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \frac{\partial V}{\partial x} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \\
& \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \left(\eta'_{x} \right)^{i,1} \cdot \left(H \int_0^{\xi} \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,3} - \frac{3}{2 \Delta y} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^{\xi} \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2 \Delta y} \cdot \left(\eta'_{y} \right)^i \cdot \left(H \frac{dy_1}{dx} \int_0^{\xi} \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2 \Delta y} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^{\xi} \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2 \Delta y} \cdot \left(\eta'_{y} \right)^i \cdot \left(H \frac{dy_1}{dx} \int_0^{\xi} \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2 \Delta y} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \frac{dy_1}{dx} \int_0^{\xi} \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,1} + \frac{4}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \left(\eta'_{y} \right)^i \cdot \left(H \int_0^{\xi} \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \left(\eta'_{y} \right)^i \cdot \left(H \int_0^{\xi} \frac{\partial U}{\partial x} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \left(H \int_0^{\xi} \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,2} + \frac{4}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,2} - \frac{1}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \left(\eta'_{y} \right)^i \cdot \left(H \int_0^{\xi} \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \left(\eta'_{y} \right)^i \cdot \left(H \int_0^{\xi} \frac{\partial U}{\partial x} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \left(H \int_0^{\xi} \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,3} - \frac{1}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{y} \right)^i \cdot \\
& \left(H \int_0^{\xi} \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,3} + \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_x \frac{dy_1}{dx} \frac{\partial U}{\partial x} + H \xi'_x \frac{dy_1}{dx} \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_x \frac{dy_1}{dx} \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,1} + \frac{2\nu}{1-2\nu} \cdot \\
& \left\{ H \xi'_x \frac{dy_1}{dx} \eta'_y \frac{\partial V}{\partial \eta} + H \xi'_x \frac{dy_1}{dx} \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_x \frac{dy_1}{dx} \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} + \left\{ H \xi'_y \frac{dy_1}{dx} \eta'_y \frac{\partial U}{\partial \eta} + H \xi'_y \frac{dy_1}{dx} \xi'_y \frac{\partial U}{\partial \xi} + \right. \\
& \left. H \xi'_y \frac{dy_1}{dx} \frac{\partial V}{\partial x} + H \xi'_y \frac{dy_1}{dx} \eta'_x \frac{\partial V}{\partial \eta} + H \xi'_y \frac{dy_1}{dx} \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,1} - \left\{ \frac{dy_1}{dx} \xi'_z \frac{\partial U}{\partial \xi} + \frac{dy_1}{dx} \frac{\partial W}{\partial x} + \frac{dy_1}{dx} \eta'_x \frac{\partial W}{\partial \eta} + \right. \\
& \left. \frac{3}{2 \Delta y} \left(\eta'_{x} \right)^{i,1} \left(\int_0^{\xi} \tilde{f}_y^1(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,1} - \frac{3}{2 \Delta y} \left(\eta'_{x} \right)^{i,1} \left(\int_0^{\xi} \tilde{f}_x^1(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,1} - \right. \\
& \left. \left(\xi'_{x} \tilde{f}_y^1(\tilde{\xi}) H \right)_k^{i,1} + \left(\xi'_{y} \tilde{f}_y^1(\tilde{\xi}) H \right)_k^{i,1} + \rho \left(\int_0^{\xi} \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{i,1} \cdot \frac{2(1+\nu)}{E} \right); \tag{50}
\end{aligned}$$

where $\tilde{f}_x^1(\xi)$ and $\int_0^\xi \tilde{f}_x^1(\tilde{\xi}) H d\tilde{\xi}$ are defined by Eq (48) and Eq (49), respectively, and

$$\frac{\partial}{\partial x} \int_0^\xi \tilde{f}_x^1(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\rho_w g(h_s - \xi H) \cdot \left\{ \frac{\partial h_s}{\partial x} - \xi \frac{\partial H}{\partial x} \right\} \cdot \frac{dy_1}{dx} \cdot \frac{2(1+\nu)}{E} - \frac{1}{2} \rho_w g (h_s - \xi H)^2 \cdot \frac{d^2 y_1}{dx^2} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \quad (51)$$

$$\int_0^\xi \tilde{f}_y^1(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \frac{1}{2} \rho_w g (h_s - \xi H)^2 \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}. \end{cases} \quad (52)$$

c) **the third equation on the ice-shelf lateral edge $y = y_1(x)$ (lines 9240-11150 in the program code)** is

$$\begin{aligned} & \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,1} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,1} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,1} + \\ & \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,1} + \\ & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_y)^i \cdot \\ & \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_y)^i \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \frac{\partial W}{\partial x} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_y)^i \cdot \\ & \left(H \frac{dy_1}{dx} \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \cdot (\eta'_y)^i \cdot \left(H \frac{dy_1}{dx} \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,1} + \frac{4}{2\Delta y} \cdot (\eta'_y)^i \cdot \\ & \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot (\eta'_y)^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,2} + \frac{4}{2\Delta y} \cdot (\eta'_y)^i \cdot \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,2} - \frac{1}{2\Delta y} \cdot \\ & (\eta'_y)^i \cdot \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot (\eta'_y)^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,3} - \frac{1}{2\Delta y} \cdot (\eta'_y)^i \cdot \\ & \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,3} + \left\{ H \xi'_x \xi'_z \frac{\partial U}{\partial \xi} + H \xi'_x \frac{\partial W}{\partial x} + H \xi'_x \eta'_x \frac{\partial W}{\partial \eta} + H (\xi'_x)^2 \frac{\partial W}{\partial \xi} \right\}_k^{i,1} + \left\{ H \xi'_y \frac{dy_1}{dx} \xi'_z \frac{\partial U}{\partial \xi} + \right. \end{aligned}$$

$$\begin{aligned}
& H\xi'_y \frac{dy_1}{dx} \frac{\partial W}{\partial x} + H\xi'_y \frac{dy_1}{dx} \eta'_x \frac{\partial W}{\partial \eta} + H\xi'_y \frac{dy_1}{dx} \xi'_x \frac{\partial W}{\partial \xi} \Big|_k^{i,1} - \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,1} - \frac{2\nu}{1-2\nu} \cdot \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \right. \\
& \left. \xi'_x \frac{\partial U}{\partial \xi} + \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,1} \approx (\rho g H \xi)_k^{i,1} \cdot \frac{2(1+\nu)}{E} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,1} \cdot \frac{2(1+\nu)}{E}; \quad (53)
\end{aligned}$$

As in the case of obtaining of Eqs (46), accounting for Eqs (26)-(28) and for the momentum equations (5) we can write the following **equations on the ice-shelf lateral edge $y = y_2(x)$:**

$$\left\{
\begin{aligned}
& \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{xx} H d\xi \right)_k^{i,1} + \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{xx} H d\xi \right)_k^{i,1} + \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{xy} H d\xi \right)_k^{i,N_\eta-2} + \\
& - \frac{4}{2\Delta y} \left(\eta'_y \right)^{N_\eta} \left(\int_0^\xi \sigma_{xy} H d\xi \right)_k^{i,N_\eta-1} + \frac{3}{2\Delta y} \left(\eta'_y \right)^{N_\eta} \left(\int_0^\xi \sigma_{xx} \frac{dy_2}{dx} H d\xi \right)_k^{i,N_\eta} + \left(\xi'_x \sigma_{xx} H \right)_k^{i,N_\eta} + \\
& \left(\xi'_y \sigma_{xx} \frac{dy_2}{dx} H \right)_k^{i,N_\eta} - \left(\sigma_{xz} \right)_k^{i,N_\eta} \approx - \frac{3}{2\Delta y} \left(\eta'_y \right)^{N_\eta} \left(\int_0^\xi f_x^2(\tilde{\xi}) H d\xi \right)_k^{i,N_\eta} - \\
& \left(\xi'_y f_x^2(\tilde{\xi}) H \right)_k^{i,N_\eta} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{i,N_\eta}; \\
\\
& \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{xx} \frac{dy_2}{dx} H d\xi \right)_k^{i,N_\eta} + \frac{1}{2\Delta y} \left(\eta'_x \right)^{i,N_\eta} \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{i,N_\eta-2} - \frac{4}{2\Delta y} \left(\eta'_x \right)^{i,N_\eta} \left(\int_0^\xi \sigma_{yx} H d\xi \right)_k^{i,N_\eta-1} + \\
& \frac{3}{2\Delta y} \left(\eta'_x \right)^{i,N_\eta} \left(\int_0^\xi \sigma_{xx} \frac{dy_2}{dx} H d\xi \right)_k^{i,N_\eta} + \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yy} H d\xi \right)_k^{i,N_\eta-2} - \frac{4}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yy} H d\xi \right)_k^{i,N_\eta-1} + \\
& \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{yy} \frac{dy_2}{dx} H d\xi \right)_k^{i,N_\eta} + \left(\xi'_x \sigma_{xx} \frac{dy_2}{dx} H \right)_k^{i,1} + \left(\xi'_y \sigma_{yx} \frac{dy_2}{dx} H \right)_k^{i,1} - \left(\sigma_{zx} \frac{dy_2}{dx} \right)_k^{i,1} \approx \\
& \approx - \left(\frac{\partial}{\partial x} \int_0^\xi f_x^2(\tilde{\xi}) H d\xi \right)_k^{i,1} - \frac{3}{2\Delta y} \left(\eta'_x \right)^{i,N_\eta} \left(\int_0^\xi f_x^2(\tilde{\xi}) H d\xi \right)_k^{i,N_\eta} - \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi f_y^2(\tilde{\xi}) H d\xi \right)_k^{i,N_\eta} - \\
& \left(\xi'_x f_x^2(\tilde{\xi}) H \right)_k^{i,N_\eta} - \left(\xi'_y f_y^2(\tilde{\xi}) H \right)_k^{i,N_\eta} + \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{i,N_\eta}; \\
\\
& \left(\frac{\partial}{\partial x} \int_0^\xi \sigma_{zx} H d\xi \right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial}{\partial \eta} \int_0^\xi \sigma_{zx} H d\xi \right)_k^{i,N_\eta} + \frac{1}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zy} H d\xi \right)_k^{i,N_\eta-2} - \\
& - \frac{4}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zy} H d\xi \right)_k^{i,N_\eta-1} + \frac{3}{2\Delta y} \left(\eta'_y \right)^i \left(\int_0^\xi \sigma_{zx} \frac{dy_2}{dx} H d\xi \right)_k^{i,N_\eta-2} + \left(\xi'_x \sigma_{zx} H \right)_k^{i,N_\eta} + \\
& \left(\xi'_y \sigma_{zx} \frac{dy_1}{dx} H \right)_k^{i,N_\eta} - \left(\sigma_{zz} \right)_k^{i,N_\eta} \approx (\rho g H \xi)_k^{i,N_\eta} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,N_\eta};
\end{aligned}
\right. \quad (54)$$

where indices, " $N_\eta - 2$ ", " $N_\eta - 1$ ", and " N_η ", denote respectively the numbers of the grid layers, moving from " $N_\eta - 2$ " in the horizontal transverse direction and ending at the ice lateral edge $y = y_2(x)$.

In terms of the displacements equations (54) are expressed as

- a) **the first equation on the ice-shelf lateral edge $y = y_2(x)$** ([lines 11635-14507 in the program code](#)) is

$$\begin{aligned}
 & \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
 & \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \\
 & \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
 & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \\
 & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \\
 & \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{1}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
 & \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
 & \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta-2} - \frac{4}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
 & \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2\Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot
 \end{aligned}$$

$$\begin{aligned}
& \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2 \Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2 \Delta y} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
& \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} + \frac{3}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \left(\eta'_{\text{y}} \right)^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \frac{\partial U}{\partial x} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_{\text{y}} \right)^i \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \left(\eta'_{\text{y}} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{\text{y}} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_x \frac{\partial U}{\partial x} + H \xi'_x \eta'_x \frac{\partial U}{\partial \eta} + H (\xi'_x)^2 \frac{\partial U}{\partial \xi} \right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_x \eta'_y \frac{\partial V}{\partial \eta} + \right. \\
& \left. H \xi'_x \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_x \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_y \frac{dy_2}{dx} \frac{\partial U}{\partial x} + H \xi'_y \frac{dy_2}{dx} \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_y \frac{dy_2}{dx} \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \\
& \left\{ H \xi'_y \frac{dy_2}{dx} \eta'_y \frac{\partial V}{\partial \eta} + H \xi'_y \frac{dy_2}{dx} \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_y \frac{dy_2}{dx} \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} - \left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} \approx \\
& - \frac{3}{2 \Delta y} \left(\eta'_{\text{y}} \right)^i \left(\int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,N_\eta} - \left(\xi'_{\text{y}} \tilde{f}_x^2(\xi) H \right)_k^{i,N_\eta} + \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{i,N_\eta} \cdot \frac{2(1+\nu)}{E}; \\
\end{aligned} \tag{55}$$

where

$$\tilde{f}_x^2(\xi) = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\rho_w g (h_s - \xi H) \cdot \frac{dy_2}{dx} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \tag{56}$$

$$\int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \frac{1}{2} \rho_w g (h_s - \xi H)^2 \cdot \frac{dy_2}{dx} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}. \end{cases} \tag{57}$$

- b) **the second equation at the ice-shelf lateral edge $y = y_2(x)$ (lines 14511-17265 in the program code)** is

$$\begin{aligned}
& \left(\eta'_{\cdot y} \right)^i \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2\Delta y} \cdot \frac{2\nu}{1-2\nu} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} + \frac{3}{2\Delta y} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right)_k^{i,N_\eta} + \frac{3}{2\Delta y} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{3}{2\Delta y} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \frac{\partial V}{\partial x} d\xi \right)_k^{i,N_\eta} + \frac{3}{2\Delta y} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right)_k^{i,N_\eta} + \frac{3}{2\Delta y} \cdot \left(\eta'_{\cdot y} \right)^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_x \frac{dy_2}{dx} \frac{\partial U}{\partial x} + H \xi'_x \frac{dy_2}{dx} \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_x \frac{dy_2}{dx} \xi'_x \frac{\partial U}{\partial \xi} \right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu} \cdot \\
& \left\{ H \xi'_x \frac{dy_2}{dx} \eta'_y \frac{\partial V}{\partial \eta} + H \xi'_x \frac{dy_2}{dx} \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_x \frac{dy_2}{dx} \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} + \left\{ H \xi'_y \frac{dy_2}{dx} \eta'_y \frac{\partial U}{\partial \eta} + H \xi'_y \frac{dy_2}{dx} \xi'_y \frac{\partial U}{\partial \xi} + \right. \\
& \left. H \xi'_y \frac{dy_2}{dx} \frac{\partial V}{\partial x} + H \xi'_y \frac{dy_2}{dx} \eta'_x \frac{\partial V}{\partial \eta} + H \xi'_y \frac{dy_2}{dx} \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} - \left\{ \frac{dy_2}{dx} \xi'_z \frac{\partial U}{\partial \xi} + \frac{dy_2}{dx} \frac{\partial W}{\partial x} + \frac{dy_2}{dx} \eta'_x \frac{\partial W}{\partial \eta} + \right. \\
& \left. \frac{dy_2}{dx} \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} \approx - \left(\frac{\partial}{\partial x} \int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,N_\eta} - \frac{3}{2\Delta y} \left(\eta'_{\cdot x} \right)^{i,N_\eta} \left(\int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,N_\eta} - \\
& \frac{3}{2\Delta y} \left(\eta'_{\cdot x} \right)^{i,N_\eta} \left(\int_0^\xi \tilde{f}_y^2(\tilde{\xi}) H d\tilde{\xi} \right)_k^{i,N_\eta} - \left(\xi'_{\cdot x} \tilde{f}_x^2(\xi) H \right)_k^{i,N_\eta} - \left(\xi'_{\cdot y} \tilde{f}_y^2(\xi) H \right)_k^{i,N_\eta} + \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\tilde{\xi} \right)_k^{i,N_\eta}. \\
& \frac{2(1+\nu)}{E}; \tag{58}
\end{aligned}$$

where $\tilde{f}_x^2(\xi)$ and $\int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi}$ are defined by Eq (56) and Eq (57), respectively, and

$$\begin{aligned}
& \frac{\partial}{\partial x} \int_0^\xi \tilde{f}_x^2(\tilde{\xi}) H d\tilde{\xi} = \\
& \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ \rho_w g(h_s - \xi H) \cdot \left\{ \frac{\partial h_s}{\partial x} - \xi \frac{\partial H}{\partial x} \right\} \cdot \frac{dy_2}{dx} \cdot \frac{2(1+\nu)}{E} + \frac{1}{2} \rho_w g(h_s - \xi H)^2 \cdot \frac{d^2 y_2}{dx^2} \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}; \end{cases} \tag{59}
\end{aligned}$$

$$\int_0^\xi \tilde{f}_y^2(\tilde{\xi}) H d\tilde{\xi} = \begin{cases} 0, & \xi < \frac{h_s}{H}; \\ -\frac{1}{2} \rho_w g(h_s - \xi H)^2 \cdot \frac{2(1+\nu)}{E}, & \xi \geq \frac{h_s}{H}. \end{cases} \tag{60}$$

c) the third equation on the ice-shelf lateral edge $y = y_2(x)$ ([lines 17269](#)-

[19180](#) in the program code) is

$$\begin{aligned}
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,N_\eta} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,N_\eta} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,N_\eta} + \frac{1}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \\
& \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,N_\eta-2} + \frac{1}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \\
& \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,N_\eta-2} - \frac{4}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \\
& \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,N_\eta-1} - \frac{4}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,N_\eta-1} + \frac{3}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \frac{\partial W}{\partial x} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \\
& \left(H \frac{dy_2}{dx} \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right)_k^{i,N_\eta} + \frac{3}{2 \Delta y} \cdot (\eta'_{y'})^i \cdot \left(H \frac{dy_2}{dx} \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right)_k^{i,N_\eta} + \left\{ H \xi'_x \xi'_z \frac{\partial U}{\partial \xi} + H \xi'_x \frac{\partial W}{\partial x} + \right. \\
& H \xi'_x \eta'_x \frac{\partial W}{\partial \eta} + H (\xi'_x)^2 \frac{\partial W}{\partial \xi} \Big)_k^{i,N_\eta} + \left\{ H \xi'_y \frac{dy_2}{dx} \xi'_z \frac{\partial U}{\partial \xi} + H \xi'_y \frac{dy_2}{dx} \frac{\partial W}{\partial x} + H \xi'_y \frac{dy_2}{dx} \eta'_x \frac{\partial W}{\partial \eta} + \right. \\
& H \xi'_y \frac{dy_2}{dx} \xi'_x \frac{\partial W}{\partial \xi} \Big)_k^{i,N_\eta} - \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,N_\eta} - \frac{2\nu}{1-2\nu} \cdot \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi} + \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,N_\eta} \approx \\
& (\rho g H \xi')_k^{i,N_\eta} \cdot \frac{2(1+\nu)}{E} + \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,N_\eta} \cdot \frac{2(1+\nu)}{E}; \tag{61}
\end{aligned}$$

Appendix D: Equations (5) in terms of the displacements

In terms of the displacements Eqs (5) are expressed as

a) **the first equation (lines 22304-23431 in the program code)** is

$$\begin{aligned}
& \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left\{ H \xi'_x \frac{\partial U}{\partial x} + H \xi'_x \eta'_x \frac{\partial U}{\partial \eta} + H (\xi'_x)^2 \frac{\partial U}{\partial \xi} \right\}_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_x \eta'_y \frac{\partial V}{\partial \eta} + H \xi'_x \xi'_y \frac{\partial V}{\partial \xi} + H \xi'_x \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,j} + \\
& \left\{ H \xi'_y \eta'_y \frac{\partial U}{\partial \eta} + H (\xi'_y)^2 \frac{\partial U}{\partial \xi} + H \xi'_y \frac{\partial V}{\partial x} + H \xi'_y \eta'_x \frac{\partial V}{\partial \eta} + H \xi'_y \xi'_x \frac{\partial V}{\partial \xi} \right\}_k^{i,j} - \left\{ \xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi} \right\}_k^{i,j} = \rho \left(\int_0^\xi \frac{\partial^2 U}{\partial t^2} H d\xi \right)_k^{i,j} \cdot \frac{2(1+\nu)}{E}; \tag{62}
\end{aligned}$$

b) **the second equation (lines 23437-24511 in the program code)** is

$$\begin{aligned}
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial V}{\partial x} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial V}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2(1-\nu)}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial U}{\partial x} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial U}{\partial \eta} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left\{ H \xi'_x \eta'_y \frac{\partial U}{\partial \eta} + H \xi'_x \xi'_y \frac{\partial U}{\partial \xi} + H \xi'_x \frac{\partial V}{\partial x} + H \xi'_x \eta'_x \frac{\partial V}{\partial \eta} + H (\xi'_x)^2 \frac{\partial V}{\partial \xi} \right\}_k^{i,j} + \\
& \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ H \xi'_y \eta'_y \frac{\partial V}{\partial \eta} + H (\xi'_y)^2 \frac{\partial V}{\partial \xi} \right\}_k^{i,j} + \frac{2\nu}{1-2\nu} \cdot \left\{ H \xi'_y \frac{\partial U}{\partial x} + H \xi'_y \eta'_x \frac{\partial U}{\partial \eta} + H \xi'_y \xi'_x \frac{\partial U}{\partial \xi} + H \xi'_y \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,j} - \\
& \left\{ \xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi} \right\}_k^{i,j} = \rho \left(\int_0^\xi \frac{\partial^2 V}{\partial t^2} H d\xi \right)_k^{i,N_\eta} \cdot \frac{2(1+\nu)}{E}; \tag{63}
\end{aligned}$$

c) **the third equation (lines 24517-25274 in the program code)** is

$$\begin{aligned}
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,j} + \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \\
& \left(\frac{\partial}{\partial x} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial U}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \frac{\partial W}{\partial x} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_x \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_x \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_x \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_z \frac{\partial V}{\partial \xi} d\xi \right) \right)_k^{i,j} + \\
& \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \eta'_y \frac{\partial W}{\partial \eta} d\xi \right) \right)_k^{i,j} + \left(\eta'_y \frac{\partial}{\partial \eta} \left(H \int_0^\xi \xi'_y \frac{\partial W}{\partial \xi} d\xi \right) \right)_k^{i,j} + \left\{ H \xi'_x \xi'_z \frac{\partial U}{\partial \xi} + H \xi'_x \frac{\partial W}{\partial x} + H \xi'_x \eta'_x \frac{\partial W}{\partial \eta} + \right. \\
& \left. H (\xi'_x)^2 \frac{\partial W}{\partial \xi} + H \xi'_y \xi'_z \frac{\partial V}{\partial \xi} + H \xi'_y \eta'_y \frac{\partial W}{\partial \eta} + H (\xi'_y)^2 \frac{\partial W}{\partial \xi} \right\}_k^{i,j} - \frac{2(1-\nu)}{1-2\nu} \cdot \left\{ \xi'_z \frac{\partial W}{\partial \xi} \right\}_k^{i,j} - \frac{2\nu}{1-2\nu} \cdot \left\{ \frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \right. \\
& \left. \xi'_x \frac{\partial U}{\partial \xi} + \eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi} \right\}_k^{i,j} = \rho \left(\int_0^\xi \frac{\partial^2 W}{\partial t^2} H d\xi \right)_k^{i,j} \cdot \frac{2(1+\nu)}{E}; \tag{64}
\end{aligned}$$

References

1. Lamb, H.: Hydrodynamics. (6th ed.). Cambridge: Cambridge University Press, 1994.
2. Landau, L.D., E.M. Lifshitz: Theory of Elasticity. (3rd ed.). Oxford: Butterworth-Heinemann, (Vol. 7), 1986.
3. Pattyn, F.: Ice-sheet modeling at different spatial resolutions: focus on the grounding zone, *Ann. Glaciol.*, 31, 211–216, 2000.
4. Pattyn, F.: Transient glacier response with a higher-order numerical ice-flow model, *J. Glaciol.*, 48, 467–477, 2002.
5. Holdsworth, G. & J. Glynn: Iceberg calving from floating glaciers by a vibrating mechanism. *Nature*, 274, 464-466, 1978.
6. Konovalov Y.V. (2019) Ice-shelf vibrations modeled by a full 3-D elastic model. *Annals of Glaciology*, 60(79), 68-74. doi:[10.1017/aog.2019.9](https://doi.org/10.1017/aog.2019.9) ([http://dx.doi.org/10.1017/aog.2019.9](https://dx.doi.org/10.1017/aog.2019.9))
7. Lurie, A.I.: Theory of Elasticity. Berlin: Springer, (Foundations of Engineering Mechanics), 2005.
8. Tikhonov, A.N., Samarskii, A.A.: Equations of Mathematical Physics. Pergamon Press Ltd., USA, 1963
9. Freed-Brown, J., Amundson J., MacAyeal, D., & Zhang, W.: Blocking a wave: Frequency band gaps in ice shelves with periodic crevasses. *Ann. Glaciol.*, 53(60), 85-89, doi: [10.3189/2012AoG60A120](https://doi.org/10.3189/2012AoG60A120), 2012
10. Ashcroft, N.W., & Mermin, N.D.: Solid state physics. Books Cole, Belmont, CA, 1976
11. Balmforth, N.J., & Craster, R.V.: Ocean waves and ice sheets. *J. Fluid Mech.*, 395, 89-124, doi: [10.1017/S0022112099005145](https://doi.org/10.1017/S0022112099005145), 1999

12. Bassis, J.N., Fricker, H.A., Coleman, R., Minster, J.-B.: An investigation into the forces that drive ice-shelf rift propagation on the Amery Ice Shelf, East Antarctica. *J. Glaciol.*, 54 (184), 17-27, doi: 10.3189/002214308784409116, 2008
13. Bennetts, L.G., Biggs, N.R.T., Porter, D.: The interaction of flexural-gravity waves with periodic geometries. *Wave Motion*, 46 (1), 57-73, doi.org/10.1016/j.wavemoti.2008.08.002, 2009
14. Bennets, L., Squire, V.: Wave scattering by multiple rows of circular ice floes. *J. Fluid Mech.*, 639, 213-238. doi:10.1017/S0022112009991017, 2009
15. Bennets, L., Williams, T.: Wave scattering by ice floes and polynyas of arbitrary shape. *J Fluid Mech.*, 662, 5-35. doi:10.1017/S0022112010004039, 2010
16. Bennetts, L.G., Squire V.A.: On the calculation of an attenuation coefficient for transects of ice-covered ocean. *Proc. R. Soc. A.*, 468, 136–162, doi:10.1098/rspa.2011.0155, 2012
17. Bromirski, P.D., Sergienko, O.V., MacAyeal, D.R.: Transoceanic infragravity waves impacting Antarctic ice shelves. *Geophys. Res. Lett.*, 37, L02502. doi:10.1029/2009GL041488, 2009
18. Bromirski, P., Stephen, R.: Response of the Ross Ice Shelf, Antarctica, to ocean gravity-wave forcing. *Ann. Glaciol.*, 53(60), 163-172. doi:10.3189/2012AoG60A058, 2012
19. Bromirski, P. D., Diez, A., Gerstoft, P., Stephen, R. A., Bolmer, T., Wiens, D. A., Aster, R. C., and Nyblade, A.: Ross ice shelf vibrations. *Geophys. Res. Lett.*, 42, 7589–7597, doi:10.1002/2015GL065284, 2015
20. Gerstoft P., Bromirski P., Chen Z., Stephen R.A., Aster R.C., Wiens D.A., Nyblade, A.: Tsunami excitation of the Ross Ice Shelf, Antarctica. *The Journal of the Acoustical Society of America*, 141(5), 3526, doi:10.1121/1.4987434, 2017

21. Chen, Z., Bromirski, P., Gerstoft, P., Stephen, R., Wiens, D., Aster, R., & Nyblade, A.: Ocean-excited plate waves in the Ross and Pine Island Glacier ice shelves. *J. Glaciol.*, 64(247), 730-744, doi:10.1017/jog.2018.66, 2018
22. Chen, Z., Bromirski, P., D, Gerstoft, P., Stephen, R. A., Lee, W. S., Yun, S., Olinger, S.D., Aster R.C., Wiens D.A., Nyblade A.A.: Ross Ice Shelf icequakes associated with ocean gravity wave activity. *Geophys. Res. Lett.*, 46, 8893– 8902, doi:10.1029/2019GL084123, 2019
23. Chou, T.: Band structure of surface flexural-gravity waves along periodic interfaces. *J. Fluid Mech.*, 369, 333-350, 1998.
24. Gerstoft, P., Bromirski, P., Chen, Z., Stephen, R.A, Aster, R.C., Wiens, D.A., Nyblade, A.: Tsunami excitation of the Ross Ice Shelf, Antarctica. *The Journal of the Acoustical Society of America*, 141(5), 3526, doi:10.1121/1.4987434, 2017
25. Godin, O. A, Zabotin, N. A.: Resonance vibrations of the Ross Ice Shelf and observations of persistent atmospheric waves. *J. Geophys. Res. Space Phys.*, 121, 10157-10171, doi:10.1002/2016JA023226, 2016
26. Goodman, D.J., Wadhams, P., & Squire, V.A.: The flexural response of a tabular ice island to ocean swell. *Ann. Glaciol.*, 1, 23–27, 1980
27. Holdsworth, G.: Tidal interaction with ice shelves. *Ann. Geophys.*, 33, 133-146, 1977
28. Hughes, T. J.: West Antarctic ice streams. *Reviews of Geophysics and Space Physics*, 15(1), 1-46, 1977
29. Ilyas, M., Meylan, M.H., Lamichhane, B., Bennetts, L.G.: Time-domain and modal response of ice shelves to wave forcing using the finite element method. *J. Fluids and Structures*, 80, 113-131, doi:10.1016/j.jfluidstructs.2018.03.010, 2018
30. Kalyanaraman B., Bennetts L.G., Lamichhane B., Meylan M.H.: On the shallow-water limit for modelling ocean-wave induced ice-shelf vibrations. *Wave Motion*, 90, 1-16, doi: 10.1016/j.wavemoti.2019.04.004, 2019

31. Kalyanaraman B., Meylan M.H., Bennetts L.G., Lamichhane B.P.: A coupled fluid-elasticity model for the wave forcing of an ice-shelf. *J. Fluids and Structures*, 97, 103074, doi: 10.1016/j.jfluidstructs.2020.103074, 2020
32. Lingle, C. S., Hughes, T. J., Kollmeyer, R. C.: Tidal flexure of Jakobshavn Glacier, West Greenland. *J. Geophys. Res.*, 86(B5), 3960-3968, 1981
33. MacAyeal, D.R., Okal, E.A., Aster, R.C., Bassis, J.N., Brunt, K.M., Cathles, L.M., Drucker, R., Fricker, H.A., Kim, Y.-J., Martin, S., Okal, M.H., Sergienko, O.V., Sponsler, M.P., & Thom, J.E.: Transoceanic wave propagation links iceberg calving margins of Antarctica with storms in tropics and Northern Hemisphere. *Geophys. Res. Lett.*, 33, L17502. doi:10.1029/2006GL027235, 2006
34. MacAyeal, D., Sergienko, O., Banwell, A.: A model of viscoelastic ice-shelf flexure. *J. Glaciol.*, 61(228), 635-645, doi:10.3189/2015JoG14J169, 2015
35. Massom, R.A.; Scambos, T.A.; Bennetts, L.G.; Reid, P.; Squire, V.A.; Stammerjohn, S.E. Antarctic ice shelf disintegration triggered by sea ice loss and ocean swell. *Nature*, 558, 383–389, 2018
36. Mei, C.C.: Resonant reflection of surface water waves by periodic sandbars. *J. Fluid Mech.*, 152, 315-335, doi: S0022112085000714, 1985
37. Meylan, M., Squire, V.A., & Fox, C.: Towards realism in modelling ocean wave behavior in marginal ice zones. *J. Geophys. Res.*, 102(C10), 22981–22991, 1997
38. Meylan, M., Bennetts, L., Hosking, R., Catt, E. On the calculation of normal modes of a coupled ice-shelf/sub-ice-shelf cavity system. *J. Glaciol.*, 63(240), 751-754. doi:10.1017/jog.2017.27, 2017
39. Papathanasiou, T. K., Karperaki, A. E., Theotokoglou, E. E., and Belibassakis, K. A.: Hydroelastic analysis of ice shelves under long wave excitation, *Nat. Hazards Earth Syst. Sci.*, 15, 1851–1857, doi:10.5194/nhess-15-1851-2015, 2015

40. Papathanasiou T.K., Karperaki, A.E., Belibassis K.A.: On the resonant hydroelastic behaviour of ice shelves, *Ocean Modelling*, 133, 11-26, doi:10.1016/j.ocemod.2018.10.008, 2019
41. Reeh, N., Christensen, E.L., Mayer, C., Olesen, O.B.: Tidal bending of glaciers: a linear viscoelastic approach. *Ann. Glaciol.*, 37, 83–89, 2003
42. Robin, G. de Q.: Seismic shooting and related investigations. In Norwegian-British-Swedish Antarctic Expedition, *Sci. Results* 5, *Glaciology* 3, Norsk Polarinstitutt (pp. 1949-1952). Oslo: University Press, 1958
43. Rosier, S.H.R., Gudmundsson, G.H., & Green, J.A.M.: Insights into ice stream dynamics through modeling their response to tidal forcing. *The Cryosphere*, 8, 1763–1775, 2014
44. Scambos, T.A., Hulbe, C., Fahnestock, M., Bohlander, J.: The link between climate warming and break-up of ice shelves in the Atlantic Peninsula. *J. Glaciol.*, 46(154), 516-530, doi:10.3189/172756500781833043, 2000
45. Shearman, E.D.R.: Radio science and oceanography, *Radio Sci.*, 18(3), 299–320, doi:10.1029/RS018i003p00299, 1983
46. Sheng, P.: Introduction to wave scattering, localization and mesoscopic phenomena. Springer, Berlin, 2006
47. Schmeltz, M., Rignot, E., & MacAyeal, D.R.: Tidal flexure along ice-sheets margins: Comparison of InSAR with an elastic plate model. *Ann. Glaciol.*, 34, 202-208, 2001
48. Schulson, E.M.: The Structure and Mechanical Behavior of Ice. *JOM*, 51 (2), 21-27, 1999
49. Sergienko, O.V.: Elastic response of floating glacier ice to impact of long-period ocean waves. *J. Geophys. Res.*, 115, F04028. doi:10.1029/2010JF001721, 2010
50. Sergienko, O. Normal modes of a coupled ice-shelf/sub-ice-shelf cavity system. *J. Glaciol.*, 59, 76–80, 2013

51. Sergienko, O.V. Behavior of flexural gravity waves on ice shelves: Application to the Ross Ice Shelf. *J. Geophys. Res. Oceans*, **122**, 6147–6164, 2017
52. Smith, A.M.: The use of tiltmeters to study the dynamics of Antarctic ice shelf grounding lines. *J. Glaciol.*, **37**, 51–58, 1991
53. Squire, V.A., Dugan, J.P., Wadhams, P., Rottier, P.J., & Liu, A.K.: Of ocean waves and sea ice. *Annu. Rev. Fluid Mech.*, **27**, 115–168, 1995
54. Stephenson, S.N.: Glacier flexure and the position of grounding lines: measurements by tiltmeter on Rutford Ice Stream, Antarctica. *Ann. Glaciol.*, **5**, 165-169, 1984
55. Turcotte, D.L., Schubert, G.: *Geodynamics*. (3rd ed.). Cambridge: Cambridge University Press, 2002
56. Van der Veen C.J.: Fracture mechanics approach to penetration of bottom crevasses on glaciers. *Cold Reg. Sci. Technol.*, **27**(3), 213-223, 1998
57. Vaughan, D.G.: Tidal flexure at ice shelf margins. *J. Geophys. Res.*, **100**(B4), 6213–6224, doi:10.1029/94JB02467, 1995
58. Wadhams, P.: The seasonal ice zone. In Untersteiner, N. (Ed.), *Geophysics of sea ice* (pp. 825–991), London: Plenum Press, 1986
59. Walker, R.T., Parizek, B.R., Alley, R.B., Anandakrishnan, S., Riverman, K.L., Christianson, K.: Ice-shelf tidal flexure and subglacial pressure variations. *Earth and Planetary Science Letters*, **361**, 422–428, doi: 10.1016/j.epsl.2012.11.008, 2013