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ABOUT THE PROPERTIES OF NETWORKS

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ABSTRACT

In this paper, we investigate the following (1) the product of cs-networks, the image of cs-network by sequence-covering map is cs-network, the product of cs-networks is cs*-networks, the product of k-networks is a k-network, the image of cs*-network by 1-sequence-covering map is cs*-network, the image of k-network by compact-covering map is a k-network.*

Introduction

To determine preserving topological properties of topological spaces by product and continuous map is one of the central question of general topology. The networks (cs, cs*, k) are characterized by important properties of topological spaces. Soma properties of networks (cs, cs*, k) and of covering maps (sequence, 1-sequence, compact) are discussed in [1, 3-12].

Main results

Let X be a T_1 topological space and $P = \{P_\alpha : P \subset X\}$ be a family with $x \in \bigcap P_\alpha$.

Definition-1. A sequence $\{x_n\}$ in X is called eventually in P if $\{x_n\}$ converges to x, and there exists $m \epsilon N$ such that $\{x\} \cup \{x_n : n \geq m\} \subset P$.

Definition-2. The family P is called a network at point $x \in X$ if for any sequence $\{x_n\}$ converging to x and a neighborhood U of x, there exists $P \in P$such that $P \subset U$ and $\{x_n\}$ is eventually in P.

Definition-3. The family P is called a network at point $x \in X$ if for each neighborhood of x there exists $P \in P$ such that $P \in U$.

Definition-4. The family P is called a cs*-network at a point $x \in X$ if whenever $\{x_n\}$ is a sequence converging to a point $x \in U$ with U open is X, then $\{x_{n_i} : i \in N\} \subset P \subset U$ for some subsequence $\{x_{n_i}\}$ of $\{x_n\}$ and some $P \in P$.

Proposition-1. If the families P and T are cs-networks respectively at points $x \in X$ and $y \in Y$, then the family $P \times T$ is cs-network too at point $(x, y) \in X \times Y$.

Prof. Let G be a neighborhood of point (x, y) and $\{x_n\}$, $\{y_n\}$ are some sequences converging to points x and y respectively. It is easy to see that there exist neighborhoods U, V of points x and y respectively, such that $U \times V \subset G$. Moreover, there exist $P \in P, T \in T$ and $n_0 \in N$, $m_0 \in N$ that $\{x_n\} \subset P \subset U$ and $\{y_k\} \subset T \subset V$ for each $n > n_0$, $k > m_0$. We take $m = \max(n_0, m_0)$, then $\{(x_n, y_n)\} \subset P \times T \subset G$ for each $n > m$. Hence, $P \times T$ is cs-network at point (x, y).

Corollary-1. The families $P_i, i = \overline{1, n}$ are cs-networks at points $x_i \in X_i$ respectively, then their product $\prod_{i=1}^{n} P_i$ is a cs-network too at point $(x_1, x_2, \dots, x_n) \in \prod_{i=1}^{n} X_i$.

Example. Let $X = [= 3, 3]$ be space. It is easy to see the family $P = \{\cup \left(1 - \frac{1}{n}, 1 + \frac{1}{n}\right)\}$ is a cs-network at point $x = 1$ and $T = \{\cup \left(2 - \frac{1}{n}, 2 + \frac{1}{n}\right)\}$ is a cs-network at point $y = 2$, where $n \in N$. For each neighborhood G of point $A(x, y)$ we take $r = min_{B \in \partial G}\{d(A, B)\}$, where d is metric in X. Next we take $U = \left(1 - \frac{r}{3}, 1 + \frac{r}{3}\right)$, $\left(V = (2 - \frac{r}{3}, 2 + \frac{r}{3}\right)$, then $U \times V \subset G$. We can find $n_0 \in N$ such that for $P = \left(1 - \frac{1}{n_0}, 1 + \frac{1}{n_0}\right)$, $T = \left(2 - \frac{1}{n_0}, 2 + \frac{1}{n_0}\right)$ this attitude $P \times T \subset U \times V \subset G$ is understandable. Therefore, $P \times T$ is a cs-network too.

Proposition-2. If the families P and T are cs*-networks respectively at points $x \in X$, $y \in Y$ then a family $P \times T$ is cs*-network too at point $(x, y) \in X \times Y$.

Proof. In this case again let G be a neighborhood of point (x, y) and $\{x_n\}$ and $\{y_n\}$ are some sequences converging to points x and y respectively and is known there exists neighborhoods U, V of points x and y respectively, such that $U \times V \subset G$. Moreover, by definition of cs*-network there exist $P \in P$, $T \in T$ and subsequences $\{x_{n_i}: i \in N\}$ and $\{y_{n_j}: j \in N\}$ of sequences $\{x_n\}$ and $\{y_n\}$ respectively, such that $\{x_{n_i}: i \in N\} \subset P \subset U$ and $\{y_{n_j}: j \in N\} \subset T \subset V$. Afterward we re-numbered subsequences and we have $\{(x_{n_k}, y_{n_k}): k \in N\} \subset P \times T \subset G$.

Hence, $P \times T$ is a cs*-network at the point (x, y) and we have proved the proposition-2.

Corollary-2. The families $P_i, i = \overline{1, n}$ are cs*-networks respectively at points $x_i \in X_i$, then their product $\prod_{i=1}^{n} P_i$ is a cs*-network at the point $(x_1, x_2, \dots, x_n) \in \prod_{i=1}^{n} X_i$.

Definition-5. [8]. Let $f: X \to Y$ be a map continuous and onto

f is a sequence-covering map if each convergent sequence (includes its limit point) of Y is the image of some convergent sequence of X.

f is a 1-sequence-covering map if for each $y \in Y$, there is $x \in f^{-1}(y)$ such that whenever $\{y_n\}$ is a sequence converging to y in Y there is sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.

Remark. 1-sequence-covering map ⇒ sequence-covering map.

Proposition-3. If $f: X \to Y$ is sequence-covering map and P is a cs-network at point $x_0 \in X$, then $f(P) = \{f(P): P \in P\}$ is a cs-network at the point $y_0 = f(x_0)$.

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Proof. By definition of continuous map for each neighborhood V of point y_0 there exists a neighborhood U of points x_0 such that $f(U) \subset V$. Since the family P is cs-network at the point x_0, there exists $P \in P$ such that $P \subset U$. Therefore, there exists $T = f(P) \in f(P)$ such that $T \subset V$. Now we will show that for each sequence $\{y_n\}$ converging to y_0 there is $m \in N$ such that $\{y_n\} \subset T$ for every $n > m$. We have that f is sequence-covering map, so the sequence $\{y_n\}$ is the image of some sequence $\{x_n\}$ of X converging to x_0. Then there exists $m \in N$ such that $\{x_n\} \subset P$ for every $n > m$, so $\{f(x_n)\} = \{y_n\} \subset f(P) = T$ for every $n > m$. So $f(P)$ is cs-network at point y_0.

Proposition-4. If $f: X \to Y$ 1-sequence covering map and P is a cs*-network at point $x_0 \in X$, then $f(P) = \{f(P): P \in P\}$ is cs*-network at point $y_0 = f(x_0)$.

Proof. Us sufficient show that for every sequence $\{y_n\}$ converging to point $y_0 \in V$ with V open in Y there exists subsequence $\{y_{n_i}: i \in N\}$ and $T \in f(P)$ such that $\{y_{n_i}: i \in N\} \subset T \subset V$. We have that f is 1-sequence covering map. Therefore, there exist $z_0 \in f^{-1}(y_0)$ and $x_n \in f^{-1}(y_n)$ such that $\{x_n\}$ is a converging sequence to z_0. In addition, P is a cs*-network at a point x_0, so there exists subsequence $\{x_{n_i}: i \in N\}$ of $\{x_n\}$ and $P \in P$ such that $\{x_{n_i}\} \subset P$, therefore, $\left\{f\left(x_{n_i}\right) = y_{n_i}\right\} \subset \{y_n\} \subset f(P) = T$. Hence, $f(P) = \{f(P): P \in P\}$ is cs*-network at the point y_0.

Definition-6. P is called k-network if whenever $K \subset U$ with K compact and U open in X, then $K \subset \bigcup P' \subset U$ for some finite $P' \subset P$.

Let $f: X \to Y$ be a map continuous and onto.

Definition-7. The map f is called *compact-covering map* if each compact subset of Y is the image of some compact subset of X.

Definition-8. If the families P and T are k-networks respectively in X and Y, then the family $P \times T$ is k-network in $X \times Y$.

Proof. Let K be a compact subset of $X \times Y$ and $K \subset U$ with U open in $X \times Y$. We denote by K_1 and K_2 the projects of K to X and Y respectively. It is easy to see K_1 and K_2 are compact subsets of X and Y respectively. Let be $K_1 \subset U_1$ and $K_2 \subset U_2$, for some open subsets U_1, U_2. We have that P and T are k-networks. So there exist finite subfamilies $P' = \{P_i: P_i \in P, i = \overline{1,n}\}$ and $T' = \{T_j: T_j \in T, j = \overline{1,m}\}$ of P and T respectively such that $K_1 \subset \{\bigcup_{i=1}^{n} P_i\} \subset U_1$ and $K_2 \subset \{\bigcup_{j=1}^{m} T_j\} \subset U_2$. Then it is easy to see $K \subset (K_1 \times K_2) \cap U \subset (\{\bigcup_{i=1}^{n} P_i\} \times \{\bigcup_{j=1}^{m} T_j\}) \cap U \subset (U_1 \times U_2) \cap U \subset U$.

As you know, $\{\bigcup_{i=1}^{n} P_i\} \times \{\bigcup_{j=1}^{m} T_j\} = \bigcup_{i=1}^{n} \bigcup_{j=1}^{m} P_i \times T_j$, where $P_i \times T_j \in P \times T$. Hence, $P \times T$ is k-network in $X \times Y$ too.

Corollary-3. The families $P_i, i = \overline{1,n}$ are k-networks respectively in X_i then their product $\prod_{i=1}^{n} P_i$ is k-network in $\prod_{i=1}^{n} X_i$.

Proposition-9. If $f: X \to Y$ is compact-covering map and P is a k-network in X, then $f(P) = \{f(P): P \in P\}$ is k-network in Y.

Proof. Let be F is compact and V is open with $F \subset V$. By definition of compact-covering map there exists compact subset K of X such that $f(K) = F$. We have that f is continuous map so $f^{-1}(V)$ is open in X and $K \subset f^{-1}(V)$. Otherwise, P is a k-network so there exists finite $P' \subset P$ such that $K \subset \bigcup P' \subset f^{-1}(V)$. Thus implies $F \subset f(\bigcup P') =$

$\cup f(P') \subset V$. Therefore, $f(P)$ is k-network in Y.

REFERENCES:

1. Arhangel'skii A.V., Mappings and spaces / A.V.Arhangel'skii // Russian Math. Surveys 21, 1966, no. 4-P. 115-162.
2. Engelking. R. General topology/ R.Engelking – Moscow:MIR, 1986.
3. Bozarov D. U. Функтор единичного шара пространства знакопеременных мер с конечным носителем и бесконечной степенью Бозаров Дилмурод Уралович Каршинский инженерно-экономический институт e-mail: d. bozorov@ inbox. ru //MINISTRY OF HIGHER AND SECONDARY SPECIAL EDUCATION OF THE REPUBLIC OF UZBEKISTAN. – C. 107.
4. Franklin. S.P. Spaces in which sequences suffice / S.P.Franklin // Fund. Math. 57, 1965 – P. 107-115
5. Gu, J. On 1-sequential quotient mappings / J.Gu // J.Math. Study, 44, 2003.-P. 305-308.
6. Lin, S. Sequence-covering maps of metric spaces / S.Lin. P.Yan // Topology Appl., 109. 2001.-P.301-314.
7. Nguyen Van Dung. On 1-sequence-covering π – S-Images of Locally Separable Metric Spaces / Nguyen Van Dung / Evropean Journal Of Pure And Applied Mathematics, Vol. 2, No. 2, 2009.- P.182-194.
8. Rongxing Shen. On generalized metrizable properties in quasitopological groups / Rongxing Shen // Topology and in Applications, 173, 2014.- P.219-226.
9. Shou Lin. Sequence-covering maps of metric spaces / Shou Lin, Pengfei Yan // Topology and its Applications, 109,2001- P.301-314.
10. Zhaowen Li. On sequence-covering msss-maps / Zhaowen Li, Qingguo Li, Xiangnan Zhou // Matematiki vesnik, 59, 2007- P.15-21