Pricing Asian Option

Product

Asian Option

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Asian Option Introduction

- An Asian option or *average* option is a special type of option contract where the payoff depends on the average price of the underlying asset over a certain period of time
- The payoff is different from the case of a European option or American option, where the payoff of the option contract depends on the price of the underlying stcok at exercise date.
- Asian options allow the buyer to purchase (or sell) the underlying asset at the average price instead of the spot price.
- Asian options are commonly seen options over the OTC markets.
- Average price options are less expensive than regular options and are arguably more appropriate than regular options for meeting some of the needs of corporate treasurers.

Asian Option

- One advantage of Asian options is that they reduce the risk of market manipulation of the underlying instrument at maturity.
- Because of the averaging feature, Asian options reduce the volatility inherent in the option; therefore, Asian options are typically cheaper than European or American options.
- Asian options have relatively low volatility due to the averaging mechanism. They are used by traders who are exposed to the underlying asset over a period of time
- The Asian option can be used for hedging and trading Equity Linked Notes issuance.
- The arithmetic average price options are generally used to smooth out the impact from high volatility periods or prevent price manipulation near the maturity date.

Pricing

- The payoff of an average price call is max(0, S_{avg} - K) and that of an average price put is max(0, K- S_{avg}), where S_{avg} is the average value of the underlying asset calculated over a predetermined averaging period.
- If the underlying asset price S is assumed to be lognormally distributed and S_{ave} is a geometric average of the *S's*, analytic formulas are available for valuing European average price options. This is because the geometric average of a set of lognormally distributed variables is also lognormal.
- When, as is nearly always the case, Asian options are defined in terms of arithmetic averages, exact analytic pricing formulas are not available. This is because the distribution of the arithmetic average of a set of lognormal distributions does not have analytically tractable properties.
- However, the distribution of arithmetic average can be approximated to be lognormal by moment matching technical.

Asian Option

Pricing (Cont.)

- One calculates the first two moments of the probability distribution of the arithmetic average in a risk-neutral world exactly and then fit a lognormal distribution to the moments.
- Consider a newly issued Asian option that provides a payoff at time *T* based on the arithmetic average between time zero and time *T*. The first moment, M_1 and the second moment, *M2*, of the average in a risk-neutral world can be shown to be

$$M_1 = \frac{e^{(r-q)T} - 1}{(r-q)T} S_0$$

$$M_2 = \frac{2e^{[2(r-q)+e^2]T} S_0^2}{(r-q+\sigma^2)(2r-2q+\sigma^2)T^2} + \frac{2S_0^2}{(r-q)T^2}\left(\frac{1}{2(r-q)+\sigma^2} - \frac{e^{(r-q)T}}{r-q+\sigma^2}\right)$$

where r is the interest rate and q is the devidend yield and $q \neq r$.

Pricing (Cont.)

- By assuming that the average asset price is lognormal, an analyst can use Black's model.
- The present value of an Asian call option is given by

$$PV_C = (M_1 N(d_1) - KN(d_2))D$$

$$d_{1,2} = \frac{ln(M_1/K) \pm \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$\sigma^2 = \frac{1}{T}\ln(\frac{M_2}{M_1^2})$$

where

D	the discount factor
N	the cumulative standard normal distribution function
T	the maturity date

Pricing (Cont.)

- The present value of an Asian put option is given by

$$PV_P = \big(KN(-d_2) - F_0N(-d_1)\big)D$$

- We can modify the analysis to accommodate the situation where the option is not newly issued and some prices used to determine the average have already been observed.
- Suppose that the averaging period is composed of a period of length T_1 over which prices have already been observed and a future period of length T_2 (the remaining life of the option).

Pricing (Cont.)

- The payoff from an average price call is

$$max\left(\frac{\bar{S}T_1 + S_{avg}T_2}{T_1 + T_2} - K, 0\right)$$

where

S_{avg}	the average asset price of period T_2 (future period)
$\bar{S}$	the spent average asset price of p)eriod T_1 (realized period

- This is the same as

$$\frac{T_2}{T_1 + T_2} max\left(S_{avg} - K^*, 0\right)$$

where

$$K^* = \frac{T_2}{T_1 + T_2}K - \frac{T_1}{T_2}\bar{S}$$

Pricing (Cont.)

- When *K** > 0, the option can be valued in the same way as a newly issued Asian option provided that we change the strike price from *K* to *K** and multiply the result by $t_2/(t_1+t_2)$

$$PV_C = \frac{T_2}{T_1+T_2}(M_1N(d_1) - K^*N(d_2))D$$

$$PV_P = \frac{T_2}{T_1+T_2}\left(K^*N(-d_2) - M_1N(-d_1)\right)D$$

- When *K** < 0 the option is certain to be exercised and can be valued as a forward contract. The value is

$$PV_C = \frac{T_2}{T_1+T_2}(M_1 - K^*)D$$

$$PV_P = \frac{T_2}{T_1+T_2}(K^* - M_1)D$$

Practical Guide

- First calculate the spent average based on realized spot price.
- Then compute the adjusted strike using the spent average
- After that obtain the first and second moments.
- Use the moments to get the adjusted volatility.
- Finally calculate the present value via BlackScholes formula.
- FinPricing is using the Turnbull-Wakeman model. Another well-known model is the Levy Model

American Equity Option

Example

Face Value	3361.12
Currency	CAD
Start Date	1/10/2017
Maturity Date	7/10/2017
Call or Put	Call
Buy or Sell	Sell
Underlying Assets	.GSPTXBA
Position	-2790.764362
Spent Average	4104.9327

Reference:

https://finpricing.com/lib/EqRangeAccrual.html