Identifying efficient ensemble perturbations for initializing subseasonal-to-seasonal prediction

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Initialization of S2S predictions

- S2S predictions is beyond atmosphere predictability limit
  - Coupled Earth system models must be used
- Usually done with ensemble:
  - How to initialize them consistently to obtain reliable results?
- Already tested, use of the:
  - Bred vectors (Peña & Kalnay, 2004; Yang et al., 2008; O'Kane et al., 2019)
  - Backward Lyapunov Vectors (Vannitsem & Duan, 2020)

In the present work, we study the projections of the initial conditions on mainly:

- Covariant Lyapunov vectors and their adjoint
- Dynamical Mode Decomposition: adjoint modes
- Perron-Frobenius Mode Decomposition: adjoint modes

for a low-order coupled ocean-atmosphere model (MAOOAM)

Dynamical Mode Decomposition (DMD)

- Considering 2 collections of states of the dynamical system X=[$\mathbf{x}_0$ … $\mathbf{x}_{K-1}$] and Y=[$\mathbf{x}_1$ … $\mathbf{x}_K$], then one define

$$\mathrm{M}^{\mathrm{DMD}} = \mathrm{Y}\,\mathrm{X}^{+}$$

where X^{+} is the pseudo-inverse

Same decomposition exists for the Perron-Frobenius (PF) operator.

- as the DMD decomposition of the observable $\mathbf{g}(\mathbf{x})=\mathbf{x}$. Related to Linear Inverse Modeling (Penland, 1989)
- The left eigenvectors $\mathbf{w}_i$ of $\mathrm{M}^{\mathrm{DMD}}$ provides approximation of the system's Koopman operator eigenfunctions and are called *adjoint* DMD modes. (Tu et al., 2014)
- The Koopman K operator is an ∞-dimensional operator propagating the observable of the system. For an observable *g*:

$$\mathcal{K}^{\tau}\, g(\boldsymbol{x}) = g\left(\boldsymbol{\Phi}^{\tau}(\boldsymbol{x})\right)$$

where ϕ is the flow of the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$

$$\Rightarrow \sum_{i=1}^{P} \boldsymbol{c}_i^{\mathrm{DMD}}\, \lambda_i^{\mathrm{DMD}}\, \boldsymbol{w}_i^{*}\ \boldsymbol{x}$$

- The action of the Koopman operator can then be approximated with the DMD where the $\mathbf{c}_i^{\mathrm{DMD}}$ are modes depending on the observable *g and the* $\mathbf{w}_i$ define approximate invariant manifolds for the Koopman op.

Lyapunov vectors

Considering a dynamical system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$

and the evolution of perturbations in its tangent space:

$$\dot{\delta\boldsymbol{x}}(\tau) = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}(\tau)} \delta\boldsymbol{x}(\tau) \quad \text{given by} \quad \delta\boldsymbol{x}(t) = \mathrm{M}(t,t_0)\,\delta\boldsymbol{x}_0 \quad , \quad \delta\boldsymbol{x}_0 = \delta\boldsymbol{x}(t_0)$$

where M is the propagator (of the perturbations).

- The Backward Lyapunov Vectors (BLVs) are the eigenvectors of

$$\left(\mathrm{M}(t,t_0)\mathrm{M}(t,t_0)^{\mathsf{T}}\right)^{1/(2(t-t_0))} \quad \text{in the limit } t_0 \to \infty$$

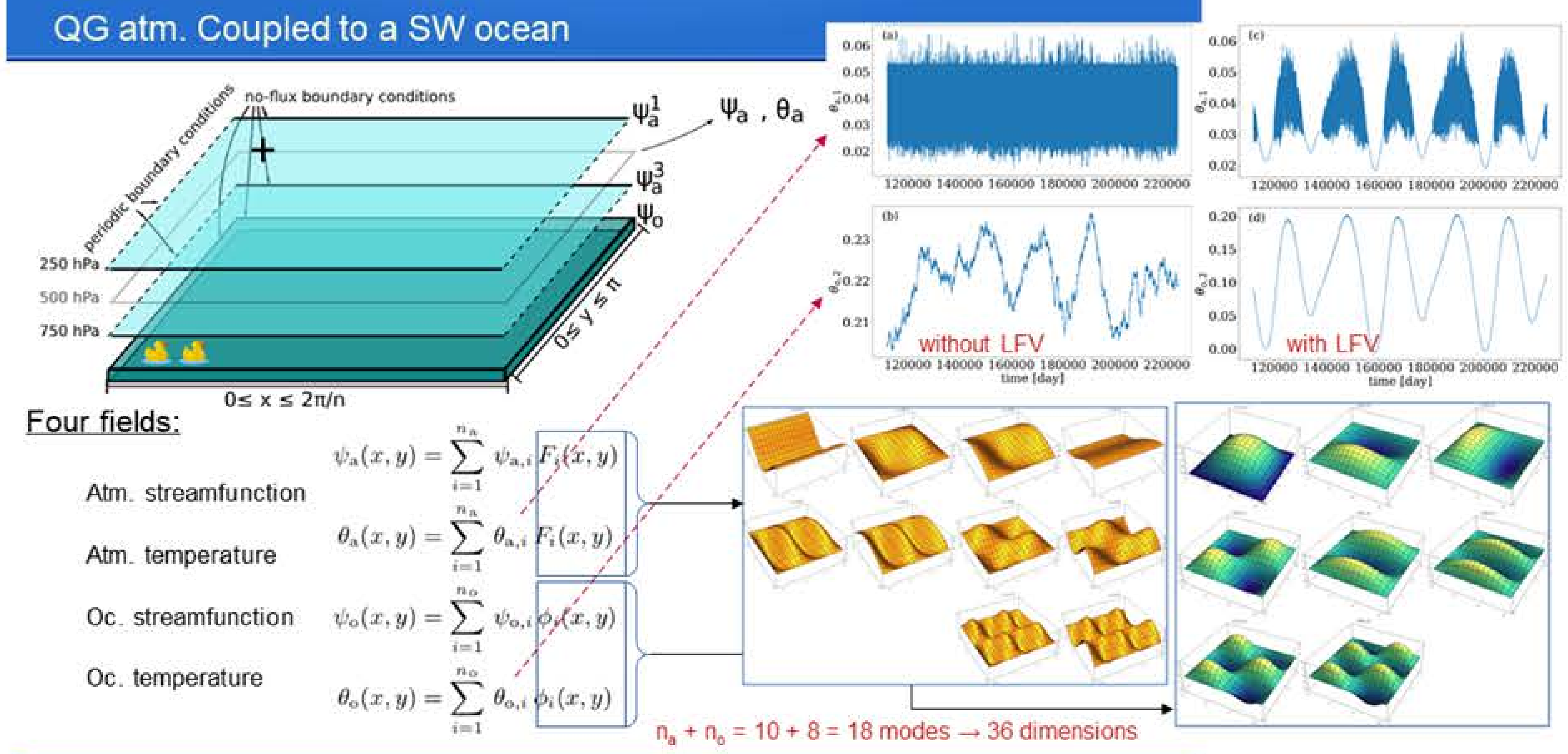
can be interpreted as an orthonormal basis defining volumes covariant with the dynamics.

- The Covariant Lyapunov Vectors (CLVs) are such that: $\mathrm{M}(t,t_0)\,\boldsymbol{\varphi}_i(t_0) = \Lambda_i(t,t_0)\,\boldsymbol{\varphi}_i(t)$
- The adjoint CLVs $\tilde{\boldsymbol{\varphi}}_i$ are adjoint (biorthonormal) to the CLVs: $\tilde{\boldsymbol{\varphi}}_i^{\mathsf{T}}\,\boldsymbol{\varphi}_j = \delta_{i,j}$

Both can be interpreted as directions covariant with the dynamics.

MAOOAM: an ocean-atmosphere coupled model

QG atm. Coupled to a SW ocean



Experimental design

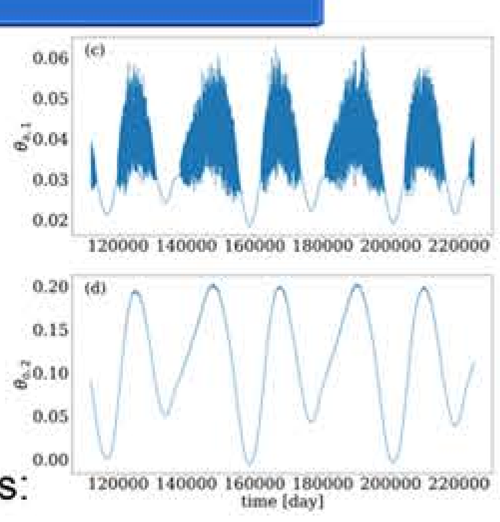
N ensemble forecasts along a reference trajectory:

Control run IC: $\boldsymbol{x}_n^{\mathrm{ctrl}}(0) = \boldsymbol{x}_n(0) + \delta\boldsymbol{x}_0^{\mathrm{ctrl}}$ n = 1,…,N

Perfect ensemble IC: $\boldsymbol{y}_{m,n}(0) = \boldsymbol{x}_n^{\mathrm{ctrl}}(0) + \delta\boldsymbol{x}_0^{m}$ $m = 1,\ldots,M-1$

$\delta\mathbf{x}_0^{M} = 0$

Same distribution U[-ε/2,ε/2]



Experiments

Projection onto subspaces spanned by selected vectors:

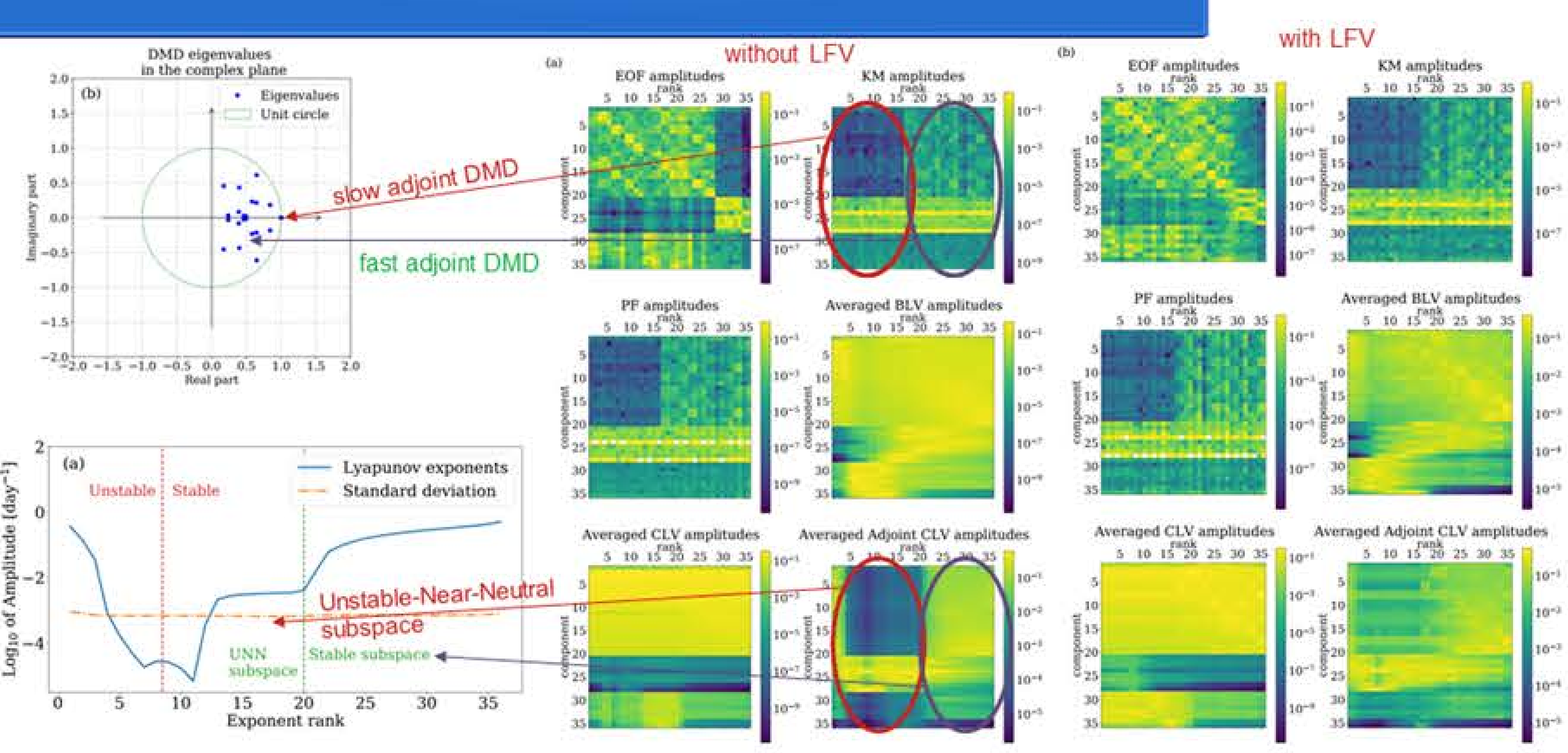
Projector: $\Pi = \mathrm{B}(\mathrm{B}^{*}\,\mathrm{B})^{-1}\,\mathrm{B}^{*}$ where B is the column matrix of the selected vectors

Projected perturbations: $\delta\boldsymbol{x}_0'^{m} = \Pi\,\delta\boldsymbol{x}_0^{m}$

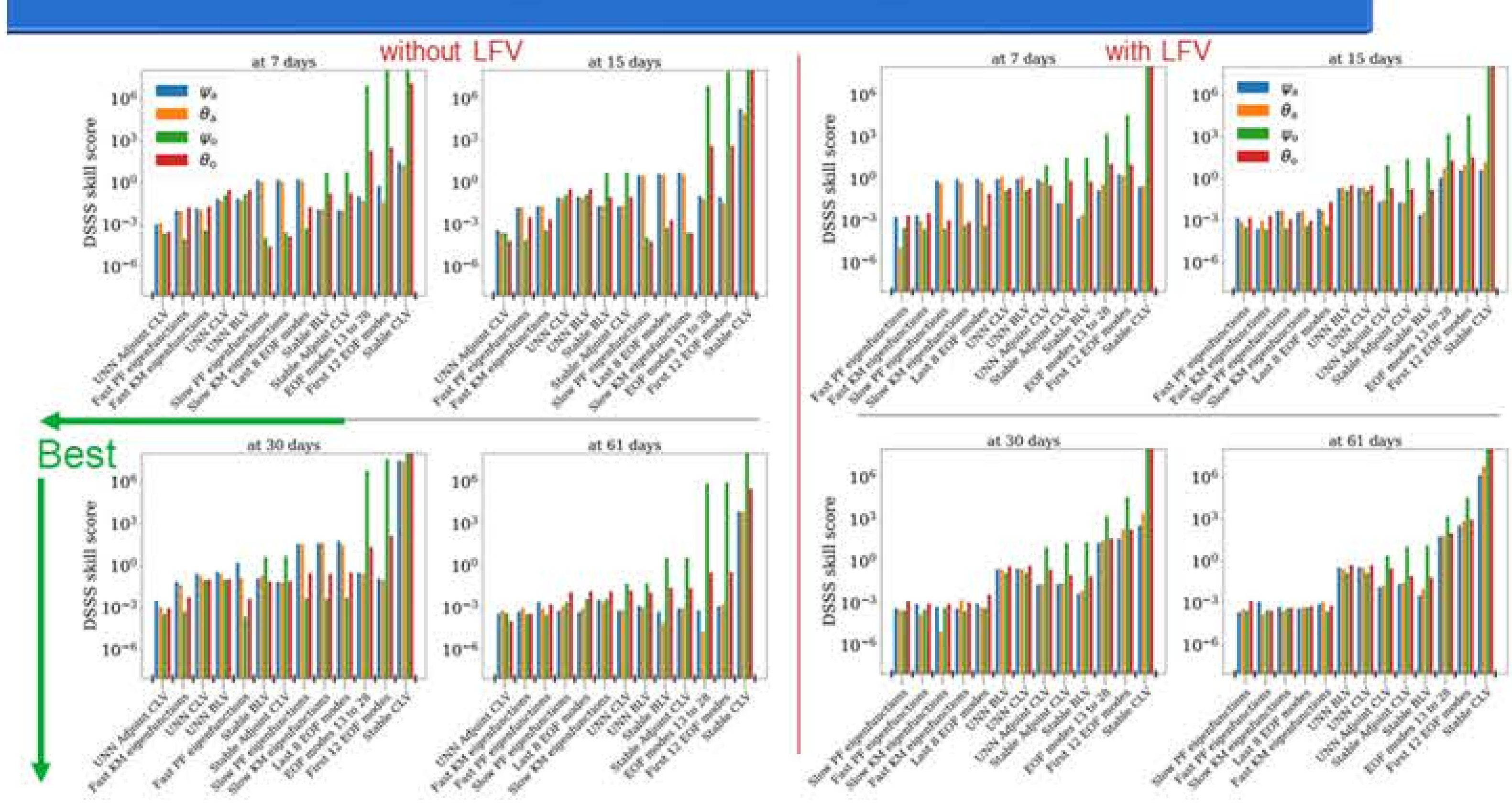
New ensembles: $\boldsymbol{y}_{m,n}'(0) = \boldsymbol{x}_n^{\mathrm{ctrl}}(0) + \delta\boldsymbol{x}_0'^{m}$, $m = 1,\ldots,M$

Goal:
→ Obtain forecasts as reliable as the ones provided by the perfect ensembles.

Selected bases: analysis of the reference trajectory



Results: DSSS skill scores at different lead times



DSSS:
Skill score based on the Dawid-Sebastiani score. Equal to zero if same reliability as the perfect ensemble.

Dawid & Sebastiani (1999)

Leutbecher (2019)

Conclusions

Key results:

- Approximated KM and PF eigenfunctions obtained using DMD provide reliable ensemble forecasts, and are "easy" to compute.
- Adjoint CLVs also provide reliable forecasts (sometimes the most reliable ones), but are notably hard to compute.
- A consistent link exists between the two frameworks, which explains the results.
- Results seem to not depend on the regime (with or without LFV)
- Results of Vannitsem & Duan (2020) with the BLVs can also be explained with the DMDs

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Forthcoming developments:

- Many, but most notably, replication of the study with a higher-dimensional system, with a non-trivial dimensionality reduction to make DMD tractable.
- Ultimately, development of the approach in a realistic S2S framework.

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