Existence of Prime Numbers

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Abstract

Prime numbers are quite fascinating, considering their very existence seems to have no identifiable patterns that can give insight as to how and why prime numbers exist in the first place. In particular, for a world that has enjoyed the benefits of mathematics for thousands of years, it might prove difficult to look at the truths that have been constructed over the years from a different perspective.

This paper details some observable properties between prime and even numbers based on Goldbach's conjecture. Sections 2 and 3 details the simulations conducted for this experiment that attempts to explain how and the reasons why prime numbers exist. Section 4 details the observable properties of prime and even numbers based on the results from these simulations.

Section 5 provides proof for the inextricable link between prime and even numbers. Section 6 details how prime numbers are created, whereas section 7 provides observable properties that give insights as to the reason why prime numbers exist. Section 8 offers some observable properties of prime numbers that proves Goldbach's conjecture was correct all along. Section 9 provides proofs that the number 1 is a prime number, whereas section 10 provides proofs that the number 2 is not a prime number at all. Section 11 provides proofs that prime numbers are not directly linked to composite numbers, while section 12 provides proofs that nonprime numbers are created by means of addition.

1. Introduction

What is currently known about prime numbers are as follows:

a. 1 is not considered a prime number
b. 2 is the only even prime number that can exist
c. 4 is the only even number that is the sum of two even prime numbers
d. All prime numbers are positive whole numbers
e. All prime numbers are only evenly divisible by itself and 1
f. All prime numbers cannot be divided by any smaller number without leaving any remainders

Goldbach's conjecture states:

a. Even numbers greater than 2 equates to the sum of two prime numbers.

2. Simulating the world of prime numbers

Proving even numbers greater than 2 is the sum of two prime numbers has eluded humanity for hundreds of years. Prime numbers are quite unique in the sense that no number other than itself and 1 can evenly divide it.

Attempting to verify Goldbach's conjecture through an equation alone might prove difficult since there are no discernable patterns between each prime number. Therefore, a different approach and a simple shift in perspective might reveal not only how prime numbers are created, but why prime numbers exist in the first place.

Usually, a simple what if question can provide a huge paradigm shift:

What if Goldbach's conjecture is not really a problem, but is a piece of the prime number puzzle instead? Can it give new insights about prime numbers?

The conditions that will be implemented in the computer simulations are based on Goldbach's conjecture. The simulations will attempt to discover how prime numbers are created and for what reason they exist. The results from these computer simulations might provide some insights and a clearer picture as to the true nature of prime numbers.

Simulating new worlds are quite easy with computer algorithms, where rules can be changed and specific sets of instructions are carried out almost instantaneously. Using the python language to construct a world of prime numbers with the following conditions. See python code in Appendix A.

Simulation conditions**:** Create a world where

a. Only odd prime numbers are allowed to add with other odd prime numbers or to itself
b. All odd numbers, both prime and composite numbers, can multiply with each other
    - The product is excluded if it is higher than the last found even number to avoid unnecessary confusion
    - The results are separated for proof
c. 1, 2 and 4 are not included because:
    - The properties of odd and even numbers states that:
        - odd + even = odd, so 2 will not be included
    - 4 is not included because 2 is the only prime number that can make 4 exist

- 1 is not considered a prime number

d. The starting_prime_numbers list in the python code will only contain prime numbers
e. All found even numbers will be subtracted by 1 to identify the odd number that came before it
    - The difference is only considered a prime number if the number is not in the list of odd composite numbers
    - Subtraction was used because the number 4 was not included in the experiment, thus this is the only method to get the odd number 5

Objectives of the computer simulations:

a. Prove that even numbers higher than 4 requires odd prime numbers to exist
b. Prove that even numbers are not required for all even numbers greater than 4 to exist
c. Prove that odd composite numbers are not required for all even numbers to exist
d. Prove that all even numbers greater than 2 is the sum of at least one pair of prime numbers
e. Determine how prime numbers are created
f. Reasons why prime numbers exist

Possibly also prove that:

a. 1 is a prime number
b. 2 is not a prime number, but was mistakenly included because it looks and behaves like a prime number
c. Composite numbers were mistakenly linked to prime numbers
d. Nonprime numbers are created by means of addition
e. All prime numbers are odd
f. All even numbers are the sum of at least one pair of prime numbers
g. Prime numbers only exist to fill the void between two even numbers that no other odd nonprime number can fill
h. Prime numbers can create all other numbers greater than 1
i. Can be considered as another proof that an infinite number of prime numbers exist

3. Entering the world of prime numbers

Simulations 1 to 4 documents the creation of prime numbers and explores the relation between prime and even numbers. Simulation 5 to 9 proves that some even numbers can be the sum of more than one pair of prime numbers. Simulations 10 to 13 provides proof that some even numbers cannot exist without a specific prime number. Simulation 14 proves that a sequence of all possible even numbers can still exist even if four consecutive odd prime numbers were excluded from the simulation.

First, to establish the logic of this simulation, let's create a story:

In the beginning, the only number that exist is the number 2. Later on, the number 4 was found when 2 was added to another 2, but there was something missing between 2 and 4. Sometime later, someone suggested subtracting 1 from 4, thus 3 was revealed.

For the upcoming simulations, numbers 1, 2, and 4 are excluded from the experiment as stated in the simulation conditions. See python code in Appendix A.

Simulation 1: Starting with 3

```
Starting number is: [3]
Find numbers until: 2000
Found numbers are: [3, 6]

Found even numbers: [6]

Missing even numbers: []
Found odd composite numbers: []
```

As expected, 6 is the only even number found. To get the next prime number, all even numbers found will be subtracted by 1. Thus, $6 - 1 = 5$. Since 5 is not in the list of odd composite numbers, the newly found number is a prime number.

Simulation 2: Start with 3 and 5

```
Starting number is: [3, 5]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 8, 10]

Found even numbers: [6, 8, 10]

Missing even numbers: []
Found odd composite numbers: [9]
```

As shown by the results, two new even numbers appeared by simply including the number 5 in the list of prime numbers. To get the next set of odd numbers, all even numbers are subtracted by 1.

6 – 1 = 5	8 – 1 = 7	10 – 1 = 9

Since 5 was already used as a prime number and 9 is an odd composite number, only 7 can be considered as a prime number. Proceeding to the next simulation using the three prime numbers found so far.

Simulation 3: Start with 3, 5, and 7

```
Starting number is: [3, 5, 7]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 12, 14]

Found even numbers: [6, 8, 10, 12, 14]

Missing even numbers: []
Found odd composite numbers: [9]
```

As can be seen from the results, all even numbers from 6 to 14 is complete just by using prime numbers 3, 5, and 7, but the rest of the even numbers from 16 to 2,000 cannot exist without additional prime numbers. To get the next set of odd numbers, all even numbers will be subtracted by 1 except 6, 8, and 10, since all three numbers has already been computed in simulation 2.

12 – 1 = 11	14 – 1 = 13

Since 11 and 13 are not in the list of odd composite numbers, both will be included in the list of prime numbers. Proceeding to the next simulation using the five prime numbers found so far.

Simulation 4: Start with 3, 5, 7, 11, and 13

Starting number is: [3, 5, 7, 11, 13] Find numbers until: 2000 Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 20, 22, 24, 26] Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26] Missing even numbers: [] Found odd composite numbers: [9, 15, 21, 25]

As can be seen in the output, all even numbers from 6 to 26 are complete just by using prime numbers 3, 5, 7, 11, and 13. Since there are only five prime numbers, there will never be any even numbers beyond 26 that can exist in this simulated world.

Subtracting 1 from the newly found even numbers as shown below:

16 – 1 = 15 18 – 1 = 17	20 – 1 = 19 22 – 1 = 21	24 – 1 = 23 26 – 1 = 25

Since 15, 21, and 25 are in the list of odd composite numbers, only 17, 19, and 23 will be included in the list of prime numbers. Proceeding to the next simulation using the eight prime numbers found so far.

Simulation 5: Start with 3, 5, 7, 11, 13, 17, 19, and 23

Starting number is: [3, 5, 7, 11, 13, 17, 19, 23] Find numbers until: 2000 Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 46] Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 46] Missing even numbers: [44] Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45]

Now things got more interesting, 44 is missing from the list of even numbers found between 6 to 46. So why is 44 missing and which prime number is required for 44 to exist? To find other prime numbers, subtract all even numbers by 1 and if the difference is not in the list of odd composite numbers, then include it to the list of prime numbers.

28 – 1 = 27 30 – 1 = 29 32 – 1 = 31	34 – 1 = 33 36 – 1 = 35 38 – 1 = 37	40 – 1 = 39 42 – 1 = 41 46 – 1 = 45

The numbers that were not in the list of odd composite numbers are 29, 31, 37, and 41, so these numbers must be prime numbers. To find which prime number is required for 44 to exist, the simulation will include each newly found prime numbers one at a time.

Simulation 6: Find 44 by including only the prime number 29

Result: **Not Found**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 29

```
Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 29]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29,
30, 32, 34, 36, 38, 40, 42, 46, 48, 52, 58]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 46,
48, 52, 58]

Missing even numbers: [44, 50, 54, 56]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57]
```

As can be seen from the output, 44 is still missing, but three new even numbers were created just by including the prime number 29. Since 44 was not available, the results will be ignored for now.

Simulation 7: Find 44 by including only the prime number 31

Result: **Found**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 31

Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 31]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 30, 31, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 54, 62]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 54, 62]

Missing even numbers: [52, 56, 58, 60]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57]

Finally, the algorithm was able to find 44 by simply adding 31 in the list of prime numbers, but 52 and 58 are now missing even though both numbers were found in simulation 6. It can also be observed that 48 is still there even though 29 was not included in the list of prime numbers.

Simulation 8: Find 44 by including only the prime number 37

Result: **Found**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 37

Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 37]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 30, 32, 34, 36, 37, 38, 40, 42, 44, 46, 48, 50, 54, 56, 60, 74]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 54, 56, 60, 74]

Missing even numbers: [52, 58, 62, 64, 66, 68, 70, 72]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69]

The output of the simulation shows that 44 has also been found, but 56 has suddenly appeared even though it was missing in simulation 6 and 7. This will be explored more in detail later in simulation 10.

Simulation 9: Find 44 by including only the prime number 41

Result: **Found**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 41

```
Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 41]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 30,
32, 34, 36, 38, 40, 41, 42, 44, 46, 48, 52, 54, 58, 60, 64, 82]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44,
46, 48, 52, 54, 58, 60, 64, 82]

Missing even numbers: [50, 56, 62, 66, 68, 70, 72, 74, 76, 78, 80]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69,
75, 77, 81]
```

The output of the simulation shows that 44 was also found, but 56 vanished again even though it showed up in simulation 8. Is there a relation between the even number 56 and the prime number 37? This will be explored more in detail later in simulation 10.

Since 44 appeared for 31, 37, and 41, it can be inferred at this point that more than one odd prime number is directly linked to an even number's existence.

4. Observations so far:

In Simulation 5, where 44 was the missing even number, the algorithm was able to find it when either 31, 37, or 41 was included in the list of prime numbers. Although 29 was not able to create 44, it did not matter because the even number 44 is the sum of more than one pair of prime numbers. Through this observation alone, it is now possible to infer that some even numbers can exist through the sum of more than one pair of prime numbers.

It was also observed that 56 only appeared in simulation 8, where the only new prime number introduced since simulation 5 was 37. Thus, is it possible that if all prime numbers from 3 to 41 were used except for 37 that 56 will simply not exist?

Simulation 10: Will 56 exist without the prime number 37?

Result: **Does not exist**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 29, 31, and 41
Except: 37

```
Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29,
30, 31, 32, 34, 36, 38, 40, 41, 42, 44, 46, 48, 50, 52, 54, 58, 60, 62, 64, 70, 72, 82]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44,
46, 48, 50, 52, 54, 58, 60, 62, 64, 70, 72, 82]

Missing even numbers: [56, 66, 68, 74, 76, 78, 80]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69,
75, 77, 81]
```

As shown above, it is now confirmed that 56 will simply not exist without the prime number 37. This statement is valid if and only if the list of prime numbers is from 3 to 41. But what if the next prime number is introduced?

The next even number is 44, which was missing in simulation 5. Thus, $44 - 1 = 43$. Since 43 is not in the list of odd composite numbers, then it must be a prime number.

Simulation 11: Will 56 exist when the prime number 43 is introduced, but not the prime number 37?

Result: **Does exist**
Previous Prime numbers 3, 5, 7, 9, 11, 13, 17, 19, 23
New Prime number: 29, 31, 41, and 43
Except: 37

```
Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41, 43]
Find numbers until: 2000

Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29,
30, 31, 32, 34, 36, 38, 40, 41, 42, 43, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 70, 72, 74,
82, 84, 86]
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Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 70, 72, 74, 82, 84, 86]

Missing even numbers: [68, 76, 78, 80]
Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69, 75, 77, 81, 85]

As can be seen from the results, 56 does exist as soon as 43 was introduced, even though 37 was still excluded from the list of prime numbers. Therefore, this further solidifies the proof that even numbers can exist through the sum of more than one pair of prime numbers.

5. Proof of the inextricable link between prime and even numbers

Since it has been established that some even numbers require a specific odd prime number to exist, it is time to prove whether this statement is true for some of the odd prime numbers. Simulations 12 and 13 will explore scenarios where a particular odd prime number doesn't exist, in order to determine if there are other observable properties between prime and even numbers.

Simulation 12: Will an even number cease to exist without the prime number 3?

Prime numbers: 5 to 109
Exclude Prime Numbers: 3
Highest even number possible: 218 from $109 + 109$

Starting number is: [5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109]
Find numbers until: 2000

Found numbers are: [5, 7, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29, 30, 31, 32, 34, 36, 37, 38, 40, 41, 42, 43, 44, 46, 47, 48, 50, 52, 53, 54, 56, 58, 59, 60, 61, 62, 64, 66, 67, 68, 70, 71, 72, 73, 74, 76, 78, 79, 80, 82, 83, 84, 86, 88, 89, 90, 92, 94, 96, 97, 98, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Found even numbers: [10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 102, 104, 106, 108, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Missing even numbers: [6, 8]
Found odd composite numbers: [25, 35, 49, 55, 65, 77, 85, 91, 95, 115, 119, 121, 125, 133, 143, 145, 155, 161, 169, 175, 185, 187, 203, 205, 209, 215, 217]

As shown in the results, both even numbers 6 and 8 ceases to exist as soon as the prime number 3 was taken out of the list of prime numbers. Even if new prime numbers are introduced, without 3 as part of the prime numbers, both even numbers 6 and 8 will simply not exist at all.

Simulation 13: Will an even number cease to exist without the prime number 5?

Prime numbers: 3 to 109
Exclude Prime Numbers: 5
Highest even number possible: 218 from $109 + 109$

Starting number is: [3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109]
Find numbers until: 2000
Found numbers are: [3, 6, 7, 10, 11, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29, 30, 31, 32, 34, 36, 37, 38, 40, 41, 42, 43, 44, 46, 47, 48, 50, 52, 53, 54, 56, 58, 59, 60, 61, 62, 64, 66, 67, 68, 70, 71, 72, 73, 74, 76, 78, 79, 80, 82, 83, 84, 86, 88, 89, 90, 92, 94, 96, 97, 98, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Found even numbers: [6, 10, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 102, 104, 106, 108, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Missing even numbers: [8, 12]
Found odd composite numbers: [9, 21, 27, 33, 39, 49, 51, 57, 63, 69, 77, 81, 87, 91, 93, 99, 111, 117, 119, 121, 123, 129, 133, 141, 143, 147, 153, 159, 161, 169, 171, 177, 183, 187, 189, 201, 203, 207, 209, 213, 217]

As can be expected, 6 is now back in the list of found even numbers, whereas 8 and 12 is now missing from the established set of even numbers seen from simulation 11. Therefore, these results now present a conclusive proof that the very existence of some even numbers greater than 2 depends on the existence of a corresponding odd prime number. The results from the simulations also proves that an even number greater than 2 is the sum of at least one pair of prime numbers.

6. Existence of prime numbers

Interestingly, whole numbers follow a strict pattern of sequence that alternates between even and odd numbers. Starting with 0 as an even number, numbers from 0 to 10 will alternate between even and odd numbers. Whenever there are no available odd composite number that can fill the gap between two even numbers, an odd prime number is created out of the necessity to complete the pattern of the sequence. The famous quote attributed to Aristotle about how nature will always fill any absence with something similar perfectly fits how odd prime numbers are created.

As can be observed from simulations 1 to 3, an odd prime number always appears whenever there are no existing odd composite number able to fill the void between two even numbers.

Using simulation 2 to further explain this concept.

Given 3 and 5 as prime numbers:

Even numbers found	6, 8, and 10
Composite numbers	9, 15, and 25

The only odd numbers besides 3 and 5 are 9, 15, and 25, which are composite odd numbers derived from multiplying 3 and 5 by themselves and with each other. The odd number 9 fills the void between even numbers 8 and 10, but no other odd number exists that can fill the void between even numbers 6 and 8. Thus, a new prime number 7 was created to fill the missing odd number between even numbers 6 and 8.

7. Other reasons for the existence of prime numbers

a. Continue to find other even numbers if one prime number simply vanishes

Prime numbers ensure that other even and odd numbers can still be found if one of the prime numbers goes missing. In simulation 4 where 3, 5, 7, 11, and 13 are prime numbers, if the prime number 3 simply disappears, only even numbers 6 and 8 will vanish, but even numbers from 10 onwards will still exist without requiring the prime number 3 or any of its multiples as shown in simulation 12.

b. To find even numbers without relying on other odd composite or even numbers

Results From Simulation 3: Starting number is: [3, 5, 7] Found numbers are: [3, 5, 6, 7, 8, 10, 12, 14] Found even numbers: [6, 8, 10, 12, 14] Missing even numbers: [] Found odd composite numbers: [9]	Results From Simulation 4: Starting number is: [3, 5, 7, 11, 13] Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 20, 22, 24, 26] Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26] Missing even numbers: [] Found odd composite numbers: [9, 15, 21, 25]

Prime numbers exist to be able to find other even numbers without having to rely on other odd composite or even numbers. In simulation 4, where two additional odd prime numbers were included, six new even numbers were found that did not simply exist in simulation 3.

A good example is the even number 26:

If the prime number 3 simply ceases to exist, the odd composite number 21 from (3 x 7) will also vanish. Thus, the odd prime number 5 will no longer have any other odd composite number to pair with to have the sum that equates to the even number 26. Since 1 was also not included in the list of odd numbers, the odd composite number 25 from (5 x 5) will not be able to pair with any other odd numbers to have the sum that equates to the even number 26. Therefore, the only way to find the even number 26 is by adding the prime number 13 to itself.

c. The building block for all odd composite and even numbers

Odd prime numbers are able to create the entire sequence of even numbers by simply adding an odd prime number to itself or to another odd prime number. Odd prime numbers are also able to create the entire sequence of odd composite numbers by multiplying with other odd numbers or by itself. Therefore, it can be inferred that odd prime numbers alone are the building blocks for both odd composite and even numbers greater than 4.

d. Forms a strong foundation to support even numbers

Even numbers greater than 2 is the sum of at least one pair of prime numbers as shown by the results of the simulations. This statement can easily be proven by showing what happens when four consecutive odd prime numbers are taken out of the simulation. As can be seen in simulation 14, all possible even numbers in the sequence still exists even if four consecutive odd prime numbers were not included in the list of prime numbers.

Simulation 14: Exclude four consecutive odd prime numbers

Prime numbers: 3 to 109
Exclude Prime Numbers: 41, 43, 47, and 53
Highest even number possible: 218 from $109 + 109$

Starting number is: [3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109]
Find numbers until: 2000
Found numbers are: [3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26, 28, 29, 30, 31, 32, 34, 36, 37, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 59, 60, 61, 62, 64, 66, 67, 68, 70, 71, 72, 73, 74, 76, 78, 79, 80, 82, 83, 84, 86, 88, 89, 90, 92, 94, 96, 97, 98, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Found even numbers: [6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 102, 104, 106, 108, 110, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218]

Missing even numbers: []

Found odd composite numbers: [9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69, 75, 77, 81, 85, 87, 91, 93, 95, 99, 105, 111, 115, 117, 119, 121, 125, 133, 135, 143, 145, 147, 153, 155, 161, 165, 169, 171, 175, 177, 183, 185, 187, 189, 195, 201, 203, 207, 209, 213, 217]

e. Odd prime numbers are spread in a way that supports more than one even number

As shown by the results from simulation 14, this provides a conclusive proof that even numbers greater than 4 can exist without any of the odd composite or other even numbers. Furthermore, even numbers greater than 2 are the sum of at least one pair of prime numbers since the even numbers from 6 to 218 can still exist without the four excluded consecutive odd prime numbers or any of the odd composite numbers derived from these prime numbers.

8. Reasons why Goldbach's Conjecture was correct all along

Goldbach's conjecture is not really a problem that requires an equation to be proven, but is more of a hint about prime numbers themselves. The conjecture offers a detailed observation about some of the properties of prime numbers, which when explored can give detailed insights as to how and why prime numbers exists in the first place.

Through these simulations, it is now possible to infer the following statements:

a. Even numbers are not required for other even numbers to exist except 4
b. Odd composite numbers are not required for even numbers to exist
c. All even numbers except 2, simply will not exist without the corresponding prime number directly related to it provided that 1 is not a prime number.
d. A specific even number can be the sum of more than one pair of prime numbers.
e. All odd prime numbers alone are able to produce all even numbers except 2 and 4, since 1 is not considered a prime number.
f. An infinite number of prime numbers exists in order to fill the void between two even numbers that an odd composite number cannot fill.

The results from the simulations provide a conclusive proof that Goldbach's conjecture about prime numbers was correct all along simply because it is one of the properties of prime numbers.

9. The number 1 is really a prime number

The number 1 is quite interesting because it was excluded from the list of prime numbers. All known prime numbers are odd except 2, which really makes it a thought-provoking issue worth looking into.

a. Odd prime numbers fill a void left by odd composite numbers

Since all other odd prime numbers appear from the need to fill a void between two even numbers, the number 1 should be considered a prime number as well. Exploring this statement from another perspective:

If everything started from nothing and the even number 2 was suddenly discovered, the void between the two even numbers 0 and 2 should be filled by an odd composite number. But since there exist no other odd composite number that can fill this void, a prime number should be created in its place. Therefore, the number 1 should be a prime number by all accounts.

b. The number 2 should not exist

It is a bit unusual that the even number 2 simply exists out of nowhere, irrespective of it being a prime number. If the simulation is proven correct that even numbers require prime numbers to exist, but 1 is not considered as a prime number, then the even number 2 should not exist as well.

c. All odd prime numbers are connected to at least two even numbers

All other odd prime numbers are usually directly linked to at least two even numbers as shown in simulation 12 and 13, where an odd prime number was removed to prove that a specific even number will simply cease to exist.

In this regard, 1 can be directly linked to the even numbers 2 and 4:

$1 + 1 = 2$	$1 + 3 = 4$

Removing 1 from the list of prime numbers will make 2 and 4 cease to exist as well. Thus, if 1 is included as a prime number, then all even numbers including 2 are the sum of at least one pair of odd prime numbers. Therefore, the very definition of a prime number is not only being divisible by itself and 1, but also that no even numbers can exist without prime numbers.

d. Can create other numbers greater than 1 using prime numbers alone

Goldbach's conjecture (Strong) Prime + Prime = Even	Goldbach's conjecture (Weak) Prime + Prime + Prime = Odd
$1 + 1 = 2$	$1 + 1 + 1 = 3$

$1 + 3 = 4$	$1 + 1 + 3 = 5$
$1 + 5 = 6$	$1 + 1 + 5 = 7$ $1 + 3 + 3 = 7$
$1 + 7 = 8$	$1 + 1 + 7 = 9$ $1 + 3 + 5 = 9$

If 1 is considered as a prime number, then prime numbers exist as a foundation for all other numbers to exist.

e. Provides a possible alternative story for how whole numbers exists

Following the logic of the simulation with a slight modification. Instead of subtraction, add 1 instead to all even numbers.

Beginning with nothing 0 is considered as an even number, thus add 1 to it	$0 + 1 = 1$
1 is not an odd composite number, so it is considered a prime number Prime numbers can be added to itself	$1 + 1 = 2$
2 is even, thus add 1 to it	$2 + 1 = 3$
3 is not an odd composite number, so it is considered a prime number Prime numbers can be added to itself and other odd prime numbers	$1 + 3 = 4$ $3 + 3 = 6$

10. Why the number 2 is not a prime number

Whole numbers follow a strict pattern of sequence that alternates between odd and even numbers, but whenever there is a void between two even numbers that an odd composite number cannot fill in a sequence, an odd prime number appears. This property alone for all other prime numbers except 2 proves that the number 2 is not a prime number.

Other reasons why the number 2 is not a prime number:

a. The number 4 cannot exist without the prime number 2 since 1 is not considered a prime number, but all other even numbers are the sum of at least one pair of prime numbers except 2. Why is 2 different from all other even numbers? Where did 2 come from?

b. All other odd prime numbers exist to fill the void between two even numbers, whereas the number 2 does not have any voids to fill
    - If everything started with 1:
        - 0, 1, 2, 3, 4, and so on
        - Then 2 does not need to fill anything

    - If everything started with an odd number like 3:
        - 0, 3, 6, 9, and so on
        - Then there is nothing to fill

    - But if everything started with 2, then adding 2 to all even numbers will result to:
        - 0, 2, 4, 6, 8, 10, 12, 14 and so on
        - Prime numbers will fill up all the voids where no composite odd number exists, namely 1, 3, 5, 7, 11, and 13 since 9 is an odd composite number
            - The properties of prime numbers make more sense if at the beginning, only even number exists

c. The number 2 is the only prime number that cannot be added with any other odd prime number to create even numbers
    - the properties of even and odd number states:
        - Odd + Even = Odd

d. Similarities with the number 1 as a divisor
    - The number 1 can evenly divide all other numbers including itself
    - The number 2 can evenly divide all even numbers including itself
        - Is it because the number 2 is the sum of two number 1?

e. The number 2 behaves like how number 1 behaves if it is a prime number
   - Creates the very first even number greater than 0
     - If number 1 is considered as a prime number, the number 2 is the first even number that a prime number is able to create
       - $1 + 1 = 2$

     - Since 2 is considered as a prime number and 1 is not, the number 4 is the first even number that a prime number is able to create
       - $2 + 2 = 4$
       - But the number 4 is not the first even number greater than 0

   - Can create all other odd numbers except 1:

Number 1 can create all odd numbers except 1 by adding itself to any even numbers $2 + 1 = 3$ $4 + 1 = 5$ $6 + 1 = 7$ $8 + 1 = 9$	Number 2 can create all odd numbers except 1 by adding itself to any odd numbers $2 + 1 = 3$ $2 + 3 = 5$ $2 + 5 = 7$ $2 + 7 = 9$	Can both be rewritten using Goldbach's conjecture (weak) 2 can be rewritten as $1 + 1$ $1 + 1 + 1 = 3$ $1 + 1 + 3 = 5$ $1 + 1 + 5 = 7$ $1 + 1 + 7 = 9$

Although the number 2 shares the same characteristic as other prime numbers, where it cannot be evenly divided by any other number except by itself and 1 does not automatically make the number 2 a prime number. Another reason why the number 2 looks like a prime number is because of the fact that it behaves like how the number 1 behaves if it is a prime number as shown in section 10.e.

All prime numbers have other similar characteristics that the number 2 simply does not share such as supporting more than one even number and not requiring other even numbers to create more even numbers. Furthermore, the fact that the number 2 is the only even prime number out of an infinite number of odd prime numbers should have raised some questions as to the reason why the number 2 is remarkably different from all other odd prime numbers.

As shown in simulation 14, all odd prime numbers support more than one even number, where removing four consecutive prime numbers did nothing to disrupt the sequence of even numbers ranging from 6 to 218. But the prime number 2 alone only supports the even number 4, which also does not explain where 2 came from if 1 is not a prime number. Therefore, the very existence of the number 2 as a prime number strongly contradicts the existence of all other prime numbers.

11. Prime numbers are not directly linked to composite numbers

(Ang, 2021) provided some proofs that the currently accepted definition for prime numbers contradicts how prime numbers exists in the first place. He further stated that if prime numbers are only evenly divisible by itself and 1, then no new prime numbers will exist by means of multiplication between two numbers. Moreover, he also showed that all odd and even composite numbers derived from prime numbers will cease to exist if no new prime numbers are found.

If his statements are proven correct, it is now safe to infer that:

Prime numbers are not directly linked to composite numbers because prime numbers are created by means of addition, whereas composite numbers are created by means of multiplication.

The fascinating thing about prime numbers are its ability to create other numbers by simply adding a prime number to itself or with another prime number. To demonstrate the proof that composite numbers are not actually directly linked to prime numbers, a simple reconstruction of the multiplication table for all odd numbers will reveal the pattern for how other odd nonprime numbers are created. Reconstructing the multiplication table for odd numbers using a modified formula based on Goldbach's conjecture (weak):

$7 + 7 + 7 = 21$ (7 x 3)	$23 + 23 + 23 = 69$ (23 x 3)
$21 + 7 + 7 = 35$ (7 x 5)	$69 + 23 + 23 = 115$ (23 x 5)
$35 + 7 + 7 = 49$ (7 x 7)	$115 + 23 + 23 = 161$ (23 x 7)
$49 + 7 + 7 = 63$ (7 x 9)	$161 + 23 + 23 = 207$ (23 x 9)
$63 + 7 + 7 = 77$ (7 x 11)	$207 + 23 + 23 = 253$ (23 x 11)
$77 + 7 + 7 = 91$ (7 x 13)	$253 + 23 + 23 = 299$ (23 x 13)
$91 + 7 + 7 = 105$ (7 x 15)	$299 + 23 + 23 = 345$ (23 x 15)
$105 + 7 + 7 = 119$ (7 x 17)	$345 + 23 + 23 = 391$ (23 x 17)
$and\ so\ on\ \ldots$	$and\ so\ on\ \ldots$

To further illustrate the proof:

Given prime numbers: 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
Missing odd numbers: 9, 15, 21, 25, 27, 33, 35

$3 + 3 + 3 = 9$	$5 + 5 + 5 = 15$	$7 + 7 + 7 = 21$
$9 + 3 + 3 = 15$	$15 + 5 + 5 = 25$	$21 + 7 + 7 = 35$
$15 + 3 + 3 = 21$	$25 + 5 + 5 = 35$	$and\ so\ on \ldots$
$21 + 3 + 3 = 27$	$35 + 5 + 5 = 45$	$11 + 11 + 11 = 33$
$27 + 3 + 3 = 33$	$45 + 5 + 5 = 55$	$13 + 13 + 13 = 39$
$33 + 3 + 3 = 39$	$55 + 5 + 5 = 65$	$17 + 17 + 17 = 51$
$and\ so\ on \ldots$	$and\ so\ on \ldots$	$and\ so\ on \ldots$

Using the odd nonprime number to find other odd nonprime numbers:

$9 + 9 + 9 = 27$ (9 x 3)	$15 + 15 + 15 = 45$ (15 x 3)	$21 + 21 + 21 = 63$ (21 x 3)
$27 + 9 + 9 = 45$ (9 x 5)	$45 + 15 = 15 = 75$ (15 x 5)	$63 + 21 + 21 = 105$ (21 x 5)
$and\ so\ on \ldots$	$and\ so\ on \ldots$	$and\ so\ on \ldots$

As shown in the examples above, odd prime numbers are able to create odd nonprime numbers that looks exactly like odd composite numbers. By using the sum derived from adding the prime number onto itself, an entire multiplication table that looks exactly like odd composite numbers are created. This likeness is the reason why composite numbers were mistaken to be a part of the property of prime numbers in the first place.

12. Nonprime numbers are created by means of addition

Prime numbers are able to produce nonprime numbers that looks exactly like composite numbers using modified formulas derived from Goldbach's conjectures. By using the sum derived from adding the prime number onto itself, an entire multiplication table containing both odd and even numbers can be created that looks exactly like composite numbers.

Reconstructing the multiplication table for the prime number 3 using modified formulas derived from Goldbach's conjectures:

Modified formula based on Goldbach's conjecture (Strong)	Modified formula based on Goldbach's conjecture (Weak)	Using Nonprime Numbers: 9 from (6 + 3) or (3 x 3)
$3+3=6$ (3 x 2)	$3+3+3=9$ (3 X 3)	$9+9=18$ (9 x 2)
$6+3=9$ (3 x 3)	$9+3+3=15$ (3 X 5)	$18+9=27$ (9 x 3)
$9+3=12$ (3 x 4)	$15+3+3=21$ (3 X 7)	$27+9=36$ (9 x 4)
$12+3=15$ (3 x 5)	$21+3+3=27$ (3 X 9)	$36+9=45$ (9 x 5)
$15+3=18$ (3 x 6)	$27+3+3=33$ (3 X 11)	$45+9=54$ (9 x 6)
$18+3=21$ (3 x 7)	$33+3+3=39$ (3 X 13)	$54+9=63$ (9 x 7)
$21+3=24$ (3 x 8)	$39+3+3=45$ (3 X 15)	$63+9=72$ (9 x 8)
$24+3=27$ (3 x 9)	$45+3+3=51$ (3 X 17)	$72+9=81$ (9 x 9)
and so on …	*and so on …*	*and so on …*

As can be seen from the given table, prime numbers follow a specific pattern when creating odd and even nonprime numbers. This very same pattern can also be used on other nonprime numbers to create other nonprime numbers. Thus, this provides a conclusive proof that nonprime numbers are derived from addition, while composite numbers are derived from multiplication.

Composite numbers are created by means of multiplication, whereas nonprime numbers are created by means of addition.

The only link that composite numbers actually have with prime numbers is that both composite and nonprime numbers look very identical to each other. This similarity between composite and nonprime numbers is the main reason why composite numbers were mistakenly associated with prime numbers in the first place. All prime numbers are created when there are no odd nonprime number that can fill the void between two even numbers, therefore it will never be divisible by any other number except by itself and 1.

This very definition that all prime numbers are not evenly divisible by any other number except by itself and 1 is the cause of a lot of confusion when it comes to prime numbers. Since there are not a lot of information about prime numbers, it was assumed that this is true for all other numbers including even numbers. Moreover, the fact that the number 1 is not considered a prime number further solidified the notion that the number 2 is a prime number.

The results provide a conclusive proof that prime numbers are not directly related to composite numbers, but is a fortunate happenstance that gave us the ability to find prime numbers more efficiently. Therefore, by proving that composite numbers are not directly linked to any of the prime numbers, it is now safe to infer that all prime numbers are odd and that 1 is also a prime number.

13. Conclusion

Some problems are quite difficult to solve because of the preconceived notions that has already been instilled deep within each one of us. Studying the very reason for its existence can usually shed light to some of the problems related to it. Looking at prime numbers from a different perspective gave some new insights as to the reason why prime numbers exist in the first place. The simulations showed that both prime and nonprime numbers are inextricably linked to each other, since both prime and nonprime numbers require each other to exist. Moreover, if 1 is accepted as a prime number, then prime numbers alone are the building blocks for all other numbers greater than 1 by means of addition.

The results from the simulations also gave a reasonable explanation as to why the number 1 is the perfect number to start the list of prime numbers. It also proved that the number 2 is not a prime number, but was mistaken to be a prime number because it shares some of the characteristics that other odd prime numbers have. Moreover, it was also proven that prime numbers are not directly linked to composite numbers, but were mistakenly associated with prime numbers because of the resemblance it has with nonprime numbers.

Goldbach’s conjecture opened the window for everyone to be able to take a closer look as to the reasons why and how prime numbers exist. The conjecture has also been instrumental in revealing the reason why all prime numbers are odd and that all nonprime numbers simply cannot exist without prime numbers.

Based on all the results from the simulations, it is now possible to infer that:

If 1 is accepted as a prime number:

- All prime numbers are odd
- Prime numbers are not directly linked to composite numbers
- Prime numbers fill the void between two even numbers that no other odd nonprime number can fill
- Both prime and nonprime numbers are created by means of addition

- Prime numbers form a foundation that supports all even numbers
- Prime numbers are the building blocks for all other numbers greater than 1
    - Prime numbers alone can create all numbers greater than 1 using modified formulas based on Goldbach's conjectures

Acknowledgements

To my parents, thank you for the love, support, and belief in me as I navigate my way in life. This research paper would have not been possible without your support.

To my wife Natassia, thank you for the love, understanding, and constant support through this chapter in my life. Thank you for always being there for me when I need it the most.

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Reference:

Ang, Glenn Patrick King. (2021). The Definition of Prime Numbers Contradicts Its Very Own Existence. Zenodo. https://doi.org/10.5281/zenodo.5710380

Appendix A:

Link to the GitHub repository containing the python code used in this research paper:

https://github.com/07231985/primenumbers

```
# find the truth about why prime numbers exist
# created by Glenn Patrick King Ang 10/21/2021

all_even_numbers = []
found_even_numbers = []
find_numbers_until = 2000
starting_prime_numbers = [3, 5, 7]
find_odd_composite_nums = []
found_numbers = starting_prime_numbers.copy()
missing_even_numbers = []
loop = 0

# designed for 2 as a prime number
for even_nums in range(6, find_numbers_until, 2):
    all_even_numbers.append(even_nums)

while loop < 3:
    for current_num in starting_prime_numbers:
        if current_num % 2 == 1:
            for next_num in starting_prime_numbers:
                if next_num % 2 == 1:
                    sum_of_odd_nums = current_num + next_num
                    if sum_of_odd_nums not in found_numbers and
sum_of_odd_nums < find_numbers_until:
                        loop = 0
                        found_numbers.append(sum_of_odd_nums)

    loop += 1
    if loop > 3:
        break

# identify all the even integers and put it inside
found_even_integer

for all_even in all_even_numbers:
    if all_even in found_numbers:
        found_even_numbers.append(all_even)

# find the missing even integers if any

for all_even in all_even_numbers:
    if all_even not in found_numbers:
        last_even_found = found_even_numbers[-1]
        index_last_even_found =
all_even_numbers.index(last_even_found)

        if all_even < all_even_numbers[index_last_even_found]:
```

```
            missing_even_numbers.append(all_even)

# Multiply all prime numbers by itself and  with each other:
# ignore product if it is greater than the last even number + 1
for prime_num in starting_prime_numbers:
    for prime_num_two in starting_prime_numbers:
        product_itself = prime_num * prime_num_two
        if len(found_even_numbers) > 1:
            if product_itself < found_even_numbers[-1]:
                if product_itself not in
find_odd_composite_nums:

find_odd_composite_nums.append(product_itself)

# Multiply all odd numbers by itself and  with each other:
# ignore product if it is greater than the last even number + 1
all_odd_nums = starting_prime_numbers.copy()

for odd_composite in find_odd_composite_nums:
    if odd_composite not in all_odd_nums:
        all_odd_nums.append(odd_composite)
for odd_nums in all_odd_nums:
    for odd_nums_two in all_odd_nums:
        product_all_odds = odd_nums * odd_nums_two
        if len(found_even_numbers) > 1:
            if product_all_odds < found_even_numbers[-1]:
                if product_all_odds not in
find_odd_composite_nums:

find_odd_composite_nums.append(product_all_odds)

found_numbers.sort()
find_odd_composite_nums.sort()
found_even_numbers.sort()

print(f"Starting number is: {starting_prime_numbers}")
print(f"Find numbers until: {find_numbers_until}")
print(f"Found numbers are: {found_numbers}")
print(f"\nFound even numbers: {found_even_numbers}")
print(f"\nMissing even numbers: {missing_even_numbers}")
print(f"Found odd composite numbers:
{find_odd_composite_nums}")
```