Coarse scale quantities

Base quantities A, B, C

Note: $U_k(T)$ – full patch
T – Corrector el.
T' – local patch el.

$$A_{TT'} = 2^d \overset{2^d}{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}$$

$$B_{TT'} = 2^d \overset{2^d}{\begin{bmatrix} | & | & | \\ | & | & | \end{bmatrix}}$$

$$C_{TT'} = 2^d \overset{2^d}{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}$$

$$A_{Tij} = (\hat{A}_T \nabla \lambda_j, \nabla \lambda_i)_T$$
$$B_{TT'ij} = (\bar{A}_T \nabla \tilde{Q}_T \lambda_j, \nabla \lambda_i)_{T'}$$
$$C_{TT'ij} = (\bar{A}_T \nabla \bar{Q}_T \lambda_j, \nabla \hat{Q}_T \lambda_i)_{T'}$$

1. K_{Tij} – Petrov-Galerkin stiffness matrix

$$K_{Tij} = (\bar{A}_T \nabla (\chi_T \lambda_j - \hat{Q}_T \lambda_j), \nabla \lambda_i)_{U_k(T)}$$
$$= A_{Ti\sigma_T(j)} - \sum_{T'} B_{TT'i\sigma_{T'}(j)}$$

where $\sigma_T(j)$ maps dofs from T' to $U_k(T)$

Note: only $\lambda_j|_T \neq 0$ give nonzero contributions. Only $\lambda_i|_{U_k} \neq 0$ gives nonzero contributions.

$$K_T = k^d 2^d \overset{2^d}{\begin{bmatrix} | & | & | \\ | & | & | \end{bmatrix}}$$

2. $L_{TT'ij}$ – Local error matrix

$$L_{TT'ij} = (\hat{A}_T \nabla (\chi_T - \bar{Q}_T)\lambda_j, \nabla(\chi_T - \hat{Q}_T)\lambda_i)_{T'}$$
$$= \begin{cases} C_{TT'ij}, & T \neq T' \\ C_{TT'ij} - B_{TT'ij} - B_{TT'ji} + A_{Tij}, & T = T' \end{cases}$$

Note: Only $\lambda_i|_T \neq 0$ and $\lambda_j|_T \neq 0$ gives contributions.

$$L_{TT'} = 2^d \overset{2^d}{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}$$