[Daiya et al., 3(6): June, 2016] ISSN 2349-4506
Impact Factor: 2.545

Global Journal of Engineering Science and Research Management

TRIPlE DIRICHLET AVERAGE AND FRACTIONAL DERIVATIVE INVOLVING WITH GENERALIZED K-MITTAG-LEFFLER FUNCTION

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KEYWORDS: Triple Dirichlet Average, Generalized K-Mittag-Leffler Function, Fractional Derivative, Gamma and Beta Function.

Mathematics Subject Classification: - 33E12, 33C60, 26A33.

ABSTRACT
The object of this article is to establish some results of Triple Dirichlet averages of Generalized K- Mittag-Leffer functions introduced by Saxena, daiya and Singh [16]. Representations of such relations are obtained in terms of fractional derivative. Some interesting special cases findings.

INTRODUCTION
The Dirichlet average of a function is certain kind of integral average with respect to Dirichlet measure. The concept of Dirichlet average was introduced by Carlson [1,2,4] It is studied, among others, by zu Castell [5], Massopust and Forster [13], Numan [15], Neuman and Van Fleet [16], Daiya [6] and others. A detailed and comprehensive account of various types of Dirichlet averages has been given by Carlson [3] in his monograph.. Deora and Banerji [7] have shown that the Triple Dirlchlet average is equivalent to fractional derivative.

Definition1: Let $k \in R; \alpha, \beta, \gamma \in C;\ \mathrm{Re}(\alpha) > 0\ and\ \tau \in C,$ then the generalized k-Mittag-Leffler function is given by Saxena, Daiya and Singh [17]

$$E_{k,\alpha,\beta}^{\gamma,\tau}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)} \frac{z^t}{t!} \tag{1}$$

Where $(x)_\tau, (x,\tau \in C)$ denotes the Pochhammer symbol with $(1)_n = n!$ for $n \in N = N \cup \{0\}$, which is defined in terms of gamma function as

$$(x)_\tau = \frac{\Gamma(x+\tau)}{\Gamma(x)} = \begin{cases} 1 & (\tau = 0;\ x \in C \setminus \{0\}) \\ x(x+1)\ldots(x+\tau-1) & (\tau = n \in N;\ x \in C) \end{cases}$$

Special cases of $E_{k,\alpha,\beta}^{\gamma,\tau}(z)$

(i) For τ = q, equation (1) yields generalized K-Mittag-Leffler function defined by Saxena, Daiya and Singh [17]

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(n\alpha+\beta)} \frac{z^t}{t!} = E_{k,\alpha,\beta}^{\gamma,q}(z) \tag{2}$$

(ii) For k = 1, equation (2) yields generalized Mittag-Leffler function defined by Shukla and Prajapati [19]

$$E_{1,\alpha,\beta}^{\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq}}{\Gamma(n\alpha+\beta)} \frac{z^t}{t!} = E_{\alpha,\beta}^{\gamma,q}(z) \quad (3)$$

(ii) When q = 1, equation (2) gives the Mittag-Leffler function defined by Doorego & Cerutti [9]

$$E_{k,\alpha,\beta}^{\gamma,1}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{\Gamma_k(n\alpha+\beta)} \frac{z^t}{t!} = E_{k,\alpha,\beta}^{\gamma}(z) \quad (4)$$

Note 2: A detailed account of Mittag-Leffler function and their application can be found in the survey paper by Haubold et al.[12], Mathai et al. [14], Saxena et al.[18] and Daiya and Ram [6]

I will need some more notations in the further exposition. In the sequel, the symbol E_{n-1} will denote the Euclidean simplex, defined by

$$E_{n-1} = \left[(u_1, \ldots, u_{n-1});\ u_j \geq 0,\ j = 1, 2, \ldots n,\ u_1, \ldots, u_{n-1} \leq 1\right]. \quad (5)$$

The concept of the Dirichlet average. Following[8] let W be a convex set in £ and let $z = (z_1, \ldots, z_n) \in \Omega^n;\ n \geq 2$ and let f be a measurable function on W.
Define

$$f(b; z) = \int_{E_{n-1}} f(u \cdot z)\, d\mu_b(u) \quad (6)$$

And

$$(u \cdot z) = \sum_{i=1}^{n-1} u_i z_i + (1 - u_1 -, \ldots, u_{n-1})\, z_n \quad (7)$$

and $d\mu_b$ is the Dirichlet measure defined

$$du_b(u) = \frac{1}{B(b)}\, u_1^{b_1-1}, \ldots, u_{n-1}^{b_{n-1}-1} \left(1 - u_1 - \ldots - u_{n-1}\right)^{b_n-1} du_1, \ldots, du_{n-1}$$

(8)

With the multivariable Beta function

$$B(b) = \frac{\Gamma(b_1), \ldots, \Gamma(b_k)}{\Gamma(b_1 +, \ldots, b_k)} \qquad \operatorname{Re}(b_j) > 0,\ (j = 1, 2, \ldots, k) \quad (9)$$

For n = 2, we have

$$dm_{\eta,\eta'}(u) = \frac{\Gamma(\eta+\eta')}{\Gamma(\eta)\Gamma(\eta')}\, u^{\eta-1}(1-u)^{\eta'-1}\, du \quad (10)$$

Carlson[2] investigated the average (6) for $f(z) = z^k;\ k \in R$ in the form

$$\operatorname{Re}(b; z) = \int_{E_{n-1}} (u \cdot z)^k\, d\mu_b(u) \quad (11)$$

If n = 2 . Carlson[2, 3] proved that

$$R_k(\beta,\beta';x,y)=\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\int_0^1[ux+(1-u)y]^k u^{\beta-1}(1-u)^{\beta'-1}du$$
(12)

Where $\beta,\beta'\in C;\min[\text{Re}(\beta),\text{Re}(\beta')]>0$; $x,y\in R$

Let z be species with complex elements Z_{ijk}. let $u=(u_1,...,u_l),v=(v_1,...,v_m)$ and $w=(w_1,...,w_n)$ be an ordered l-tuple, m-tuple and n-tuple of real non-negative weights $\sum u_i=1\sum v_j=1$ and $\sum w_k=1$ respectively (see Deora and Banerji [7])

Define

$$(u\cdot z\cdot v\cdot w)=\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n}u_i\, z_{ijk}\, v_j w_k \tag{13}$$

If Z_{ijk} is regarded as a point of the complex plane all these convex combinations are points in the convex hull denote by $H(z)$.

Let $u=(u_1,...,u_l)$ be an ordered l-tuple of complex numbers with positive real part $\text{Re}(\mu)>0$ and similarly for $\alpha=(\alpha_1,...,\alpha_m)$ and $\beta=(\beta_1,...,\beta_n)$ then define $dm_\mu(u)$, $dm_\alpha(v)$ and $dm_\beta(w)$ as (13).

Let f be the holomorphic on a domain D in the complex plane if $\text{Re}(\mu)>0,\text{Re}(\alpha)>0,\text{Re}(\beta)>0$ *and* $H(z)\subset D$, define

$$F(\mu,z,\alpha,\beta)=\int\int\int f(u\cdot z\cdot v\cdot w)\,dm_\mu(u)\,dm_\alpha(v)\,dm_\beta(w) \tag{14}$$

Triple Dirichlet average for $(l=m=n=2)$ of $(u\cdot z\cdot v\cdot w)^t$ is defined by Deora and Banerji [7].

$$\Re_t(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\int_0^1\int_0^1\int_0^1(u\cdot z\cdot v\cdot w)^t\,dm_{(\mu,\mu')}(\mu)\,dm_{(\alpha,\alpha')}(v)\,dm_{(\beta,\beta')}(w)$$
(15)

where $\text{Re}(\mu)>0$, $\text{Re}(\mu')>0$, $\text{Re}(\alpha)>0,\text{Re}(\alpha')>0,\text{Re}(\beta)>0$ *and* $\text{Re}(\beta')>0$,

and

$$(u\cdot z\cdot v\cdot w)=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}(u_i\cdot z_{ijk}\cdot v_j\cdot w_k)$$

$$=\begin{bmatrix}u_1z_{111}v_1w_1+u_1z_{112}v_1w_2+u_1z_{121}v_2w_1+u_1z_{122}v_2w_2+u_2z_{211}v_1w_1\\ +u_2z_{212}v_1w_2+u_2z_{221}v_2w_1+u_2z_{222}v_2w_2\end{bmatrix} \tag{16}$$

Assume in first species

$z_{111}=a,\ z_{112}=b,\ z_{121}=c,\ z_{122}=d$

and in second species

$z_{211} = e,\ z_{212} = f,\ z_{221} = g,\ z_{222} = h$

and $\begin{cases} u_1 = u & u_2 = 1-u \\ v_1 = v & v_2 = 1-v \\ w_1 = w & w_2 = 1-w \end{cases}$

Therefore

$$(u\cdot z\cdot v\cdot w)=\begin{bmatrix} uvw(a-b-c+d-e+f+g-h)+uv(b-d-f+h) \\ +vw(e-f-g+h)+wu(c-d-g+h) \\ +u(d-h)+v(f-h)+w(g-h)+h \end{bmatrix}$$

(17)

and

$$dm_{\mu,\mu'}(u)=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\ u^{\mu-1}(1-u)^{\mu'-1}\ du \tag{18}$$

$$dm_{\alpha,\alpha'}(v)=\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\ v^{\alpha-1}(1-v)^{\alpha'-1}\ dv \tag{19}$$

$$dm_{\beta,\beta'}(w)=\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\ w^{\beta-1}(1-w)^{\beta'-1}\ dw \tag{20}$$

Using equation (15)

$$\Re_t(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\int_0^1\int_0^1\int_0^1[uvw(a-b-c+d-e+f+g-h)$$
$$+uv(b-d-f+h)+vw(e-f-g+h)+wu(c-d-g+h)+u(d-h)+v(f-h)+w(g-h)+h]^t$$
$$u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\ du\ dv\,dw.$$

(21)

Let consider the triple average for $(l=m=n=2)$ of $E_{k,\alpha,\beta}^{\gamma,\tau}\ [u\cdot z\cdot v\cdot w]$.

$$J_t(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)n!}$$
$$\int_0^1\int_0^1\int_0^1[uvw(a-b-c+d-e+f+g-h)+uv(b-d-f+h)+vw(e-f-g+h)+wu(c-d-g+h)$$
$$+u(d-h)+v(f-h)+w(g-h)+h]^t\ u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\ du\ dv\,dw.$$

(22)

FRACTIONAL DERIVATIVE

Fractional derivative with respect to an arbitrary function has bee used by Erdelyi [10]. The general definition for the fractional derivative of order $\alpha \in C_\infty$, $\mathrm{Re}(\alpha)>0$ the Riemann-Liouville integral is defined as

$$\left(D_x^\alpha F\right)(x)=\frac{1}{\Gamma(-\alpha)}\int_0^x \frac{F(t)}{(x-t)^{1-\alpha}}\,dt$$

(23)

and

$$\left(D_{x-x_0}^\alpha F\right)(x)=\frac{1}{\Gamma(-\alpha)}\int_{x_0}^x \frac{F(t)}{(x-t)^{1-\alpha}}\,dt, \quad \mathrm{Re}(\alpha)<0 \tag{24}$$

where $F(t)$ is of the form $x^p f(x)$ and $f(x)$ is analytic at $x=0.$

I need Gamma and beta function in the further expression

$$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

(25)

We know that

$$\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},\ (m,n>0) \tag{26}$$

MAIN RESULTS

Theorem 1: Let $k\in R;\ \alpha,\beta,\gamma\in C;\ \mathrm{Re}(\alpha)>0\ and\ \tau\in C,$ then Triple Dirichlet Average is established for $(l=m=n=2)$ of $E_{k,\alpha,\beta}^{\gamma,\tau}\,[u\cdot z\cdot v\cdot w].$

$$J_n\left(\mu,\mu',z,\alpha,\alpha',\beta,\beta'\right)=\frac{(\alpha)_n(\beta)_n}{(\alpha+\alpha')_n(\beta+\beta')_n}\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)}\ (y-x)^{1-\mu-\mu'}\ D^{-\mu'}\left[E_{k,\alpha,\beta}^{\gamma,\tau}(x)^n(y-x)^{\mu-1}\right] \tag{27}$$

Proof :- using equation (22)

$$J_t\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)n!}$$

$$\int_0^1\int_0^1\int_0^1\Big[uvw(a-b-c+d-e+f+g-h)+uv\left(b-d-f+h\right)$$
$$+vw\left(e-f-g+h\right)+wu(c-d-g+h)+u\left(d-h\right)+v\left(f-h\right)+w(g-h)+h\Big]^t$$
$$u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\ du\ dv\,dw.$$

For $a=x;e=y;b=c=d=f=g=h=0$ and $t=n$

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)\Gamma\left(\mu'\right)}\frac{\Gamma\left(\alpha+\alpha'\right)}{\Gamma\left(\alpha\right)\Gamma\left(\alpha'\right)}\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma\left(\beta\right)\Gamma\left(\beta'\right)}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k\left(n\alpha+\beta\right)n!}$$

$$\int_0^1\int_0^1\int_0^1\left[uvw(x-y)+vwy\right]^n\ u^{\mu-1}\left(1-u\right)^{\mu'-1}v^{\alpha-1}\left(1-v\right)^{\alpha'-1}w^{\beta-1}\left(1-w\right)^{\beta'-1}\ du\ dv\,dw.$$

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)\Gamma\left(\mu'\right)}\frac{\Gamma\left(\alpha+\alpha'\right)}{\Gamma\left(\alpha\right)\Gamma\left(\alpha'\right)}\frac{\Gamma\left(\beta+\beta'\right)}{\Gamma\left(\beta\right)\Gamma\left(\beta'\right)}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k\left(n\alpha+\beta\right)n!}$$

$$\int_0^1\int_0^1\int_0^1(vw)^n\left[ux+(1-u)y\right]^n\ u^{\mu-1}\left(1-u\right)^{\mu'-1}v^{\alpha-1}\left(1-v\right)^{\alpha'-1}w^{\beta-1}\left(1-w\right)^{\beta'-1}\ du\ dv\,dw.$$

Using Beta function, Gamma function and suitable adjustments.

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{(\alpha)_n(\beta)_n}{(\alpha+\alpha')_n(\beta+\beta')_n}\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)\Gamma\left(\mu'\right)}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k\left(n\alpha+\beta\right)n!}$$

$$\int_0^1\left[ux+(1-u)y\right]^n u^{\mu-1}\left(1-u\right)^{\mu'-1}du.$$

Using definition of fraction derivative (23), we get

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{(\alpha)_n(\beta)_n}{(\alpha+\alpha')_n(\beta+\beta')_n}\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)}\ \left(y-x\right)^{1-\mu-\mu'}\ D^{-\mu'}\left[E_{k,\alpha,\beta}^{\gamma,\tau}(x)^n(y-x)^{\mu-1}\right]$$

This completes the proof of Theorem 1.

Corollary 1.1 Put $\tau=q$ equation (27) reduce in the following from

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{(\alpha)_n(\beta)_n}{(\alpha+\alpha')_n(\beta+\beta')_n}\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)}\ \left(y-x\right)^{1-\mu-\mu'}\ D^{-\mu'}\left[E_{k,\alpha,\beta}^{\gamma,q}(x)^n(y-x)^{\mu-1}\right]\tag{28}$$

Corollary 1.2 Put $k=1$ equation (28) reduce in the following from

$$J_n\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{(\alpha)_n(\beta)_n}{(\alpha+\alpha')_n(\beta+\beta')_n}\frac{\Gamma\left(\mu+\mu'\right)}{\Gamma\left(\mu\right)}\ \left(y-x\right)^{1-\mu-\mu'}\ D^{-\mu'}\left[E_{\alpha,\beta}^{\gamma,q}(x)^n(y-x)^{\mu-1}\right]$$

(29)

Theorem 2: Let $k\in R;\ \alpha,\beta,\gamma\in C;\ \mathrm{Re}(\alpha)>0\ and\ \tau\in C,$ then Triple Dirichlet Average is established for $(l=m=n=2)$ of $E_{k,\alpha,\beta}^{\gamma,\tau}\left[u\cdot z\cdot v\cdot w\right].$

$$J_{-v}\left(\mu,\mu';z;\alpha,\alpha',\beta,\beta'\right)=\frac{\Gamma\left(\mu+\mu'\right)\Gamma\left(\alpha+\alpha'\right)\Gamma\left(\beta+\beta'\right)}{\Gamma\left(\mu\right)\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}(1-d)^{1-\mu-\mu'}(1-f)^{1-\alpha-\alpha'}\ (1-g)^{1-\beta-\beta'}$$

$$D_{1-d}^{-\mu'}D_{1-f}^{-\alpha'}D_{1-g}^{-\beta'}\left[E_{k,\alpha,\beta}^{\gamma,\tau}\left(dfg\right)\right](1-g)^{\beta-1}(1-f)^{\alpha-1}(1-d)^{\mu-1}\tag{30}$$

Proof: using equation (22)

$$J_t(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)n!}$$

$$\int_0^1\int_0^1\int_0^1[uvw(a-b-c+d-e+f+g-h)+uv(b-d-f+h)$$
$$+vw(e-f-g+h)+wu(c-d-g+h)+u(d-h)+v(f-h)+w(g-h)+h]^t$$
$$u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\,du\,dv\,dw.$$

Set $a=dfg;b=df;c=dg;e=fg;h=1$ and $t=-v$ in (22)

$$J_{-v}(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)(-v)!}$$

$$\int_0^1\int_0^1\int_0^1[uvw(dfg-df-fg-dg+d+f+g-1)+uv(df-f-d+1)+vw(fg-f-g+1)$$
$$+wu(dg-d-g+1)+u(d-1)+v(f-1)+w(g-1)+1]^{-v}$$
$$u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\,du\,dv\,dw.$$

on suitable adjustments of terms,

$$J_{-v}(\mu,\mu',z,\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)(-v)!}$$

$$\int_0^1\int_0^1\int_0^1\{[1-u(1-d)][1-v(1-f)][1-w(1-g)]\}^{-v}$$
$$u^{\mu-1}(1-u)^{\mu'-1}v^{\alpha-1}(1-v)^{\alpha'-1}w^{\beta-1}(1-w)^{\beta'-1}\,du\,dv\,dw.$$

For $u=\frac{p}{1-d};v=\frac{q}{1-f};\ w=\frac{r}{1-g}$

$$J_{-v}(\mu,\mu';z;\alpha,\alpha',\beta,\beta')=\frac{\Gamma(\mu+\mu')}{\Gamma(\mu)\Gamma(\mu')}\frac{\Gamma(\alpha+\alpha')}{\Gamma(\alpha)\Gamma(\alpha')}\frac{\Gamma(\beta+\beta')}{\Gamma(\beta)\Gamma(\beta')}\sum_{n=0}^{\infty}\frac{(\gamma)_{n\tau,k}}{\Gamma_k(n\alpha+\beta)(-v)!}$$

$$(1-d)^{1-\mu-\mu'}(1-f)^{1-\alpha-\alpha'}(1-g)^{1-\beta-\beta'}\int_0^{1-g}\int_0^{1-f}\int_0^{1-d}[(1-p)(1-q)(1-r)]^{-v}$$
$$r^{\beta-1}(1-g-r)^{\beta'-1}q^{\alpha-1}(1-f-q)^{\alpha'-1}p^{\mu-1}(1-d-p)^{\mu'-1}\,dp\,dq\,dr.$$

Now using definition of fractional derivatives,

$$=\frac{\Gamma(\mu+\mu')\Gamma(\alpha+\alpha')\Gamma(\beta+\beta')}{\Gamma(\mu)\Gamma(\alpha)\Gamma(\beta)}(1-d)^{1-\mu-\mu'}(1-f)^{1-\alpha-\alpha'}(1-g)^{1-\beta-\beta'}$$

$$D_{1-d}^{-\mu'} D_{1-f}^{-\alpha'} D_{1-g}^{-\beta'} \left[E_{k,\alpha,\beta}^{\gamma,\tau}(dfg) \right] (1-g)^{\beta-1} (1-f)^{\alpha-1} (1-d)^{\mu-1}$$

This completes the proof of Theorem 2.

Corollary 2.1 Put $\tau = q$ equation (30) reduce in the following from

$$J_{-\nu}(\mu,\mu',z,\alpha,\alpha',\beta,\beta') = \frac{\Gamma(\mu+\mu')\Gamma(\alpha+\alpha')\Gamma(\beta+\beta')}{\Gamma(\mu)\Gamma(\alpha)\Gamma(\beta)} (1-d)^{1-\mu-\mu'} (1-f)^{1-\alpha-\alpha'} (1-g)^{1-\beta-\beta'}$$

$$D_{1-d}^{-\mu'} D_{1-f}^{-\alpha'} D_{1-g}^{-\beta'} \left[E_{k,\alpha,\beta}^{\gamma,q}(dfg) \right] (1-g)^{\beta-1} (1-f)^{\alpha-1} (1-d)^{\mu-1} \quad (31)$$

Corollary 2.2 Put $k = 1$ equation (31) reduce in the following from

$$J_{-\nu}(\mu,\mu',z,\alpha,\alpha',\beta,\beta') = \frac{\Gamma(\mu+\mu')\Gamma(\alpha+\alpha')\Gamma(\beta+\beta')}{\Gamma(\mu)\Gamma(\alpha)\Gamma(\beta)} (1-d)^{1-\mu-\mu'} (1-f)^{1-\alpha-\alpha'} (1-g)^{1-\beta-\beta'}$$

$$D_{1-d}^{-\mu'} D_{1-f}^{-\alpha'} D_{1-g}^{-\beta'} \left[E_{\alpha,\beta}^{\gamma,q}(dfg)^{-\nu} \right] (1-g)^{\beta-1} (1-f)^{\alpha-1} (1-d)^{\mu-1} \quad (32)$$

ACKNOWLEDGEMENT

This work is supported by Post-Doctoral Fellowship of the National Board of Higher Mathematics (NBHM), Department of Atomic Energy, India.

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