Upper ocean statistics from super-resolved SST imagery

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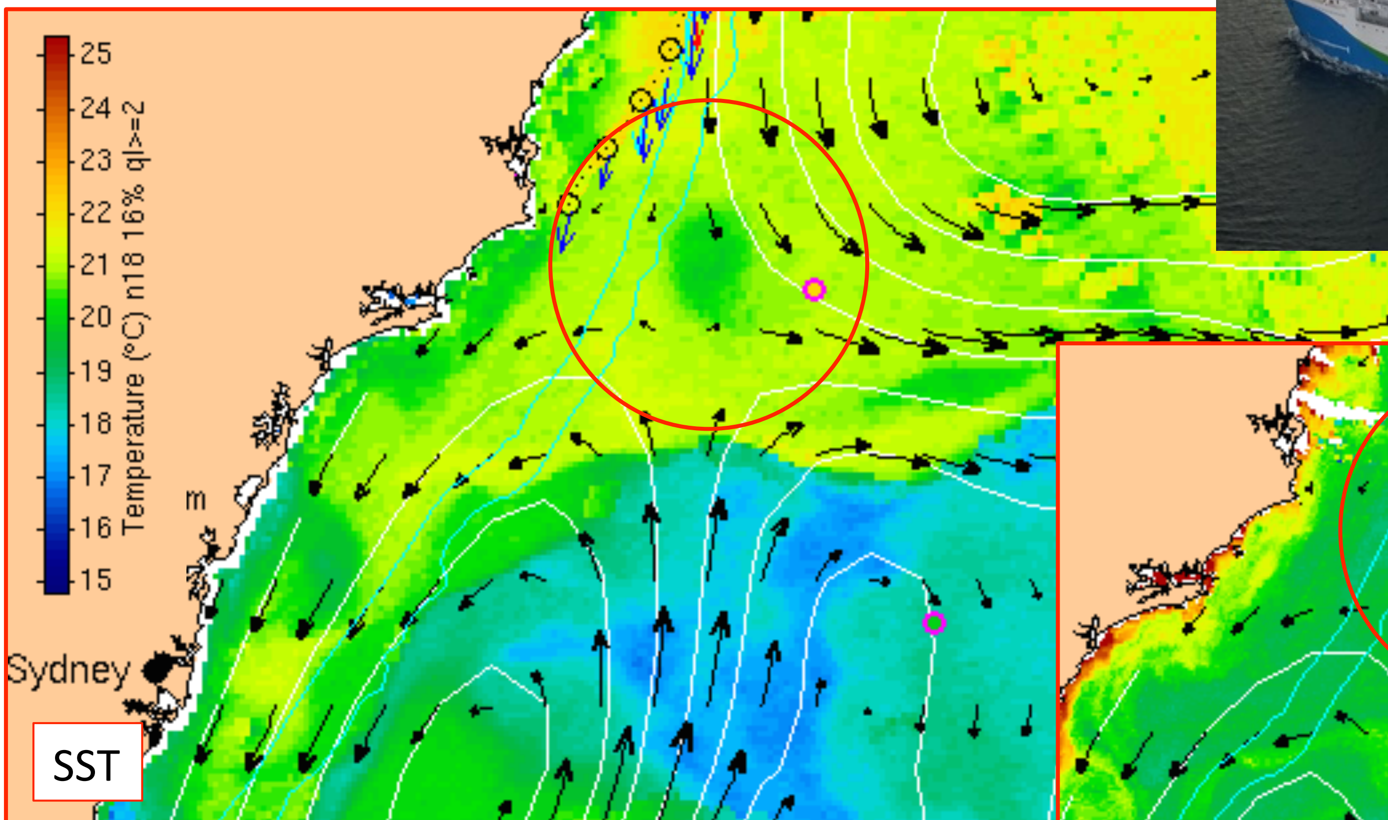
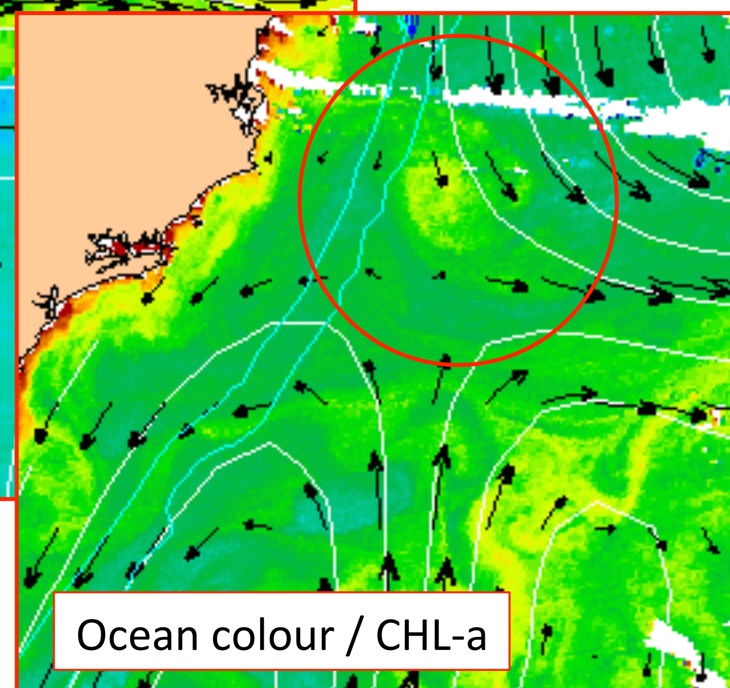
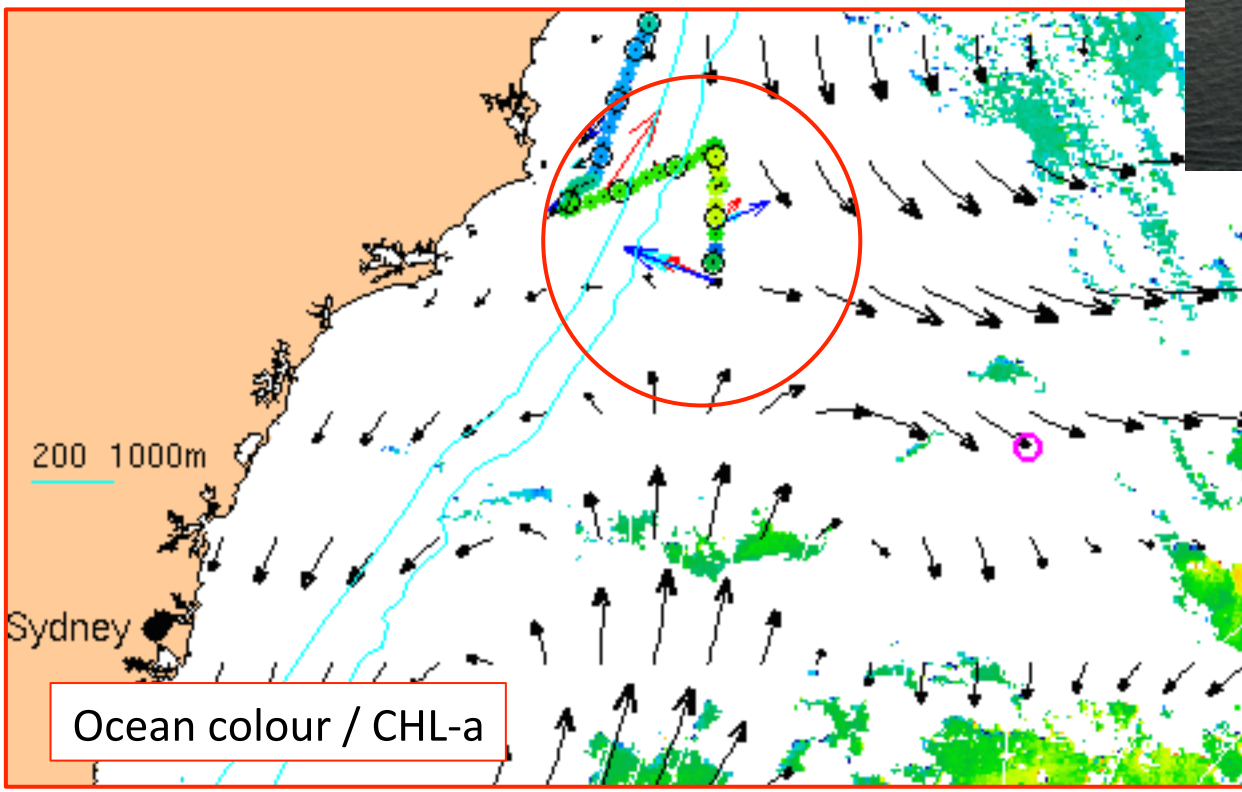
Hunting submesoscale eddies in the EAC

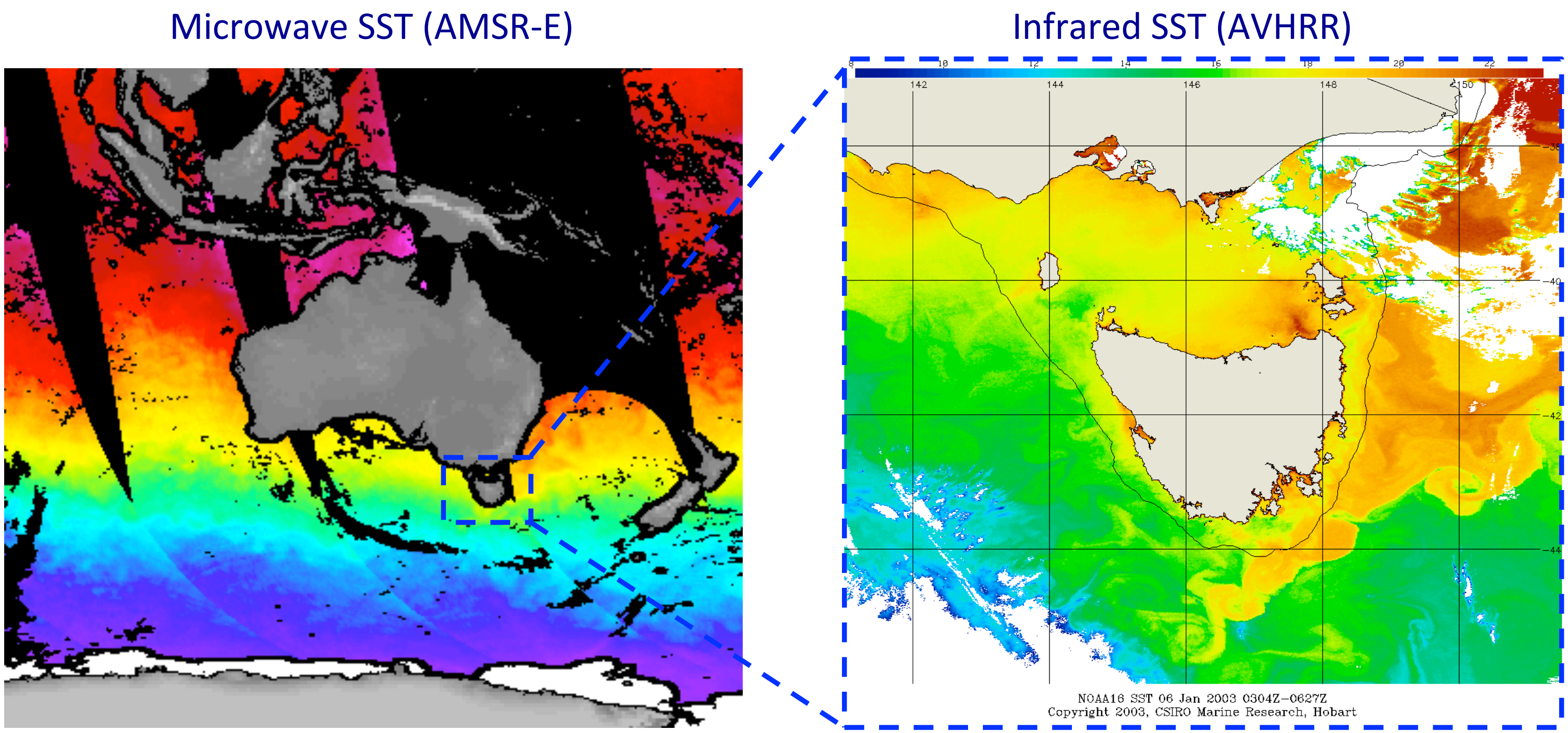




oceancurrent.imos.org.au
(Thanks to David Griffin)

Hunting submesoscale eddies in the EAC





Sea-surface temperature reveal footprint of submesoscale flow:

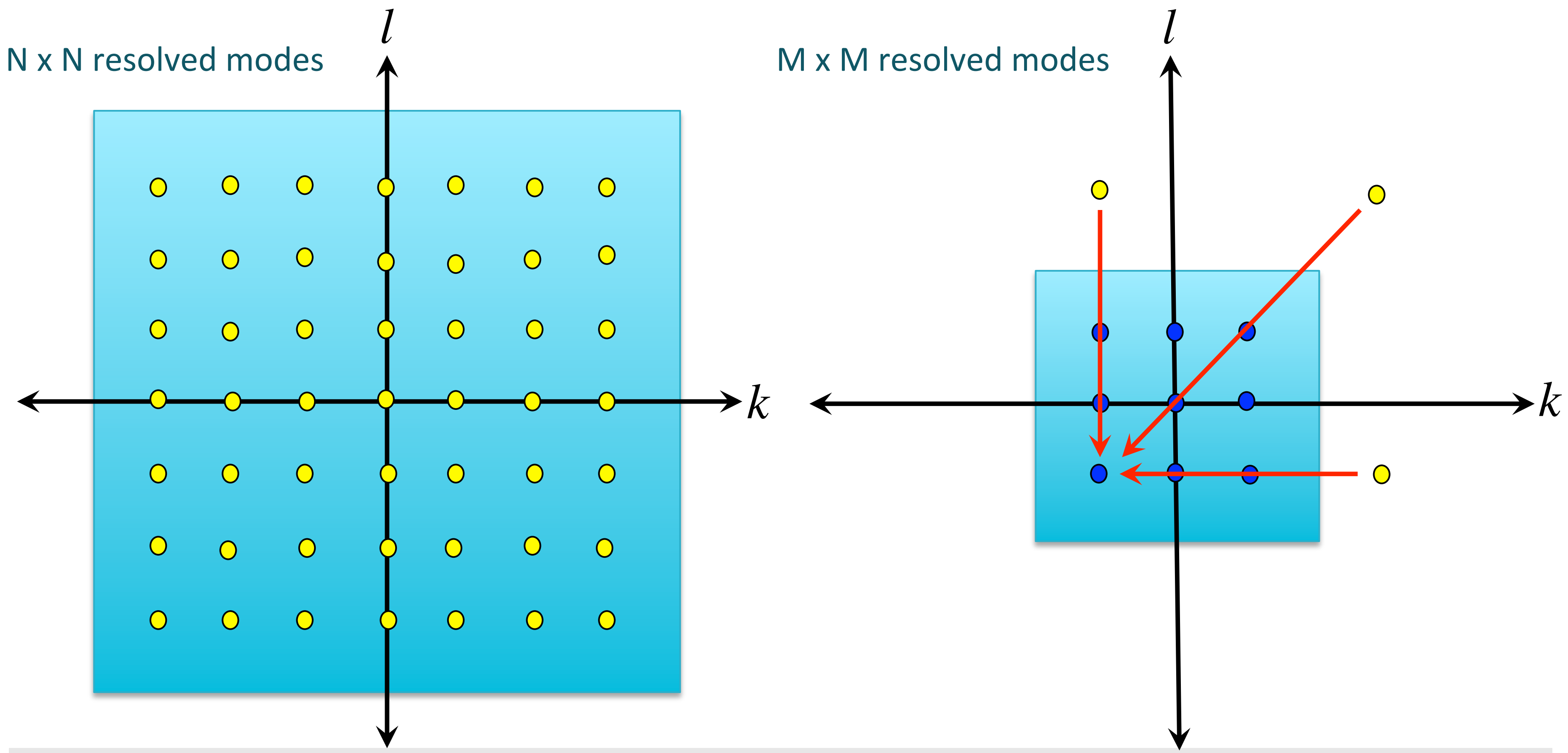
- **Passive microwave observations** (subskin temperature) have spatial resolutions of **20-50 km** and can **penetrate clouds**
- **Infrared observations** (skin temperature) have spatial resolutions of **1 km** but are **obscured by clouds**

- Derive **super-resolved SST images** by combining microwave images with **statistical knowledge** from infrared observations
- Exploit **spatial aliasing** of small scales by **coarse observations**

Original image

Subsampled image

Aliasing of sparse observations



Coarse-grid modes are superposition of fine-grid modes in **same aliasing set.**

$$\psi_{k,l}^{coarse} = \sum_{\tilde{k},\tilde{l}} \psi_{\tilde{k},\tilde{l}}^{fine}$$

$$\tilde{k} \text{ mod M} = k$$
$$\tilde{l} \text{ mod M} = l$$

Filtering sparse observations

Observation: Low-resolution observations with aliased information about small scales

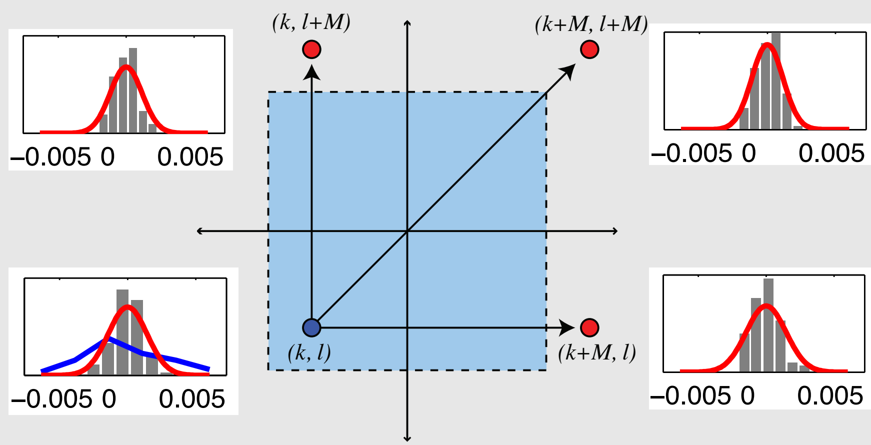
$$\psi^{obs}_{k,l} = \sum_{\tilde{k},\tilde{l}} \psi^{true}_{\tilde{k},\tilde{l}} + \sigma^{obs}_{k,l}$$

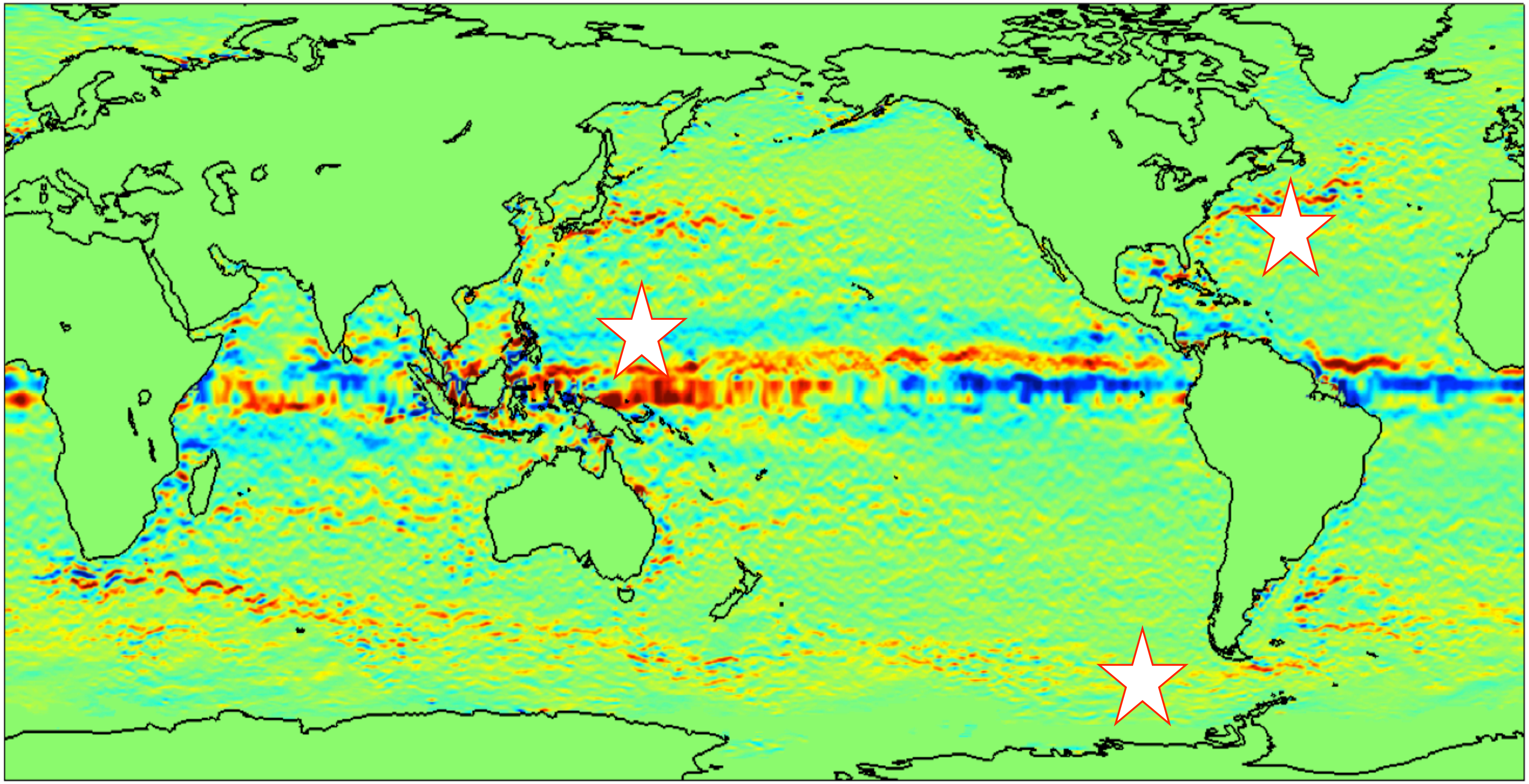
Forecast: Quasi-linear stochastic model using parameters from available high-resolution obs

$$\partial_t \hat{\theta} = -(\gamma - \mathrm{i}\omega)\hat{\theta}(t) + \sigma \dot{W}(t)$$

Kalman Filter

Analysis: Super-resolved SST image with resolution given by high-resolution forecast model



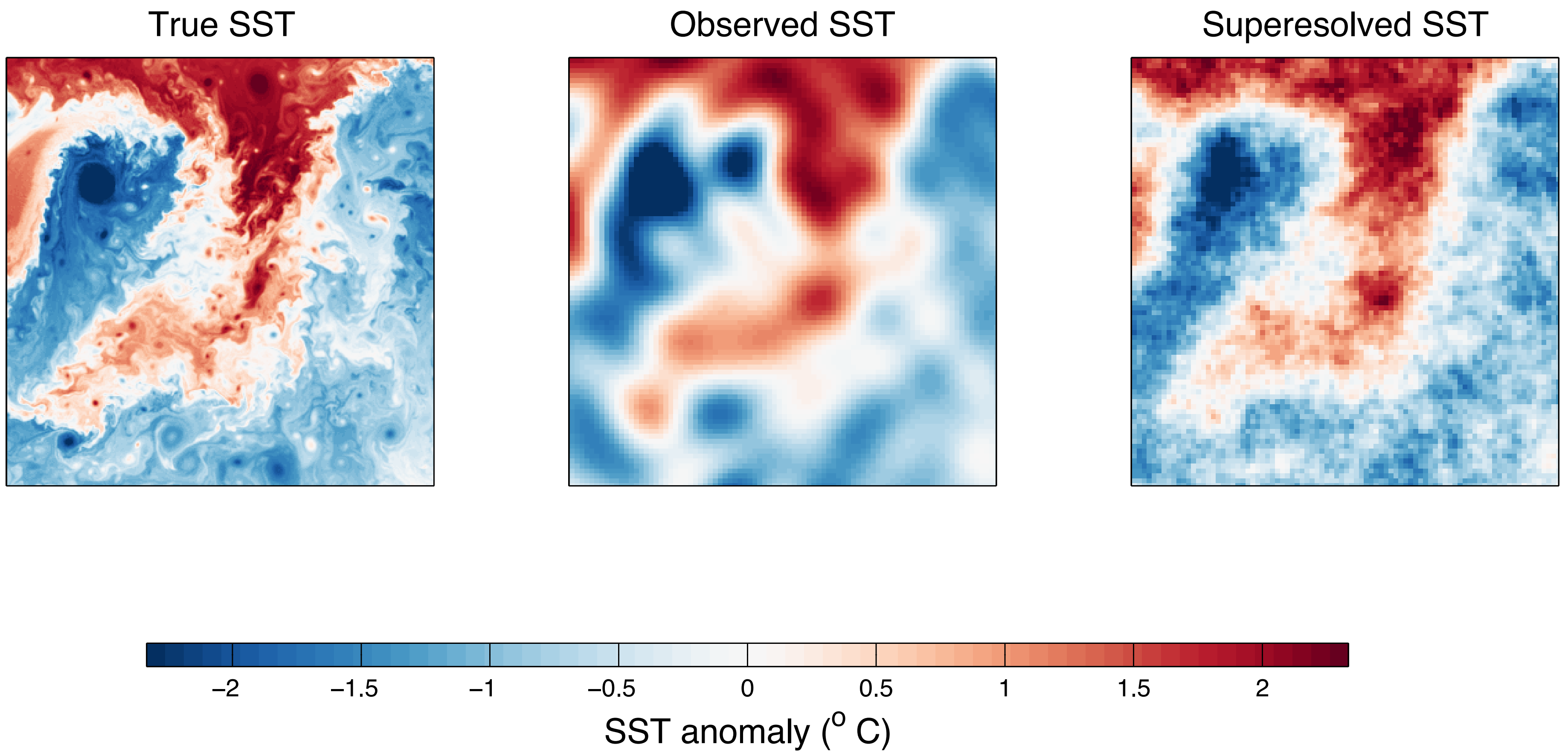


Zonal velocity

- **Quasigeostrophic model** driven by Forget (2010) hydrography.
- Assume that **surface density anomalies** are dominated by SST.
- Synthetic daily temperature observations over a 90-day period with both **microwave (40 km) and infrared (5 km) resolutions**.
- Infrared observations used to learn **stochastic parameters**.

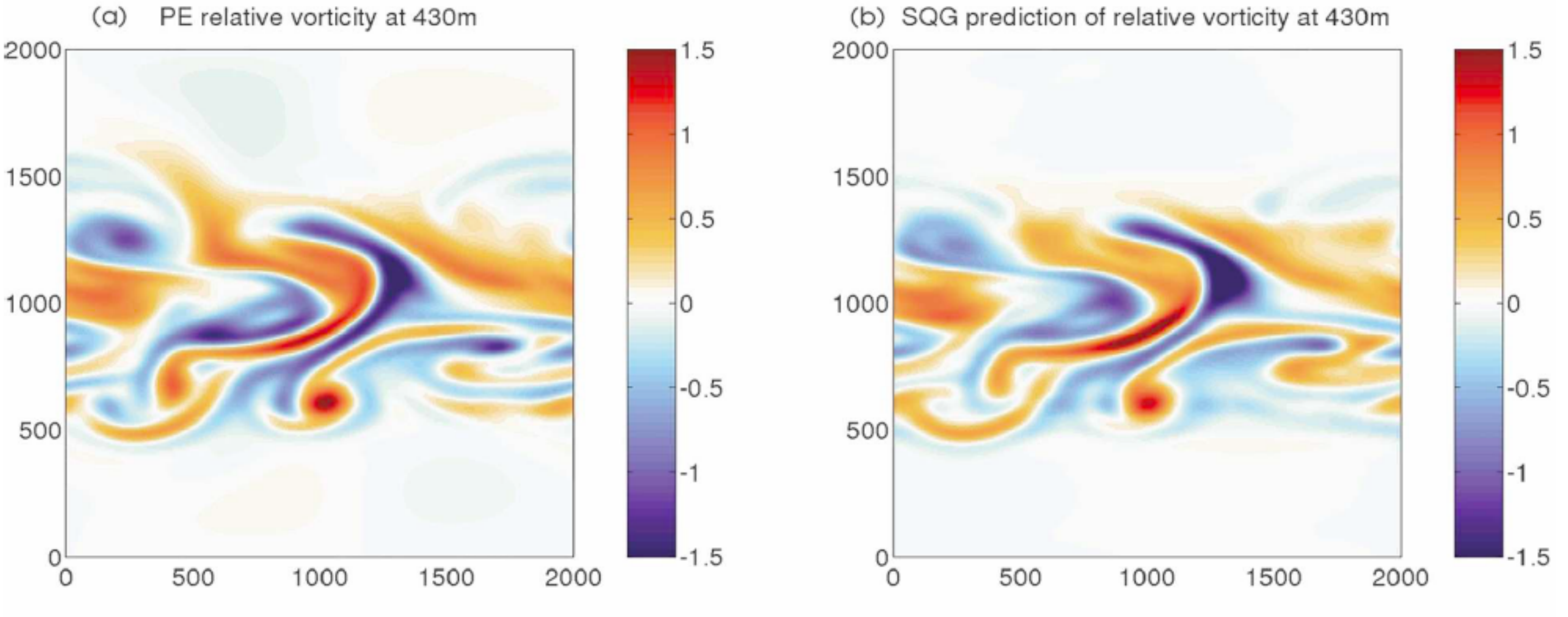
Super-resolved SST

SST snapshots: Subtropical Pacific



$$\theta_{kl} = \langle \theta_{kl} \rangle + A(k,l)X, \qquad A^*(k,l)A(k,l) = R(k,l)$$

Upper ocean flow reconstruction



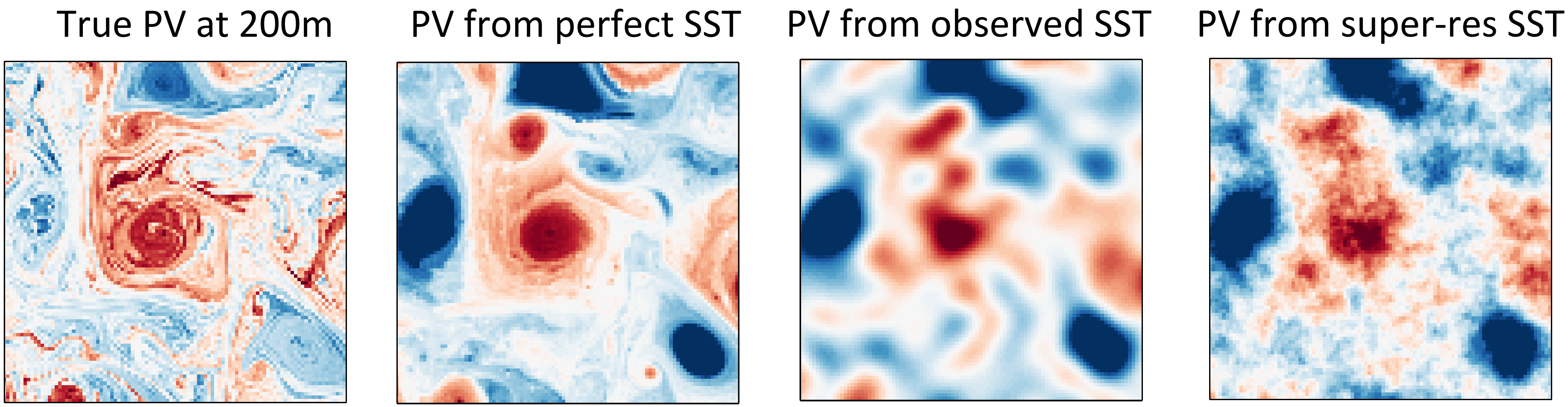
Lapeyre & Klein (2006)

- **Surface quasigeostrophic (SQG) model**: Interior streamfunction slaved to surface density/temperature (Lapeyre & Klein 2006)

$$\hat{\psi}_{k,l}(z) \sim \frac{\hat{\theta}_{k,l}(0)}{K}\exp\left(\sigma K z\right), \qquad z \le 0, \quad K^2 = k^2 + l^2$$

- Streamfunction is **smoothed version** of temperature: Microwave SST reconstructs flow with resolution of O(100) km.

Upper ocean flow reconstruction



- Lapeyre and Klein (2006): model interior PV with empirically derived vertical profile function: $q(x,y,z) = \alpha(z)\ \theta_{surf}(x,y)$
- Even with **perfect observations of SST,** Surface QG methods have a **depth of validity varies regionally**.
- Super-resolved SST results in improved **subsurface stream-function reconstruction** compared with raw observations.

Conclusions

AGU PUBLICATIONS

JGR

Journal of Geophysical Research: Oceans

RESEARCH ARTICLE

10.1002/2014JC010357

Key Points:

- The resolution of microwave SST images is increased using a statistical model
- The model is based upon statistics learned from intermittent infrared

Upper ocean flow statistics estimated from superresolved sea-surface temperature images

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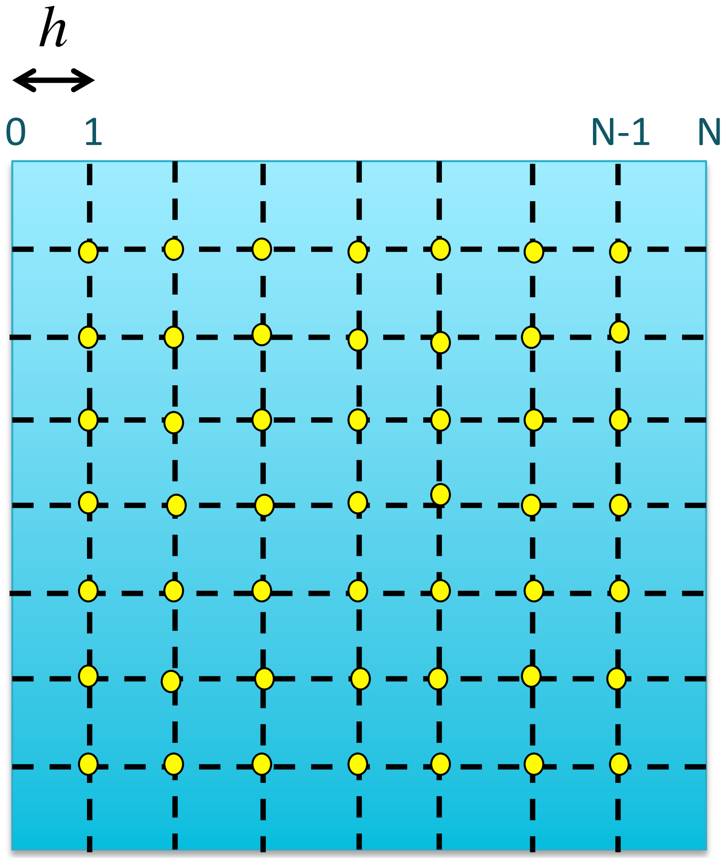
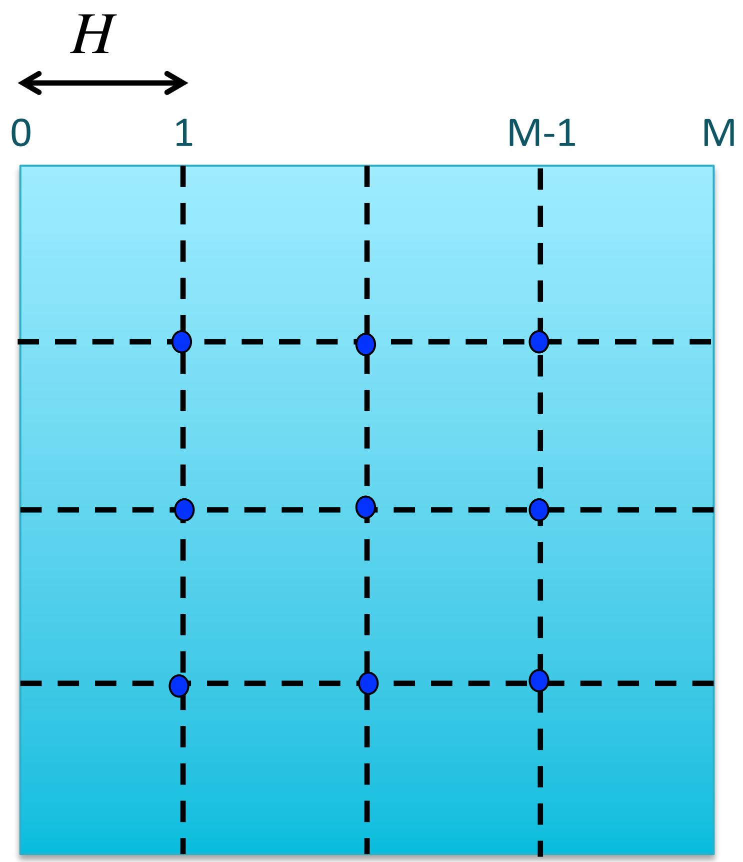
- Combine coarse-resolution microwave SST images with a simple statistical model to construct **super-resolved SST images.**
- **Upper ocean flow statistics** can be derived by projecting SST onto subsurface streamfunction
- Plan to implement this with satellite data and compare with existing SST products. **Collaborators wanted!**

Aliasing of sparse observations



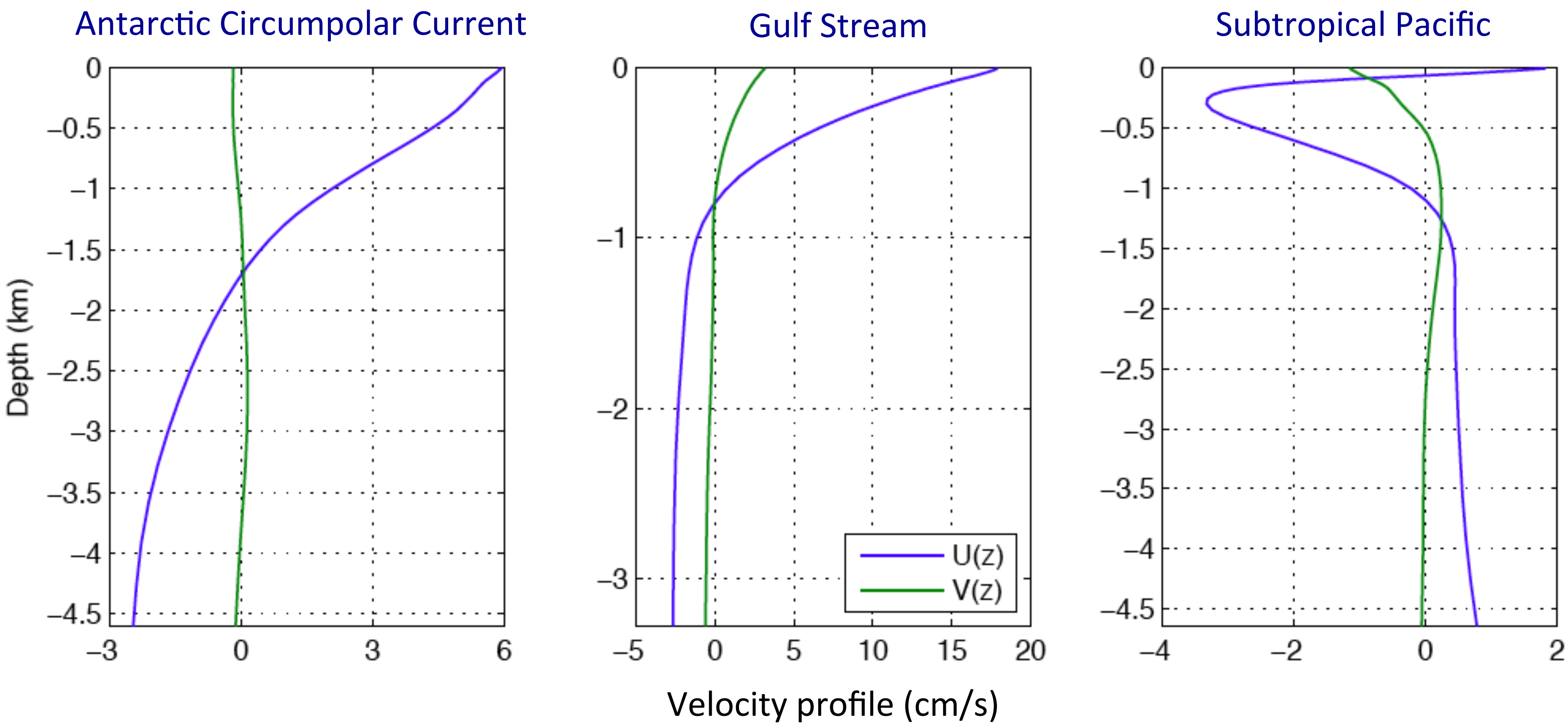


Fourier transform on **fine** grid:

$$\psi^{fine}_{\tilde{k},\tilde{l}} = \frac{1}{N^2} \sum_{m,n=1}^{N} \psi\left(mh, nh\right) e^{ih(m\tilde{k}+n\tilde{l})}$$

Fourier transform on **coarse** grid:

$$\psi^{coarse}_{k,l} = \frac{1}{M^2} \sum_{m,n=1}^{M} \psi\left(mH, nH\right) e^{iH(mk+nl)}$$



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- Assume that **surface density anomalies** are dominated by SST.
- Synthetic daily temperature observations over a 90-day period with both **microwave (40 km) and infrared (5 km) resolutions**.
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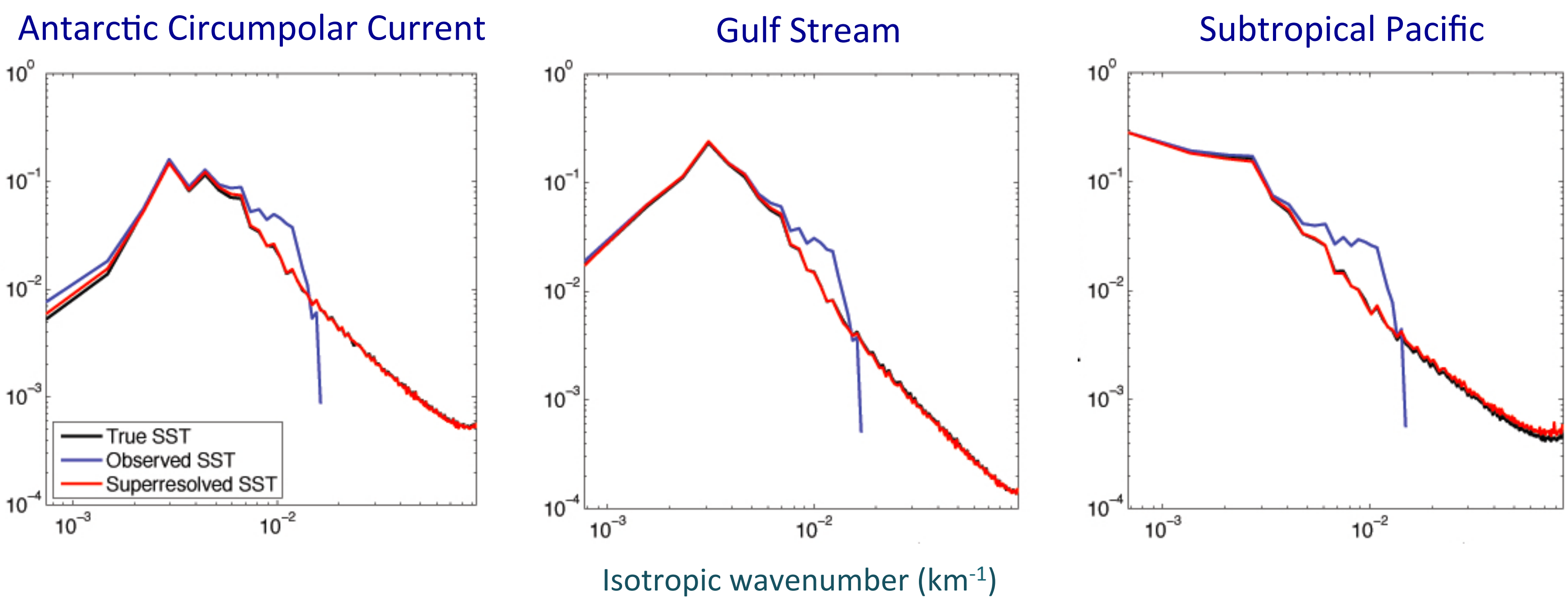
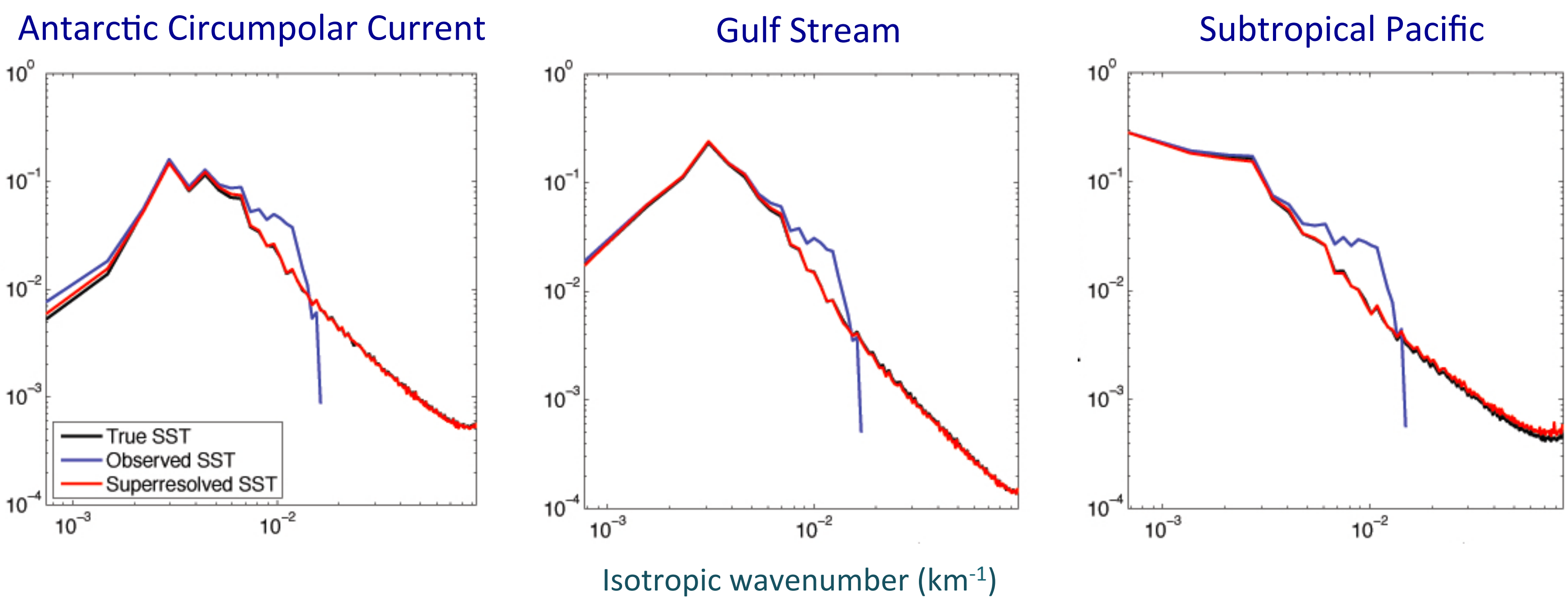
Temperature variance spectrum: $\left\langle \left|\theta(k)\right|^2 \right\rangle$



- **Effect of aliasing** can be seen in spurious variance in observations near the limit of resolution
- Super-resolved estimate correctly **redistributes variance** to small scales

Super-resolved SST



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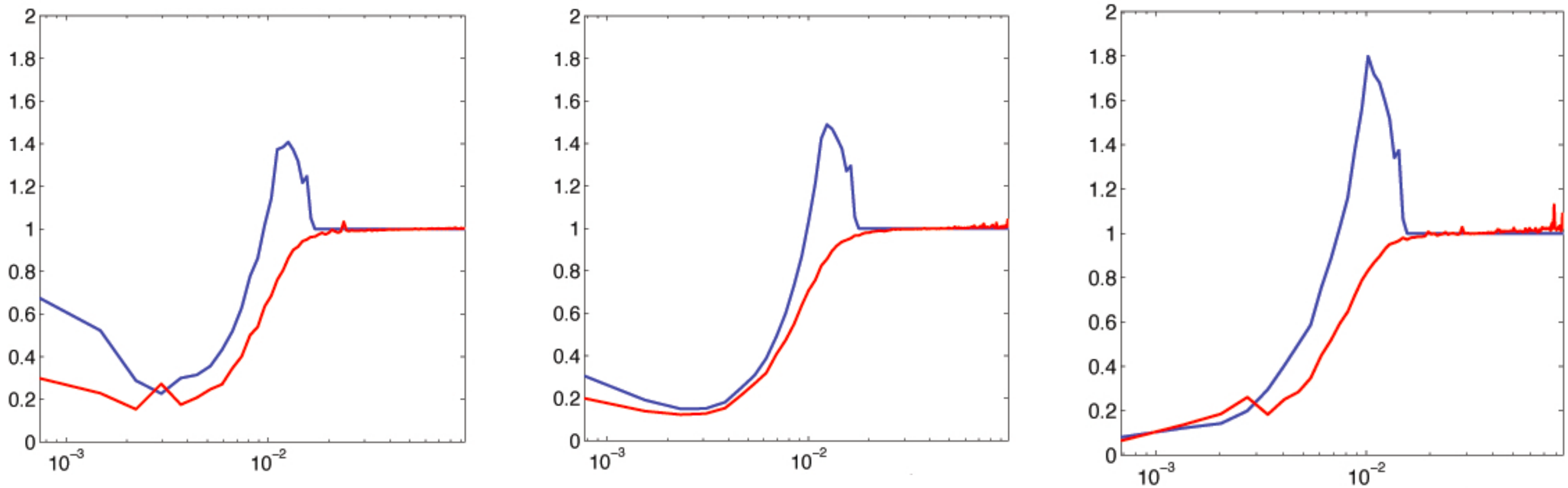
RMS error: $\left\langle \left| \theta(k) - \theta^{true}(k) \right|^2 \right\rangle^{1/2} \Big/ \left\langle \left| \theta^{true}(k) \right|^2 \right\rangle^{1/2}$

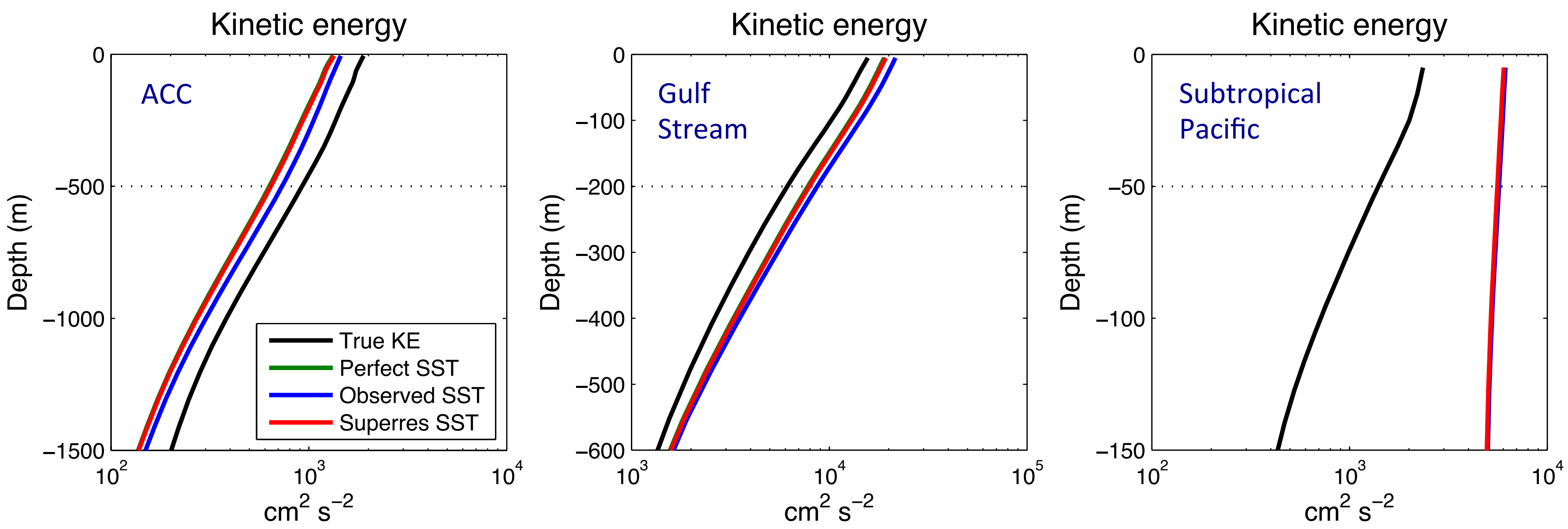
Antarctic Circumpolar Current — Gulf Stream — Subtropical Pacific



Isotropic wavenumber (km^{-1})

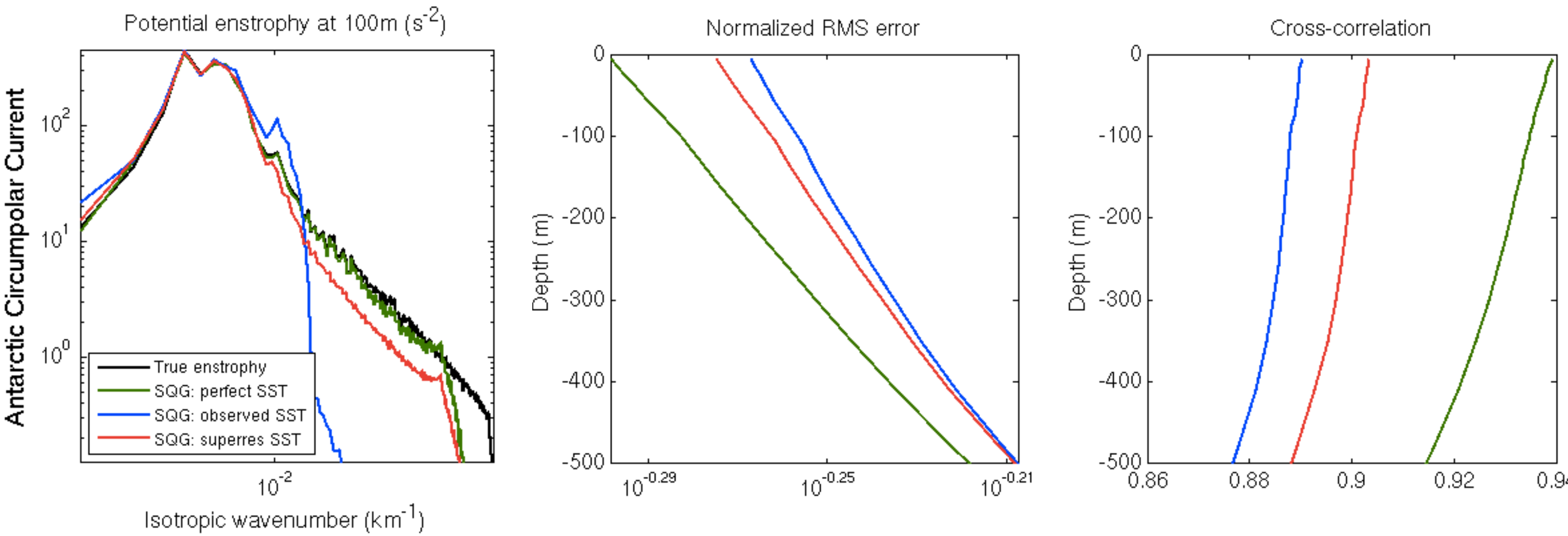
- **Effect of aliasing** can be seen in spurious variance in observations near the limit of resolution
- Super-resolved estimate correctly **redistributes variance** to small scales

Upper ocean flow reconstruction



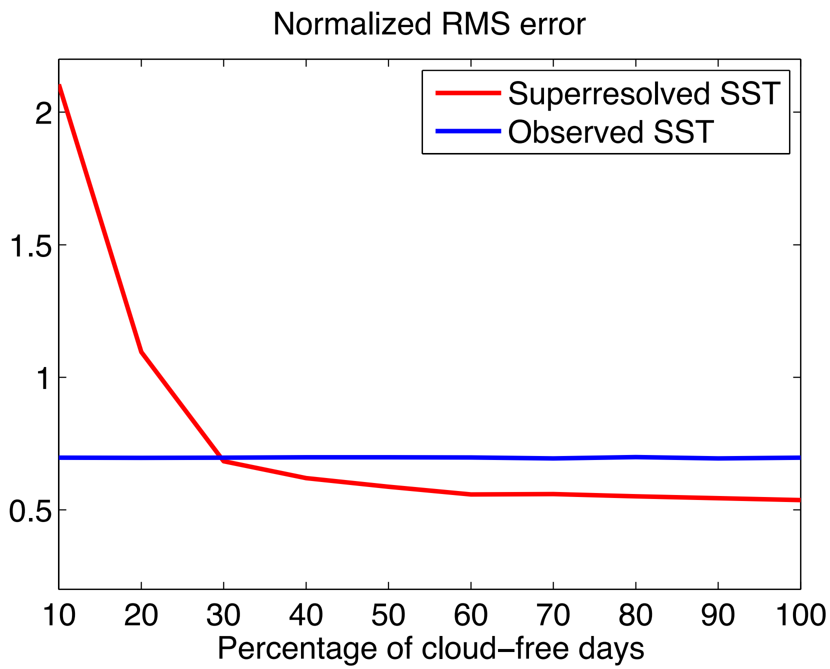
- Even with **perfect observations of SST,** SQG methods have a **depth of validity that varies regionally**.
- Argues for **inclusion of interior dynamics**: Lapeyre (2009), Ponte and Klein (2013), Wang et al. (2013).
- However, super-resolved SST results in improved **surface mode reconstruction** compared with raw observations.

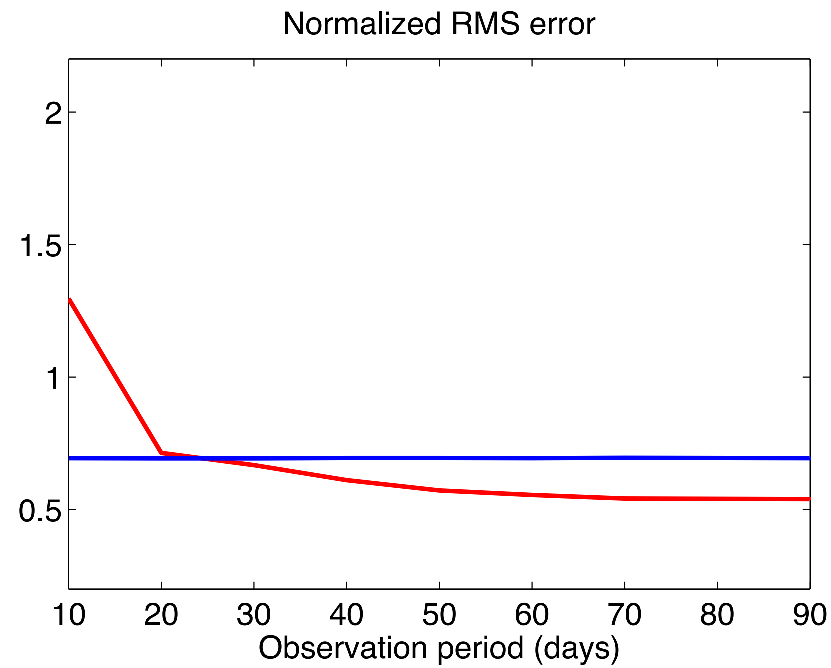
Upper ocean flow reconstruction



- Lapeyre and Klein (2006): model interior PV with empirically derived vertical profile function: $q(x,y,z) = \alpha(z)\ \theta_{surf}(x,y)$
- Even with **perfect observations of SST,** Surface QG methods have a **depth of validity varies regionally**.
- Super-resolved SST results in improved **subsurface stream-function reconstruction** compared with raw observations.

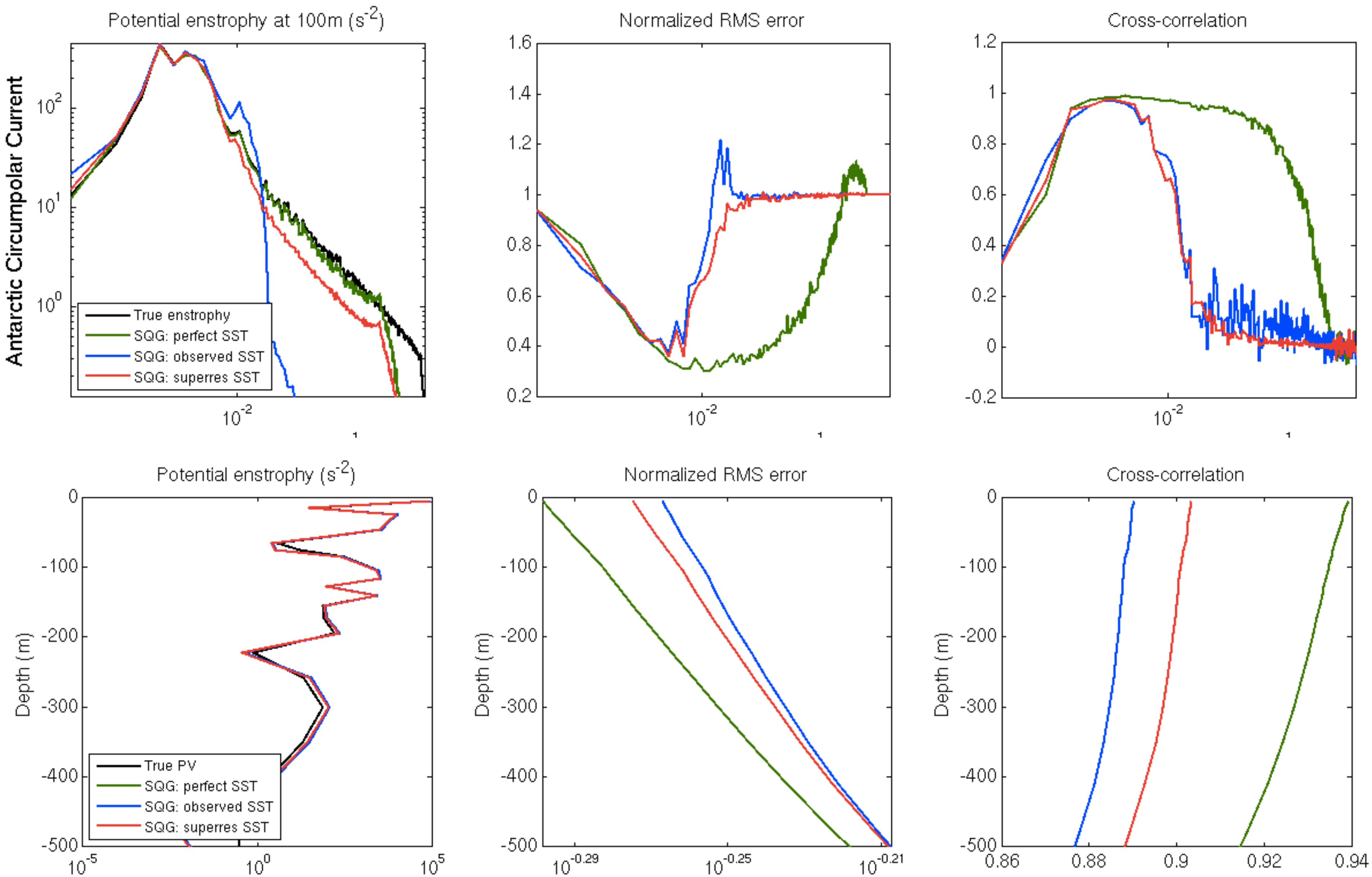
Sensitivity to **clouds and observing period**:





- **Accuracy of small-scale statistics** calculated using high-resolution images depends on quality of data
- **Model effect of imperfect data** by randomly discarding frames ("clouds") or shortening observing period

Upper ocean flow reconstruction



In general, observation will sample over a footprint given by sampling weight *G(x,y)*

$$\theta^{\mathrm{obs}}(x,y)=\int G(x',y')\,\theta(x-x',y-y')\,dx'dy'.$$

Coarse-grid Fourier transform is weighted by *spectral transfer function*

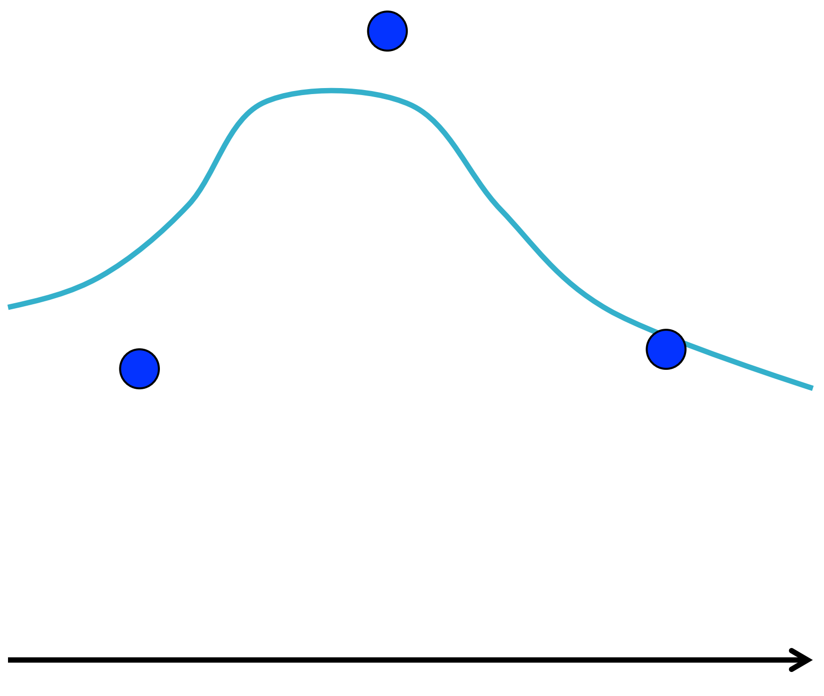
$$\tilde{\theta}^{\mathrm{obs}}(k,l)=\sum_{i,j=-\infty}^{\infty}\hat{G}(k+iM,l+jM)\,\hat{\theta}(k+iM,l+jM),$$

For a Gaussian sampling footprint of width ℓ, transfer function is a Gaussian of width $1/\ell$

$$G(x,y)=\frac{1}{2\pi\ell^2}e^{-(x^2+y^2)/2\ell^2},\ \hat{G}(p,q)=e^{-2\pi^2(p^2+q^2)\ell^2/L^2}.$$

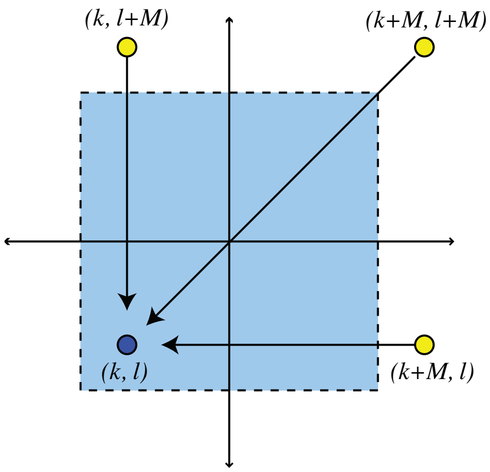
Filtering sparse observations

Data assimilation or filtering seeks the *best-guess estimate* of the *state of the system* by combining *noisy, incomplete observations* with an *internal forecast model.*



M x M observations of each resolved mode + aliased modes

$$\psi^{obs}_{k,l} = \sum_{\tilde{k},\tilde{l}} \psi^{true}_{\tilde{k},\tilde{l}} + \sigma^{obs}_{k,l}$$



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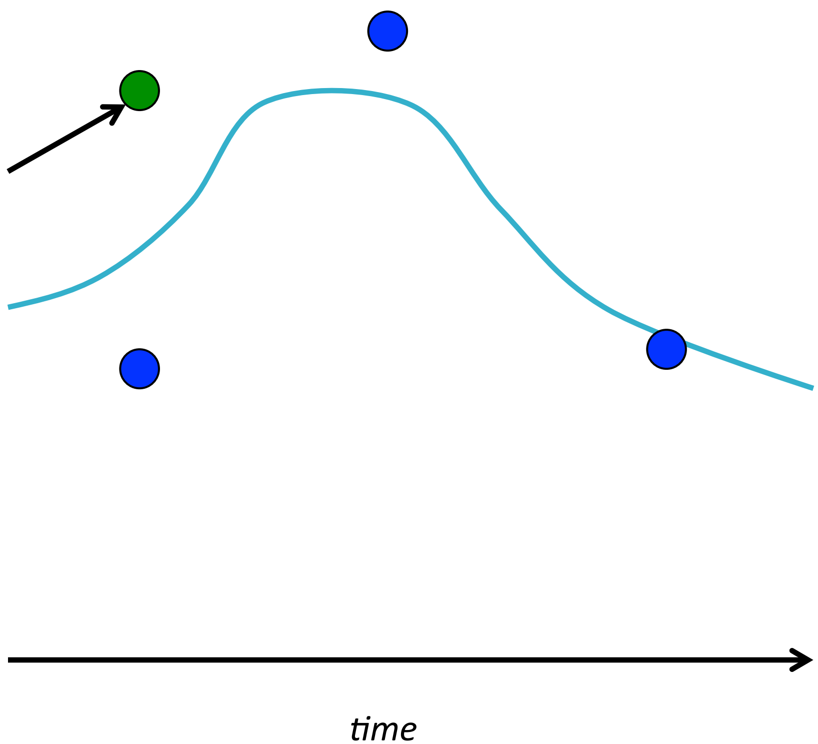

1. Forecast step:

Make prediction for *N x N* modes using quasi-linear stochastic model.

$$\partial_t \hat{\theta} = -\left(\gamma - \mathrm{i}\omega\right)\hat{\theta}(t) + \sigma \dot{W}(t)$$

Forecast mean and covariance:

$$\langle \theta \rangle, \quad R_{pq} = \langle \theta_p^* \theta_q \rangle$$

Tune parameters to give correct energy and timescales estimated from **infrared observations**.

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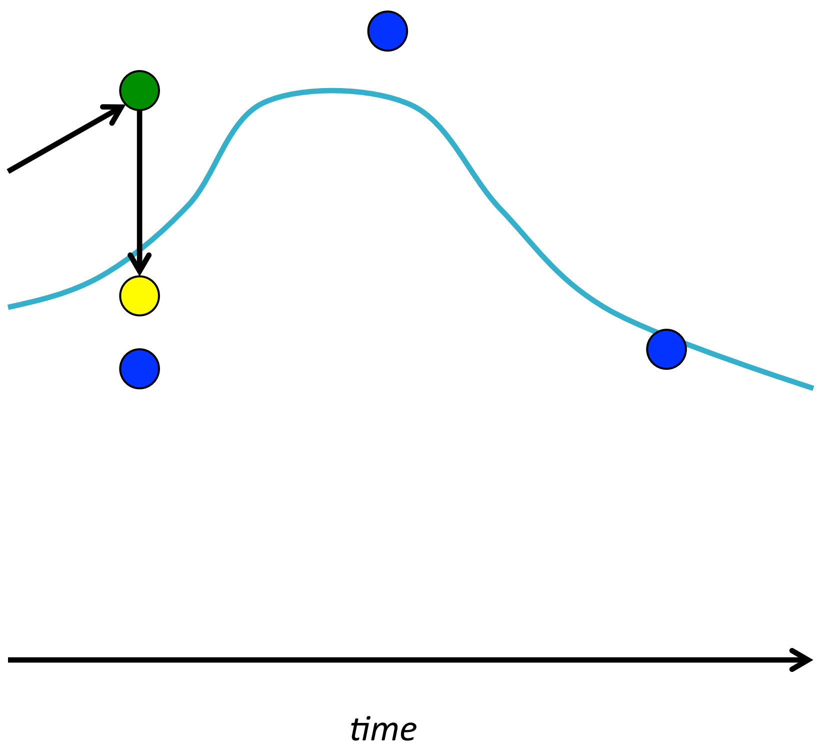

2. Update step:

Combine *N x N* prediction (-) with *M x M* observation (~) using **Kalman filter** solution:

$$\langle \theta_+ \rangle = (1 - KG)\langle \theta_- \rangle + K\tilde{\theta}$$

$$R_+ = (1 - KG) R_-$$

Optimal solution when dynamics and observation operator are linear with unbiased uncorrelated Gaussian noise.

Filtering sparse observations

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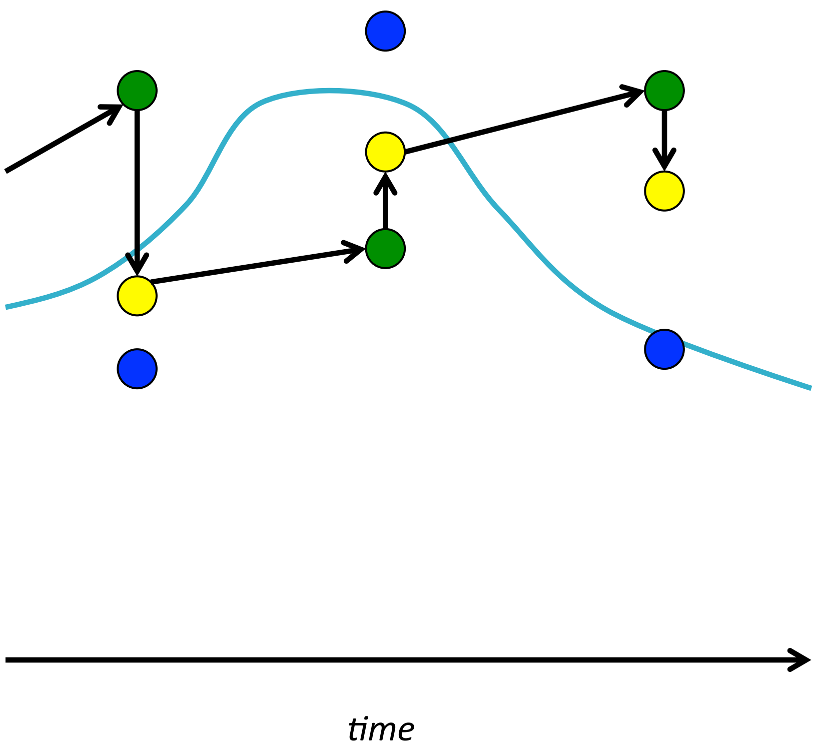

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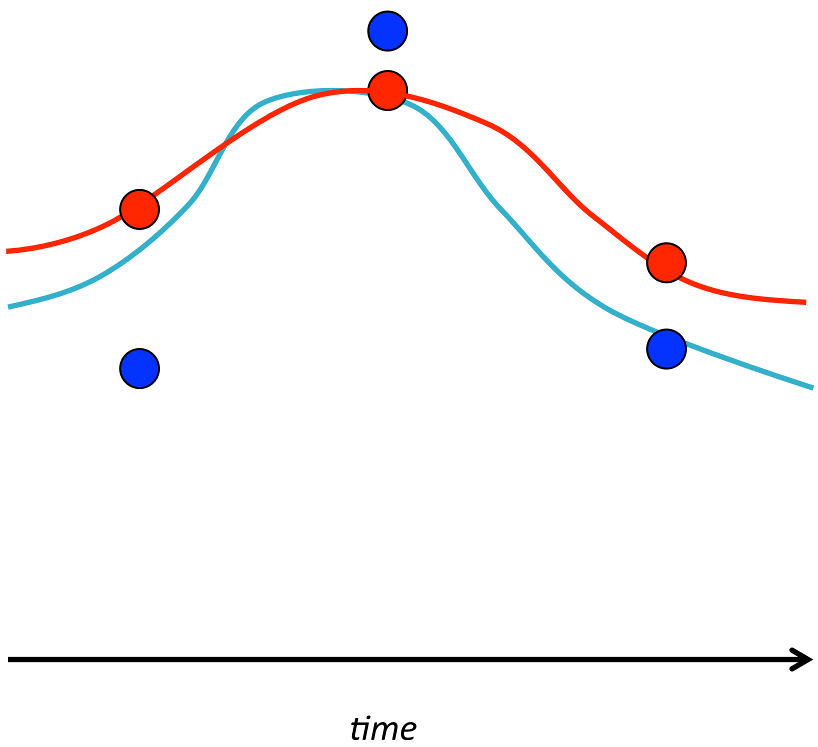
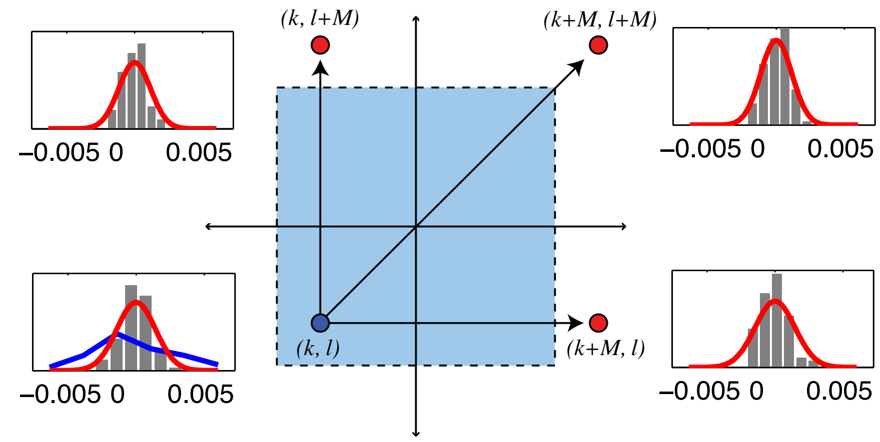


3. Smoothing step:

Apply **Rauch-Tung-Straub smoother** to remove unphysical jumps.



Resulting **superresolved SST estimate** is a pdf with an **effective resolution** given by model, not observations.