Multiplexing and transient estimates in lung EIT instruments

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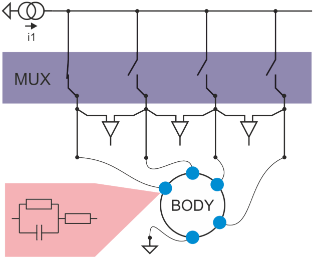
Multiplexing in EIT instruments I

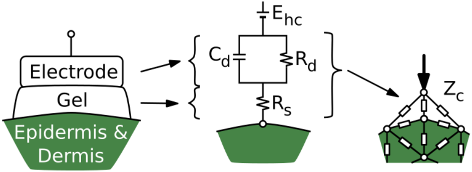
- Multiplexing is inherent to EIT. During current change, voltage amplitudes change and a transient can appear.



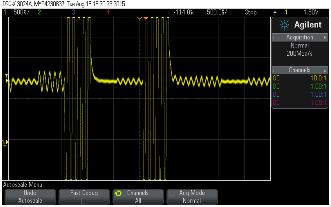
- Traditional approach is to settle to steady state. To achieve high frame rates, settling time after multiplexing is significant.
- Transient may not only take up significant amount of signal power, but also contain useful information about electrodes.

Contact impedance modelling I

- Model electrode as lumped capacitance (dead skin layer) C, in parallel with lumped resistance (ionic gel) R i.e. a parallel RC circuit [1, 2].



- 64 electrode system, $f > 10$ kHz injection and 30 fps, implies < 0.5 ms measurement time (5 full cycles per measurement).
- Typically $\tau \approx 0.1$-0.2 ms, and significant amount of useful energy is lost.



- In practice, contact may have several characteristic time constants, inductive effects, or imperfect capacitors ($Z(w) = \frac{1}{iC(\omega)^{\beta}}, 0 < \beta < 1$), which will not have single exponential behaviours.
- Explore fitting exponential and polynomial transient models

$$U_{\mathbf{A}}^{\mathrm{EXP}}(t) = A_1 \cos(\omega t) + A_2 \exp(-\frac{t}{A_3}) \tag{1}$$

$$U_{\mathbf{A}}^{\mathrm{POLY}}(t) = A_1 \cos(\omega t) + \sum_{i=2}^{N} A_i t^i. \tag{2}$$

Simulating transients with Fourier series

- Assuming linear constitutive relations $D = \epsilon E$, $J = \sigma E$, and negligible magnetic induction. Maxwell's equations:

$$\nabla \cdot (\epsilon \frac{\partial E}{\partial t} + \sigma E) = 0, \qquad E = \nabla u \quad \text{for some} \quad u.$$

  Let $u(x,t) = \sum_\omega u(x,\omega) e^{\mathrm{i}\omega t}$,

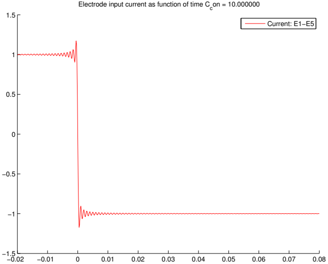
$$\nabla \cdot ((\sigma + \mathrm{i}\omega\epsilon)\nabla u(x,\omega)) = 0, \quad \text{for each} \quad \omega \tag{3}$$

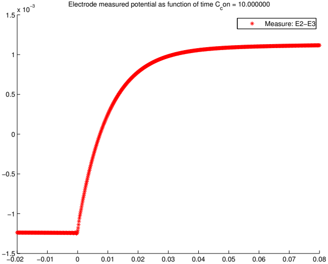
- Generalise CEM with frequency dependent impedance e.g. $\frac{1}{z(w)} = \frac{1}{R} + i(\omega)^\beta C$, $\beta \geq 0$.
- Given current $I(t) : [-T, T] \to \mathbb{R}$, decompose using Fourier series into modes $\tilde{I}(w)$, solve (static) (3) with $\tilde{I}(\omega)$, and sum resulting $u(x,\omega)$ weighted by $e^{\mathrm{i}\omega t}$

Square wave current simulation

- Conductive ($\sigma = 1$)/permittive ($\epsilon = 0.01$) medium and current

$$I(t) = \begin{cases} 1 & t \leq 0 \\ -1 & t > 0. \end{cases}$$



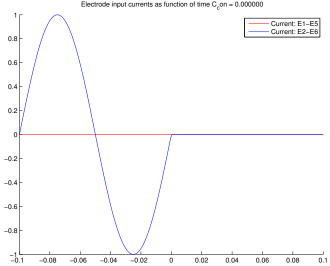


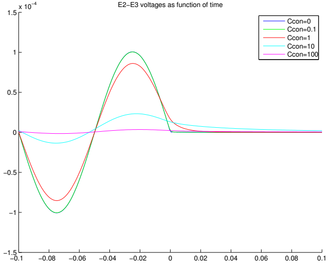
- Current discontinuity at $t = 0$ generates exponential decay in typical measured voltage, due to permittivity.

- Conductive ($\sigma = 1$)/permittive ($\epsilon = 0.01$) medium and current

$$I(t) = \begin{cases} \sin(10\pi t/T) & t \leq 0 \\ 0 & t > 0. \end{cases}$$

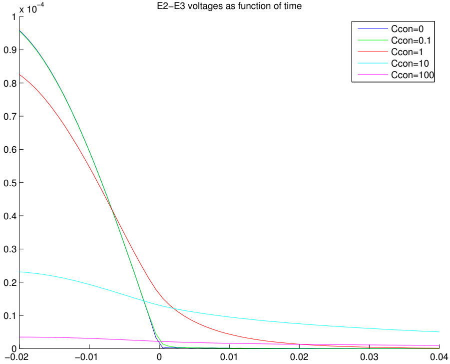
- Left/Right represents input current/measurement at $t = 0$. Note decay of measurement at $t = 0$.





Sinusoidal current simulation II

- Smooth current transition at $t = 0$, hence decay of measured voltages is due to non-zero electrode capacitance.



- Smaller capacitances have higher amplitude and faster decay, since $\tau \approx CR$ (approximate since effective τ will depend on bulk medium permittivity/conductivity.)

Signal Processing and data reduction

- Denote $U(t)$ be raw voltage data, sampled uniformly (2 MHz), and passed through 16-bit ADC.
- $\approx$ 3200 bits per cycle(0.1ms). 64 electrodes implies $\approx$ 2 Gbits/s data rate.
- Instead, typically compress data at FPGA e.g. at i^{th}-cycle

$$I_i = \sum_{n=iT}^{(i+1)T} V(t_n)\sin(\omega t_n), \qquad Q_i = \sum_{n=iT}^{(i+1)T} V(t_n)\cos(\omega t_n),$$

$$D_i = \sum_{n=iT}^{(i+1)T} V(t_n), \qquad SS_i = \sum_{n=iT}^{(i+1)T} V(t_n)^2.$$

where $T = \frac{2\pi}{\omega}$. Assume saturation flag for each cycle.

- $\approx$ 22 bytes $=$ 176 bits per cycle (0.1ms). 64 electrodes implies $\approx$ 0.1 Gbits/s data rate (50 times data reduction.)

Amplitude estimation - Full data course I

- **Classical approach and FFT:** Reduce gain on current driven electrode just after measurement, and wait for a number of cycles until transient has effectively disappeared.
- **Exponential/Polynomial model:** Denote $\mathrm{M} = \mathrm{EXP}$ or POLY from in (2). Objective is to determine parameter $\{A_i\}_{i=1}^{N}$ and formulate as (non-linear) optimisation problem [3]

$$\mathbf{A} = \arg\min_{\mathbf{A}'} \sum_{i=1}^{N} (U(t_i) - U_{\mathbf{A}}^{\mathrm{M}}(t_i))^2$$

- MATLAB function `lsqnonlin.m` is used to solve this (and subsequent) optimisation problems.

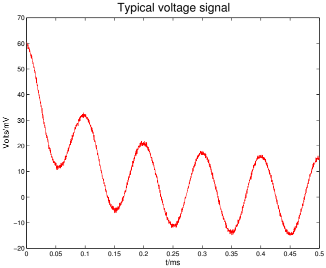
Simulated transient signal

- Without loss of generality consider phase-corrected signal

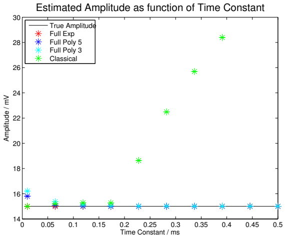
$$V(t) = A\cos(\omega t) + f(t) + n(t),$$

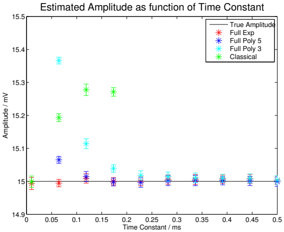
where $n(t)$ is white Gaussian noise and $f(t)$ is transient (assumed here to be $B\exp(-t/C)$).

- **Simulation parameters**: Frequency $f = 10$ kHZ, 5 full cycles (0.5ms measurement time), 2 MHz sampling rate, with 16-bit ADC. Initial/final amplitude 60/15 mV and noise variance $\sigma^2 = 0.5$ mV.



- Sinusoidal amplitude estimation as a function of $\tau \in [0, 0.5]$ over 20 noise realisations.
- Red/Blue/Magenta/Green - Exponential/3^{rd} order polynomial/ 5^{th} order polynomial models/FFT respectively.

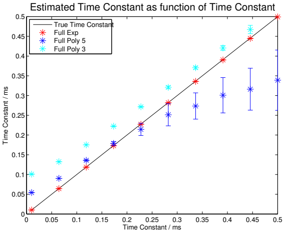




- FFT gives poor estimates for $\tau > 0.05$. Exponential model estimates very well, and polynomial model estimates well for $\tau > 0.1$.
- Amplitude estimates at $\tau = 0.2278$s:
  - Exponential - 15.001 ± 0.013mV. 5^{th} order polynomial - 14.997 ± 0.014mV. FFT - 18.637 ± 0.015mV.

Full data results - Time constant estimation

- Time constant estimation as a function of $\tau \in [0, 0.5]$.



- Time constants also estimated very well with exponential model. 3^{rd} and 5^{th} order polynomial estimates time constants better for $\tau > 0.3$ and $\tau < 0.3$ respectively.
- Time constant estimates at $\tau = 0.2278$s:
  - Exponential - 0.2272 ± 0.0002ms. 5^{th} order - 0.2144 ± 0.0156ms.

Data reduction optimisation

- What information can be obtained given I, Q, S, D values (and saturation flags) from FPGA?
- Assume transient is purely exponential, integrating exponential model (2) analytically over i^{th} cycle gives

$$D(\mathbf{A})_i = -\frac{A_2}{A_3}(\exp(-\frac{T(i+1)}{A_3}) - \exp(-\frac{Ti}{A_3}))$$

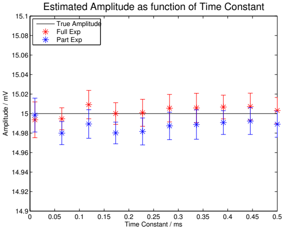
$$I(\mathbf{A})_i = A_2 A_3 \exp(-\frac{iT}{C})(1 - \exp(-\frac{T}{A_3}))/((1/A_3)^2 + \omega^2) + A_1 T/2$$

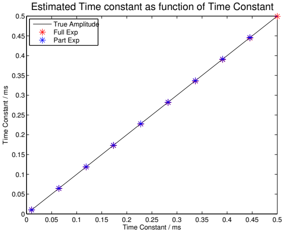
- Equivalent integrals computable for n^{th} order polynomial model. However, n extra integrals $Vt_i^n := \int_{\text{cycle}_i} V(t)t^n\,dt$ must be computed at FPGA, which will require, ≈ 48 bits per cycle per moment (≈ 30 Mbits/s).
- Let $M(\mathbf{A})_i = [D(\mathbf{A})_i, I(\mathbf{A})_i]$, and $M_i = [D_i, I_i]$ be measured data and s first cycle without saturation, then

$$\mathbf{A} = \arg\min_{\mathbf{A}'} \sum_{i=s} (M(\mathbf{A}')_i - M_i))^2$$

Partial data results - Amplitude estimation

- Left/Right represent amplitude/time constant estimation for exponential model over 20 noise realisations.
- Red/Blue represent Full/Partial data respectively.





- For exponential model partial data results are only marginally worse than full data results.
- Amplitude estimate for $\tau = 0.2278$s:
  - Full - 15.001 ± 0.013mV. Partial - 14.982 ± 0.018mV.
- Time constant estimate for $\tau = 0.2278$s:
  - Full - 0.2272 ± 0.0002ms. Partial - 0.2277 ± 0.0002ms.

Conclusions and Future Work

Conclusions

- Transient behaviour is significant when attempting high frame rates at kHZ frequencies.
- Given full time samples of voltage data, one can extract both sinusoidal amplitude and information of exponential transient, both with an exponential and polynomial model.
- Typically small set of data (e.g. I, Q, S, D) are computed per clock cycle, and sinusoidal amplitude can be extracted almost as well with this ≈ 50 times smaller data set.

Further Work

- Improved models of electrode-skin interfaces e.g. Jossinet *et al.* [2] to improve model fits.
- Algorithm presented here is inherently offline. Can fitting be performed on FPGA to extract sinusoidal component?

References I

- Boyle A J S 2010, The Effect of Boundary Shape Deformation on Two-Dimensional Electrical Impedance Tomography *MSc Thesis*
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