Spectrum Allocation by Sealed Bid Game Theory

Aritra De, Tirthankar Datta

Abstract*: Wireless communication subscribers are increasing day by day specially in fifth generation (5G) wireless communication where multiple number of users (Multiple Input Multiple Output or MIMO) can be served in a specific time. The heavy data usage is also enhanced with the increasing the number of subscribers, this data transfer speed depends on the amount of spectrum allocation to the specific subscriber. Thus, spectrum allocation is a major criterion for wireless communication performance improvement. The spectrum allocation efficiency can be observed by Game Theory, which is a popular decision maker of modern era. Sealed Bid Game theory is one of the popular segment of the game theory. The spectrum allocation can be done by using Sealed Bid Game theory and spectrum equilibrium can be observed by using different sub division of Sealed Bid Game theory.

***Keywords:*5G, MIMO, Game Theory, Sealed Bid Game Theory, Spectrum Allocation.**

I. INTRODUCTION

Wireless communication spectrum allocation is major criteria for the performance improvement of the system [1]. Multiple number of subscribers can be served by modern mobile communication generation [2].The multiple number of subscribers data speed and voce quality can be improved by using efficient spectrum allocation technique [3].The spectrum can be done by simulation method or experimental method [4].

The spectrum also can be done by using different optimization technique [5].The optimization technique is difficult to understand and mathematical calculation is time consuming [6].

Game Theory is less time consuming and it is used heavily in the modern era[7].Game theory does not assume any knowledge of its players[8].The only way to appreciate game theory is to see it in action, or better still to put in into action[9].

The user of the mobile subscriber can be static or dynamic [10].

In this work is done by considering spectrum allocation to two static user as shown in the figure (1) or with a macro cell in fifth generation wireless communication as shown in the figure (2) or spectrum allocation to a moving user[11].

Revised Manuscript Received on December 08, 2019

Aritra De.* is with MAKAUT INDIA. He is now with the Department of Electronics & Communication Engineering, as a Research Scholar

Tirthankar Datta is with the Department of Electronics & Communication Engineering, Netaji Subhash Engineering College INDIA, as a Professor and Principal

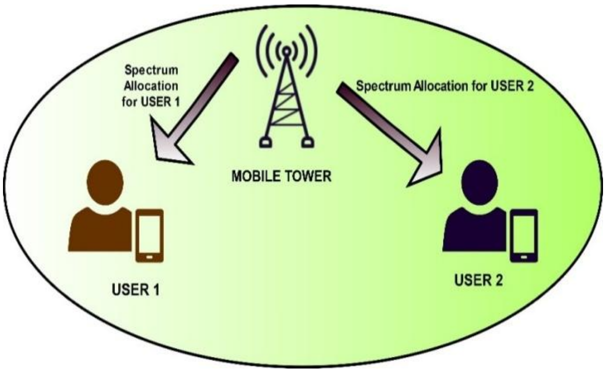

Figure 1: Spectrum Allocation for the Mobile User

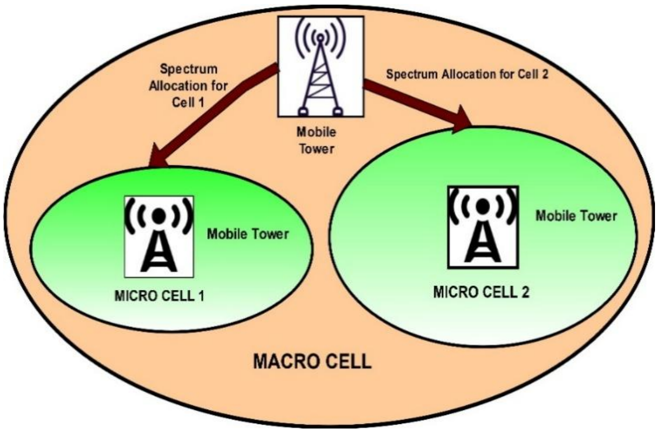

Figure 2: Spectrum Allocation for the Micro Cell

In this work wireless generation is compared in the section II, Bayesian second price auction game theory with average value calculation is discussed in the section III and IV respectively, sealed bid first price auction is discussed with average value calculation is discussed in the section V and VI respectively, two player all pay auction is discuss with average value calculation is discussed in the section VII and VIII respectively. Conclusion of the work is discussed in the section IX.

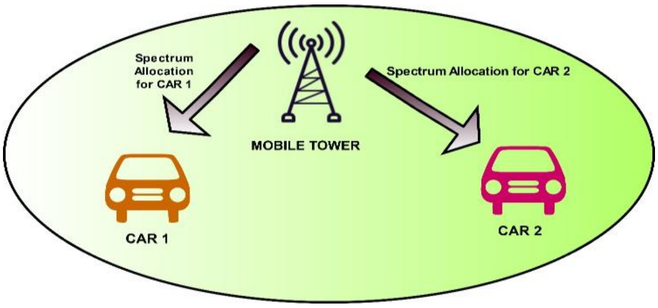

Figure 3: Spectrum Allocation for the Moving User

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

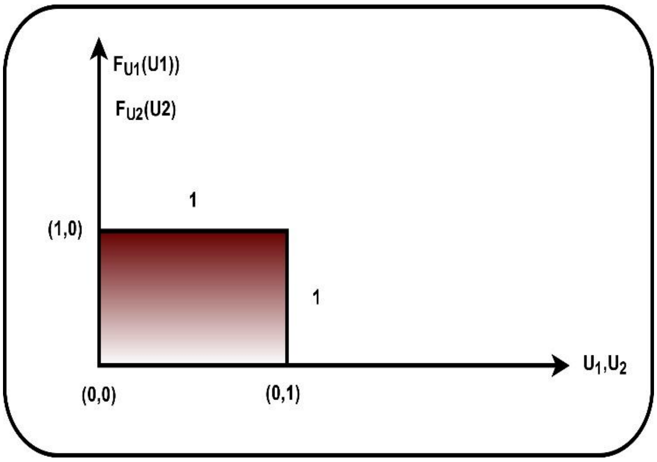

Figure 4: Probability Density Function of Spectrum allowed (sealed bid first price auction)

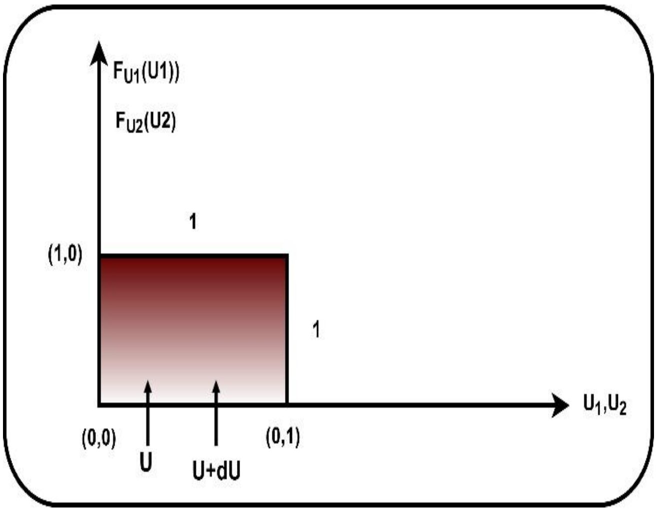

Figure 5: Probability Density Function of Spectrum allowed (extended revenue of the first price auction)

II. WIRELESS GENERATION COMPARISON

Table 1: Mobile generation comparison

Generation / Feature	1G	2G	3G	4G	5G
Deployment	1980	1990	2001	2010	2020 and beyond
Frequency Band	800 MHz	900 MHz	2100 MHz	2600 MHz	3-90GHz
Speed	2 kbps	64 kbps	2Mbps	1Gbps	Higher than 1Gbps

III. BAYESIAN SECOND PRICE AUCTION

The spectrum is allocated to two user Q_1 and Q_2 .The spectrum is allocated to the user Q_1 is C_1 and maximum spectrum is allowed is U_1,the spectrum is allocated to the user Q_2 is C_2 and maximum spectrum is allowed is U_2.

If $C_1 \geq C_2$ then Q_1 will win else if C_2>C_1 then Q_2 wins so it mean player with highest bid wins the auction.

If $C_1 \geq C_2$ then Q_1 will win and pays the second highest bid C_2.

ifC_2>C_1 then Q_2 wins and pays the second highest bid C_1.

Now each and every player has a private valuation, consider the valuation of Q_1,Q_2 is U_1,U_2 respectively.

U_1,U_2 are independent, distributed and random variable as shown in figure (4).

The Nash Equilibrium of second price auction is

C_1=U_1……………………………………….(1A)

C_2=U_2……………………………………….(1B)

Now start with the assumption that

C_2=U_2……………………………………….(1C)

A. Case 1

Consider $U_1 \geq U_2$, the bidding of player 2 is $C_2 = U_2$ also if C≥ U_2,Q_1 wins the auction and pays the second highest bid $C_2 = U_2$.Net payoff=$U_1 - U_2$ ≥0.

If he bids, C≤ $C_2 = U_2$ then player Q_1 loses the auction and his net payoff is 0.

Therefore, any bid C≥ U_2 is a best response that mean C=U_1 is a best response.

B. Case 2

If $U_1 \leq U_2$ player Q_2 is bidding $C_2 = U_2$ .If Q_1 bids C≥ $C_2 = U_2$,then he wins the auction and pays second highest bid $C_2 = U_2$ .Net payoff=$U_1 - U_2$ ≤0.

If he bids C<$C_2 = U_2$ then he loses the auction and his payoff is 0.Therefore any bid C<$C_2 = U_2$ is a best response. In particular C=U_1 is a best response.

If player Q_2 is bidding $C_2 = U_2$,then $C_1 = U_1$ is a best response for player Q_1.

Similarly it can be shown that if Q_1 is bidding C_1=U_1 then C=U_2 is a best response forQ_2.

Hence the Nash equilibrium of second price auction is

C_1=U_1……………………………………….(1D)

C_2=U_2……………………………………….(1E)

So, each player bidding his true valuation is the Nash equilibrium for the second price auction.

IV. EXPECTED REVENUE OF SECOND PRICE AUCTION

C_1=U_1……………………………………….(2A)

C_2=U_2……………………………………….(2B)

Noted that $U_1 \geq U_2$then $C_1 \leq C_2$,soQ_1 wins the auction and pays second highest bid C_2=U_2,if $U_1 \geq U_2$,revenue=U_2.

If $U_1 < U_2$then $C_1 > C_2$, so Q_2 wins the auction and pays second highest bid U_1,therefore revenue=U_1.

So, it can be concluded that the revenue to the auctioneer in the Bayesian second price auction is minimum $\{U_1, U_2\}$.

$U_1, U_2$are independent and Probability Density Function (PDF) $F_{U_1}(U_1)$and $F_{U_2}(U_2)$of Maximum spectrum allowed is distributed uniformly in the interval [0, 1] as shown in the figure 5.

A. Case 1

$U_1 \leq U_2$ ………………………………….(2C)

$U_1$is lies in [U,U+dU]

$U_2$is lies in [U+dU,1]

P_r=$P_r(U_1 \in [U, U + dU])\times$ $P_r(U_2 \in [U + dU, 1])$

=dU×(1-U-dU)

=dU(1-U)

Retrieval Number: B3439129219/2019©BEIESP
DOI: 10.35940/ijeat.B3439.129219

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

=(1-U)dU……………………………….(2D)

***B.* Case 2**

$U_1 > U_2$ ………………………………….(2E)

$U_1$is lies in [U+dU,1]

$U_2$is lies in [U,U+dU]

P_r=$P_r(U_2 \in [U, U+dU])\times$ $P_r(U_1 \in [U+dU,1])$

=dU×(1-U-dU)

=dU(1-U)………………………………….(2F)

=(1-U)dU……………………………….(2G)

The total probability minimum $\{U_1, U_2\}$ lies in between the interval [U,U+dU} is 2(1-U).

Revenue to the auctioneer=minimum $\{U_1, U_2\}$

Since minimum lies in [U,U+dU],revenue=U. Expected revenue=P_r ×U

=2(1-U)dU……………………………….(2H)

Expected revenue

=$\int_0^1 2(1-U)udU$…………………………..(2I)

=1/3………………………………………..(2K)

Hence expected revenue =1/3 also the revenue is independent.

V. SEALED BID FIRST PRICE AUCTION

The spectrum is allocated to two user Q_1 and Q_2 .The spectrum is allocated to the user Q_1 is C_1 and maximum spectrum is allowed is U_1,the spectrum is allocated to the user Q_2 is C_2 and maximum spectrum is allowed is U_2.

The Probability Density Function (PDF) $F_{U_1}(U_1)$ and $F_{U_2}(U_2)$ of Maximum spectrum allowed is distributed uniformly in the interval [0, 1] as shown in the figure 4.

The Game Theory bidding strategy is as following

C_1=$\frac{1}{2}U_1$……………………………………….(1)

C_2=$\frac{1}{2}U_2$……………………………………….(2)

π(C) Denotes the payoff to the user Q_1as a function of C where C is maximum spectrum allocated to the mobile tower.

User Q_1 wins the auction game i.e. C≥ C_2,then the payoff is =Valuation of the user 1-Bid paid on winning the auction

=U_1-C……………………………………… (3)

Average payoff to player 1 is given by,

P_r(win)×(U_1-C)+P_r(loss)×0

So,

π(C)=P_r(win)×(U_1-C).......(4)

The winning condition for the player 1 is

C≥ C_2=$\frac{1}{2}U_2$………………………………..(5)

C≥ $\frac{1}{2}U_2$…………………………………….(6)

U_2 ≤2C…………………………………….(7)

Since U_2 is distributed uniformly in [0,1],U_2 must have in [0,2C].

Probability U_2 lies in [0,2C]

=$\int_0^{2C} F_{U_2}(U_2)\, dU_2$……………………………..(8)

=$\int_0^{2C} dU_2$……………………………………..(9)

=$U_2|_0^{2C}$ ………………………………………..(10)

=2C…………………………………………..(11)

Hence,P_r(win) for player 1 is 2C,therefore

π(C)=P_r(win)×(U_1-C)………………………..(12)

=2C×(U_1-C)………………………………..(13)

π(C)=2CU_1-2C^2…………………………….(14)

$\frac{d\pi(C)}{dC}$=2U_1-4C=0…………………………..(15)

C^*=$\frac{1}{2}U_1$……………………………………(16)

So, If C_2=$\frac{1}{2}U_2$, then the bid C_1=$\frac{1}{2}U_1$ is the best response for the user 1.

By following the same procedure it can be proved that if C_1=$\frac{1}{2}U_1$ then C_2=$\frac{1}{2}U_2$ is the best response for the user 2.

VI. EXPECTED REVENUE OF THE FIRST PRICE AUCTION

Nash equilibrium is given by

C_1=$\frac{1}{2}U_1$……………………………………….(17)

C_2=$\frac{1}{2}U_2$……………………………………….(18)

Now the player wins who called for maximum bid. Hence Revenue=maximum $\{C_1, C_2\}$………………….(19)

=maximum $\{\frac{1}{2}U_1$,$\frac{1}{2}U_2\}$……………………….(20)

=$\frac{1}{2}$maximum $\{U_1$,$U_2\}$………………………..(21)

U_1,U_2 are uniform distributed in [0, 1] and probability for equation (21) lies in the infinitesimal interval [U,U+dU] as shown in the figure (4).

A. Case 1

$U_1$is the maximum U_1lies in [U, U+dU] and U_2 lies in [0,U]

So,

P_r=$P_r(U_1 \in [U, U+dU])\times$ $P_r(U_2 \in [0,U])$…………(22)

=dU×U…………………………………………….(23)

=UdU……………………………………………….(24)

B. Case 2

$U_2$is the maximum U_2lies in [U, U+dU] and U_1 lies in [0,U]

So,

P_r=$P_r(U_1 \in [0,U])\times$ $P_r(U_2 \in [U, U+dU])$…………(22)

=U×dU…………………………………………….(23)

=UdU……………………………………………….(24)

C. Equations

Probability that maximum $\{U_1$,$U_2\}$ is lies in [U,U+dU]

=UdU+UdU…………………………………..(25)

=2UdU………………………………………..(26)

So, Average revenue corresponding to maximum $\{U_1$,$U_2\}$∈[U,U+dU]

=$\frac{1}{2}$U×2UdU……………………………………(27)

=U^2dU………………………………………….(28)

So, total average revenue to the auctioneer is

=$\int_0^1 U^2$dU……………………………………..(29)

=$\frac{1}{3}U^3|_0^1$…………………………………………..(30)

=$\frac{1}{3}$…………………………………………………(31)

The expected revenue of the auctioneer is =1/3.

VII. TWO PLAYER ALL PAY PRICE AUCTION

The spectrum is allocated to two user Q_1 and Q_2.The spectrum is allocated to the user Q_1 is C_1 and maximum spectrum is allowed is U_1,the spectrum is allocated to the user Q_2 is C_2 and maximum spectrum is allowed is U_2.

User with highest spectrum allocation wins the game. Both the player pay their bid irrespective of their outcome.

Now, assume that U_1 and U_2 denotes the valuations of $Q_1$and Q_2.

The Probability Density Function (PDF) $F_{U_1}(U_1)$ and $F_{U_2}(U_2)$ of Maximum spectrum allowed is distributed uniformly in the interval [0, 1] as shown in the figure 4.

The Nash Equilibrium is as following

C_1=½U_1^2……………………………………….(32)

C_2=½U_2^2……………………………………….(33)

Now, assume that player Q_2 is bidding C_2=½U_2^2, also assume player Q_1 bids C.

π(C) is expected payoff to player 1 as a function C.

π(C)=P_r(win)×(U_1-C)+P_r(loss)×(-C)………..(34)

Q_1will win if C≥ C_2=½U_2^2……………….................(35)

=½U_2^2 ≤C………………………………………….(36)

$U_2 \leq \sqrt{2C}$……………………………………………(37)

P_r(win)=P_r($U_2 \leq \sqrt{2C}$)……………………………(38)

P_r(win)=P_r($U_2 \epsilon [0,\sqrt{2C}$)…………………………..(39)

P_r(loss)=1-P_r(win)…………………………………..(40)

P_r(loss)=1-$\sqrt{2C}$………………………………………(41)

So, from equation (34) it can be derived that

π(C)=$\sqrt{2C}$(U_1-C)+(1-$\sqrt{2C}$)*(-C)……….......….(42)

π(C)=$\sqrt{2C}U_1$-C…………………………………..(43)

By differentiating equation (43) with respect to C

$\frac{\partial \pi(C)}{\partial C}$=$\sqrt{2C}U_1$*$\frac{1}{\sqrt{2C}}$ − 1=0……………………………(44)

C=½U_1^2………………………………………………(45)

Similarly, it can be shown that if C_1=½U_1^2, then C=½U_2^2 is a best response bid for user Q_2.

C_1=½U_1^2……………………………………….(46)

C_2=½U_2^2……………………………………….(47)

So, equation (46) and (47) are Nash Equilibrium.

VIII. EXPECTED REVENUE OF TWO PLAYER ALL PAY PRICE AUCTION

Revenue=C_1+C_2…………………………………(48)

=$\frac{1}{2}U_1^2$+$\frac{1}{2}U_2^2$……………………………………….(49)

Expected Revenue

=$\frac{1}{2}E\{U_1^2\}$+$\frac{1}{2}E\{U_2^2\}$…………………………………(50)

=$\frac{1}{2}\int_0^1 U_1^2 F_{U_1}(U_1)dU_1$+$\frac{1}{2}\int_0^1 U_2^2 F_{U_2}(U_2)dU_2$…………(51)

=$\frac{1}{2}\int_0^1 U_1^2 dU_1$+$\frac{1}{2}\int_0^1 U_2^2 dU_2$…………………………..(52)

=$\frac{1}{2}\frac{U_1^3}{3}|_0^1$+$\frac{1}{2}\frac{U_2^3}{3}|_0^1$……………………………………….(53)

=$\frac{1}{2}$*$\frac{1}{3}$+$\frac{1}{2}$*$\frac{1}{3}$…………………………………………….(54)

=$\frac{1}{3}$…………………………………………………(55)

Equation (55) is the revenue .

IX. CONCLUSION

This work is completed by using different bidding game theoretical approach. The Nash equilibrium is calculated in each sub part of sealed bid game theory, which is nothing but best approach of spectrum allocation.

ACKNOWLEDGMENT

This work is acknowledged to National Program Technical Enhanced Learning (NPTEL) an initiative by Government of INDIA (GOI).This work is dedicated to Prof (Dr.)Vimal Kumar and Prof(Dr.)Aditya K.Jagannatham of Indian Institute of Technology, Kanpur for their impressive lecture series in NPTEL.

REFERENCES

1. Zhi Chen ; Xinying Ma ; Bo Zhang ; Yaxin Zhang ; ZhongqianNiu ; NingyuanKuang ; Wenjie Chen ; Lingxiang Li ; Shaoqian Li, "A survey on terahertz communications,"China Communications 08 March 2019, pp. 1–35.
2. AritraDe ;TirthankarDatta"Improvement of Performance of MIMO System Using Different Protocols" Fifteenth International Conference on Wireless and Optical Communications Networks (WOCN), pp. 1–5.
3. AritraDe ;TirthankarDatta "Some Aspects of Massive MIMO Spectrum Sharing" Fifteenth International Conference on Wireless and Optical Communications Networks (WOCN), pp.34-39.
4. Lifeng. Wang, Hien. Quoc Ngo, Magnad. Elkashlan, Trung Q. Duong, Kai-Kit. Wong, "Massive MIMO in spectrum haring networks: Achievable rate and power efficiency", IEEE System Journal, vol. 11, no. 1, pp. 20-31, March. 2017.
5. Y Li, N Li, H Li, W Xie et al., "Spectrum Sharing Based on Overlay Cognitive Full-Duplex Two-Way OFDM Relaying", IEEE. T. Veh. Technol, vol. 67, pp. 2324-2334, 2017.
6. H Li, X Zhao, "Joint resource allocation for OFDM-based cognitive two-way multiple AF relays networks with imperfect spectrum sensing", IEEE. T. Veh. Technol, vol. 67, pp. 6286-6300, 2018.
7. Bond et al., "A game theory perspective on environmental assessment: What games are played and what does this tell us about decision making rationality and legitimacy?", Environmental Impact Assessment Review, vol. 57, pp. 187-194, Feb. 2016.
8. F.F. Folami, "Gender Inequality and Role-strained among Male Nursing Students in Selected Nursing Institution Lagos Nigeria", Journal of Education and Training Studies, vol. 5, pp. 214-219, Jun. 2017.
9. William Poundstone, Prisoner's Dilemma/John von Neumann Game Theory and the Puzzle of the Bomb, Anchor, 1993.
10. A. Ali, W. Hamouda, "Advances on spectrum sensing for cognitive radio networks: theory and applications", IEEE Commun. Surv. Tutorials, vol. 19, no. 2, pp. 1277-1304, 2017.
11. A. Nath, N. Sarma, "A distributed solution for cooperative spectrum sensing scheduling in multiband cognitive radio networks", J. Netw. Comput. Appl., vol. 94, pp. 69-77, 2017.

AUTHORS PROFILE

Aritra De received his B. Tech from Techno India, Salt Lake, Kolkata MTech from Institute of Engineering & Management, Salt Lake Kolkata. He has 9years of teaching experience. He is now pursuing his PhD from the MaulanaAbulKalam Azad University of Technology formerly known as the West Bengal University of Technology.

TirthankarDattareceived his PhD (Engg.) Degree in Microelectronics & Nano Technology under the Department of Electronics & Telecommunication Engineering of Jadavpur University. Prior to that, he received M. Tech. Degree in Radio Physics & Electronics from the Institute of Radio Physics and Electronics,University of Calcutta.He is associated with Netaji Subhash Engineering College(NSEC)as Professor of Electronics & Communication Engineering Department and Principal.