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Spacing and stability of compacts systems

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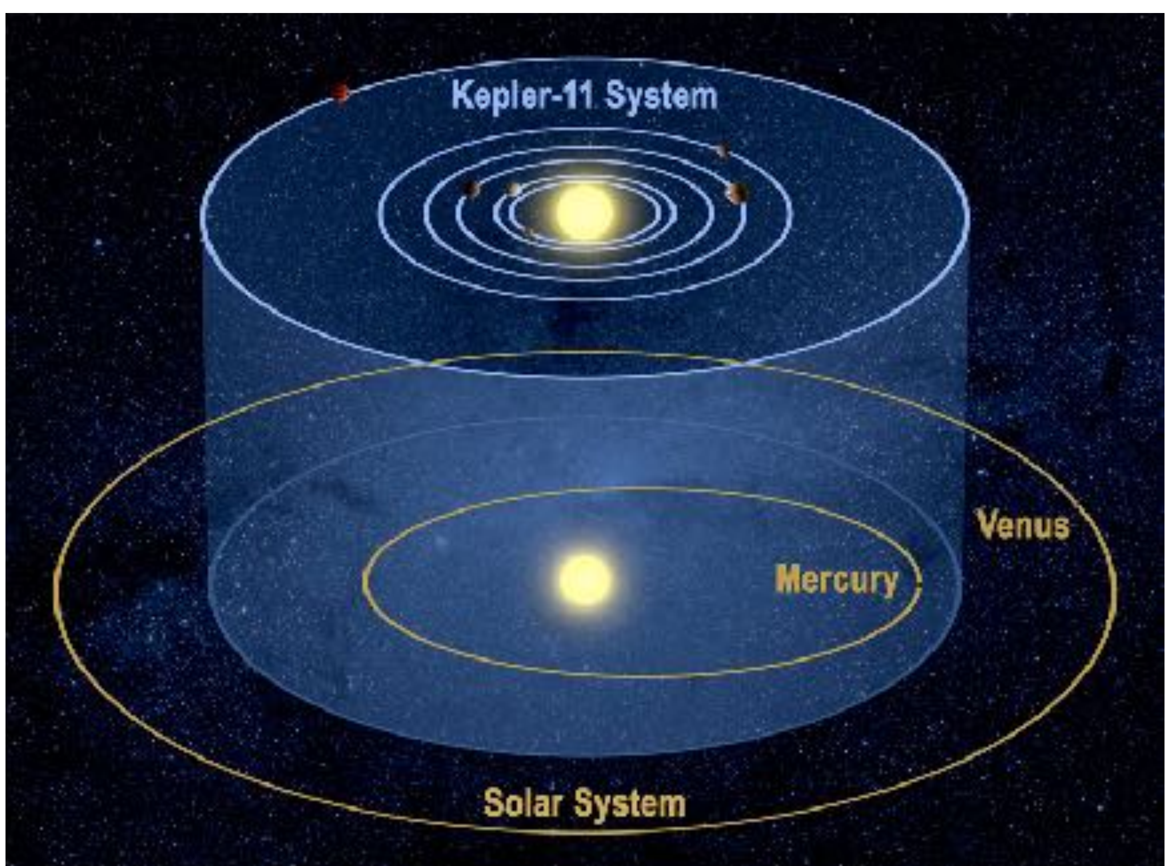
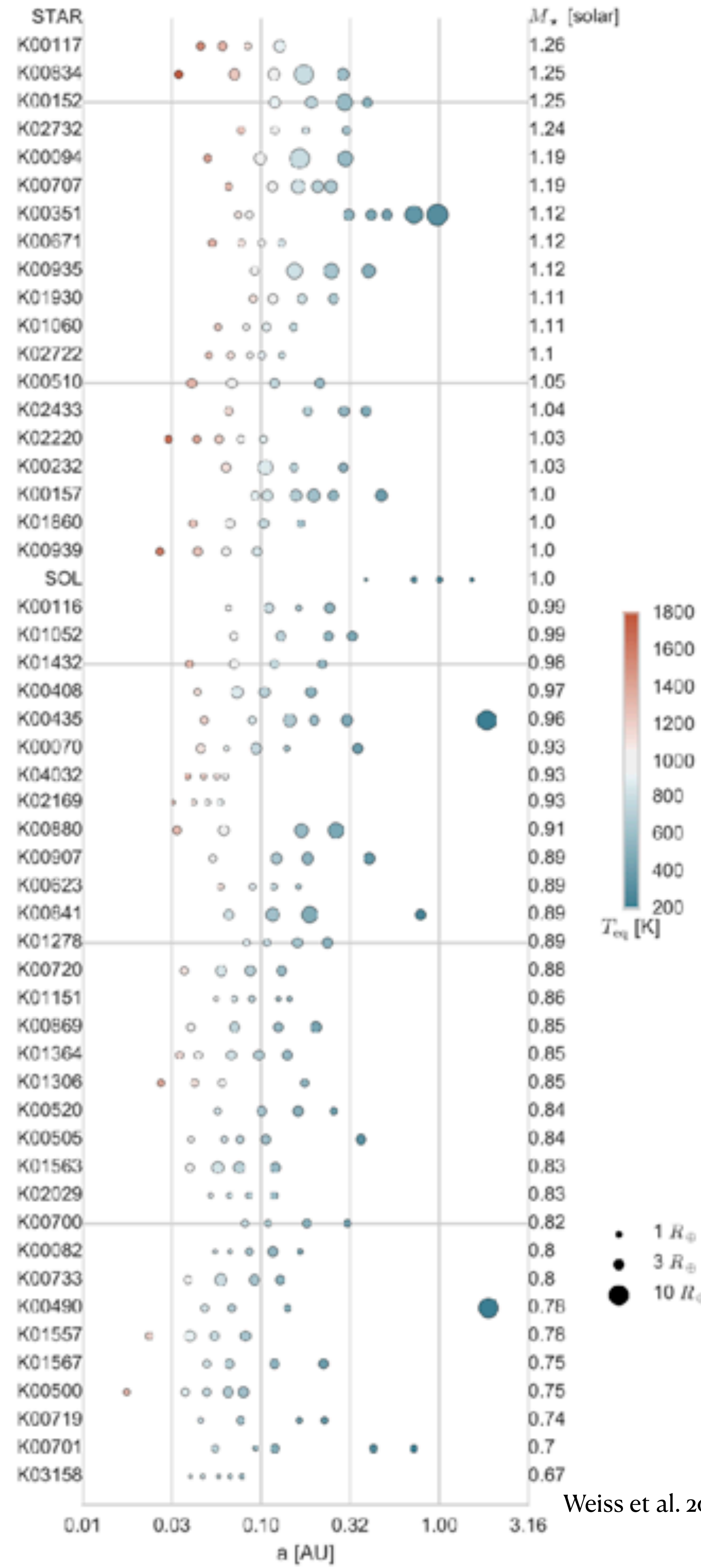
Compact systems?

- Majority multi-transit systems (3+ planets)
- Comparable masses and spacings; almost circular and coplanar orbits
- **Close to the stability limit**
- Excellent “laboratories” to test planet formation



NASA



Weiss et al. 2018

How close can you pack?

The Equal Mass and Spacing (EMS) systems

Simplest possible setting:

- 3 or more planets
- Coplanar and circular orbit
- Equal mass planets
- Spacing measured in units of Hill radius

$$R_H = \left(\frac{a_k + a_{k+1}}{2}\right)\left(\frac{m_k + m_{k+1}}{3m_S}\right)^{1/3}$$

Findings:

- Survival time scales exponentially with spacing measured in Hill radii

$$\log T = b\Delta + c$$

- Number of planets is not (that) important
- Effective stability (billion of stable orbits reached for ~10 Hill radii)

Chambers et al. 1996

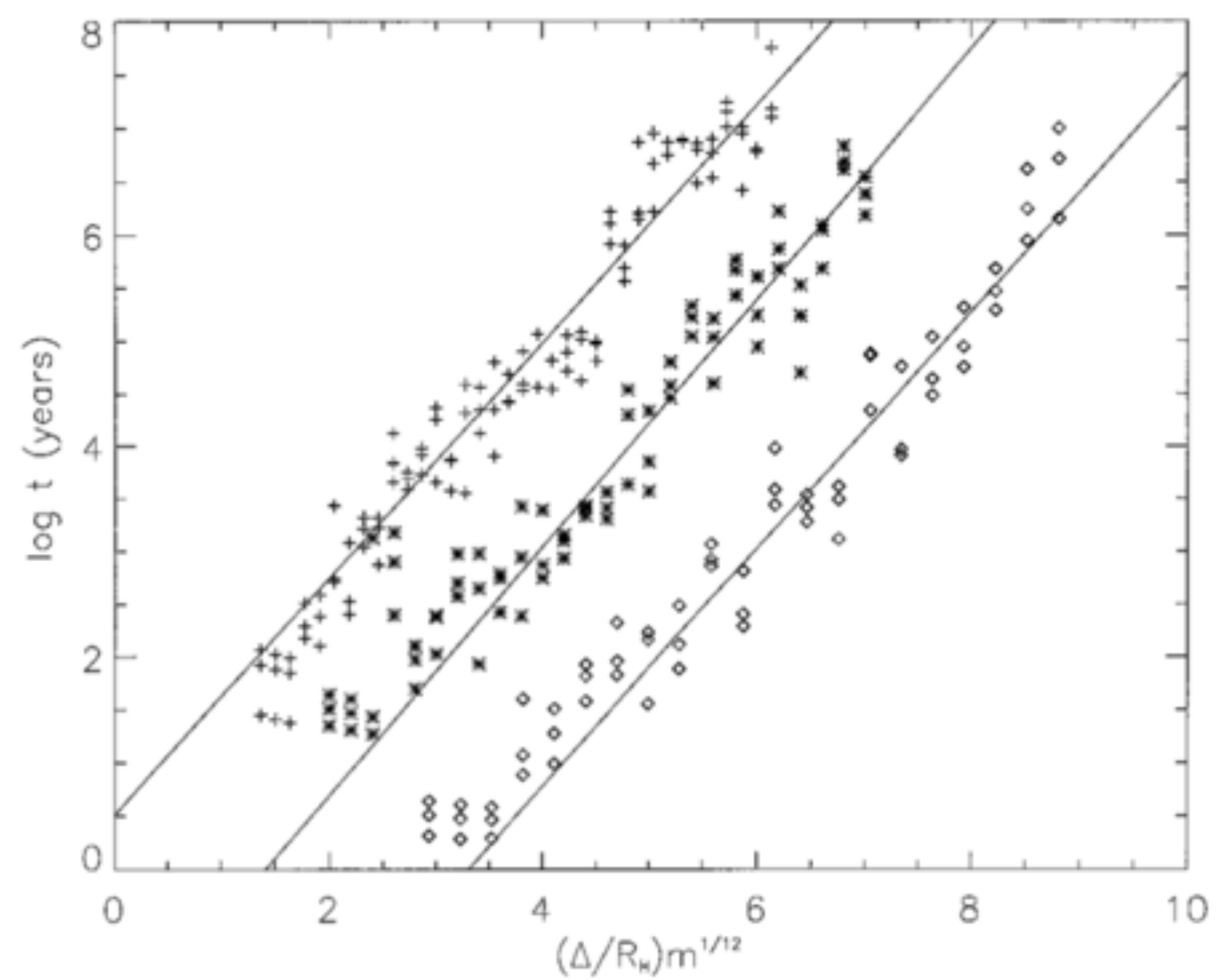


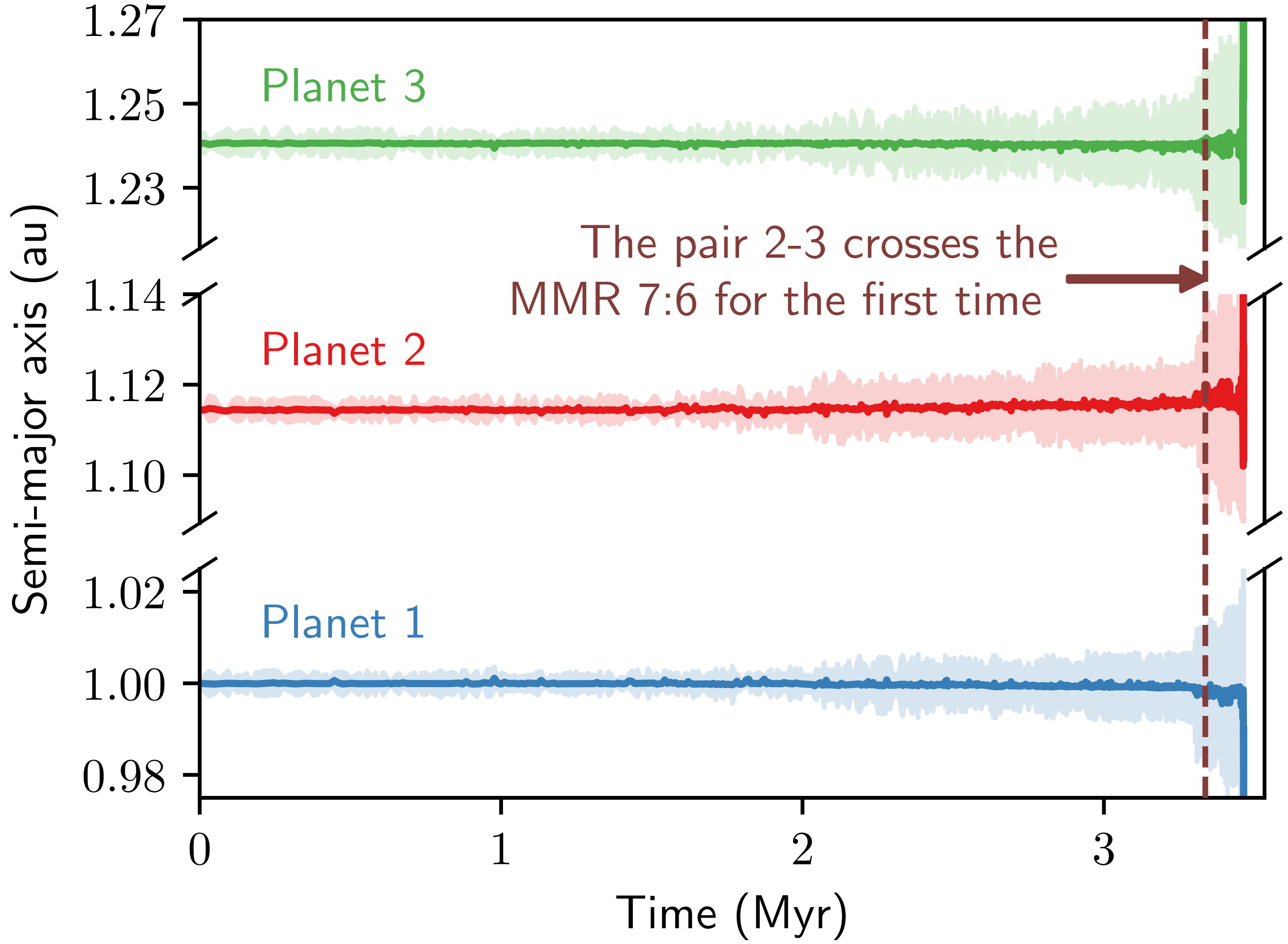
FIG. 4. The data of Fig. 3 replotted with Δ measured in units proportional to $m^{1/4}$. The crosses are data from runs with $m = 10^{-9}M_\odot$, the asterisks for $m = 10^{-7}M_\odot$, and the diamonds for $m = 10^{-5}M_\odot$. Note the similar slopes in each case.

Goal of this work

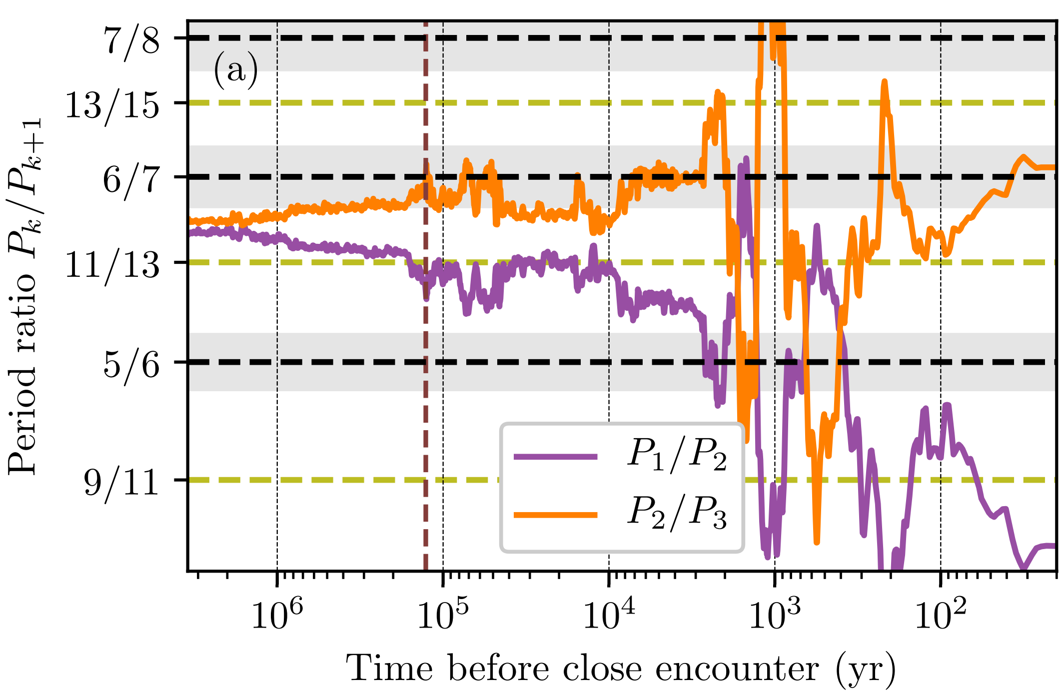
Provide an analytical model explaining quantitatively the compact system instability.

What happens?

Phenomenology of the instability



Problem: The system starts outside of the two-planet MMR, how does it moves?



- Planets disrupt each other but "quietly" (3+ planets are necessary)
- No evolution of the eccentricities for most of the time before the burst

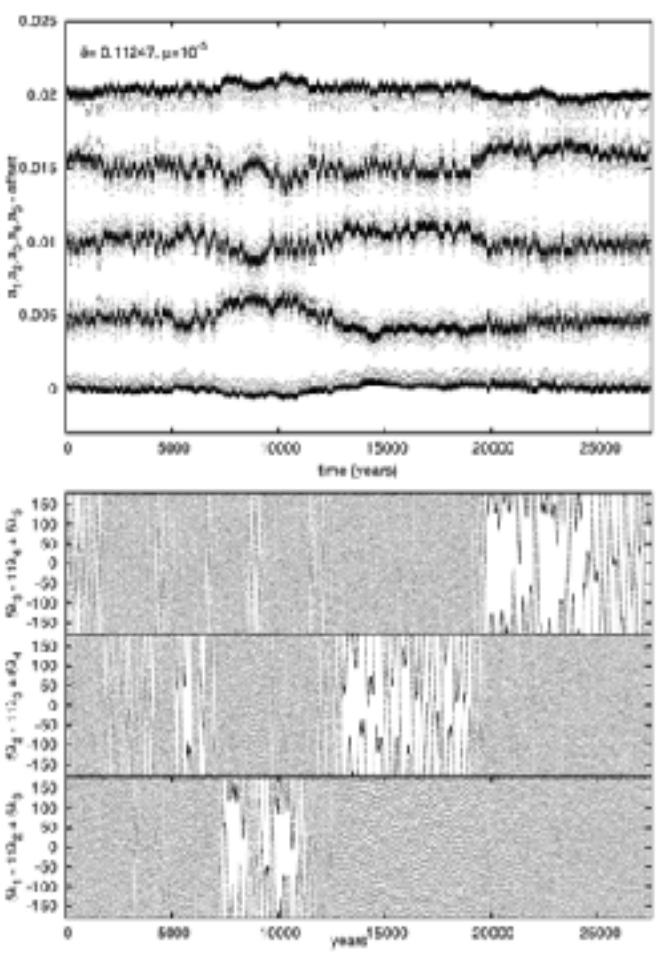
(Zeroth-order) Three body resonances

A.k.a Generalised Laplace resonances

$$\theta_{p,q} = p\lambda_1 - (p+q)\lambda_2 + q\lambda_3$$

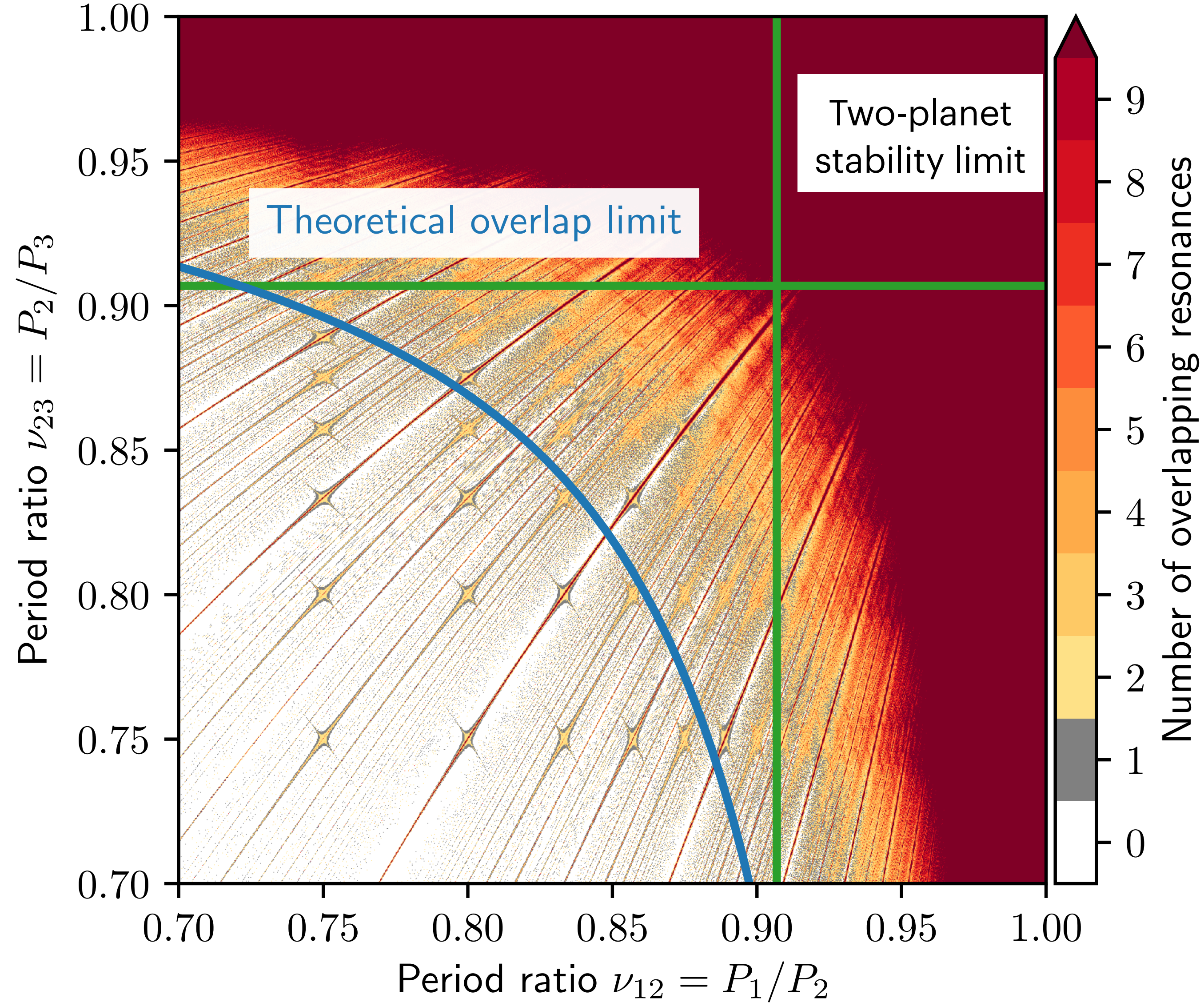
Zeroth-order since sum of coefficients $p - (p+q) + q = 0$

No dependency in eccentricities



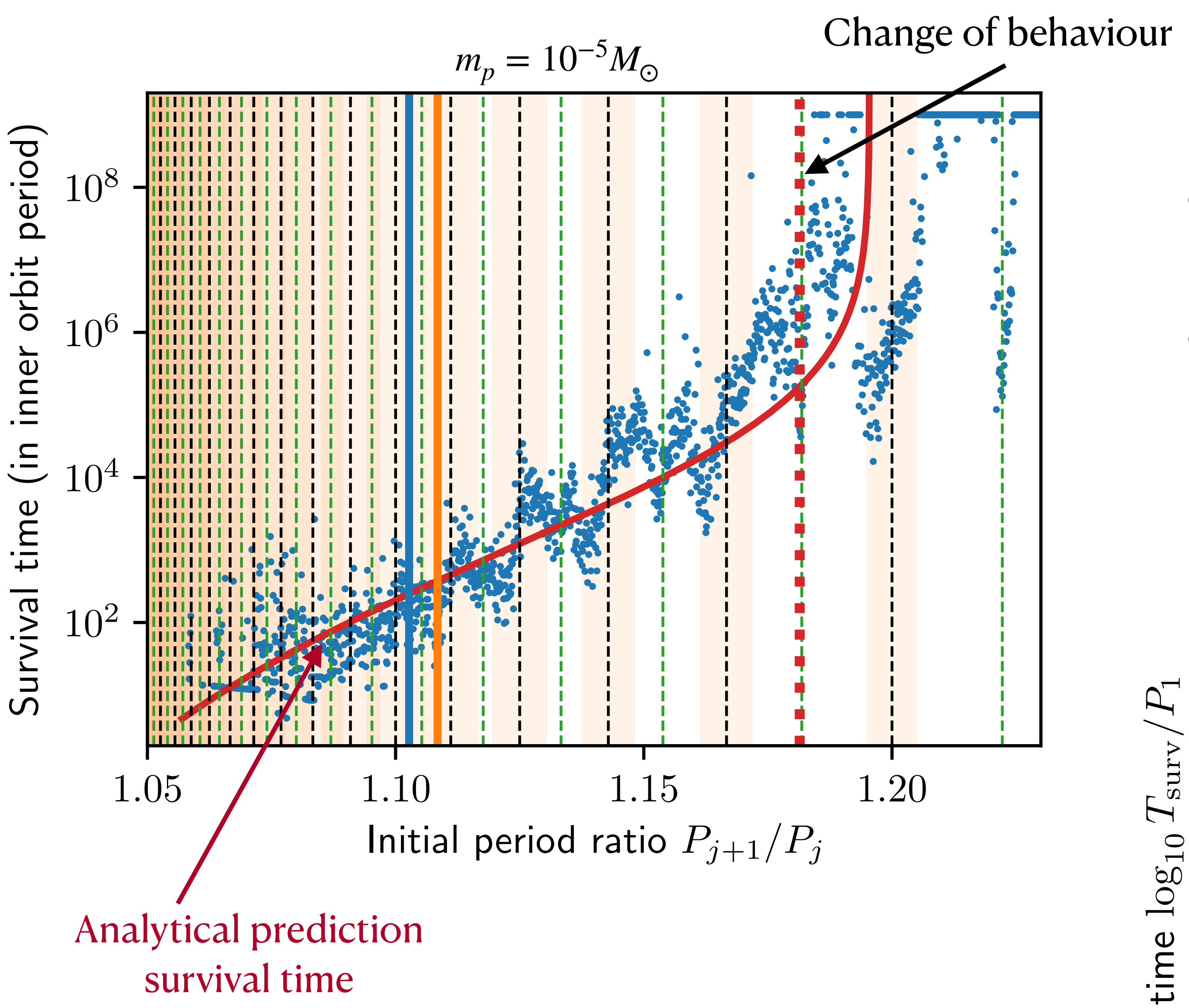
Quillen 2011

3-planet MMRs are everywhere!

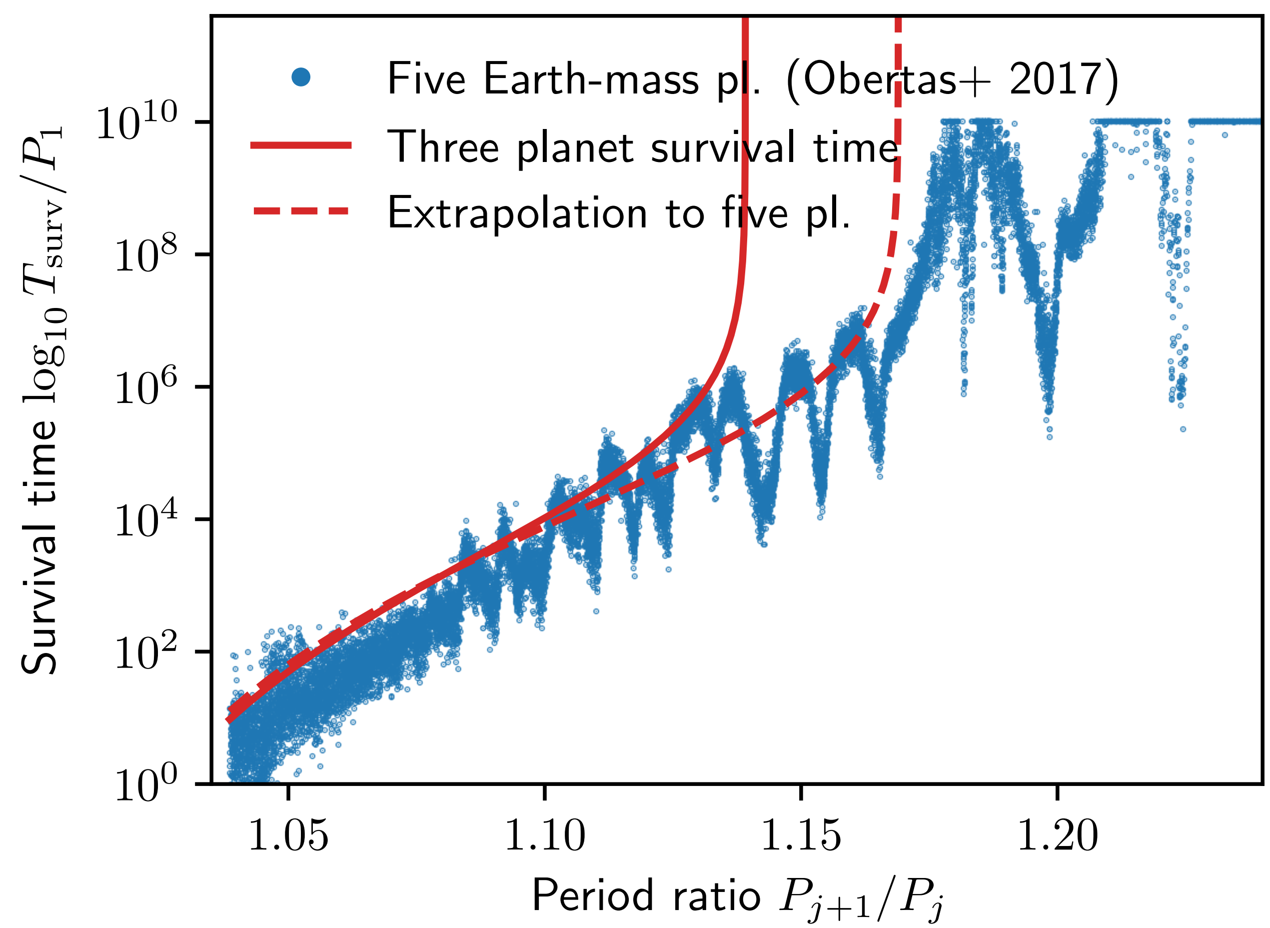


Stability limit scales as $\propto (m_{\rm pl}/m_*)^{1/4}$

Comparison to numerical results

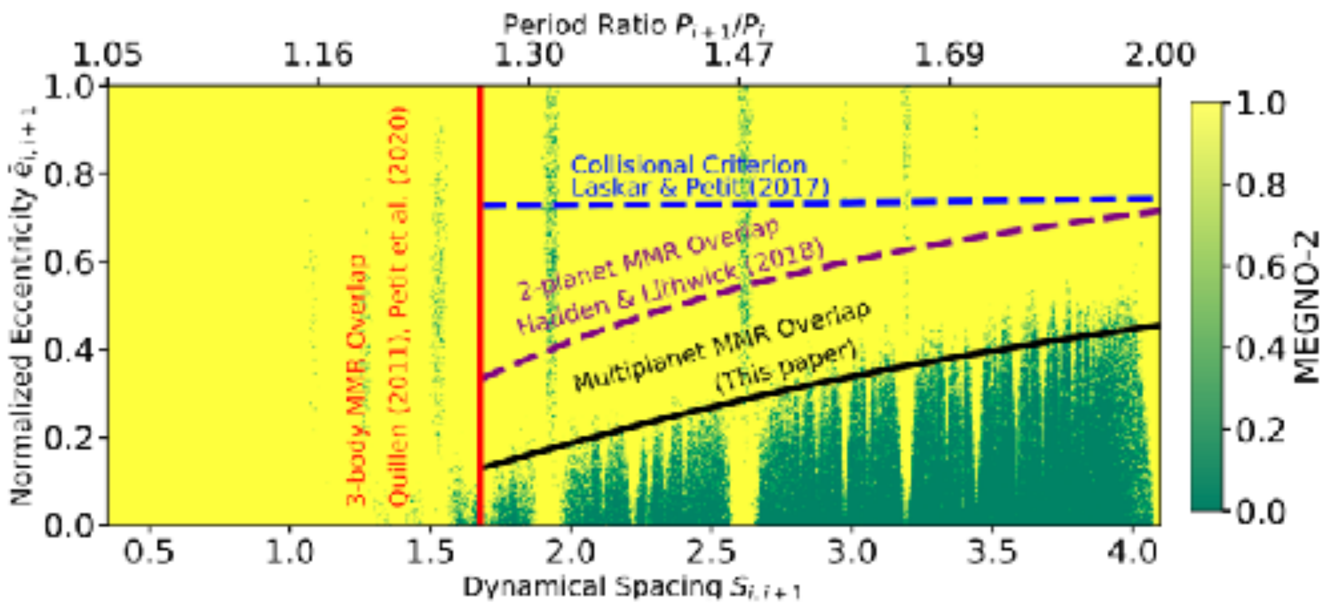


- Valid also in non-EMS cases
- Model can be generalised to more planets



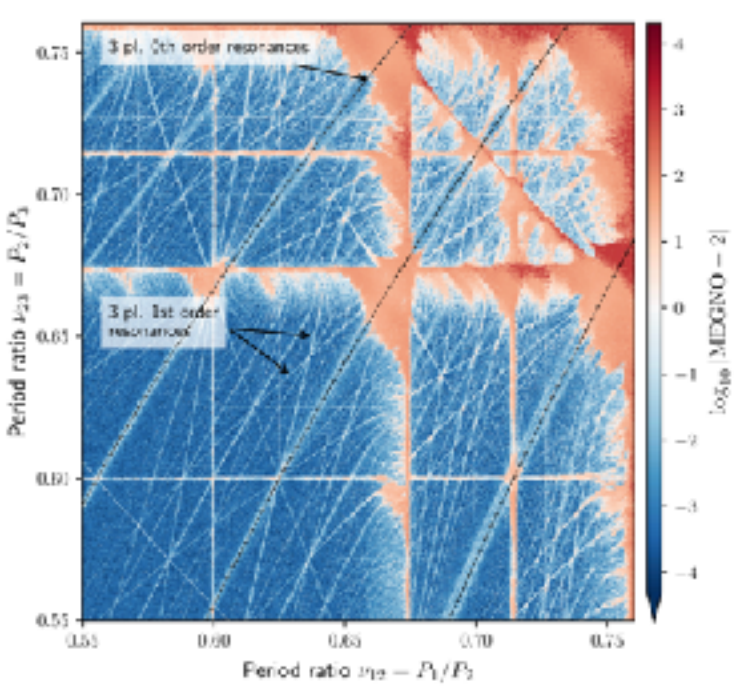
Eccentric orbits?

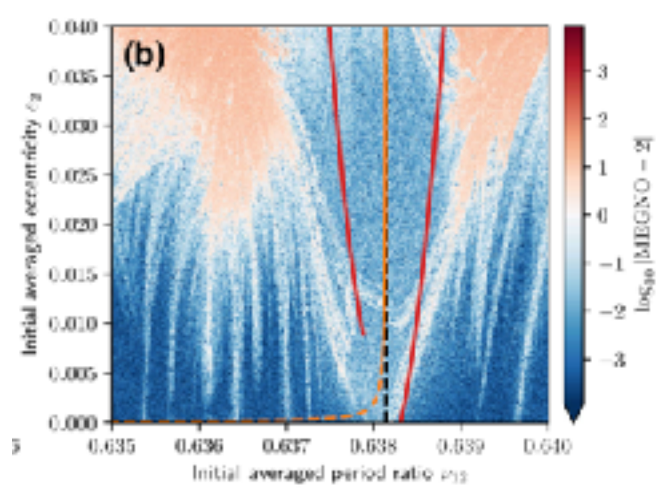
Tamayo et al. 2021



Combining the 3-body MMR overlap with known two-planet stability criteria (Petit et al. 2017, Hadden&Lithwick 2018) may be enough

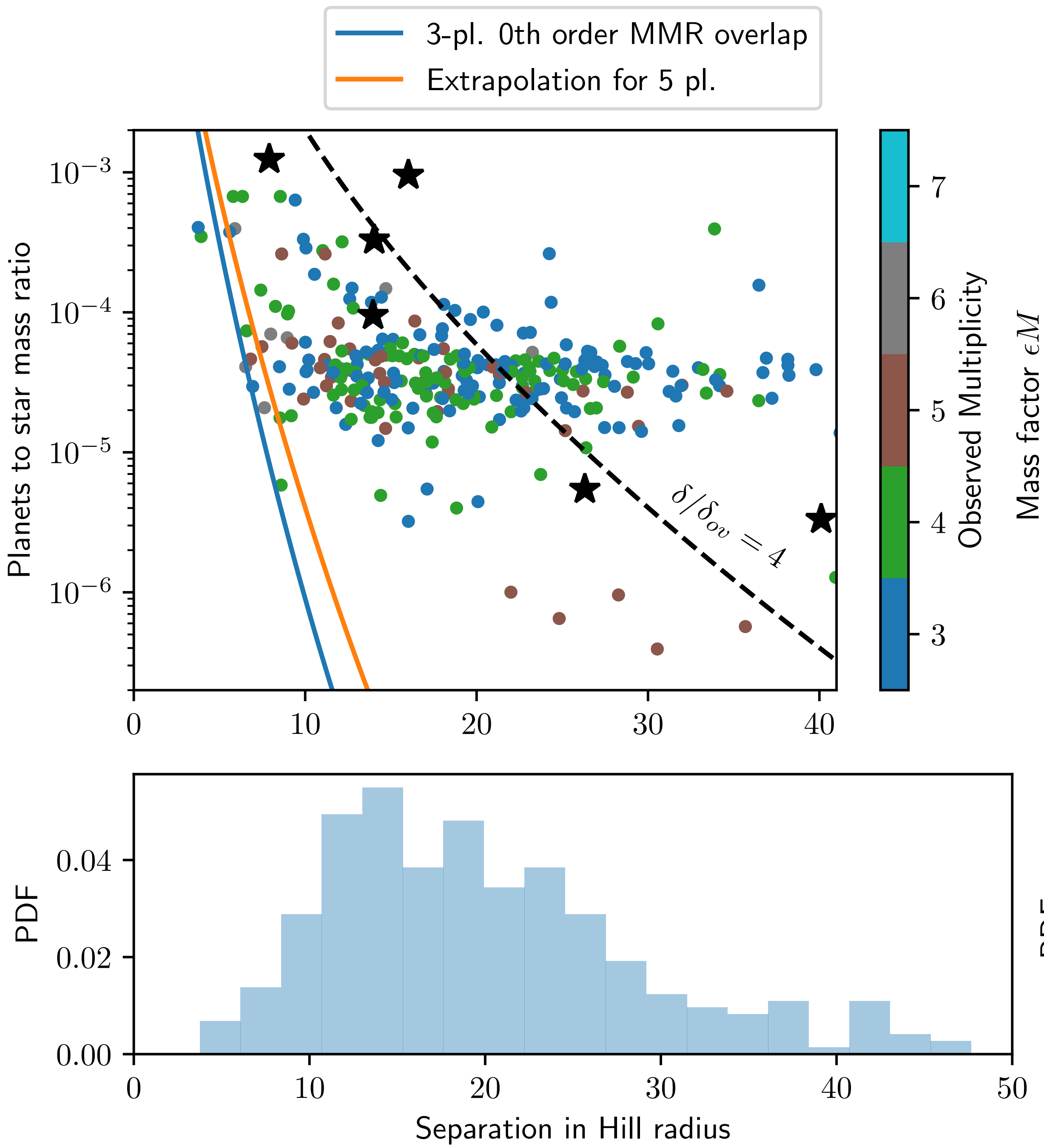
Petit 2021



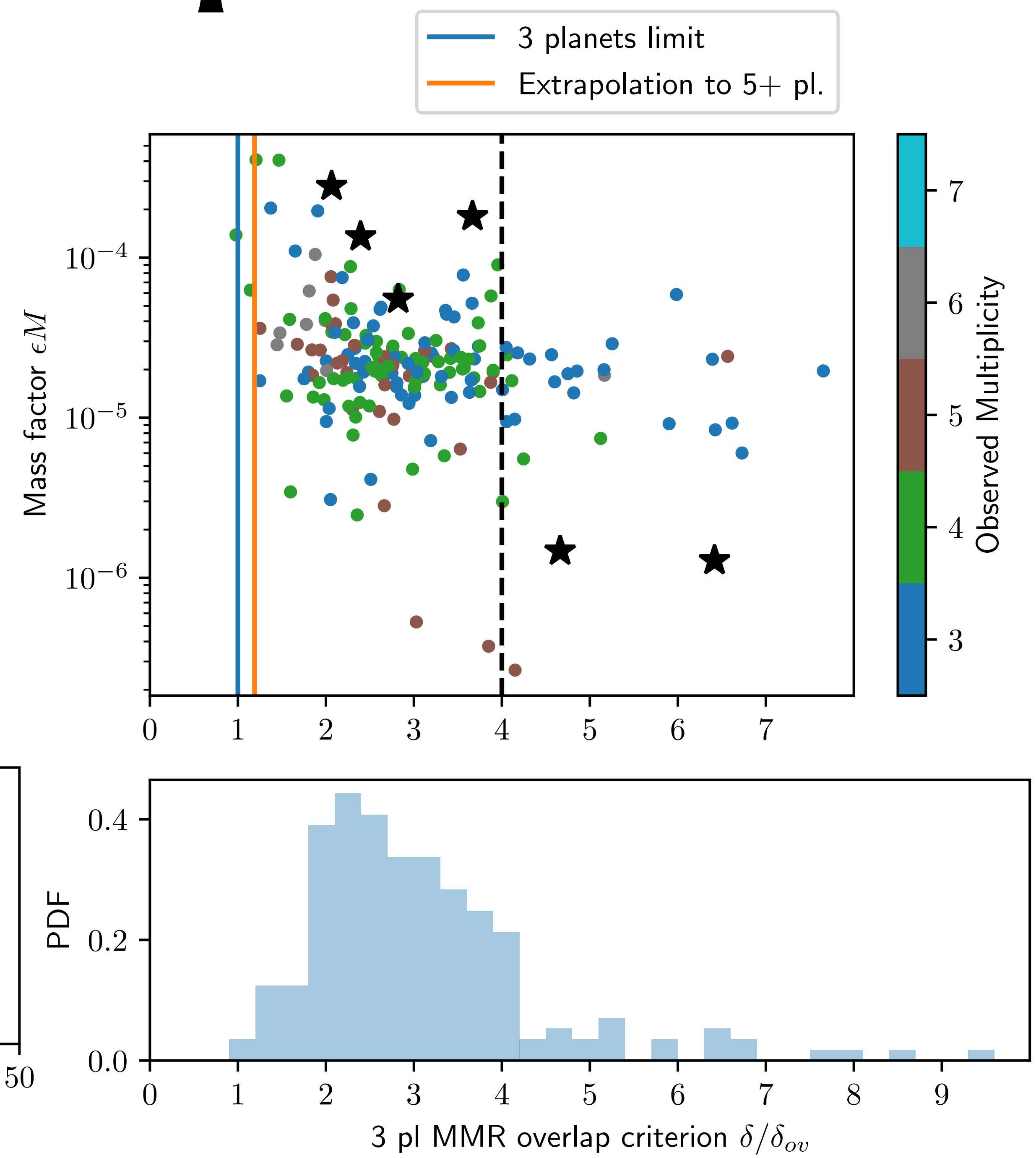


Higher-order 3-body MMR can also be treated analytically.

Observational implications



All adjacent pairs of planets



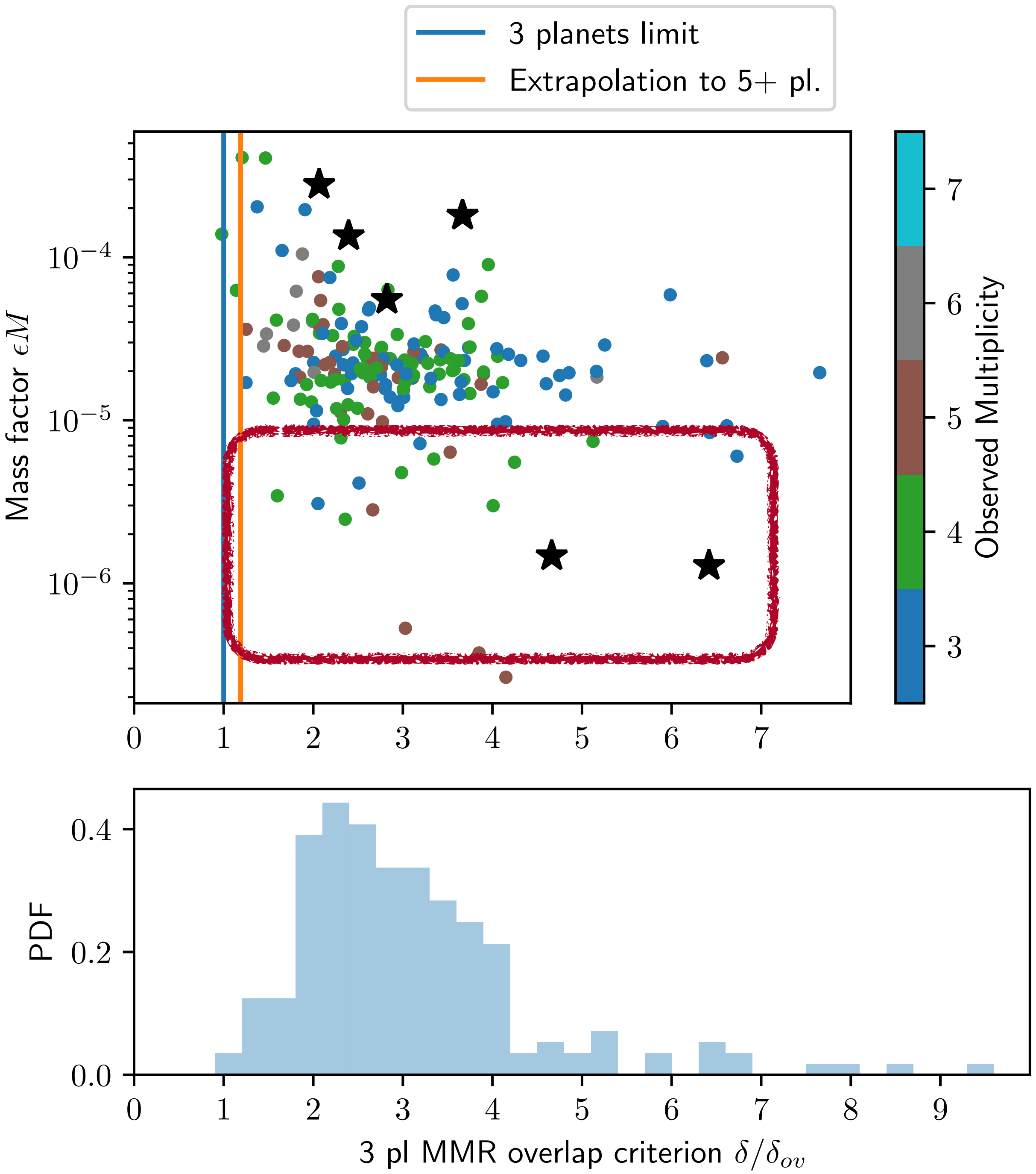
All adjacent triplets of planets

In the right units, systems are clustered closer to the stability limit

Only 3+ planet systems
California Kepler Survey catalog (Petitgura+ 2017, Johnson+ 2017, Weiss+ 2018)
Mass-radius relationship from (Weiss & Marcy 2014)

Plato and tightly packed systems

- More data for Super-Earths
- Fill the gap (maybe?): Terrestrial planets systems extend the range of mass ratios for exoplanets
- Are terrestrial systems scaled down Super-Earths? (Lambrechts et al. 2019)
- Or do they have a different formation channel?
- Why is the inner solar system so far from the stability limit?



Conclusions

- Compact systems are destabilised by the combinations of three-planet MMRs and two-planet MMRs.
- Instability time is given by the diffusion time along the three planet MMR network.
- Analytic model is consistent with numerical simulations (beyond the EMS case).
- Super-Earths systems are closer to the stability limit than expected from Hill spacing
- Spacing of terrestrial planet systems will provide insights onto their formation mechanism