Relaxed Skolem Mean Labeling of Four Star Graphs $K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ Where $l_1 \leq l_2 \leq l_3$ With $|l - l_1 - l_2 - l_3| = 1$

Abraham K Samuel, J. Vinolin, D. S. T. Ramesh

***Abstract*: The four star graphs G = $K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l} K_{1,l}$ where $l_1 \leq l_2 \leq l_3$ l_3 will be relaxed skolem mean if $|l - l_1 -$ $l2-l3=1$ 1 is the main purpose of this article.**

***Keywords*: Relaxed Skolem mean graph, relaxed skolem mean labeling, star graphs.**

I. INRODUCTION

Relaxed skolem mean labeling was first conceived by V.Balaji et. al.[3] in the year 2010. In that paper [3] he introduced the definition of relaxed skolem mean labeling for the first time and also some basic properties for a graph to be relaxed skolem mean. The most important properties are (i) If G is a graph with n vertices and m edges then G is said to be a relaxed skolem mean graph only if n ≥ m ,(ii) G = $K_{1,n}$ where n ≥ 5 is not a relaxed skolem mean graph.

II. PRELIMINARIES

Definition 1: A graph label is the assigning of labels to edges and also vertices of a graph or simply either edges or vertices of a graph only by integers.

Notation: If x is a real number, the integral part of x is denoted by $[x]$ which is the largest integer less than or equal to x.

We define the relaxed skolem mean labeling of a graph G of order p and size q as follows:

Definition 2: The vertex labeling f : V⟶{1, 2, …, p+1} and the induced edge labeling f^*: E⟶{2,3…, p+1} is a relaxed skolem mean labeling if both f and f^* are one – one functions and f^*(e = uv) = [(f(u)+f(v)+1)/2].

Note: Graph G has p vertices and the available vertex labels are p+1. Therefore, one number from the set {1, 2, 3, …, p + 1} will not be used to label any vertex of G. We call that number as the relaxed label. If the relaxed label is p + 1, the relaxed mean labeling becomes the Skolem mean labeling.

III. MAIN RESULT

Theorem 1: Four star graph $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ where l_1, l_2, l_3 are in ascending order is a relaxed skolem mean graph if $|l - l_1 - l_2 - l_3|$ =1.

Proof: Let $L_k = \sum_{j=1}^{k} l_j$; $1 \leq k \leq 3$. Hence we have $L_1 = l_1; L_2 = l_1 + l_2$ and $L_3 = l_1 + l_2 + l_3$.

Consider the graph $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$.

Let $V = V_1 \cup V_2 \cup V_3 \cup V_4$ be the set of vertices of G where $V_k = \{v_{k,i} : 0 \leq i \leq l_k\}$ for $1 \leq k \leq 3$, $V_4 = \{v_{4,i} : 0 \leq i \leq l\}$.

Let $E = \bigcup_{k=1}^{3} \{v_{k,0} v_{k,i} : 1 \leq i \leq l_k\} \cup \{v_{4,0} v_{4,i} : 1 \leq i \leq$ l be the set of edges of G. The condition

$|l - l_1 - l_2 - l_3|$=1$\Longrightarrow \left|l - \sum_{j=1}^{3} l_j\right| = 1 \Longrightarrow |l - L_3| =$ 1That is, there are two cases viz. $l = L_3$ + 1 and $l = L_3$ - 1.

Case I: $l = L_3$ + 1.

G has $L_3 + l + 4 = 2L_3 + 5$ vertices and $L_3 + l = 2L_3 +$ 1edges. The vertex labeling

$f: V \rightarrow \{1, 2, 3, \ldots, p + 1 = L_3 + l + 4 + 1 = 2L_3 + 6\}$ given as:

$f(v_{1,0}) = 1;\ f(v_{2,0}) = 2;\ f(v_{3,0}) = 4$

$f(v_{4,0}) = L_3 + l + 5 = 2L_3 + 6$

$f(v_{1,k}) = 2k + 4 \qquad 1 \leq k \leq l_1$

$f(v_{2,k}) = 2L_1 + 2k + 4 \qquad 1 \leq k \leq l_2$

$f(v_{3,k}) = 2L_2 + 2k + 4 \qquad 1 \leq k \leq l_3$

$f(v_{4,k}) = 2k + 1 \qquad 1 \leq k \leq l = L_3 + 1$

Here $2L_3 + 5$ is the relaxed label.

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,k}$ is 3 + k where$1 \leq k \leq l_1$ (edge labels are 4,5, …., l_1+ 3 = L_1 + 3), $v_{2,0}v_{2,k}$ is $L_1 + 3 + k$ for $1 \leq k \leq l_2$ (edge labels are $L_1 + 4, L_1 + 5, \ldots, L_1 + l_2 + 3$ = L_2 + 3), $v_{3,0}v_{3,k}$ is $L_2 + 4 + k$ for $1 \leq k \leq l_3$

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(edge labels are $L_2 + 5$, $L_2 + 6$, …, $L_2 + l_3 + 4 = L_3 + 4$), $v_{4,0}v_{4,k}$ is $L_3 + 4 + k$ for $1 \le k \le l$ (edge labels are $L_3 + 5$, $L_3 + 6$, …, $L_3 + l + 5 = L_3 + (L_3 + 1) + 4 = 2L_3 + 5$).

The labels of edges induced by the labels of vertices of graph G are distinct. This shows that G is a relaxed skolem mean .

Case II: $l = L_3$ - 1

G has $L_3 + l + 4 = 2L_3 + 3$ vertices and $L_3 + l = 2L_3 - 1$edges. The vertex labeling $f: V \to \{1, 2, 3, \dots, p + 1 = L_3 + l + 4 + 1 = 2L_3 + 5\}$ given as:

$f(v_{1,0}) = 1$; $f(v_{2,0}) = 2$; $f(v_{3,0}) = 3 f(v_{4,0}) = L_3 + l + 5 = 2L_3 + 4$

$f(v_{1,k}) = 2k + 2 \qquad 1 \le k \le l_1$

$f(v_{2,k}) = 2L_1 + 2k + 2 \qquad 1 \le k \le l_2$

$f(v_{3,k}) = 2L_2 + 2k + 2 \qquad 1 \le k \le l_3$

$f(v_{4,k}) = 2k + 3 \qquad 1 \le k \le l = L_3 - 1$

Here $2L_3 + 3$ is the relaxed label.

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,k}$ is $2 + k$ where$1 \le k \le l_1$ (edge labels are 3,4,…., l_1+ 2 = L_1 + 2), $v_{2,0}v_{2,k}$ is $L_1 + 2 + k$ for $1 \le k \le l_2$ (edge labels are $L_1 + 3$, $L_1 + 4$, …, $L_1 + l_2 + 2 = L_2 + 2$), $v_{3,0}v_{3,k}$ is $L_2 + 3 + k$ for $1 \le k \le l_3$ (edge labels are $L_2 + 4$, $L_2 + 5$, …, $L_2 + l_3 + 3 = L_3 + 3$), $v_{4,0}v_{4,k}$ is $L_3 + 4 + k$ for $1 \le k \le l$ (edge labels are $L_3 + 5$, $L_3 + 6$, …, $L_3 + l + 5 = L_3 + (L_3 - 1) + 4 = 2L_3 + 3$).

The labels of edges induced by the labels of vertices of graph G are distinct.

This shows that G is a relaxed skolem mean . is skolem mean

We illustrate the above two cases with the following four star graphs.

Fig.1. is an illustartion of Case I where $l = L_3 + 1$

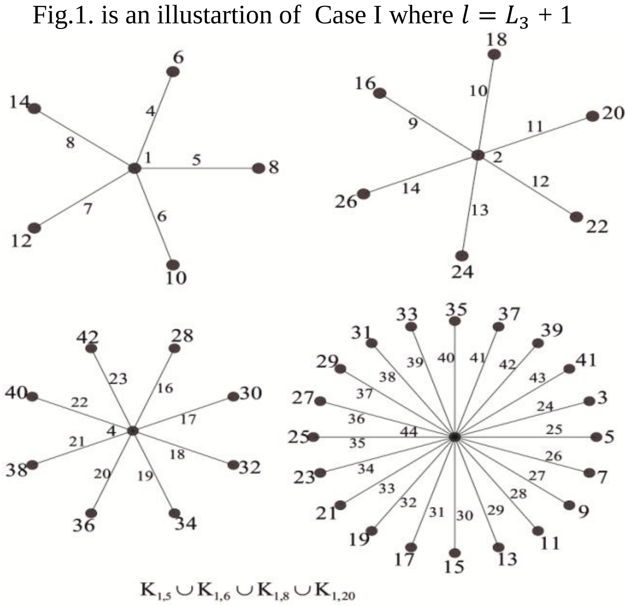

Fig. 1

Fig. 2. is an illustration of Case II where $l = L_3$ - 1

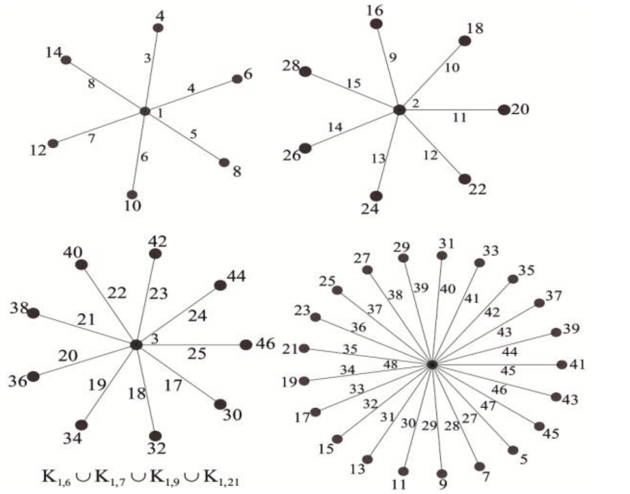

Fig. 2

IV. CONCLUSION

Relaxed skolem mean labeling of a four star graph $G = K_{1,l_1} \cup K_{1,l_2} \cup K_{1,l_3} \cup K_{1,l}$ is discussed in this paper . We mainly discussed the two cases $l = l_3$ +1 and $l = l_3 - 1$ and gave the labeling which satisfies the condition of relaxed skolem mean labeling which exists for graph G. We are in further research to find upto how many cases the graph G will allow relaxed skolem mean labeling.

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