Fuzzy Metric Dimension of Fuzzy Hypercube Q_n and Fuzzy Boolean Graphs

M.Thusleem Furjana, M.Bhanumathi

Abstract: *Let G = (V, E, μ) be a fuzzy graph. Let $\tilde{M}$ be a subset of V. $\tilde{M}$ is said to be a fuzzy metric basis of G if for every pair of vertices x, y $\in V - \tilde{M}$, there exists a vertex w $\in \tilde{M}$ such that $\tilde{d}$ (w, x) $\neq$ $\tilde{d}$ (w, y). The number of elements in $\tilde{M}$ is said to be fuzzy metric dimension (FMD) of G and is denoted by $\tilde{\beta}$ (G). The elements in $\tilde{M}$ are called as source vertices. In this paper, we study the fuzzy metric dimension of fuzzy hypercube Q_n, fuzzy Boolean Graph $BG_2(G)$ and fuzzy Boolean Graph $BG_3(G)$.*

Keywords: *fuzzy Boolean graph $BG_2(G)$, fuzzy Boolean graph $BG_3(G)$, fuzzy Hypercube Q_n, fuzzy metric dimension.*

I. INTRODUCTION

A **fuzzy graph[7]** G is a 2-tuple (V, E) where V is a non empty set of vertices $\{v_1, v_2, \ldots, v_n\}$ and E is the nonempty finite set of edges such that μ: V $\to$ [0,1] and σ: V×V $\to$ [0,1] where $\sigma(v_i, v_j) \leq \min(\mu(v_i), \mu(v_j))$ for i $\neq$ j.

= 0 for i = j.

For any v $\in$ V, if $\mu(v) > 0$ then we call v as an active vertex. If $\mu(v) = 0$ then we call v as an inactive vertex. We assume that all the vertices as active vertices. We use the notation e_{ij} to denote the edge connecting the vertices v_i and v_j. The weight of the edge e_{ij} is given by $\sigma(v_i, v_j)$ and is denoted by $w(e_{ij})$.

A **fuzzy path** [7] from a vertex v_i to a vertex v_j in a fuzzy graph is a sequence of distinct vertices and edges starting from v_i and ending at v_j. This is denoted by $P(v_i, v_j) = P$.

If v_i and v_j coincide in a fuzzy path P then we call this sequence as a **fuzzy cycle**. Let P_{ij} be the set of all fuzzy paths P from v_i to v_j. For $v_i, v_j \in V$, we define the fuzzy set μ_{ij}: $P_{ij} \to$ [0, 1] by $\mu_{ij}(P) = \min_{e\in P}(w(e))$ where $P \in P_{ij}$. Here $\mu_{ij}(P)$ is called the weight of the path P. The fuzzy path $P \in P_{ij}$ for which $\mu_{ij}(P)$ is minimum, is called as a fuzzy shortest path (FSP) between v_i and v_j. The weight of this FSP is denoted by $d^*(v_i, v_j)$. Thus, d^* can be viewed as a fuzzy set, d^*: V×V$\to$[0,1] where $d^*(v_i, v_j) = \min_{P\in P_{ij}} (\mu_{ij}(P))$ and $d^*(v_i, v_i) = 0$.

For any two fuzzy shortest paths P and Q between v_i and v_j, we consider the path with lesser number of intermediate vertices.

In 1995, George and Veeramani defined the 3-tuple (V, d^*, t) as $\tilde{d}(v_i, v_j, t) = \dfrac{t}{t + d^*(v_i, v_j)}$, where t is the number of intermediate vertices in the shortest path from which d^* is calculated [5]. $N(v_i, v_j)$ is defined as the number of intermediate vertices between v_i and v_j in fuzzy shortest path (FSP) and $\tilde{d}$ (v_i, v_j, t) is denoted as $\tilde{d}$ (v_i, v_j).

Let G = (V, E, μ) be a fuzzy graph. Let $\tilde{M}$ be a subset of V. $\tilde{M}$ is said to be a fuzzy metric basis of G if for every pair of vertices x, y $\in V - \tilde{M}$, there exists a vertex w $\in \tilde{M}$ such that $\tilde{d}$ (w, x) $\neq$ $\tilde{d}$ (w, y). The number of elements in $\tilde{M}$ is said to be **fuzzy metric dimension** (FMD) of G and is denoted by $\tilde{\beta}$ (G). The elements in $\tilde{M}$ are called as **source vertices.**

In 2012, Praba et.al introduced and defined the fuzzy metric dimension of fuzzy graphs [7]. In 2016, Bhanumathi and Thusleem furjana studied the fuzzy metric basis of some standard fuzzy graphs G, fuzzy metric basis of Total graph, middle graph and subdivision graph of some standard fuzzy graphs G [1], [2]. Also they have determined the fuzzy metric basis of fuzzy Cartesian product of some fuzzy graphs [3]. In this paper, we determine some new bounds for the fuzzy metric dimension of fuzzy hypercube Q_4, Q_6 and Q_n. Also, we study the fuzzy metric basis of fuzzy Boolean graph $BG_2(G)$ for some standard fuzzy graphs G and fuzzy Boolean graph $BG_3(G)$ for some standard fuzzy graphs G.

Theorem: 1.1[5] d^* is a metric

Theorem: 1.2[7] If G is a path then $\tilde{\beta}$ (G) = 1.

Theorem: 1.3[7] If P_n is a path on n vertices and v_k is an intermediate vertex in P_n, v_i and v_j are two vertices on either side of v_k then $\tilde{d}$ (v_k, v_i) = $\tilde{d}$ (v_k, v_j) if and only if $\dfrac{N(v_k, v_i)}{N(v_k, v_j)} = \dfrac{d^*(v_k, v_i)}{d^*(v_k, v_j)}$.

Theorem: 1.4[7] Let P_n be a path on n vertices and v_k is an intermediate vertex in P_n . If v_i and v_j are two vertices on either side of v_k such that $N(v_k, v_i) = N(v_k, v_j)$ then $\tilde{d}$ (v_k, v_i) = $\tilde{d}$ (v_k, v_j) if and only if $d^*(v_k, v_i) = d^*(v_k, v_j)$.

Theorem: 1.5[7] If C_n is fuzzy cycle then $\tilde{\beta}$ (C_n) $\leq$ 2.

Definition: 1.6 A graph G is said to be **decomposable [4]** into Hamiltonian cycles if its edge set can be partitioned into Hamiltonian cycles. A graph is said to admit cycle decomposition (respectively Hamiltonian decomposition) if its edge set can be partitioned into cycles (respectively Hamiltonian cycles).

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Let C_n denote the cycle of length $n \geq 3$. If C_m and C_n have vertex sets $\{u_1, u_2, \ldots, u_m\}$ and $\{v_1, v_2, \ldots, v_n\}$ respectively, we denote the vertices and edges of $C_m \times C_n$ by $\{u_iv_j / i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$ and $|E(C_m \times C_n)| = 2nm$. Thus, if $C_m \times C_n$ admits Hamiltonian decomposition then the number of cycles in such decomposition is two. Two Hamiltonian cycles in a graph are said to be **edge-disjoint** if there exists no common edges in them.

Theorem: 1.7[8] The binary n-cube, with n even or equivalently the product of (n/2) cycles, $C_4 \times C_4 \times \ldots \times C_4$ can be partitioned into (n/2) Hamiltonian cycles.

Definition: 1.8 A **star** [6] in a fuzzy graph consist of two node sets V and U with $|V| = 1$ and $|U| > 1$, such that $\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0$, $1 \leq i \leq n$. It is denoted by $S_{1,n}$.

II. FUZZY METRIC DIMENSION OF FUZZY HYPERCUBE.

In this section we determine fuzzy metric basis of fuzzy Hypercube Q_n for n = 4 and n = 6.

Definition: 2.1

The fuzzy hypercube or n-fuzzy cube Q_n is the graph whose vertex set is the set of all n-dimensional Boolean vectors in which two vertices are joined if and only if they differ in exactly one coordinate.

A. *Fuzzy Metric Dimension of Hypercube Q_4.*

Theorem: 2.1 If $G = Q_4$, then $2 \leq \tilde{\beta}(G) \leq 4$.

Proof: $G = Q_4 = K_2 \times K_2 \times K_2 \times K_2 = C_4 \times C_4$. Let $V_1 = \{u_1, u_2, u_3, u_4\}$ be the vertex set of one C_4 and $V_2 = \{v_1, v_2, v_3, v_4\}$ be the vertex set of another C_4. Then $V(G) = V_1 \times V_2 = \{u_1v_1, u_1v_2, u_1v_3, u_1v_4, u_2v_1, u_2v_2, u_2v_3, u_2v_4, \ldots, u_4v_1, u_4v_2, u_4v_3, u_4v_4\}$. Q_4 can be partitioned into two Hamiltonian fuzzy cycles as follows:

C_1: u_1v_1 u_1v_4 u_4v_4 u_3v_4 u_2v_4 u_2v_3 u_1v_3 u_4v_3 u_3v_3 u_3v_2 u_2v_2 u_1v_2 u_4v_2 u_4v_1 u_3v_1 u_2v_1 u_1v_1.

C_2: u_1v_1 u_1v_2 u_1v_3 u_1v_4 u_2v_4 u_2v_1 u_2v_2 u_2v_3 u_3v_3 u_3v_4 u_3v_1 u_3v_2 u_4v_2 u_4v_3 u_4v_4 u_4v_1 u_1v_1.

We will write $Q_4 = C_4 \times C_4$ as the union of two Hamiltonian fuzzy cycles, that is $C_4 \times C_4 = C_1 \cup C_2$

In C_1, take u_1v_1 as a source vertex, let P_1 be the path u_1v_1 u_1v_4 u_4v_4 u_3v_4 u_2v_4 u_2v_3 u_1v_3 u_4v_3 u_3v_3 and P_2 be the path u_3v_3 u_3v_2 u_2v_2 u_1v_2 u_4v_2 u_4v_1 u_3v_1 u_2v_1 u_1v_1. In C_2, take u_1v_2 as a source vertex. Let P_3 be the path u_1v_1 u_1v_2 u_1v_3 u_1v_4 u_2v_4 u_2v_1 u_2v_2 u_2v_3 u_3v_3 and P_4 be the path u_3v_3 u_3v_4 u_3v_1 u_3v_2 u_4v_2 u_4v_3 u_4v_4 u_4v_1 u_1v_1.

Here we calculate the fuzzy metric dimension of $C_4 \times C_4$.

Case i:

In C_1, let u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) be two vertices on C_1 such that both u_iv_j and $u_jv_i \in P_1$ or P_2 (i, j = 1, 2, 3, 4 and $i = j \neq 1$). If both u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) have the same FSP (fuzzy shortest path) from source vertex u_1v_1 then u_1v_1, u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) will be in same path. Thus, $\tilde{\beta}(C_1) = 1$. In C_2, let u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) be two vertices on C_2 such that both u_iv_j and $u_jv_i \in P_3$ (or P_4) (i, j = 1, 2, 3, 4 and $i = j \neq 1$) and If both u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) have the same FSP (fuzzy shortest path) from source vertex u_1v_2 then u_1v_2, u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) will be in same path. Thus, $\tilde{\beta}(C_4) = 1$, $\tilde{\beta}(C_4 \times C_4) = \tilde{\beta}(C_1 \cup C_2)$ and $\tilde{M} = \{u_1v_1, u_1v_2\}$. Therefore, $\tilde{\beta}(Q_4) = 2$.

Case ii:

In C_1, if the two vertices u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) belongs to either P_1 (or P_2), then by case (i) we get, $\tilde{\beta}(C_1) = 1$.

In C_2, if u_iv_j and u_jv_i (i, j = 1, 2, 3, 4 and $i = j \neq 1$) such that the FSP for u_iv_j from source vertex u_1v_2 is through P_4 and FSP for u_jv_i from source vertex u_1v_2 is through P_3 then $\tilde{d}(u_1v_2, u_iv_j) = \tilde{d}(u_1v_2, u_jv_i)$ (i, j = 1, 2, 3, 4 and $i = j \neq 1$) if and only if $N(u_1v_2, u_iv_j) = N(u_1v_2, u_jv_i)$ (i, j = 1, 2, 3, 4 and $i = j \neq 1$). This implies that, $\tilde{\beta}(C_2) \neq 1$.

Include u_4v_1 as another source vertex so that $N(u_4v_1, u_iv_j) \neq N(u_4v_1, u_jv_i)$, $\tilde{d}(u_4v_1, u_iv_j) \neq \tilde{d}(u_4v_1, u_jv_i)$ (i, j = 1, 2, 3, 4 and $i = j \neq 1$). Then metric basis of C_2 is $\{u_1v_2, u_4v_1\}$. Hence, $\tilde{\beta}(C_2) = 2$ and $\tilde{\beta}(C_4 \times C_4) = \tilde{\beta}(C_1 \cup C_2)$ Hence, $\tilde{M} = \{u_1v_1, u_1v_2, u_4v_1\}$. Therefore, $\tilde{\beta}(C_4 \times C_4) = 3$.

Case iii:

In C_1, if $u_iv_j \in P_1$ and $u_jv_i \in P_2$ (i, j = 1, 2, 3, 4 and $i = j \neq 1$) such that the FSP for u_iv_j from source vertex u_1v_1 is through P_2 and FSP for u_jv_i from source vertex u_1v_1 is through P_1 then $\tilde{d}(u_1v_1, u_iv_j) = \tilde{d}(u_1v_1, u_jv_i)$ if and only if $N(u_1v_1, u_iv_j) = N(u_1v_1, u_jv_i)$. This implies that, $\tilde{\beta}(C_1) \neq 1$. Include u_2v_1 as another source vertex so that $N(u_2v_1, u_iv_j) \neq N(u_2v_1, u_jv_i)$, $\tilde{d}(u_2v_1, u_iv_j) \neq \tilde{d}(u_2v_1, u_jv_i)$ (i, j = 1, 2, 3, 4 and $i = j \neq 1$). Then $\tilde{M} = \{u_1v_1, u_2v_1\}$ and $\tilde{\beta}(C_1) = 2$.

Similarly, we get, metric basis of C_2 as $\{u_1v_1, u_2v_1\}$ and $\tilde{\beta}(C_4 \times C_4) = \tilde{\beta}(C_1 \cup C_2)$. Hence, $\tilde{M} = \{u_1v_1, u_2v_1, u_1v_2, u_4v_1\}$. Therefore, $\tilde{\beta}(Q_4) = \tilde{\beta}(C_4 \times C_4) = 4$.

Theorem: 2.2 If $G = Q_4$, then $\tilde{\beta}(G) \leq 3$.

Proof: $G = Q_4 = K_2 \times K_2 \times K_2 \times K_2 = C_4 \times C_4$. Let $V_1 = \{u_1, u_2, u_3, u_4\}$ be the vertex set of one C_4 and $V_2 = \{v_1, v_2, v_3, v_4\}$ be the vertex set of another C_4. Then $V(G) = V_1 \times V_2 = \{u_1v_1, u_1v_2, u_1v_3, u_1v_4, u_2v_1, u_2v_2, u_2v_3, u_2v_4, \ldots, u_4v_1, u_4v_2, u_4v_3, u_4v_4\}$. Q_4 can be partitioned into three fuzzy paths as follows:

P_1: u_1v_1 u_1v_4 u_4v_4 u_3v_4 u_2v_4 u_2v_3 u_1v_3 u_4v_3 u_3v_3 u_3v_2 u_2v_2 u_1v_2 u_4v_2 u_4v_1 u_3v_1 u_2v_1.

P_2: u_1v_1 u_1v_2 u_1v_3 u_1v_4 u_2v_4 u_2v_1 u_2v_2 u_2v_3 u_3v_3 u_3v_4 u_3v_1 u_3v_2 u_4v_2 u_4v_3 u_4v_4 u_4v_1.

P_3: u_2v_1 u_1v_1 u_4v_1.

We will write $Q_4 = C_4 \times C_4$ as the union of two Hamiltonian cycles, that is $C_4 \times C_4 = P_1 \cup P_2 \cup P_3$.

In two paths P_1 and P_2, take u_1v_1 as a source vertex. If two vertices u_iv_j or $u_jv_i \in P_1$, where (i, j = 1, 2, 3, 4 and $i = j \neq 1$) and u_iv_j or $u_jv_i \in P_2$ such that fuzzy shortest path from source vertex u_1v_1 for u_iv_j or u_jv_i is through P_2 and fuzzy shortest path from source vertex u_1v_1 for u_iv_j or u_jv_i is through P_1, then $\tilde{d}(u_1v_1, u_iv_j) = \tilde{d}(u_1v_1, u_iv_j)$ or $\tilde{d}(u_1v_1, u_jv_i) = \tilde{d}(u_1v_1, u_jv_i)$ if and only if $N(u_1v_1, u_iv_j) = N(u_1v_1, u_iv_j)$ or $N(u_1v_1, u_jv_i) = N(u_1v_1, u_jv_i)$.. This implies that, $\tilde{\beta}(Q_4) \neq 1$.

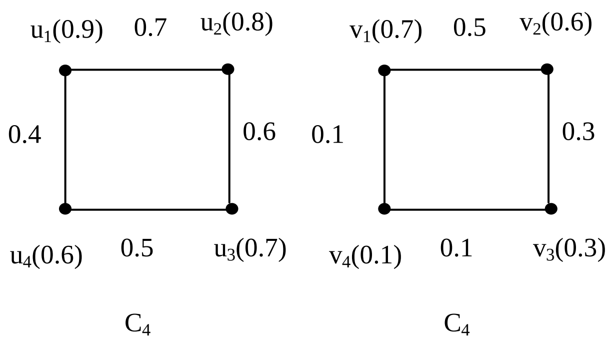
Include u_4v_1 as another source vertex so that $N(u_4v_1, u_iv_j) \neq N(u_4v_1, u_iv_j)$ or $N(u_4v_1, u_jv_i) \neq N(u_4v_1, u_jv_i)$, $\tilde{d}\,(u_4v_1, u_iv_j) \neq \tilde{d}\,(u_4v_1, u_iv_j)$ or $\tilde{d}\,(u_4v_1, u_jv_i) \neq \tilde{d}\,(u_4v_1, u_jv_i)$. Continuing this process for all three paths in Q_4, we get three source vertices for Q_4. $\tilde{M} = \{u_1v_1, u_4v_1, u_2v_1\}$. Hence, $\tilde{\beta}\,(Q_4) \leq 3$.

B. *Fuzzy Metric Dimension of Hypercube Q_6.*

Theorem: 2.3 If $G = Q_6$, then $\tilde{\beta}\,(G) \leq 4$.

Proof: $G = Q_2 \times Q_2 \times Q_2 = C_4 \times C_4 \times C_4$. If $V_1 = \{u_1, u_2, u_3, u_4\}$, $V_2 = \{v_1, v_2, v_3, v_4\}$ and $V_3 = \{x_1, x_2, x_3, x_4\}$, then $V(G) = V_1 \times V_2 \times V_3 = \{u_1v_1x_1, u_1v_2x_1, u_1v_3x_1, u_1v_4x_1, u_2v_1x_1, u_2v_2x_1, u_2v_3x_1, u_2v_4x_1, \ldots, u_4v_1x_1, u_4v_2x_1, u_4v_3x_1, u_4v_4x_1, u_1v_1x_2, u_1v_2x_2, u_1v_3x_2, u_1v_4x_2, u_2v_1x_2, u_2v_2x_2, u_2v_3x_2, u_2v_4x_2, \ldots, u_4v_1x_2, u_4v_2x_2, u_4v_3x_2, u_4v_4x_2, u_1v_1x_3, u_1v_2x_3, u_1v_3x_3, u_1v_4x_3, u_2v_1x_3, u_2v_2x_3, u_2v_3x_3, u_2v_4x_3, \ldots, u_4v_1x_3, u_4v_2x_3, u_4v_3x_3, u_4v_4x_3, u_1v_1x_4, u_1v_2x_4, u_1v_3x_4, u_1v_4x_4, u_2v_1x_4, u_2v_2x_4, u_2v_3x_4, u_2v_4x_4, \ldots, u_4v_1x_4, u_4v_2x_4, u_4v_3x_4, u_4v_4x_4\}$. Q_6 can be partitioned into four paths as follows:

P_1: $u_1v_1x_1\ u_4v_1x_1\ u_3v_1x_1\ u_2v_1x_1\ u_2v_1x_4\ u_1v_1x_4\ u_4v_1x_4\ u_3v_1x_4\ u_3v_1x_3\ u_2v_1x_3\ u_1v_1x_3\ u_4v_1x_3\ u_4v_1x_2\ u_3v_1x_2\ u_2v_1x_2\ u_1v_1x_2\ u_1v_2x_2\ u_4v_2x_2\ u_3v_2x_2\ u_2v_2x_2\ u_2v_2x_1\ u_1v_2x_1\ u_4v_2x_1\ u_3v_2x_1\ u_3v_2x_4\ u_2v_2x_4\ u_1v_2x_4\ u_4v_2x_4\ u_4v_2x_3\ u_3v_2x_3\ u_2v_2x_3\ u_1v_2x_3\ u_1v_3x_3\ u_4v_3x_3\ u_3v_3x_3\ u_2v_3x_3\ u_2v_3x_2\ u_1v_3x_2\ u_4v_3x_2\ u_3v_3x_2\ u_3v_3x_1\ u_2v_3x_1\ u_1v_3x_1\ u_4v_3x_1\ u_4v_3x_4\ u_3v_3x_4\ u_2v_3x_4\ u_1v_3x_4\ u_1v_4x_4\ u_4v_4x_4\ u_3v_4x_4\ u_2v_4x_4\ u_2v_4x_3\ u_1v_4x_3\ u_4v_4x_3\ u_3v_4x_3\ u_3v_4x_2\ u_2v_4x_2\ u_1v_4x_2\ u_4v_4x_2\ u_4v_4x_1\ u_3v_4x_1\ u_2v_4x_1\ u_1v_4x_1$.



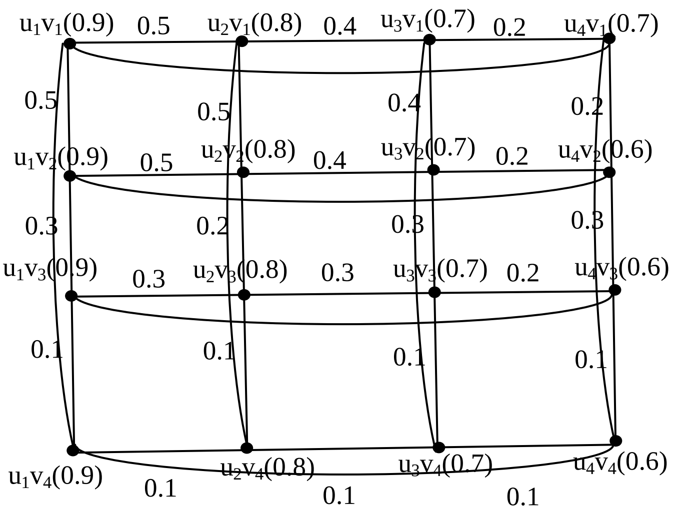


Figure: 3.3.1 Fuzzy hypercube $Q_4 = C_4 \times C_4$

P_2: $u_2v_1x_1\ u_2v_4x_1\ u_2v_3x_1\ u_2v_2x_1\ u_3v_2x_1\ u_3v_1x_1\ u_3v_4x_1\ u_3v_3x_1\ u_4v_3x_1\ u_4v_2x_1\ u_4v_1x_1\ u_4v_4x_1\ u_1v_4x_1\ u_1v_3x_1\ u_1v_2x_1\ u_1v_1x_1\ u_1v_1x_2\ u_4v_1x_2\ u_4v_2x_2\ u_4v_3x_2\ u_4v_4x_2\ u_3v_4x_2\ u_3v_1x_2\ u_3v_2x_2\ u_3v_3x_2\ u_2v_3x_2\ u_2v_4x_2\ u_2v_1x_2\ u_2v_2x_2\ u_1v_2x_2\ u_1v_2x_3\ u_4v_2x_3\ u_4v_3x_3\ u_4v_4x_3\ u_4v_1x_3\ u_3v_1x_3\ u_3v_2x_3\ u_3v_3x_3\ u_3v_4x_3\ u_2v_4x_3\ u_2v_1x_3\ u_2v_2x_3\ u_2v_3x_3\ u_1v_3x_3\ u_1v_3x_4\ u_4v_3x_4\ u_4v_4x_4\ u_4v_1x_4\ u_4v_2x_4\ u_3v_2x_4\ u_3v_3x_4\ u_3v_4x_4\ u_3v_1x_4\ u_2v_1x_4\ u_2v_2x_4\ u_2v_3x_4\ u_2v_4x_4\ u_1v_4x_4\ u_1v_1x_4\ u_1v_2x_4$.

P_3: $u_1v_1x_4\ u_1v_1x_3\ u_1v_1x_2\ u_1v_2x_2\ u_1v_2x_1\ u_1v_2x_4\ u_1v_2x_3\ u_1v_3x_3\ u_1v_3x_2\ u_1v_3x_1\ u_1v_3x_4\ u_1v_4x_4\ u_1v_4x_3\ u_1v_4x_2\ u_1v_4x_1\ u_1v_1x_1\ u_2v_1x_1\ u_2v_2x_1\ u_2v_2x_4\ u_2v_2x_3\ u_2v_2x_2\ u_2v_3x_2\ u_2v_3x_1\ u_2v_3x_4\ u_2v_3x_3\ u_1v_4x_3\ u_1v_4x_2\ u_1v_4x_1\ u_1v_4x_4\ u_2v_1x_4\ u_3v_1x_4\ u_3v_2x_4\ u_3v_2x_3\ u_3v_2x_2\ u_3v_2x_1\ u_3v_3x_1\ u_3v_3x_4\ u_3v_3x_3\ u_3v_3x_2\ u_3v_4x_2\ u_3v_4x_1\ u_3v_4x_4\ u_3v_4x_3\ u_3v_1x_3\ u_3v_1x_2\ u_3v_1x_1\ u_4v_1x_1\ u_4v_1x_2\ u_4v_4x_2\ u_4v_4x_3\ u_4v_4x_4\ u_4v_4x_1\ u_4v_3x_1\ u_4v_3x_2\ u_4v_3x_3\ u_4v_3x_4\ u_4v_2x_4\ u_4v_2x_1\ u_4v_2x_2\ u_4v_2x_3\ u_4v_1x_3\ u_4v_1x_4$.

P_4: $u_1v_1x_2\ u_1v_4x_2\ u_1v_3x_2\ u_1v_2x_2\ u_1v_2x_3\ u_1v_1x_3\ u_1v_4x_3\ u_1v_3x_3\ u_1v_3x_4\ u_1v_2x_4\ u_2v_2x_4\ u_2v_3x_4\ u_2v_4x_4\ u_1v_4x_4\ u_1v_4x_1\ u_1v_3x_1\ u_1v_2x_1\ u_1v_1x_1\ u_1v_1x_4\ u_2v_1x_4\ u_2v_1x_3\ u_2v_1x_2\ u_2v_1x_1\ u_3v_1x_1\ u_3v_1x_4\ u_4v_1x_4\ u_4v_1x_1$.

In two paths P_1 and P_2, take $u_1v_1x_1$ as a source vertex. If two vertices $u_iv_jx_k$ or $u_jv_ix_k \in P_1$, where $i, j, k = 1, 2, 3, 4$ and $i = j = k \neq 1$) and $u_iv_jx_k$ or $u_jv_ix_k \in P_2$ such that fuzzy shortest path from source vertex $u_1v_1x_1$ for $u_iv_jx_k$ or $u_jv_ix_k$ is through P_2 and fuzzy shortest path from source vertex $u_1v_1x_1$ for $u_iv_jx_k$ or $u_jv_ix_k$ is through P_1, then $\tilde{d}\,(u_1v_1x_1, u_iv_jx_k) = \tilde{d}\,(u_1v_1x_1, u_iv_jx_k)$ or $\tilde{d}\,(u_1v_1x_1, u_jv_ix_k) = \tilde{d}\,(u_1v_1x_1, u_jv_ix_k)$ if and only if $N(u_1v_1x_1, u_iv_jx_k) = N(u_1v_1x_1, u_iv_jx_k)$ or $N(u_1v_1x_1, u_jv_ix_k) = N(u_1v_1x_1, u_jv_ix_k)$. This implies that, $\tilde{\beta}\,(Q_6) \neq 1$. Include $u_2v_1x_1$ as another source vertex so that $N(u_2v_1x_1, u_iv_jx_k) \neq N(u_2v_1x_1, u_iv_jx_k)$ or $N(u_2v_1x_1, u_jv_ix_k) \neq N(u_2v_1x_1, u_jv_ix_k)$, $\tilde{d}\,(u_2v_1x_1, u_iv_jx_k) \neq \tilde{d}\,(u_2v_1x_1, u_iv_jx_k)$ or $\tilde{d}\,(u_2v_1x_1, u_jv_ix_k) \neq \tilde{d}\,(u_2v_1x_1, u_jv_ix_k)$.

Continuing this process for all four paths in Q_6, we get four source vertices for Q_6. $\tilde{M} = \{u_1v_1x_1, u_2v_1x_1, u_1v_1x_4, u_1v_1x_2\}$. Hence, $\tilde{\beta}\,(Q_6) \leq 4$.

C. *Fuzzy Metric Dimension of Hypercube Q_n.*

Theorem: 2.4. If $G = Q_n$, then $\frac{n}{2} \leq \tilde{\beta}\,(G) \leq n$.

Proof: Q_n can be decomposed into $(n/2)$ Hamiltonian cycles, by Theorem 1.7. We get, $\frac{n}{2} \leq \tilde{\beta}\,(Q_n) \leq n$, by Theorem 1.5.

III. FUZZY METRIC DIMENSION OF FUZZY BOOLEAN GRAPHS $BG_2(G)$ AND $BG_3(G)$

Let $G:(\sigma, \mu)$ be a fuzzy graph with its underlying set V and graph $G^* = (\sigma^*, \mu^*)$. Let $V(G)$ and $E(G)$ be the vertex set and edge set of G^* respectively. The pair $BG_2(G)$: $\left(\sigma_{BG_2(G)}, \mu_{BG_2(G)}\right)$ of G is defined as follows: Let the vertex set of $BG_2(G)$ be $V(G) \cup E(G)$. The fuzzy subset $\sigma_{BG_2(G)}$ is defined on $V(G) \cup E(G)$ as

$\sigma_{BG_2(G)}(u) = \sigma(u)$ if $u \in V(G)$

$\sigma_{BG_2(G)}(e) = \mu(e)$ if $e \in E(G)$

The fuzzy relation $\mu_{BG_2(G)}$ is defined as

$\mu_{BG_2(G)}(u,v)$ = μ(u, v) if u, v ∈ V(G), e = uv ∈ E(G)

$\mu_{BG_2(G)}(u,e)$ = 0 if e = uv ∉ E(G)

= μ(e), e ∈ E(G) and e is incident with u in G.

= 0, otherwise.

$\mu_{BG_2(G)}(e_i,e_j)$ = μ(e_i) ∧ μ(e_j), if the edges e_i and e_j have no common incident vertex in G.

= 0, otherwise.

By the definition, $\mu_{BG_2(G)}(x,y) \le \sigma_{BG_2(G)}(x) \wedge \sigma_{BG_2(G)}(y)$ for all x, y in V(G) ∪ E(G). Hence $\mu_{BG_2(G)}$ is a fuzzy relation on the fuzzy subset $\sigma_{BG_2(G)}$. Hence, the pair $BG_2(G)$: ($\sigma_{BG_2(G)}$, $\mu_{BG_2(G)}$) is a fuzzy graph and is termed as **Boolean fuzzy graph BG_2** of G - Second kind.

Similarly, the pair $BG_3(G)$: $\left(\sigma_{BG_3(G)}, \mu_{BG_3(G)}\right)$ of G is defined as follows. The fuzzy subset $\sigma_{BG_3(G)}$ is defined on V(G) ∪ E(G) as

$\sigma_{BG_3(G)}(u)$ = σ(u) if u ∈ V(G)

$\sigma_{BG_3(G)}(e)$ = μ(e) if e ∈ E(G)

The fuzzy relation $\mu_{BG_3(G)}$ is defined as

$\mu_{BG_3(G)}(u,v)$ = 0, if u, v ∈ V(G)

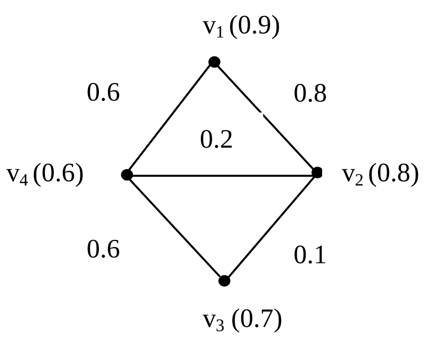
$\mu_{BG_3(G)}(u,e)$ = μ(e), e ∈ E(G) and e is incident with u in G.

= 0, otherwise.

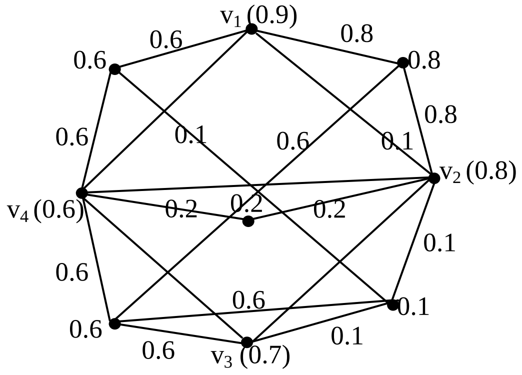
$\mu_{BG_3(G)}(e_i,e_j)$ = μ(e_i) ∧ μ(e_j), if the edges e_i and e_j have no common incident vertex in G.

= 0, otherwise.

By the definition, $\mu_{BG_3(G)}(x,y) \le \sigma_{BG_3(G)}(x) \wedge \sigma_{BG_3(G)}(y)$ for all u, v in V(G) ∪ E(G). Hence $\mu_{BG_3(G)}$ is a fuzzy relation on the fuzzy subset $\sigma_{BG_3(G)}$. Hence, the pair $BG_3(G)$: ($\sigma_{BG_3(G)}$, $\mu_{BG_3(G)}$) is a fuzzy graph and is termed as **Boolean fuzzy graph BG_3** of G - Third Kind.



Fuzzy graph G



Fuzzy Boolean Graph $BG_2(G)$

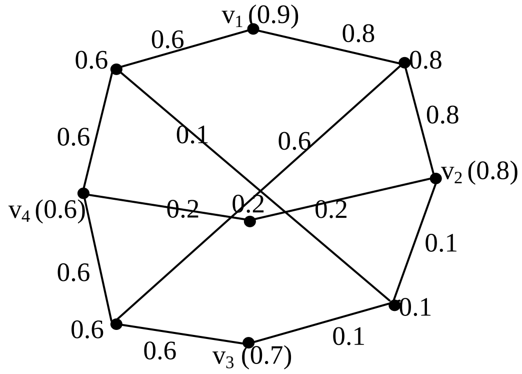


Figure: 3.1 Fuzzy Boolean Graph $BG_3(G)$

A. *Fuzzy Metric Dimension of Fuzzy Boolean Graph $BG_2(G)$.*

In this section, we determine fuzzy metric basis of Fuzzy Boolean Graph $BG_2(G)$ for some standard graphs of G.

Fuzzy Metric Dimension of $BG_2(P_n)$.

Theorem: 3.1 If G = $BG_2(P_n)$ (n > 3), then $\tilde{\beta}$ (G) ≤

$$\begin{cases} \dfrac{n+2}{2}, \text{when } n \text{ is odd.} \\ \dfrac{n+3}{2}, \text{when } n \text{ is even.} \end{cases}$$

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n and let $v_1v_2 = e_{12}$, $v_2v_3 = e_{23}, \ldots, v_{n-1}v_n = e_{n-1\,n}$ be the edges of P_n. We denote $e_{12} = e_1$, $e_{23} = e_2, \ldots, e_{n-1\,n} = e_n$. Edges of $BG_2(P_n)$ can be decomposed into P_n, P_{2n-1}, $\overline{P}_{n-1}$.

Case i: n is odd

Edges of $BG_2(P_n)$ can be decomposed into (n/2)+1 fuzzy paths as follows:

P_1: $v_n\, v_{n-1}\, v_{n-2}\, v_{n-3}\, v_{n-4} \ldots v_1\, e_1\, e_n\, e_2\, e_{n-1}\, e_3\, e_{n-2} \ldots e_{(n+4)/2}\, e_{n/2}$.

P_2: $e_1\, e_3\, e_n\, e_4\, e_{n-1} \ldots e_{(n+6)/2}\, e_{(n+2)/2}$.

P_3: $e_2\, e_4\, e_1\, e_5\, e_n \ldots e_{(n+8)/2}\, e_{(n+4)/2}$.

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$P_{n/2}$: $e_{(n-2)/2}\, e_{(n+2)/2}\, e_{(n-4)/2}\, e_{(m+4)/2}\, e_{(n-6)/2} \ldots e_{(n-1)}\, e_1$.

$P_{(n/2)+1}$: $v_n\, e_n\, v_{n-1}\, e_{n-1}\, v_{n-2}\, e_{n-2} \ldots e_1$.

In two paths P_1 and P_2, take $e_{n/2}$ as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from source vertex $e_{n/2}$ for v_k or e_l is through P_2 and fuzzy shortest path from source vertex $e_{n/2}$ for v_l or e_k is through P_1, then $\tilde{d}\,(e_{n/2}, v_k) = \tilde{d}\,(e_{n/2}, e_k)$ or $\tilde{d}\,(e_{n/2}, e_l) = \tilde{d}\,(e_{n/2}, v_l)$ if and only if $N(e_{n/2}, v_k) = N(e_{n/2}, e_k)$ or $N(e_{n/2}, e_l) = N(e_{n/2}, v_l)$. This implies that, $\tilde{\beta}\,(BG_2(P_1 \cup P_2)) \neq 1$. Include $e_{(n+2)/2}$ as another source vertex so that $N(e_{(n+2)/2}, v_k) \neq N(e_{(n+2)/2}, e_k)$ or $N(e_{(n+2)/2}, e_l) \neq N(e_{(n+2)/2}, v_l)$, $\tilde{d}\,(e_{(n+2)/2}, v_k) \neq \tilde{d}\,(e_{(n+2)/2}, e_k)$ or $\tilde{d}\,(e_{(n+2)/2}, e_l) \neq \tilde{d}\,(e_{(n+2)/2}, v_l)$.

Continuing this process for all (n/2) + 1 paths in $BG_2(P_n)$, we get (n/2)+1 source vertices for $BG_2(P_n)$. $\tilde{M} = \{e_{n/2}, e_{(n+2)/2}, e_{(n+4)/2}, \ldots, e_1, v_n\}$.

Hence, $\tilde{\beta}\,(BG_2(P_n)) \leq (n+2)/2$.

Case ii: n is even

Edges of $BG_2(P_n)$ can be decomposed into ((n+1)/2)+1 fuzzy paths as follows:

P_1: $v_n\ v_{n-1}\ v_{n-2}\ v_{n-3}\ v_{n-4}\ \ldots.\ v_1\ e_1\ e_n\ e_2\ e_{n-1}\ e_3\ e_{n-2}\ \ldots..\ e_{(n-1)/2}\ e_{(n+3)/2}$.

P_2: $e_1\ e_3\ e_n\ e_4\ e_{n-1}\ \ldots..\ e_{(n+1)/2}\ e_{(n+5)/2}$.

P_3: $e_2\ e_4\ e_1\ e_5\ e_n\ \ldots..\ e_{(n+3)/2}\ e_{(n+7)/2}$.

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$P_{(n+1)/2}$: $e_{(n-1)/2}\ e_{(n+3)/2}\ e_{(n-3)/2}\ e_{(n+5)/2}\ e_{(n-5)/2}\ \ldots..\ e_{(n-1)}\ e_1$.

$P_{((n+1)/2)+1}$: $v_n\ e_n\ v_{n-1}\ e_{n-1}\ v_{n-2}\ e_{n-2}\ \ldots..\ e_1$.

which has the same characterization as mentioned in the previous case.

Therefore, $\tilde{M} = \{e_{(n+3)/2}, e_{(n+5)/2}, e_{(n+7)/2}, \ldots, e_n, v_n\}$.

Hence, $\tilde{\beta}\,(BG_2(P_n)) \leq (n+3)/2$.

Fuzzy Metric Dimension of $BG_2(C_n)$.

Theorem: 3.2 If G = $BG_2(C_n)$, then $\tilde{\beta}\,(BG_2(C_n)) \leq$

$$\begin{cases} \dfrac{n+3}{2}, \text{when } n \text{ is odd.} \\ \dfrac{n+4}{2}, \text{when } n \text{ is even.} \end{cases}$$

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of C_n and let $v_1v_2 = e_{12}$, $v_2v_3 = e_{23}, \ldots, v_{n-1}v_n = e_{n-1\,n}$, $v_1v_n = e_{1\,n}$ be the edges of C_n.

Case i: n is even

Edges of $BG_2(C_n)$ can be decomposed into $\frac{n}{2}+2$ fuzzy paths as follows:

P_1: $v_1\ v_2\ e_{23}\ e_{45}\ e_{78}\ e_{11\ 12}\ \ldots..\ e_{n/2\ (n+2)/2}\ v_{(n+2)/2}\ v_{(n+4)/2}\ e_{(n+4)/2\ (n+6)/2}\ e_{(n+8)/2\ (n+10)/2}\ e_{(n+14)/2\ (n+16)/2}\ e_{(n+22)/2\ (n+24)/2}\ \ldots..\ e_{n1}$.

P_2: $v_2\ v_3\ e_{34}\ e_{56}\ e_{89}\ e_{12\ 1}\ \ldots..\ e_{(n+2)/2\ (n+4)/2}\ v_{(n+4)/2}\ v_{(n+6)/2}\ e_{(n+6)/2\ (n+8)/2}\ e_{(n+10)/2\ (n+12)/2}\ e_{(n+16)/2\ (n+18)/2}\ e_{(n+24)/2\ (n+26)/2}\ \ldots..\ e_{12}$.

P_3: $v_3\ v_4\ e_{45}\ e_{67}\ e_{9\ 10}\ e_{1\ 2}\ \ldots..\ e_{(n+4)/2\ (n+6)/2}\ v_{(n+6)/2}\ v_{(n+8)/2}\ e_{(n+8)/2\ (n+10)/2}\ e_{(n+12)/2\ (n+14)/2}\ e_{(n+18)/2\ (n+20)/2}\ e_{(n+26)/2\ (n+28)/2}\ \ldots..\ e_{23}$.

P_4: $v_4\ v_5\ e_{56}\ e_{78}\ e_{10\ 11}\ e_{2\ 3}\ \ldots..\ e_{(n+6)/2\ (n+8)/2}\ v_{(n+8)/2}\ v_{(n+10)/2}\ e_{(n+10)/2\ (n+12)/2}\ e_{(n+14)/2\ (n+16)/2}\ e_{(n+20)/2\ (n+22)/2}\ e_{(n+28)/2\ (n+30)/2}\ \ldots..\ e_{34}$.

P_5: $v_5\ v_6\ e_{67}\ e_{89}\ e_{1112}\ e_{34}\ \ldots..\ e_{(n+8)/2\ (n+10)/2}\ v_{(n+10)/2}\ v_{(n+12)/2}\ e_{(n+12)/2\ (n+14)/2}\ e_{(n+16)/2\ (n+18)/2}\ e_{(n+22)/2\ (n+24)/2}\ e_{(n+30)/2\ (n+32)/2}\ \ldots..\ e_{45}$.

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$P_{n/2}$: $v_{n/2}\ v_{(n+2)/2}\ e_{((n+2)/2)((n+4)/2)}\ e_{((n+6)/2)((n+8)/2)}\ e_{((n+12)/2)((n+14)/2)}\ e_{((n+20)/2)((n+22)/2)}\ \ldots..\ v_{(n+((n+6)/2))/2}\ v_{(n+((n+8)/2))/2}\ e_{(n+((n+8)/2))/2\ (n+((n+12)/2))/2}\ e_{(n+((n+12)/2))/2\ (n+((n+16)/2))/2}\ e_{(n+((n+16)/2))/2\ (n+((n+20)/2))/2}\ e_{(n+((n+24)/2))/2\ (n+((n+28)/2))/2}\ e_{(n+((n+36)/2))/2\ (n+((n+40)/2))/2}\ e_{(n+((n+52)/2))/2\ (n+((n+56)/2))/2}\ \ldots..\ e_{(n-2)/2\ n/2}$.

$P_{(n/2)+1}$: $e_{12}\ e_{(n+2)/2\ (n+4)/2}\ e_{23}\ e_{(n+4)/2\ (n+6)/2}\ e_{34}\ e_{(n+6)/2\ (n+8)/2}\ e_{45}\ e_{(n+8)/2\ (n+10)/2}\ e_{56}\ e_{(n+10)/2\ (n+12)/2}\ e_{n/2\ (n+2)/2}\ \ldots..\ e_{n1}$.

$P_{(n/2)+2}$: $e_{n1}\ v_1\ e_{12}\ v_2\ e_{23}\ v_3\ e_{34}\ v_4\ e_{45}\ v_5\ e_{56}\ v_6\ \ldots..\ e_{(n-4)/2\ (n-2)/2}\ v_{(n-2)/2}\ e_{(n-2)/2\ n/2}\ v_{n/2}$.

In two paths P_1 and P_2, take v_1 as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from v_1 for v_k or e_l is through P_2 and fuzzy shortest path from v_1 for v_l or e_k is through P_1, then $\tilde{d}\,(v_1, v_k) = \tilde{d}\,(v_1, e_k)$ or $\tilde{d}\,(v_1, e_l) = \tilde{d}\,(v_1, v_l)$ if and only if $N(v_1, v_k) = N(v_1, e_k)$ or $N(v_1, e_l) = N(v_1, v_l)$. This implies that, $\tilde{\beta}\,(BG_2(C_n)) \neq 1$. Include v_2 as another source vertex so that $N(v_2, v_k) \neq N(v_2, e_k)$ or $N(v_2, e_l) \neq N(v_2, v_l)$, $\tilde{d}\,(v_2, v_k) \neq \tilde{d}\,(v_2, e_k)$ or $\tilde{d}\,(v_2, e_l) \neq \tilde{d}\,(v_2, v_l)$.

Continuing this process for all (n/2) + 2 paths in $BG_2(C_n)$, we get (n/2)+2 source vertices for $BG_2(C_n)$. $\tilde{M} = \{v_1, v_2, v_3, \ldots, v_{n/2}, e_{n1}, e_{(n-2)/2\ n/2}\}$.

Hence, $\tilde{\beta}\,(BG_2(C_n)) \leq (n+4)/2$.

Case ii: n is odd

Edges of $BG_2(C_n)$ can be decomposed into (n+3)/2 fuzzy paths as follows:

P_1: $v_1\ v_2\ e_{23}\ e_{45}\ e_{78}\ e_{11\ 12}\ \ldots..\ e_{(n-1)/2\ (n+1)/2}\ v_{(n+1)/2}\ v_{(n+3)/2}\ e_{(n+3)/2\ (n+5)/2}\ e_{(n+7)/2\ (n+9)/2}\ e_{(n+13)/2\ (n+15)/2}\ e_{(n+21)/2\ (n+23)/2}\ \ldots..\ e_{n-11}$.

P_2: $v_2\ v_3\ e_{34}\ e_{56}\ e_{89}\ e_{12\ 1}\ \ldots..\ e_{(n+1)/2\ (n+3)/2}\ v_{(n+3)/2}\ v_{(n+5)/2}\ e_{(n+5)/2\ (n+7)/2}\ e_{(n+9)/2\ (n+11)/2}\ e_{(n+15)/2\ (n+17)/2}\ e_{(n+23)/2\ (n+25)/2}\ \ldots..\ e_{12}$.

P_3: $v_3\ v_4\ e_{45}\ e_{67}\ e_{9\ 10}\ e_{1\ 2}\ \ldots..\ e_{(n+3)/2\ (n+5)/2}\ v_{(n+5)/2}\ v_{(n+7)/2}\ e_{(n+7)/2\ (n+9)/2}\ e_{(n+11)/2\ (n+13)/2}\ e_{(n+17)/2\ (n+19)/2}\ e_{(n+25)/2\ (n+27)/2}\ \ldots..\ e_{23}$.

P_4: $v_4\ v_5\ e_{56}\ e_{78}\ e_{10\ 11}\ e_{2\ 3}\ \ldots..\ e_{(n+5)/2\ (n+7)/2}\ v_{(n+7)/2}\ v_{(n+9)/2}\ e_{(n+9)/2\ (n+11)/2}\ e_{(n+13)/2\ (n+15)/2}\ e_{(n+19)/2\ (n+21)/2}\ e_{(n+27)/2\ (n+29)/2}\ \ldots..\ e_{34}$.

P_5: $v_5\ v_6\ e_{67}\ e_{89}\ e_{1112}\ e_{34}\ \ldots..\ e_{(n+7)/2\ (n+9)/2}\ v_{(n+9)/2}\ v_{(n+11)/2}\ e_{(n+11)/2\ (n+13)/2}\ e_{(n+15)/2\ (n+17)/2}\ e_{(n+21)/2\ (n+23)/2}\ e_{(n+29)/2\ (n+31)/2}\ \ldots..\ e_{45}$.

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$P_{(n-1)/2}$: $v_{(n-1)/2}\ v_{(n+1)/2}\ e_{((n+1)/2)((n+3)/2)}\ e_{((n+5)/2)((n+7)/2)}\ e_{((n+11)/2)((n+13)/2)}\ e_{((n+19)/2)((n+21)/2)}\ \ldots..\ v_{(n+((n+5)/2))/2}\ v_{(n+((n+7)/2))/2}\ e_{(n-1+((n+7)/2))/2\ (n-1+((n+11)/2))/2}\ e_{(n-1+((n+11)/2))/2\ (n-1+((n+15)/2))/2}\ e_{(n-1+((n+15)/2))/2\ (n-1+((n+19)/2))/2}\ e_{(n-1+((n+23)/2))/2\ (n-1+((n+27)/2))/2}\ e_{(n-1+((n+35)/2))/2\ (n-1+((n+39)/2))/2}\ e_{(n-1+((n+51)/2))/2\ (n-1+((n+55)/2))/2}\ \ldots..\ e_{(n-3)/2\ (n-1)/2}$.

$P_{(n+1)/2}$: $v_n\ v_1\ e_{12}\ e_{34}\ e_{67}\ \ldots..\ e_{lm}\ v_m$

$P_{(n+3)/2}$: $e_{n1}\ v_1\ e_{12}\ v_2\ e_{23}\ v_3\ e_{34}\ v_4\ e_{45}\ v_5\ e_{56}\ v_6\ \ldots..\ e_{(n-5)/2\ (n-3)/2}\ v_{(n-3)/2}\ e_{(n-3)/2\ (n-1)/2}\ v_{(n-1)/2}$.

which has the same characterization as mentioned the previous case.

Therefore, $\tilde{M} = \{v_1, v_2, v_3, \ldots, v_{(n-1)/2}, v_n, e_{12}\}$. Hence, $\tilde{\beta}\,(BG_2(C_n)) \leq (n+3)/2$.

Fuzzy Metric Dimension of $BG_2(nK_2)$.

Theorem: 3.3 If G = $BG_2(nK_2)$ then $\tilde{\beta}\,(BG_2(G)) \leq n$.

Proof: Let $v_1, v_2, v_3, \ldots, v_{2n}$ be the vertices of nK_2 and let $v_1v_2 = e_{12}$, $v_3v_4 = e_{34}, \ldots, v_{2m-1}v_{2m} = e_{2m-1\ 2m}$ be the edges of nK_2. We denote $e_{12} = e_1$, $e_{34} = e_2, \ldots, e_{2m-1\ 2m} = e_m$. Edges of $BG_2(nK_2)$ can be decomposed into K_n and n triangles.

Case i: n is even

We know that K_n ($n \geq 4$) is decomposable into two fuzzy paths as follows:

(i) n/2 Hamiltonian fuzzy paths of length n – 1.(or)

(ii) n – 1 fuzzy paths of length n/2.

Thus, Edges of $BG_2(nK_2)$ can be decomposed into n fuzzy paths as follows:

P_1: $v_{2i-1}\ v_{2i}\ e_1\ e_2\ e_n\ e_3\ e_{n-1}\ e_4\ e_{n-2}\ e_5 \ldots\ e_{(n+4)/2}\ e_{(n+2)/2}\ v_{2m-1}$.

P_2: $v_{2i-1}\ v_{2i}\ e_2\ e_3\ e_1\ e_4\ e_n\ e_5\ e_{n-1}\ e_6 \ldots\ e_{(n+6)/2}\ e_{(n+4)/2}\ v_{2m-1}$.

P_3: $v_{2i-1}\ v_{2i}\ e_3\ e_4\ e_2\ e_5\ e_1\ e_6\ e_n\ e_7 \ldots\ e_{(n+8)/2}\ e_{(n+6)/2}\ v_{2m-1}$.

P_4: $v_{2i-1}\ v_{2i}\ e_4\ e_5\ e_3\ e_6\ e_2\ e_7\ e_1\ e_8 \ldots\ e_{(n+10)/2}\ e_{(n+8)/2}\ v_{2m-1}$.

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$P_{n/2}$: $v_{2i-1}\ v_{2i}\ e_{n/2}\ e_{(n/2)+1}\ e_{(n-2)/2}\ e_{(n+4)/2}\ e_{(n-4)/2}\ e_{(n+6)/2}\ e_{(n-6)/2} \ldots\ e_1\ e_n\ v_{2m-1}$, where i = 1, 2, 3, …, n/2.

$P_{(n/2)+1}$: $v_{2i-1}\ v_{2i}\ e_{(n/2)+1}\ e_{(n/2)+2}\ v_{2j-1}\ v_{2j}$.

$P_{(n/2)+2}$: $v_{2i-1}\ v_{2i}\ e_{(n/2)+2}\ e_{(n/2)+3}\ v_{2j-1}\ v_{2j}$.

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P_{n-1}: $v_{2i-1}\ v_{2i}\ e_{(n/2)+((n/2)-1)}\ e_{(n/2)+(n/2)}\ v_{2j-1}\ v_{2j}$, where $i < j = i+1$, i = 1, 2, 3, …, n/2.

P_n: $v_{2i-1}\ e_{(n/2)+1}\ e_{(n/2)+2} \ldots\ e_{(n/2)+(n/2)}\ v_{2j}$, where $i < j = i + ((n/2) - 1)$.

In two paths P_1 and P_2, v_1 is fixed as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from v_1 for v_k or e_l is through P_2 and fuzzy shortest path from v_1 for v_l or e_k is through P_1, then $\tilde{d}(v_1, v_k) = \tilde{d}(v_1, e_k)$ or $\tilde{d}(v_1, e_l) = \tilde{d}(v_1, v_l)$ if and only if $N(v_1, v_k) = N(v_1, e_k)$ or $N(v_1, e_l) = N(v_1, v_l)$. This implies that, $\tilde{\beta}(BG_2(nK_2)) \neq 1$. Include v_3 as another source vertex so that $N(v_3, v_k) \neq N(v_3, e_k)$ or $N(v_3, e_l) \neq N(v_3, v_l)$, $\tilde{d}(v_3, v_k) \neq \tilde{d}(v_3, e_k)$ or $\tilde{d}(v_3, e_l) \neq \tilde{d}(v_3, v_l)$ Continuing this process for all n paths in $BG_2(nK_2)$, we get n source vertices for $BG_2(nK_2)$. $\tilde{M} = \{v_1, v_2, v_3, \ldots, v_n\}$. Hence, $\tilde{\beta}(BG_2(nK_2)) \leq n$.

Case ii: n is odd.

We know that K_n ($n \geq 3$) is decomposable into n fuzzy paths of length (n–1)/2.

P_j: $e_j\ e_{j+1}\ e_{j+3} \ldots\ e_{j+((n-1)(n+1)/8)}$, where j = 1, 2, 3, …, n.

And also $BG_2(G)$ can be decomposed into n fuzzy paths of length (n+5)/2 as follows:

P_j: $v_{2j-1}\ v_{2j}\ e_j\ e_{j+1}\ e_{j+3} \ldots\ e_{j+((n-1)(n+1)/8)}\ v_{2m-1}$, where j = 1, 2, 3, …, n.

which has the same characterization as mentioned in the previous case.

Therefore, $\tilde{M} = \{e_1, e_2, e_3, \ldots, e_n\}$. Hence, $\tilde{\beta}(BG_2(nK_2)) \leq n$.

Fuzzy Metric Dimension of $BG_2(S_{1,n})$.

Theorem: 3.4 If $G = BG_2(S_{1,n})$ then $\tilde{\beta}(BG_2(G)) \leq n$.

Proof: Let $S_{1,n}$ be a star fuzzy graph with n +1 vertices and n edges. Let $v_1, v_2, v_3, \ldots, v_n$ be the n pendant vertices of $S_{1,n}$ and let $v_1v = e_1$, $vv_2 = e_2, \ldots, vv_n = e_n$ be the edges of $S_{1,n}$, where v is the central vertex of $S_{1,n}$. $BG_2(S_{1,n})$ can be decomposed into $S_{1,n}$, subdivision graph of $S_{1,n}$.

Case i: n is even

Edges of $BG_2(S_{1,n})$ can be decomposed into n/2 fuzzy paths of length two and n/2 fuzzy paths of length of four as follows:

P_1: $v_1\ v\ v_2$.

P_2: $v_3\ v\ v_4$.

P_3: $v_5\ v\ v_6$.

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$P_{n/2}$: $v_{n-1}\ v\ v_n$.

$P_{(n/2)+1}$: $v_1\ e_1\ v\ e_2\ v_2$.

$P_{(n/2)+2}$: $v_3\ e_3\ v\ e_4\ v_4$.

$P_{(n/2)+3}$: $v_5\ e_5\ v\ e_6\ v_6$.

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P_n: $v_{n-1}\ e_{n-1}\ v\ e_n\ v_n$.

In two paths P_1 and P_2, v_1 is fixed as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from v_1 for v_k or e_l is through P_2 and fuzzy shortest path from v_1 for v_l or e_k is through P_1, then $\tilde{d}(v_1, v_k) = \tilde{d}(v_1, e_k)$ or $\tilde{d}(v_1, e_l) = \tilde{d}(v_1, v_l)$ if and only if $N(v_1, v_k) = N(v_1, e_k)$ or $N(v_1, e_l) = N(v_1, v_l)$. This implies that, $\tilde{\beta}(BG_2(S_{1,n})) \neq 1$. Include v_3 as another source vertex so that $N(v_3, v_k) \neq N(v_3, e_k)$ or $N(v_3, e_l) \neq N(v_3, v_l)$, $\tilde{d}(v_3, v_k) \neq \tilde{d}(v_3, e_k)$ or $\tilde{d}(v_3, e_l) \neq \tilde{d}(v_3, v_l)$. Continuing this process for all n paths in $BG_2(S_{1,n})$, we get n source vertices for $BG_2(S_{1,n})$.

Therefore, $\tilde{M} = \{v_1, v_3, \ldots, v_{n-1}, e_1, e_3, \ldots, e_{n-1}\}$. Hence, $\tilde{\beta}(BG_2(S_{1,n})) \leq n$.

Case ii: n is odd.

Edges of $BG_2(S_{1,n})$ can be decomposed into n fuzzy paths of length three as follows:

P_1: $v_1\ e_1\ v\ v_2$.

P_2: $v_2\ e_2\ v\ v_3$.

P_3: $v_3\ e_3\ v\ v_4$.

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P_n: $v_n\ e_n\ v\ v_1$.

In two paths P_1 and P_2, e_1 is fixed as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from e_1 for v_k or e_l is through P_2 and fuzzy shortest path from e_1 for v_l or e_k is through P_1, then $\tilde{d}(e_1, v_k) = \tilde{d}(e_1, e_k)$ or $\tilde{d}(e_1, e_l) = \tilde{d}(e_1, v_l)$ if and only if $N(e_1, v_k) = N(e_1, e_k)$ or $N(e_1, e_l) = N(e_1, v_l)$. This implies that, $\tilde{\beta}(BG_2(S_{1,n})) \neq 1$. Include e_2 as another source vertex so that $N(e_2, v_k) \neq N(e_2, e_k)$ or $N(e_2, e_l) \neq N(e_2, v_l)$, $\tilde{d}(e_2, v_k \text{ or } e_l) \neq \tilde{d}(e_2, e_k \text{ or } v_l)$ or $\tilde{d}(e_2, v_k \text{ or } e_l) \neq \tilde{d}(e_2, e_k \text{ or } v_l)$.

Continuing this process for all n paths in $BG_2(S_{1,n})$, we get n source vertices for $BG_2(S_{1,n})$. $\tilde{M} = \{e_1, e_2, e_3, \ldots, e_n\}$. Hence, $\tilde{\beta}(BG_2(S_{1,n})) \leq n$.

B. *Fuzzy Metric dimension of fuzzy Boolean graph $BG_3(G)$.*

In this section, We determine the fuzzy Metric basis of fuzzy Boolean graph $BG_3(G)$ for some standard fuzzy graphs G.

Fuzzy Metric Dimension of $BG_3(P_n)$.

Theorem: 3.5 If $G = BG_3(P_n)$ ($n > 3$), then $\tilde{\beta}(G) \leq$

$$\begin{cases} \frac{n+2}{2}, \text{when } n \text{ is odd.} \\ \frac{n+3}{2}, \text{when } n \text{ is even.} \end{cases}$$

Proof: Let $v_1, v_2, v_3, \ldots, v_n \in V(P_n)$ and let $v_1v_2 = e_{12}$, $v_2v_3 = e_{23}, \ldots, v_{n-1}v_n = e_{n-1\,n} \in E(P_n)$. We denote $e_{12} = e_1$, $e_{23} = e_2, \ldots, e_{n-1\,n} = e_n$.

Edges of $BG_3(P_n)$ can be partitioned into P_{2n-1}, $\overline{P}_{n-1}$

Case i: n is odd

Edges of $BG_3(P_n)$ can be decomposed into (n/2) + 1 fuzzy paths as follows:

P_1: $v_1\ e_1\ e_n\ e_2\ e_{n-1}\ e_3\ e_{n-2} \ldots\ldots e_{(n+4)/2}\ e_{n/2}$.

P_2: $e_1\ e_3\ e_n\ e_4\ e_{n-1} \ldots\ldots e_{(n+6)/2}\ e_{(n+2)/2}$.

P_3: $e_2\ e_4\ e_1\ e_5\ e_n \ldots\ldots e_{(n+8)/2}\ e_{(n+4)/2}$.

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$P_{n/2}$: $e_{(n-2)/2}\ e_{(n+2)/2}\ e_{(n-4)/2}\ e_{(n+4)/2}\ e_{(n-6)/2} \ldots\ldots e_{(n-1)}\ e_1$.

$P_{(n/2)+1}$: $v_n\ e_n\ v_{n-1}\ e_{n-1}\ v_{n-2}\ e_{n-2} \ldots\ldots e_1\ v_1$.

In two paths P_1 and P_2 of $BG_3(G)$, take e_m as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from e_m for v_k or e_l is through P_2 and fuzzy shortest path from e_m for v_l or e_k is through P_1, then $\tilde{d}$ (e_m, v_k) = $\tilde{d}$ (e_m, e_k) or $\tilde{d}$ (e_m, e_l) = $\tilde{d}$ (e_m, v_l) if and only if $N(e_m, v_k)$ = $N(e_m, e_k)$ or $N(e_m, e_l)$ = $N(e_m, v_l)$. This implies that, $\tilde{\beta}$ $(BG_3(G)) \neq 1$. Include e_1 as another source vertex so that $N(e_1, v_k) \neq N(e_1, e_k)$ or $N(e_1, e_l) \neq N(e_1, v_l)$, $\tilde{d}$ $(e_1, v_k$ or $e_l) \neq$ $\tilde{d}$ $(e_1, e_k$ or $v_l)$ or $\tilde{d}$ $(e_1, v_k$ or $e_l) \neq$ $\tilde{d}$ $(e_1, e_k$ or $v_l)$

Continuing this process for all (n/2) + 1 paths in $BG_3(P_n)$, we get (n/2) + 1 source vertices for $BG_3(P_n)$. $\tilde{M} = \{e_n, e_1, e_2, \ldots, e_{n-2}, v_n\}$. Hence, $\tilde{\beta}$ $(BG_3(P_n)) \leq (n+2)/2$.

Case ii: n is even

Edges of $BG_3(P_n)$ can be decomposed into ((n+3)/2) fuzzy paths as follows:

P_1: $e_1\ e_n\ e_2\ e_{n-1}\ e_3\ e_{n-2} \ldots\ldots e_{(n-1)/2}\ e_{(n+3)/2}$.

P_2: $e_1\ e_3\ e_n\ e_4\ e_{n-1} \ldots\ldots e_{(n+1)/2}\ e_{(n+5)/2}$.

P_3: $e_2\ e_4\ e_1\ e_5\ e_n \ldots\ldots e_{(n+3)/2}\ e_{(n+7)/2}$.

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$P_{(n+1)/2}$: $e_{(n-1)/2}\ e_{(n+3)/2}\ e_{(n-3)/2}\ e_{(n+5)/2}\ e_{(n-5)/2} \ldots\ldots e_{(n-1)}\ e_1$.

$P_{(n+3)/2}$: $v_n\ e_n\ v_{n-1}\ e_{n-1}\ v_{n-2}\ e_{n-2} \ldots\ldots e_1\ v_1$.

which has the same characterization as mentioned in the previous case.

Therefore, $\tilde{M} = \{e_n, e_1, e_2, \ldots, e_{(n-1)/2}, v_n\}$. Hence, $\tilde{\beta}$ $(BG_3(P_n)) \leq (n+3)/2$.

Fuzzy Metric Dimension of $BG_3(C_n)$.

Theorem: 3.6 If $G = BG_3(C_n)$, then $\tilde{\beta}$ $(BG_3(C_n)) \leq$

$$\begin{cases} \frac{n+5}{2}, \text{when } n \text{ is odd.} \\ \frac{n+6}{2}, \text{when } n \text{ is even.} \end{cases}$$

Proof: Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of C_n and let $v_1v_2 = e_{12}$, $v_2v_3 = e_{23}, \ldots, v_{n-1}v_n = e_{n-1\,n}$, $v_1v_n = e_{1\,n}$ be the edges of C_n.

Case i: n is even

Edges of $BG_3(C_n)$ can be decomposed into $\frac{n}{2}+3$ fuzzy paths as follows:

P_1: $v_2\ e_{23}\ e_{45}\ e_{78}\ e_{11\,12} \ldots\ldots e_{n/2\,(n+2)/2}\ e_{(n+4)/2\,(n+6)/2}\ e_{(n+8)/2\,(n+10)/2}\ e_{(n+14)/2\,(n+16)/2}\ e_{(n+22)/2\,(n+24)/2} \ldots\ldots e_{n1}$.

P_2: $v_3\ e_{34}\ e_{56}\ e_{89}\ e_{12\,1} \ldots\ldots e_{(n+2)/2\,(n+4)/2}\ e_{(n+6)/2\,(n+8)/2}\ e_{(n+10)/2\,(n+12)/2}\ e_{(n+16)/2\,(n+18)/2}\ e_{(n+24)/2\,(n+26)/2} \ldots\ldots e_{12}$.

P_3: $v_4\ e_{45}\ e_{67}\ e_{9\,10}\ e_{1\,2} \ldots\ldots e_{(n+4)/2\,(n+6)/2}\ e_{(n+8)/2\,(n+10)/2}\ e_{(n+12)/2\,(n+14)/2}\ e_{(n+18)/2\,(n+20)/2}\ e_{(n+26)/2\,(n+28)/2} \ldots\ldots e_{23}$.

P_4: $v_5\ e_{56}\ e_{78}\ e_{10\,11}\ e_{2\,3} \ldots\ldots e_{(n+6)/2\,(n+8)/2}\ e_{(n+10)/2\,(n+12)/2}\ e_{(n+14)/2\,(n+16)/2}\ e_{(n+20)/2\,(n+22)/2}\ e_{(n+28)/2\,(n+30)/2} \ldots\ldots e_{34}$.

P_5: $v_6\ e_{67}\ e_{89}\ e_{1112}\ e_{34} \ldots\ldots e_{(n+8)/2\,(n+10)/2}\ e_{(n+12)/2\,(n+14)/2}\ e_{(n+16)/2\,(n+18)/2}\ e_{(n+22)/2\,(n+24)/2}\ e_{(n+30)/2\,(n+32)/2} \ldots\ldots e_{45}$.

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$P_{n/2}$: $v_{(n+2)/2}\ e_{((n+2)/2)((n+4)/2)}\ e_{((n+6)/2)((n+8)/2)}\ e_{((n+12)/2)((n+14)/2)}\ e_{((n+20)/2)((n+22)/2)} \ldots\ldots e_{(n+((n+8)/2))/2\ (n+((n+12)/2))/2}\ e_{(n+((n+12)/2))/2\ (n+((n+16)/2))/2}\ e_{(n+((n+16)/2))/2\ (n+((n+20)/2))/2}\ e_{(n+((n+24)/2))/2\ (n+((n+28)/2))/2}\ e_{(n+((n+36)/2))/2\ (n+((n+40)/2))/2}\ e_{(n+((n+52)/2))/2\ (n+((n+56)/2))/2} \ldots\ldots e_{(n-2)/2\ n/2}$.

$P_{(n/2)+1}$: $e_{12}\ e_{(n+2)/2\ (n+4)/2}\ e_{23}\ e_{(n+4)/2\ (n+6)/2}\ e_{34}\ e_{(n+6)/2\ (n+8)/2}\ e_{45}\ e_{(n+8)/2\ (n+10)/2}\ e_{56}\ e_{(n+10)/2\ (n+12)/2}\ e_{n/2\ (n+2)/2} \ldots\ldots e_{n1}$.

$P_{(n/2)+2}$: $e_{n1}\ v_1\ e_{12}\ v_2\ e_{23}\ v_3\ e_{34}\ v_4\ e_{45}\ v_5\ e_{56}\ v_6 \ldots\ldots e_{(n-1)n}\ v_n$.

$P_{(n/2)+3}$: $v_{(n+4)/2}\ e_{(n+4)/2\ (n+6)/2}\ v_{(n+6)/2}\ e_{(n+6)/2\ (n+8)/2}\ v_{(n+8)/2}\ e_{(n+8)/2\ (n+10)/2}\ v_{(n+10)/2}\ e_{(n+10)/2\ (n+12)/2}\ v_{(n+12)/2}\ e_{(n+12)/2\ (n+14)/2} \ldots\ldots v_{(n+(n+8)/2)/2}\ e_{(n+(n+8)/2)/2\ (n+(n+12)/2)/2}$.

In two paths P_1 and P_2 of $BG_3(G)$, v_2 is fixed as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from v_2 for v_k or e_l is through P_2 and fuzzy shortest path from v_2 for v_l or e_k is through P_1, then $\tilde{d}$ (v_2, v_k) = $\tilde{d}$ (v_2, e_k) or $\tilde{d}$ (v_2, e_l) = $\tilde{d}$ (v_2, v_l) if and only if $N(v_2, v_k)$ = $N(v_2, e_k)$ or $N(v_2, e_l)$ = $N(v_2, v_l)$. This implies that, $\tilde{\beta}$ $(BG_3(C_n)) \neq 1$. Include v_3 as another source vertex so that $N(v_3, v_k) \neq N(v_3, e_k)$ or $N(v_3, e_l) \neq N(v_3, v_l)$, $\tilde{d}$ $(v_3, v_k) \neq$ $\tilde{d}$ (v_3, e_k) or $\tilde{d}$ $(v_3, e_l) \neq$ $\tilde{d}$ (v_3, v_l).

Continuing this process for all (n/2) + 3 paths in $BG_3(C_n)$, we get (n/2) + 3 source vertices for $BG_3(C_n)$. $\tilde{M} = \{v_2, v_3, \ldots, v_{(n+2)/2}, e_{12}, e_{n1}, v_{(n+4)/2}\}$. Hence, $\tilde{\beta}$ $(BG_3(C_n)) \leq (n+6)/2$.

Case ii: n is odd

Edges of $BG_3(C_n)$ can be decomposed into (n+5)/2 fuzzy paths as follows:

P_1: $v_2\ e_{23}\ e_{45}\ e_{78}\ e_{11\,12} \ldots\ldots e_{(n-1)/2\,(n+1)/2}\ e_{(n+3)/2\,(n+5)/2}\ e_{(n+7)/2\,(n+9)/2}\ e_{(n+13)/2\,(n+15)/2}\ e_{(n+21)/2\,(n+23)/2} \ldots\ldots e_{n-11}$.

P_2: $v_3\ e_{34}\ e_{56}\ e_{89}\ e_{12\,1} \ldots\ldots e_{(n+1)/2\,(n+3)/2}\ e_{(n+5)/2\,(n+7)/2}\ e_{(n+9)/2\,(n+11)/2}\ e_{(n+15)/2\,(n+17)/2}\ e_{(n+23)/2\,(n+25)/2} \ldots\ldots e_{12}$.

P_3: $v_4\ e_{45}\ e_{67}\ e_{9\,10}\ e_{1\,2} \ldots\ldots e_{(n+3)/2\,(n+5)/2}\ e_{(n+7)/2\,(n+9)/2}\ e_{(n+11)/2\,(n+13)/2}\ e_{(n+17)/2\,(n+19)/2}\ e_{(n+25)/2\,(n+27)/2} \ldots\ldots e_{23}$.

P_4: $v_5\ e_{56}\ e_{78}\ e_{10\,11}\ e_{2\,3} \ldots\ldots e_{(n+5)/2\,(n+7)/2}\ e_{(n+9)/2\,(n+11)/2}\ e_{(n+13)/2\,(n+15)/2}\ e_{(n+19)/2\,(n+21)/2}\ e_{(n+27)/2\,(n+29)/2} \ldots\ldots e_{34}$.

P_5: $v_6\ e_{67}\ e_{89}\ e_{1112}\ e_{34} \ldots\ldots e_{(n+7)/2\,(n+9)/2}\ e_{(n+11)/2\,(n+13)/2}\ e_{(n+15)/2\,(n+17)/2}\ e_{(n+21)/2\,(n+23)/2}\ e_{(n+29)/2\,(n+31)/2} \ldots\ldots e_{45}$.

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$P_{(n-1)/2}$: $v_{(n+1)/2}\ e_{((n+1)/2)((n+3)/2)}\ e_{((n+5)/2)((n+7)/2)}\ e_{((n+11)/2)((n+13)/2)}\ e_{((n+19)/2)((n+21)/2)} \ldots\ldots e_{(n-1+((n+7)/2))/2\ (n-1+((n+11)/2))/2}$
$e_{(n-1+((n+11)/2))/2\ (n-1+((n+15)/2))/2}$
$e_{(n-1+((n+15)/2))/2\ (n-1+((n+19)/2))/2}$

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$e_{(n-1+((n+23)/2))/2}$ $_{(n-1+((n+27)/2))/2}$ $e_{(n-1+((n+35)/2))/2}$ $_{(n-1+((n+39)/2))/2}$ $e_{(n-1+((n+51)/2))/2\ (n-1+(n+55)/2))/2}$ ….. $e_{(n-3)/2\ (n-1)/2}$.

$P_{(n+1/2)}$: $v_1\ e_{12}\ e_{34}\ e_{67}$ ….. $e_{lm}\ v_m$.

$P_{(n+3/2)}$: $e_{n1}\ v_1\ e_{12}\ v_2\ e_{23}\ v_3\ e_{34}\ v_4\ e_{45}\ v_5\ e_{56}\ v_6$ ….. $e_{(n-1)\ n}\ v_n$.

$P_{(n+5)/2}$: $v_{(n+3)/2}\ e_{(n+3)/2\ (n+5)/2}\ v_{(n+5)/2}\ e_{(n+5)/2\ (n+7)/2}\ v_{(n+7)/2}\ e_{(n+7)/2\ (n+9)/2}\ v_{(n+9)/2}\ e_{(n+9)/2\ (n+11)/2}\ v_{(n+11)/2}\ e_{(n+11)/2\ (n+13)/2}$ ….. $v_{(n+(n+7)/2)/2}$ $e_{((n-1)+(n+7)/2)/2\ ((n-1)+(n+11)/2)/2}$.

which has the same characterization as mentioned in the previous case. Therefore, $\tilde{M} = \{v_2, v_3, \ldots, v_{(n+1)/2}, v_1, v_n\}$. Hence, $\tilde{\beta}$ $(BG_3(C_n)) \le (n+5)/2$.

Fuzzy Metric Dimension of $BG_3(nK_2)$.

Theorem: 3.7 If G = $BG_3(nK_2)$ then $\tilde{\beta}$ $(BG_2(G)) \le n$.

Proof: Let $v_1, v_2, v_3, \ldots, v_{2n}$ be the vertices of nK_2 and let $v_1v_2 = e_{12}$, $v_3v_4 = e_{34}, \ldots, v_{2n-1}v_{2n} = e_{2n-1\ 2n}$ be the edges of nK_2. We denote $e_{12} = e_1$, $e_{34} = e_2, \ldots, e_{2n-1\ 2n} = e_n$.

Edges of $BG_3(nK_2)$ can be decomposed into K_n and n paths of length two.

Case i: n is even

We know that K_n $(n \ge 4)$ is decomposable into two fuzzy paths as follows:

(i) n/2 Hamiltonian fuzzy paths of length n – 1.(or)

(ii) n – 1 fuzzy paths of length n/2.

Thus, Edges of BG_3 (nK_2) can be decomposed into n fuzzy paths as follows:

P_1: $v_{2i}\ e_1\ e_2\ e_n\ e_3\ e_{n-1}\ e_4\ e_{n-2}\ e_5$ ….. $e_{(n+4)/2}\ e_{(n+2)/2}\ v_{2m-1}$.

P_2: $v_{2i}\ e_2\ e_3\ e_1\ e_4\ e_n\ e_5\ e_{n-1}\ e_6$ ….. $e_{(n+6)/2}\ e_{(n+4)/2}\ v_{2m-1}$.

P_3: $v_{2i}\ e_3\ e_4\ e_2\ e_5\ e_1\ e_6\ e_n\ e_7$ ….. $e_{(n+8)/2}\ e_{(n+6)/2}\ v_{2m-1}$.

P_4: $v_{2i}\ e_4\ e_5\ e_3\ e_6\ e_2\ e_7\ e_1\ e_8$ ….. $e_{(n+10)/2}\ e_{(n+8)/2}\ v_{2m-1}$.

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$P_{n/2}$: $v_{2i}\ e_{n/2}\ e_{(n/2)+1}\ e_{(n-2)/2}\ e_{(n+4)/2}\ e_{(n-4)/2}\ e_{(n+6)/2}\ e_{(n-6)/2}$ ….. $e_1\ e_n\ v_{2m-1}$, where i = 1, 2, 3, …, n/2.

$P_{(n/2)+1}$: $v_{2i}\ e_{(n/2)+1}\ e_{(n/2)+2}\ v_{2j-1}$.

$P_{(n/2)+2}$: $v_{2i}\ e_{(n/2)+2}\ e_{(n/2)+3}\ v_{2j-1}$.

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P_{n-1}: $v_{2i}\ e_{(n/2)+((n/2)-1)}\ e_{(n/2)+(n/2)}\ v_{2j-1}$, where i < j = i + 1, i = 1, 2, 3, …, n/2.

P_n: $v_{2i-1}\ e_{(n/2)+1}\ e_{(n/2)+2}$ ….. $e_{(n/2)+(n/2)}\ v_{2j}$, where i < j = i +((n/2) – 1).

In two paths P_1 and P_2 of $BG_3(G)$, take e_1 as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from e_1 for v_k or e_l is through P_2 and fuzzy shortest path from e_1 for v_l or e_k is through P_1, then $\tilde{d}$ (e_1, v_k) = $\tilde{d}$ (e_1, e_k) or $\tilde{d}$ (e_1, e_l) = $\tilde{d}$ (e_1, v_l)if and only if $N(e_1, v_k) = N(e_1, e_k)$ or $N(e_1, e_l) = N(e_1, v_l)$. This implies that, $\tilde{\beta}$ $(BG_2(nK_2)) \ne 1$. Include e_2 as another source vertex so that $N(e_2, v_k) \ne N(e_2, e_k)$ or $N(e_2, e_l) \ne N(e_2, v_l)$, $\tilde{d}$ $(e_2, v_k) \ne \tilde{d}$ (e_2, e_k) or $\tilde{d}$ $(e_2, e_l) \ne \tilde{d}$ (e_2, v_l).

Continuing this process for all n paths in $BG_3(nK_2)$, we get n source vertices for $BG_3(nK_2)$. $\tilde{M} = \{e_1, e_2, e_3, \ldots, e_{n/2}, e_{(n/2)+1}, \ldots, e_n\}$. Hence, $\tilde{\beta}$ $(BG_3(nK_2)) \le n$.

Case ii: n is odd.

We know that K_n $(n \ge 3)$ is decomposable into n fuzzy paths of length (n–1)/2.

P_j: $e_j\ e_{j+1}\ e_{j+3}$ ….. $e_{j+((n-1)(n+1)/8)}$, where j = 1, 2, 3, …, n.

Also Edges of $BG_3(G)$ can be decomposed into n fuzzy paths of length (n+3)/2 as follows:

P_j: $v_{2j}\ e_j\ e_{j+1}\ e_{j+3}$ ….. $e_{j+((n-1)(n+1)/8)}\ v_{2m-1}$, where j = 1, 2, 3, …, n.

which has the same characterization as mentioned in the previous case.

Therefore, $\tilde{M} = \{e_1, e_2, e_3, \ldots, e_n\}$. Hence, $\tilde{\beta}$ $(BG_3(nK_2)) \le n$.

Fuzzy Metric Dimension of $BG_3(S_{1,n})$.

Theorem: 3.8 If G = $BG_3(S_{1,n})$ then $\tilde{\beta}$ $(BG_3(G)) \le$

$$\begin{cases} \frac{n+1}{2}, \textit{when n is odd.} \\ \frac{n}{2}, \textit{when n is even.} \end{cases}$$

Proof: Let $S_{1,n}$ be a star fuzzy graph with n +1 vertices and n edges. Let $v_1, v_2, v_3, \ldots, v_n$ be the n pendant vertices of $S_{1,n}$ and let $v_1v = e_1$, $vv_2 = e_2, \ldots, vv_n = e_n$ be the edges of $S_{1,n}$, where v is the central vertex of $S_{1,n}$. Edges of $BG_3(S_{1,n})$ can be decomposed into subdivision graph of $S_{1,n}$.

Case i: n is even

Edges of $BG_3(S_{1,n})$ can be decomposed into n/2 fuzzy paths of length of four as follows:

P_1: $v_1\ e_1\ v\ e_2\ v_2$.

P_2: $v_3\ e_3\ v\ e_4\ v_4$.

P_3: $v_5\ e_5\ v\ e_6\ v_6$.

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$P_{n/2}$: $v_{n-1}\ e_{n-1}\ v\ e_n\ v_n$.

In two paths P_1 and P_2, v_1 is fixed as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from v_1 for v_k or e_l is through P_2 and fuzzy shortest path from v_1 for v_l or e_k is through P_1, then $\tilde{d}$ (v_1, v_k) = $\tilde{d}$ (v_1, e_k) or $\tilde{d}$ (v_1, e_l) = $\tilde{d}$ (v_1, v_l)if and only if $N(v_1, v_k) = N(v_1, e_k)$ or $N(v_1, e_l) = N(v_1, v_l)$. This implies that, $\tilde{\beta}$ $(BG_3(S_{1,n})) \ne 1$. Include v_3 as another source vertex so that $N(v_3, v_k) \ne N(v_3, e_k)$ or $N(v_3, e_l) \ne N(v_3, v_l)$, $\tilde{d}$ $(v_3, v_k) \ne \tilde{d}$ (v_3, e_k) or $\tilde{d}$ $(v_3, e_l) \ne \tilde{d}$ (v_3, v_l).

Continuing this process for all n/2 paths in $BG_3(S_{1,n})$, we get n/2 source vertices for $BG_3(S_{1,n})$. $\tilde{M} = \{v_1, v_3, \ldots, v_{n-1}\}$. Hence, $\tilde{\beta}$ $(BG_3(S_{1,n})) \le n/2$.

Case ii: n is odd.

Edges of $BG_3(S_{1,n})$ can be decomposed into n/2 fuzzy paths of length four and one fuzzy path of length two as follows:

P_1: $v_1\ e_1\ v\ e_2\ v_2$.

P_2: $v_3\ e_3\ v\ e_4\ v_4$.

P_3: $v_5\ e_5\ v\ e_6\ v_6$.

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$P_{(n-1)/2}$: $v_{n-2}\ e_{n-2}\ v\ e_{n-1}\ v_{n-1}$.

$P_{(n+1)/2}$: $v\ e_n\ v_n$.

In two paths P_1 and P_2, take e_1 as a source vertex. If two vertices v_k or $e_l \in P_1$ and v_l or $e_k \in P_2$ such that fuzzy shortest path from e_1 for v_k or e_l is through P_2 and fuzzy shortest path from e_1 for v_l or e_k is through P_1, then $\tilde{d}$ (e_1, v_k)

$= \tilde{d}\,(e_1, e_k)$ or $\tilde{d}\,(e_1, e_l) = \tilde{d}\,(e_1, v_l)$ if and only if $N(e_1, v_k) = N(e_1, e_k)$ or $N(e_1, e_l) = N(e_1, v_l)$. This implies that, $\tilde{\beta}\,(BG_3(S_{1,n})) \neq 1$. Include e_3 as another source vertex so that $N(e_3, v_k) \neq N(e_3, e_k)$ or $N(e_3, e_l) \neq N(e_3, v_l)$, $\tilde{d}\,(e_3, v_k) \neq \tilde{d}\,(e_3, e_k)$ or $\tilde{d}\,(e_3, e_l) \neq \tilde{d}\,(e_3, v_l)$.

Continuing this process for all n paths in $BG_3(S_{1,n})$, we get n source vertices for $BG_3(S_{1,n})$. $\tilde{M} = \{e_1, e_3, e_5, \ldots, e_{n-2}, e_n\}$. Hence, $\tilde{\beta}\,(BG_3(S_{1,n})) \leq (n+1)/2$.

IV CONCLUSION

We have determined the fuzzy metric dimension of fuzzy Hypercube Q_4 and Q_6, obtained some new bounds for fuzzy metric dimension of fuzzy Hypercube Q_n.

We have calculated fuzzy metric dimension of fuzzy Boolean graph $BG_2(G)$ of fuzzy path, fuzzy cycle, star fuzzy graph and nK_2. We have also determined the fuzzy metric dimension of fuzzy Boolean graph $BG_3(G)$ of fuzzy path, fuzzy cycle, star fuzzy graph and nK_2.

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