A Reliability Model for a Two Dissimilar Units Series System with Repair Time-Dependent Standby

Jai Bhagwan, Amit Manocha, Anil Taneja

Abstract*: *The present paper stochastically analyze a system comprising two dissimilar units (unit-1/unit-2) working in series configuration. System fails completely when either of the units gets failed. The repair time of unit-2 is considered to be much more as compared to the repair time of unit-1. So, to minimize the breakdown period of the system, a standby unit is provided against the second unit. Regenerative point technique (RPT) is used to develop a semi-markovian reliability model for the mentioned system. Optimum cut-off points concerning the profitability of the system have also been obtained. The model has applications in industries, particularly in aluminum industry.

Keywords: Dissimilar units, Optimum cut-off points, Repair time dependent standby, semi-Markov process, Series configuration

I. INTRODUCTION

The industries/organizations are now being modernized and focused on producing more reliable systems with increased availability and lesser break down time to achieve the set target. Redundancy is one of the most effective techniques, which may be used to enhance the performability of industrial systems and such systems have been analyzed by various researchers. Mokaddis et al. [1] analyzed standby system with three different operative stages. Parashar and Taneja [2] dealt with PLC hot standby system. A standby system with general life and repair time distribution was studied by Bieth et al. [3].Mahmoud and Mosherf [4] discussed different types of failure and preventive maintenance in their study. Malhotra and Taneja [5] stochastically analysed a system wherein operability of more than one unit depends upon requirement. Manocha and Taneja [6] took arbitrary distribution for all random variables. El- Sherbeny [7] studied such systems with the concept of random change of units. Manocha et al. [8] investigated database system keeping hot standby unit under constant observation.

Dissimilar units system may also be observed in the industrial sector. Mokkadis et al. [9] analysed a system by considering two types of repair and inspection of failed unit. Sadeghi and Roghanian [10] studied two unit warm standby system by considering two dissimilar units with imperfect switching mechanism. Rahbi et al. [11] did the reliability analysis of rodding anode plant consisting of eight dissimilar units used in aluminum industry. Chopra and Ram[12] carried out reliability analysis of two dissimilar units parallel system using Gumbel-Hougaard family copula. In a two dissimilar unit series system, it may be observed that one of the two dissimilar units, whenever gets failed, may require more time to get repaired as compared to the other. These types of systems are used at a large scale in network communication, textile industry, aluminum industry etc. For such systems, if a unit gets failed, the whole system becomes non-functional and hence introduction of a standby unit may reduce the frequency of breakdowns. However, using standby units against both the units may be a costly affair. Therefore, to keep a balance between the cost of using standby units and breakdown time of the systems, one may use single standby unit against that unit whose recovery time after failure is more than the other.

The present study is an attempt to stochastically analyse a system comprising two dissimilar units connected in series (unit-1/unit-2), where a standby unit is kept against the second unit. In the system under consideration, let us assume that unit-2 takes more time to repair on failure as compared to the repair time of the first unit and hence breakdown period of system is much more in case of the failure of second unit. To reduce breakdown period of the system a standby unit is installed against unit-2.The product being manufactured by such a system is assumed to be first processed on unit-1 and then on unit-2. System fails completely if either unit-1 or unit-2 along with its standby after putting it into operation gets failed. The technique and the other assumptions taken in the present study are same as that taken in [2].Optimum cut-off points for various costs which affect the profitability of the system have also been obtained.

II. NOTATIONS

O_1/ O_2	operative unit-1 / unit-2
S_2	standby unit for unit-2
ω_1 /ω_2	constant failure rate of unit-1and 2
F_{r1}/ F_{Wr1}	unit-1 under repair/ waiting for repair
F_{r2}/F_{R2}/ F_{Wr2}	unit-2 under repair/repair from previous state/ waiting for repair
D_1 / D_2	down unit-1/unit-2
$g_1(t)$ / $g_2(t)$	density function of repair time for unit-1and 2

Note: For some other notations one may refer to [2] and [5].

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III. TRANSITION DENSITIES & MEAN SOJOURN TIMES

Possible transitions for the model are shown in Fig.1.All the states except 3 and 4 are regenerative states.
States 0 and 2 are up, whereas 1, 3 and 4 are failed states.

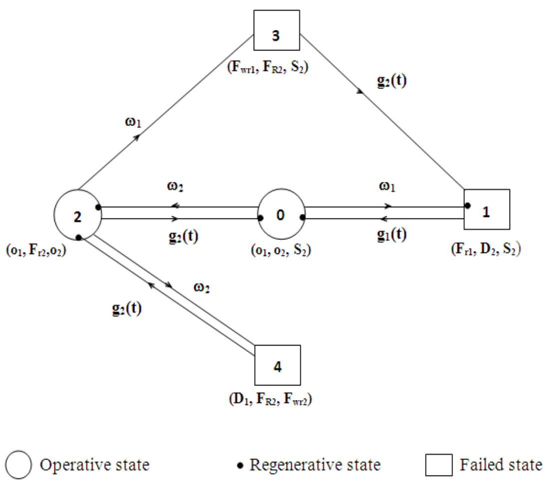

Fig.1. Transition diagram

Transition densities are:

$q_{01}(t)=\omega_1 e^{-(\omega_1+\omega_2)t}$, $q_{02}(t)= \omega_2 e^{-(\omega_1+\omega_2)t}$,

$q_{10}(t)= g_1(t)$, $q_{20}(t)=e^{-(\omega_1+\omega_2)t} g_2(t)$,

$q_{21}^{(3)}(t)=(\omega_1 e^{-(\omega_1+\omega_2)t} ©1)g_2(t)$,

$q_{22}^{(4)}(t)=(\omega_2 e^{-(\omega_1+\omega_2)t} ©1)g_2(t)$,

$q_{23}(t)=\omega_1 e^{-(\omega_1+\omega_2)t}\overline{G}_2(t)$. (1-7)

By the probabilistic argument, the non-zero elements p_{ij} are obtained as: $p_{ij}=\lim_{s\to 0} q_{ij}^{*}(s)$ (8)

For the developed model, mean sojourn time and contribution to sojourn time are mathematically expressed as:

$$\mu_0=\int_0^\infty e^{-(\omega_1+\omega_2)t}dt,\quad \mu_2=\int_0^\infty e^{-(\omega_1+\omega_2)t}\overline{G_2(t)}dt \qquad (9\text{-}10)$$

Thus, $m_{01} + m_{02} = \mu_0$, $m_{10} = -g_1^{*'}(0) = K_1$ (say),

$m_{20} + m_{23} + m_{24} = \mu_2$, $m_{20} + m_{21}^{(3)} + m_{22}^{(4)} = -g_2^{*'}(0) = K_2$.

(11-14)

IV. RELIABILITY AND MTSF

If $rt_0(t)$ and $rt_2(t)$ denotes the CDF of first passage time from states 0 and 2 to a failed state respectively, then we have

$rt_0(t) = Q_{01}(t) + Q_{02}(t) ® rt_2(t)$

$rt_2(t) = Q_{23}(t) + Q_{24}(t) + Q_{20}(t) ® rt_0(t)$ (15-16)

Thus, the reliability of the system

$R(t)=L^{-1}[\{D(s)–N(s)\}/sD(s)]$ (17)

and

$$MTSF=\int_0^\infty R(t)dt= N/D \qquad (18)$$

where

$N(s)=\{\omega_1/(s+\omega_1+\omega_2)\}$

$+\{\omega_2(\omega_1+\omega_2)/(s+\omega_1+\omega_2)^2\}\{1-g_2^{*}(s+\omega_1+\omega_2)\}$

$D(s)=1-\{\omega_2 g_2^{*}(s+\omega_1+\omega_2)/(s+\omega_1+\omega_2)\}$

$N=\mu_0+p_{02}\mu_2$,

$D = 1– p_{02}p_{20}$. (19-22)

V. AVAILABILITY ANALYSIS

The recursive relations for point-wise availability $up_i(t)$, i=0,1,2 are:

$up_0(t) = a_0(t) + q_{01}(t)© up_1(t) + q_{02}(t)© up_2(t)$

$up_1(t) = q_{10}(t) © up_0(t)$

$up_2(t) = a_2(t)+ q_{20}(t) © up_0(t) + q_{21}^{(3)} © up_1(t)$

$+ q_{22}^{(4)} © up_2(t)$

where $a_0(t)=e^{-(\omega_1+\omega_2)t}$, $a_2(t)=e^{-(\omega_1+\omega_2)t}\overline{G_2(t)}$ (23-27)

Thus, as time t approaches to infinity the availability is $up_0=\lim_{s\to 0} s\,up_0^{**}(s)=\lim_{s\to 0} s\,N_1(s)/D_1(s)= N_1/D_1$

where $N_1(s) = \{1– q_{22}^{(4)*}(s)\}\, a_0^{*}(s) + q_{02}^{*}(s)\, a_2^{*}(s)$,

$D_1(s) = \{1– q_{01}^{*}(s)\, q_{10}^{*}(s)\}\, \{1– q_{22}^{(4)*}(s)\}$

$– q_{02}^{*}(s)\, \{q_{10}^{*}(s)\, q_{21}^{(3)*}(s) + q_{20}^{*}(s)\}$,

$N_1= (1– p_{22}^{(4)})\, \mu_0 + p_{02}\mu_2$,

$D_1= (1– p_{22}^{(4)})\, (\mu_0 + p_{01} K_1) + p_{02}K_2$. (28-32)

VI. BUSY PERIOD ANALYSIS

The system of equations obtained for evaluating busy period of repairman $bt_i(t)$, i=0,1,2 are:

$bt_0(t) = q_{01}(t) © bt_1(t) + q_{02}(t) © bt_2(t)$

$bt_1(t) = l_1(t) + q_{10}(t) © bt_0(t)$

$bt_2(t) = l_2(t) + q_{20}(t) © bt_0(t) + q_{21}^{(3)} © bt_1(t)$

$+ q_{22}^{(4)} © bt_2(t)$

where $l_1(t)=\overline{G}_1(t)$, $l_2(t)=\overline{G}_2(t)$ (33-37)

Thus, in steady-state, we have

$bt_0=\lim_{s\to 0}\{s\,N_2(s)/D_1(s)\}= N_2/D_1$

where

$N_2(s) = \{q_{01}^{*}(s) – q_{01}^{*}(s)\, q_{22}^{(4)*}(s)+ q_{02}^{*}(s)\, q_{21}^{(3)*}(s)\}\, l_1^{*}(s)$

$+ q_{02}^{*}(s)\, l_2^{*}(s)$,

$N_2 = \{(1– p_{22}^{(4)})\, p_{01} + p_{02}\, p_{21}^{(3)}\}\, K_1+ p_{02} K_2$. (38-40)

EXPECTED NUMBER of VISITS BY REPAIRMAN

The equations for obtaining expected number of visits by repairman $ev_i(t)$, i=0,1,2 in specific unit of time, are:

$ev_0(t) = Q_{01}(t) ® \{ev_1(t)+1\}+ Q_{02}(t) ® \{ev_2(t)+1\}$

$ev_1(t) = Q_{10}(t) ® ev_0(t)$

$ev_2(t)= Q_{20}(t)®ev_0(t) + Q_{21}^{(3)}(t) ® ev_1(t) + Q_{22}^{(4)}(t) ®ev_2(t)$

In long run

$$ev_0 = \lim_{s \to 0}\{s\, N_3(s)/D_1(s)\} = N_3/D_1$$

where $N_3(s) = \{1 - Q_{22}^{(4)**}(s)\}\{Q_{01}^{**}(s) + Q_{02}^{**}(s)\}$

$$N_3 = (1 - p_{22}^{(4)}) \qquad (41\text{-}46)$$

VII. PROFIT ANALYSIS

The profit equation, therefore, is

$$P_0 = C_R up_0 - C_B bt_0 - C_V ev_0 \qquad (47)$$

where C_R = Revenue per unit up time
C_B = Cost per unit time for engaging repair facility
C_V= Repairman charges for each visit

VIII. RESULT AND DISCUSSION

Letting $g_1(t)=\alpha_1 e^{-\alpha_1 t}$, $g_2(t)=\alpha_2 e^{-\alpha_2 t}$; considering ω_2= 0.005 per hr, α_1= 0.2 per hr; and varying ω_1 , α_2 ; the values of MTSF and availability are tabulated as:

Table-I: Values of MTSF and Availability (up_0) w.r.t. ω_1 and α_2

ω_1 (per hr)	α_2=0.1(per hr)		α_2=0.15 (per hr)		α_2=0.2 (per hr)	
	MTSF (In hrs)	Availability (up_0)	MTSF (In hrs)	Availability (up_0)	MTSF (In hrs)	Availability (up_0)
0.001	828.77	0.9925	865.6	0.9939	894.07	0.9944
0.002	453.53	0.9874	464.18	0.9890	472.16	0.9895
0.003	312.18	0.9823	317.12	0.9840	320.78	0.9846
0.004	238	0.9773	240.82	0.9792	242.91	0.9798
0.005	192.31	0.9724	194.12	0.9744	195.45	0.9750
0.006	161.33	0.9675	162.59	0.9696	163.51	0.9703

Graphs of profit (P_0) with respect to the following have been plotted in Figs. 2 and 3.

i) C_R for varied ω_1
ii) C_B for varied C_V

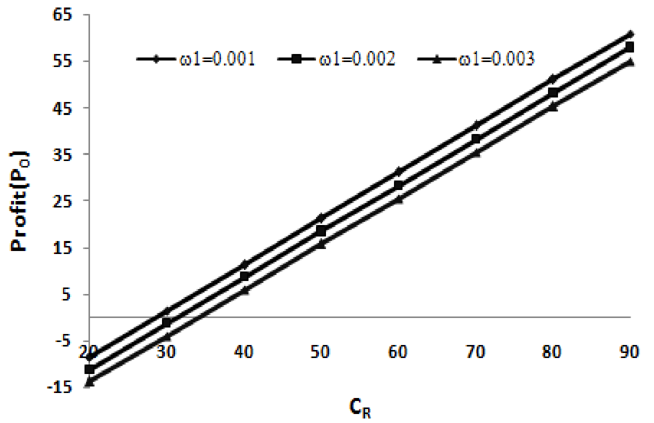

Fig.2. Profit (P_0) versus (C_R)

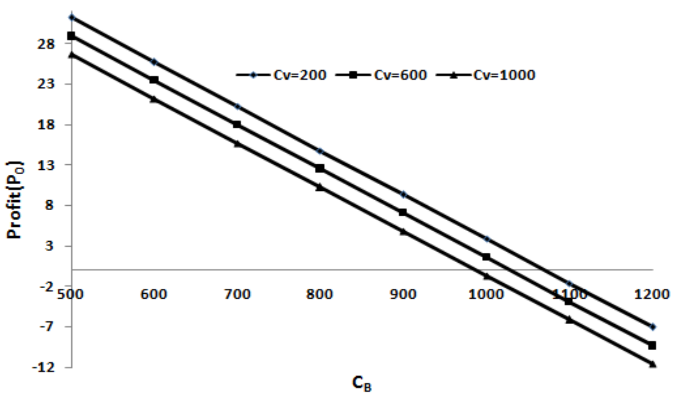

Fig.3. Profit (P_0) versus cost (C_B)

The above figures reveal what has been tabulated as follows:

Table-II: Conditions for the profitability of the system

Assumed parametric values	Varied parameter	Condition for the system to be profitable(Optimum cut-off points)	Remark
ω_2 = 0.005, α_1 = 0.2, α_2 = 0.1, C_B = 500, C_V = 200	ω_1 = 0.001	C_R >28.64	Otherwise, system will put to a loss
	ω_1 = 0.002	C_R >31.33	
	ω_1 = 0.003	C_R >34.03	
ω_1 = 0.001, ω_2 = 0.005, α_1 = 0.2, α_2 = 0.1, C_R = 60	C_V = 200	C_B <1070.12	Otherwise, system will put to a loss
	C_V = 600	C_B <1028.54	
	C_V = 1000	C_B <986.96	

IX. CONCLUSION

The stochastic analysis is carried out for a system comprising two dissimilar units connected in series with a standby unit against that unit which has more recovery time after failure. Cost is always a crucial factor for any industry/organization and hence cut-off points for revenue/cost have been determined, finding numerical results for a particular case, which may be used to assess as to what value of the parameter under consideration should be taken in order to have a profitable system. Cost analysis may be done for other parameters of interest also in the similar way by the users of such systems.

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