l_1-Norm Constrained Minimum Eror Entropy Algorithm

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***Abstract:** This work proposes a linear phase sparse minimum error entropy adaptive filtering algorithm. The linear phase condition is obtained by considering symmetry or anti symmetry condition onto the system coefficients. The proposed work integrates linear constraint based on linear phase of the system and l_1-norm for sparseness into minimum error entropy adaptive algorithm. The proposed l_1-norm linear constrained minimum error entropy criterion (l_1-CMEE) algorithm makes use of high-order statistics, hence worthy for non-Gaussian channel noise. The experimental results obtained for linear phase sparse system identification in the presence of non-Gaussian channel noise reveal that the proposed algorithm has lower steady state error and higher convergence rate than other existing MEE variants.*

***Keywords:** Constrained adaptive filtering, Information theory, non-Gaussian noise, sparse system*

I. INTRODUCTION

Constrained adaptive filtering has now become a topic of deep interest due to substantial advancements in linear constrained applications. A linear constraint based on some advance information about the filter coefficients is utilized in developing constrained adaptive algorithm. For example, linear phase of system is utilized as constraint in developing constrained adaptive algorithm [1]. Similarly information about pseudorandom code in spread spectrum and direction of interest in adaptive beam forming are utilized as constraints in developing adaptive algorithm [2-3].

In this paper, we are considering the linear phase adaptive filtering problem. Some examples of linear phase adaptive filtering are: system identification, channel equalization, spectral estimation, line enhancement [4]. These applications need to preserve the constant phase delay among the frequency components to restrict the phase distortion in pass band. Hence, several constrained adaptive filtering algorithm have been developed in the past. Constrained least mean square (CLMS) algorithm is well known adaptive algorithm because of simplicity and low computational cost [5]. Several other constrained adaptive algorithms have been developed, for example: constrained affine projection (CAP) for colored input, least square algorithm for linear phase filtering, fast least square algorithm for linear phase system, constrained recursive least square [6-9]. However, these algorithms are based on minimum mean square error (MSE) criterion by considering only second order statistics. Hence, these algorithms perform well in the presence of Gaussian observation noise. But the performance is degraded in the presence of non-Gaussian observation noise due to higher order statistics.

Meanwhile information theory based adaptive algorithms have been developed to deal with non-Gaussian noise [10]. Some examples of information theory based adaptive filtering algorithms are: maximum correntropy criterion (MCC) adaptive algorithm, minimum error entropy adaptive algorithm (MEE), mutual information based adaptive algorithm [11-13]. . Recently, Siyuan Peng et al. have proposed constrained MEE (CMEE) algorithm for constrained adaptive filtering in the presence of impulsive channel noise [14]. CMEE adaptive algorithm is developed by adding a linear constraint on the system coefficients into cost function of minimum error entropy adaptive algorithm (MEE). The idea behind developing any MEE based adaptive algorithm is to lower the entropy of error between desired output and unidentified system output. As entropy is of higher order statistics, hence suitable for non-Gaussian channel noise. The entropy considered in MEE based algorithms is quadratic i.e. Renyi entropy.

The aforesaid CMEE algorithm performs imperfectly in sparse system. Recently Jos´e F. de Andrade Jr. and Marcello L. R. de Campos have proposed l_1-norm linear constrained LMS algorithm to consider the linear constraint and sparsity of the system [15]. Based on the same approach, the proposed work integrates the l_1-norm based sparsity penalty and linear constraint into MEE adaptive algorithm to take into account the sparseness of constrained system in the presence of non-Gaussian noise. l_1 -norm based adaptive algorithm increases the convergence speed of small coefficients and reduce the bias of large coefficients. Hence, the proposed l_1-CMEE algorithm excels in constrained sparse system identification in the presence of non-Gaussian noise.

The rest of the paper is organized as follows. In section 2, we review constrained minimum error entropy (CMEE) algorithm. In section 3, we propose l_1-CMEE algorithm by integrating l_1-norm into the cost function of CMEE algorithm. In addition, we derive the update equation of l_1-CMEE algorithm by using the concept of Lagrange multiplier. In section 4, the estimation performance of proposed l_1-CMEE algorithm is examined by the experiments carried out in MATLAB and compared with existing MEE, CMEE, l_1-CLMS algorithms. Finally the conclusion of the proposed work is drawn in section 5.

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II. REVIEW OF CONSTRAINED MINIMUM ERROR ENTROPY ALGORITHM (CMEE)

Consider an unknown linear phase system with coefficient vector $\mathbf{w_0} \in \mathbf{R}^{NX1}$. Let $\mathbf{x}(k) \in \mathbf{R}^{NX1}$ is input vector to unknown system and adaptive filter and $\widehat{\mathbf{w}}(k) \in \mathbf{R}^{NX1}$ represents coefficient vector of adaptive filter.
Now we can write the instantaneous estimation error $e(k)$ between adaptive filter output and unknown system output as:

$$e(k) = \boldsymbol{w}_0^T \mathbf{x}(k) + p(k) - \widehat{\boldsymbol{w}}^T(k)\mathbf{x}(k) \quad (1)$$

where p(k) represents the channel noise.

The information potential $\widehat{V}(e)$ in term of quadratic Renyi's entropy $\widehat{H}_{R_2}(e)$ can be written as [12]:

$$\widehat{H}_{R_2}(e) = -\log \frac{1}{M^2} \sum_{i=k-M+1}^{k} \sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) - e(j)) = -\log V(e) \quad (2)$$

where g_σ is kernel function having bandwidth σ and M is available sample length,

and $$\widehat{V}(e) = \frac{1}{M^2} \sum_{i=k-M+1}^{k} \sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) - e(j)) \quad (3)$$

The most widely used kernel is the Gaussian kernel defined as:

$$g_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \quad (4)$$

Hence, the constrained minimum error entropy (CMEE) algorithm can be derived by solving following optimization criterion.

$$\arg\max_{\boldsymbol{w}}(\widehat{V}(e)) \text{ subject to } \boldsymbol{A}^T \widehat{\boldsymbol{w}} = \boldsymbol{b} \quad (5)$$

where $\boldsymbol{A}$ is constraint matrix of NxL dimension and $\boldsymbol{b}$ is the corresponding L constraint values. In this work, the matrix $\boldsymbol{A}$ imposes a linear phase constraint on the system coefficients. The linear phase of FIR filter is obtained by symmetry or anti symmetry property of system coefficients.

We can write linear phase condition for system coefficients as:

$$\widehat{w}_i = \pm \widehat{w}_{N-i-1} \quad (6)$$

where $+$ sign represents symmetric condition and $-$ sign represents anti-symmetric condition.

Hence, the constraint on system coefficients for linear phase will be:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \mp 1 \\ \vdots & \vdots & \iddots & \vdots \\ 0 & \mp 1 & \dots & 0 \\ \mp 1 & 0 & \dots & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{\frac{N-1}{2}} \\ \boldsymbol{0}^T \\ \mp \boldsymbol{J}_{(N-1)/2} \end{bmatrix} \text{ for N odd} \quad (7)$$

and

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & \mp 1 \\ \vdots & \vdots & \iddots & \vdots \\ 0 & \mp 1 & \dots & 0 \\ \mp 1 & 0 & \dots & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{\frac{N}{2}} \\ \mp \boldsymbol{J}_{N/2} \end{bmatrix} \text{ for N even} \quad (8)$$

$$\mathbf{b} = [\mathbf{0}\ \mathbf{0} \ldots \mathbf{0}]^T \quad (9)$$

where I is an identity matrix of order $\frac{N-1}{2}$ and $\boldsymbol{J}$ represents an identity matrix in which all rows are written in reverse order
Using the Lagrange multipliers approach, we have the unconstrained cost function for optimization problem represented by eq. (5) as:

$$J(w) = \widehat{V}(e) + \boldsymbol{\lambda}_1^T(\boldsymbol{A}^T \widehat{\boldsymbol{w}} - \boldsymbol{b}) \quad (10)$$

Here $\boldsymbol{\lambda}_1$ is a vector of Lagrange multipliers of dimension Lx1.

By Gradient ascent approach, the weight update equation of CMEE becomes [14]:

$$\widehat{\boldsymbol{w}}(k+1) = \boldsymbol{R}\left[\widehat{\boldsymbol{w}}(k) + \frac{\mu}{2M^2}\left[\sum_{i=k-M+1}^{k} \sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) - e(j))(e(i) - e(j))(\mathbf{x}(i) - \mathbf{x}(j))\right]\right] + \boldsymbol{S} \quad (11)$$

where $= (\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T)$, $\boldsymbol{S} = \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{b}$, and $\boldsymbol{I}$ is an identity matrix of dimension NXN.

III. l_1-NORM CONSTRAINED MINIMUM ERROR ENTROPY (l_1-CMEE) ALGORITHM

In this section, we combine the effect of zero attraction based on l_1-norm and linear phase constraint with MEE algorithm to consider sparse system identification in the presence of impulsive channel noise. The zero attraction is based on l_1-norm.
.
The optimization problem of l_1-CMEE algorithm becomes:

$$\arg\max_{\boldsymbol{w}}(\widehat{V}(e))$$

$$\text{subject to} \quad \begin{cases} \boldsymbol{A}^T \widehat{\boldsymbol{w}} = \boldsymbol{b} \\ \|\widehat{\boldsymbol{w}}\|_1 = a \end{cases} \quad (12)$$

where a $\| \quad \|_1$ is l_1 norm and a is constraint value.

The unconstrained optimization function of l_1-CMEE becomes:

$$J(\widehat{\boldsymbol{w}}) = \widehat{V}(e) + \boldsymbol{\lambda}_1^T(\boldsymbol{A}^T \widehat{\boldsymbol{w}} - \boldsymbol{b}) + \lambda_2(\|\widehat{\boldsymbol{w}}\|_1 - a) \quad (13)$$

$$J(\widehat{\boldsymbol{w}}) = \frac{1}{2M^2} * \left[\sum_{i=k-M+1}^{k} \sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) - e(j))(e(i) - e(j))(\mathbf{x}(i) - \mathbf{x}(j))\right] + \mathbf{A}\,\boldsymbol{\lambda}_1 +$$

$\lambda_2(\mathrm{F}_{l1}(\widehat{\mathbf{w}}(\mathrm{k}+1)))$ (14)

Using Gradient ascent approach, the weight update equation of l_1-CMEE becomes:

$\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mathbf{A}\ \boldsymbol{\lambda 1}+\lambda 2(\mathrm{F}l1\mathbf{w}\mathrm{k}+1)$
(15)

where $\mathrm{F}_{l1}(\widehat{\mathbf{w}}) = \frac{\partial\|\widehat{\mathbf{w}}\|_1}{\partial(\widehat{\mathbf{w}})} = \mathrm{sign}(\widehat{\mathbf{w}})$ (16)

In steady state condition,

$\mathrm{F}_{l1}(\widehat{\mathbf{w}}(\mathrm{k}+1)) = \mathrm{F}_{l1}(\widehat{\mathbf{w}}(\mathrm{k}))$ (17)

Now pre multiplying eq, (15) by $\mathbf{A}^{\mathrm{T}}$, we have:

$\boldsymbol{A}^T\ \widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\boldsymbol{A}^T\ (\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mu\ \mathbf{A}\ \boldsymbol{\lambda 1}+\mu\lambda 2(\mathrm{F}l1\mathbf{w}\mathrm{k}$
(18)

From eq. (18), we let
$\boldsymbol{\lambda_1} = -\frac{1}{2M^2}(\boldsymbol{A}^T\ \mathbf{A})^{-1}\boldsymbol{A}^T\ [\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)-\lambda 2\boldsymbol{A}T\mathbf{A}-1\boldsymbol{A}T(\mathrm{F}l1\mathbf{w}\mathrm{k}$
(19)

Hence
$\widehat{\mathbf{w}}(\mathrm{k}+1) = \widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mu\mathbf{A}-\mathbf{1}2M2\boldsymbol{A}T\mathbf{A}-1\boldsymbol{A}T$
$i=k-M+1kj=k-M+1kg\sigma 2(e(i)-ej) ei-ej(\mathbf{x}i-\mathbf{x}(j)-$
$\lambda 2\boldsymbol{A}T\mathbf{A}-1\boldsymbol{A}T(\mathrm{F}l1\mathbf{w}\mathrm{k}+\lambda 2(\mathrm{F}l1\mathbf{w}\mathrm{k}+1)$ (20)

$\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}\boldsymbol{R}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mu\ \lambda 2\boldsymbol{R}\mathrm{F}l1\mathbf{w}\mathrm{k}+1$
(21)

Pre multiplying eq. (21) by $F_{lp}^T(\widehat{\mathbf{w}}(\mathrm{k}))$, we let:
$F_{lp}^T(\widehat{\mathbf{w}}(\mathrm{k}))\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$F_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\widehat{\mathbf{w}}(\mathrm{k}) +$
$\frac{\mu}{2M^2}F_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\boldsymbol{R}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e j) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mu\ \lambda 2Fl1T(\mathbf{w}\mathrm{k})\boldsymbol{R}\mathrm{F}l1\mathbf{w}\mathrm{k}+1$
(22)

Considering
$e_{l1}(k) = \mathrm{F}_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\widehat{\mathbf{w}}(\mathrm{k}+1) - \mathrm{F}_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\widehat{\mathbf{w}}(\mathrm{k})$ (23)

$q = F_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\boldsymbol{R}\mathrm{F}_{l1}(\widehat{\mathbf{w}}(\mathrm{k}))$ (24)

Hence,
$\lambda_2 =$
$\frac{e_{lp}(k)}{\mu q} - \frac{1}{2qM^2}F_{l1}^T(\widehat{\mathbf{w}}(\mathrm{k}))\boldsymbol{R}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$e(j))(e(i)-e(j))(\mathbf{x}(i)-\mathbf{x}(j)]$ (25)

$\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}\boldsymbol{R}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
ej
$) ei-ej(\mathbf{x}i-\mathbf{x}(j)+\mu el1k\mu q-12qM2Fl1T\mathbf{w}\mathrm{k}\boldsymbol{R}i=k-M+1kj$
$=k-M+1kg\sigma 2(e(i)-ej) ei-ej(\mathbf{x}i-\mathbf{x}(j)\boldsymbol{R}\mathrm{F}l1\mathbf{w}\mathrm{k}$
(26)
$\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}\boldsymbol{R}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) - e(j))(\mathbf{I} -$
$\mathrm{F}l1\mathbf{w}\mathrm{k}Fl1T\mathbf{w}\mathrm{k}\boldsymbol{R}qei-ej(\mathbf{x}i-\mathbf{x}(j)+el1kq\boldsymbol{R}\mathrm{F}l1\mathbf{w}\mathrm{k}$
(27)

We can rewrite eq. (27) as:

$\widehat{\mathbf{w}}(\mathrm{k}+1) =$
$\boldsymbol{R}[\widehat{\mathbf{w}}(\mathrm{k}) + \frac{\mu}{2M^2}[\sum_{i=k-M+1}^{k}\sum_{j=k-M+1}^{k} g_{\sigma\sqrt{2}}(e(i) -$
$ej)\mathbf{I}-\mathrm{F}l1\mathbf{w}\mathrm{k}Fl1T\mathbf{w}\mathrm{k}\boldsymbol{R}qei-ej(\mathbf{x}i-\mathbf{x}(j)+el1kq\mathrm{F}l1\mathbf{w}\mathrm{k}+\boldsymbol{S}.$

IV. SIMULATION RESULTS

This section discusses the estimation performance of the proposed work. The unknown system and adaptive filter are considered to be of same length N having linear phase feature. The location and values of non-zero coefficients are considered to be of Gaussian distribution having zero mean and unity variance.

In this work, we have considered Gaussian distribution for the input signal having zero mean and unity variance. At first, we have considered impulsive channel noise to compare the performance of proposed l_1-CMEE with CMEE, l_1-MEE and l_1-CLMS algorithms. The alpha stable noise as impulsive chanel noise is considered in this work [12].

In the first experiment, we demonstrate the transient behavior of the proposed l_1-CMEE algorithm for odd N=13 for different values of sparsity constant T={3,7}. Here, we have considered symmetric condition for linear phase of unknown system. We compare the convergence analysis of the proposed l_1-CMEE algorithm with CMEE, l_1-MEE and l_1-CLMS algorithms. The parameter, a $=\|\mathbf{w_0}\|_1$ is taken for l_1-CMEE and l_1-CLMS algorithms. We have taken the step size $\mu = 0.02$ for CMEE and l_1-CMEE algorithms and $\mu = 0.05$ for l_1-CLMS algorithm. The kernel width σ=0.8 is adopted here.

Figure1 and figure 2 demonstrate the estimation performance for N=13 for different value of T={3,7}..

In the second experiment, we have taken N=16 and tested the performance of the proposed algorithm for sparsity constant T={2,6}.

Figure 3and figure 4 demonstrate the estimation performance for N=16for for different values of T.

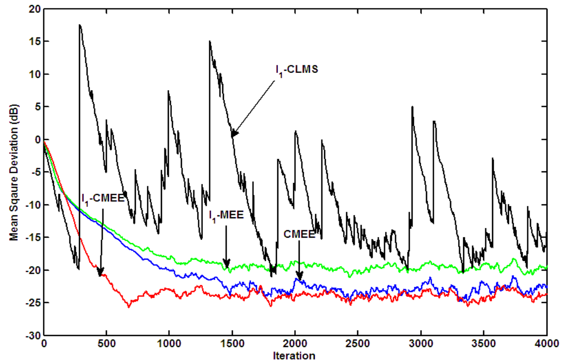

Fig. 1. Comparison of the proposed l_1-CMEE algorithm with other MEE variants for sparsity level T=3 and N=13 in the presence of impulsive channel noise

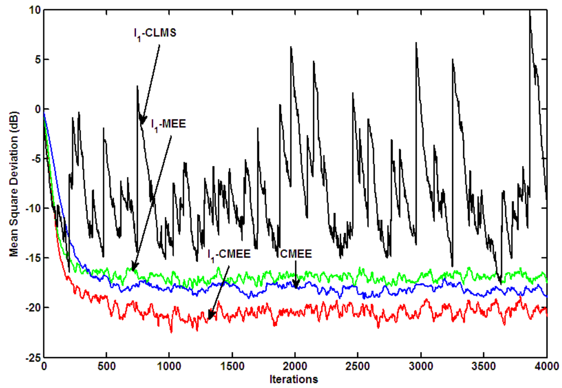

Fig. 2. Comparison of the proposed l_1-CMEE algorithm with other MEE variants for sparsity level T=7 and N=13 in the presence of impulsive channel noise

From fgs. 1, 2, 3 and 4, it is clear that the proposed algorithm has lower mean square deviation error and higher convergence rate than other MEE based algorithms for any value of sparsity level T in even or odd length linear phase system. The existing l_1-CLMS algorithm performs very poorly in the presence of impulsive noise, as this is based on second order statistics of error signal and impulsive noise is of higher order statistics.
The same can be inferred from the results given in table I that is based on Fig. 1.

Table- I: Convergence behavior of the proposed algorithm extracted from fig. 1

Algorithms	Minimum Mean Square Deviation Error (dB)	Iteration Number
l_1-CMEE	-25.29	689
CMEE	-23.2	1489
l_1-MEE	-20.19	1930
l_1-CLMS		Does not converge

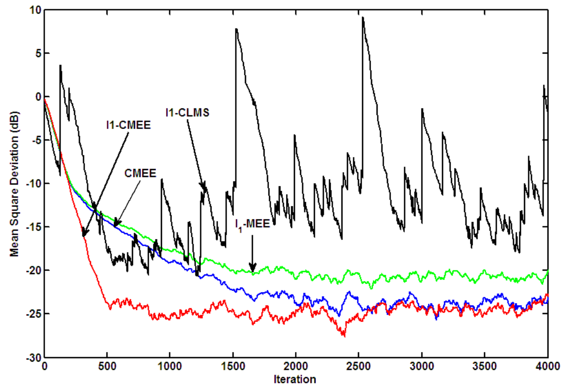

Fig. 3. Comparison of the proposed l_1-CMEE algorithm with other MEE variants for sparsity level T=2 and N=14 in the presence of impulsive channel noise

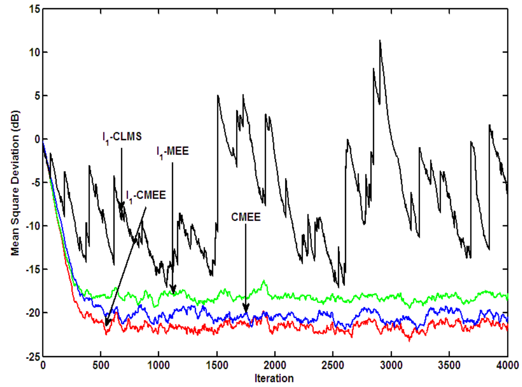

Fig. 4. Comparison of the proposed l_1-CMEE algorithm with other MEE variants for sparsity level T=6 and N=14 in the presence of impulsive channel noise

In the next experiment, we compare the proposed l_1-CMEE with CMEE, l_1 -MEE and l_1 -CLMS algorithms in the presence of Gaussian noise having channel SNR=20 dB.
The other parameters taken are: N=14, T=6, the step size $\mu = 0.02$ for l_1-MEE, CMEE and l_1-CMEE algorithms and $\mu = 0.05$ for l_1-CLMS algorithm.
However, the performance of l_1-CLMS improves in the presence of Gaussian channel noise. Still the proposed l_1 -CMEE algorithm performs better than l_1 -CLMS algorithm. Fig.5 confirms the same.

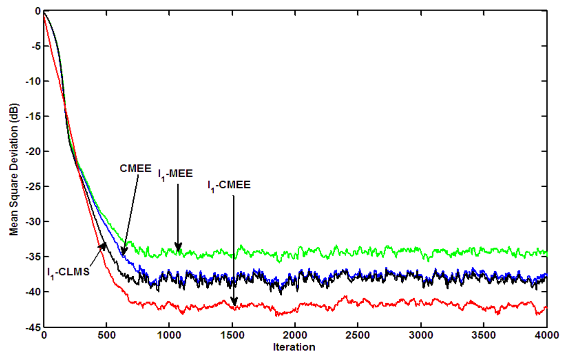

Fig. 5. Comparison of the proposed l_1-CMEE algorithm with other MEE variants for sparsity level T=6 and N=14 in the presence of Gaussian channel noise.

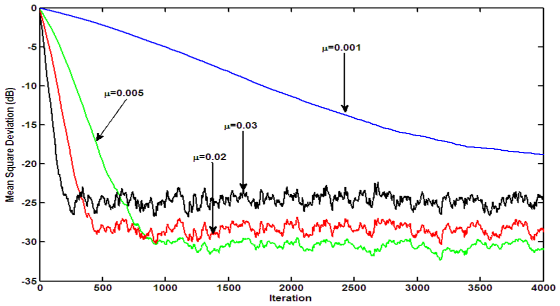

Fig. 6. Performance of the proposed l_1-CMEE algorithm under different values of step size

The impact of step size on the performance of MEE algorithm is shown in fig. 6. Here the channel noise is impulsive as taken in the first experiment. As the step size increase, the convergence speed increases but mean square deviation error also increases. Hence, the step size should be chosen very carefully to improve the performance of the proposed algorithm so that the balance between convergence speed and mean square deviation error should be maintained. The other parameters taken are: N=14, T=2, kernal width σ=0.8.

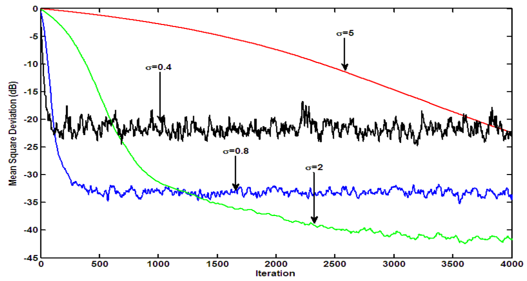

Fig. 7. Performance of the proposed l_1-CMEE algorithm under different values of kernal width σ

Fig. 7 shows the effect of kernel width of l_1-CMEE on its performance. The kernel width affects both the convergence speed and MSD error. Hence it should be selected very carefully. Other parameters taken are: N=14, T=2, $\mu = 0.02$ for CMEE and l_1-CMEE algorithms and $\mu = 0.05$ for l_1-CLMS algorithm.

V. CONCLUSION

This paper presents an information theory based l_1-CMEE algorithm for sparse linear phase system identification. The proposed algorithm performs better in the presence of both impulsive and Gaussian channel noise. As higher order statistics of error is utilized in developing the proposed algorithm, hence performs better in the presence of impulsive noise which is of higher order statistics. The performance of the proposed algorithm is examined for different values of sparsity constant (number of non-zero coefficients). The proposed algorithm has higher convergence speed and lower MSD than other MEE algorithms in sparse system identification. The effects of other parameters are also tested in the proposed work.

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