Complex mathematical models of inertial navigation system errors

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Abstract

The article is devoted to mathematical models of errors of inertial navigation systems (INS). The main advantages of autonomous inertial navigation systems are their resistance to horizontal accelerations and the ability to work autonomously under any conditions. However, over time, the errors of autonomous INS, due to the drift of gyroscopes, zero offset and drift of accelerometers, as well as other factors, reach significant values. Therefore, research to compensate for these errors is an important and urgent task in the autonomous mode of operation of the aircraft.

To establish the connection between the output and input errors of autonomous INS, the equation of errors of autonomous INS is made. In this case, two models of errors of autonomous INS are investigated: nonlinear and linear.

Depending on the requirement for accuracy and time of calculation of navigation parameters choose different models of INS errors. The linear model is simpler and requires less computational time. But the development of modern technologies allows to solve complex problems at an acceptable time interval. Therefore, it is possible to use nonlinear models.

Key words: aircraft, inertial navigation system, error, error, correction, mathematical model, nonlinear, linear.

Introduction

The operation of the aircraft in conditions of active and passive interference is complicated. Determination of navigation parameters and orientation parameters of the aircraft is carried out using the INS installed on board the aircraft, which is autonomous and invariant to horizontal accelerations. To increase the accuracy of autonomous INS with the help of mathematical models of errors predictive models of errors of autonomous INS are built.

The purpose of the article is to consider and analyze the linear and nonlinear mathematical model of INS errors.

Results and discussion

1. Nonlinear error models of autonomous INS. Equation of horizontal orientation errors.

In real conditions, the platform always deviates from the navigation coordinate system (in our case, the navigation coordinate system – a geographical triangle) at some angles Φ_E, Φ_N, Φ_{up} (see Fig. 1). These angles are called orientation errors. The following discusses how to identify these orientation errors.

The Poisson equation is known (Giroskopicheskiye sistemy, 1971; Meleshko V.V., Nesterenko O.I., 2011):

$$\dot{C}_m^n = C_m^n \breve{\omega}_m - \breve{\omega}_n C_m^n, \quad (1)$$

where $\breve{\omega}_m, \breve{\omega}_n$ – skew-matrix matrices; C_m^n – conversion matrix from coordinate system “m” to coordinate system “n”.

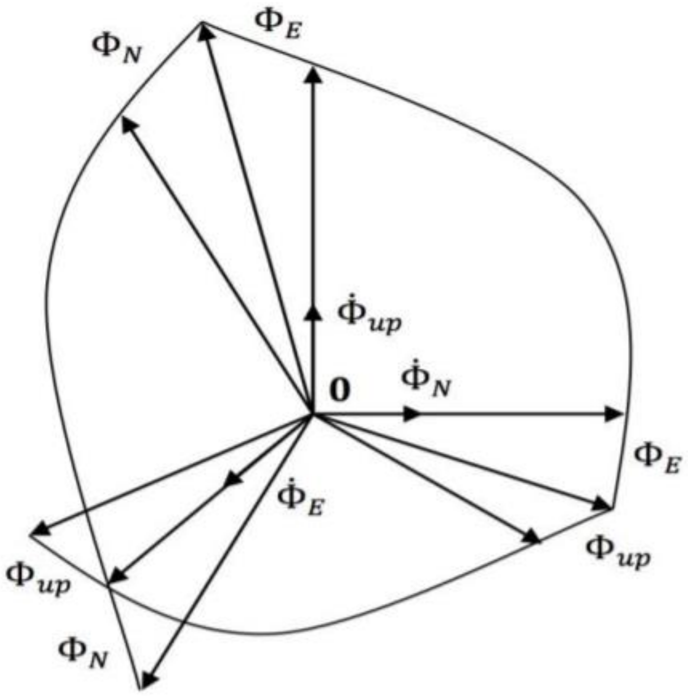

Figure 1 – Deviation angles between platform and geographical triangles

The problem considers two coordinate systems: the platform coordinate system (is affected "p") and i geographical triangular (is affected "LL"). In this case, the Poisson equation has the form:

$$\dot{C}_{LL}^{p} = C_{LL}^{p}\breve{\omega}_{LL} - \breve{\omega}_{p}C_{LL}^{p}, \qquad (2)$$

where C_{LL}^{p} – matrix of transformation from a geographical trihedron into a platform coordinate system:

$$C_{LL}^{p} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \qquad (3)$$

where:

$c_{11} = \cos\Phi_N \cos\Phi_{up} - \sin\Phi_E \sin\Phi_N \sin\Phi_{up};$
$c_{12} = \cos\Phi_N \sin\Phi_{up} + \sin\Phi_E \sin\Phi_N \cos\Phi_{up};$
$c_{13} = -\cos\Phi_E \sin\Phi_N;$
$c_{21} = -\cos\Phi_E \sin\Phi_{up};$
$c_{22} = \cos\Phi_E \cos\Phi_{up};$
$c_{23} = \sin\Phi_E;$
$c_{31} = \sin\Phi_N \cos\Phi_{up} + \sin\Phi_E \cos\Phi_N \sin\Phi_{up};$
$c_{32} = \sin\Phi_N \sin\Phi_{up} - \sin\Phi_E \sin\Phi_N \cos\Phi_{up};$
$c_{33} = \cos\Phi_E \cos\Phi_N;$

and skew-symmetric matrices $\breve{\omega}_{LL}, \breve{\omega}_p$ acquire the form:

$$\breve{\omega}_{LL} = \begin{bmatrix} 0 & -\omega_{up}^{LL} & \omega_N^{LL} \\ \omega_{up}^{LL} & 0 & -\omega_E^{LL} \\ -\omega_N^{LL} & \omega_E^{LL} & 0 \end{bmatrix}_{LL}. \qquad (4)$$

$$\breve{\omega}_{p} = \begin{bmatrix} 0 & -\omega_{up}^{p} & \omega_N^{p} \\ \omega_{up}^{p} & 0 & -\omega_E^{p} \\ -\omega_N^{p} & \omega_E^{p} & 0 \end{bmatrix}_{p}, \qquad (5)$$

where $\omega_i^{LL}, \omega_i^{p}$ – absolute angular velocities of geographical and platform triangles. The absolute angular velocities of a geographical triangle are determined by formulas (Giroskopicheskiye sistemy,1971; Bromberg P. V., 1979; Salychev O.S., 1998):

$$\begin{cases} \omega_E^{LL} = -\frac{v_N}{R} \\ \omega_N^{LL} = \frac{v_E}{R} + u\cos\varphi \\ \omega_{up}^{LL} = \frac{v_E}{R} tg\varphi + u\sin\varphi \end{cases} \qquad (6)$$

The difference between the absolute angular velocities ω_p та ω_{LL} caused by a calculation error $\Delta\omega_{i,i=E,N,up}$ and the drift rate of the gyroscope $\omega_{i,i=E,N,up}^{dr}$. Let $\varepsilon = \Delta\omega + \omega^{dr}$ – the sum of calculation errors and the drift rate of the gyroscope, in this case:

$$\omega_p = \omega_{LL} + \varepsilon, \qquad (7)$$

where $\omega_i^{LL}, \omega_i^{p}$

$$\varepsilon = \begin{bmatrix} \varepsilon_E \\ \varepsilon_N \\ \varepsilon_{up} \end{bmatrix} = \begin{bmatrix} \Delta\omega_E + \omega_E^{dr} \\ \Delta\omega_N + \omega_N^{dr} \\ \Delta\omega_{up} + \omega_{up}^{dr} \end{bmatrix}. \qquad (8)$$

When varying equation (6), you can find calculation errors:

$$\begin{cases} \Delta\omega_E = -\frac{\omega_E^{dr}}{R} \\ \Delta\omega_N = \frac{\delta v_E}{R} - u \sin\varphi\, \delta\varphi \\ \Delta\omega_{up} = \frac{\delta v_E}{R} tg\varphi + (u\cos\varphi + \frac{v_E}{R} sec^2\varphi)\delta\varphi \end{cases} \quad (9)$$

Substitute (9) in (8) and get:

$$\varepsilon = \begin{bmatrix} \varepsilon_E \\ \varepsilon_N \\ \varepsilon_{up} \end{bmatrix} = \begin{bmatrix} \Delta\omega_E + \omega_E^{dr} \\ \Delta\omega_N + \omega_N^{dr} \\ \Delta\omega_{up} + \omega_{up}^{dr} \end{bmatrix} = \begin{bmatrix} -\frac{\delta v_N}{R} + \omega_E^{dr} \\ \frac{\delta v_E}{R} - u\sin\varphi\,\delta\varphi + \omega_N^{dr} \\ \frac{\delta v_E}{R} tg\varphi + (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)\delta\varphi + \omega_{up}^{dr} \end{bmatrix} \quad (10)$$

Similarly, for skew-symmetric matrices:

$$\breve{\omega}_p = \breve{\omega}_{LL} + \breve{\varepsilon}, \quad (11)$$

where

$$\breve{\varepsilon} = \begin{bmatrix} 0 & -\varepsilon_{up} & \varepsilon_N \\ \varepsilon_{up} & 0 & -\varepsilon_E \\ -\varepsilon_N & \varepsilon_E & 0 \end{bmatrix}. \quad (12)$$

Taking into account equation (11) we rewrite equation (2):

$$\dot{C}_{LL}^p = C_{LL}^p \breve{\omega}_{LL} - (\breve{\omega}_{LL} + \breve{\varepsilon})C_{LL}^p. \quad (13)$$

Substitute equations (3), (4), (10) and (12) into equation (13) and obtain nonlinear equations of errors of horizontal orientation:

$$\dot{\Phi}_E = (-\frac{v_N}{R} - \frac{\delta v_N}{R} + \omega_E^{dr})\cos\Phi_N + \frac{v_N}{R}\cos\Phi_{up} - (\frac{v_E}{R} + u\cos\varphi)\sin\Phi_{up} +$$
$$+(\frac{v_E}{R} tg\varphi + u\sin\varphi + \frac{\delta v_E}{R} tg\varphi + u\cos\varphi\,\delta\varphi + \frac{v_E}{R}sec^2\varphi\,\delta\varphi + \omega_{up}^{dr})\sin\Phi_N. \quad (14)$$

$$\dot{\Phi}_N = \frac{v_E}{R} + u\cos\varphi + \frac{\delta v_E}{R} - u\sin\varphi\,\delta\varphi + \omega_N^{dr} - \frac{v_N\sin\Phi_{up}}{R\cos\Phi_E} -$$
$$-(\frac{v_E}{R} + u\cos\varphi)\frac{\cos\Phi_{up}}{\cos\Phi_E} + (-\frac{v_N}{R} - \frac{\delta v_N}{R} + \omega_E^{dr}) tg\Phi_E \sin\Phi_N -$$
$$-(\frac{v_E}{R} tg\varphi + u\sin\varphi + \frac{\delta v_E}{R} tg\varphi + u\cos\varphi\,\delta\varphi + \frac{v_E}{R}sec^2\varphi\,\delta\varphi$$
$$+\omega_{up}^{dr}) tg\Phi_E \cos\Phi_N, \quad (15)$$

Here: v_N, v_E, v_{Up} – projections of the velocity of the aircraft on the axis of the geographical trihedron; $\delta v_N, \delta v_E, \delta v_{Up}$ – projections of the velocity of the aircraft on the axis of the geographical trihedron; $\Phi_N, \Phi_E, \Phi_{Up}$ – angles of deviation between the platform and geographic trihedra; $\omega_N^{dr}, \omega_E^{dr}, \omega_{Up}^{dr}$ – projections of the GSP drift velocity on the axis of the geographical trihedron; φ – the latitude of the area; $\delta\varphi$ – error determining latitude; u – angular velocity of rotation of the Earth; R – radius of the Earth;

Equation of errors of horizontal accelerometers.

To derive the error equations of horizontal accelerometers, first consider the basic equation of navigation.

The basic equation of inertial navigation. In the inertial coordinate system, Newton's second law is known (Inertsial'nyye, 2012):

$$m\frac{d^2r}{dt^2} = F, \quad (16)$$

where m – the mass of the material point; r – radius vector of the material point; F – equal effect of all forces applied to the material point is equal to:

$$F = F_{акт} + G_r, \quad (17)$$

where $F_{акт}$ – active non-gravity forces acting on the point; G_r – gravitational forces acting on a point in the gravitational field of the Earth.

Put equation (17) in (16) we get:

$$m\frac{d^2r}{dt^2} = F_{акт} + G_r. \quad (18)$$

Let's redo equation (18) and get:

$$\frac{d^2r}{dt^2} = \frac{F_{акт}}{m} + \frac{G_r}{m}$$

or

$$a = f + g_m, \quad (19)$$

where α – absolute acceleration; f – acceleration, measured by an accelerometer; g_m – gravitational acceleration.

Next we will consider how to use equation (19) to determine the navigation parameters of objects. We know the Coriolis formula:

$$\left.\frac{dr}{dt}\right|_I = \left.\frac{dr}{dt}\right|_n + \omega_n \times r = v + u + r, \quad (20)$$

where the index "n" denotes the Earth's coordinate system; v – the speed of the object relative to the terrestrial coordinate system; u – angular velocity of rotation of the Earth.

Integrate equation (20) with respect to the inertial coordinate system and we obtain the absolute acceleration of the object:

$$a = \frac{d}{dt}[v + u + r]_I = \left.\frac{dv}{dt}\right|_I + u \times \left.\frac{dr}{dt}\right|_I. \quad (21)$$

Decompose the first term of the right-hand side of the equation (21):

$$\left.\frac{dv}{dt}\right|_I = \left.\frac{dv}{dt}\right|_{LL} + \omega_{LL} \times v, \quad (22)$$

where the index "LL" denotes a geographical trihedron.

Put equations (20), (22) in equation (21) and obtain:

$$a = \left.\frac{dv}{dt}\right|_{LL} + \omega_{LL} \times v + u \times v + u \times u \times r. \quad (23)$$

Let us rewrite equation (19) taking into account equation (23):

$$f + g_m = \left.\frac{dv}{dt}\right|_{LL} + \omega_{LL} \times v + u \times v + u \times u \times r,$$

or

$$f = \left.\frac{dv}{dt}\right|_{LL} + \omega_{LL} \times v + u \times v + u \times u \times r -$$
$$g_m = \left.\frac{dv}{dt}\right|_{LL} + \omega_{LL} \times v + u \times v - g, \quad (24)$$

where $g = g_m - u \times u \times r$ – acceleration of gravity.

Equation (24) is the basic equation of navigation.

Let us decompose the second and third terms of the right-hand side of equation (24):

$$\omega_{LL} \times v = det\begin{bmatrix} i & j & k \\ \omega_E & \omega_N & \omega_{up} \\ v_E & v_N & v_{up} \end{bmatrix} = (\omega_N v_{up} - \omega_{up} v_N)i + (\omega_{up} v_E - \omega_E v_{up})j + (\omega_E v_N - \omega_N v_E)k, \quad (25)$$

$$u \times v = det\begin{bmatrix} i & j & k \\ u_E & u_N & u_{up} \\ v_E & v_N & v_{up} \end{bmatrix} = (u_N v_{up} - u_{up} v_N)i + (u_{up} v_E - u_E v_{up})j + (u_E v_N - u_N v_E)k, \quad (26)$$

where u_E, u_N, u_{up} – projections of the angular velocity of rotation of the Earth on the axes of the geographical trihedron, defined by the formulas:

$$u_E, u_N, u_{up} \begin{cases} u_E = 0 \\ u_N = u\cos\varphi \\ u_{up} = u\sin\varphi \end{cases}. \quad (27)$$

In this case, taking into account equations (24), (25), (26), the projection of the accelerations of the object on the axis of the geographical trihedron are determined by the formulas:

$$\begin{cases} f_E = \frac{dv_E}{dt} + \omega_N v_{up} - \omega_{up} v_N + u_N v_{up} - u_{up} v_N \\ f_N = \frac{dv_N}{dt} + \omega_{up} v_E - \omega_E v_{up} + u_{up} v_E - u_E v_{up} \\ f_{up} = \frac{dvup}{dt} + \omega_E v_N - \omega_N v_E + u_E v_N - u_N v_E + g \end{cases}. \quad (28)$$

Let us rewrite equation (29) in matrix form:

$$f = \dot{v} + \breve{\omega} v + \breve{u} v + g^*, \quad (29)$$

where

$$f = \begin{bmatrix} f_E \\ f_N \\ f_{up} \end{bmatrix}; v = \begin{bmatrix} v_E \\ v_N \\ v_{up} \end{bmatrix}; \breve{\omega} = \begin{bmatrix} 0 & -\omega_{up} & \omega_N \\ \omega_{up} & 0 & -\omega_E \\ -\omega_N & \omega_E & 0 \end{bmatrix};$$

$$\breve{u} = \begin{bmatrix} 0 & -u_{up} & u_N \\ u_{up} & 0 & -u_E \\ -u_N & u_E & 0 \end{bmatrix}; g^* = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}.$$

Due to the deviation of the gyroplatform from the geographical trihedron at the corners $\Phi_E, \Phi_N, \Phi_{up}$ accelerometers do not measure the projection of accelerations along the axes of the geographical trihedron. The relationship between the projections of accelerations along the axes of the gyroplatform ("p") and the geographical trihedron ("LL") is expressed by the formula:

$$f^p = c_{LL}^p f^{LL} + \Delta f, \quad (30)$$

where Δf – accelerometer errors (this includes zero offset and scale factor error). The difference between f^p and f^{LL} can be found by varying equation (29) and we obtain:

$$f^p - f^{LL} = \delta f = \delta \dot{v} + \breve{\omega} \delta v + \delta \breve{\omega} v + \breve{u} \delta v. \quad (31)$$

From equation (31) it follows:

$$f^p = f^{LL} + \delta f = f^{LL} + \delta \dot{v} + \breve{\omega} \delta v + \delta \breve{\omega} v + \breve{u} \delta v. \quad (32)$$

Let's put equation (30) in equation (32) and get:

$$f^{LL} + \delta \dot{v} + \breve{\omega} \delta v + \delta \breve{\omega} v + \breve{u} \delta v = C_{LL}^p f^{LL} + \Delta f, \quad (33)$$

it follows that:

$$\delta \dot{v} = [C_{LL}^p - I] f^{LL} - [\breve{\omega} + \breve{u}] \delta v - \delta \breve{\omega} v + \Delta f. \quad (34)$$

We will decompose the equation (34):

$$\begin{bmatrix} \delta \dot{v}_E \\ \delta \dot{v}_N \\ \delta \dot{v}_{up} \end{bmatrix} = [C_{LL}^p - I] \begin{bmatrix} f_E \\ f_N \\ f_{up} \end{bmatrix} - \begin{bmatrix} 0 & -\omega_{up} - u_{up} & \omega_N + u_N \\ \omega_{up} + u_{up} & 0 & -\omega_E - u_E \\ -\omega_N - u_N & \omega_E + u_E & 0 \end{bmatrix} \begin{bmatrix} \delta v_E \\ \delta v_N \\ \delta v_{up} \end{bmatrix} - $$
$$- \begin{bmatrix} 0 & -\delta\omega_{up} & \delta\omega_N \\ \delta\omega_{up} & 0 & -\delta\omega_E \\ -\delta\omega_N & \delta\omega_E & 0 \end{bmatrix} \begin{bmatrix} v_E \\ v_N \\ v_{up} \end{bmatrix} + \begin{bmatrix} \mu_E f_E \\ \mu_N f_N \\ \mu_{up} f_{up} \end{bmatrix} + \begin{bmatrix} B_E \\ B_N \\ B_{up} \end{bmatrix}. \quad (35)$$

After converting expression (35), we obtain the error equation of horizontal accelerometers:

$$\delta \dot{v}_E = -f_E - v_{up}(\frac{\delta v_E}{R} - u \sin\varphi\, \delta\varphi) + v_N(\frac{\delta v_E}{R} tg\varphi + u \cos\varphi\, \delta\varphi + \frac{v_E}{R} \sec^2\varphi\, \delta\varphi) -$$
$$-\delta v_{up}(2u \cos\varphi + \frac{v_E}{R}) + \delta v_N(2u \sin\varphi + \frac{v_E}{R} tg\varphi) - f_{up} \cos\Phi_E \sin\Phi_N +$$
$$+f_E \cos\Phi_N \cos\Phi_{up} + f_N(\cos\Phi_N \sin\Phi_{up} + \sin\Phi_E \sin\Phi_N \cos\Phi_{up}) -$$
$$-f_E \sin\Phi_E \sin\Phi_N \sin\Phi_{up} + f_E \mu_E + B_E; \quad (36)$$

$$\delta\dot{v}_N = -f_N - v_{up}\frac{\delta v_N}{R} - v_E(\frac{\delta v_E}{R} - u\cos\varphi\delta\varphi + \frac{v_E}{R}\sec^2\varphi\delta\varphi) - \delta v_{up}\frac{v_N}{R} -$$
$$-\delta v_E(2u\sin\varphi + \frac{v_E}{R}tg\varphi) + fv_N\cos\Phi_E\cos\Phi_{up} - f_E\cos\Phi_E\sin\Phi_{up} + \quad (37)$$
$$+f_{up}\sin\Phi_E + f_N\mu_N + B_N.$$

Here: f_N, f_E, f_{up} – projections of the apparent acceleration of the aircraft on the axis of the geographic trihedron; μ_N, μ_E – accelerometer scale factor errors; B_N, B_E – zero offset accelerometers;

As a result, we obtain 4 nonlinear equations of horizontal errors of autonomous INS (14), (15), (36) and (37). We see that these equations are complex, so to use these equations they need to be simplified.

2. Linear error models of autonomous INS.

Orientation error equation. In an ideal system, the GSP accurately simulates a reference trihedron and the axes closely connected to the stabilized platform of the instrument trihedron $x_p y_p z_p$ are parallel to the axes of the reference trihedron $x_{LL} y_{LL} z_{LL}$. Axes of the instrument trihedron $x_p y_p z_p$ coincide with the axes of sensitivity of gyroscopes in their neutral position (angles of precession are zero).

In real systems, the absolute angular velocities of the reference and instrument trihedron will be inconsistent at small angles $\Phi_E, \Phi_N, \Phi_{up}$, which change over time and characterize the errors of orientation of the GSP. Corners Φ_E, Φ_N determine the errors of the instrument vertical, and the angle Φ_{up} – error in azimuth orientation of the platform.

The matrix of transition from the navigation coordinate system to the platform coordinate system has the form (taking into account the small number of angles $\Phi_E, \Phi_N, \Phi_{up}$) [6]:

$$C_{LL}^p = \begin{bmatrix} 1 & \Phi_{up} & -\Phi_N \\ -\Phi_{up} & 1 & \Phi_E \\ \Phi_N & -\Phi_E & 1 \end{bmatrix}, \quad (38)$$

where "p" – platform trihedron; "LL" – geographical trihedron. The linear equations of INS orientation errors are based on the relation:

$$\begin{bmatrix} \omega_E \\ \omega_N \\ \omega_{up} \end{bmatrix}_p = C_{LL}^p \begin{bmatrix} \omega_E \\ \omega_N \\ \omega_{up} \end{bmatrix}_{LL} + \begin{bmatrix} \Phi_E \\ \Phi_N \\ \Phi_{up} \end{bmatrix}. \quad (39)$$

The first term is responsible for the skew of the platform SC relative to the navigation SC, the second – for departure errors.

Substitute (38) into (39) and obtain a system of equations in scalar form:

$$\begin{cases} \omega_E^P = \omega_E^{LL} + \omega_N^{LL}\Phi_{up} - \Phi_N\omega_{up}^{LL} + \dot{\Phi}_E \\ \omega_N^P = \omega_N^{LL} + \omega_{up}^{LL}\Phi_E - \Phi_{up}\omega_E^{LL} + \dot{\Phi}_E \\ \omega_{up}^P = \omega_{up}^{LL} + \omega_E^{LL}\Phi_N - \Phi_E\omega_N^{LL} + \dot{\Phi}_{up} \end{cases}. \quad (40)$$

Let's find out the reasons of difference ω_i^P i ω_i^{LL}:

$$\begin{cases} \omega_E^P - \omega_E^{LL} = \omega_E^{dr} + \Delta\omega_E \\ \omega_N^P - \omega_N^{LL} = \omega_N^{dr} + \Delta\omega_N \\ \omega_{up}^P - \omega_{up}^{LL} = \omega_{up}^{dr} + \Delta\omega_{up} \end{cases}, \quad (41)$$

where ω_i^{dr} – drift of the computing platform; $\Delta\omega_i$ – errors in calculating platform control signals ω_i:

$$\begin{cases} \Delta\omega_E = -\frac{\delta V_N}{R} \\ \Delta\omega_N = \frac{V_E}{R} - u\sin\varphi\delta\varphi \\ \Delta\omega_{up} = \frac{\delta V_E}{R} + u\cos\varphi\delta\varphi + \frac{V_E}{R}\sec^2\varphi\delta\varphi \end{cases}. \quad (42)$$

Substitute (41), (42) into (40) and obtain:

$$\begin{cases} \dot{\Phi}_E + \omega_N \Phi_{up} - \Phi_N \omega_{up} = -\frac{\delta v_N}{R} + \omega_E^{dr} \\ \dot{\Phi}_N - \omega_E \Phi_{up} + \Phi_E \omega_{up} = \frac{\delta v_E}{R} - u \sin\varphi \, \delta\varphi + \omega_N^{dr} \\ \dot{\Phi}_{up} + \omega_N \Phi_N - \Phi_E \omega_N = \frac{\delta v_E}{R} tg\varphi + (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi)\delta\varphi + \omega_{up}^{dr} \end{cases}. \quad (43)$$

The system of equation (43) describes the errors of the orientation of the platform in the horizon and in azimuth.

Equation of errors of horizontal accelerometers. Linear error equations of horizontal INS accelerometers are based on the relation [7]:

$$\begin{bmatrix} a_E \\ a_N \\ a_{up} \end{bmatrix}_p = C_{LL}^{Pl} \begin{bmatrix} a_E \\ a_N \\ a_{up} \end{bmatrix}_{LL} + \begin{bmatrix} B_E \\ B_N \\ B_{up} \end{bmatrix} + \begin{bmatrix} a_E \cdot \mu_E \\ a_N \cdot \mu_N \\ a_{up} \cdot \mu_{up} \end{bmatrix}, \quad (44)$$

where $B_{E,N,Up}$ – zero offset accelerometers; μ_i – accelerometer scale factor errors; $a_{E,N,Up}$ – projections of the apparent acceleration on the axis of a geographical trihedron.

Substitute (38) into (44) and obtain a system of equations in scalar form:

$$\begin{cases} a_E^p - a_E^{LL} = a_N^{LL} \Phi_{up} - a_{up}^{LL} \Phi_N + B_E + a_E \mu_E \\ a_N^p - a_N^{LL} = -a_E^{LL} \Phi_{up} + a_{up}^{LL} \Phi_E + B_N + a_N \mu_N \end{cases}. \quad (45)$$

Difference a_i^p from a_i^{LL} due to the values $\delta\dot{v}_E, \delta\dot{v}_N$ and errors in the calculation of Coriolis amendments, which are determined by varying the nominal values of these amendments. Finally, we get:

$$\begin{cases} \delta\dot{v}_E = a_N \Phi_{up} - a_{up} \Phi_N + B_E + a_E \mu_E + (2u \sin\varphi + \frac{v_E}{R} tg\varphi)\delta v_N + \\ + v_N u \cos\varphi + \frac{\delta v_E}{R} tg\varphi v_N + (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi) v_N \delta\varphi \\ \delta\dot{v}_N = -a_E \Phi_{up} + a_{up} \Phi_E + B_N + a_N \mu_N - (2u \sin\varphi + \frac{v_E}{R} tg\varphi)\delta v_E - \\ - v_E u \cos\varphi \, \delta\varphi - \frac{\delta v_E}{R} tg\varphi v_E - (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi) v_E \delta\varphi \end{cases}, \quad (46)$$

where

$$\delta\Delta a_E^C = (2u \sin\varphi + \frac{v_E}{R} tg\varphi)\delta v_N + v_N u \cos\varphi + \frac{\delta v_E}{R} tg\varphi v_N + (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi) v_N \delta\varphi;$$

$$\delta\Delta a_N^C = -(2u \sin\varphi + \frac{v_E}{R} tg\varphi)\delta v_E - v_E u \cos\varphi \, \delta\varphi - \frac{\delta v_E}{R} tg\varphi v_E - (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi) v_E \delta\varphi -$$

errors from the calculation of Coriolis corrections.

In addition, the two connection equations have the form [8]:

$$\begin{cases} \delta\dot{\varphi} = \frac{\delta v_N}{R} \\ \delta\dot{\lambda} = \frac{\delta v_E}{R \cos\varphi} + \frac{v_E}{R \cos\varphi} tg\varphi \delta\varphi \end{cases}. \quad (47)$$

Equations (43), (46) and (47) make up the system of equations of INS errors:

$$1) \dot{\Phi}_E + \omega_N \Phi_{up} - \omega_{up} \Phi_N = -\frac{\delta v_N}{R} + \omega_E^{dr};$$

$$2) \dot{\Phi}_N - \omega_E \Phi_{up} - \omega_{up} \Phi_E = \frac{\delta v_E}{R} - u \sin\varphi \, \delta\varphi + \omega_N^{dr};$$

$$3) \dot{\Phi}_{up} + \omega_E \Phi_N - \omega_N \Phi_E = \frac{\delta v_E}{R} tg\varphi + (u \cos\varphi + \frac{v_E}{R} sec^2 \varphi)\delta\varphi + \omega_{up}^{dr};$$

$$4)\delta\dot{v}_E = a_N\Phi_{up} - a_{up}\Phi_N + a_E\mu_E + b_E + (2u\sin\varphi + \frac{v_E}{R}tg\varphi)\delta v_N +$$

$$v_N u\cos\varphi\,\delta\varphi + \frac{\delta v_E}{R}tg\varphi v_N + (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)v_N\delta\varphi;$$

$$5)\delta\dot{v}_N = -a_E\Phi_{up} + a_{up}\Phi_E + a_N\mu_N + b_N - (2u\sin\varphi + \frac{v_E}{R}tg\varphi)\delta v_E -$$

$$-v_E u\cos\varphi\,\delta\varphi - \frac{\delta v_E}{R}tg\varphi v_E - (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)v_E\delta\varphi;$$

$$6)\delta\dot{\varphi} = \frac{\delta v_N}{R};$$

$$7)\delta\dot{\lambda} = \frac{\delta v_E}{R\cos\varphi} + \frac{v_E}{R\cos\varphi}tg\varphi\delta\varphi.$$

We write down the system of INS error equations in discrete form and get:

$$1)\frac{\Phi_{E_{k+1}} - \Phi_{E_k}}{T} + \omega_{N_k}\Phi_{up_k} - \omega_{up_k}\Phi_{N_k} = -\frac{\delta v_{N_k}}{R} + \omega_E^{dr};$$

$$2)\frac{\Phi_{N_{k+1}} - \Phi_{N_k}}{T} - \omega_{N_k}\Phi_{up_k} - \omega_{up_k}\Phi_{N_k} = \frac{\delta v_{E_k}}{R} - u\sin\varphi\,\delta\varphi_k + \omega_N^{dr};$$

$$3)\frac{\Phi_{up_{k+1}} - \Phi_{up_k}}{T} + \omega_{E_k}\Phi_{N_k} - \omega_{N_k}\Phi_{E_k} = (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)\delta\varphi_k + \frac{\delta v_{E_k}}{R}tg\varphi + \omega_{up}^{dr};$$

$$4)\frac{\delta_{E_{k+1}} - \delta_{E_k}}{T} = a_{N_k}\Phi_{up_k} - a_{up_k}\Phi_{N_k} + a_{E_k}\mu_E + b_E + v_N u\cos\varphi\,\delta\varphi_k + (2u\sin\varphi$$

$$+\frac{v_E}{R}tg\varphi)\delta v_{N_k} + \frac{\delta v_{E_k}}{R}tg\varphi v_N + (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)v_N\delta\varphi_k;$$

$$5)\frac{\delta v_{N_{k+1}} - \delta v_{N_k}}{T} = -a_{E_k}\Phi_{up_k} + a_{up_k}\Phi_{E_k} + a_{N_k}\mu_N + b_N - \frac{\delta v_{E_k}}{R}tg\varphi v_E -$$

$$-(u\sin\varphi + \frac{v_E}{R}tg\varphi)\delta v_{E_k} - v_E u\cos\delta\,\varphi_k - (u\cos\varphi + \frac{v_E}{R}sec^2\varphi)v_E\delta\varphi_k;$$

$$6)\frac{\delta\varphi_{k+1} - \delta\varphi_{N_k}}{T} = \frac{\delta v_{N_k}}{R};$$

$$7)\frac{\delta\lambda_{k+1} - \delta\lambda_k}{T} = \frac{\delta v_{E_k}}{R\cos\varphi} + \frac{v_E}{R\cos\varphi}tg\varphi\delta\varphi_k.$$

Let's rewrite the system of INS error equations in matrix form:

$$x_k = Fx_{k-1} + w_{k-1},$$

where $x_k = \begin{bmatrix} \Phi_E \\ \Phi_N \\ \Phi_{up} \\ \delta v_E \\ \delta v_N \\ \delta\varphi \\ \delta\lambda \end{bmatrix}_k$; $w_{k-1} = \begin{bmatrix} T.\omega_E^{dr} \\ T.\omega_N^{dr} \\ T.\omega_{up}^{dr} \\ T.a_E.\mu_E + T.b_E \\ T.a_N.\mu_N + T.b_N \\ 0 \\ 0 \end{bmatrix}_{k-1}$;

$$F = \begin{bmatrix} 1 & T\omega_N & -T\omega_N & 0 & -\frac{T}{R} & 0 & 0 \\ -T\omega_{up} & 1 & T\omega_E & \frac{T}{R} & 0 & -Tu\sin\varphi & 0 \\ T\omega_N & -T\omega_E & 1 & \frac{T}{R}tg\varphi & 0 & T(u\cos\varphi + \frac{v_E}{R}\sec^2\varphi) & 0 \\ 0 & -Ta_{up} & Ta_N & 1+\frac{T}{R}tg\varphi v_N & T(2u\sin\varphi + \frac{v_E}{R}tg\varphi) & T(2v_N u\cos\varphi + \frac{v_N^2}{R}\sec^2\varphi) & 0 \\ Ta_{up} & 0 & -Ta_E & -2T(2\sin + \frac{v_E}{R}tg\varphi) & 1 & -T(\frac{v_E^2}{R}\sec^2\varphi + 2v_E u\cos\varphi) & 0 \\ 0 & 0 & 0 & 0 & \frac{T}{R} & 1 & 0 \\ 0 & 0 & 0 & \frac{T}{R\cos\varphi} & 0 & \frac{Tv_E}{R\cos\varphi}tg\varphi & 1 \end{bmatrix}.$$

In practice, when considering horizontal channels of information, as a rule, simplified equations of errors of autonomous INS are used. At the same time, we neglect cross-links and errors from calculation of Coriolis corrections, and then we can write down the equation of errors of INS separately for each horizontal information channel:

– East Channel:

$$\begin{cases} \delta\dot{v}_E = -g\Phi_N + a_N\Phi_{up} + B_E + \mu_E a_E \\ \dot{\Phi}_N = \frac{\delta v_E}{R} + \omega_N^{dr} \\ \dot{\omega}_N^{dr} = -\beta\omega_N^{dr} + A\sqrt{2\beta}w \end{cases}. \quad (48)$$

– North Channel:

$$\begin{cases} \delta\dot{v}_N = g\Phi_E - a_E\Phi_{up} + B_N + \mu_N a_N \\ \dot{\Phi}_E = -\frac{\delta v_N}{R} + \omega_E^{dr} \\ \dot{\omega}_E^{dr} = -\beta\omega_E^{dr} + A\sqrt{2\beta}w \end{cases}. \quad (49)$$

Here: $\omega_N^{dr}, \omega_E^{dr}$ – projections of the GSP drift velocity on the axis of the geographical trihedron; *A* – root mean square deviation of random drift; *β* – the average frequency of random changes in drift; *w* – white noise.

Equations (48) and (49) include two components: stationary (Schuler's), which contains terms $g\Phi_E, g\Phi_N, B_E, B_N$, which are independent of the motion of objects and non-stationary, which contains members $a_N\Phi_{up}, \mu_E a_E, a_E\Phi_{up}, \mu_N a_N$ and depends on the movement of objects. Consider the stationary equations of INS errors (Bromberg P. V., 1979; Salychev O.S., 2012; Neusypin K.A., 2009):

- East Channel:

$$\begin{cases} \delta\dot{v}_E = -g\Phi_N + B_E \\ \dot{\Phi}_N = \frac{\delta v_E}{R} + \omega_N^{dr} \\ \dot{\omega}_N^{dr} = -\beta\omega_N^{dr} + A\sqrt{2\beta}w \end{cases}. \quad (50)$$

Rewrite equation (50) in matrix form:

$$x_k = Fx_{k-1} + w_{k-1}, \quad (51)$$

where

$$x_k = \begin{bmatrix} \delta v_E \\ \Phi_N \\ \omega_N^{dr} \end{bmatrix}_k; F = \begin{bmatrix} 1 & -Tg & 0 \\ \frac{T}{R} & 1 & T \\ 0 & 0 & 1-T\beta \end{bmatrix}; w_{k-1} = \begin{bmatrix} TB_E \\ 0 \\ TA\sqrt{2\beta}w \end{bmatrix}_{k-1}.$$

– North Channel:

$$\begin{cases} \delta\dot{v}_N = g\Phi_E + B_N \\ \dot{\Phi}_E = -\frac{\delta v_N}{R} + \omega_E^{dr} \\ \dot{\omega}_E^{dr} = -\beta\omega_E^{dr} + A\sqrt{2\beta}w \end{cases}. \quad (52)$$

Rewrite equation (52) in matrix form:

$$x_k = Fx_{k-1} + w_{k-1}, \quad (53)$$

where

$$x_k = \begin{bmatrix} \delta v_N \\ \Phi_E \\ \omega_E^{dr} \end{bmatrix}_k; F = \begin{bmatrix} 1 & Tg & 0 \\ -\frac{T}{R} & 1 & T \\ 0 & 0 & 1 - T\beta \end{bmatrix}; w_{k-1} = \begin{bmatrix} TB_N \\ 0 \\ TA\sqrt{2\beta}w \end{bmatrix}_{k-1}.$$

Thus, linear models of INS errors in discrete form are obtained, which are used later in the development of algorithms.

Conclusions

1. Linear and nonlinear mathematical models of INS errors are considered. Depending on the requirement for accuracy and time of calculation of navigation parameters, different models of INS errors are selected.

2. It is advisable to use a linear mathematical model because of its simplicity, while the calculation time is quite short.

3. Currently, with the development of modern computing technologies, there are powerful onboard digital computer system (ODCS) that can solve complex problems in an acceptable time interval. Therefore, as a model of INS errors, you can use nonlinear models and thus get better results compared to the use of a linear model.

The direction of further research is practical modeling in the *MatLab* environment of linear and nonlinear model of INS errors.

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