Triple Diffusive Surface Tension Driven Convection in a Composite Layer in the Presence of Vertical Magnetic Field

Manjunatha N., Sumithra R.

Abstract: ***The problem of triple diffusive surface tension driven convection is investigated in a composite layer in the presence of vertical magnetic field. A closed form solution is obtained under microgravity condition. The parameters suitable for fluid layer dominant and porous layer dominant composite layers are determined. The parameters appropriate for controlling the convection are determined which are useful to manufacture pure crystals.***

Keywords: ***Triple diffusive, Species concentration, Magnetic field, Surface tension, Composite layer.***

I. INTRODUCTION

The presence of more than one chemical dissolved in fluid mixtures is very often requested for describing natural phenomena such as contaminant transport, warming of stratosphere, magmas and sea water. The multi component has wide applications in crystal growth, geothermally heated lakes, earth core, solidification of molten alloys, underground water flow, acid rain effects and so on. For single fluid layer, Chand [1] has applied the linear stability analysis and a normal mode analysis to study the triple-diffusive convection in a micropolar ferromagnetic fluid layer heated and saluted from below. Suresh Chand [10] has investigated the triple-diffusive convection in a micropolar ferrofluid layer heated and saluted below subjected to a transverse uniform magnetic field in the presence of uniform vertical rotation. In porous medium, the triply diffusive convection in a Maxwell viscoelastic fluid is mathematically investigated in the presence of uniform vertical magnetic field through porous medium studied by Pawan Kumar Sharma *et al.* [8] using linearized stability theory and normal mode analysis.

For the composite layers, Sumithra [9] has studied the triple-diffusive Marangoni convection in a two layer system and obtained the analytical expression for the thermal Marangoni Number. Manjunatha and Sumithra [3-6] have investigated the combined effects of magnetic field and non uniform basic temperature gradients on two and three component convection in two layer system.

In this paper the lower rigid surface of the porous layer and the upper free surface are considered to be insulating to temperature, insulating to both salute concentration perturbations. At the upper free surface, the surface tension effects depending on temperature and salinities are considered. At the interface, the normal and tangential components of velocity, heat and heat flux, mass and mass flux are assumed to be continuous and intended for Darcy-Brinkman model. The resulting eigenvalue problem is solved exactly and an analytical expression for the thermal Marangoni number is obtained for composite layer.

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II. FORMULATION OF THE PROBLEM

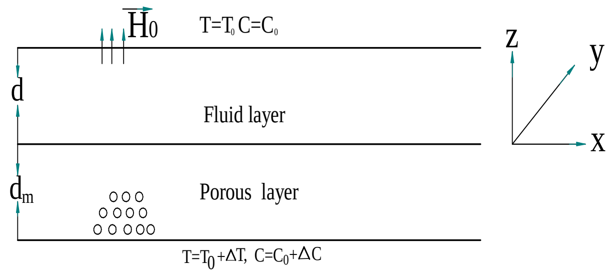

Figure 1: Physical configuration

Consider a three different diffusing components with different molecular diffusivities, electrically conducting fluid layer of thickness d horizontal above the isotropic sparsely packed porous layer saturated with same fluid of thickness d_m in the presence of magnetic field H_0 in the vertical Z-direction. The lower surface of the porous layer is considered to be rigid and the upper surface of the fluid layer is free at which the surface tension effects depending on temperature and both the species concentrations is considered. Both the boundaries are kept at different constant temperatures and salinities. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the Z- axis, vertically upwards.

The basic equations for fluid and porous layer respectively as,

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\nabla \cdot \vec{H} = 0 \tag{2}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla P + \mu \nabla^2 \vec{q} + \mu_p \left(\vec{H} \cdot \nabla \right) \vec{H} \tag{3}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{4}$$

$$\frac{\partial C_1}{\partial t}+(\vec{q}\cdot\nabla)C_1=\kappa_1\nabla^2 C_1 \quad (5)$$

$$\frac{\partial C_2}{\partial t}+(\vec{q}\cdot\nabla)C_2=\kappa_2\nabla^2 C_2 \quad (6)$$

$$\frac{\partial \vec{H}}{\partial t}=\nabla\times\vec{q}\times\vec{H}+\nu_m\nabla^2\vec{H} \quad (7)$$

$$\nabla_m\cdot\vec{q}_m=0 \quad (8)$$

$$\nabla_m\cdot\vec{H}_m=0 \quad (9)$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}_m}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}_m\cdot\nabla_m)\vec{q}_m\right]=-\nabla_m P_m+\mu_m\nabla^2\vec{q}_m-\frac{\mu}{K}\vec{q}_m-\mu_p\left(\vec{H}_m\cdot\nabla_m\right)\vec{H}_m \quad (10)$$

$$A\frac{\partial T_m}{\partial t}+(\vec{q}_m\cdot\nabla_m)T_m=\kappa_m\nabla_m^2 T_m \quad (11)$$

$$\varepsilon\frac{\partial C_{m1}}{\partial t}+(\vec{q}_m\cdot\nabla_m)C_{m1}=\kappa_{m1}\nabla_m^2 C_{m1} \quad (12)$$

$$\varepsilon\frac{\partial C_{m2}}{\partial t}+(\vec{q}_m\cdot\nabla_m)C_{m2}=\kappa_{m2}\nabla_m^2 C_{m2} \quad (13)$$

$$\varepsilon\frac{\partial \vec{H}}{\partial t}=\nabla_m\times\vec{q}_m\times\vec{H}_m+\nu_{em}\nabla_m^2\vec{H}_m \quad (14)$$

Here $\vec{q}=(u,v,w)$ is the velocity vector, $\vec{H}$ is the magnetic field, t is the time, μ is the fluid viscosity, $P=p+\frac{\mu_p H^2}{2}$ is the total pressure, ρ_0 is the fluid density, μ_p is the magnetic permeability, ε is the porosity, $A=\frac{(\rho_0 C_p)_m}{(\rho_0 C_p)_f}$ is the ratio of heat capacities, C_p is the specific heat, K is the permeability of the porous medium, T is the temperature, κ is the thermal diffusivity of the fluid, κ_1 and κ_2 are the solute1 and solute2 diffusivity of the fluid in the fluid layer, C_1 and C_2 are the concentration1 and concentration2 for the fluid in the fluid layer, $\nu_m=\frac{1}{\mu_p\sigma}$ is the magnetic viscosity, μ_m is the effective viscosity of the fluid in the porous layer, C_{m1} and C_{m2} are the concentration1 and concentration2 for the fluid in porous layer, $\nu_{em}=\frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts 'm' and 'f ' refer to the porous and the fluid layer respectively.

The basic steady state is assumed to the quiescent and consider the solution of the form,

$$\vec{q}=\vec{q}_b,\ P=P(z),\ T=T_b(z),\ C_1=C_{1b}(z),\ C_2=C_{2b}(z),\ \vec{H}=H_0(z) \quad (15)$$

$$\vec{q}_m=\vec{q}_{mb}, P_m=P_{mb}(z_m), T_m=T_{mb}(z_m), C_{m1}=C_{1mb}(z_m), C_{2m}=C_{2mb}(z_m) \quad (16)$$

The temperature and species concentration distributions respectively are found to be

$$T_b(z)=T_0-\frac{(T_0-T_u)z}{d} \qquad 0\le z\le d \quad (17)$$

$$T_{mb}(z_m)=T_0-\frac{(T_l-T_u)z_m}{d_m} \qquad 0\le z_m\le d_m \quad (18)$$

$$C_{1b}(z)=C_{10}-\frac{(C_{10}-C_{1u})z}{d} \qquad 0\le z\le d \quad (19)$$

$$C_{1mb}(z_m)=C_{10}-\frac{(C_{1l}-C_{1u})z_m}{d_m} \qquad 0\le z_m\le d_m \quad (20)$$

$$C_{2b}(z)=C_{20}-\frac{(C_{20}-C_{2u})z}{d} \qquad 0\le z\le d \quad (21)$$

$$C_{2mb}(z_m)=C_{20}-\frac{(C_{2l}-C_{20})z_m}{d_m} \qquad 0\le z_m\le d_m \quad (22)$$

where

$T_0=\frac{\kappa d_m T_u+\kappa_m d T_l}{\kappa d_m+\kappa_m d}$, $C_{10}=\frac{k_1 d_m C_{1u}+k_{1m} d C_{1l}}{k_1 d_m+k_{1m} d}$, $C_{20}=\frac{k_2 d_m C_{2u}+k_{2m} d C_{2l}}{k_2 d_m+k_{2m} d}$ are the interface temperature and concentrations and the subscript 'b' denotes the basic state.

To examine the stability of the system, we give a small perturbation to the system as

$$\vec{q}=\vec{q}_b+\vec{q}',\ P=P_b(z)+P',\ T=T_b(z)+\theta,$$
$$C_1=C_{1b}(z)+S_1,\ C_2=C_{2b}(z)+S_2,\ \vec{H}=H_0(z)+\vec{H}' \quad (23)$$

$$\vec{q}_m=\vec{q}_{mb}+\vec{q}_m',\ P_m=P_{mb}(z_m)+P_m',\ T_m=T_{mb}(z_m)+\theta_m,$$
$$C_{1m}=C_{1mb}(z_m)+S_{m1},\ C_{2m}=C_{2mb}(z_m)+S_{m2},$$
$$\vec{H}_m=H_0(z_m)+\vec{H}_m' \quad (24)$$

Where the primed quantities are the dimensionless ones. Equations (23) & (24) are substituted into the (1) to (14), apply curl twice to eliminate the pressure term from (3) & (10) and then the variables are nondimensionalized.

To render the equations nondimensional, we choose different scales for the two layers (Chen and Chen [2], Nield [7]), so that both layers are of unit length such that $(x,y,z)=d(x',y',z')$ and $(x_m,y_m,z_m)=d_m(x_m',y_m',z_m'-1)$.

Omitting the primes for simplicity, we get in $0\le z\le 1$ and $0\le z_m\le 1$ respectively

$$\frac{1}{\Pr}\frac{\partial(\nabla^2 w)}{\partial t}=\nabla^4 w+Q\tau_{fm}\frac{\partial(\nabla^2 H_z)}{\partial z} \quad (25)$$

$$\frac{\partial\theta}{\partial t}=W+\nabla^2\theta \quad (26)$$

$$\frac{\partial S_1}{\partial t}=W+\tau_1\nabla^2 S_1 \quad (27)$$

$$\frac{\partial S_2}{\partial t}=W+\tau_2\nabla^2 S_2 \quad (28)$$

$$\frac{\partial H_z}{\partial t}=\frac{\partial w}{\partial t}+\tau_{fm}\nabla^2 H_z \quad (29)$$

$$\frac{\beta^2}{\Pr_m}\frac{\partial \nabla_m^2 w_m}{\partial t} = \hat{\mu}\beta^2\nabla_m^4 w_m - \nabla_m^2 w_m + Q_m \tau_{mm}\beta^2 \frac{\partial \nabla_m^2 H_{zm}}{\partial z_m} \quad (30)$$

$$A\frac{\partial \theta_m}{\partial t} = w_m + \nabla_m^2 \theta_m \quad (31)$$

$$\varepsilon \frac{\partial S_{m1}}{\partial t} = w_m + \tau_{m1}\nabla_m^2 S_{m1} \quad (32)$$

$$\varepsilon \frac{\partial S_{m2}}{\partial t} = w_m + \tau_{m2}\nabla_m^2 S_{m2} \quad (33)$$

$$\varepsilon \frac{\partial H_{zm}}{\partial t} = \frac{\partial w_m}{\partial t} + \tau_{mm}\nabla_m^2 H_{zm} \quad (34)$$

Where, for the fluid layer $\Pr = \frac{\nu}{\kappa}$ is the Prandtl number, $Q = \frac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ is the Chandrasekhar number, $\tau_1 = \frac{\kappa_1}{\kappa}$ is the ratio salinity1 diffusivity to thermal diffusivity, $\tau_2 = \frac{\kappa_2}{\kappa}$ is the ratio salinity2 diffusivity to thermal diffusivity. For the porous layer, $\Pr_m = \frac{\varepsilon \nu_m}{\kappa_m}$ is the Prandtl number, $\beta^2 = \frac{K}{d_m^2} = Da$ is the Darcy number, β porous parameter, $\hat{\mu} = \frac{\mu_m}{\mu}$ is the viscosity ratio, $Q_m = \frac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}} = Q\varepsilon\hat{d}^2$ is the Chandrasekhar number , $\tau_{m1} = \frac{\kappa_{m1}}{\kappa_m}$ is the ratio salinity1 diffusivity to thermal diffusivity, $\tau_{m2} = \frac{\kappa_{m2}}{\kappa_m}$ is the ratio salinity2 diffusivity to thermal diffusivity, θ and θ_m are the temperature in fluid and porous layers respectively, S_1, S_2 and S_{m1}, S_{m2} are the concentrations in fluid and porous layer respectively and W and W_m are the dimensionless vertical velocities in fluid and porous layer respectively.

We apply normal mode expansion on dependent variables as follows,

$$\begin{bmatrix} w \\ \theta \\ S_1 \\ S_2 \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ S_1(z) \\ S_2(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \quad (35)$$

$$\begin{bmatrix} w_m \\ \theta_m \\ S_{m1} \\ S_{m2} \\ H_m \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \Theta_m(z_m) \\ S_{m1}(z_m) \\ S_{m2}(z_m) \\ H_m(z_m) \end{bmatrix} f_m(x_m,y_m)e^{n_m t} \quad (36)$$

With $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the nondimensional horizontal wavenumbers, n and n_m are the frequencies. Since the dimensional horizontal wavenumbers must be the same for the fluid and porous layers, we must have $\frac{a}{d} = \frac{a_m}{d_m}$ and hence $a_m = \hat{d}a$.

Introducing Eqs. (35) and (36) into the Eqs. (25) to (34) then we get an Eigen value problem consisting of the following ordinary differential equation in $0 \le z \le 1$ and $0 \le z_m \le 1$ respectively

$$\left(D^2 - a^2 + \frac{n}{\Pr}\right)\left(D^2 - a^2\right)W = -Q\tau_{fm}D\left(D^2 - a^2\right)H \quad (37)$$

$$\left(D^2 - a^2 + n\right)\Theta + W = 0 \quad (38)$$

$$\left[\tau_1\left(D^2 - a^2\right) + n\right]S_1 + W = 0 \quad (39)$$

$$\left[\tau_2\left(D^2 - a^2\right) + n\right]S_2 + W = 0 \quad (40)$$

$$\left[\tau_{fm}\left(D^2 - a^2\right) + n\right]H + DW = 0 \quad (41)$$

$$\left[\left(D_m^2 - a_m^2\right)\hat{\mu}\beta^2 + \frac{n_m\beta^2}{\Pr_m} - 1\right]\left(D_m^2 - a_m^2\right)W_m = Q_m\tau_{mm}\beta^2 D_m\left(D_m^2 - a_m^2\right)H_m \quad (42)$$

$$\left(D_m^2 - a_m^2 + An_m\right)\Theta_m + W_m = 0 \quad (43)$$

$$\left[\tau_{m1}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]S_{m1} + W_m = 0 \quad (44)$$

$$\left[\tau_{m2}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]S_{m2} + W_m = 0 \quad (45)$$

$$\left[\tau_{mm}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]H_m + DW_m = 0 \quad (46)$$

It is known that the principle of exchange of instabilities holds for triple diffusive magneto convection in both fluid and porous layers separately for certain choice of parameters. Therefore, we assume that the principle of exchange of instabilities holds even for the composite layers. In other words, it is assumed that the onset of convection is in the form of steady convection and accordingly we take $n = n_m = 0$. Eliminating the magnetic field in Eqs. (41) and (46). The Eigen value problem becomes, in $0 \le z \le 1$ and $0 \le z_m \le 1$ respectively.

$$\left(D^2 - a^2\right)^2 W = QD^2 W \quad (47)$$

$$\left(D^2 - a^2\right)\Theta + W = 0 \quad (48)$$

$$\tau_1\left(D^2 - a^2\right)S_1 + W = 0 \quad (49)$$

$$\tau_2\left(D^2 - a^2\right)S_2 + W = 0 \quad (50)$$

$$\left[\left(D_m^2 - a_m^2\right)\hat{\mu}\beta^2 - 1\right]\left(D_m^2 - a_m^2\right)W_m = Q_m\beta^2 D_m^2 W_m \quad (51)$$

$$\left(D_m^2 - a_m^2\right)\Theta_m + W_m = 0 \quad (52)$$

$$\left[\tau_{m1}\left(D_m^2 - a_m^2\right)\right]S_{m1} + W_m = 0 \quad (53)$$

$$\left[\tau_{m2}\left(D_m^2 - a_m^2\right)\right]S_{m2} + W_m = 0 \quad (54)$$

III. BOUNDARY CONDITIONS

The boundary conditions are nondimensionalized then subjected to normal mode analysis and finally they take the form

$$D^2W(1)+a^2M\Theta(1)+a^2M_{s1}S_1(1)+a^2M_{s2}S_2(1)=0$$
$$W(1)=0, D\Theta(1)=0,\ DS_1(1)=0, DS_2(1)=0,$$
$$\hat{T}W(0)=W_m(1),\ \hat{T}\hat{d}DW(0)=D_mW_m(1),$$
$$\hat{T}\hat{d}^2\left(D^2+a^2\right)W(0)=\hat{\mu}\left(D_m^2+a_m^2\right)W_m(1),$$
$$\hat{T}\hat{d}^3\beta^2\left(D^3W(0)-3a^2DW(0)\right)=$$
$$-D_mW_m(1)+\hat{\mu}\beta^2\left(D_m^3W_m(1)-3a_m^2D_mW_m(1)\right),$$
$$\Theta(0)=\hat{T}\Theta_m(1),\quad D\Theta(0)=D_m\Theta_m(1),$$
$$S_1(0)=\hat{S}_1S_{m1}(1),\quad DS_1(0)=D_mS_{m1}(1),$$
$$S_2(0)=\hat{S}_2S_{m2}(1),\quad DS_2(0)=D_mS_{m2}(1),$$
$$W_m(0)=0, D_mW_m(0)=0, D_m\Theta_m(0)=0,$$
$$D_mS_{m1}(0)=0, D_mS_{m2}(0)=0 \qquad (55)$$

Where $M=-\dfrac{\partial\sigma_t}{\partial T}\dfrac{(T_0-T_u)d}{\mu\kappa}$ is the thermal Marangoni number, $M_{s1}=-\dfrac{\partial\sigma_t}{\partial C_1}\dfrac{(C_{10}-C_{1u})d}{\mu\kappa}$ is the solute1 Marangoni number, $M_{s2}=-\dfrac{\partial\sigma_t}{\partial C_2}\dfrac{(C_{20}-C_{2u})d}{\mu\kappa}$ is the solute2 Marangoni number, $\hat{T}=\dfrac{\kappa}{\kappa_m}$ is the ratio of thermal diffusivities of fluid to porous layer , $\hat{d}=\dfrac{d_m}{d}$ is the depth ratio, $\hat{S}_1=\dfrac{\kappa_{s1}}{\kappa_{s1m}}$ is the ratio of solute1 diffusivities of fluid to porous layer , $\hat{S}_2=\dfrac{\kappa_{s2}}{\kappa_{s2m}}$ is the ratio of solute2 diffusivities of fluid to porous layer.

IV. METHOD OF SOLUTION

From eqs (47) and (51), we get velocity distributions for fluid and porous layer respectively

$$W(z)=A_1cosh\delta z+A_2sinh\delta z+A_3cosh\xi z+A_4sinh\xi z \qquad (56)$$

$$W_m(z_m)=A_5coshc_4z_m+A_6sinhc_4z_m+A_7coshc_5z_m+A_8sinhc_5z_m \qquad (57)$$

Where

$$\delta=\frac{\sqrt{Q}-\sqrt{Q+4a^2}}{2},\ \xi=\frac{\sqrt{Q}+\sqrt{Q+4a^2}}{2}$$

$C_4=\sqrt{\dfrac{C_1+C_3}{2}}, C_5=\sqrt{\dfrac{C_1-C_3}{2}}$ and $A_i's$ $(i=1,2,....8)$ are arbitrary constants are obtained by using velocity boundary conditions of (55). The expressions for $W(z)$ and $W_m(z_m)$ are appropriately written as

$$W(z)=A_1\left[cosh\delta z+a_1sinh\delta z+a_2cosh\xi z+a_3sinh\xi z\right] \qquad (58)$$

$$W_m(z_m)=A_1\begin{bmatrix}a_4\cosh c_4z_m+a_5sinhc_4z_m+a_6coshc_5z_m\\+a_7sinhc_5z_m\end{bmatrix} \qquad (59)$$

We get the species concentration for fluid layer S_1, S_2 from Eqs. (49) & (50) also from Eqs. (53) & (54), we get the species concentration for porous layer S_{m1}, S_{m2} using the species concentration boundary conditions of (55) as

$$S_1(z)=A_1\left[a_8coshaz+a_9sinhaz+\frac{f(z)}{\tau_1}\right] \qquad (60)$$

$$S_{m1}(z_m)=A_1\left[a_{10}cosha_mz_m+a_{11}sinha_mz_m+\frac{f_m(z_m)}{\tau_{m1}}\right] \qquad (61)$$

$$S_2(z)=A_1\left[a_{16}coshaz+a_{17}sinhaz+\frac{f(z)}{\tau_2}\right] \qquad (62)$$

$$S_{m2}(z_m)=A_1\left[a_{18}cosha_mz_m+a_{19}sinha_mz_m+\frac{f_m(z_m)}{\tau_{m2}}\right] \qquad (63)$$

Where

$$f(z)=-\frac{(a_1\sinh\delta z+\cosh\delta z)}{(\delta^2-a^2)}-\frac{(a_3\sinh\xi z+a_2\cosh\xi z)}{(\xi^2-a^2)}$$

$$f_m(z_m)=-\frac{(a_5\sinh c_4z_m+a_4\cosh c_4z_m)}{(c_4^2-a_m^2)}-\frac{(a_7\sinh c_5z_m+a_6\cosh c_5z_m)}{(c_5^2-a_m^2)}$$

$$a_8=\hat{S}_1(a_{10}\cosh a_m+a_{11}\sinh a_m)-\Delta_{47}$$

$$a_9=\frac{1}{a}(a_{11}a_m\cosh a_m+a_{10}a_m\sinh a_m-\Delta_{48})$$

$$a_{10}=-\frac{\Delta_{51}}{\Delta_{50}},\ a_{11}=\frac{\Delta_{49}}{a_m}$$

$$a_{16}=\hat{S}_2(a_{18}\cosh a_m+a_{19}\sinh a_m)-\Delta_{59}$$

$$a_{17}=\frac{1}{a}(a_{19}a_m\cosh a_m+a_{18}a_m\sinh a_m-\Delta_{60})$$

$$a_{18}=\frac{\Delta_{63}}{\Delta_{62}},\ a_{19}=\frac{\Delta_{61}}{a_m}$$

$$\Delta_{46}=\frac{1}{\tau_1}\left[\frac{\delta(\sinh\delta+a_1\cosh\delta)}{(\delta^2-a^2)}+\frac{\xi(a_2\sinh\xi+a_3\cosh\xi)}{(\xi^2-a^2)}\right]$$

$$\Delta_{47}=\frac{\hat{S}_1\Delta_{470}}{\tau_{m1}}-\frac{\Delta_{471}}{\tau_1}$$

$$\Delta_{470}=\frac{(a_5\sinh c_4+a_4\cosh c_4)}{(c_4^2-a_m^2)}+\frac{(a_7\sinh c_5+a_6\cosh c_5)}{(c_5^2-a_m^2)}$$

$$\Delta_{471}=\frac{1}{(\delta^2-a^2)}+\frac{a_2}{(\xi^2-a^2)},\ \Delta_{48}=\frac{\Delta_{480}}{\tau_{m1}}-\frac{\Delta_{481}}{\tau_1}$$

$$\Delta_{480}=\frac{c_4(a_4\sinh c_4+a_5\cosh c_4)}{(c_4^2-a_m^2)}+\frac{c_5(a_6\sinh c_5+a_7\cosh c_5)}{(c_5^2-a_m^2)}$$

$$\Delta_{481} = \frac{a_1\delta}{\left(\delta^2 - a^2\right)} + \frac{a_3\xi}{\left(\xi^2 - a^2\right)},$$

$$\Delta_{49} = \frac{1}{\tau_{m1}}\left[\frac{a_5c_4}{\left(c_4^2 - a_m^2\right)} + \frac{c_5a_7}{\left(c_5^2 - a_m^2\right)}\right]$$

$$\Delta_{50} = a\hat{S}_1 \sinh a \cosh a_m + a_m \cosh a \sinh a_m$$

$$\Delta_{51} = \frac{\Delta_{49}}{a_m}\left(a\hat{S}_1 \sinh a \sinh a_m + a_m \cosh a \cosh a_m\right) - \Delta_{510}$$

$$\Delta_{510} = a \sinh a\ \Delta_{47} + \Delta_{48} \cosh a + \Delta_{46}$$

$$\Delta_{58} = \frac{\tau_1\Delta_{46}}{\tau_2},\ \Delta_{59} = \frac{\hat{S}_2\Delta_{470}}{\tau_{m2}} - \frac{\Delta_{471}}{\tau_2}$$

$$\Delta_{60} = \frac{\Delta_{480}}{\tau_{m2}} - \frac{\Delta_{481}}{\tau_2},\ \Delta_{61} = \frac{1}{\tau_{m2}}\left[\frac{a_5c_4}{\left(c_4^2 - a_m^2\right)} + \frac{c_5a_7}{\left(c_5^2 - a_m^2\right)}\right]$$

$$\Delta_{62} = a\hat{S}_2 \sinh a \cosh a_m + a_m \cosh a \sinh a_m$$

$$\Delta_{63} = -\frac{\Delta_{61}}{a_m}\left(a\hat{S}_2 \sinh a \sinh a_m + a_m \cosh a \cosh a_m\right) + \Delta_{630}$$

$$\Delta_{630} = a \sinh a\ \Delta_{59} + \Delta_{60} \cosh a + \Delta_{58}$$

V. THERMAL MARANGONI NUMBER

From eqs (48) & (52), we get temperature distributions for fluid and porous layers using temperature boundary conditions of (55) and they are

$$\Theta(z) = A_1\left[a_{12} coshaz + a_{13} sinhaz - f(z)\right] \quad (64)$$

$$\Theta_m(z_m) = A_1\left[a_{14} cosha_m z_m + a_{15} sinha_m z_m - f_m(z_m)\right] \quad (65)$$

Where

$$a_{12} = \hat{T}(a_{14} \cosh a_m + a_{15} \sinh a_m) - \Delta_{52}$$

$$a_{13} = \frac{1}{a}(a_{15} a_m \cosh a_m + a_{14} a_m \sinh a_m - \Delta_{53})$$

$$a_{14} = -\frac{\Delta_{57}}{\Delta_{56}},\ a_{15} = \frac{\Delta_{55}}{a_m}$$

$$\Delta_{52} = \hat{T}\Delta_{480} - \Delta_{470}, \Delta_{53} = \Delta_{480} - \Delta_{481}$$

$$\Delta_{54} = \frac{\delta\left(\sinh\delta + a_1\cosh\delta\right)}{\left(\delta^2 - a^2\right)} + \frac{\xi\left(a_2\sinh\xi + a_3\cosh\xi\right)}{\left(\xi^2 - a^2\right)}$$

$$\Delta_{55} = \frac{a_5c_4}{\left(c_4^2 - a_m^2\right)^2} + \frac{c_5a_7}{\left(c_5^2 - a_m^2\right)^2},\ \Delta_{56} = \Delta_{38},$$

$$\Delta_{57} = \frac{\Delta_{55}}{a_m}\left(a\hat{T} \sinh a \sinh a_m + a_m \cosh a \cosh a_m\right) - \Delta_{570},$$

$$\Delta_{570} = a \sinh a\ \Delta_{52} + \Delta_{53} \cosh a + \Delta_{54}$$

Now the thermal Marangoni number is obtained by the boundary condition (55) as

$$M = \frac{-1}{a^2}\left[\frac{D^2W(1) + M_{s1}a^2S_1(1) + M_{s2}a^2S_2(1)}{\Theta(1)}\right]$$

$$M = -\left[\frac{\Lambda_1 + \Lambda_2 + \Lambda_3}{\Lambda_4}\right] \quad (66)$$

Where

$$\Lambda_1 = \delta^2 \cosh\delta + a_1\delta^2 \sinh\delta + a_2\xi^2 \cosh\xi + a_3\xi^2 \sinh\xi$$

$$\Lambda_2 = M_{s1}a^2\left[a_8 \cosh a + a_9 \sinh a - \frac{R_1}{\tau_1}\right]$$

$$\Lambda_3 = M_{s2}a^2\left[a_{16} \cosh a + a_{17} \sinh a - \frac{R_1}{\tau_2}\right]$$

$$\Lambda_4 = a^2\left[a_{12} \cosh a + a_{13} \sinh a - R_1\right]$$

$$R_1 = \frac{\left(a_1\sinh\delta + \cosh\delta\right)}{\left(\delta^2 - a^2\right)} + \frac{\left(a_3\sinh\xi + a_2\cosh\xi\right)}{\left(\xi^2 - a^2\right)}$$

VI. RESULT AND DISCUSSION

The thermal Marangoni number M obtained as a function of the parameters is drawn versus the depth ratio $\hat{d} = \frac{d_m}{d}$ and the results are represented graphically showing the effects of the variation of one physical quantity fixing the other parameters. The dimensionless fixed values are

$\hat{T} = 1.0,\ a = 1.2,\ \hat{\mu} = 2.0, \beta = 0.3,\ \varepsilon = 1.0, Q = 50,\ M_{s1} = 10$
$M_{s2} = 10, \tau_1 = \tau_2 = \tau_{m1} = \tau_{m2} = \hat{S}_1 = \hat{S}_2 = 0.75$.

The effects of the parameters $a, \beta, \varepsilon, \hat{\mu}, Q, \tau_1, M_{s1}, M_{s2}$ and τ_{m1} on thermal Marangoni number are depicted in figures 2 to 10.

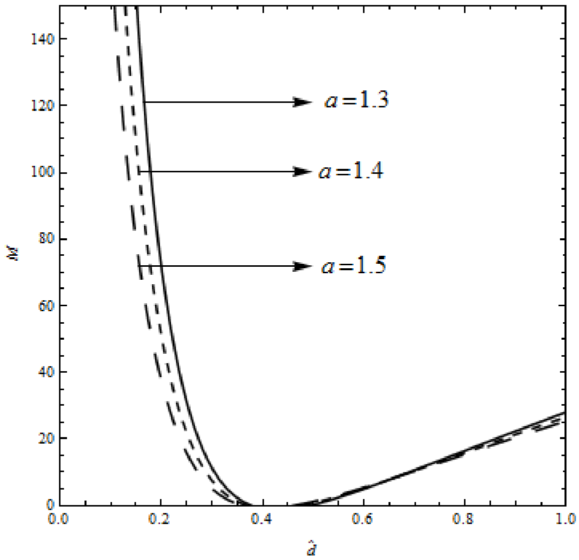

Figure 2: Effects of horizontal wave number a

Figure 2 show the effects of a, horizontal wave number on the thermal Marangoni number M for the values a =1.3, 1.4, 1.5. It is evident from the graph that an increase in the value of a, the thermal Marangoni number decreases and its effect is to destabilize the system. Also the curves are converging indicating that the effect of horizontal wave number is drastic for fluid layer dominant composite layers.

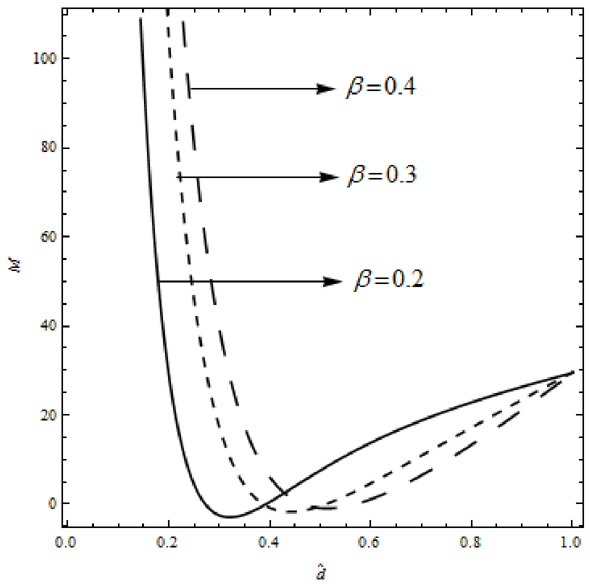

Figure 3: Effects of porous parameter β

Figure 3 show the variations of the porous parameter $\beta = \sqrt{K/d_m^2}$ on the thermal Marangoni number for the values β =0.2, 0.3, 0.4. Increase in the value of β , that is, increasing the permeability, the thermal Marangoni number increases. Hence the surface tension driven triple diffusive magneto convection sets in earlier on increasing the porous parameter, this may be due to presence of diffusing components. Also, for $\hat{d} \geq 0.4$ the thermal Marangoni number decreases to destabilize the system.

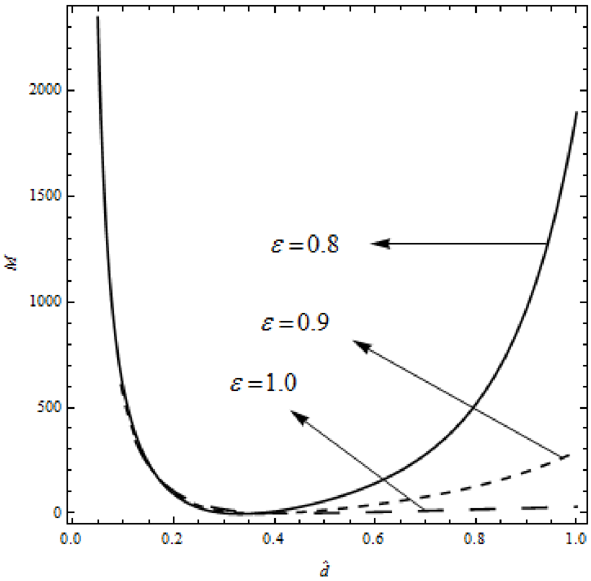

Figure 4: Effects of porosity ε

Figure 4 show the effects of porosity ε for the values ε =0.8, 0.9, 1.0. It is observed that there is no effect of porosity for smaller value of depth ratio up to $\hat{d} \geq 0.4$. For $\hat{d} \geq 0.4$ the curves are diverging indicating that, its effect is drastic for larger depth ratios, hence its effect is immense for porous layer dominant composite layer. Whereas ε increases, the thermal Marangoni number decreases i.e., to destabilize the system.

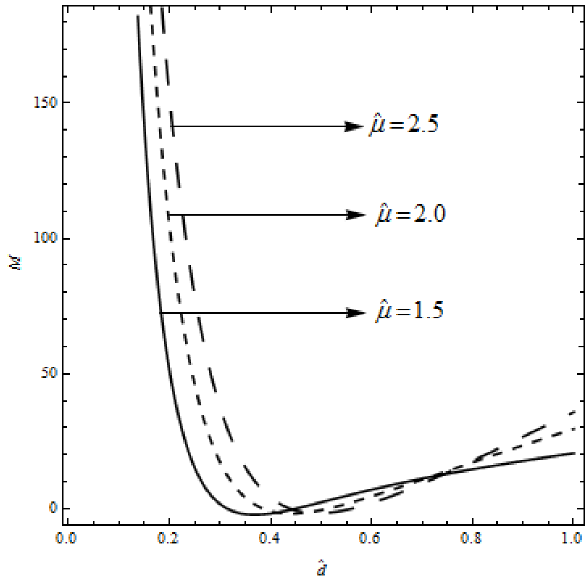

Figure 5: Variations of viscosity ratio $\hat{\mu}$

Figure 5 show the variations of viscosity ratio $\hat{\mu}$ for the values $\hat{\mu}$ = 1.5, 2.0, 2.5. Increase in the value of $\hat{\mu}$, the values of the thermal Marangoni number M increases for $\hat{d} \leq 0.4$. Also, $\hat{d} \geq 0.4$ the increase in the values of viscosity ratio decreases the thermal Marangoni number. By increasing the viscosity ratio the system can be stabilized or destabilized and hence the surface tension driven triple diffusive magneto convection is delayed or faster.

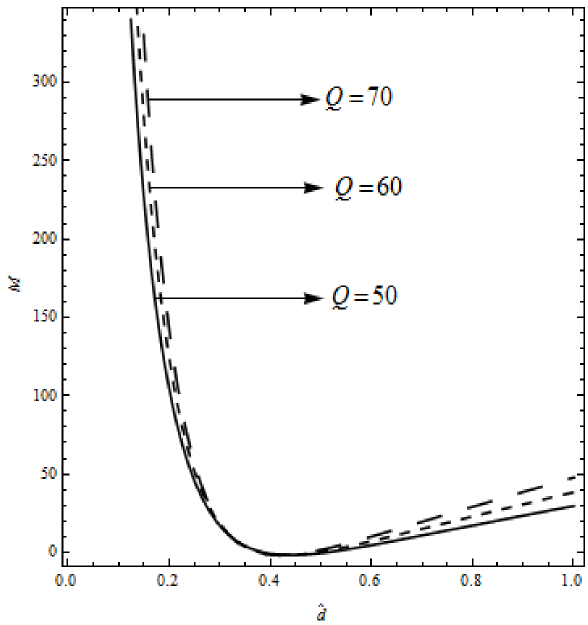

Figure 6: Effects of Chandrasekhar number Q

Figure 6 exhibits the effects of the magnetic field on the onset of triple diffusive surface tension driven magneto convection by the Chandrasekhar number Q for the values Q= 50, 60, 70 .When the value of the Q is increasing, the thermal Marangoni number increases for smaller depth ratio.

The curves are converging between the $0 \le \hat{d} \le 0.5$, which is evident that the effect of Q is drastic for fluid layer dominant composite layer. Also, $\hat{d} \ge 0.5$ the curves are diverging indicating that the effect of Q is effective for porous layer dominant composite layer.

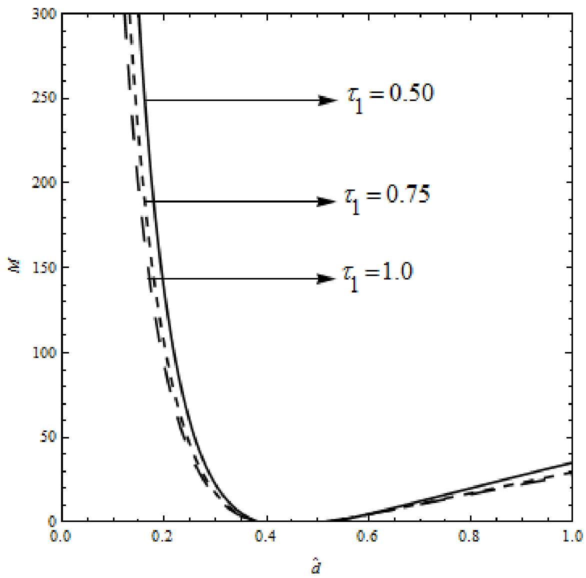

Figure 7: Effects of τ_1

Figure 7 display the effects of τ_1 is the ratio of solute1 diffusivity to thermal diffusivity fluid in fluid layer for the values τ_1 = 0.50, 0.75, 1.0. As increase in the value of τ_1, there is a decrease in the values of the thermal Marangoni number. Increasing the value of τ_1 the surface tension driven triple diffusive magneto convection sets in earlier i.e., system can be destabilized.

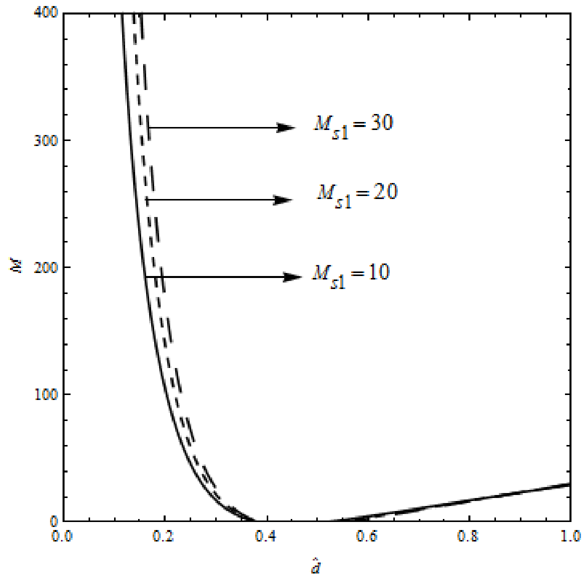

Figure 8: Effects of M_{s1} solute1 Marangoni number

Figure 8 show the effects of M_{s1} is the solute1 Marangoni number for M_{s1}=10, 20, 30. By increasing the values of solute1 Marangoni number, the thermal Marangoni number increases. The surface tension driven triple diffusive magneto convection can be delayed by increasing solute1 Marangoni number, hence the system can be stabilized. Also the curves are converging which is evident that the effect of M_{s1} is drastic for fluid layer dominant composite layer.

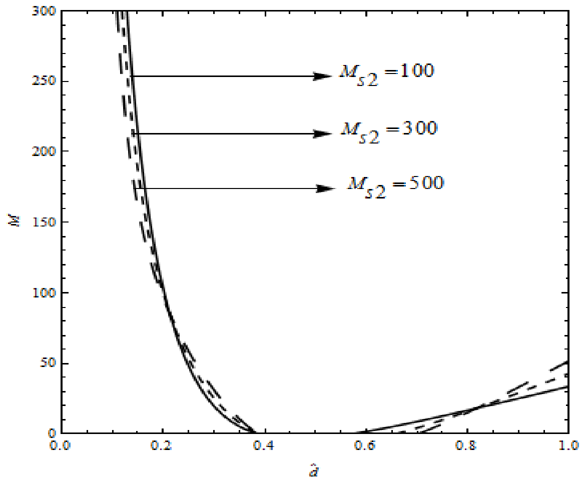

Figure 9. Effects of M_{s2} solute2 Marangoni number

Figure. 9 illustrates the effects of M_{s2} is the solute2 Marangoni number for M_{s2}=100, 300, 500. From the graph it is evident that, by increasing the values of solute2 Marangoni number the thermal Marangoni number decreases also for smaller depth ratio solute2 Marangoni number increases to stabilize the system. So, the surface tension driven triple diffusive magneto convection can be preponed by increasing solute2 Marangoni number, hence the system can be stabilized or destabilized.

Figure 10 display the variations of the value of τ_{m1} is the ratio of salute1 diffusivity to thermal diffusivity of the porous layer for the values τ_{m1} = 0.50, 0.75, 1.0. Increasing this ratio, thermal Marangoni number decreases. So, the surface tension driven triple diffusive magneto convection is preponed i.e., system can be destabilized. The converging curves indicating that τ_{m1} parameter is effective for the fluid layer dominant composite layer.

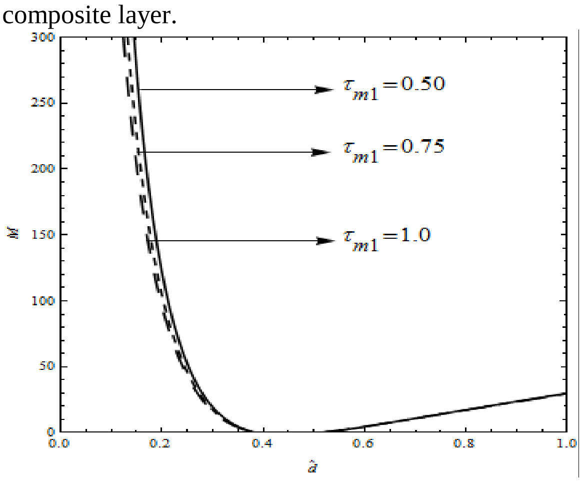

Figure 10: Effects of τ_{m1}

VII. CONCLUSION

(i) By decreasing horizontal wave number, porosity, ratio of solute1 diffusivity to thermal diffusivity fluid in fluid layer, solute2 Marangoni number, ratio of solute1 diffusivity to thermal diffusivity fluid in porous layer and by increasing the porous parameter, viscosity ratio, Chandrasekhar number, solute1 Marangoni number, the surface tension driven triple diffusive magneto convection can be delayed and hence the system can be stabilized.

(ii) The parameters $\beta, \varepsilon, \hat{\mu}, Q, \tau_1$ and M_{s2} are effective for porous layer dominant composite layers.

(iii) The parameters a, M_{s1} and τ_{m1} are effective for fluid layer dominant composite layers.

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