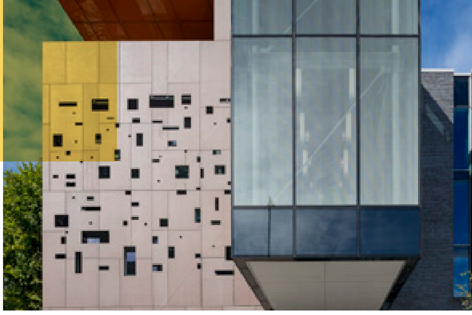

Ollscoil Chathair
Bhaile Átha Cliath
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Proceedings

Eighth Conference on Research in Mathematics Education in Ireland MEI 8

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Theme:
Accessing Mathematics: Inspiring Engaged Communities

CASTeL — Centre for the Advancement of STEM Teaching and Learning

MEI 8

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Foreword

It is our pleasure to present the proceedings form the Eighth Conference on Research in Mathematics Education in Ireland (MEI 8), taking place online for the first time, and hosted by DCU Institute of Education, Dublin City University. After a challenging 18 months, we acknowledge the dedication of all contributors for their commitment to mathematics education research and for dedicating their time and efforts in preparing insightful and important papers for this conference.

Our conference theme, *Accessing Mathematics: Inspiring Engaged Communities*, aims to situate mathematics as a pursuit that is accessible for all learners at all ages. Mathematics is a collaborative pursuit, and all around the globe communities of learners engage in mathematics in different ways. Since the start of 2020 accessing mathematics, and collaboration in mathematics, has been a challenge for many. While mathematics educators utilise remote or distance learning when necessary, this is not always ideal for all learners. We must endeavour to ensure that all members of mathematics learning communities can be encouraged and inspired to have a meaningful learning experience, and that these learning communities support the learning of all of those who participate in them.

In these proceedings of MEI 8, we present papers that reflect a broad variety of mathematical research that is taking place in Ireland and further afield. Collectively, the authors seek to solidify and progress the research field of mathematics education and seek to further understand how we can provide meaningful access and experiences for all learners of mathematics.

We acknowledge the work of the organising and scientific committees for their support and commitment to making sure that MEI continues to run smoothly and successfully. We also acknowledge CASTeL for their continuous support of the MEI conference.

We look forward to your participation and to meeting all participants (virtually) throughout the two days. We hope you all enjoy two days of engaging presentation, discussion and debate, and hopefully we will meet again in person at MEI 9!

Mary Kingston and Paul Grimes

Equity, Access, And Justice in Mathematics Education Research: A Personal-Professional Journey of Perspective

Mark Hoover

University of Michigan

I argue that we are morally obliged to consider the significant privilege of our lives and the whiteness of mathematics education institutions. Further, to determine implications for our work will require: (i) questioning things we've long taken for granted; (ii) developing means for stepping outside our training and standards; and (iii) learning to work together and in communities of those who are not like us but with whom we find alignment. Failing to take up this imperative confirms that we are the problem and existing injustice is our choice, not a reality imposed on us. I mean to argue this with both fervour and humility, using my own personal and professional journey to raise possibilities for us to consider, as a community, in charting the future of mathematics education research.

Attention to issues of equity, access, and social justice in mathematics education has increased in recent years, but justice is not a topic just to add to a list or raise up as a major domain in the field. It is a fundamental reorientation, for most of us individually and for the mathematics education research community collectively — for us. A challenge we face is that it is profoundly context dependent. Social groups, cultural histories, and political power differ from place to place. Concerns are different in Ireland than in Britain, different in South Africa than in China. They are different within countries — among regions and among communities. Attention to equity, access, and justice also depends on the scholar and scholarship. It is different for each of us, as people with personal histories and as professionals with our training and scholarly niches.

I am a white man from the United States. I taught mathematics, kindergarten to university, for about 10 years, and have been studying teaching and mathematical knowledge for teaching for another 25. I have my own context and scholarship. Although not an expert, I have spent several decades trying to understand the issues of equity, access, and social justice, their relationships to each other, and their significance for mathematics education. I would like to share my story of bringing these concerns to bear on my research and invite you to reflect on your own story, that we may reflect on our collective story, both past and future. I begin with background about myself and the evolution of my thinking. Then I describe how my concerns have re-shaped my own and my colleagues' mathematics education research, in both obvious and, for me, surprising ways. I close with reflections and questions to help us continue the conversation.[1] I apologize if my talking about myself seems self-indulgent or

[1] I want to thank several colleagues who provided invaluable feedback on an early draft: Deborah Ball, Matthew Dahlgren, Maisie Gholson, Nadine Hoover, Siún Nic Mhuirí, and Darrius Robinson. This article is based on work supported by the National Science Foundation (grant numbers 1502778 and 1760788).

uninteresting, but my aim is to use my experience to make visible some of the terrain we need to traverse, challenges we need to face, and possibilities ahead.

My privileged well intentioned whiteness

I grew up in a farming community in western New York State, where my mother taught English and my father taught mathematics at a small liberal arts college. We built a house, without electricity, and cultivated a farm with 40 sheep, 150,000 Christmas trees, and smaller numbers of other plants and animals. People of diverse backgrounds sojourned comfortably in our home. I grew up as a post-1960s universalist Quaker — relatively naïve about the world but committed to peace, education, community, simplicity, integrity, and equality. As a young adult, I drafted a statement of conscientious objection to war, attended a Quaker Meeting for Worship at a maximum-security prison for several years, and engaged in *Alternatives to Violence Project* workshops.[2]

Who was I? I grew up comfortable and safe in rural, white America. My childhood was unusual in many ways, but it was also a version of white-liberal normalcy. I carried this background with me as I moved professionally into mathematics, teaching, teacher education, and education research.

Working collectively with Deborah Ball, Hyman Bass, and a shifting group of graduate students and colleagues, I have studied teaching, learning to teach, and the mathematical demands of teaching. In 2003, we presented a paper at the annual conference of the American Educational Research Association titled, *In Attention to Equity in Teaching Elementary Mathematics*. We argued:

- Inequality is routinely reproduced inside instructional practice.
- Breaking this cycle depends on joining concerns for equity with the daily and minute-to-minute work of teaching.
- Teachers can have leverage at strategic points in the intersection of concerns for equity and the work of teaching.

Reception was tepid. Colleagues who generally engaged enthusiastically with our work at the time seemed to think we had lost our way. They knew us for our research on the work of teaching and mathematical knowledge for teaching. Many colleagues either did not attend or were disappointed and left quietly. Others who attended due to interest in equity did not seem to understand our work or were sceptical of our motives and methods. Grant proposals to further this work did not review well. We were uncertain how to interpret reactions and came to realize that the issues and politics were subtle. I have come to understand how reasonable these responses were given my underdeveloped understanding of privilege, our research group's positionality, and the history of education research with marginalized groups.

[2] For information on AVP, see https://avpusa.org/.

Three additional experiences influenced my awareness of justice issues in important ways. Each represents my well-intentioned engagement with justice, yet with a lack of understanding of race and racism and the nature of privilege and power. The ways I took them up may reflect my privileged whiteness, which often goes through the motions without achieving meaningful change.

First, I began working with 1960s civil-rights activist Bob Moses and the Algebra Project to extend earlier work on political access (voting rights) to economic access (through success in mathematics).[3] I collaborated with the Algebra Project on its launch of a dozen cohort classrooms across the country. To support the community's efforts to learn to teach, I designed and documented an intensive two-week summer public teaching program. For three summers, Bob taught young people who had scored in the bottom quartile on state exams, students whom the system and the country was treating as disposable.

Second, the University of Michigan School of Education where I worked sought to change its culture. It launched a speaker series to support greater attention to diversity, inclusion, and equity. It developed a strategic plan aimed at changing practice, for example via hiring protocols, diversity training, and community conversations.

Third, I took part in peace work my sister was doing in Indonesia.[4] She has split her time equally between the United States and Indonesia since the early 1980s. After the tsunami of 2004, she worked in East Aceh, which other non-profits avoided because of the ongoing civil war. Over time, she developed an approach that combined nonviolence training, trauma recovery, building economic opportunities, and education. In my notes, I found the following minute. It conveys the importance of building connections among people, in which people work with mutual investment and benefit:

> We believe that the establishment of right relationships among people provides a powerful means of gaining personal and national security — ours, and others'. We seek to make the military obsolete by increasing efforts to meet people's basic needs around the world. We believe that to effectively meet people's needs, these efforts must simultaneously provide needed resources and build connections among people despite their differences.

Each of these experiences gave me clearer notions of challenges and foundational orientations for addressing injustice. At the same time, in retrospect, they seem simply additions and elaborations of what I carried with me from childhood. I incorporated them into my life without changing course.

[3] See https://algebra.org/ and Moses and Cobb (2002).

[4] See https://consciencestudio.com/ and Hoover (2018).

Insights from more invested and challenging consideration of race and racism

In the past five years, I have invested more actively in understanding racism — in part due to circumstance (personal, professional, and societal idiosyncrasies) and in part due to a sense that I was not understanding something important. In 2016, I worked with others to plan a retreat, *Quakers and Race: A Spiritual Journey*. Nine of us formed a bi-monthly discussion group. I began to read more widely about racism and talk with others about what I was learning. I offer four insights that have profoundly reoriented my thinking.

Race results from exploitation; it is not the cause

I came to understand that a common underlying line of thinking in white society is backwards. In this thinking, race and racism start with prejudice, where people see others who look or act different and look down on them. This perceived inferiority then makes it okay to exploit them. This leads to thinking that the solution is to overcome negative attitudes, to value difference, to see the good in everyone. The idea is that doing so would end race and racism. I do not remember explicitly thinking this way, but I see that my actions, largely unexamined, were consistent with this thinking. Several sources convinced me that the cause-and-effect relationship is the other way around. Racist ideas and stereotypes result from exploitation and systemic oppression: they do not cause them. Race and racism rationalize exploitation. I read in Ibram Kendi's (2016) well-researched history of racist ideas, *Stamped from the Beginning*, that the use of "black" as a racial category was introduced in the 1400s by a biographer of Prince Henry of Portugal to lump together and paint as inferior ethnically diverse African groups in an effort to make the lucrative slave trade palatable and that "white" as a racial category first appears in legal documents of the United States in the late 1600s to maintain available labour, undermine collective resistance, and limit access to land and rights. I came to understand that race and racism are social constructs designed to justify and protect ongoing exploitation.

Racism and other forms of oppression are systemic

A second point reiterated in what I heard and read was that the historic and systemic nature of racism is fundamental but often overshadowed by attention to attitudes and behaviours. Chenjerai Kumanyika (2017) argues that discussions about race and oppression tend to focus on individual attitudes as if racism were a disease (who has it?) or a puzzle to solve (what do we do to avoid it?). I came to realize that the focus needs to be, instead, on how racism permeates our institutions — how whiteness is established as the norm and how racism is produced and reproduced in everyday interactions and in who has what rights and how resources are distributed. As Kumanyika puts it, racism is not about your distant cousin being a bigot; it is about housing policy, educational funding, credit scores, hiring practices, skewed representation, and misrepresentation that we participate in and accept as normal. I came to see how systemic racism is baked into my life and the world. It is differential access to goods, services, and opportunities that becomes common practice and integral to institutions. It dominates public bodies, private corporations, and public and private schools and universities and is reinforced by the actions of conformists and newcomers. Writing in

1967, Stokely Carmichael and Charles Hamilton drew attention to this distinction between vicious, ugly acts of prejudice and the structural, systemic character of racism:

> When white terrorists bomb a black church and kill five black children, that is an act of individual racism, widely deplored by most segments of the society. But when in that same city — Birmingham, Alabama — five hundred black babies die each year because of the lack of power, food, shelter and medical facilities, and thousands more are destroyed and maimed physically, emotionally and intellectually because of conditions of poverty and discrimination in the black community, that is a function of institutional racism (Carmichael and Hamilton, 1967, p. 4).

Yes, the systemic nature of racism is, in a sense, obvious. I had heard ideas about institutional and systemic oppression throughout my life and nodded with a degree of understanding. Yet, I was in my fifties before realizing how profoundly central this baked-in character was — in institutions, laws, policies, cultural stories, and worldviews — and consequently how blind and complicit I was, simply by growing up white. Most black people in the United States, without my privilege are forced to face social realities and become aware of all of this and more at an early age.

Investment in white privilege maintains a system of exploitation

I also came to understand that, analogous to privilege afforded to men, white privilege is an institutional set of benefits that affords disproportionate power and resources to white people. I could see what Cory Collins (2018, p.3) meant when he says that the term inspires pushback because the word *white* creates discomfort among those not used to being defined or described by their race and the word *privilege* gets interpreted as suggesting they have never struggle. Collins identifies three forms of white privilege: (i) where "normal" is defined by white characteristics; (ii) where white people are extended greater compassion and benefit of the doubt; and (iii) where white people receive greater opportunities to accumulate and inherit power. George Lipsitz (2006) takes the analysis of white privilege further, identifying an ongoing investment in being white — a possessive investment, literally and figuratively. White supremacy then is a system for maintaining differentiated benefits. Lipsitz argues that there is an investment of time and energy given to the creation and re-creation of a system designed to protect the privileges of whites by denying communities of colour opportunities, including opportunities for asset accumulation. Again, this leads me to see my world and myself in a new light, where good intentions and white benevolence that does not fundamentally alter the system is as much the problem as is bigotry and racist hatred.

This first set of three ideas, that racism results from exploitation, is fundamentally systemic, and is actively maintained by people and institutions, represents a paradigm shift for me. I came to realize that racism is everywhere present and that I am everywhere involved. There are no sidelines. If I am not part of the solution, I am the problem. The next question for me was to understand what exploitation is and where it comes from.

Exploitation is rooted in patriarchy and a western worldview of dominion

My exploration of the historical why and how of exploitation led me to see connections in the United States among racism, confiscation of indigenous land, and patriarchy. It led me back to colonialism and Europe. Instead of exploitation being an unavoidable part of human character or the result of a few greedy, self-serving individuals, I came to see that it has been the defining policy of European societies and their territories. Three sites for my learning stand out: The Doctrine of Discovery; contrasts between indigenous and "industrial" thinking; and conceptions of patriarchy.

The Doctrine of Discovery. The doctrine is not a document per se but an evolving way of thinking in European history and colonies that promoted and sanctioned the conquest, colonization, dehumanization, and exploitation of non-Christian territories and peoples. Leveraging the social construction of race, the doctrine rationalized global exploitation — the unprovoked plundering of others. It has a long history, from papal decrees to legal arguments to international law. I elaborate on its history here because it has significantly expanded my understanding of the context and problem of racism and because it connects the European and U.S. contexts.

What I have found so astounding is how far back its roots extend, how prominent its role has been throughout our history, how painstakingly it has been crafted over time, and how visible it remains in today's laws, policies, and attitudes. Robert Miller traces the evolution of the doctrine from early arguments about natural rights and a shift in the Catholic Church from shepherding one's flock to guardianship over all earthly flock (in Miller et al., 2010). He weaves connections among what, for me, have always been a series of descriptive events that simply unfolded. I came to see that arguments for exploitation and domination for colonial Europe began with papal bulls stretching from the crusades of 1096-1271 to the Church's sanctioning of conquest first by Portuguese and then Spanish monarchs. Authority then shifts from the Church to states to international law. The arguments involve nuances of *dominium*, governmental sovereignty, and property. They use conceptions of "natural" law (as defined by European standards) to justify plundering, as if it had moral integrity. At times, the rhetoric is subtle. At other times, it is not. Pope Nicholas (1455) authorized Portugal, "to invade, search out, capture, vanquish, and subdue all Saracens and pagans." Sighting and symbolic possession (flags or markings) was often taken as sufficient warrant for claims to land and people.

For me, the role of the Christian Church, the sophistication of arguments designed to defend exploitation as justified, and my obliviousness to this history have been eye opening. Miller also examines how England mixed papal authority with the imposition of English law, ignoring Irish legal and property rights, in its colonization of Ireland (1155-1603). Miller argues that England's experience in Ireland and its development of legal arguments to rationalize its actions in Ireland provided a foundation for later arguments justifying global expansion. Two significant pivots occurred when early English legal scholars argued that claims to territory not yet claimed by Portugal or Spain avoided infringement on papal authorization, and then in 1580 Elizabeth I and her legal advisors argued that "discovery" required occupation to be justified. As Miller points out, Elizabeth I, who had been excommunicated in 1570, was not concerned with papal approval, but with establishing international law that would recognize and respect English claims.

I have little sense of how different ones of you may look upon this history. Many of you may view this history as common knowledge, not as revelation, but it has given me a fuller understanding of connections and developments shaping injustice and its justification in our world. Growing up in the United States, I learned about British settlement of North America as reflecting their democratic industriousness and Protestant work ethic, not as highly crafted social policy developed from centuries of nationally competitive exploitation.

Furthermore, the history of the Doctrine of Discovery is not simply about the past. The U.S. *Johnson v M'Intosh* decision of 1823 is a stunning articulation of it that remains with us today. The decision includes explicit, detailed formulations of its central tenets, that the "first discover" has significant property and sovereignty rights, as well as sole authority to buy land, that indigenous people retain limited occupancy and use rights, that non-Christians are inferior to Christian Europeans, who are responsible for civilizing them, and that discoverers have rights to contiguous or vacant lands or land seized in just wars (all defined in European terms). And the Doctrine of Discovery, as articulated here and elsewhere, continues to serve as legal precedent as recent as the 1990s and 2000s in Canada, Australia, New Zealand, and the United States (Miller et al., 2010). Learning that my country of origin was built from such principles and that these principles are codified in our laws and continue to serve as the basis for court rulings today shocks me.

What I have come to understand is that the Doctrine of Discovery is not an anomaly. Indeed, it reflects a defining feature of western civilization, visibly and invisibly present in my thought, life, and society. Its history permeates my and our institutions and cultural views. As a child growing up in the United States, I learned in school about the "Age of Discovery," not the Doctrine of Discovery. Growing up as a Quaker, I learned about Quakers' pacifism, support for abolition of slavery, and good relations with indigenous people, not about their complicity in society's use of the doctrine to wantonly plunder. At the retreat I mentioned earlier, *Quakers and Race: A Spiritual Journey*, I learned instead about Paula Palmer's re-examination of American Quakers' role in assimilation practices at native American boarding schools.[5] I began to learn about different ways of seeing the world and the narrowness and distortion in many of the stories of my youth and my education.

Contrasts between indigenous and "industrial" thinking. In 1993 at the Schumacher Lecture at Harvard University, Winona LaDuke, an Anishinaabekwe from the White Earth Reservation in northern Minnesota and a U.S. vice-presidential candidate in 1996 and 2000, was asked to provide the mostly white audience with a sense of an indigenous worldview (LaDuke, 1993). She wonders how to communicate across perspectives and offers contrasts as

[5] See https://friendspeaceteams.org/trr/.

a means. Her insights expand my understanding of the why and how of exploitation. She contrasts indigenous thinking with "industrial thinking" (Figure 1).

Figure 1

Contrasts between indigenous and industrial thinking suggested by Winona LaDuke (1993)

Indigenous Thinking	*"Industrial Thinking"*
natural law and a state of balance	entitled to full dominion over nature
cyclical structure of nature	linear thinking
cultural and biological diversity; in relationship	superiority of civilized (tame) over primitive (wild)
more verbs; things are animate/alive	inanimate nouns; commodification of the sacred
goal of conspicuous distribution; honour in giving	goal of accumulation and consumption

Earlier in my life, I studied social-cultural anthropology at the University of North Carolina. Many of LaDuke's contrasts are not new to me, such as differences between cyclical and linear thinking and her descriptions of balance, relationships, and reciprocity. There is a power, though, in how she packages them and considers their implications for the world as it is today. And some contrasts are new for me, or are visible in a new light, for instance accumulation. To what extent are my life goals about accumulation, where accumulation is the end (not the means to a greater good, such as to then give generously), where accumulation is the priority (once I have accumulated enough, then I can attend to other matters), or where accumulation determines value (the worth of people, businesses, and societies)? LaDuke identifies accumulation as central to "industrial" thinking, which for me echoes the prominence of accumulation in Lipsitz's (2006) possessive investment in whiteness. I have come to understand that accumulation plays a prominent role in nearly all forms of oppression and that it is foundational to European and western thought.

To convey the difference between indigenous and industrial thinking, LaDuke relates a story of the origin of the Lakota word for a white person.

> There was a white man out on the prairie in the Black Hills, and he was starving. He came into a Lakota camp in the middle of the night, and the Lakota of course were astonished to see him. They began to watch him to see what he was doing. He went over to the food, took something, and ran away. A little while later, the Lakota looked to see what he had taken: he had stolen a large amount of fat. So the Lakota word for a white person, wasichu, means "he who steals the fat."

The Doctrine of Discovery rationalizes stealing the fat. I have come to see stealing the fat as central to a western, European worldview. Unabashed colonization may be in the past, but key components of our worldview are still in place. I have come to see that racism is just one of many mechanisms developed for stealing the fat.

Conceptions of patriarchy. I have also come to understand better the dynamics of exploitation from the work of Riane Eisler (1988, 2003, 2019). She distinguishes between domination systems:

Figure 2

Contrasts between partnership and domination systems suggested by Riane Eisler (2018)

Configuration of Partnership Systems	*Configuration of Domination Systems*
Democratic and egalitarian structure in both the family and the state or tribe, and all institutions in between. Equal partnership between women and men and high valuing of "soft" or feminine traits and activities in both women and men, and in social and economic policy. Low degree of built-in violence (not *needed* to maintain domination) and hierarchies of actualization, where power is not power over, but rather power to and power with.	Hierarchies of domination, not only in the state, but also in the family, and all institutions in between. Gendered system of values, ranking male over female, with rigid gender stereotypes of femininity and masculinity, and devaluing anything considered "soft" or feminine, such as caring, caregiving, and nonviolence, which are considered inappropriate for "real men" and are not part of the guiding social and economic system of values. Socially condoned and idealized violence, from child and wife beating to pogroms and chronic warfare, maintaining rigid top-down rankings of domination — man over woman, man over man, race over race, religion over religion, and so forth.

For Eisler (2018), a systems scientist, the dynamics of racism, patriarchy, economic oppression, and more are part of a system. As she puts it, "the struggle for our future is not between religion and secularism, right and left, East and West, capitalism and socialism, but in all these sectors between traditions of domination and a partnership way of life." Eisler's framing has helped me understand why regressive regimes focus on retaining or restoring domination in gender and parent-child relations and why all modern progressive movements challenge one thing: traditions of domination.

Eisler closes her remarks at the 2018 Safe Ireland Summit by offering four key cornerstones for a more equitable, sustainable, and caring world: childhood relations, gender relations, economic relations, and new narratives and new language. I have come to see the pivotal role children play in reproducing exploitive systems. Just as I have learned when I was young that prisons are our society's most effective training ground for violence, I have come to see that child abuse is not just an outcome of domination systems — it is adaptive; it assures reproduction of the system. Eisler focuses on gender, but I find her work equally applicable to race relations and our relationship with the environment.

As I read more, I could see more fully how systemic the challenges are, how baked in the problems are, how problematic many of my stories are, and how different the world can look through the eyes of others. For example, in a fascinating analysis of patriarchy, Carol Christ (2016) argues that male dominance is enforced through systemic violence and threats of violence. She defines patriarchy as:

> … a system of male dominance, rooted in the ethos of war which legitimates violence, sanctified by religious symbols, in which men dominate women through the control of female sexuality, with the intent of passing property to male heirs, and in which men who are heroes of war are told to kill men, and are permitted to rape women, to seize land and treasures, to exploit resources, and to own or otherwise dominate conquered people. (p. 214)

She analyses warfare, private property, and control of women's sexuality, putting the commonplace in new light. She speaks of being stunned by Merlin Stone's statement that, "in matrilineal societies there are no illegitimate children, because all children have mothers" (p. 216). The point being that the language of "illegitimate child" only makes sense in the context of male ownership of children. Christ conveys how different the world could be by describing how a Mosuo woman of the Himalayas explained:

> … that in her culture women and men define themselves through their connections to maternal clans. When a girl reaches the age of sexual maturity, her mother prepares a room where she can invite a man to dine with her. If she chooses, she invites him to spend the night with her. Children produced from such unions become part of their mother's maternal clan. The 'fathering' role is assumed by the uncles and brothers of the mother, while the mothering role is shared among sisters. If either member of a couple tires of their sexual relationship, they end it and seek other partners. (p. 218)

Christ uses the Mosuo practice to reveal the many ways our western practices are designed to establish and maintain male control over women's sexuality. She goes on, using patriarchy to explain why we have warfare and private property. Throughout her analysis, she reveals design and coherence for what have always appeared to me to be a haphazard assortment of practices, merely circumstance.

Over time, I have come to see that the systemic exploitation of people of colour, women, children, and the environment are related. I have also come to see that racism is not simply a U.S. dynamic. While the United States has its distinctive form, racism is a global issue, with deep European roots, perpetrated through colonialism. Consider for a moment a growing mathematics education literature on racism in Brazil, India, Australia, and other countries. Indeed, my impression is that Europe's taboo on speaking about racism since the second world war is losing its grip and that Marxist claims that racism is just a misnomer for classism are fading. I am not trying to convince you that my emerging view is right or that the scholars I have referenced have cornered the market on truth, but I am trying to give you a sense of my expanding awareness of the world and the different ways people, from different communities, see it.

Implications for my mathematics education research

An obvious question here is what this personal journey has to do with mathematics education. For me, mathematics education is entangled and deeply implicated. Mathematics education is a major mechanism in our society for including and excluding, designating status, and controlling economic access. As Danny Martin (2019) and Dan Battey and Luis Leyva (2016) argue, it is a white institutional space that maintains a legacy of privilege, where violence and dehumanization characterize the experience of the less powerful. I am compelled to explore how this might be true, how mathematics education might be complicit in the rationalization and defence of exploitation, how I have likely been blind to its role in the oppression of non-white, non-male members of society, and what I might do to change this.

In recent studies, I have been examining what my new awareness implies for my understanding of the work of teaching and its mathematical demands. In addition to shifting what I study, it has shifted how I study, with whom and how I collaborate, and the orientation and sensibilities I bring to the work. I begin by describing the work of two of my close colleagues: Deborah Ball and Maisie Gholson. Each of us has independent work, but we also shape and are shaped by each other's work, and we invest in ongoing work together. I begin with them because their thinking is central to what I am coming to understand about how mathematics education research might address the challenges we face. I discuss recent relatively independent work we each have been doing, but my goal is to provide examples of what I see as implications for mathematics education research in light of the insights I have described above.

As a white Jewish woman with institutional power, Deborah's awareness of racial and intersectional issues in the United States has grown in different yet parallel ways to my own. This growth has led her to develop her thinking about the "power" of teaching — for harm and for good, in a society with its history of enslavement, oppression, and racism, where systemic oppression finds its way into everyday micro-moments of teaching. This is evident in her recent Klein Lecture at the 14th International Congress on Mathematics Education (Ball, 2021). She points out that systems and people are connected and argues three points:

- Teaching is powerful. When it is done with care and judgment, students can thrive — learn mathematics, develop positive identities, learn to value others and work collectively.
- Teaching also involves enormous discretion.
- How that discretion is exercised can either reinforce racialized and oppressive patterns of social, personal, and epistemic injustice and harm, or it can disrupt these patterns.

She explores the nature of teaching as practice and the need for research on "practicing (in)justice," in other words becoming aware of how and when injustice happens in practice and developing practices that disrupt injustice, and are more just. She identifies five challenges for such research:

1. Combining the embodied and relational dimensions with the cognitive and knowledge entailments.

2. Building theory and insight while contextualizing the work and centering identities.
3. Connecting the dots between macro-structures and micro-interactions.
4. Using care to distinguish prescription from detail.
5. Representing the work in a usable discourse of practice.

She unpacks what is meant by each of these, the logic that connects them to an imperative to disrupt rather than perpetuate patterns of oppression, and the broadened collective needed for their meaningful study.

As a black woman engaged in mathematics education research in an inhospitable institutional context, Maisie Gholson works in a black feminist framework to understand how children's identities and relational ties to mathematics, peers, and teachers create different developmental trajectories and learning opportunities. She foregrounds children's humanity and the visceral contexts that shape their experiences. In her invited lecture at the 14th International Congress on Mathematics Education, she argues that:

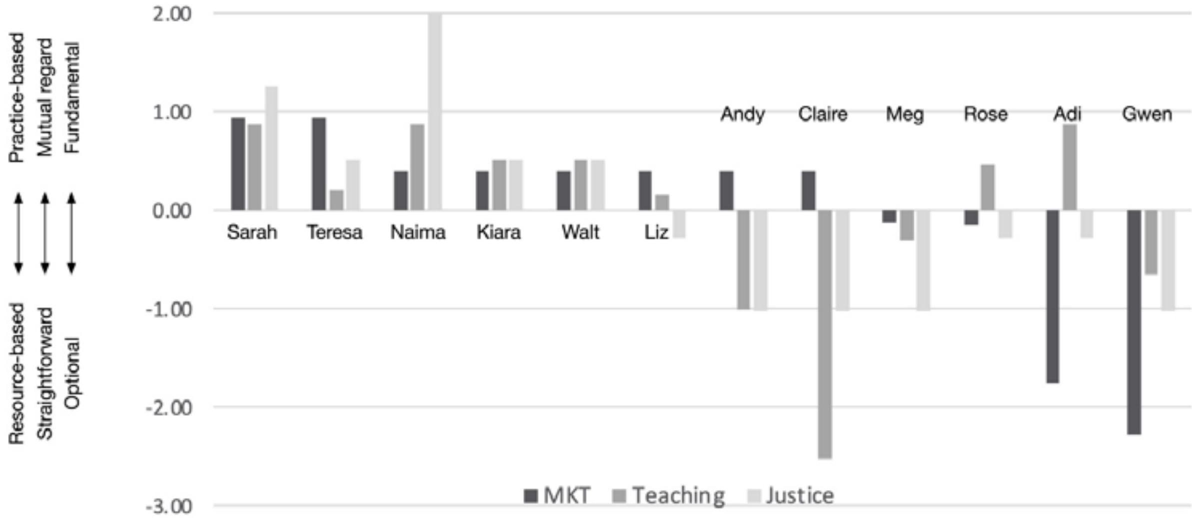
> There is a moral imperative to study the phenomenology of marginalized learners, like Black American children, to protect and promote their physical, socio-emotional, and intellectual well-being in relation to mathematics education." (Gholson, 2021)

She calls for mathematics education research that promotes black life and well-being. This means dealing with ecological complexity at all levels — the neuro-physical, cognitive, interpersonal, community, societal, and socio-historical levels. She argues that doing so requires development of tools for studying the mundane and everyday, in ways that help us see "the consequential in the inconsequential, the significant in the taken for granted." She engages lived experience through a hermeneutic circle, a reflexive process of questioning text through different horizons of understanding. She draws on Gadamer's description of horizons as contexts of meaning and refers to Laverty as noting that, "a person with no horizon, in Gadamer's view, does not see far enough and they overvalue what's nearest at hand." Mathematics education does not see far enough. It needs to see beyond its trained, institutional vision — of mathematics education and of research. In contrast, "marginalized folks [including children] have a unique perspective that allows them to see oppression in ways that others with more privileged identities do not." Maisie specifically argues that mathematics education researchers need better tools for seeing what happens in mathematics teaching and learning through the experiences and perspectives of children. Maisie is finding new ways to gain insight into the experiences of black girls and shed new and clearer light on what is happening in mathematics teaching and learning.

In my own work, I have been examining what my new awareness implies for my understanding of the work of teaching and its mathematical demands. Imani Goffney and I have been studying nuanced aspects of the work of attending to justice in teaching and associated mathematical demands (Goffney and Hoover, 2021). Elaborating this work has allowed us to see and prioritize mathematical knowledge and skill that, if routinely addressed in the mathematical education of teachers, would increase teachers' capacity to provide positive learning experiences for students currently marginalized. In a different study, Reidar Mosvold, Matthew Dahlgren, and I have been exploring how mathematics teacher educators think about the mathematical understandings needed to teach mathematics to learners (Hoover et al., 2021). We have found that their thinking about mathematical knowledge for teaching is shaped by how they think about teaching and how they think about justice. In a sample of 12 mathematics teacher educators, we found that their thinking in these three domains tended to align and that misalignment provided important insight into how professional development might serve them better (Figure 3).

Figure 3

Profiles of the extent to which participants think of mathematical knowledge for teaching as practice based, teaching as mutually involved, and justice as fundamental and consequential



Before engaging in this study, we would not have thought that developing one's thinking about justice would be an important tool for developing an understanding of the practice-based nature of mathematical knowledge for teaching. Now we do, and we propose giving the development of thinking about both teaching and justice more prominence in the professional development of mathematics teacher educators and the mathematical education of teachers. Our analysis suggests that individuals who understand the mutual character of teaching or the fundamental character of justice are assets for collective work on mathematical knowledge for teaching and should be recognized.

Maisie, Deborah, and I work in education because we value education and see it as a hope for a better future. Our central professional goals are personal goals. Currently, we are studying what it takes to communicate across difference in mathematics classrooms. In combining Maisie's abiding attention to students and their experiences within and beyond the classroom with Deborah's and my concern for articulating the work of teaching in ways that support learning to teach, we hope to learn from each other about how to attend better to justice in mathematics teaching and learning.

Reflections for me: Questions for us

From my colleagues and my efforts to consider the systemic harm done and ways of doing better in mathematics education and mathematics education research, I offer three broad reflections. The first is about *the need for "other" perspectives — perspectives different from our own, of those most harmed by current systems and practices that perpetuate these systems*. Maisie calls us to have a deeper, more profound regard for students' experiences and those of their communities, not simply to consider them while standing on our own ground but pressing ourselves to sense the experience of the other, to the extent we can. Stretching ourselves. In an essay for teachers, *On a Certain Blindness in Human Beings*, William James talks about our inner worlds, where significance and eagerness for life abide (James, 2010/1899). Each of us has this sense of significance and eagerness regarding our life. It is our joy, and to miss it, to go through life without heeding it, is to miss all. From this, though, James describes how immensely difficult it is to know the inner world of another. The essay is about the blindness that afflicts us all, "in regard to the feelings of creatures and people different from ourselves" and the "injustice of our opinions, so far as they presume to deal with the significance of alien lives" (p. 146). James draws attention to the near impossibility of knowing others and yet the vital necessity and the profound reward of glimpsing, through others, "the vast world of inner life beyond us" (p. 152). It is our efforts to see from another's perspective that affords us an ever-emerging new centre and a more meaningful life. Although James is not writing about "social justice" per se, at least not as a topic as we might identify it today, he refers to those "different from ourselves" and to the "injustice" of our views. My growing sense is that we need to heed the systemic challenges we face, but as these are constituted and reconstituted by our daily interactions, our everyday practices, it matters that we attend to our blindness as people and as researchers. This requires investment in "other" perspectives.

Mathematics education research as a field needs to expand its Gadamerian horizons. Maisie suggests ways of doing this in both our methods and our empathy. For me, the need for exploring perspective and expanding my horizon combines the personal and professional. My research does not have meaning apart from who I am and the contexts in which I work, a point that many other scholars have noted before my noticing it. The insights I have written about above are from my ongoing exploration of perspective, what some might call my political education. In addition, I find myself actively working to find and bridge perspective in my research. As a small example, when I now write something, I invest in rereading with other specific perspectives in mind. Of course, this is what I have always done in many ways, but I do it now with specific, deliberately chosen views, at both the sentence and manuscript level. How might Imani Goffney, a black female colleague, read this sentence? How might Eve Tuck, a Unangax̂ scholar of critical race and indigenous studies, who does not know me and may question the significance of land and place in my work, view this paper? Increasingly, I ask non-white colleagues to review my work, with a focus on noticing my perspective and offering their own. And I look for ways to compensate them for their expertise and vital contributions to my work. Another version of this practice is to ask myself if what I have written is from an institutional perspective, which is inevitably a white perspective. What might I be taking for granted, defaulting to because it is part of the story that I have been steeped in throughout my professional training and career? This exercising of perspective is reshaping me and reshaping my work.

This example leads to a second reflection, *that meaningful regard for justice requires we carry out our work more collectively than has historically been the norm in mathematics education research, and that, in this, each of us be engaged in justice both personally and professionally*. I have been studying collective work as crucial for mathematics teaching and learning for over twenty years. What is it, how is it done, to what ends? In part, from early in my life, I have seen an important collective aspect to doing mathematics. I have also always sensed that collective mathematical work in classrooms was essential for the teaching and learning of mathematics in a democracy. In recent years, though, in relation to seeking to understand (in)justice and act on that understanding, I have come to see that collective work is key for attending to (in)justice. Mathematics education is about the lives of all of us in this world. It is not a disciplinary study of learning or a sociological study of policies and institutions. It is a professional field of study, practical and political in nature, immediately concerned with our collective life and making it better. It is not only about mathematics education researchers' lives, or teachers' lives. It is very much about students' lives and the lives of those who live in our communities and societies. And when power is involved, when power becomes a problem, it is essential that change come from within. It requires that all of us find our voice and speak our truth. This is not a simple matter of having everyone working together as equals. It has no single form. It requires reorganizing the work in ways that groups find their work, that is meaningful to them, and that their work is valued by those who have power, or more importantly, by and within the system.

Bob Moses learned to organize for change from Ella Baker, a civil rights activist with unflinching faith in the power of ordinary people and a collective approach to leadership. Bob always insisted that students have a significant role in any meeting or event of the Algebra Project or any activity in which it was involved. And he would press the young people to figure out what matters to them and what they want to say and do. He encouraged young people to form the Young People's Project, which uses math literacy work to develop the abilities of elementary through high school students to succeed in school and in life.[6] For Bob, the people most affected needed to be central to solving the problem. This means having authentic places for students, teachers, and communities in the work and holding that space for them. It also means holding space for the people most affected in the academy. For instance, Maisie Gholson uses a black feminist framework, which acknowledges the value of black women and sees their work as an expression of their autonomy rather than an adjunct to the work of others. For me, this is an example of the kind of earned insurgency of which Bob Moses spoke and which our field needs to recognize, not just as legitimate, but as essential to efforts to address equity, access, and justice in mathematics education.

I have no straightforward answers for what this means for my work as a mathematics education researcher or our work together, but I know I need to consider and act on its implications. There may be a place for individual scholarship, mine or others, but collective

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[6] See https://www.typp.org/.

engagement is a key resource for attending to justice in all aspects of research. To be a resource, though, engagement needs to stretch me, and such work is not always easy.

The third reflection I offer is *our need to rely on our human faculties of conscience, discernment, and spiritual experience to direct our research*. I can imagine many reservations about doing so. Instead of speaking to those, I focus my comments on the need. First, the problems we face regarding equity, access, and justice require a deeper listening, one fostered by attending to conscience. Perhaps one way to think about this is that when the institutional structure is the problem, including that of the mathematics education research community, then we cannot rely on it alone to guide our work. We must find a means of seeing outside of our institutionally established frames and defaults. Yet it needs to be a reliable means. Our spiritual foundations seem the most likely source. I do not mean the institutions of religion or the particulars of individual faith. I have in mind a rather universalist experience of right and wrong, of what is true for me, true for you, perhaps true for us, and a drive to make decisions based on our best sense of what is right or true. John Woolman writes in his journal about how, inside each of us, there is that which is true and pure, where the heart stands in complete sincerity.

> There is a principle which is pure placed in the human mind, which in different places and ages hath had different names; it is, however, pure, and proceeds from God. It is deep and inward, confined to no forms of religion, nor excluded from any, when the heart stands in perfect sincerity. In whomsoever this takes root and grows, they become brethren. (Woolman, 1720-1771/1871/1914, p. 36-37)

I am not trying to make a religious or philosophical point. Mine is a practical point. I find I need to question much of my training, my conceptions of disciplined research, journal standards, everything. I am not inclined to throw them all to the wind. They hold a great deal of wisdom. But I have come to understand that they are also ill and need healing. I have learned in my life that when I still myself, free myself of distress, hold the world in my heart, and listen patiently to that still small voice within, I find good guidance — imperfect, but with practice, helpful guidance. I also find it helpful to know that others know this for themselves, by whatever name, and that, when we call it forth from each other, we see better, see beyond the problematic stories, practices, and trauma of our past. As my sister, Nadine, has written, discernment is the human capacity to grasp the inner nature and relationship of things, especially when obscure, that leads to keen insight and judgement (Hoover, 2018, p. 47). This is the foundation of all science and knowledge.

The flip side of seeing that institutional structures are part of the problem is that the people who are less fully part of the white institutional space of mathematics education research have, of necessity, needed to find other foundations from which to draw than the standard-bearers of mathematics education research. When I consider for a moment who in our field is afforded the most say in what counts as legitimate research, I see how conservative the field is in decisions about important problem spaces and legitimate ways of knowing. I also see hope in complementary resources in those from communities most harmed in systems of oppression. Most prominent are ways of knowing that draw from religious traditions and spiritual wisdom. For instance, indigenous scholars have drawn on indigenous knowledge, worldviews, and spiritual wisdom in ways that have exposed the narrowness of scientific thought and science education in a democratic society suffering from environmental disasters and profound social trauma. Black scholars often draw from spiritual traditions — even more so when out of view of the establishment and its frequent disdain. Likewise, women and women's ways of knowing have reshaped both the content and methods of the biological sciences, in recognized ways, over many decades. We live in a world that needs these groundings. We work in a profession that needs them.

Looking back on my journey, one important take-away is that I have learned to question things I have long taken for granted. Even though I was concerned with justice and sought to live accordingly, I have had a certain blindness that comes with growing up in this world, especially growing up privileged. What can we each do to overcome our blindness? Of course, my narrative is my narrative. Perhaps you will find parts of it helpful, but I am not trying to "sell" it to you. Instead, I call on each of you to develop your own narrative regarding oppression in the world, its many forms, connections among them, and implications for mathematics education research. It is not okay to not have a narrative, to not have struggled to make sense of oppression or to listen to others' perspectives. A second take-away is that I find I must work in community and communities of those who are not like me but with whom I find alignment. In this, I must recognize and acknowledge the value of what those others contribute to the work. In addition, for those of us in positions of power, of all kinds, it may be important to hold space for those who do not have power. They need us to hold space for them to do their work, as they are led, without our inserting ourselves and taking over. Finally, I invite us to each find a reliable means of stepping outside our training and standards, so that we might know when our training and standards are the problem, so that we have a moral compass for reflecting on our profession and the work we do, and so that we have a reservoir that keeps us going and committed.

From where does the onus for change come if not from us?

References

Ball, D. L. (2021, July 14). *Understanding the power of teaching and its role (in)justice.* Felix Klein Award Lecture at the 14th International Congress on Mathematical Education, East Normal University, Shanghai, China.

Battey, D., & Leyva, L. A. (2016). A framework for understanding whiteness in mathematics education. *Journal of Urban Mathematics Education, 9*(2), 49–80.

Carmichael, S., & Hamilton, C. (1967). *Black power: The politics of liberation in America.* Vintage.

Christ, C. P. (2016). A new definition of patriarchy: Control of women's sexuality, private property, and war. *Feminist Theology*, *24*(3), 214-225.

Collins, C. (2018). What Is White Privilege, Really?. *Teaching Tolerance*, *60*, 1-11.

Eisler, R. T. (1988). *The chalice and the blade: Our history, our future*. Harper & Row.

Eisler, R. (2003). *The power of partnership: Seven relationships that will change your life*. New World Library.

Eisler, R. (2018). Breaking out of the domination trance: Remarks at the 2018 Safe Ireland Summit, Dublin, Ireland. *Kosmos Quarterly, winter.* Retrieved August 31, 2021, from https://www.kosmosjournal.org/kj_article/breaking-out-of-the-domination-trance-building-foundations-for-a-safe-equitable-caring-world/.

Eisler, R., & Fry, D. P. (2019). *Nurturing our humanity: How domination and partnership shape our brains, lives, and future*. Oxford University Press.

Gholson, M. (2021, July 17). *Recognizing the invisibilized labor of Black learners in the U.S.: A case for the study of the mundane in mathematics education*. Invited lecture at the 14th International Congress on Mathematical Education, East Normal University, Shanghai, China.

Goffney, I. M., & Hoover, M. (2021). Equitably teaching and mathematically preparing. Manuscript in preparation.

Hoover, M., Dahlgren, M., Mosvold, R., & Goffney, I. M. (2021). Mathematics teacher educators' thinking about teaching and justice as pivotal to thinking about mathematical knowledge for teaching. Manuscript in preparation.

Hoover, N. C. (2018). *Creating cultures of peace: A movement of love and conscience*. Conscience Studio. ISBN 978-0-9828492-2-4.

James, W. (2010). On a certain blindness in human beings. In R. Richardson (Ed.), *The heart of William James* (pp. 145-163). Harvard University Press.

Johnson v M'Intosh, 21 U.S., 543 (1823), https://supreme.justia.com/cases/federal/us/21/543/.

Kendi, I. X. (2019). *How to be an antiracist*. One world.

Kumanyika, C. (Guest). (2017, February 15). Turning the lens (S2: E1) [Audio podcast episode]. In *Scene on Radio: Seeing White* (John Biewen, Host). Center for Documentary Studies. https://www.sceneonradio.org/seeing-white/.

LaDuke, W. (1993, October 23). Voices from White Earth: Gaa-waabaabiganikaag. Schumacher Center for a New Economics. https://centerforneweconomics.org/publications/voices-from-white-earth-gaa-waabaabiganikaag/.

Lipsitz, G. (2006). *The possessive investment in whiteness: How white people profit from identity politics*. Temple University Press.

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education*, *22*(4), 459–478.

Miller, R. J., Ruru, J., Behrendt, L., & Lindberg, T. (2010). *Discovering indigenous lands: The doctrine of discovery in the English colonies*. Oxford University Press.

Moses, R., & Cobb, C. E. (2002). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Beacon Press.

Woolman, J. (1914). *The journal of John Woolman: With an introduction by John G. Whittier*. Houghton Mifflin Company.

Developing mathematics teaching in 'traditional' instruction environments

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The international literature base offers a range of aspects that can fall within the remit of high quality mathematics teaching practices: the use of cognitively demanding tasks, the inclusion of openings for student reasoning, and working with multiple solution methods among these. The issue at the heart of this paper is whether these aspirations are universally practical. Located in a South African context where 'traditional' forms of instruction predominate, and where large classes and poor levels of resourcing are common, my focus is on the versions of ambitious practices that might need to be developed to be responsively useful: versions that would be recognizable as aspirations in the international field, and simultaneously agreed as useful and aspirational goals in such contexts. Strands of the Mediating Primary Mathematics framework, developed in South Africa, with staging points towards contextually sensitive, aspirational goals for ambitious teaching, are shared and discussed in the paper.

Setting the scene

In a recent review of frameworks considering the quality of mathematics teaching, Charalambous and Praetorius (2018) noted both the multiplicity of offerings in the field and differences in their points of focus. However, they also noted common elements, among these, attention to aspects of what Cohen (2011) has described as 'ambitious instruction'. Many of these aspects are linked with the 'reform' agenda that is widely associated with the American National Council of Teachers of Mathematics (NCTM) Standards (NCTM, 2000). Among these aspects are pedagogic practices involving the inclusion of high cognitive demand tasks that invite student reasoning (Stein, Smith, Henningsen, & Silver, 2000), encouraging students to work with multiple solution methods and explore each other's explanations (McClain, 2002), with more limited emphasis on procedural working in mathematics in some frameworks (Charalambous & Litke, 2018).

These visions tend to be presented as 'universal'. In this lecture I am not setting out to dispute the value of these practices. But my work in South Africa is located in a context at some distance from the conditions, culture and classroom norms of the American classroom settings that are the focus of NCTM-advocated practices. Primary level classrooms often have fifty or more children. In the last decade, national workbooks have been rolled out for literacy and numeracy, improving the level of access to texts, but student and teacher resources beyond these texts in terms of manipulatives continue to be limited. Only a small minority of high status schools have any technology available in classrooms; some 'township' schools have a usually rather dilapidated room with older computers that is infrequently used. Studies in primary mathematics education have identified substantial gaps in primary teachers' mathematical knowledge (Venkat & Spaull, 2015) and problems with coherence, and limited attention to connection and progression in mathematics teaching (Askew et al, 2019). Rote choral chanting of responses is a common classroom norm (Hoadley, 2018), and classroom culture in sub-Saharan Africa has been described as highly authoritarian in mode, reflecting aspects of broader societal cultures (Tabulawa, 2013). An emphasis on low level procedural working in instruction has been documented by the middle grades (Noor & Christensen, 2013). There is also evidence of limited success with trying to import reforms centred on learner-centred instructional practices.

My focus in this paper, and in the lecture that accompanies it, is to ask what this context and these conditions mean for what counts as 'ambitious practice' in primary mathematics instruction. Specifically, what might be viable and useful foci for developing the quality mathematics teaching in such contexts? And how do these foci overlap with and differ from what is seen as constituting high quality teaching in more advantaged settings?

Practical theory

The question about viable and useful foci for mathematics teaching development in traditional and authoritarian settings is an empirical one. But answering empirical questions involves assumptions about the world and human change within it, and therefore requires some leaning on theory. My position on teacher change is fundamentally a constructivist one: that development has to take into account teachers' current knowledge bases and repertoires of practice, and build from these bases. Given the brief outline of context and conditions above, the list of ambitious practices derived from the international literature stands at a large distance from the existing state of play, making it difficult to achieve these lofty aims within the medium term.

A second aspect of interest in my work is finding levers with the potential to work for development at scale. This point is of importance in a South African education field in which smaller-scale qualitative studies abound (Deacon, Osman & Buchler, 2010), and where difficulties with moving to scale have also been described in the international literature (Maass et al., 2019). The implications for interventions seeking teacher development are to focus on mechanisms that are cost- and capacity-effective enough for moving beyond the confines and timelines of the research and development projects in which they were initially trialled.

Putting both of these imperatives together led us to the development of a framework for thinking about the development of primary mathematics teaching (the phase in focus of the work of the Wits Maths Connect-Primary project): the Mediating Primary Mathematics (MPM) framework (Venkat & Askew, 2018). The framework has a subject-specific and a rather 'instructional' bias, linked with the authoritarian, rather than dialogic, forms of teaching that predominate on the ground. Within instruction, staging points in terms of the mathematics that is offered for learning are built in a hierarchy that – at its base, involves teaching that displays some incoherence or error, into coherence, and then connection, and – at the top level – generality. Variation theory (Marton & Booth, 1997), elaborated with additional concepts developed by Watson and Mason (2005) and Watson and Mason (2006) is built into key strands of the MPM framework to consider the example spaces worked with in instruction, with development work seeking to expand these example spaces.

Earlier writing details the framework itself (Venkat & Askew, 2018), ways of using the framework to evaluate teaching (Askew et al, 2019), and the ways in which generality as a goal fits within Vygotskian socio-cultural views of mathematics as a network of scientific concepts (Venkat & Adler, 2021). In this paper, I offer examples of the ways in which we have used strands of the MPM framework to consider episodes of teaching and then have conversations with teachers in traditional pedagogy settings, that take current practices into account and seek to build from them.

Strands of the MPM framework

The MPM framework breaks down attention to the incoherence > coherence > connection > generality trajectory across a number of strands built upon the tasks and example spaces related to tasks that teachers enact mathematical instruction upon. In line with the sociocultural framing and connected to the teacher-led instruction format, mediating between the student and the mathematics to be learned is seen as occurring via the teachers' work with a range of mediating tools – artefacts, inscriptions and classroom talk/gesture, with the latter broken down further into three sub-strands related to the methods for generating and validating solutions, building mathematical connections and building responsive connections with student inputs and offers.

By way of example, in Figure 1 below, I detail the indicators for two of the strands in Tables 1 and 2 (from Venkat & Askew, 2018):

Table 1

Levels of artefact use, with indicators/illustrative excerpts

No artefacts or artifacts that are problematic/ inappropriate	*Unstructured artifacts used in unstructured ways*	*Structured artefacts used in unstructured ways*	*Structured or unstructured artefacts used in structured ways*
0	1	2	3
Lesson is conducted purely orally, with no artifacts or inscriptions	Pairs of numbers adding to nine are explored and counters used to check that a pair of numbers totals to 9.	Abaci, 100 squares, etc., used with unit counting, and without reference to structural properties. Beads on the abacus used to add 4 and 8 by counting along 4 beads on the top row, 8 beads on the second row and counting all.	Abacus, 100 square/place value blocks/cards, number lines, etc. 10s strips and unit squares used to support identification of value of underlined digit in several 2-digit numbers.

Table 2

Levels of generating/validating solutions, with indicators/illustrative excerpts

No method or problematic generation/validation	*Singular method/validation*	*Localized method/validation*	*Generalized method/validation*
0	1	2	3
Mixing of knowns and unknowns; error/ambiguity. Solution to 20 ÷ 4 begins with the teacher talking about the need for five groups to share across.	Provides a method that generates/validates the immediate answer; enables learner to produce the answer in the immediate example space. Teacher tells learners to use counters to find the answer to 4 + 5.	Provides a method that can generate/validate answers beyond the particular example space. Teacher shows how adding 10 on a 1–100 square involves moving one row down.	Provides/validates a strategy/ method that can be generalized to both other example spaces and without restriction to a particular artifact/inscription). Teacher works on adding 9 by adding 10 (as a quick fact) and then subtracting 1.

It is worth noting, in relation to the socio-cultural framing, that the trajectory goes from highly empirical ways of working with mathematics towards increasingly working with mathematics viewed as a connected network of scientific concepts in the Vygotskian sense. Seen through the lens of variation theory, this trajectory also embodies highly localised ways of working with mathematics in instruction at the lower reaches, to instruction that focuses attention on mathematical structure and properties. And layered upon these two theories, we found it useful – in the South African context – to consider the trajectory in relation to the incoherence > coherence > connection > generality path as it offered a development route that could be recognised as powerful and useable in a traditional pedagogy terrain.

In the following section, I offer extracts dealing with the ways in which teachers' ways of working with tasks and examples can be considered within particular strands of the MPM framework, and also the ways in which the model suggests responsive conversations with teachers that build on their current repertoires of practice in expansive ways. These excerpts are drawn from classroom observations in our broader research and development work in South Africa.

Analytical work coupled with developmental work

Excerpt 1

In an episode drawn from Venkat and Naidoo (2012), where a Grade 2 teacher had asked her class for a pair of numbers with a sum of 16, 8 + 8 was the first student offer. The teacher responded by writing 8 + 8 = on the board, and then asked another student to check if this was true by using bottle tops. This second student proceeded to arrange the bottle tops in two rows of eight bottle tops, and then counted the bottle tops one by one to get 16. The teacher wrote 16 in as the answer on the board, and then asked for other pairs of numbers that would give a sum of 16. 10 + 6 was then offered, and written on the board by the teacher. Students were asked to check if this gave 16 by making the two addends on their individual abaci. Once again, following counting in ones, 16 was written in as the answer. 9 + 9 was then offered. Once again, the same process was followed, with children asked to make the two numbers on their abaci, and count the total.

Analytically, we see here, the side-lining of attention to connections between examples. The result for 9 + 9 is calculated empirically from scratch, rather than seen as connected with, or derivable from 8 + 8. In fact, all the sums offered in this excerpt were calculated in this way, through repeated one-by-one counting. Of further interest is the mode of use of artefacts in this excerpt. No distinction is made between the use of bottle tops and working with abaci – both are used for counting in ones, and the arrangements of objects from one example are ‘cleared’ with subsequent examples starting again from scratch. The potential of artefacts such as abaci and the ‘doubles’ arrangement of the bottle tops to draw attention to number relationships that are useful for understanding number structure and properties, is – once again – negated in this kind of instruction. In coding terms, this excerpt suggests a Level 1 mode of working on both the ‘artefacts’ and ‘solutions’ strands.

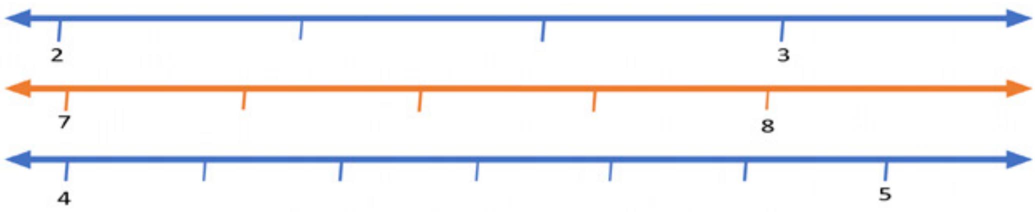
In developmental terms, the coding at the level of a basic coherence leads to a focus on connection as a next step that is likely to be met with what Schweisfurth and Elliott (2019) term as ‘local receptivity’, as an aspiration well aligned with context and culture on the ground. One possible conversation here with a teacher might consist of discussing ways of ‘annotating’ the 8 + 8 bottle tops arrangement to make 9 + 9, rather than starting over, and adapting the first result to reflect this annotation. There are also other responsive pedagogic possibilities – asking about the children who were able to answer immediately without any overt counting, and whether there are responsive ways of working that might help to keep these students’ learning trajectories moving forward, rather than pulling them back into more rudimentary calculation strategies.

Excerpt 2

In a project working to develop the capacity of local district Subject Advisers to offer tailored mathematically oriented feedback, we observed a Grade 5 lesson on fractions in which a teacher began with some number lines drawn on the board – see Figure 1. She asked the students in her class to figure out the numbers of the unmarked tick marks on the lines. After a minute, asking the students to respond, she accepted chorused whole class responses for each number line. As one of the lesson observers, I noted that several children were able to offer appropriate answers, and noted also that on the 7 to 8 line, the chorused response tended to be: ‘*Seven, seven and one quarter, seven and two quarters, seven and three quarters, eight’* – alongside the teacher gesturing to each tick mark along this line.

Figure 1

Teacher-drawn number lines on the board



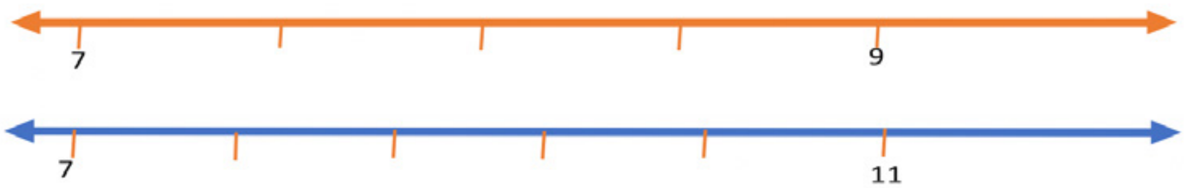
Somewhat unusually, the teacher asked for explanations of how the children decided on what number to say. A few children put their hands up, and one offered: '*Three spaces, thirds; four spaces, quarters; five spaces, fifths'* before trailing off. The teacher accepted this offer and repeated it, and then proceeded to move on to the next task.

Analytically, the artefacts 'pre-prepared' for use in this lesson are the number lines that have been inscribed on the board. The fraction number lines do have the potential to point to both 'part-whole' and 'measure' interpretations of fractions (Lamon, 2012), and the ways in which fractional parts are related to unit wholes. There are pointers to a Level 3 coding, with the caveat that all the examples drawn included a single unit gap between the start and end numbers on the number line, limiting the generality of the fraction-related thinking that children were invited to engage with. The example space on offer did produce a rationale with generality for the range of variation that was made available: that the fraction to count in should be linked to the number of intervals on the line. This solution method 'holds' for all cases with a single unit gap between start and end numbers with equally spaced intervals, and thus, has some 'reach' beyond the specific examples in this set. Broader generality is not solicited by the teacher; nor are the limitations of the reach of this rule broached or questioned: the rule is appropriate for the example space, but it remains localised to a particular category of fraction-oriented number lines.

Our conversation with the teacher in this instance was based on discussing the rule offered in this class, acknowledging its efficacy for the examples providing and noting that it encouraged the class to consider and verbalise how they produced their answers, and then asking about the rule in relation to a couple of additional examples – see below:

Figure 2

Additional examples



This conversation allowed for a focus, again, on teaching that allows for an expansion of current student understandings, while – once again – building from the teacher's observed practice repertoire. Underlying this choice is Watson and Mason's (2005) notion of finding counter-examples, problems for which a given rule or property does 'not' work. However, the choice was not to frame the teacher conversation in this way, as this framing was likely to be more unfamiliar in a context where the teaching of low-level procedures for solving given sets of problems has been described as predominant (Ally & Christiansen, 2013). Instead, we offered and discussed an example that threatened the appropriateness of the rule, sensing that expanding the example space to include a more varied set of number ranges would be more likely to be incorporated into future practice. Other options are possible here too:

- What happens to the rule if there are unequal intervals?
- Can the problem be looked at if one or both of the number line bounds are rational numbers rather than integers?

Watson's examples of questions that can be used to encourage children to attend to structure and generality are typically more open than the approaches we have tended to use, but the aspiration for development of pedagogy to encompass more opportunities for children to engage with mathematical thinking remains, in ways that are attuned to classroom conditions and pedagogic cultures in our ground.

Final Comments

Coming back to the question of whether visions of 'ambitious instruction' are universal, my sense is that this is not the most useful to ask. Rather, my interest is a more pragmatic one: can we set out a version of ambitious instruction that would, simultaneously, be recognised and seen as valid in the international field in mathematics education, while also being recognised as useful and reachable in staged ways in the local ground. An important point to note about the MPM model is that it is centred on mathematical expansion within pedagogy, rather than aiming for expansions in teachers' pedagogic forms towards reform-oriented practices per se. For some reading our work, our ambitions may not feel ambitious enough. For us, the expansions we seek remain aspirational and practicable to work with on the ground, and at some scale. Askew et al (2019) have reported on improvement seen in the practices of a group of teachers over time in the context of our professional development intervention activity. In a context of high poverty, high inequality and low performance in mathematics, such evidence of moves towards making more expansive opportunities to learn mathematics available are important to take into account in a country where studies lamenting the lack of move towards learner-centred instruction abound. Essentially, our adapted versions of ambitious instruction, are – for now – sufficient to keep our hands full.

References

Ally, N. & Christiansen, I. (2013). Opportunities to develop mathematical proficiency in Grade 6 mathematics classrooms in KwaZulu-Natal. *Perspectives in Education, 31*, 106-121.

Askew, M., Venkat, H., Abdulhamid, L., Mathews, C., Morrison, S., Ramdhany, V., & Tshesane, H. (2019). Teaching for structure and generality: Assessing changes in teachers mediating primary mathematics. In M. Graven, H. Venkat, A. Essien, &

P.Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education (Vol. 2)*, 41–48. Pretoria, South Africa: PME.

Charalambous, C.Y. & Litke, E. (2018). Studying instructional quality by using a content-specific lens: the case of the Mathematical Quality of Instruction framework. *ZDM Mathematics Education, 50*, 445-460.

Charalambous, C.Y. & Praetorius, A.K. (2018). Studying mathematics instruction through different lenses: setting the ground for understanding instructional quality more comprehensively. *ZDM Mathematics Education, 50*, 355-366.

Cohen, D. K. (2011). *Teaching and its predicaments.* Cambridge: Harvard University Press.

Deacon, R., Osman, R., & Butler, M. (2010). Education policy studies in South Africa, 1995–2006. *Journal of Education Policy, 25*(1), 95-110.

Hoadley, U. (2018). *Pedagogy in poverty*. London: Routledge.

Maass, K., Cobb, P., Krainer, K. & Potari, D. (2019). Different ways to implement innovative teaching approaches at scale. *Educational Studies in Mathematics, 102*, 303-318.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Schweisfurth, M. and Elliott, J. (2019) When ‘best practice’ meets the pedagogical nexus: recontextualisation, reframing and resilience. *Comparative Education, 55*(1), 1-8. (doi: 10.1080/03050068.2018.1544801)

Stein, Smith, Henningsen, & Silver, (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teacher College.

Tabulawa, R. (2013). *Teaching and Learning in Context: why pedagogical reforms fail in Sub-Saharan Africa.* Oxford: African Books Collective.

Venkat, H. & Adler, J. (2021). Mediating mathematics in instruction: Trajectories towards generality in ‘traditional’ teaching. In Zehetmeier, S., Potari, D. & M. Ribeiro (Eds.). *Mathematics teaching and mathematics teacher professional development*. 5-23. London: Routledge.

Venkat, H., & Askew, M. (2018). Mediating primary mathematics: Theory, concepts and a framework for studying practice. *Educational Studies in Mathematics , 97* (1), 71–92.

Venkat, H., & Naidoo, D. (2012). Analyzing coherence for conceptual learning in a Grade 2 numeracy lesson. *Education as Change , 16* (1), 21–33.

Venkat, H. & Spaull, N. (2015) What do we know about primary teachers’ mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development, 41*, 121–130.

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Learners generating examples*. New York: Lawrence Erlbaum Publishers.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning , 8* (2), 91–111.

Conceptualisations of Inclusion in the Context of Primary Mathematics Curriculum Policy and Professional Development

Realising an inclusive environment for all learners requires the specialised expertise of all three organisations. This symposium aims to make explicit the conceptualizations of inclusion that underpin primary mathematics curriculum policy and professional development in Ireland; and the collaborative efforts employed by PDST, NCSE and NCCA to realise this vision for inclusion and equity in the context of primary mathematics.

The National Council for Curriculum and Assessment (NCCA) paper provides an inside glance into the curriculum development process with a spotlight on inclusion. It illuminates where the curriculum vision for inclusion stems from and is situated in the context of broader policy and research, along with the incorporation of stakeholder voice and agency in shaping and refining developments.

The National Council for Special Education (NCSE) paper provides an overview of relevant research and guiding principles in the area of inclusion. It focuses on how this research informs future professional development for teachers in relation to both curriculum reform and the inclusion of students with special educational needs. In the context of primary curriculum reform, this paper examines the key considerations required to achieve an inclusive mathematics classroom for all learners in all contexts.

The Professional Development Service for Teachers (PDST) paper notes that in Ireland there is a gap between the policies and theories underpinning inclusive practices for all learners and the actual practices realised in primary mathematics. This paper highlights how effective teacher professional development aims to support teachers to co-construct inclusive practices for all learners in primary mathematics.

Developing a Draft Primary Mathematics Curriculum: A Vision for Equitable, Accessible and Inclusive Learning Experiences for Every Child

Tracy Curran, John Behan, Margaret Flood and Jacqueline Fallon

National Council for Curriculum and Assessment (NCCA)

The National Council for Curriculum and Assessment (NCCA) advises the Minister for Education on curriculum and assessment for early childhood education, primary and post-primary schools. Inclusion and the promotion of equitable access, engagement and challenge are core considerations for NCCA's development of curriculum and assessment in Ireland. At primary level, a draft mathematics curriculum is in development and this paper provides an inside glance into the curriculum development process with a spotlight on inclusion. It is situated in broader policy and research contexts and describes the incorporation of stakeholder voice and agency in shaping and refining developments. The paper also references some key ideas and considerations which guide the curriculum development process in service of more equitable, accessible and inclusive mathematical learning experiences for every child.

Introduction

The National Council for Curriculum and Assessment (NCCA) advises the Minister for Education on curriculum and assessment for early childhood education, primary and post-primary schools. As part of Primary Curriculum review and redevelopment, a draft Primary Mathematics Curriculum (PMC) is currently in development. Central to this curriculum is a vision for equitable, accessible and inclusive learning experiences for every child. This paper begins by describing key contexts for the curriculum development process. Following this, some aspects of the research which underpinned this vision for inclusion and equity in the curriculum are presented. The paper then shows how inclusion and equity have been incorporated into the process through stakeholder voice and agency. In the latter part of this paper, some of the key ideas and considerations that inform the development of the draft PMC are discussed and summarised, and their application to the design and development process is described.

A Fresh Vision for Children's Mathematical Learning

In the proposed draft PMC, mathematics is characterised as the study of the relationships, connections and patterns that surround us, which in turn allows us to understand and engage fully with our world. Every child, without exception, is considered to have an innate, intuitive and instinctive sense of mathematics; is capable of using these tools and engaging with mathematical concepts and ideas from birth; and can deepen and develop her/his learning over time. The overarching aim of the proposed draft PMC is for every child to develop mathematical proficiency, namely: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence and productive disposition. As children engage with increasingly sophisticated mathematical learning experiences their mathematical proficiency is developed and refined.

The draft PMC prioritises and promotes equity and access for all children, irrespective of their cognitive ability, cultural context, or socio-economic background. Indeed, the development of the draft PMC is situated in the revision and redevelopment of the broader primary curriculum, where inclusion, access and equity are central to the vision for children's learning in primary school (NCCA, 2020). The Draft Primary Curriculum Framework strongly recognises the agency and professionalism of teachers in managing the complexity of learning in the classroom and the diversity of children's learning journeys. It holds that inclusion, access and equity for children in the classroom is realised when teachers engage children in appropriately playful and engaging learning experiences which are tailored to their individual needs, strengths and interests.

Context for Developments

Policy Context

In broad policy terms, inclusion is described as a process of addressing and responding to the diverse needs of learners, whilst simultaneously removing barriers so that each child can gain the maximum benefit from his or her school experience (National Council for Special Education [NCSE], 2011). In the last decade, a number of developments have occurred that are relevant to curriculum development. For example, the increased numbers of children entering primary schools has led to increased provision for children with special educational needs (SEN) (NCSE, 2019) and the Irish Government's ratification of the Convention on the Rights of Persons with Disabilities in March 2018 has prompted policy change. Further, the new model of special education teaching allocation (Department of Education and Skills [DES], 2017) aims to support and promote a whole-school approach to SEN provision, as well as a commitment to the inclusion of pupils with SEN in mainstream schools. However, it also recognises that in some instances, a minority of pupils with significant and enduring needs require more specialist settings such as special school placements or special classes in mainstreams schools.

The characterisation of an inclusive school culture set out in the most recent guidelines for primary schools (DES, 2017a) as well as other policies such as the Inclusive Framework for Schools (NCSE, 2011) and the DEIS (Delivering Equality of Opportunity in Schools) Action Plan (DES, 2017b) provides an important context for curriculum that serves the needs of all children in all school settings. Ambitions for promoting active participation and engagement, belonging and community, as well as high aspirations for all children's learning contribute to the development of curriculum policy in support of this vision.

Research Base

It was initially planned that the PMC would be published in two parts, the first publication being the specification for junior infants to second class, followed by the specification for third to sixth class. Subsequently, it was decided by the then Minister of Education, Richard Bruton, that the new curriculum should be published as a single specification. As a consequence of the initial plan, the research base for curriculum developments was collected in two phases. In the first phase, a systematic review of the literature was conducted, concentrating on teaching and learning for children aged three to eight years. This comprised an international audit of mathematics curriculum policy (Burke, 2014); Research Report 17 (Dunphy et al., 2014) which focused on definitions, theories, development and progression in primary mathematics; and Research Report 18 (Dooley et al., 2014) which looked at pedagogy and learning more specifically. Drawing on this research base, a background paper and brief for development of the draft PMC (NCCA, 2016) was produced. Following the publication of the first draft specification of the PMC for junior infants to second class (NCCA, 2017), consultation took place between October 2017 to March 2018. The report from this consultation (NCCA, 2018) added significantly to the research base.

The second phase of research reports served to complement the existing research base by focusing on the senior classes of primary school. A research addendum to Research Reports 17 and 18 was compiled (Dooley, 2019) which looked at broad teaching and learning considerations for children in the upper years of primary school. This was further supplemented with five short research papers which examined core mathematical concepts, skills and processes with which children engage across the five mathematical domains (Delaney, 2020; Leavy, 2020; Nic Mhuirí, 2020a, 2020b; Twohill, 2020).

Throughout this research, myriad references are made to the importance of promoting inclusion and addressing diversity. For example, in Research Report 17 (Dunphy et al., 2014), the authors point to high-quality learning experiences that are critical to closing existing equity gaps and ensuring that every child can realise their mathematical potential. In providing such experiences, the report suggested that rather than needing distinctive teaching approaches or even distinctive curricula, what is required is a focus on addressing individual and specific needs of children. In Research Report 18 (Dooley et al, 2014), it is stressed that while learning paths are useful to illustrate a general developmental continuum of children's learning, individual children actually progress their learning in diverse and non-linear ways. Indeed, appropriately sequencing learning according to children's individual developmental paths is highlighted as a feature of 'good pedagogy'. In the research addendum to these reports (Dooley, 2019), it is again acknowledged that children have diverse ways of making sense of mathematics. Moreover, considerable attention is also paid in this report to offering an equitable curriculum through the provision of a culturally sensitive pedagogy. To do so, Sleeter's (2012, p.571) advice to educators is highlighted, "[w]hat makes more sense is for teachers to bring to the classroom an awareness of diverse cultural possibilities that might relate to their students, but then to get to know the students themselves".

Accordingly, the background paper and brief for PMC developments (NCCA, 2016) articulates a clear commitment to design an inclusive PMC for every child that promotes the principles of inclusion, equity and access. It states:

> *The curriculum will be developed in line with the principles of universal design for learning and as such, promote the principles of equity and access for children with a diverse range of abilities. For children with special educational needs and in particular, those with severe and profound and low moderate*

> *needs, the curriculum will outline what is appropriate and relevant for them to know and provide differentiated support so they can access this learning. The curriculum will support children who attend Irish- and English-medium schools, and acknowledge and support children from different language backgrounds where neither English nor Irish is their first language. It will be considerate of the wide range of diverse backgrounds that children come from and their differing starting points as they enter primary school, including children from socio-economically disadvantaged backgrounds. (p.57).*

Voice for Inclusion and Equity

To provide a strong, representative, and responsive basis for its curriculum and development work, NCCA has established Development Groups to undertake specific tasks in curriculum areas or subjects. The Early Childhood and Primary Mathematics Development Group includes stakeholder representatives who advocate for inclusive curriculum. In consultation with the Development Group, NCCA has worked with a number of 'critical friends' groups with specific expertise on inclusion to gather feedback during the drafting process.

Consultation is a critical opportunity to gather feedback from children, teachers, school leaders, parents and the wider public on curriculum developments. In the context of the PMC, a draft specification for junior infants to second class was published for consultation in October 2017. As the consultation report (NCCA, 2018) noted, NCCA took a number of steps to gather feedback on how well the draft PMC addressed the issue of inclusion. For example, a diverse range of school settings was included in the school network strand of the consultation. Consultation findings show that the draft specification was recognised by schools as being very inclusive, with a number of participants acknowledging efforts made to include every child. In particular, teachers working in SEN settings welcomed the outcomes-based approach of the draft curriculum. In time, a draft of the full specification will again be published for consultation and this will provide a further opportunity for NCCA to listen and learn and subsequently improve and refine the draft in terms of equity and inclusion.

Key Considerations for Developments

The challenge of supporting and meeting the needs of all children is well recognised. In the years since the 1999 primary curriculum was introduced, there have been a number of research-based frameworks that have been devised in response to this challenge. One such prominent framework is Universal Design for Learning (UDL) (CAST, 2018). Derived from the principles of universal design in architecture, UDL is built upon the premise that designing a building or indeed a curriculum with the needs of diverse users in mind from the outset has positive outcomes for all users. In the past two decades, cultural responsiveness has also gained increasing attention and prominence in scholarly work and policy outputs. Like UDL, it is an approach to reach and include learners who may traditionally have been more marginalised. In the context of curriculum development and enactment, culturally responsive practices are important in ensuring that children's cultural identity and references are recognised and honoured in all aspects of teaching and learning. Growing scholarly work (e.g.

Hammond, 2015; Ladson-Billings, 2011; Richards et al., 2007; Rose and Meyer, 2002) offers important considerations for developing an inclusive and equitable PMC. These include:

- Every child is capable of learning mathematics.
- Every child should have equitable access to rich, meaningful and challenging learning opportunities.
- The teacher / child relationship is central to the learning experience.
- Learning goals and outcomes should be clear and accessible, and offer an appropriate level of support, challenge and interest to every child.
- Providing children with the opportunity to engage, present and express their learning in multiple ways enriches the learning experience of every child.
- Methods and approaches should be flexible and diverse enough to provide appropriate learning experiences, challenges, and supports for every child.
- Assessment should help children to see their progress, to identify challenges and areas for support and to plan their next steps, as well as serving to help teachers adjust instruction and maximise learning.
- Inclusive and culturally inviting classrooms provide the optimal learning space where every child can appreciate his/her sameness and difference and feel like they belong.

These key considerations offer the potential to provide increased equity of access and opportunity for all children, and in doing so, presents an opportunity to address the challenges faced in meeting the needs of an increasingly diverse Irish school population.

Design and Development Process

Following the publication of the primary school mathematics curriculum in 1999, a suite of guidelines was produced by NCCA with a focus on supporting children with SEN, including guidelines for teachers of students with mild general learning disabilities, moderate general learning disabilities, and severe and profound learning difficulties. The vision for the draft PMC is that the curriculum will attend to the learning experiences for every child, notwithstanding the setting that the child might attend. Accordingly, as stated in the background paper and brief (NCCA, 2016, p.20):

> *The development of the new primary mathematics curriculum will be cognisant of the myriad factors impacting schools in Ireland currently, as well as new theoretical perspectives offered in the literature.*

Naturally, specific supports should and will be developed to accompany the curriculum so that the individual needs of children can be supported. Like the Primary Language Curriculum / Curaclam Teanga na Bunscoile (NCCA, 2019), the draft PMC will have a Teacher Toolkit. Sample support materials, including those that highlight, support and promote opportunities for inclusive learning, are being created as part of the curriculum development process. It is intended that in the next phase of consultation, these support materials will be reviewed and trialled in a variety of school contexts. Finally, in order to support teachers and school leaders in creating inclusive learning experiences, NCCA continues to collaborate with our partners in the Professional Development Service for Teachers (PDST) and the NCSE. Together, we are committed to ensuring that policy aspirations for inclusion and equity translate to the lived experiences of children in our schools.

Conclusion

Curriculum development in Ireland is a comprehensive process that aspires to the highest standards of rigour and integrity. Research, deliberation, consultation and networks underpin the work of the NCCA and serve to ensure that curriculum policy is inclusive of all voices. As strongly evidenced in the Draft Primary Curriculum Framework (NCCA, 2020) equitable, accessible and inclusive learning experiences are central to the vision for children's learning experiences, not least in terms of their learning in primary mathematics. Realising a vision for equity, accessibility and inclusion for every child is a challenge which requires commitment and collaboration from all stakeholders. As primary curriculum review and redevelopment continues to progress in Ireland, there is much cause for optimism that policy aspirations for greater inclusion will be realised in the lived experiences of every child.

References

Burke, D. (2014). *Audit of mathematics curriculum policy across 12 jurisdictions: Commissioned report for National Council for Curriculum and Assessment.* Retrieved June 23, 2021. https://ncca.ie/media/2031/audit-mathematics-curriculum-policy.pdf

CAST. (2018). *Universal design for learning guidelines, version 2.2.* Retrieved June 23, 2021. http://udlguidelines.cast.org

Delaney, S. (2020). *Number in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment.* Retrieved June 23, 2021. https://ncca.ie/media/4622/primary_maths_research_number_seniorclasses.pdf

Department of Education and Skills, DES. (2017a). *Circular 0013/2017. Special education teaching allocation in mainstream primary schools.* Retrieved June 23, 2021. https://www.education.ie/en/Circulars-and-Forms/Active-Circulars/cl0013_2017.pdf

Department of Education and Skills, DES. (2017b). *DEIS Plan 2017: Delivering equality of opportunity in schools.* Retrieved June 23, 2021. https://www.education.ie/en/Publications/Policy-Reports/DEIS-Plan-2017.pdf

Dooley, T. (2019). *Learning and teaching primary mathematics: An addendum to NCCA research reports 17 and 18*. Retrieved June 23, 2021. https://ncca.ie/media/4087/primary_maths_research_addendum_2019.pdf

Dooley, T., Dunphy, E, & Shiel, G. (2014). *Mathematics in early childhood and primary education. Research report 18: Commissioned report for National Council for*

Curriculum and Assessment. Retrieved June 23, 2021. https://ncca.ie/media/2147/ncca_research_report_18.pdf

Dunphy, E. Dooley, T., & Shiel, G. (2014). *Mathematics in early childhood and primary education. Research report 17: Commissioned report for National Council for Curriculum and Assessment*. Retrieved June 23, 2021. https://ncca.ie/media/1494/maths_in_ecp_education_theories_progression_researchreport_17.pdf

Hammond, Z. (2015). *Culturally responsive teaching and the brain: Promoting authentic engagement and rigor among culturally and linguistically diverse students*. Corwin.

Ladson-Billings, G. (2011). Is meeting the diverse needs of all students possible? *Kappa Delta Pi Record,* 47, 13-15.

Leavy, A. (2020). *Data and Chance in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment.* Retrieved June 23, 2021. https://ncca.ie/media/4620/primary_maths_research_data_and_chance_seniorclasses.pdf

National Council for Curriculum and Assessment, NCCA. (2016). *Background paper and brief for development of a new primary mathematics curriculum*. Retrieved June 23, 2021. https://ncca.ie/media/1341/maths_background_paper_131016_tc.pdf

National Council for Curriculum and Assessment, NCCA. (2017). *Primary mathematics curriculum, draft specification Junior infants to second class: For consultation*. Retrieved June 23, 2021. https://ncca.ie/media/3148/primary_mathsspec_en.pdf

National Council for Curriculum and Assessment, NCCA. (2018). *Consultation report on the primary mathematics curriculum for junior infants to second class*. Retrieved June 23, 2021. https://ncca.ie/media/3605/pmc_consultation_report_july2018.pdf

National Council for Curriculum and Assessment, NCCA. (2020). *Draft primary curriculum framework: For consultation*. Retrieved June 23, 2021. https://ncca.ie/media/4456/ncca-primary-curriculum-framework-2020.pdf

National Council for Special Education, NCSE. (2011). *Inclusive education framework: A guide for schools on the inclusion of pupils with special educational needs*. Retrieved June 23, 2021. https://ncse.ie/wpcontent/uploads/2014/10/InclusiveEducationFramework_InteractiveVersion.pdf

National Council for Special Education, NCSE. (2019). *Policy advice on special schools and classes: An inclusive education for an inclusive society?* Retrieved June 23, 2021. https://ncse.ie/wp-content/uploads/2019/11/Progress-Report-Policy-Advice-on-Special-Schools-Classes-website-upload.pdf?fbclid=IwAR2Bbn2jdqhH0fHcexBD6HZP5eEPd9cxDRD05k5Nx5etFo3sC_YSSKdeR8U

Nic Mhuirí, S. (2020a). *Measures in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment*. Retrieved June 23, 2021. https://ncca.ie/media/4626/primary_maths_research_measures_seniorclasses.pdf

Nic Mhuirí, S. (2020b). *Shape and space in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment.* Retrieved June 23, 2021. https://ncca.ie/media/4624/primary_maths_research_shape_and_space_seniorclasses.pdf

Richards, H., Brown, A., & Forde, T. (2007). Addressing diversity in schools: Culturally responsive pedagogy. *Teaching Exceptional Children*, *39*(3), 64-68.

Rose, D. & Meyer, A. (2002). *Teaching every student in the digital age: Universal design for learning*. Association for Supervision and Curriculum Development.

Sleeter, C. (2012). Confronting the marginalization of culturally responsive pedagogy. *Urban Education, 47*(3), 562-584.

Twohill, A. (2020). *Algebra in the senior primary classes: Commissioned research paper for National Council for Curriculum and Assessment*. Retrieved June 23, 2021. https://ncca.ie/media/4619/primary_maths_research_algebra_seniorclasses.pdf

The Impact of Continuing Professional Development (CPD) on the Inclusion of Students with Special Education Needs (SEN) in the Teaching and Learning of Mathematics in Primary Schools

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National Council for Special Education (NCSE)

The National Council for Special Education (NCSE) was set up to improve the delivery of education services to persons with special educational needs with particular emphasis on children. A fundamental aspect in the delivery of this objective includes the provision of high-quality Continuous Professional Development (CPD) for teachers. This paper aims to provide readers with an overview of relevant research in this area and the guiding principles which will inform future CPD provisions for teachers in relation to both curriculum reform and the inclusion of students with special educational needs. In the context of Primary curriculum reform, this paper also sets out to examine the key considerations required to achieve an inclusive Mathematics classroom for all learners in all contexts.

Introduction

In Ireland, almost one in four children have Special Educational Needs (SEN) that can impact their learning (Banks and McCoy, 2011). This prevalence rate is derived using the broad definition of SEN in the Education for Persons with Special Educational Needs Act (2004) (EPSEN Act) which states

> in relation to a person, a restriction in the capacity of the person to participate in and benefit from education on account of an enduring physical, sensory, mental health or learning disability, or any other condition which results in a person learning differently from a person without that condition. (p.6).

Teachers have a fundamental impact on whether students learn (Fullen, 2006) and, therefore, in driving curriculum reform, effective and meaningful CPD for teachers is essential. Research highlights that provision of decontextualized, once off in-service seminars to introduce new curricula is ineffective (Fung, 2000) and instead positive impactful CPD opportunities are needed for teachers (Crawford et al., 2007). The Draft Primary Curriculum Framework (NCCA, 2020) emphasises the importance of "ongoing access to, and opportunities for, high-quality and school-based continuing professional development." (p. 28). It also addresses the idea of teacher and leader collaboration which is highlighted as "enabling and supporting teachers and school leaders to identify and prioritise school-based CPD needs alongside national priorities." (NCCA, 2020, p.28).

Research findings consistently support the central role teachers play in the education of learners with SEN. There is evidence that the quality of teaching is one of the most important factors in learner outcomes (NCSE, 2013). The central role of the teacher in moving towards an inclusive education system is widely acknowledged; the World Report on Disability (WHO, 2011) stressed that appropriate training of mainstream teachers is crucial for them to be confident and competent in teaching children with diverse needs. In order to promote inclusion in the redeveloped Mathematics curriculum, all teachers need to be equipped to meet the increasingly diverse needs of learners. Provision of CPD opportunities for teachers is integral to successful inclusion of all learners. The NCSE recognises the importance of providing quality and tailored support for teachers in the embedding of new curricula, as evidenced by the roll out of CPD and supports for the Primary Language Curriculum (PLC). The NCSE Primary Curriculum Team developed in-service courses and continue to provide in-school support. The PLC represents the first of the curricula to move from content objectives to learning outcomes. In the rollout of the Primary Mathematics Curriculum CPD will be needed to help teachers enact this change in their classrooms.

Supporting Inclusion through Teacher Continuing Professional Development

The Cosán Framework (Teaching Council, 2016) has highlighted inclusion as one of the key areas that should be addressed in schools through teacher CPD. There have been many important legislative and policy developments in the movement towards inclusion, both nationally and internationally, including the Salamanca Statement and Framework for Action on Special-Needs Education (1994), the Council of Europe Political Declaration (2003) and Action Plan (2006), the EPSEN Act (2004), and the United Nations International Convention on the Rights of Persons with Disabilities (2006) which Ireland signed in 2007 and ratified in 2018.

More recently, Guidelines for Primary and Post-Primary Schools; Supporting Students with Special Educational Needs in Mainstream Schools (Department of Education, 2017), highlights the central role the mainstream class teacher has in ensuring the progress and care of all students in the classroom, including students with special educational needs. It is essential that CPD with a focus on supporting students with special educational needs is made available for mainstream class teachers and not just for Special Education Teachers, Special School Teachers and Special Classes Teachers. Teacher CPD can have a positive impact on inclusion (Rose et al, 2010) but there is a need to look at teacher's knowledge as well as their attitudes and beliefs. Teacher expectations are essential to a child's success (Shevlin and Rose, 2003) and thus teacher attitudes and beliefs can be barriers to the inclusion of students with SEN in the classroom. Hodkinson (2009) argues that successful inclusion may be dependent first upon teachers' attitudes and beliefs and secondly their competence to deliver.

The NCSE offers online support and in-person training days as well as in-school support. However, very few mainstream teachers attend these courses. For example, only 7% of those who attended NCSE Term 1 Primary Teacher Professional Learning National seminars in 2020 were mainstream teachers. This was in a Covid-19 context however; that aside, it does highlight the idea of possible mandatory CPD in the area of SEN for all teachers. CPD for teachers is a common theme across a majority of NCSE Policy Advice papers (2011, 2012, 2013, 2015 and 2018). These papers often refer to research studies which recommend that teachers should have access to CPD relevant to their student's needs. NCSE acknowledges that teachers in different settings require CPD with different foci to best meet the needs of their students (NCSE Policy Advice 2, 2011). Highlighted in these policy advice papers is the need for mandatory CPD for all teachers so they can gain the knowledge, skills and competencies which empower them to enable students with special educational needs to fulfil their potential in all contexts.

Supporting Inclusion in the Teaching and Learning of Mathematics in the Primary Classroom

Children who fail to acquire competence in basic mathematical facts tend to have negative attitudes towards mathematics and often avoid this in day-to-day living in later life. (Doherty et al., 2011). To promote and advance the teaching of mathematics for students with SEN, initial and sustained support for teachers needs to be provided so that all teachers feel confident and skilled to include all learners in the mathematics class. Effective and meaningful CPD enables teachers to negotiate each individual student's learning strengths and unique needs. It is important that teachers understand how the diverse needs of their students impact on teaching and learning in the classroom. An over emphasis on teaching specifically to the category of need can mean the focus becomes about the label and not the holistic strengths and needs of the child (Cologon, 2014). In the context of mathematics, it is known that low arithmetical attainment can be associated with a general learning difficulty, a readiness lag, or a specific learning difficulty (SLD) in numeracy, also known as dyscalculia (Neville, 2012). For example, 3–7% of all children, adolescents, and adults suffer from dyscalculia, which is a severe, persistent difficulty performing arithmetical calculations and can lead to marked difficulty in school, at work and in everyday life (Haberstroh and Schulte-Körne, 2019). Within the classroom, unfamiliarity with dyscalculia may lead to unrealistic expectations regarding accessing the class curriculum and number fact recall. Examples such as these further highlight that teachers require support to understand the different supports that students require in order to access the mathematics curriculum.

Approached to Mathematics Education for Students with SEN

Students with learning disabilities require a structured approach to mathematics as some students may learn inappropriate or incorrect strategies through incidental learning. Approaches may include direct explicit teaching and opportunities to practice different skills and strategies to consolidate learning; accommodations; universal design for learning; and differentiated instruction.

Explicit Teaching

Direct teaching, using explicit strategies, is a well recommended technique for teaching all students and works well for students with learning difficulties in Mathematics (Westwood, 2000). This approach is described as the systematic delivery of Mathematics lessons, within a structured class, using the following specific procedures; 'introducing objectives, reviewing previously learned concepts, modelling new skills, and providing guided and independent practice' (McKenna et al., 2015, p. 8). This supports teachers to apply procedure-based mathematics instruction to support all students in the classroom.

Accommodations

Accommodations in the Mathematics classrooms using assistive technology can range from low to high technology. For example, students who are deaf or hard of hearing may use an FM system, interpreter, real-time captioning and visual warning devices and there is a vast array of technology-based accommodation such as voice recognition software, switches and augmentative communication devices. It is important that effective and appropriate CPD is made available to ensure teachers know how to utilise the accommodations to support access to the Mathematics Curriculum by students with SEN.

Universal Design for Learning (UDL)

The UDL framework is one which provides multiple methods of presentation, multiple methods of expression and multiple options for engagement (Meyer and Rose, 1998). UDL enables the educator to remove barriers by anticipating the needs of all students. “Accessibility and inclusion are embedded within all three UDL principles” (Dyjur, Ferreira, and Clancy, 2021, p.73). The broader goal of accessible pedagogy can benefit students with and without SEN (Moon et al., 2012). Connecting UDL to Mathematics curriculum development is an important goal in promoting the teaching of Mathematics for students with SEN. “Universal design for learning (UDL) is a valuable tool for the proactive planning of engaging, accessible lessons in today's diverse classrooms” (Sally, 2011, p10). Targeted CPD for teachers in this aspect of inclusive practice is a key element for determining success in applying the principles of UDL.

Differentiated Instruction

Differentiated Instruction and UDL are not mutually exclusive. Differentiation is a process by which all pupils are enabled to engage in the curriculum by the provision of learning tasks and activities that are tailored to their needs and abilities. Willis and Mann (2000, p. 1) state that “differentiation is a teaching philosophy based on the premise that teachers should adapt instruction to pupil differences”. Differentiation should be seen in terms of different styles and strengths of pupils and not on a hierarchy of abilities. Activities, methodology, environment, resources and outcomes can be varied to take into account the diverse range of interests, needs and experience of the pupils. Widodo, Prihatiningsih and Taufiq (2021) point to the importance of a range of resources and the use of various learning media in learning mathematics. Creating multiple pathways for learning through differentiated instruction as part of the daily learning experience is essential in advancing the teaching of mathematics for students with SEN. Blending the concepts of UDL and differentiated instruction with an inclusive curriculum for all is the ultimate goal in terms of engaging students in opportunities to access mathematics.

Conclusion

It is important that teachers have a full understanding of the principles of both UDL and differentiated instruction and how to implement these practices to realise the goal of inclusion. All learners in all school contexts can benefit from engagement and participation in inclusive learning environments within Mathematics. There are many considerations in promoting and advancing the teaching of Mathematics for students with SEN. One of these is, an inclusive curriculum where there is a shift from content objectives to learning outcomes. Secondly, the delivery of sustained high-quality CPD and support to all teachers should be prioritised to include accommodations, UDL and differentiated instruction, ensuring that CPD targets all teachers in all contexts. The importance of teacher professional development is clear in effecting not just curriculum reform but also including all students in the mathematics classroom.

References

Banks, J. & McCoy, S. (2011). *A student on the prevalence of special educational needs.* NCSE.

Cologon, K. (2014b). Preface. In K. Cologon (Ed.), *Inclusive education in the early years: Right from the start* (pp. xviii - xix). Oxford University Press.

Crawford, K. (2007). Continuing professional development in higher education: Voices from below. *The International Journal of Knowledge, Culture, and Change Management:* Annual Review *6.* https://doi.org/10.18848/1447-9524/CGP/v07i08/50415

Doherty, U., Egan, M., Daly, P., Coady, M., Holland, M., Kelleher, D., Long, S., McCarthy, E. & O'Sullivan, S. (2011) *Introduction to: Strands: Strategies for teachers to respond actively to the needs of children with Down syndrome*. Mary Immaculate College.

Dyjur, P., Ferreira, C., & Clancy, T. (2021). Increasing accessibility and diversity by using a UDL framework in an infographics assignment. *Currents in Teaching & Learning, 12*(2), 71–83.

European Agency for Development in Special Needs Education (2010). Teacher Education for Inclusion (TE4I); *International Literature Review*. European Agency for Development in Special Needs Education.

Evans, L (2008). Professionalism, professionality and the development of education professionals. *British Journal of Educational Studies*, *56*(1), 20-38.

Fung, Y. (2000). A constructivist strategy for developing teachers for change: A Hong Kong experience. *Journal of In-Service Education, 26*(1), 153-167.

Government of Ireland. (1984). *Report of the committee on in-service education*. Stationery Office.

Government of Ireland. (1992). *Green Paper, education for a changing world.* Stationery Office.

Government of Ireland. (1995). *White Paper, charting our education future.* Stationery Office.

Government of Ireland. (2001). *Teaching council act.* Stationery Office.

Guidelines for Primary Schools. (2017). *Supporting students with special educational needs in mainstream schools.* Department of Education.

Government of Ireland (2004). *Education for persons with special educational needs.* Government of Ireland.

Hodkinson, A., & Vickerman, P. (2009). *Key issues in special educational needs and inclusion.* Sage.

Heacox, D. (2002). *Differentiating instruction in the regular classroom: How to reach and teach all learners, grades 3–12.* Free Spirit.

Haberstroh, S., Schulte-Körne, G. (2019). The Diagnosis and Treatment of Dyscalculia. *Dtsch Arztebl Int. 116*(7), 107-114. https://doi.org/10.3238/arztebl.2019.0107

Hitchcock, C., Meyer, A., Rose, D., & Jackson, R. (2002). Providing new access to the general curriculum: Universal design for learning. *Teaching exceptional Children, 35*(2), 8-17. https://doi.org/10.1177/004005990203500020

Lauchlan, F., & Boyle, C. (2007). Is the use of labels in special education helpful? *Support for Learning, 22*(1), 36-42.

McKenna, J. W., Shin, M., & Ciullo, S. (2015). Evaluating reading and mathematics instruction for students with learning disabilities: A synthesis of observation research. *Learning Disability Quarterly, 22(January),* 1-13. https://doi.org/10.1177/0731948714564576

Meyer, A., & Rose, D. H. (1998). Learning to read in the computer age. In J. Chall & J. Onofrey (Eds.), *From reading research to practice.* Brookline Books.

Moon, N. W., Todd, R., Morton, D.& Ivery, E. (2012) *Accommodating students with disabilities in science, technology, engineering, and university education.* National Science Foundation.

National Council for Special Education [NCSE]. (2011). *The future role of special schools and classes in Ireland.* NCSE

National Council for Special Education. (2011). *NCSE policy advice paper: The education of deaf and hard of hearing children in Ireland.* NCSE

National Council for Special Education. (2012). *The education of children with challenging behaviour arising from severe emotional disturbance/behavioural disorders.* NCSE

National Council for Special Education. (2013). *Supporting children with special educational needs in schools.* NCSE

National Council for Special Education. (2015*). Supporting children with autism spectrum disorder in schools.* NCSE

National Council for Special Education. (2018). *Comprehensive review of the special needs assistants scheme.* NCSE

Neville, A. (2012). Dyscalculia/Specific learning difficulty in mathematics: Identification and intervention in Irish primary schools. *REACH Journal of Special Needs Education in Ireland, 26*(1), 315.

Westwood, P. (2000). *Numeracy and learning difficulties: Approaches to teaching and assessment.* ACER Press.

Rose, R., Shevlin, M., Winter, E., & O'Raw, P. (2010). Special and inclusive education in the Republic of Ireland: Reviewing the literature from 2000 to 2009. *European Journal of Special Needs Education,* 25(4), 359-373. https://doi.org/10.1080/08856257.2010.513540

Shevlin,M. & Rose,R.(2003). *Encouraging voices: respecting the insights of young people who have been marginalised.* National Disability Authority.

Spencer, S. (2011). Universal design for learning: Assistance for teachers in today's inclusive classrooms. *Interdisciplinary Journal of Teaching and Learning, 1(1),10-22*.

Teaching Council (2011). *Policy on the continuum of teacher education.* Teaching Council of Ireland.

Ward, S., & Eden, C. (2009). *Key issues in education policy.* SAGE.

WHO. (2011). *World Report on Disability*. Retrieved from http;//www.who.int/disacilities/world_report/2011/report.pdf

Widodo, S. A., Prihatiningsih, A., & Taufiq, I. (2021). Single subject research: use of interactive video in children with developmental disabilities with dyscalculia to introduce natural numbers. *Participatory Educational Research (PER), 8*(2), 94-108.

Willis, S., & Mann, L. (2000). Differentiating instruction: Finding manageable ways to meet individual needs (excerpt). *Curriculum Update, ASCD*. Retrieved from http://www.ascd.org/readingroom/cupdate/2000/1win.html106

An Analysis of Effective Teacher Professional Development Models that Support Teachers in Delivering an Inclusive Primary Mathematics Pedagogy for All

Professional Development Service for Teachers (PDST)

Inclusion has significantly shaped the composition of educational settings at all levels in the Republic of Ireland since the early 2000s. While the definition of inclusive education is continually evolving, the PDST primary STEM team supports teachers to facilitate inclusive practices. Recent policies have resulted in allocating additional resources to schools, however, despite this level of investment, there are significant challenges as schools continue to engage with the process that is inclusion. This paper notes that there appears to be a gap between the theories underpinning inclusive practices for all learners and the actual practices realised in primary mathematics. Analysis examines how effective teacher professional development supports teachers to co-construct inclusive practices for all learners in primary mathematics. This professional development is aligned to the bespoke Sustained Support Model (PDST, 2017) and the role of Collaborative Professionalism (PDST, 2021) in facilitating the conditions and cultures necessary to develop teachers' reflective and professional skill and autonomy in delivering an inclusive primary mathematics pedagogy for all.

Introduction

The Professional Development Service for Teachers (PDST) is Ireland's largest teacher professional development service supporting teachers and school leaders in a range of pedagogical, curricular and educational areas. It is funded by the Teacher Education Section of the Department of Education (DE) and managed by Dublin West Education Centre. As a key priority of the DE, inclusion is integral to the work of individual teams across the organisation. This professional development is informed by the PDST's bespoke Sustained School Support (2017) and Collaborative Professionalism (2021) models. In particular, this paper will examine how effective professional development supports teachers in co-constructing inclusive environments for all learners in primary mathematics. The paper concludes citing key considerations arising from the current gap that exists between the conceptualisation of inclusion policy and its enactment in the primary mathematics classroom.

Inclusion

The definition of inclusive education is continually evolving. Traditionally inclusive education concerned pupils with Special Educational Needs (SEN), however this definition has since broadened. Spratt and Florian (2014, p. 90) argue that inclusive education now encompasses "all learners who may be excluded or marginalised by the processes of schooling." Brennan et al. (2019) support this perspective, arguing that inclusive pedagogy avoids the exclusion of any learner. The PDST Primary STEM team is tasked with supporting teachers in realising inclusive pedagogy for all pupils in primary mathematics, science, and the STEM disciplines.

Policies for Inclusion

Schools across Ireland are required to engage with the process of inclusion as outlined in the policies and circulars issued by the DE and their supporting documents from the National Council for Special Education (NCSE) and the National Educational Psychological Service (NEPS). The *Learning Support Guidelines* (2000) promoted the planned implementation of shared teaching approaches, involving the class teacher and the special education teacher (SET), in the pupil's classroom. These guidelines outlined the disadvantages of frequent and prolonged withdrawal of pupils with SEN from their classrooms. Although substantial progress has been made, Project IRIS (2015) revealed strategies for promoting differentiated teaching were limited in most schools, and teachers often reported inadequate knowledge of specific teaching approaches. The dominant use of withdrawal was identified as a limiting approach to providing effective support. The *Special Education Circular 0013/2017* and *The Guidelines for Primary Schools - Supporting Pupils with Special Educational Needs* (DES, 2017) sought to address this and changed the landscape of how schools allocate special education teaching resources. These Guidelines emphasised the importance of co-operative or team teaching as an inclusive pedagogical approach where appropriate, for the holistic development of all pupils. The PDST Primary STEM team is entrusted with realising this inclusive pedagogy at a macro level through engagement with key stakeholders during policy development, and at a micro level by supporting mainstream class teachers, Special Education Teachers (SETs), and school leaders in improving the learning outcomes and experiences of all pupils.

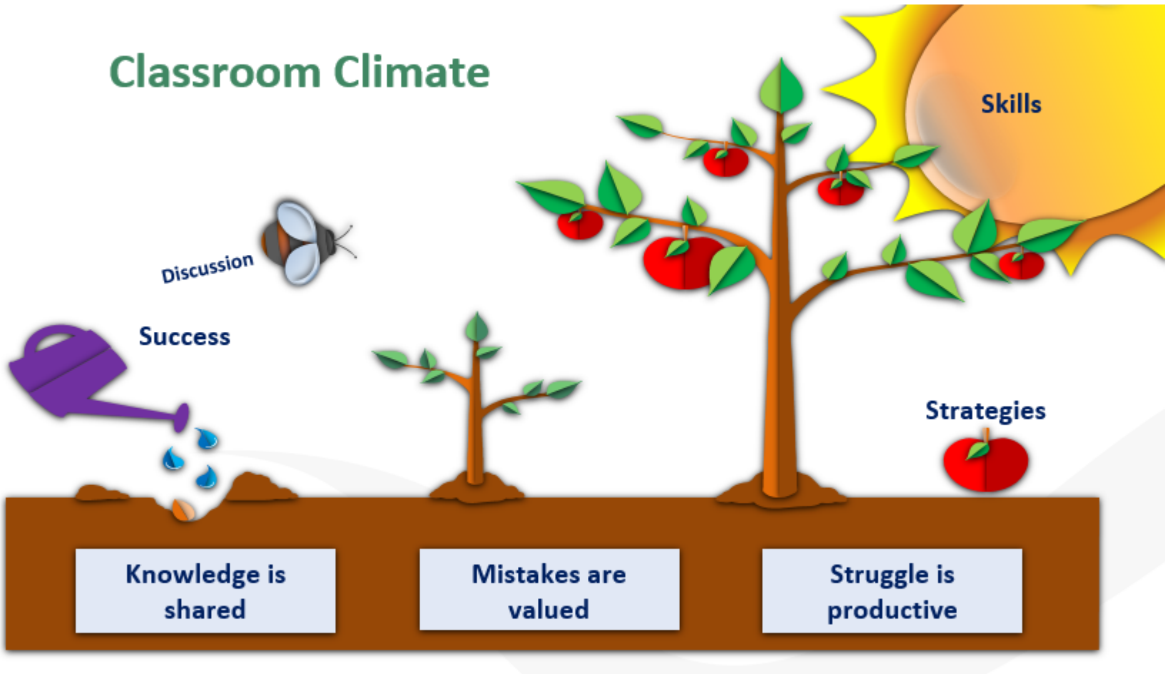
Inclusive Pedagogy

Rouse (2009) outlines three aspects involved in becoming an inclusive practitioner. They are 'knowing' (theory, policy and legislation), 'doing' (turning knowledge into action) and 'believing' (in their capacity to teach all children). Meaningful shift in pedagogical practice requires teachers to progress their understanding in all three of these domains. Professional development is central to supporting teachers in understanding and implementing inclusive pedagogy in primary mathematics. Brennan, King and Travers (2019) affirm this perspective when they assert that "teachers need to be effectively supported in developing their understanding of inclusive pedagogy in order to challenge hegemonic assumptions about difference and to develop inclusive practice." (p.4).

Enhancing teacher's belief in an inclusive environment may see a shift in emphasis from the more didactic teacher-led methods to more child-centred discovery, constructivist, or problem-solving and cooperative learning (Borko, et al., 2003). Teachers' ability to implement varying instructional strategies may be dependent on confidence in their self-efficacy to cater for diverse needs, as well as knowledge of their pupils' needs. We can assume, therefore, that "differentiated instruction is 'responsive' teaching rather than 'one-size-fits-all' teaching" (Tomlinson, 2003, p.151). This responsive teaching requires a child-centred approach where teachers have high expectations for all pupils, along with an in-depth knowledge of the curriculum and a pedagogical approach that is inclusive of all learners. Carefully chosen tasks in response to priority learning needs, where appropriate, enable all pupils to experience success, while also providing stretch opportunities for other pupils. The PDST Primary STEM team developed the graphic below to help teachers visualise the elements of an inclusive classroom climate for primary mathematics.

Figure 1

PDST Primary STEM Inclusive Classroom Climate (2017)



Effective Professional Development for Inclusive Pedagogy

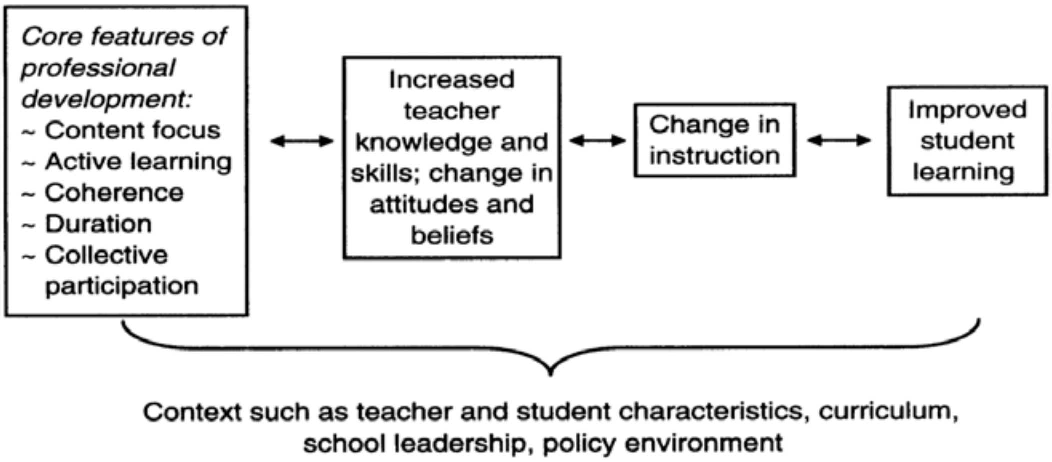
PDST Advisors support teachers and school leaders in developing inclusive practices through professional development models such as seminars, sustained school support and professional communities or collaboratives. These models encourage reflective practice through the school self-evaluation process. In response to Project IRIS (2015) and the SEN Guidelines (2017), the PDST Primary STEM team have facilitated seminars focused on team teaching in mathematics in Education Centres across the country. Advisors explore a range of models of team teaching which include lead and support, alternative teaching, parallel teaching, teaming and station teaching. These pedagogical models are intended to meet the targeted needs of pupils with SEN while improving outcomes, skills and experiences for all learners. In-class support models have a number of advantages, including the transfer of skills to the classroom teacher, increased collaborative planning and greater opportunities for pupils to keep pace with classroom work (Griffin and Shevlin, 2007). Arranging pupils in temporary mixed ability groups can lead to both improved student engagement and achievement compared to groups where pupils are tracked, streamed or grouped by ability (OECD, 2012). Team teaching reduces pupil teacher ratio and can enable pupils to focus on tasks that require them to rely on each other's skills, which tends to work equally well for all pupils (Slavin, 2010). Effective team teaching strives to provide pupils with SEN greater access to the wider curriculum and a positive classroom environment that maximises the learning experiences and outcomes of all pupils.

The aim of these stand-alone seminars was to deepen SETs', mainstream class teachers' and school leaders' understanding of policy developments and inclusive teaching and learning practices. However contemporary research is consistent in its call for professional development that is sustained and contextualised while promoting continuous enquiry and problem-solving embedded in the daily life of schools (PDST, 2021). Sustained and effective support moves beyond the idea of singular "CPD" events and instead facilitates change in schools by empowering and enabling teachers to identify and collaboratively address the needs of their school context (PDST, 2021). Teachers should therefore be encouraged to engage with sustained school support as outlined by PDST (2017), during which they can develop inclusive approaches such as Team Teaching and embed pedagogical change over a period of time. Effective and sustained professional development is central to the continuous and cyclical nature of planning for the inclusion process. Sustained school support empowers teachers to develop their competence and confidence in relation to the inclusion of all learners (Travers et al., 2010), through building trust in collaboration with colleagues. As the literature suggests, teacher collaboration is a central facet to inclusive education (Ainscow, 2014; Friend et al., 2010; Nevin, et al., 2009).

Desimone's (2009) core conceptual framework for studying the effect of professional development on teachers and students (Figure 2) illustrates the bi-directional effects of effective professional development on improved pupil learning through continuous reflection and collaboration. Collaborative professional development can take many forms. The PDST Primary STEM team enables collaborative professionalism through sustained school support and in particular their Connecting Classrooms series of online collaborative communities. These models of professional development support teachers in a sustained and contextualised manner allowing them to co-construct inclusive pedagogies for mathematics over an extended period with the support of a PDST Primary STEM advisor. Sustained school support and the Connecting Classrooms series encourages a more collaborative approach to professional development compared to isolated individual school visits and stand-alone online events (PDST, 2021).

Figure 2

Proposed core conceptual framework for studying effects of professional development on teachers and students, Desimone (2009).



Key Considerations to Realise Inclusive Practices for All Learners in Primary Mathematics

There are a number of key considerations and challenges with regard to developing inclusive practices for all learners in primary mathematics. These exist both at a macro (systemic) and a micro (school/classroom) level.

At a macro level, policy provides structure and standards supported by research and theory within the particular field, in this case, inclusion for all pupils in the mathematics classroom. It is therefore essential that policy conceptualisation happens in conjunction with those best placed to inform and deliver the professional development models needed for its enactment. A unified development of policy alongside planned and sustained professional development benefits all stakeholders, and ultimately enhances the inclusion practices in primary school mathematics. Our ongoing consultation and engagement with NCCA and NCSE enables all stakeholders to establish a shared vision for the new Primary Mathematics curriculum, leading to better inclusion for all pupils in primary mathematics. This vision should be mindful of the time needed for change to become embedded in practice and how this change is impacted by other policies and curriculum development at primary level.

A significant challenge for professional development services is supporting teachers in understanding the complexity of implementing change(King, 2014), and to employ effective pedagogies for teacher learning that develop the knowledge, beliefs and practices to support inclusive pedagogy (Florian, 2008). Developing this collaborative culture takes time and conscious effort from all parties involved; teachers and, perhaps most importantly, school management (Hipp and Huffman, 2007).

At a micro level, sustained school support is most effective when teachers are empowered by leadership to spend time engaging with advisors and the sustained support process in a meaningful way. In an effective inclusive school, school leaders work diligently at leading this sustained support process and prioritize supporting professional development which responds to needs of teachers (Philpott et al, 2010*).* It is therefore incumbent upon professional development services to support school leaders in fostering a culture of collaboration between teachers, encouraging the sharing of practice, ideas and approaches, and thus empowering teachers to become more effective inclusive practitioners.

Conclusion

Since the turn of the millennium, inclusion has held a prominent position in policy and practice guidelines. Despite the consistency of this messaging, a gap remains between inclusive policy and practice, between vision and reality, between knowing and doing. Through professional development, the PDST endeavours to bridge the gap between the policies on inclusion and the everyday reality for teachers and pupils in classrooms.

In this paper, the sustained school support and collaborative models are recommended as an approach to address this gap. Collaboration and consultation with key stakeholders at the policy writing stage, is needed to enact a supportive and transformative plan for sustained school support. This plan should afford school leaders and teachers the time and support needed to work collaboratively, build confidence and competence to enhance their inclusive practice in primary mathematics.

References

Ainscow, M. (2014). From special education to effective schools for all: Widening the agenda. In L. Florian (Ed.), *The Sage handbook of special education* (pp.172–185). Sage.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389.

Brennan., A. (2017). *Exploring the impact of a professional learning community on teacher professional learning for inclusive practice.* Unpublished doctoral thesis. DCU. http://doras.dcu.ie/21956/

Brennan, A., King, F., & Travers, J. (2019). Supporting the enactment of inclusive pedagogy in a primary school. *International Journal of Inclusive Education*. https://doi.org/10.1080/13603116.2019.1625452

Department of Education and Skills. (DES). (2017a). *Circular to the management authorities of all mainstream primary schools special education teaching allocation*, Circular 0013/2017, Department of Education and Skills, Ireland, Retrieved from https://www.education.ie/en/Circulars-and-Forms/Active-Circulars/cl0013_2017.pdf

Department of Education and Skills. (DES). (2017b). *Guidelines for primary schools supporting pupils with special educational needs in mainstream schools*. Department of Education and Skills. Retrieved from https://www.education.ie/en/The-Education-System/Special-Education/Guidelines-for-Primary-Schools-Supporting-Pupils-with-Special-Educational-Needs-in-Mainstream-Schools.pdf

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*, 181-200.

Florian, L. (2008). Inclusion: Special or inclusive education: Future trends. *British Journal of Special Education*, *35*(4), 202–208. https://doi.org/10.3102/0013189X08331140

Florian, L. (2014). What counts as evidence of inclusive education? European Journal of Special Needs Education, 29(3), 286–294. https://www.tandfonline.com/doi/full/10.1080/08856257.2014.933551

Government of Ireland. (2000). *Learning-support guidelines.* Stationery Office.

Government of Ireland. (2004). *Education for persons with special educational needs act* (EPSEN). Stationery Office.

Griffin, S., & Shevlin, M. (2007). *Responding to special educational needs: An Irish perspective.* Gill and MacMillan.

Griffiths, M. (2001). *Action for social justice in education. Fairly different.* Open University Press.

Hipp, K. K., & Huffman, J.B. (2007). *Professional learning communities*. Open University Press.

King, F. (2014). Evaluating the impact of teacher professional development: An evidence- based framework. *Professional Development in Education, 40*(1), 89-111. https://doi.org/10.1080/19415257.2013.823099

OECD. (2012). *Equity and quality in education: Supporting disadvantaged students and schools.* OECD Publishing.

Philpott, D. E., Furey, E., & Penney, S. C. (2010). Promoting leadership in the ongoing professional development of teachers: Responding to globalization and inclusion. *Exceptionality education international*, *20*(2).

Professional Development Service for Teachers. (2017). *Sustained support.* PDST.

Professional Development Service for Teachers. (2021). *Civitas parium*. PDST. (publication pending)

Rose, R., Shevlin, M., Winter, E., & O'Raw, P. (2015). *Project IRIS–inclusive research in Irish schools. A longitudinal study of the experiences of and outcomes for pupils with special educational needs (SEN) in Irish schools.* National Council for Special Education (NCSE).

Rouse, M. (2009). Developing inclusive practice: a role for teachers and teacher education. *Education in the North, 16*(1), 6-13.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Education Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Shevlin, M., Kenny, M., & Loxley, A. (2008). A time of transition: Exploring special educational provision in the republic of Ireland. *Journal of Research in Special Educational Needs, 8*(3), 141-152.

Shevlin, M., Kearns, H., Ranaghan, M., Twomey, M., Smith, R., & Winter, E. (2009). *Creating inclusive learning environments in Irish schools: Teacher perspectives.* National Council for Special Education (NCSE).

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*,4-14.

Slavin, R. (2010). 'Co-operative learning: what makes group-work work?' In H. Dumont, D. Istance & F. Benavides (eds.), *The nature of learning: Using research to inspire practice* (pp. 161-178). OECD.

Travers, J., Balfe, T., Butler, C., Day, T., McDaid, R., O'Donnell, M., & Prunty, A. (2010). *Addressing the challenges and barriers to inclusion in Irish schools: Report to Research and Development Committee of the Department of Education and Skills.* St Patrick's College.

"To Develop the Whole Child, We Must Develop the Mathematical Child" (Clements & Sarama, 2014, p.2)

In recent years, interest in early childhood mathematics has grown both nationally and internationally. Within the Irish context, policy documents highlight the important role played by early childhood educators in laying the foundations of mathematics education (e.g. DES, 2011). However, little is known about mathematics in Irish preschool classrooms and the ways in which mathematics is fostered in children's earliest years (Dunphy, 2018). The papers presented here aim to contribute to this developing area of Irish research.

In our first paper, Sandra O'Neill (DCU), reviews DES inspection reports in relation to mathematics in preschool settings. Findings from this review suggest that while reference to numeracy appears in the vast majority of reports, advice to improve practice is less frequent. When offered, advice often advocates for the discontinuation of practice deemed to be inappropriate rather than suggesting actions that could enhance teaching and learning in relation to mathematics

Our second paper by Córa Gillic (DCU), reports on findings from a study which explored the beliefs and practices of Irish early childhood educators with respect to mathematical activity in early childhood settings. Findings show that while educators do note children's mathematical activity in play, they do not always respond, and when they do, it is not always in a mathematical way.

The availability of professional development supports in the field of early childhood mathematics is lacking within the Irish preschool education context. However, our final paper, presented by Claire O'Buachain (NCIRL), reports on the findings of a collaborative initiative developed to support parents, pre-school educators and primary school teachers in supporting young children's engagement with mathematics. Findings show that during the programme both parental and child involvement in mathematical activity was high and that mathematics was perceived as an enjoyable endeavour by all participants. A key finding is that cross-sectoral collaboration is a benefit to young children's mathematical development.

References

Clements, D.H. & Sarama, J. (2014). *Learning and teaching mathematics The learning trajectories Approach.* Routledge.

Department of Education and Skills (2011). *Literacy and numeracy for learning and for life. The national strategy to improve literacy and numeracy among children and young people 2011-2020.* Department of Education and Skills.

Dunphy, E. (2018). Transition from preschool to primary school; Optimising opportunities for mathematics learning. *Education Matters, 1*(2018), 105-110

An Investigation into the Incidence of Numeracy Advice in Early Years Education-Focused Inspections in the Republic of Ireland

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Since 2015 Early Childhood Care and Education settings in the Republic of Ireland have been subject to Early Years Education- focused Inspections (EYEI). These inspections have been identified as a way of identifying good practice and improving outcomes for children in relation to numeracy. This study undertook a textual analysis of the most recent EYEI reports to explore how often numeracy is referred to, and what type of actions are advised to promote and support improvements in practice. The results show that while numeracy is referred to regularly within the reports, advice is less frequent. When offered, advice often advocates for the discontinuation of practice deemed inappropriate rather than suggesting actions that could enhance teaching and learning in relation to mathematics.

Introduction

Children possess an innate capacity for mathematics (National Research Council [U.S.], 2009) and demonstrate an interest in math in their play, daily routines and interactions. Early mathematics skills and knowledge have been found to be the greatest predictor of later academic achievement regardless of the sex or socio-economic background of the child (Duncan et al., 2007) and are stronger predictor of later school success than tests of intelligence or memory (Krajewski & Schneider, 2009). Consequently, numeracy has emerged as a policy focus in early childhood care and education (ECCE). In the Republic of Ireland (RoI), Early Years Education-focused Inspections (EYEI) carried out by the Department of Education and Skills (DES) began in 2015. Early years settings providing the State-funded ECCE scheme for children from 2 years 8 months until they enrol in primary school, are subject to these inspections. This study is concerned with the regularity with which numeracy is commented on in EYEI reports, and the type and depth advice that is given to improve practice. Research questions posed include: How often do inspection reports refer to numeracy? How often is advice issued? How detailed is the advice? and finally, what level of uniformity exists in advice and comments?

The Emergence of Early Childhood Mathematics Policy

Since 2017 a number of policy developments have led to a focus on early childhood mathematics (ECM) in RoI. The interim review of the national literacy and numeracy strategy has laid out updated targets for ECCE, claiming that a stronger focus on numeracy is warranted and the re-invigoration of numeracy through everyday practice in ECCE settings is required (DES, 2017a). A key priority action is to 'support practitioners in ECCE settings… to gain a deeper understanding of numeracy concepts, the sequence in which children learn early mathematical ideas and identifying and providing materials and activities which further promote learning in this area' (DES, 2017a, p. 21). The strategy identifies the EYEI as a mechanism to support and promote improvements in the ECCE sector, and targets the upskilling of ECCE educators through multiple means including; the Better Start National Quality Development Service; the Aistear Síolta Practice Guide (www.aistearsiolta.ie) and EYEIs. Specifically, the strategy states that the 'Inspection team evaluates current numeracy provision in early years settings…[and] provides guidance and advice to practitioners to ensure that children have daily exposure to key early numeracy concepts, experiences and materials' (DES, 2017a, p.28). In addition, since 2017 a focus on STEM policy (DES, 2017b; DES, 2017c; DES, 2020) has brought further attention to ECM. The EYEI tool was updated in 2018 and now includes explicit criteria related to numeracy, math thinking and learning (DES, 2018). Settings are now required to provide numeracy opportunities and are inspected accordingly.

The Role of the Educator

ECCE educators in Ireland must hold a minimum level 5 qualification[1]. Numeracy is often included at level 5 and 6, and in degree programmes. However, anecdotal evidence suggests that attention to ECM in initial education is rudimentary. Within the social pedagogy tradition of ECCE 'discovery maths' is the preferable approach. This is where maths experiences are informal in nature, embedded in children's free play and where learning occurs incidentally rather than intentionally (Thiel & Perry, 2018). Many educators, including those lecturing in higher education, are uncomfortable with the concept of teaching as traditionally, their role is as that of a play partner. This impacts how and what ECCE educators are taught in their initial education. While graduates report greater confidence, educators holding all levels of qualifications in Ireland state that their initial training is not preparing them to support children's numeracy skills (DES, 2016). This is unsurprising, as teaching mathematics in pre-school is a complex task. Math-related pedagogical content knowledge (PCK) in preschool involves a number of components. ECCE educators must be able to notice mathematical situations in which children engage, interpret the nature of the math activity and have the knowledge to enhance children's mathematical thinking and understanding (Lee, 2017). These complex skills cannot be gained through a focus on discovery math in initial education. There is, therefore, considerable scope for developments in thinking about the numeracy preparation of ECCE educators (Dunphy, 2018).

Method

To explore how the inspection process affirms good practice and provides advice on the development or improvement of numeracy, a textual analysis of the 200 most recent EYEI inspection reports was carried out in early 2021. The reports are publically available via DES' website (www.education.ie) and relate to inspection visits that occurred between 22nd November 2019 and 28th February 2020. EYEIs are based on a quality framework against which inspections are conducted (DES, 2018). The framework includes four broad areas: 1) Quality of the context to support children's learning and development; 2) Quality of the

[1] See https://www.qqi.ie/Articles/Pages/National-Framework-of-Qualifications-(NFQ).aspx for details

processes to support children's learning and development; 3) Quality of children's learning experiences and achievements; 4) Quality of management and leadership for learning. Inspection reports are structured using these headings, broken down further into 20 outcomes and 'signposts for practice'. Table 1 identifies the outcomes and the signposts for practice that relate to numeracy. Area 1 and 4 are excluded as they do not contain explicit outcomes or signposts in relation to this topic. Each report was analysed in the following stages; 1) Review of Area 2 of to identify reference to numeracy; 2) Review of Area 3 of to identify reference to numeracy; 3) Automated search of the full report for key terms; numeracy, math*, shape, space, measure, count, pattern, number; 4) a final scan of each report searching for reference to numeracy. Bullet points that referenced numeracy, math or key terms were copied and stored for further analysis, noting the area in the report from which this information was drawn. A secondary thematic analysis was carried out on this data to uncover uniformity within comments and advice (using Braun & Clarke, 2006).

Table 1

Numeracy –related outcomes and signpost for practice from the EYEI Framework

Area	Outcome	Signpost for Practice
2 - Quality of processes to support children's learning and development	Outcome 9 – Emergent language, literacy and numeracy skills are fostered	Practitioners model appropriate language, including mathematical language, and encourage an expanded use of vocabulary through the use of open-ended questioning and language enrichment during interactions
		Mathematical thinking and learning is promoted through the use of open-ended resources and games, linked to the everyday lives of children
		It is evident that children have opportunities to engage with activities that build early positive dispositions towards science, technology, engineering, the arts and mathematics
3 – Quality of children's learning experiences and achievements	Outcome 15 - Children	Children explore sound, pattern, rhythm and repetition in language
	Communicate their experiences, thoughts, ideas and feelings with others in a variety of ways	Children demonstrate an awareness and emergent understanding of the meaning and uses of symbols, pictures, print and numbers as a means of communication
		Children have a growing understanding of the meaning and use of mathematical language

Note. Area 1 and 4 are excluded as neither contain specific outcomes or signposts relating to numeracy

Findings

80% of the sample reports include reference to numeracy. As expected, comments appear most regularly in in area 2 and area 3, with two examples in area 4. Often the comments have significant uniformity and use similar language. For example, reference to literacy and numeracy are often grouped together in comments

> The children's early literacy and numeracy skills are well supported through the use of songs, rhymes and the range of play materials (area 2, report 2)

> The children's language, literacy and numeracy skills are fostered well through the use of songs, rhymes and action games (area 2, report 72)

46% of the sample were found to provide specific examples from practice rather than the standardised comments listed above. Examples identified both positive and negative practice

> The practitioners use effective strategies to support the development of the children's early numeracy skills. For example, during inspection two children were supported to guess how many blocks it would take to create an arch and then they were supported to count the blocks upon completion, testing their hypothesis (report 164)

Montessori resources were regularly referred to as a way of supporting numeracy, and appeared in approximately 11% of sample reports. It is difficult to decipher in some examples what exactly the named materials are supporting; language, literacy or numeracy

> Pre-literacy and pre-numeracy skills as well as emergent language skills are fostered very effectively through the use of the Montessori materials (area 2, report 41)

> Montessori mathematical materials promote high-quality mathematical thinking and learning (area 2, report 115)

Actions for improvement were found in 22% of reports. A number of themes emerged when this subset was analysed. Firstly, almost half of the actions advised (44%) related to discontinuing formal teaching practices. Again, literacy and numeracy are grouped in these comments

> Formal teaching of literacy and numeracy concepts needs to be replaced by alternative approaches. (report 116)

> The practitioners are advised to discontinue the use of worksheets to promote children's literacy and numeracy skills. (report 160)

In a small number of instances, advice is phrased more positively suggesting actions that can improve practice

> Numeracy skills are fostered naturally in play. Practitioners focus on concepts such as height when the children are playing with bricks and volume when they are playing with water and sand (report 45)

A second theme that emerged from this subset was the guidance to add further materials that could support numeracy (34% of reports)

> The addition of a range of resources such as weighing scales, cookery books and measuring tapes would support the development of early numeracy skills through playful strategies. (report 200)

Overall, there were very few references to mathematical content areas in the reports. No reference was found to pattern or space in a mathematical context. Reference to shape appeared in 12% of reports; capacity appeared in 1%; and sorting, matching and classifying appeared in 1%. Finally, it should be noted that 1% of the sample could not be analysed as the incorrect report was uploaded to education.ie.

Discussion and Conclusion

Reference to numeracy appears in 80% of the reports sampled. However, the vague uniform comments, coupling of literacy and numeracy, and negative framing of some comments does little to provide guidance to ensure children's 'daily exposure to key early numeracy concepts, experiences and materials' (DES, 2017a, p.28). Specific examples from practice that identify everyday math are more powerful and demonstrative when educators are unsure about ECM. The comment 'during water play practitioners could discuss concepts such as weight, temperature, sinking and floating, during their engagement with children. This will support meaningful opportunities for children to engage with experiences that build positive dispositions towards mathematical understanding and skills. (report 27)'is a better illustration of how practice can be improved than 'practitioners are encouraged to increase their use of mathematical vocabulary during their interactions' (report 118). The Literacy and Numeracy Strategy Interim report states that maths concepts such as 'number words and symbols, shapes, counting, patterns, spatial awareness, measurement and data analysis' (DES, 2017, p.28) should be supported, but there was very little reference to these concepts in the reports. It is unclear whether this indicates that practice related to these concepts aren't being demonstrated or simply not being captured. However, with many ECCE educators stating that they are not prepared to support numeracy, more explicit advice is warranted in all reports.

Most early years inspectors hold an ECCE qualification within the social pedagogy tradition that favours discovery math. The concept of teacher as a play partner places a higher value on the learning environment as a 'third teacher' and could explain why over 33% of advice relates to the addition of further numeracy materials. However, this advice will do little to support educators to understand how and why materials are used to improve numeracy practice and outcomes. The relatively high number of references to Montessori materials could be explained in a number of ways; items such as the pink tower or number rods are immediately identifiable in the environment; inspectors are Montessori trained; or a further indication of the focus on materials rather than pedagogy or educators' PCK. Research suggests that ECCE settings in RoI are increasingly using formal approaches to support the development of academic skills (Ring et al., 2016). This study provides further evidence that this is the case. A significant proportion of the advice included in the reports dictate that formal approaches to numeracy are discontinued. It is evident that ECCE educators are trying to support numeracy in the best way they know how, but many educators require more explicit supports. The DES (2017a) also identified the Aistear Síolta Practice Guide (ASPG) as a support to upskill educators as it includes resources and self-study guides on mathematics. Reference to ASPG appears in relation to environment, assessment and transition but not to numeracy. It could be argued that neglecting to advise settings to access and use these materials is a further missed opportunity to help educators upskill.

Finally, changes in complex domains such as ECM rarely happen without inducement (Newton & Alexander, 2013). The inspectorate is in a privileged position to influence numeracy practice in ECCE settings. More explicit examples, with clear reference to numeracy *independent of literacy* will help identify good practice. A focus on pedagogy and PCK will lead to a more effective inspection process and improve outcomes for children.

References

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101.

DES (2016). *Survey of early years practitioners.* DES.

DES (2017a). *National strategy: Literacy and numeracy for learning and life interim review: 2011-2016 New targets: 2017-2020*. DES.

DES (2017b). *STEM education policy statement 2017-2026*. DES.

DES (2017c). *STEM education implementation plan 2017-2019*. DES

DES (2018). *A guide to early years education inspection (EYEI)*. DES.

Duncan, G., Dowsett, C. J., Claessens, A., Magnuson, K., …Duckworth, K. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446

Dunphy, E. (2018). Transition from preschool to primary school. Optimising opportunities for mathematics learning. *Education Matters, 2017-2018, 105-110.*

Krajewski, K., & Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties. *Learning and Instruction, 19*(6), 513–526.

Lee, J. E. (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood, 49*(4), 229–243

National Research Council [NRC] (2009). *Mathematics learning in early childhood: paths toward excellence and equity.* The National Academies Press.

Ring, E., Mhic Mhathúna, M., Moloney, M., …Ozonyia, M. (2016). *An examination of concepts of school readiness among parents and educators in Ireland.* Department of Children and Youth Affairs.

Thiel, O. & Perry, B. (2018). Innovative approaches in early childhood mathematics. *Early European Childhood Education Research Journal, 26*(4), 463-468.

Newton, K. & Alexander P. (2013). Early Mathematics Learning in Perspective. In L. English, & J. Mulligan (Eds.), *Reconceptualising early mathematics learning* (pp.5-28). Springer

Responding to Children's (Mathematical) Thinking in Preschool

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With an increasing focus on STEM (DES, 2017) and mathematics education (DES, 2011) in Irish preschool education contexts, this paper reports on the findings of a qualitative study which explored the beliefs and self-reported practices of eight early childhood educators in relation to mathematics in preschool settings. This paper focuses in particular on the answers of participants to questions focusing on educator 'noticing': educator recollections of observations of mathematical concepts in children's free play and their responses to them. Findings show that while participants could recall mathematical aspects of children's free play or interests, these were not always responded to, or if responded to were responded to in a non-mathematical way. These findings align to those in international research and contribute to the gap in the research context in relation to mathematics in Irish preschool settings.

Introduction

Internationally, recognition of the importance of early childhood education is growing (UNICEF, 2017). This combined with a policy focus on STEM within Irish early childhood education settings (Department of Education and Skills [DES], 2017, 2018a) has led to early childhood STEM practices coming under the spotlight. From a rights-based perspective, young children have a right to mathematics education (Cohrssen & Page, 2016) and to have their mathematical explorations or interests responded to (Dockett & Goff, 2013). However, in order to respond, one first has to notice. Noticing young children's mathematical explorations recognises not only the mathematical value of the activity, but also children's competencies in the area of mathematics (Dockett & Goff, 2013). This paper reports on eight early childhood educators' recognition of and responses to observed mathematical concepts in preschool children's free play.

Theoretical Framework

Play is considered a key context for young children's learning that facilitates children's mathematical explorations as well as providing children with opportunities to problem-solve (Dockett & Gough, 2013; Dooley et al., 2014). Dockett & Goff (2013) contend that while play is a key vehicle for mathematical exploration, children's engagement with the mathematics can only be valued and enhanced when it is supported by adults in the setting. However, in order to effectively support children's mathematical explorations and thinking, adults need to have a knowledge of the big ideas of early childhood mathematics (Dooley et al., 2014), recognise the mathematics in children's play (Dockett & Gough, 2013: Opperman et al, 2016) and be aware of pedagogical strategies to further develop young children's emerging mathematical ideas (Dooley et al., 2014). Strategies to support young children's mathematical explorations include mathematical talk and discussion, developing productive

Professional Noticing

According to Ball (2011) “to notice is to observe, realise, or attend to” (p. xx). Noticing effectively is both a complex and challenging professional skill (Jacobs et al., 2010). Dunekacke, et al. (2016) note that the skill of perceiving mathematical situations within the complex, informal environment of a preschool classroom is more challenging, given the emergent nature of the mathematics through child-led play.

Within the context of a play-based approach to early childhood mathematics, Lee (2017) applies this three-part framework to professional noticing of children’s mathematical knowledge and explorations: “noticing mathematical situations in children’s play, interpreting these mathematical episodes and enhancing the mathematical thinking therein” (p. 253).

Dockett & Gough (2013) note that observation is a common tool to document children’s activity in early childhood settings. However, children’s mathematical activity is rarely purposively observed or documented in early childhood practice (Anthony et al., 2015). McCray & Chen (2012) posit that there is a strong link between a preschool teacher’s capability to identify mathematics in play and the quality of mathematical learning in preschool classrooms. Oppermann et al. (2016) contend that being able to recognise and respond to mathematics in children’s play is dependent on the mathematical subject knowledge of the educator. However, mathematics is not often a feature of early childhood training programmes. Lee & Ginsburg (2009) suggest that the traditional focus on other aspects of child development (e.g. social, emotional and language development) in pre-service training courses impacts on educator noticing of mathematical activity. Jacobs et al. (2010) propose that a lack of training in learning to notice aspects of mathematics in children’s activity in pre-service training contributes to teacher inability to adequately notice children’s mathematical thinking.

Methods

Qualitative semi-structured interviews were employed in this study as they provide a flexible method of gaining insight into participants’ views on a topic and allow participants to expand on issues that are significant and meaningful to them (Bryman, 2016). Semi-structured interviews were chosen over questionnaire survey data collection methods, as it was thought that questionnaires would provide insufficient detail about beliefs and practices or reveal only limited aspects of participants’ thinking (Walliman, 2014) about mathematical activity in preschool.

Research Sample

The data reported in this paper is drawn from a study which explored the beliefs and self-reported practices of eight Irish preschool educators in relation to mathematics. The eight participants varied in level of qualification from those completing a level 6 qualification in Montessori teaching to those with an honours Bachelor degree in Early Childhood Care and Education. Length of experience varied from new entrant (student) to twenty-five years.

Pseudonyms were applied and ethical procedures were followed in accordance with ethical research guidelines issued by NUI Galway.

Findings

This paper focuses on the analysis of the answers of participants to the questions: can you recall some play scenarios in which you observed children's developing mathematical concepts and how did you interact with the child(ren)? Interview transcripts were analysed against Lee's (2017) three constructs of preschool educator professional noticing for mathematics: Noticing, Interpreting and Enhancing.

Louise recalled for 'Show and Tell' a child brought in a measuring tape and "measured everyone in the room…seeing who was the tallest and smallest…that was lovely…we weren't aware it had any sort of mathematical links to it…it was about understanding that everybody's different…we focussed more on accepting everyone for who they are, rather on different heights". Defending her interpretation, Louise stated, "I'm more socially minded so I focus on your feelings and your emotions and your perception of other people".

While Louise did not enhance it mathematically at the time, during the interview, she offered ways in which she could have developed the activity in a mathematical way. "I might have had height charts or something and matched the heights…gotten the children to measure themselves".

Louise also recollected an observation of children building houses with blocks, "they were talking about the shape of the windows and the height of the chimney…one was saying 'my chimney's very tall and your chimney's very small". Louise acknowledged that she didn't recognise the mathematics in that play episode, stating "I was putting it down as them communicating and exploring the world around them…I didn't see or think about the mathematical underpinning of it…until now".

Eileen remembered a play scenario where two children playing with a set of connecting blocks. "They said they were going to see how far it was to the moon and were using their 'rope' to measure the distance". Eileen added, "they were thinking about measuring, but in a way that meant something to them…I wish I had a book on Space to extend it, we could have looked at planets". Eileen had acknowledged that the children were engaged in measuring but interpreted the children's interest as being in 'Space' and not measuring, despite the children having, "repeated the activity a few times". Space would be the focus of enhancement.

Ann noticed two children hanging clothes on a washing line, "the conversation was about the amount of pegs and if there was going to be enough to hang all the clothes…there wasn't and they went back along the line and shared the pegs between the clothes so there would be enough". Ann interpreted the scenario as the children being involved in 'estimating, will there be enough pegs and I suppose sharing is dividing?' Ann also stated, "they were definitely problem-solving". However, no enhancement activity took place as Ann decided that as the children had, "worked it out by themselves", no support or enhancement was necessary on her part.

Other participants also took the view that if children were playing mathematically, there was no need for adult enhancement. Carol, noted that her role was that of facilitator of a rich environment, where children were "let to learn it themselves". This sentiment was echoed by two Montessori participants, who valued the mathematical aspects of the Montessori sensorial materials and children's self-discovery of these.

Aisling, recalled an observation of children making a car park with construction materials, where some boys were involved in "measuring space for cars and comparing the size of spaces and toy cars, then some girls joined in and the car park became a house with bedrooms…lovely teamwork". When asked how she responded to this observation, Aisling said "there was no need, they were so happy and playing well together".

Discussion

Participants could recall episodes of children's play where mathematical concepts were a feature. This suggests that, at some level, educators are noticing mathematical language or interests in children's play. However, in the case of Louise, noticing was done retrospectively during the interview process. She also considered ways to develop the mathematical thinking during the interview. Perhaps this indicates that mentor support can help educators to reflect on their observations of play episodes in a mathematical way? At present, such support is facilitated by the DES early years inspectorate (DES, 2017). While this support is welcome, few inspectors are currently employed in this role (DES, 2018b) so it may be some time before early years educators can avail of this much needed support.

While this is a small-scale study, it seems likely that some educators are still predominantly biased towards a focus on developing children's social and emotional development. This is evident in Louise's response where she clearly states that, "I'm more socially minded so I focus on your feelings and your emotions and your perception of other people". This outlook is well-documented in the research literature (e.g. Lee & Ginsburg, 2009) and is often attributed to the assumption that mathematics in preschool is not appropriate (Lee & Ginsburg, 2009) or indeed valued (Hachey, 2013).

The finding that educators do not respond to mathematics observed in children's play is problematic. Young children are not having their mathematical explorations/interests validated. This goes against the rights based perspective argued by Cohrssen & Page (2016) who express the 'ethical obligation' of educators to prepare children for life by supporting their mathematical development (p. 104). How can educators be meeting this obligation when they are not responding to the mathematics observed? Perhaps it is a question of educators not recognising this right, or perhaps, there is a tension between the importance of mathematics and the goals for social development (Lee & Ginsburg, 2009)?

This paper suggests that there is also a need to argue against the role of the educator solely as a facilitator of a mathematically rich environment, (e.g. Carol and Montessori participants) where children's mathematical ideas are self-discovered and supported by their interactions with objects in the environment. Educators need to understand the importance of their role in supporting mathematical development through interactions, through strategies such as mathematical talk and discussion, development of productive disposition, mathematical modelling, providing cognitively challenging tasks (enhancing) and engaging in formative assessment (Dooley et al., 2014).

Dockett & Gough (2013) note that in order to notice and respond to children's mathematical explorations, educators need a solid understanding of mathematics. At present, modules in mathematics /STEM education are not mandatory for pre-service early childhood education in Ireland (DES, 2019) and currently there is no formal continuing professional development in the area. Of the eight participants, only two had attended an undergraduate degree module on literacy and numeracy for early years, however, even with this knowledge, Eileen chose to focus on enhancing children's interest in Space rather than on children's explorations of measure.

This paper contributes an Irish perspective to international literature on mathematics in preschool classrooms. Findings align with well- established conclusions on educator attitudes towards mathematics in preschool. Further studies are needed to explore why, even when armed with training in early years mathematics, Irish early childhood educators choose not to focus on enhancing the mathematics observed in children's play.

References

Anthony, G., McLachlan, C., & Lim Fock Poh, R. (2015). Narrative assessment: making mathematics learning visible in early childhood settings. *Mathematics Education Research Journal, 27,* 385-400.

Ball, D. (2011). Foreward in M. G. Sherin, V. R. Jacobs & R. A. Philipp (Eds.) *Mathematics teacher noticing Seeing through teachers' eyes.* (pp. xx-xxiv). Routledge.

Bryman, A. (2016). Social research methods (5th ed.). Oxford University Press.

Cohrssen, C. & Page, J. (2016). Articulating a rights-based argument for mathematics teaching in early childhood education. *Australasian Journal of Early Childhood, 41*(3), 104-108.

Department of Education and Skills (2017). *National strategy: literacy and numeracy for learning and life 2011-2020 Interim review: 2011-2016 New targets: 2017-2020.* DES.

Department of Education and Skills (2018a). *A guide to early years education inspection (EYEI)*. DES.

Department of Education and Skills (2018b). *A review of early years inspection education-focused inspection: April 2016 – June 2017 Insights and future developments.* DES.

Department of Education and Skills (2019). *Professional award criteria and guidelines for initial professional education (level 7 and level 8) degree programmes in early learning and care (ELC) sector in Ireland.* DES.

Department of Education and Skills (2020). *STEM Education 2020: Reporting on practice in early learning and care, primary and post-primary Contexts.* DES.

Dockett, S. & Goff, W. (2013). Noticing young children's mathematical strengths and agency. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australia).* Mathematics Education Research Group of Australasia.

Dooley, T., Dunphy, E., & Shiel, G. (2014). Mathematics in early childhood and primary education (3-8) years Teaching and learning. Research report No.18. National Council for Curriculum and Assessment.

Dunekacke, S., Jenβen, L., Eilerts, K. & Blömeke, S. (2016). Epistemological beliefs of prospective preschool teachers and their relation to knowledge, perception, and planning abilities in the field of mathematics: A process model. *ZDM, 48,* 125-137.

Hachey, A. (2013). The early childhood mathematics revolution. *Early Education and Development, 24*(4), 419-430.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking, *Journal for Research in Mathematics Education, 41*(2), 169-202.

Lee, J. (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood, 49,* 229-243.

Lee, J. & Ginsburg, H. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood, 34*(4), 37-45

McCray, J.S. & Chen, J. (2012). Pedagogical content knowledge for preschool mathematics: Construct validity of a new teacher interview, *Journal of Research in Childhood Education, 26*(3), 291-307.

Oppermann, E., Anders, Y., & Hachfeld, A. (2016). The influence of preschool teachers' content knowledge and mathematical ability beliefs on their sensitivity to mathematics in children's play. *Teaching and Teacher Education,* 58*,*174-184.

United Nations Children's Fund (2017). *Early moments matter for every child.* Retrieved: April 23, 2021 https://www.unicef.org/media/files/UNICEF_Early_Moments_Matter_for_Every_Child_report.pdf

Walliman (2014). Your undergraduate dissertation the essential guide for success (2nd ed.). Sage.

Area Based Childhood Early Numeracy Programme: Numeracy in Action

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The Early Learning Initiative, Area Based Childhood, early numeracy programme aims to improve the educational outcomes for children in developing early maths skills, while increasing parental involvement in children's learning and development. The working group and pre numeracy planning meetings provide a collaborative space for early years educators and infant class teachers to reflect upon teaching practice and plan for children's learning. This approach ensures continuity and progression in mathematical learning from home to early years settings and local schools through a playful approach. The purpose of this paper is to present the collaborative work between infant class teachers in primary schools and early childhood educators from the Dublin Dockland and North East Inner City of Dublin.

Introduction

The benefit of learning about maths in early childhood has been well documented through research in Ireland and internationally. Research conducted in Ballymun by O'Kane and Hayes (2010) confirmed this with the participants identifying developing mathematical concepts and the language of Maths, along with oral language, as being the most important skills for children to possess when starting primary school.

In 2005, the Early Learning Initiative commissioned the Dartington Social Research Unit to undertake a community survey to identify the early education needs of the children in the area. Its main finding was that while parents had high aspirations for their children, they did not understand their pivotal role in enhancing their own children's learning. This finding informed the planning and designing of our programmes. This programme is funded through the Area Based Childhood (ABC) programme.

Parental confidence in helping with maths homework was measured as part of the *2009 National Assessment of Mathematics and English Reading* (NA 2009) (Eivers et al., 2010). Comparison of its findings with the findings from our recent survey would suggest that parents in the Docklands felt less confident in helping their children than parents nationally. The fact that the NA (2009) found that pupil mean test scores differed significantly by parent confidence, highlights the need for this project. The report also found that parental ratings of their children's ability tended to be overly positive and indicated that many parents lacked a clear understanding of how their child was performing. It recommended that parents be advised about practices that help their child's general academic development as well as specific curricular areas. This is very much in line with the needs as identified by the *Dartington Survey* (2006).

Similarly, the PISA Report highlighted that 1 in 6 students in Ireland is poorly prepared for future mathematical needs as students and citizens. Low socioeconomic status (SES) students are more at risk of low achievement, particularly when they attend schools in which large numbers of students are also socioeconomically disadvantaged (Shiel et al., 2007). The recent *2009 National Assessment of Mathematics and English Reading* (Eivers et al., 2010) also linked low familial SES as an indicator of lower pupil achievement. Given the recent Higher Education Authority *Study on Progression in Higher Education* showing that mathematics is a key predictor of future economic performance and maths-based subjects the trigger for non-completion at third level (HEA 2010), this project is both timely and necessary.

Internationally, research findings highlight the importance of early numeracy as an indicator of future academic success. Examining data from six studies of close to 36,000 pre-schoolers in the United States, Canada and England, researchers found that, having controlled for IQ, family income, gender, temperament, type of previous educational experience, and whether children came from single or two parent families, the mastery of early mathematical concepts on school entry predicted not only future math achievement, it also predicts future reading achievement (Northwestern University, 2007). Interestingly, the opposite -- reading skills predicting mathematical success – was not significant.

Research in the US (National Academy of Sciences 2009) indicated that opportunities for pre-schoolers to learn mathematics were currently inadequate, particularly for those in low income groups. It found that as mathematical learning was often embedded in other activities and secondary to other learning goals, it was not effective. Giving parents, care givers, early years practitioners and teachers, it argued, the tools to develop and build on children's interests and provide children with high quality Mathematical interactions was the 'foundation for future learning and would help address long-term systematic inequities in educational outcomes'.

The *Draft National Plan to Improve Literacy and Numeracy in Schools* (Department of Education and Skills [DES], 2010b) is in agreement with our consortium in thinking that the teaching and learning of mathematics in Ireland requires even greater attention than literacy. With surveys of mathematics achievement at the primary level, and patterns of participation and achievement in the State examinations and in international surveys, indicating that there are systemic issues in mathematics education that required attention, it argued that system-wide measures are needed to improve the way students engage with mathematics and develop numeracy skills. The numeracy programme provides a prototype of how local communities can promote successful learning, teaching, and assessment in numeracy.

Our consortium has been working together since 2006, with the community action research approach is approved by the National College of Ireland Ethics Committee. Beginning in 2011 with funding from the National Early Years Access Initiative (NEYAI), this programme is aimed at improving early year's numeracy and mathematical skills from birth to six years of age. With funding from the ABC Programme, this programme has grown from 16 organisations and 498 children in 2011/12 to 38 organisations and 1,265 children in 2019/20. The programme revolves around three community Early Numeracy Weeks. There is a different theme each term. The numeracy themes are, Positional Directional Language, Shape and Space, Counting, Symbols of the Environment, Time, Measurement and Capacity, Money, Number, Sequence and Pattern.

Working group meetings, pre numeracy planning workshops and onsite mentoring support educators in early years services and infant classes in primary schools to reflect on and improve the quality of the programme and their practice using the *Aistear Síolta Practice Guide* as a resource. These meetings have been hosted using virtual online platforms since April 2020 due to the Covid 19 worldwide pandemic.

Numeracy Programme and National Policy

There are two main elements of the numeracy programme which correlate with '*First 5, A Whole-of-Government Strategy for Babies, Young Children and their Families* (*2019-2028'*). Action area 2- relates to a new model of parenting support which provides guidance to parents to promote healthy behaviours, facilitating positive play-based early learning. The numeracy home-activity cards support capacity building within the home learning environment, supporting parents to realise their potential as their child's primary educator. This is achieved by providing parents with guidance on key vocabulary words to use when describing their child's actions during play. This practice promotes a positive play based home learning environment as outlined in Goal three of the strategy.

Building Block 3, refers to a skilled and sustainable workforce, the numeracy programme provides early years educators with the opportunity to share their experiences and reflect upon their learning in the working group meeting and the pre-numeracy workshops. This peer learning provides the opportunity to build capacity within the workforce and develop the quality of the early years educational provision.

The *Department of Education and Skills (*2018) education focused inspections report highlighted the need to build capacity within early years educators to provide high quality educational experiences for children from three years, prior to starting in primary school. The numeracy programme provides a space for educators to reflect upon their practice and plan for developing numeracy educational experiences. Collaboration between early years educators and primary teachers allows the educators to consider how they wish to plan for the learning experiences considering children's interests and the learning environment. How will families be invited to participate? How will children's learning opportunities be extended?

Programme Delivery

There were 1,265 children and 1,898 parents involved in the programme overall. The children and parents were predominantly associated with 11 schools, 12 early childhood care services, 5 school age childcare services, five libraries and five health centres. Participants received numeracy cards and activity packs to be used at home and in schools and services.

The service delivery moved onto a virtual platform from March to June 2020 in response to the Covid-19 pandemic. Nine schools, eight early childhood care education services and five school age childcare centres engaged virtually. The libraries actively promoted the programme on social media. The Early Learning Initiative posted play-based numeracy activities daily on social media as well as links to the numeracy cards. There was a total of 18 numeracy themed posts from March to June with a reach of 20,439 and 1,642 engagements.

Participant Learning and Feedback

On completion of each Early Numeracy Week both staff and parents were asked to provide through an evaluation. In total across the three terms in 2019/2020, 250 parents and 126 staff completed evaluations. Although these figures are both lower than those of 2018/19 (432 and 193 respectively), it must be noted that the lockdown of schools and services in March of 2020 hindered the collection of evaluations for Early Numeracy Weeks 2 and 3. The majority of both staff (89%, n=112) and parents (96%, n=210) agreed or strongly agreed that the Early Numeracy Weeks were an enjoyable experience for the children involved. Ninety-two percent (n=209) of parents also highlighted their own enjoyment in completing the activities with their child.

Staff also reported that the Early Numeracy Weeks provided valuable learning opportunities for the children (91%, n=109), parents (89%, n=105) and staff (82%, n=97). According to staff, the Early Numeracy Weeks improved children’s understanding of the numeracy theme (75%, n=94) and increased parental involvement (53%, n=67). Along with enjoyment, parents found the Early Numeracy Weeks encouraged them to become more involved with their child’s learning (92%, n=216), talk and play with their child more (88%, n=205) and improve their teaching skills/knowledge (83%, 191). Parents also reported the Early Numeracy Weeks improved their child’s numeracy skills (94%, n=217), provided their children with the opportunity to learn more about numeracy (93%, n=214), improved their child’s understanding of each numeracy theme (91%, n=207), and provided their child with the opportunity to spend more quality time with adults (84%, n=190).

Staff reported that the Early Numeracy Weeks positively impacted their own practices and learning, primarily in the areas of learning new ideas for incorporating numeracy into their teaching (29%, n=29), finding ways to make learning numeracy fun (22%, n=22) and gaining a better understanding of the children’s abilities (17%, n=17).

Conclusion

The numeracy programme provides the opportunity for infant class primary school teachers and early years educators to work together to improve the educational outcomes for the children they work with. This is a positive peer learning opportunity which benefits children’s learning opportunities and capacity building of professionals to reflect upon and build on their practice as educators. The children’s needs and interests are placed at the focus of the planning process. The role of the adult is to provide a rich play environment that supports children’s early learning and numeracy skills. Community action research supports reflecting upon and building on practice in a collaborative way involving all stakeholders.

References

Chen, X. (2009). *Students who study science, technology, engineering, and mathematics (STEM) in postsecondary education. stats in Brief. NCES 2009-161.* National Center for Education Statistics.

Department of Education and Skills (2010). *Draft national plan to improve literacy and numeracy in Schools.* DES.

Department of Education and Skills (2018). *The education focused inspection report.* DES.

Eivers, E., Close, S., Shiel, G., Millar, D., Clerkin, A., Gilleece, L., & Kiniry, J. (2010). *The 2009 national assessments of mathematics and English reading.* Stationery Office.

Government of Ireland. (2018). *First 5, A Whole-of-Government strategy for babies, young children and their families* (2019-2028). GOI.

Kennedy, E., & Shiel, G. (2010). Raising literacy levels with collaborative on-site professional development in an urban disadvantaged school. *The Reading Teacher, 63*(5), 372-383.

O'Kane, M., & Hayes, N. (2010). *Supporting early childhood educational provision within a cluster of Deis preschool and primary school settings with a specific focus on transition between the two educational settings.* Centre for Social and Educational Research, Dublin Institute of Technology.

Exploring the Sustainability of Lesson Study from Multiple Perspectives

While Lesson Study (LS), a form of collaborative teacher Professional Development (PD), offers considerable benefits such as deepening teacher knowledge and supporting teachers to enhance their practice in a collaborative manner, a question remains regarding its sustainability. This symposium explores the professional experiences of LS practitioners in Ireland and Japan who have examined LS from multiple perspectives focusing on issues of sustainability in a variety of contexts.

The first paper explores the potential of LS as a vehicle to promote and support collaborative PD in a rural, multi-grade Irish primary school over the course of one school year, where LS was utilised to design and implement integrated STEM (science, technology, engineering and maths) lessons in an early years setting. Analysis of findings also revealed insights into the knowledge-related demands of designing and implementing STEM lessons. While teachers perceived lesson study to be a beneficial form of professional development, some factors constrained their engagement, including practical, cultural and sustainability challenges.

Providing an alternative perspective, the next paper presents a critical reflection on the author's recent novel experience as a LS facilitator, where they used online LS as a boundary object to support the collaborative professional learning of a group of Irish primary teacher participants, each of whom taught in three different schools. Their schools were members of an existing inter-school Shared Education partnership. The aim of the LS in this instance was to foster participants' achievement of agency by introducing them to online LS as a sustainable model of PD which can support collaborative practice within and between schools.

Following from this, the third paper draws on a double case study that sought to adapt LS to support teacher PD regarding the inclusion of Special Educational Needs (SEN) learners in primary mathematics. This study sought to examine the influence of LS on Irish primary teachers' perceptions, understandings and actions relating to inclusive practice with regards to learners who present with SEN in primary mathematics. Underpinned by the Teaching for Robust Understanding (TRU) Framework (Schoenfeld, 2018) and Dudley's (2013) case pupil approach, teacher learning is framed by critical reflection on mathematics through the lens of SEN learner experiences.

The final paper focuses on bansho (Japanese board writing) analysis as an approach to sustain LS in Japan, a context where LS is an established practice but might face the risk of being "taken for granted". Guided by the representational system framework in mathematics education by Nakahara (1995), this study aimed to investigate how multiple representations are presented as bansho in a Japanese mathematics classroom. Subsequently, the ways these representations are facilitating (or hindering) pupils' understanding were also examined.

Through exploring LS sustainability from these different perspectives, we intend to provoke thought and dialogue around how the issues facing LS could be addressed, in order to fully leverage its potential in the Irish context.

Lesson Study as a Vehicle to Promote and Support Collaborative Professional Development in STEM Education in the Early Years of Primary School

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With increasing focus on primary curricular reform in Ireland, a growing understanding of the importance of education in the early years has led professional development organisations to consider the relevance of STEM (science, technology, engineering and maths) education for young children. This research explores the potential of Lesson Study (LS) as a vehicle to promote and support collaborative professional development in a rural, multi-grade primary school. Three teachers participated in four cycles of LS over the course of one school year. LS was utilised to design and implement integrated STEM lessons in Junior and Senior Infants (ages 4–7 years). Through an action research methodology, qualitative data were generated from interviews, lesson plans, weekly collaborative meetings, observation sheets, and the researcher's reflective journal and field notes. Analysis suggests that teachers began to develop new pedagogical practices as a result of iterative and collaborative LS processes. Findings also reveal insights into the knowledge-related demands of designing and implementing STEM lessons. Successive and collaborative cycles enabled teachers to become more confident in their teaching of STEM education, and they believed they had a greater understanding of the children's learning. While teachers perceived LS to be a beneficial form of professional development, some factors constrained their engagement, including practical, cultural and sustainability challenges. The work concludes by contemplating the place of LS and STEM education in the current educational landscape and makes recommendations to support their implementation nationally.

Introduction

Internationally, Lesson Study (LS) has been recognised as an effective model of professional development to support curriculum reform (Lewis & Takahashi, 2013) and the provision of high-quality mathematics experiences to young learners (Leavy & Hourigan, 2017). Presently in Ireland, the proposed inclusion of 'Mathematics, Science and Technology Education' as one of the five broad curricular areas within the draft primary curriculum framework (National Council for Curriculum and Assessment, 2020) offers an opportunity to provide professional development that introduces new teaching, learning, and assessment approaches that enhance STEM education. Accordingly, this study explores the reality of implementing integrated STEM practices in an infant classroom where LS was used as a form of professional development. This research examines and assesses LS as a tool to support teachers in developing their knowledge and skills in teaching STEM in the early years. The findings of this study are timely as the Irish STEM policy is in the early stages of implementation; therefore, this research can inform policy makers, teacher educators, schools and teachers concerning the constraining and enabling factors that may exist whilst attempting to implement change in Irish primary schools' STEM practices.

Literature Review

Lesson Study

LS originated as a practice in Japan in the late 1800s and has been the primary vehicle for Japanese teachers' professional development. LS integrates many of the features of effective professional development (Vermunt, Vrikki, Warwick, Mercer, & van Halem, 2019; Hourigan & Leavy, 2019; 2021). Desirable outcomes associated with LS include development of teacher knowledge (Cajkler, Wood, Norton, Pedder, & Xu, 2015; Leavy & Hourigan, 2018a; Ní Shúilleabháin, 2016; Dudley, Xu, Vermunt, & Lang, 2019), an increased focus on children's learning (Cajkler et al. 2015; Leavy & Hourigan, 2018b; Dudley et al. 2019; Vermunt et al. 2019), increased teacher collaboration (Lewis, Perry, & Hurd, 2009, Murata, 2011, Cajkler et al. 2015, Hourigan and Leavy 2021) and reduced feelings of professional isolation (Lewis et al. 2009, Cajkler et al. 2015).

Different forms of LS have developed as it has been adopted and adapted in different countries (Murata, 2011). However, numerous challenges to LS have been cited concerning the cultural contexts of settings and the structure of schools. Obstacles to LS include the cost of implementation, sustainability, insufficient teacher content knowledge and connection to student learning, teachers' lack of familiarity with the research process, teachers' already hectic work schedules, lack of solid leadership, extra stress for teachers to refine their practice, and problems in collaboration (Murata, 2011; Akiba & Wilkinson, 2016; Wolthuis, van Veen, de Vries, & Hubers, 2020; Hourigan and Leavy, 2021). Considering the various barriers schools face in adopting and adapting LS, it is unsurprising that its sustainability has been challenging in some countries. Murata (2011) adds that teachers are not accustomed to professional development being sustained from year to year; she believes that teachers may practise LS for one year and then expect to move on to their next professional development experience. Wolthuis et al. (2020) found that the LS process was too time-consuming for what it yielded: "one lesson plan or a one-time insight into student responses" (p. 10). They believe that if long-term, school-based professional development is to become the norm, a cultural shift is required involving teachers changing their perspective on professional development.

STEM Education

Recent literature reports many advantages to young children learning through STEM education (Clements & Sarama, 2016; Rosicka, 2016) and the importance of early exposure to build positive learning dispositions (Park, Dimitrov, Patterson, & Park, 2017). In Ireland, the STEM Education Policy Statement (2017–2026) recognises the importance of introducing STEM in early years education, "We need a national focus on STEM education in our early years settings and schools to ensure we have an engaged society and a highly-skilled workforce in place'' (Department of Education and Skills 2017, p. 5). The influential role of primary teachers is highlighted "as the primary educators at the foundation of the STEM pipeline, it is critical that those involved in STEM policy, curricula and teacher education are cognisant of their role of influence" (Hourigan, Dwyer, Leavy, & Corry, 2021, p. 20).

Teachers' perceptions of STEM are critical, as they influence the time and consideration given to STEM education (Simoncini & Lasen, 2018).

The literature reveals several issues that may impact on the quality of STEM teaching and learning, notably, teacher competence and confidence, inquiry-based learning, content and pedagogic knowledge (Honey, Pearson & Schweingruber, 2014). Further inconsistency and confusion are evident in the many definitions of STEM education and the many interpretations of the integration of science, technology, engineering and maths (Honey et al. 2014; Margot & Kettler, 2019; Hourigan et al. 2021). With these numerous challenges in mind, research highlights the need for collaborative, sustained and situated STEM professional development (Honey et al. 2014; Rosicka, 2016; MacCraith, 2016; Hourigan et al. 2021).

Data Collection

This study was conducted in two rural schools over an eight-month period; within the researcher's own infant classroom (School 1) and a second rural infant classroom in a neighbouring school (School 2). The LS group consisted of the researcher (B.F.), two other Special Education Teachers (SETs) and a More Knowledgeable Other (M.K.O.) in the final LS cycle. Two semi-structured interviews were conducted with both SETs at the beginning and end of the research and one semi-structured interview conducted with the principal at the end of the study. The interviews captured the value that teachers placed on LS and LS's feasibility as a vehicle of professional development. Collaborative LS group meetings were conducted weekly throughout the research process. Collaborative lesson plans and observation schedules were designed and followed by the participants for each research lesson.

Findings

The most pervasive theme revealed by the data was that the teachers highly valued the professional collaboration inherent in LS. LS created a stronger community of teachers, broke down professional isolation and created learning opportunities. This research supports findings from Dudley (2013) that LS is a vehicle for collaboration to enhance professional capital (Hargreaves & Fullan, 2012). Teachers recognised that creating an atmosphere of openness enabled them to share challenges of practice and provide solutions. Empowering leadership to nurture colleagues' collective capacity and the distribution of leadership also provided a foundation upon which LS grew.

This research suggests that LS effectively brought about positive changes in teacher practice (Corcoran, 2011; Dudley, 2013; Cajkler et al. 2015; Hourigan & Leavy, 2021). Teachers began to remove themselves from being the sole transmitters of knowledge to facilitating discussion during STEM lessons. However, this shift in practice proved challenging for some participants and this echoes findings from Margot and Kettler (2019). LS provided support to take pedagogical risks and trial new teaching approaches. However, teachers require time to engage with the meaning of inquiry-based education and their role as facilitator.

Implementing STEM education for the first time extended teachers' Subject Matter Knowledge (Shulman, 1986) and Pedagogic Content Knowledge (ibid) in how to design, deliver and teach STEM lessons, and notice how children work. Collaborative planning and reflection aided teachers in solving problems and finding new levels of understanding when grappling with STEM, thus aligning with research by Murata (2011) and Vermunt et al. (2019). Engineering was regarded as a linchpin that successfully drew the other disciplines together in a more meaningful manner. Teachers observed that Aistear (National Council for Curriculum and Assessment, 2009) supported STEM learning, in particular promoting targeted language development and positive learning dispositions.

LS enabled a sharper focus on children's learning during STEM lessons. Dudley (2013) believes that LS helps teachers subdue the classroom's intricacies, leading them to observe children anew. By observing the children's learning, teachers became more responsive to various children's needs and pitched lessons more appropriately. The children exhibited 21st century-skill development, positive learning dispositions and impressive engagement, which together demonstrate how powerful early STEM experiences can be.

Many factors affected teachers' participation in LS. International research (Murata 2011; Akiba & Wilkinson 2016; Wolthuis et al. 2020) has detailed the challenges of implementing LS outside of Japan; similar challenges were recognised in this study. There were practical challenges to implementing LS in the primary school system. Cultural challenges were evident, as LS was unlike any professional development that teachers had undertaken previously. Lastly, the sustainability of LS remains an area for further support and study.

Conclusion

Although this was a small-scale study, it provides important insights into LS's introduction in an Irish primary school. Additionally, it provides interesting understandings into the challenges faced by teachers when introducing STEM education in primary school. LS provided a context for teachers to introduce STEM education to Junior and Senior Infants. LS was a new form of professional development for the teachers and time is required for teachers to become familiar with the process. This experience of professional development may have been a stimulus for teachers to recognise that an alternative approach is possible. The positive outcomes of the research justify the conclusion that LS could be adapted for use in Irish primary schools.

References

Akiba, M. & Wilkinson, B. (2016). Adopting an international innovation for teacher professional development: State and district approaches to lesson study in Florida. *Journal of Teacher Education*, 67(1), 74–93.

Cajkler, W., Wood, P., Norton, J., Pedder, D., & Xu, H. (2015). Teacher perspectives about lesson study in secondary school departments: A collaborative vehicle for professional learning and practice development. *Research Papers in Education*, 30(2), 192–213.

Clements, D.H., & Sarama, J. (2016). Math, science, and technology in the early grades. *Future of Children*, 26(2), 75–94.

Corcoran, D. (2011). Learning from Lesson Study: Power Distribution in a Community of Practice. In L. Hart, A. Alston, A. & A. Murata (Ed.), *Lesson study research and practice in mathematics education* (pp. 251-269). Springer.

Brown, T., Jones, L., & Bibby, T. (2004). Identifying with mathematics in initial teacher training. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 161-180). Information Age.

Department of Education and Skills (2017). *Education Policy Statement: 2017–2026*. Retrieved Jan 18, 2018 https://www.education.ie/en/The-Education-System/STEM-Education-Policy/stem-education-policy-statement-2017-2026-.pdf

Dudley, P. (2013). Teacher learning in Lesson Study: What interaction-level discourse analysis revealed about how teachers utilised imagination, tacit knowledge of teaching and fresh evidence of pupils learning, to develop practice knowledge and so enhance their pupils' learning. *Teaching and Teacher Education*, *34*, 107–121.

Hargreaves, A., & Fullan, M. (2012). Professional capital: Transforming teaching in every school. Teachers College Press.

Honey, M., Pearson, G., & Schweingruber, H. A. (2014). STEM integration in K-12 education: Status, prospects, and an agenda for research. National Academies Press.

Hourigan, M. & Leavy, A. M. (2019). Learning from teaching: Pre-service elementary teachers' perceived learning from engaging in formal Lesson Study. *Irish Educational Studies, 38*(3), 283-308. https://doi.org/10.1007/s10857-018-09425-w

Hourigan, M., & Leavy, A. M. (2021). The complexities of assuming the 'teacher of teachers' role during Lesson Study. *Professional Development in Education.* https://doi.org/10.1080/19415257.2021.1895287

Hourigan, M., O'Dwyer, A., Leavy, A., & Corry, E. (2021). Integrated STEM – a step too far in primary education contexts? *Irish Educational Studies*. https://doi.org/10.1080/03323315.2021.1899027

Leavy, A., & Hourigan, M. (2018a). Using lesson study to support the teaching of early number concepts: Examining the development of prospective teachers specialized content knowledge. *Early Childhood Education Journal, 46*(1), 47-60. https://doi.org/10.1007/s10643-016-0834-6

Leavy, A., & Hourigan, M. (2018b). The role of perceptual similarity, data context and task context when selecting attributes: Examination of considerations made by 5-6 year olds in data modelling environments. *Educational Studies in Mathematics, 97*(2), 163-183. https://doi.org/10.1007/s10649-017-9791-2

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, 12(4), 285–304.

Lewis, C., & Takahashi, A. (2013). Facilitating curriculum reforms through lesson study. *International Journal for Lesson and Learning Studies*, 2(3), 207- 217.

MacCraith, B. (2016). *STEM Education in the Irish School System: Report of the STEM Education Review Group*. Retrieved 2 Feb 2017 https://www.education.ie/en/publications/education-reports/stem-education-in-the-irish-school-system.pdf

Margot, K. C. & Kettler, T. (2019). Teachers' perception of STEM integration and education: A systematic literature review. *International Journal of STEM Education*, 6(1), 1–16.

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L.C. Hart, A.S. Alston, & A. Murata (Ed.) *LS research and practice in mathematics education* (pp. 1-12). Springer.

National Council for Curriculum and Assessment. (2009). *Aistear: The early childhood curriculum Framework.* Retrieved 3 Feb 2016 https://ncca.ie/media/4151/aistear_theearlychildhoodcurriculumframework.pdf

National Council for Curriculum and Assessment. (2020). *Draft primary curriculum framework.* Retrieved 12 Jan 2021 https://ncca.ie/media/4456/ncca-primary-curriculumframework-2020.pd

Ní Shúilleabháin, A. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study: Case study findings. *International Journal for Lesson and Learning Studies*, 5(3), 212–226.

Park, M.-H., Dimitrov, D.M., Patterson, L.G., & Park, D.-Y. (2017). Early childhood teachers' beliefs about readiness for teaching science, technology, engineering, and mathematics. *Journal of Early Childhood Research*, 15(3), 275–291.

Rosicka, C. (2016). *Translating STEM education research into practice*. Retrieved 27 Jul 2017 https://research.acer.edu.au/professional_dev/10/

Simoncini, K. & Lasen, M. (2018). Ideas about STEM among Australian early childhood professionals: How important is STEM in early childhood education?'. *International Journal of Early Childhood,* 50(3), 353–369.

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher,* 15(2), 4-14.

Wolthuis, F., van Veen, K., de Vries, S., & Hubers, M. D. (2020). Between lethal and local adaptation: Lesson study as an organizational routine. *International Journal of Educational Research*, 100, 1–12.

Vermunt, J.D., Vrikki, M., Warwick, P., Mercer, N., & van Halem, N. (2019). The impact of Lesson Study professional development on the quality of teacher learning. *Teaching & Teacher Education,* 81, 61–73.

Leading Online Lesson Study: Brokering at the Boundaries

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This paper presents a critical reflection on the author's recent novel experience as a Lesson Study (LS) facilitator, where they acted as a boundary broker, using online LS as a boundary object to support the collaborative professional learning of a group of Irish primary teachers who taught in three different schools. The schools involved were part of an existing inter-school Shared Education partnership. The aim of the LS in this instance was to foster participants' achievement of agency by introducing them to online LS as a sustainable form of Professional Development (PD) which can support their collaborative practice within and between their schools. The author's critical reflection, which derives from their reflective diary and field notes, draws from Schön's (1983) notion of reflection-on-action, as well as Brookfield's (2016) critical lenses and is theoretically framed by emerging literature which proposes LS as a vehicle for teacher agency.

Introduction

Lesson Study (LS) describes a model of school-based Professional Development (PD) whereby a group of teachers research, plan, teach, observe and reflect on a lesson with a group of learners in a collaborative manner (Lewis et al., 2006). While the model originates in Japan, it has been adapted, adopted and contextualised by practitioners across the world (Seleznyov, 2018). Through its structured protocols and fundamentally collaborative nature, LS has been credited with enabling teachers to cross institutional, interpersonal and intrapersonal boundaries of practice (Akkerman & Bruining, 2016; Dudley et al., 2019), thus overcoming the relative isolation and insulation they often experience in their professional practice (Brosnan, 2014). The use of digital tools to conduct LS have become increasingly popular, particularly in instances where practitioners are dispersed (Hrastinski, 2021). During the recent COVID-19 pandemic, many LS practitioners found themselves forced to move their LS practice into the online space for the first time (e.g. Goei et al., 2021). The aim of the online LS in the present study was to foster participants' achievement of agency by introducing them to online LS as a sustainable form of PD which can support the participating teachers' collaborative practice within and between their schools.

Lesson Study as a Vehicle to Foster Teacher Agency

Teacher agency describes a teacher's ability to act with competence, purpose, autonomy and reflexivity within their own practice as well as within the wider social context (Priestly et al., 2016). Teacher agency is considered to be temporal, that is, constructed based on past beliefs, knowledge and experiences, enacted in the present and oriented towards the future (Emirbayer & Mische, 1998; Priestly et al., 2016). This paper adopts an ecological view of agency, which positions agency as contextualised and contingent on a complex interplay of individual actions within a broader social milieu (Biesta & Tedder, 2006). LS can be considered to serve as a vehicle to support teachers' achievement of agency in three key ways: 1. Pedagogical Content Knowledge (PCK); 2. Collaborative expertise and 3. Professional community membership (Holden & Fotou, in review). PCK comprises the unique expertise required by teachers to engage in effective teaching. (Shulman, 1987). PCK is considered to be developed through activities within LS, which contribute to teachers' knowledge, for example, during the research phase, teachers have the opportunity to examine research about the particular subject or topic. Professional community membership relates to the nature of relational connections within the LS group. (Coburn & Russell, 2008; Lave & Wenger, 1991). This relates to the way LS protocols help to create a sociocultural learning space for teachers, where they learn through engaging in critical reflective dialogue, in line with Lave and Wenger's (1991) and Vygotksy's (1980) theories of situated and sociocultural learning. Collaborative expertise describes the ability to effectively engage within the context of a group, in order to share knowledge and ideas (Coburn & Russell, 2008). This is evident in LS where the teachers in the LS group leverage the experience and expertise of others within the group, for example, teachers from two different schools working together.

Lesson Study as a Boundary Object and Brokering Boundary Crossings

As teachers operationalise the factors of PCK, collaborative expertise and professional community membership, they engage in boundary crossing (Akkerman & Bakker, 2011). In the case of the study described in this paper, boundaries were institutional (between school sites), interpersonal (between each other) and intrapersonal (within themselves), in that individual participants belonged to three different school communities, and through engaging in LS, were invited to these multilevel boundaries in order to interact and engage with new ideas and practices (Akkerman & Bruining, 2016; Wenger-Trayner et al., 2015). Encounters at boundaries of practice are associated with risk, but also with growth and opportunity, as is the case when a boundary is crossed in order to engage in new learning (Akkerman & Bruining, 2016). Star (2010) suggests the term "boundary object" as a tool which enables boundary encounters and supports establishment of shared meaning across a boundary. In this instance, the structured protocols of LS served as a tool to establish shared meaning. The author's role was that of a "boundary broker" (Kubiak at el., 2015, p.81) who aimed to facilitate members of the LS groups in their negotiation of the various sociocultural and cognitive boundaries of their existing practice.

Overview and Context of the Project

The LS project described in this paper took place over the course of six months and involved six primary teachers in two parallel LS groups of three, with one representative teacher from each of the three schools in each group. All six teachers involved were teaching the same class level (5th class, pupils aged 10-12 years). While the initial intention was for the LS to take place in a face-to-face context, the onset of the COVID-19 pandemic meant that the author was restricted to online facilitation only. Whilst relatively experienced in facilitating face-to-face LS as part of their role with a national support service, the project represented the author's first experience of online LS facilitation. Following receipt of ethical approval from the author's organisation and consent from relevant stakeholders, the two groups were facilitated by the author to research, plan, teach and reflect on a research lesson with their classes in a collaborative manner. Both group lessons had a STEM curricular focus. LS meetings took place weekly on the Zoom platform, with each meeting lasting 60 to 90 minutes. A shared Google Drive folder was utilised to allow teacher participants to collaboratively create and share various LS materials. Each teacher was invited to video record their research lesson, and select up to three noteworthy moments from their taught lesson to play back and discuss with the other teachers in their group. The three schools involved in the study had enjoyed a successful history of collaboration as part of their involvement in a Shared Education partnership[1]. However, in line with much of the literature pertaining to Shared Education partnerships (e.g. Donnelly, 2012; Loader & Hughes, 2017) teachers and school leaders in the three schools reported that much of the focus to date had been on pupil contact and collaboration. Part of their rationale for agreeing to take part in the LS project was to explore its potential for enhancing professional collaboration and sharing of knowledge between teachers in each of the schools.

Critical Reflections on Brokering Boundary Crossing in Online Lesson Study

In the next section, I reflect on some examples of the challenges I encountered as an online LS facilitator and multilevel boundary broker. I discuss how I tried to address these during the project, drawing on the lenses of self-dialogue and literature (Brookfield, 2016) to try to make sense of what I experienced.

Broker as a Vulnerable Marginal Stranger

As part of my facilitation of the two LS groups as earlier described, I found myself occupying multiple roles, for example, fellow teacher, researcher, facilitator, knowledgeable other (Lewis et al., 2006). Effective brokers are expected to address and articulate meanings and perspectives of the intersecting practices of daily teaching, of teacher education and of educational research (Akkerman & Bruining, 2016). This brokering role left me faced with a constant challenge of how to make the practice of one community relevant to another, whilst staying true to the LS process. In some instances, the process of negotiation was relatively straightforward. For example, reflecting on my successful initial phone calls with each school I noted “The principals’ …positive perceptions of educational research in my discussions with them was notable- they showed a great level of support and interest, despite my being essentially a stranger!” [Reflective diary entry, pre-project phone calls with principals]. This resonated with the requirement to retain the role of marginal stranger “who sort of belongs and who sort of doesn’t” (Tanggaard, 2007, p.460). Akkerman and Bakker (2011) contend that this ambiguous position demanded of brokers is necessary for them to serve as actors in innovations.

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[1] Shared Education is a programme which runs in Northern Ireland and the Border region of the Republic of Ireland, whose aim is to develop positive relations between Protestant and Catholic schools.

I felt my experience as a broker resonated with Kubiak et al. (2015, p.82) who suggest, in that "even if enough of an insider to have the legitimacy to be listened to, they must also remain outward facing so as to offer a different perspective which provides value to the community". However, following initial buy-in, the negotiation with teachers and school leaders in practice proved challenging and demanded considerable personal flexibility and compromise, as is evidenced in the following example.

> "The proposed approach [from principals] is for two parallel lesson studies to run within the partnership. This will involve two morning meetings and two afternoon meetings. I don't know how I'm going to manage my own energy levels doing that! However I was eager to facilitate the schools and felt obliged to go along with their suggestion." [Reflective diary entry, initial planning meeting with principals and teacher participants]

This example highlights the relative vulnerability of the broker, and resonates with Kubiak et al. (2015) who point out the demand for brokers at times to sacrifice their own power in order support achievement of collective agency.

Trust and Rapport in the Online Space

A further challenge I identified was the difficulty of creating the necessary micro-climate of trust and rapport for the participants to interact meaningfully in the online space (Kubiak et al., 2015). I noted that "With Zoom, you either say it to everyone or no one! …Can the online space allow for meaningful or effective facilitation? Especially when I don't know the teachers and have never met them" [Reflective diary entry, prior to first meeting with participants]. In order to establish trust, rapport and to establish a shared sense of meaning, I relied heavily on the structured protocols of LS as a boundary object. Drawing from expertise of LS colleagues who had previously engaged in LS online (e.g. Goei et al., 2021), I created an instructional online LS booklet which was shared with all participants prior to commencing the LS project. This booklet offered a clear and systematic structure to each meeting, including prompt questions to guide discussion. As part of each meeting, a timekeeper and note taker were nominated, in order to give the teacher participants a sense of ownership over the project. While I was perceived as a knowledgeable other in relation to curriculum, I was uneasy with the fact that I had never had to teach pupils remotely (due to pandemic-related school closures) the way the teachers were required to. Furthermore, due to my lack of experience in this regard, I was acutely aware of the associated risk of being perceived as a "naïve out-of-touch visitor" (Kubiak et al., 2015 p.85) who lacked legitimacy or relevance. Suchman (1994, p.25) articulates this experience as broker succinctly, where the broker enters "onto territory in which we are unfamiliar and to some significant extent therefore unqualified". In order to transform my discomfort into something constructive, the beginning of each LS meeting took the form of a structured check-in with all participants. The aim of this was to build rapport by inviting teachers to share and reflect on their ongoing experiences of remote teaching. Upon further reflection, I realised that this use of time also supported teachers to face a shared problem, start collaborative work and build a group identity. Akkerman and Bruining (2016, p. 246) term this process "transformation" and identify it as a key learning mechanism associated with boundary crossing

Concluding Thoughts

Framed by sociocultural learning theories, this paper has offered some insights into the challenges experienced by a LS practitioner in brokering multilevel boundary crossings during their facilitation of an online LS. The structured protocols of LS acted not just as a boundary object, but also as a necessary scaffold for the author to maintain the personal fortitude required to engage in effective brokering. Whilst challenging, the experience of engaging in an online LS has brought the author a sense of hope and excitement for what further potential this mode of LS holds, in terms of connecting teachers and supporting their ongoing achievement of agency.

References

Akkerman, S. & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132-169.

Akkerman, S. & Bruining, T. (2016). Multilevel boundary crossing in a professional development school partnership. *Journal of the Learning Sciences, 25*(2), 240-284.

Biesta, G., & Tedder, M. (2006). How is agency possible? Towards an ecological understanding of agency-as-achievement. *Learning lives: Learning, identity, and agency in the life course*. Working paper. University of Exeter.

Brookfield, S. (2016). *Becoming a critically reflective teacher.* Wiley.

Brosnan, A. (2014). Introducing lesson study in promoting a new mathematics curriculum in Irish post-primary schools. *International Journal for Lesson and Learning Studies, 3*(3), 236-251.

Coburn, C. E., & Russell, J. L. (2008). District policy and teachers' social networks. *Educational Evaluation and Policy Analysis*, *30*(3), 203-235.

Donnelly, C. (2012). Defending identity and ethos: An analysis of teacher perceptions of school collaboration in Northern Ireland. *Policy Futures in Education, 10*(5), 540-551.

Dudley, P., Xu, H., Vermunt, J. D., & Lang, J. (2019). Empirical evidence of the impact of lesson study on students' achievement, teachers' professional learning and on institutional and system evolution. *European Journal of Education, 54*(2), 202-217.

Emirbayer, M., & Mische, A. (1998). What is agency? *American Journal of Sociology, 103*(4), 962-1023.

Goei, S. L., van Joolingen, W.R., Goettsch, F., Khaled, A., Coenen, T., In 't Veld, S. G., de Vries, S. and Schipper, T.M. (2021). Online lesson study: Virtual teaming in a new normal. *International Journal for Lesson and Learning Studies.* https://doi.org/10.1108/IJLLS-09-2020-0078

Holden, M. & Fotou, N. (in review). Lesson study as a vehicle for teacher agency: A proposed theoretical framework to support educative practice.

Hrastinski, S. (2021). Digital tools to support teacher professional development in lesson studies: A systematic literature review. *International Journal for Lesson & Learning Studies.* https://doi.org/10.1108/IJLLS-09-2020-0062

Kubiak, C., Fenton-O'Creevy, M., Appleby, K., Kempster, M., Reed, M., Solvason, C., & Thorpe, M. (2015). Brokering boundary encounters. In E. Wenger-Trayner, M. Fenton-O'Creevy, S. Hutchinson, C. Kubiak, & B. Wenger-Trayner (Eds.), *Learning in landscapes of practice.* Routledge.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35*(3), 3-14.

Loader, R., & Hughes, J. (2017). Joining together or pushing apart? Building relationships and exploring difference through shared education in Northern Ireland. *Cambridge Journal of Education, 47*(1), 117-134.

Priestly, M. Biesta, G. & Robinson, S. (2016). *Teacher agency: An ecological approach.* Bloomsbury.

Schön, D. (1983). *The reflective practitioner: How professionals think in action.* Basic Books.

Seleznyov, S. (2018). Lesson study: an exploration of its translation beyond Japan. *International Journal for Lesson and Learning Studies, 7*(3), 217-229.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-23.

Star, S. (2010). This is not a boundary object: Reflections on the origin of a concept. *Science, Technology, & Human Values, 35*(5), 601-617.

Suchman, L. (1994). Working relations of technology production and use. *Computer Supported Cooperative Work, 2*, 21–39.

Tanggaard, L. (2007). Learning at trade vocational school and learning at work: Boundary crossing in apprentices' everyday life. *Journal of Education and Work,* 20, 453–466.

Vygotsky, L. (1980). *Mind in Society: The development of higher psychological processes*. Harvard University Press.

Wenger-Trayner, E., Fenton-O'Creevy, M., Hutchinson, S., Kubiak, C., & Wenger-Trayner. B. (2015). *Learning in landscapes of practice.* Routledge.

Does Lesson Study Influence Teachers' Perceptions, Understandings and Actions Relating to Inclusive Practice With Regards to Special Educational Needs in Mainstream Primary Mathematics?

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The pedagogical practices in mainstream settings for inclusion and the response to the identified and targeted needs for Special Educational Needs (SEN) learners appear somewhat fragile and indeed fragmented. There is a gap between the knowledge and theory of SEN and actual practice, and this is evident in primary mathematics. Teacher Professional Development (PD) plays a pivotal role in supporting schools towards the inclusion of SEN learners. It is prudent that the PD needs of teachers address this gap. This case study follows two primary schools as they develop a Professional Learning Community (PLC) and engage in the Lesson Study (LS) process to support their PD regarding the inclusion of SEN learners in mainstream primary mathematics. Underpinned by the Teaching for Robust Understanding (TRU) framework in mathematics (Schoenfeld, 2018) and focused on case study pupils (Dudley, 2013), the LS cycle is framed by the teacher participants experiencing mathematics through the eyes of the child, and in this case, the SEN learner. Early findings show that teachers developed PCK for mathematics teaching. They also collaborated effectively to understand and direct the mathematical trajectories for their SEN learners and made connections to all their learners. In this case, LS shows promise and potential as a sustainable form of teacher PD that can support the inclusion of all learners in an inclusive mainstream setting.

Introduction

Unlike in Japan, LS is not as widespread a practice in Ireland, where it is an emerging form of PD for teachers at the primary level. It is suggested that the LS approach is integral to schooling in Japan, and efforts to understand it better outside of this context require a focus on the features and the theories that underpin it. This leads to a situation in Ireland that requires greater attention to teacher PD forms and how schools can adapt approaches such as LS to meet their context and situation. According to Gero (2014), LS is much more than a set of procedures driven by fundamental themes that reflect the Japanese culture. The critical features of LS can be linked to Japan's culture of interdependence, an emphasis on continuous effort, and the practice of critical reflection (Heine et al., 1999; Stevenson & Stigler, 1992). The methods associated with LS are consistent with the implicit values and beliefs shared by the general society in Japan, and that reflects their culture and historical experience. LS echoes the collaborative approach and emphasises the interdependence between its citizens and its esteemed Japanese culture. The admirable mindset and diligence that crafts teaching practice indicate and characterise the emphasis that Japanese people tend to place on hard work (Seleznyov, 2018). Finally, the expectation that teachers think critically about their practice evokes the self-critical mindset prevalent in Japanese individuals (OECD, 2010). These signature qualities that underpin LS may prove problematic when applied in a western culture that reveres individualism, celebrates personal ability, and shelters self-esteem (Kitayama et al., 1997; Whang & Hancock, 1994). Thus, the sustainability of any adaption of

LS into western contexts such as Ireland needs to be mindful of the cultural differences within which they work. It is fundamental to understand and consider how to respond to meet teachers' PD needs in response to their context, the pupils they teach and the identified gaps and needs within each setting. As Huang, Takahashi, and Ponte (2019) assert, numerous challenges and barriers face the adoption of LS, but emphasise that the theories supporting Lesson Study and the research methods for Lesson Study are now emerging as research issues.

The purpose of LS as it is practised in Japan is much broader than is often appreciated in western cultures (Lewis et al., 2019); Huang et al. (2019). In the Japanese context, the LS process is intended to not just lead to improved teaching but to support a more profound sense of professional community among teachers (Lewis, Perry and Hurd, 2009; Sato 2008), to facilitate teacher understanding of curriculum reforms and of the national performance indicators (Takahashi & McDougal 2014). LS also assists teachers to bring about in their daily teaching, the shared long term vision for their pupils devised collaboratively by the school (Takahashi & McDougal 2016). Given this context, adapting LS to meet the PD needs of primary teachers who aim to create inclusive learning experiences in the context of mainstream mathematics for their learners seems novel and intriguing.

Statement of the Problem

Inclusive education is founded on the principle that each child, irrespective of their gender, social class, cultural background or ability level, has the right to an education in a mainstream setting (Westwood, 2007). Despite this principle informing educational policy formation in many western countries, the actual practice of the inclusion of SEN learners is not always consistent with policy. It depends on teacher perceptions and understandings of inclusion and attitudes towards SEN learners presenting in the mainstream setting (Brennan et al., 2019). Perceptions, attitudes, and understanding of SEN pupils and their needs significantly impact the success or otherwise of the inclusion process in educational contexts and, particularly, a willingness among school personnel to contribute to the inclusion process (Skidmore, 2004). An SEN learner's learning difficulties can generally influence a teacher's perceptions, beliefs, and subsequent actions regarding the inclusion process and SEN learners.

Schools respond to greater diversity today (Ainscow, 2020; Brennan et al. 2019). How schools respond to inclusion is related to teachers' attitudes, knowledge, skills, capacity and understanding (Hornby, 2010; Horne & Timmons, 2009). While Rose et al. (2015) report many teachers do not possess the knowledge, skills and understanding to craft the necessary inclusive learning environment, how teachers' access professional development to equip themselves with the required skills to embrace inclusion in their unique school context appears unclear. This research investigates if the LS model of sustained teacher professional development and learning can support teachers' understandings, perceptions and actions to co-construct the necessary inclusive primary mathematics classroom. This study seeks to track and record if a sustained model of inquiry-based teacher professional development such as

Lesson Study leads mainstream teachers and Special Education Teachers (SET) to improve their mathematical PCK collaboratively. The study aims to facilitate teacher learning as they respond to the inclusion of SEN learners in the primary mathematics classroom. The research phase supports the team to plan a lesson that promotes their inquiry. This study seeks to support a system that facilitates mainstream teachers and SETs to collaborate to meet SEN pupils' identified and targeted needs in the mainstream primary mathematics classroom.

The Context of this Study

The Inclusive Pedagogical Approach informed the Lesson Study community in Action (IPAA) framework (Florian, 2014), the Teaching for Robust Understanding framework (TRU) (Schoenfeld, 2013), the case study pupil approach (Dudley, 2013), adapted for SEN learners informed the research and planning phase of the LS cycle. The development of teacher PCK was captured using a thematic analysis (Braun & Clarke, 2006) approach to generating qualitative data generated during the study.

Pedagogical Content Knowledge

Developed by Schulman (1986), Pedagogical Content Knowledge (PCK) is identified by Ball, Thames & Phelps (2008) as Knowledge of Content and Students (KCS), such as the "knowledge that combines knowing about students and knowing about mathematics" (p. 401). In LS, both PCK on action (the knowledge, skills, reasons and planning, and beliefs) as well as PCK in action (teaching specific content in class) are addressed as in the action research paradigm of Schön (1983). PCK is constructed in a complex manner in which teachers actively make sense of the knowledge base, subject-specific knowledge, teacher and pupils' beliefs, and learning outcomes. An expert teacher has well-formed PCK for all topics taught, developed, and shaped in regular classroom practice and supported through reflection. Early career teachers need to build and expand their PCK (Ni Shúilleabháin, 2016) and models of teacher inquiry that promote collaboration, building trust and sharing of expertise facilitate this process. This is the case with all teachers when collaboratively planning to meet the individual academic needs of an SEN learner in the mainstream setting (Florian, 2014; Friend et al., 2010).

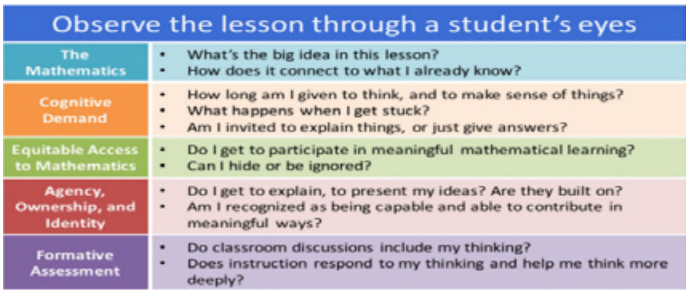
Teaching for Robust Understanding (TRU)

The five dimensions of the TRU framework describe the necessary attributes that illustrate methods of teaching which can support learners to be knowledgeable, flexible, and resourceful thinkers and problem solvers. By engaging in the TRU framework, there is a change in focus from a teacher-centred to a student-centred perspective. Teacher learning and reflections ask not: "Do I like what the teacher is doing?" instead ", What does instruction feel like, from the point of view of the student?" This emphasises how the learners have meaningful opportunities to make sense of the mathematical content. This perspective is represented in Figure 1 below. The TRU framework does not say how to teach, as it recognises the variety that constitutes effective teaching. TRU serves to problematise instruction to support rich teaching that does not impose a prescribed approach on teachers.

The TRU offers an outlook for teaching and the necessary language for dialogue regarding pedagogy in mathematics in a convincing manner.

Figure 1

Observing a mathematics lesson from the student perspective (Schoenfeld, 2018).



Case Study & Reflective Practice

A qualitative inquiry informs the data generated for this case study. The researcher sought to create a rich dialogic space to research the teacher learning among the participating teachers. This rich talk took place in two primary school settings as teachers engaged in the LS process to sustain teacher professional development and learning, which yielded qualitative data for analysis. Throughout the LS process, the research investigates teacher learning in a social space in which teachers negotiated and made sense of their learning. This study aims to capture and illuminate the learning that took place in the dialogic space (supported by the research phase and the lesson planning phase of the Lesson Study cycle), that could be described as surprising, challenging, reflective and marks changes in the perceptions and understandings of knowledge in the teaching of SEN learners in the mainstream primary mathematics setting. In this case, these experiences supported and nurtured the trust to build among the teacher participants to allow for the uniqueness of the situation to emerge. The teachers shared ideas, observations, reflections, experiences and their new learning. The case study approach included reflective practice for both the teacher participants and the researcher. Having access to this dialogic space supported teachers to critically reflect on their perceptions and understandings of the inclusion of SEN learners in their school setting. It made it clear the knowledge they acquired by engaging in the Lesson Study process.

Early Data Analysis, Tentative Findings and Discussion

LS was a novel form of teacher PD that challenged traditional approaches to mainstream mathematics classrooms. It offered participants space and time to inquire, research, experiment, reflect and collaboratively learn as a community of learners. It was a safe space to engage with new approaches to teaching mathematics and learn from one another and build on this knowledge. A key theme that is tentatively emerging at this early stage of data analysis regards supporting all the learners in the mainstream setting, not just the SEN learner.

Concluding Thoughts and Reflections

In this study, teacher participants used the research phase of LS to engage with the IPAA framework (Florian, 2014), the TRU framework (Schoenfeld, 2013), and the case study pupil approach (Dudley, 2013) adapted for SEN learners. The development of teacher PCK was captured using a thematic analysis approach (Braun & Clarke, 2006) based upon the qualitative data generated during the study. The LS cycle shows promise as the PD takes place within the school setting. It involves collaboration, requires the teachers to engage with research, reflection and inquiry while enabling them to plan with the needs of the SEN learner in mind.

References

Ainscow, M. (2020). Promoting inclusion and equity in education: lessons from international experiences. *Nordic Journal of Studies in Educational Policy, 6*(1), 7-16, https://doi.org/10.1080/20020317.2020.1729587

Ball, D. L., Thames, M. H, & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Battista, M.T. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan, 75*(6), 462-470.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-102.

Brennan, A., King, F., Travers, J. (2019). Supporting the enactment of inclusive pedagogy in a primary school. *International Journal of Inclusive Education.* https://doi.org/10.1080/13603116.2019.1625452

Dudley, P. (2013). Teacher learning in Lesson Study: What interaction-level discourse analysis revealed about how teachers utilised imagination, tacit knowledge of teaching and fresh evidence of pupils learning, to develop practice knowledge and so enhance their pupils' learning. *Teaching and Teacher Education, 34*(1), 107-121. https://doi.org/10.1016/j.tate.2013.04.006

Farrell, P. (2000). The impact of research on developments in inclusive education. *International Journal of Inclusive Education, 4*(2), 153-162. https://doi.org/10.1080/136031100284867

Florian, L. (2014). What counts as evidence of inclusive education? *European Journal of Special Needs Education, 29*(3), 286-294. https://doi.org/10.1080/08856257.2014.933551

Friend, M., Cook, L., Hurley-Chamberlain, D., & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in special education. *Journal of Educational and Psychological Consultation, 20*(1), 9–27. https://doi.org/10.1080/10474410903535380

Gero, G. (2014). The prospects of Lesson Study in the US. Teacher support and comfort within a district culture of control. *International Journal for Lesson and Learning Studies, 4*(1), 7-25. https://doi.org/10.1108/IJLLS-02-2014-0007

Griffin, S., & Shevlin, M. (2011) Responding to special educational needs: An Irish perspective, (2nd ed.). Gill & MacMillan. https://uknowledge.uky.edu/cgi/viewcontent.cgi?article=1005&context=edp_facpub

Heine, S. J., Lehman, D. R., Markus, H. R., & Kitayama, S. (1999). Is there universal need for positive self-regard? *Psychological Review, 106*(4), 766-794. Retrieved from https://eds-b-ebscohost-com.proxy.library.lincoln.ac.uk/eds/pdfviewer/pdfviewer? vid=2&sid=c900fdee-c956-4bf5-a84f-70b52e71f4bc%40pdc-v-sessmgr02

Hui, T., & Yan-jun, Y. (2016). Study on the Development of Teachers ' Reflection Ability in " Lesson Study ." (Iccse), 772–776.

Horne, P., & Timmons, V. (2009). Making it work: Teachers' perspectives on inclusion. *International Journal of Inclusive Education, 13*(3), 273–86. https://doi.org/10.1080/13603110701433964

Kitayama, S., Markus, H. R., Matsumoto, H., & Norasakkunkit, V. (1997). Individual and collective processes in the construction of the self: Self-enhancement in the United States and self-criticism in Japan. Journal of Personality and Social Psychology, 72(6), 1245-1267. Available from http://explore.bl.uk/primo_library/libweb/action/display.do?tabs=detailsTab&gathStatTab=true&ct=display&fn=search&doc=ETOCRN026889649&indx=1&recIds=ETOCRN026889649

Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through Lesson Study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285-304. https://doi.org/10.1007/s10857-009-9102-7

Lewis C., Friedkin S., Emerson K., Henn L., & Goldsmith L. (2019). How does lesson study work? Toward a theory of lesson study process and impact. In R. Huang, A. Takahashi & J. da Ponte (eds), *Theory and practice of lesson study in mathematics. Advances in mathematics education.* Springer. https://doi.org/10.1007/978-3-030-04031-4_2

Ni Shuilleabhain, A. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study. *International Journal for Lesson and Learning Studies, 5*(3), 212-226. https://doi.org/10.1108/IJLLS-11-2015-0036

Rose, R., Shevlin, M., Winter, E., & O'Raw, P. (2015). *Project IRIS–inclusive research in Irish schools. A longitudinal study of the experiences of and outcomes for pupils with special educational needs (SEN) in Irish schools*. National Council for Special Education (NCSE).

Schoenfeld, A.H. (2018). Video analyses for research and professional development: The teaching for robust understanding (TRU) framework. *ZDM Mathematics Education, 50*(3), 491–506. https://doi.org/10.1007/s11858-017-0908-y

Seleznyov, S. (2018). Lesson study: An exploration of its translation beyond Japan. *International Journal for Lesson and Learning Studies, 7*(3), 217–229. https://doi.org/10.1108/IJLLS-04-2018-0020

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Education Researcher, 15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Stevenson, H.W., & Stigler, J.W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education.* Simon and Schuster.

Westwood, P. (2007). *Common sense methods for children with special educational needs.* Taylor and Francis.

Presenting Multiple Representations at the Chalkboard: Bansho Analysis of a Japanese Mathematics Classroom

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The widespread interest in lesson study (LS) outside Japan has resulted in the effort to adapt and implement LS in different settings. As one of the key elements in LS, bansho (Japanese board writing) has also gained the attention of educators and researchers. Guided by the representational system framework in mathematics education by Nakahara (1995), this study aimed to investigate how multiple representations are presented as bansho in a Japanese mathematics classroom. Subsequently, the ways these representations are facilitating (or hindering) pupils' understanding were also examined. The findings showed that the abrupt "leap" from a lower abstraction level to a much higher abstraction level of representation could be one of the reasons why pupils were having difficulties in solving the reverse-thinking problem. The absence of representation of relations in the tape diagram could also be a potential reason for pupils' struggles. With concrete evidence based on a bansho analysis, it is hoped that teachers would develop competencies to understand and interpret pupils' learning in a LS context. This effort would be an approach in sustaining LS in Japan, a context where LS is an established practice but might face the risk of being "taken for granted".

Introduction

Inherent to the widespread adoption of lesson study (LS) is the effort to adapt and implement LS in different contexts. Parallel to that is the necessity to make features of LS in its native context explicit because it is claimed that LS is "under-theorised" (Elliott, 2012, p. 114) because it very much a cultural practice in Japan (Kuno, 2011; Makinae, 2010; Sarkar Arani, Fukaya, & Lassegard, 2010). With such long history, it is no exaggeration to say that LS has become an intrinsic part of Japanese educational culture. Consequently, much of the theory behind LS is tacit and implied. In response to this phenomenon, researchers have attempted to provide guidelines and to disclose theories underlying LS to facilitate a better understanding of the LS process.

For instance, Lewis and Hurd (2011) provided an overview of the LS cycle which includes four main stages: (1)study curriculum and formulate goals, (2)plan, (3)conduct research lesson and (4)reflect. Remarkably, one feature that is constantly present during all the stages of LS cycles in Japan is the board writing or, in its Japanese term, *bansho* (Ermeling, 2015). The fact that *bansho* in Japan has survived and accommodated new technologies has attracted much attention from researchers outside Japan. For instance, in *The Teaching Gap*, Stigler and Hiebert (1999) observed that many mathematics teachers in the United States used an overhead projector, whereas almost all teachers in Japan used chalkboard. This might be seen as a trivial difference, but Stigler and Hiebert claimed that "this superficial difference points to a deeper, more significant difference in the way teaching is conducted" (1999, p. 75).

Bansho has always been an essential part of LS because of its importance as a specialist skill and knowledge among educators in Japan. Up to the present, Japanese educators are still constantly honing *bansho* practice and integrating it with current education policies. On the other hand, studies on *bansho* outside Japan demonstrate a tendency to collect shreds of evidence of the effect on bansho on teaching and learning, subsequently investing energy to adopt and adapt *bansho* in local contexts. This is essential in ensuring that the decision to adopt *bansho* in a new context is intentional and has been given careful consideration. With that, I believe there is a need to examine the application of *bansho* in an actual classroom and comprehend how *bansho* facilitates pupils' understanding in a mathematics classroom. The findings of the study would create knowledge on teaching and learning that bridge theory and practice in the actual lesson. The development of such knowledge will be essential in sustaining LS in an evidence-based manner.

Statement of the Problem

The concept of representations is widely discussed in mathematics education because it is deemed useful in supporting mathematical thinking (Dufour-Janvier, Bednarz, & Bélanger, 1987). In addition, how representations are dealt with is also acknowledged as one of the key quality aspects of interaction processes in the mathematics classroom (Ainsworth, 2006; Dreher, Kuntze, & Lerman, 2015; Duval, 2006). Pupils' ability to handle representations and to change between representations is considered a core element of mathematical competence because mathematical concepts can only be accessed through representations, therefore making it central for the construction process of the pupils' conceptual understanding (Duval, 2006; Goldin & Shteingold, 2001). Hence, knowledge on how to use different representations merit attention.

One of the essential roles of *bansho* in Japanese classrooms is to visualise and materialise pupils' thinking processes (Tan, Fukaya, & Nozaki, 2018). Particularly in a mathematics classroom, *bansho* is employed primarily to represent mathematical concepts because "writing and the development of representational techniques are indispensable for doing and thinking mathematics" (Greiffenhagen, 2014, p. 505). Nonetheless, exploration of the representations on the chalkboard and their interactions with pupils' understanding is an area that is yet to be studied but is worth explicating. By taking the role of *bansho* in a mathematics classroom in Japan seriously, the process to improve teaching and learning through LS should consider the impact of *bansho* in an actual classroom, particularly how *bansho* can be used to facilitate (or hinder) pupils' understanding. While much research illustrates that multiple representations can be helpful in pupil's learning, it has also been cautioned that "multiple representations may fail to enhance students' learning if they are not used in the 'right' way" (Rau & Matthews, 2017, p. 531). Notably, for representations to be effective, pupils would need to properly interpret each individual representation and make connections among multiple representations. If these conditions are not met, the use of multiple representations may hinder instead of facilitating pupils' learning (ibid). However, some LS in Japan tend to seek "good practice" where teachers are overly concerned with the flow of the lesson more than the analysis and interpretation of the pupils' experiences in a classroom (Saito, 2012; Sato, 2006). Therefore, to sustain LS in Japan, a more detailed and evidence-based examination that focuses on pupils' learning is necessary. In this study, I focus on a Japanese mathematics classroom where *bansho* is used as the primary means of content visualisation and investigate how the way multiple representations are presented and dealt with could provide more information on pupils' learning processes as well as difficulties.

Research Objectives

This study aimed to investigate how multiple representations are presented as bansho in a Japanese mathematics classroom. Subsequently, the ways these representations are facilitating (or hindering) pupils' understanding were also examined.

Research Context

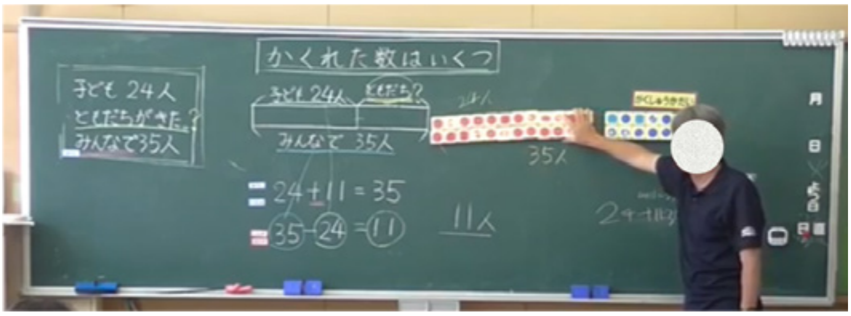
The data site for the study was a classroom in a primary school in Aichi Prefecture, Japan. The topic of the lesson under observation was "What is the hidden number?" with one *hatsumon* (key question for provoking pupils' thinking). The question was presented at the beginning of the lesson, in the form of a *hanashi* (story) as below:

> In the beginning, there were 24 children playing. Then their friends came. That makes it 35 people altogether.

Pupils presented their solutions to answer the question "what is the hidden number?", which involved different methods and representations. These solutions were recorded on the chalkboard by the teacher and by direct pupils' participation where pupils wrote on the chalkboard. The *bansho* at the end of the lesson is as follows:

Figure 1

Actual bansho at the end of the lesson



All the classroom activities and interactions were recorded with two video cameras (one placed at the front of the classroom and one at the back of the classroom) and two audio recorders. The video camera fixed at the back of the classroom was set to record the chalkboard and the process of *bansho* formation. One digital camera was also used to capture photography of pupils' learning materials, and the researcher's field notes were also used as a means of data collection. Particular attention was paid to pupils' utterances and how these were reflected on the chalkboard. Then, the *bansho* formation process (what/how/when pupils' utterances are written on the chalkboard) was reproduced.

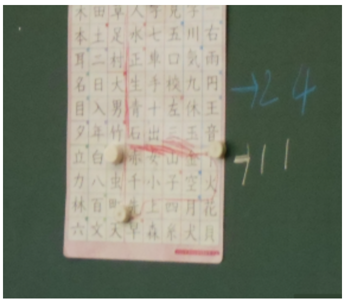
Preliminary Findings and Discussions

Guided by the representational system framework in mathematics education by Nakahara (1995), the data analysis showed that there were five representations used on the chalkboard. They include manipulative, illustrative, linguistic, and symbolic representations.

While the abstraction level in Nakahara's representational system increases from realistic→manipulative→ illustrative→ linguistic→symbolic, the representations on *bansho* did not progress in such order. The teacher started representing the question in an illustrative manner (tape diagram). Then, pupils shared their ways to arrive at the solutions, using a symbolic representation (equations). However, some of the pupils did not agree and were confused with the second equation (35-24=11) because the term "altogether" indicates addition. Therefore, the teacher employed the linguistic representation by explaining verbally how the tape diagram corresponds with the equation but without much success. Subsequently, a pupil proposed a different way of representing the solution in a manipulative-illustrative manner. She used her *kanji* grid table (a table with adopted logographic Chinese characters used in the Japanese writing system) to explain her solution to her classmates (see Figure 2). About 5 minutes before the lesson ended, the teacher realised that some of the pupils were still not convinced with the choice of solution (35-24=11). Therefore, the teacher decided to employ the manipulative representation of number blocks. From this series of events, it could be inferred that the pupils were trying to "steer" the direction of the lesson to representations of a lower level using *kanji* table grids. When the teacher noticed the pupils' difficulties and struggles, he attempted to return to representations of lower levels as well, using linguistic and manipulative representations to provide more concrete imageries of the concept. The abrupt "leap" from illustrative to symbolic representation could be one of the reasons why pupils were having difficulties in solving the reverse-thinking problem.

Figure 2

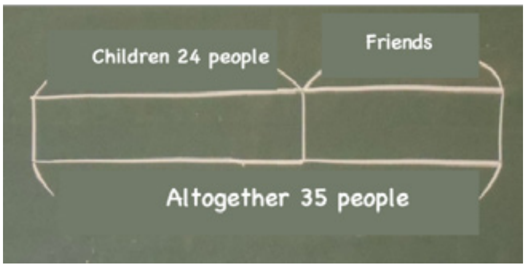
Kanji table



In addition, as Hirashima and Kurayama (2011) claimed, in a reverse-thinking problem, learners are required to think about the calculation method only after understanding the story. The sentences in the story were relatively easy, but the pupils were required to consider the relation among them to pose an adequate solution method. In the first representation, the tape diagram (see Figure 2), the teacher tried to depict the relations among the three sentences in the story. However, the tape diagram does not seem to illustrate the transformational relationship (i.e. the "before-after") in the story. In other words, the sequence of events in the story which include "in the beginning", "some friends came", and "altogether", were not represented in the tape diagram. The absence of representation of relations among sentences could also be a potential reason for pupils' struggles. Further examination on this aspect will be performed along with the use of the textbook and pupils' notebooks.

Figure 3

Tape diagram (translated from Japanese language)



Concluding Thoughts

Representations are often associated with the potential to enhance pupils' learning. However, it is crucial to determine what and how exactly are the representations facilitating or impeding pupils' understanding. In this study, a *bansho* analysis of multiple representations has been conducted to comprehend how *bansho* can be used to facilitate (or obstruct) pupils' understanding in a mathematics classroom. With concrete evidence based on a *bansho* analysis, it is hoped that teachers would develop competencies to understand and interpret pupils' learning in a LS context. This effort would be an approach in sustaining LS in Japan, a context where LS is stable but where LS could be taken for granted.

References

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183–198. https://doi.org/10.1016/j.learninstruc.2006.03.001

Dreher, A., Kuntze, S., & Lerman, S. (2015). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *Int J of Sci and Math Educ, 14*, 1–20. https://doi.org/10.1007/s10763-015-9633-6

Dufour-Janvier, B., Bednarz, N., & Bélanger, M. (1987). Pedagogical considerations concerning the problem of representation in the teaching and learning of mathematic. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109–122). Erlbaum.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Elliott, J. (2012). Developing a science of teaching through lesson study. *International Journal for Lesson and Learning Studies, 1*, 108–125. https://doi.org/10.1108/20468251211224163

Ermeling, B. (2015). *Lesson learned from a chalkboard: Slow and steady technology integration.* Teachers College Record.

Goldin, G., & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In A. Cucuo & F. Curcio (Eds.), *The role of representation in school mathematics* (pp. 1–23). Boston,Virginia: NCTM.

Greiffenhagen, C. (2014). The materiality of mathematics: Presenting mathematics at the blackboard. *British Journal of Sociology, 65*(3), 502–528. https://doi.org/10.1111/1468-4446.12037

Hirashima, T., & Kurayama, M. (2011). *Learning by problem-posing for reverse-thinking problems BT - Artificial intelligence in education*. In G. Biswas, S. Bull, J. Kay, & A. Mitrovic (Eds.) (pp. 123–130). Springer.

Kuno, H. (2011). Conceptualizing lesson study as change management recipe. In *Teacher professional development: Traditions and change* (pp. 4–12). Centre of Excellence, Nazarbayev Intellectual Schools.

Lewis, C., & Hurd, J. (2011). *Lesson study step by step: How teacher learning communities improve instruction*. Heinemann.

Makinae, N. (2010). The origin of lesson study in Japan. In *EARCOME5, Japan Society of Mathematical Education.* Tokyo.

Nakahara, T. (1995). *Study of constructivism approach in mathematics education* (Sansuu. suugaku kyouiku niokeru kousei teki apurouchi no kenkyuu). Seibunsha.

Rau, M. A., & Matthews, P. G. (2017). How to make 'more' better? Principles for effective use of multiple representations to enhance students' learning about fractions. *ZDM, 49*(4), 531–544. https://doi.org/10.1007/s11858-017-0846-8

Saito, E. (2012). Key issues of lesson study in Japan and the United States: A literature review. *Professional Development in Education, 38*, 1–13. https://doi.org/10.1080/19415257.2012.668857

Sarkar Arani, M. R., Fukaya, K., & Lassegard, J. P. (2010). Lesson study as professional culture in Japanese schools: An historical perspective on elementary classroom practices. *Japan Review, 22*, 171–200. https://doi.org/10.2307/25791346

Sato, M. (2006). *Challenge of schools (Gakko no chosen).* Shogakka.

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press. Retrieved from https://search.library.wisc.edu/catalog/999878366002121

Tan, S., Fukaya, K., & Nozaki, S. (2018). Development of bansho (board writing) analysis as a research method to improve observation and analysis of instruction in lesson study. *International Journal for Lesson and Learning Studies, 7*(3), 230–247. https://doi.org/10.1108/IJLLS-02-2018-0011

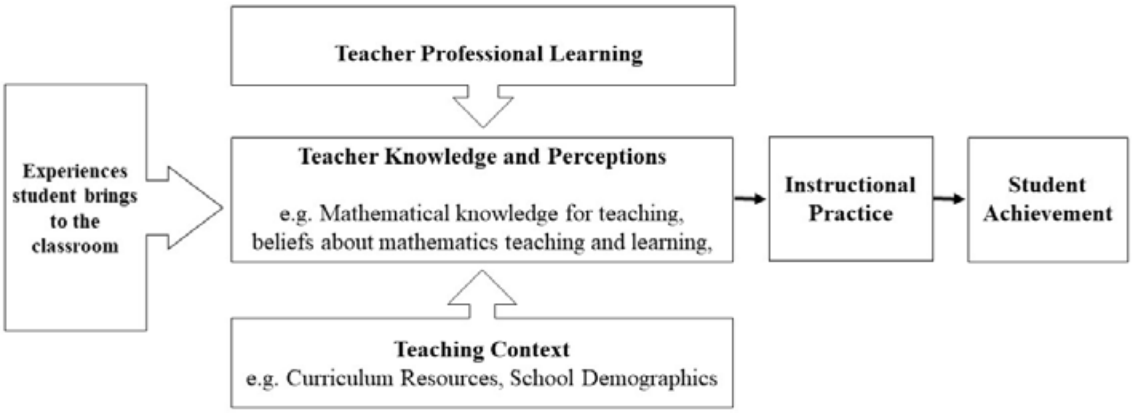
Learning Across the Transition: Bridging Mathematical Experiences from Primary to Post-Primary School

Educational transitions from primary to post-primary school have been described as the most challenging phase in a student's education. This transition involves significant changes in many aspects, such as adjusting to new school environments, increased workload, and change in teaching practices. In the context of mathematics education, a significant decline in students' mathematics achievement has been associated with this transition.

This symposium focuses on bridging the mathematical experiences for students from primary to post-primary school and has been informed by the conceptual model of Campbell et al. (2014) (Figure 1). The first paper reports on the key factors that influence students' experiences of transitions in mathematics: namely, student self-regulation, school and academic-related and social factors. These interrelated factors are situated in the Teaching Context of primary and post-primary teachers. The second paper presents a review of literature reported on the learner experience of transitions and provides insights into the Experiences Student Brings to the Classroom. The third paper reports on primary and post-primary Teacher Knowledge and Perceptions that underpin their Instructional Practices and ultimately impact on Student Achievement. The final paper proposes a model for Teacher Professional Learning that uses an Educational Design Research approach to support teachers in bridging mathematics transition and proposes design principles for a programme of professional learning for broadening and developing Teacher Knowledge and Perceptions.

Figure 1

Conceptual model adapted from Campbell et al., (2014, p. 423).



References

Campbell, P.F., Nishio, M., Smith, T.M., Clark, L.M., Conant, D.L., Rust, A.H., DePiper, J.M., Frank, T.J., Griffin, M.J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

Factors Influencing Students' Experiences of Mathematics Transitions from Primary to Post-Primary School

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This study presents the findings from a scoping review of literature (2016-2020) to explore factors that influence students' experiences of the mathematics transition from primary to post-primary education. Synthesis of findings identified three factors, namely student self-regulation, school and academic-related, and social factors that contribute to shape students' experiences. Findings suggest that no single factor can be attributed to influence students' experiences of mathematics transition and an interplay between various factors contributes to these experiences. The review also highlights a gap in research on the social factors influencing students' experiences of mathematics transitions.

Introduction

The transition from primary to post-primary school has been described as the most challenging phase in a student's education. This transition involves significant changes in many aspects, such as adjusting to new school environments, increased workload, change in teaching practices etc. Research studies mostly report on the challenges and negative experiences of students rather than the positive impacts of transition (Jindal-Snape et al., 2019). Such negative experiences impact not only on students' academic achievement but also on their psychological well-being and can affect their self- esteem and self-concept (Yao et al., 2018). In the context of mathematics education, a significant decline in students' mathematics achievement and mathematical identity has been associated with this transition. Students' interest and motivation for learning may decrease and they can develop negative attitudes towards mathematics (Yao et al., 2018). Negative transition experiences have also impacted on students' emotional health, e.g. mathematics anxiety increases in this transition to post-primary school (Suren & Kandemir, 2020). Such negative transition experiences can strongly impede the development of students' mathematical identities and impact on their academic achievement and progression in mathematics.

Many of the challenges faced by students across this transition are attributed to curricular and pedagogical discontinuities in mathematics education between levels. In particular, teachers' lack of knowledge about mathematics curriculum at the other level has been highlighted (O'Meara et al., 2020). Research on teachers' perceptions of transition-related issues identifies that teachers need additional support to facilitate students' mathematical learning during this phase and to enable collaboration between primary and post-primary school teachers (Prendergast et al., 2019). Given this, it is important that teachers are aware of the factors that contribute to students' negative or positive experiences of mathematics learning as they transition to post-primary school. This study reports on a scoping literature review of studies that focus on primary to post-primary mathematics transition and were published in the past five years (2016-2020). The key objective of this review is to address the research question:

What factors influence students' experiences of mathematics transitions from primary to post-primary school?

Methodology

This scoping review was carried out using the EBSCOhost and Web of Science databases, with searches involving the use and combination of several keywords. For example, one of the search strings used was: (school trans*) AND (primary or elementary or junior or post-primary or secondary or middle or grade 7 or grade 8). The criteria used to identify studies to be included in this review were: peer-reviewed publications in the English language from 2016 to 2020 that included students aged 10-16 years. A period of five years was chosen as this scoping review was conducted as a precursor to a wider systematic review and to confirm the relevance of study criteria and potential scope of the study. Studies focusing on specialised or vulnerable groups or ethnic or racial groups were excluded from the selection as this review focused on normative transitions in mathematics from primary to post-primary school. The search process, followed by screening of titles, abstracts and full texts yielded 22 studies for inclusion.

Findings and Discussion

The scoping review identified three key factors influencing students' positive or negative experiences of mathematics transition from primary to post-primary school. The review analysis suggests that no single factor can be attributed to influence students' transition experiences - rather there is an interplay among three key factors, namely, student self-regulation factors, school and academic related factors, and social factors (Table 1).

Table 1

Studies reporting on factors influencing students' experiences of mathematics transition.

Factor	Reference
Student self-regulation	Evans & Field, 2020a; Field et al., 2019; Klee & Miller, 2019; Madjar et al., 2018; Metsapelto et al., 2017; Murphy & Weinhardt, 2020; Semeraro et al., 2020; Skilling, Bobis & Martin, 2020; Widlund, et al., 2018
School and academic related	Arens & Moller, 2016; Deieso & Fraser, 2019; Demonty et al., 2018; Evans & Field, 2020; Fryer & Oga-Baldwin, 2019; Johnson et al., 2020; Lazarides, et al., 2019; O'Meara, Prendergast, Cantley, et al., 2020; O'Meara, Johnson, & Leavy, 2020; Prendergast et al., 2019
Social	Evans & Field, 2020b; Evans et al., 2020; Lazarides et al., 2019; Semeraro et al., 2020; Yao et al., 2018

Student self-regulation factors include aspects related to students' beliefs and/or emotions that influence an individual's experiences (positive or negative) in transitions such as self-concept, self-esteem, motivation and engagement. *School and academic factors* include aspects relating to the learning environment, curriculum content, instructional and pedagogical practices. *Social factors* relate to the home and family environment and students' relationships with family, peers and teachers. An overview of the 22 studies that reported on factors that influence students' experiences of mathematics transition from primary to post-primary education is presented in Table 1.

Student Self-Regulation Factors

Student self-regulation factors, such as students' attitudes towards mathematics learning and their beliefs about their own abilities are reported to influence their transition experiences. Studies investigating students' perceptions of transition experiences report that post-primary school students show more negative attitudes to mathematical inquiry and reduced enjoyment and engagement in mathematics as compared to primary school students (Deieso & Fraser, 2019).

The level of students' mathematics engagement also predicts the value they attach to mathematics learning. Skilling et al. (2020) reported that 'engaged' students believed mathematics to be important for their future education and valued mathematics learning. These students had a preference for understanding over performance and had high levels of self-efficacy. In contrast, 'disengaged' students placed a lower value on mathematics learning and rated performance over understanding. Such students exhibited lower self-efficacy and negative emotions such as mathematics anxiety.

Studies report that mathematics anxiety is increased as students transition to post-primary school, but it stabilises or drops to initial levels towards the end of the first year of transition (Madjar et al., 2018). Other studies have reported on gender differences in mathematics anxiety, with girls experiencing higher mathematics anxiety and lower self-efficacy than boys (Deieso & Fraser, 2019; Klee & Miller, 2019; Madjar et al., 2018). It is a matter of international concern that mathematics anxiety not only influences the school transitional phase, but it can have long lasting consequences. Field et al. (2019) report that pre-transition levels of anxiety and changes during transition are significant predictors of mathematics anxiety at age 18. Additionally, they reported that mathematics attainment (prior to transition and its trajectories across the transition) also predict later mathematics anxiety. However, the effect size was small which suggests the influence of other contextual factors.

School Related and Academic Factors

Studies examining school related factors mainly focussed on the influence of the learning environment and its implications on students' mathematics learning. Findings from these studies suggest that the aspects such as perceived teacher support, teacher enthusiasm, student perceived autonomy and perceived performance are significantly related with post-primary students' mathematics learning (Deieso & Fraser, 2019; Evans & Field, 2020a; Fryer & Oga-Baldwin, 2019; Lazarides et al., 2018). These studies highlighted that declined levels of interest in mathematics and perceived support from teachers negatively impacted on students' enjoyment and involvement in mathematics learning in post-primary school.

Students' perceptions of the instructional practices used in mathematics classrooms also influence their mathematics learning and achievement. In a study of 4926 primary and post-primary students, Arens & Moller (2016) explored the relationship between achievement in mathematics and language and two aspects of classroom environment – perceived instructional quality and student-teacher relationships. They reported that student-perceived instructional quality was more strongly (positively) associated with mathematics achievement as compared to the perceptions of student-teacher relationships. Evans & Field (2020a) found a negative association between student-reported school belonging and their mathematical attainment suggesting high-achieving students' dissatisfaction with the school climate.

Several studies have focussed on investigating the relationship between teachers' knowledge base and the instructional and pedagogical practices used in mathematics classrooms. Studies have reported curricular and pedagogical inconsistencies between primary and post-primary school mathematics. Prendergast et al. (2019) report that Irish teachers at both primary and post-primary level identified issues such as lack of knowledge of each other's curriculum and lack of communication between teachers at both levels as important factors influencing mathematics transitions. A study of 100 teachers by Demonty et al. (2018) also noted significant gaps in primary and post-primary teachers' content knowledge for teaching algebra. A mismatch of pedagogical practices in primary and post-primary mathematics classrooms has also been reported. O'Meara et al. (2020) found a more frequent use of manipulatives in primary classrooms than in the post-primary classrooms. There were also significant differences between primary and post-primary teachers' confidence in the use of manipulatives, and this was related to the different levels of support provided to teachers.

Social Factors

Social factors that influence students' experiences of mathematics transitions are less reported. Parental influence and student-teacher relationships are the most identified of these factors. Studies suggest that parental factors and home environment are strong predictors of mathematics achievement across transition (Evans & Field, 2020b; Evans et al., 2020). Using a secondary analysis of data from a national longitudinal study in the UK, Evans & Field (2020b) found that positive relationships with parents, level of parents' education and their school involvement play an influential role in mathematics attainment of 11-year-olds. The level of parental education was reported to be the strongest predictor of students' mathematical attainment trajectories from primary to post-primary school.

Another influential construct in students' experiences of mathematics transition is student-teacher relationships. The quality of these relationships have been found to have a direct influence on students' mathematics achievement and levels of mathematics anxiety (Semeraro et al., 2020). Positive relationships with teachers are also important in the development of students' socio-emotional skills and can lead to increased mathematics attainment and positive attitudes for mathematics learning (Evans & Field, 2020a; Semeraro et al., 2020). Students' relationship with peers can also influence their experiences. However, only one study that focussed on this aspect was found in this review and the authors reported a decline in peer relationships as students transition to post-primary school (Yao et al., 2018).

Conclusions and Implications

This review identifies three key factors that influence students' experiences in mathematics transition from primary to post-primary school - *student self-regulation factors, school and academic related factors* and *social factors*. Findings suggest that students' experiences of mathematics transition cannot be attributed to any single factor as various contextual factors may combine to shape these experiences. The trajectories of mathematics attainment of students as they transition to post-primary school have been reported to be impacted by a variety of factors such as mathematics attitudes, school affect, teacher characteristics and working memory (Evans et al., 2020a, 2020b). A significant impact of these combined factors is a shift in students' attitudes and motivation in mathematics which results in decline in academic achievement. More than half of the studies presented evidence of a decrease in student motivation and engagement and an increase in mathematics anxiety among post-primary school students. Long lasting impacts of the levels of mathematics attainment and anxiety from this transition have also been reported (Field et al., 2019).

Studies focussing on *student related factors* report mostly on the influence of negative student experiences. Negative experiences act as barriers to student learning and result in disinterest, disengagement and negative attitudes towards mathematics. Addressing these negative factors requires a greater focus on the continuity between the primary and post-primary mathematics curricula, improved coherence in teaching and learning approaches at both levels and enabling meaningful student engagement in mathematics through rich learning tasks. *School related factors*, such as communication and collaboration between teachers, curricular and pedagogical inconsistencies have significant influence on transitions in mathematics. Further research is needed to examine the impact of greater continuity between pedagogical approaches used in primary and post-primary mathematics. Establishing and supporting professional learning communities that bring together primary and post-primary mathematics teachers could lead to increased collaboration and communication between teachers and enable sharing of instructional and pedagogical practices in mathematics. Finally, interventions that focus on addressing *social factors* such as peer relationships, parental influences and student-teacher relationships are needed. These may include measures such as increased parental involvement in mathematical activities and measures that provide greater emotional support to students. Promoting student and teacher engagement in reflective practices may also help to develop positive mathematical identities.

This scoping review is the first part of a systematic review to identify what factors influence students' experiences of mathematics transitions. It will also examine successful interventions for supporting student learning, and provide evidence-based recommendations for the mathematics classroom and mathematics teacher education.

References

Arens, A. K., & Moller, J. (2016). Dimensional comparisons in students' perceptions of the learning environment. *Learning and Instruction, 42*, 22-30.

Deieso, D., & Fraser, B. J. (2019). Learning environment, attitudes and anxiety across the transition from primary to secondary school mathematics. *Learning Environments Research*, *22*(1), 133–152

Demonty, I., Vlassis, J., & Fagnant, A. (2018). Algebraic thinking, pattern activities and knowledge for teaching at the transition between primary and secondary school. *Educational Studies in Mathematics, 99*(1).

Evans, D., & Field, A. P. (2020a). Maths attitudes, school affect and teacher characteristics as predictors of maths attainment trajectories in primary and secondary education: Predictors of Maths Trajectories. *Royal Society Open Science, 7*(10).

Evans, D., & Field, A. P. (2020b). Predictors of mathematical attainment trajectories across the primary-to-secondary education transition: Parental factors and the home environment: Predictors of maths attainment. *Royal Society Open Science, 7*(7).

Evans, D., Gaysina, D., & Field, A. P. (2020). Internalizing symptoms and working memory as predictors of mathematical attainment trajectories across the primary–secondary education transition. *Royal Society Open Science, 7*(5).

Field, A. P., Evans, D., Bloniewski, T., & Kovas, Y. (2019). Predicting maths anxiety from mathematical achievement across the transition from primary to secondary education. *Royal Society Open Science, 6*(11), 13.

Fryer, L. K., & Oga-Baldwin, W. L. Q. (2019). Succeeding at junior high school: Students' reasons, their reach, and the teaching that h(inders)elps their grasp. *Contemporary Educational Psychology*, *59*.

Johnson, P., O'Meara, N., & Leavy, A. Factors supporting and inhibiting teachers' use of manipulatives around the primary to post-primary education transition. *International Journal of Mathematical Education in Science and Technology*, 23.

Klee, H. L., & Miller, A. D. (2019). Moving up! Or down? Mathematics anxiety in the transition from elementary school to junior high. *The Journal of Early Adolescence*, *39*(9), 1311–1336.

Lazarides, R., Gaspard, H., & Dicke, A. L. (2019). Dynamics of classroom motivation: Teacher enthusiasm and the development of math interest and teacher support. *Learning and Instruction, 60*, 126-137.

Madjar, N., Zalsman, G., Weizman, A., Lev-Ran, S., & Shoval, G. (2018). Predictors of developing mathematics anxiety among middle-school students: A 2-year prospective study. *International Journal of Psychology, 53*(6), 426-432.

Metsäpelto, R.-L., Taskinen, P., Kracke, B., Silinskas, G., Lerkkanen, M.-K., Poikkeus, A.-M., & Nurmi, J.-E. (2017). Changes in achievement values from primary to lower secondary school among students with and without externalizing problems. *Learning and Individual Differences, 58*, 75-82.

O’Meara, N., Prendergast, M., Cantley, I., Harbison, L., & O’Hara, C. (2020). Teachers’ self-perceptions of mathematical knowledge for teaching at the transition between primary and post-primary school. *International Journal of Mathematical Education in Science and Technology, 51*(4), 497-519.

O'Meara, N., Johnson, P., & Leavy, A. (2020). A comparative study investigating the use of manipulatives at the transition from primary to post-primary education. *International Journal of Mathematical Education in Science and Technology, 51*(6), 835-857.

Prendergast, M., O'Meara, N., O'Hara, C., Harbison, L., & Cantley, I. (2019). Bridging the primary to secondary school mathematics divide: Teachers' perspectives. *Issues in Educational Research, 29*(1), 243-260.

Semeraro, C., Giofrè, D., Coppola, G., Lucangeli, D., & Cassibba, R. (2020). The role of cognitive and non-cognitive factors in mathematics achievement: The importance of the quality of the student-teacher relationship in middle school. *PLoS ONE*, *15*(4), 1–22.

Skilling, K., Bobis, J., & Martin, A. J. The "ins and outs" of student engagement in mathematics: shifts in engagement factors among high and low achievers. *Mathematics Education Research Journal*, 25.

Suren, N., & Kandemir, M. A. (2020). The Effects of Mathematics Anxiety and Motivation on Students' Mathematics Achievement. *International Journal of Education in Mathematics Science and Technology, 8*(3), 190-218.

Widlund, A., Tuominen, H., & Korhonen, J. (2018). Academic well-being, mathematics performance, and educational aspirations in lower secondary education: Changes within a school year. *Frontiers in Psychology*, *9*.

Yao, Y., Kong, Q., & Cai, J. (2018). Investigating Elementary and Middle School Students’ Subjective Well-Being and Mathematical Performance in Shanghai. *International Journal of Science and Mathematics Education, 16*, 107-127.

Personal Factors Impacting Students' Mathematical Learning Across the Transition from Primary to Post-Primary: Insights from Literature

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Among the many factors influencing a student's experience of the transition from primary to post-primary are those related to the individual students themselves. This paper provides insights from literature into such personal characteristics, examining how they impact on the transition and in particular on students' mathematical learning. While a wide range of such factors exist, this paper focuses on gender, special education needs and adolescence. As evidenced in this paper, the involvement of so many factors make the transition to post-primary school a very different experience for individual students in terms of their mathematical learning, with some encountering more barriers and challenges than others. Accordingly, a single one-size-fits-all approach or support would appear difficult to conceive and unreasonable to expect.

Introduction

The transition from primary to post-primary has been identified as a significant time for student learning, particularly in the area of mathematics. The transition from primary to post-primary is viewed not merely as a physical event, but as a process spanning across the final year of primary school and the first year of post-primary. Reflecting the intricacy of this transition, Hargreaves, Earl and Ryan (1996) describes the "triple change" that learners face during this time; changes that are social, physical and intellectual in nature. Many studies highlight the significance of the primary to post-primary transition in the lives of learners; their experiences of the process are seen to influence their subsequent perspectives and outcomes (Smyth, 2017), as well as having an impact on their mental health and wellbeing (White, 2020). Set in the time of great physical and emotional change, the transition from primary to post-primary school presents adolescent students with a myriad of new experiences, emotions and potential tipping points (Tilleczek, 2007). While the exploration of all factors, whether personal, social or those related to the school environment, is beyond the scope of this paper, it is important to highlight the vast extent of change at play in the lives of learners at this time.

A Time of Challenge and Opportunity for Students

The transition to post-primary school is commonly presented in literature in terms of the challenges and difficulties experienced, with the proportion of students who have been found to experience such difficulties varying among studies. In the large-scale Australian study, almost one third of students (31%) claimed to have experienced a difficult transition (Waters et al., 2012), while in Scotland, a majority of students at the end of first year of post-primary recalled having difficulties during the transition (West et al., 2010). In Ireland, it was found that a small cohort of students experienced sustained difficulties during the transition, with certain cohorts of students more likely to experience such difficulties (Smyth et al., 2004). Factors relating to such variance will be discussed in the proceeding section.

It is important to highlight that the transition can also present opportunities for learners, having positive impacts on wellbeing and resilience (Jindal-Snape et al., 2019), as well as providing them with a solid footing in areas of achievement, behaviour and belonging (Rice et al., 2015). Further, from a learner's viewpoint, they themselves generally feel positive about the transition, attributing a high degree of importance to the move to post-primary school (Howard & Johnson, 2004). Many view the move as a "key rite of passage", with accompanying expectations of independence and of being treated as young adults (Pratt & George, 2005).

Personal Factors

Literature has identified a collage of factors which can have influence over a student's transition from primary to post-primary. Research from many countries have brought together findings which highlight the opportunities and challenges faced by students during the transition process. While many such factors relate to the school environment, this section will focus on those that are personal to the students themselves, examining how they relate to mathematical learning at this time.

Gender

Gender differences across the transition can be seen frequently in the literature. It has been shown that male students tend to settle into post-primary school more quickly, with female students experiencing more difficulties over the transition period (e.g. Smyth, 2017; Benner & Graham, 2009). With female students, on average, experiencing puberty earlier than their male counterparts, Martel (2013) highlights that this can make them more vulnerable to negative emotional outcomes in early adolescence, often resulting from social and physical comparisons with peers. Increased instances of loneliness and anxiety have also been recorded more commonly amongst female students (Benner & Graham, 2009). Symonds and Galton (2014) surmise that consequences of this phenomenon may explain the lower levels of self-esteem female students possess across the transition. The same study also shows that male students possess higher levels of disengagement from school and learning, as they seek to establish themselves in new peer groups by actively pushing against learning.

Specific to mathematics, significant gender differences are evidenced in terms of achievement and across a range of affective elements. In achievement terms, Ryan (2018) notes a significant gender disparity amongst students in first year of post-primary school, with male students outperforming girls. Further, it should be noted that where achievement levels decline across the transition, this occurs at a more rapid pace for female students (Benner & Graham, 2009). Meanwhile, Irish results from Trends in International Mathematics and Science Study (TIMSS) shows that in fourth class, girls were more likely to hold positive attitudes towards learning mathematics (Perkins, Clerkin, & Chubb, 2020). Comparing these results to students in second year of post-primary, a gap appeared according to gender, in which boys were substantially more likely to display positive attitudes than girls. While this trend is also reflected in data from other test countries, significantly, Irish results displayed a more severe decline in the attitudinal values for girls than the TIMSS average. A similar gender flip can be seen in relation to student engagement towards mathematics, with boys holding higher levels of engagement than girls in second year (Perkins et al., 2020). Interestingly, girls have been shown to hold higher levels of engagement towards non-mathematics specific areas in post-primary school, such as valuing learning, participation in extracurricular activities, school identification and compliance (Wang & Eccles, 2012). Finally, in relation to self-efficacy, data from a recent Irish study indicates that at the end of their first year in post-primary school, female students have noticeably lower levels of self-efficacy towards mathematics than their male counterparts (Ryan, 2018). Such a trend is also seen for students' self-concept, with a decline across the transition more prominent amongst female students (Smyth, 2017) and may indeed relate to the drop in self-esteem as noted previously.

Special Educational Needs

Students with SEN are more susceptible to negative outcomes across the transition process (White, 2020; Smyth, 2016). In the Irish context, a relevant comparative study in this area gathered data from two cohorts of students, one in sixth class and the other in first year (Foley, Foley & Curtin, 2016). Findings reveal that students with SEN face more obstacles than their peers during the transition process. Issues include taking longer to adjust to a new setting, establishing friendships, experiencing increased anxiety and being more vulnerable to bullying. Research also indicates concerns held by students with SEN and their parents regarding mathematical learning at this time (Barnes-Holmes et al., 2013). Specifically, within this study, it was found that students found academic subjects particularly difficult following the transition to post-primary, with mathematics cited more commonly than other subject areas. Further, parents of students with SEN expressed concerns about their children falling behind academically as they move to post-primary school and the level of communication with the post-primary school at the early stages of the transition process.

Adolescence

The transition from primary to post-primary occurs for the majority of students during the early stages of adolescence and is a time of personal change for students. This stage of life brings with it a raft of mental, physical and emotional changes, to which students must adapt. Adolescence can result in changes in relationships students hold. Bishop (2012) claims that for adolescents, social interactions are a critical aspect of identity formation within the area of mathematics. Symonds and Galton (2014) also highlight such influences on identity formation, with friendships and the role students play in friendship groups of relevance. Additionally, Browne (2012) warns that adolescents, in particular, can place higher priority on social relationships with their peers than on achieving high scores, reinforcing the view that one's mathematical identity can be indeed influenced by others, or at least concealed to a certain extent so as to undermine their achievements. All this is happening at a critical time in which students are negotiating new understandings about what it means to be a mathematical learner.

While peer relations hold particular significance at this time, changes can also be witnessed in terms of the relationships between students and their parents. Smyth (2017) found the level of closeness between children and their parents declined across a four-year period, as children went from nine to thirteen years old, coinciding with their move to post-primary school. The author points out that the improved levels of autonomy in this instance may help students as they adapt to life in post-primary school, and that signs of reduced parental involvement on items like homework can reflect good levels of academic and organisational preparedness on the student's behalf.

Final Remarks

The review of the literature around the transition from primary to post-primary reveals an intricate web of social, emotional, personal, curricular and pedagogical issues, spanning a two-year time frame. This paper provides an insight into some of the personal characteristics at play that can influence students' experience of the transition and their mathematical learning at this time. It is important to note that a band of other factors relevant to this category also exist, including student age, primary school attended, the socio-economic background of family, dispositions held by the students as well as their prior achievement, all of which can play significant roles at this time. The many factors at play can make the transition a very different experience for different students, with some encountering more obstacles than others, which can be seen to influence their experience with mathematics. In this light, a single one-size-fits-all approach or support for the transition would appear difficult to conceive and even unreasonable to expect, with the needs of individual students requiring specific consideration in any such developments.

References

Barnes-Holmes, Y., Scanlon, G., Desmond, D., Shevlin, M., & Vahey, N. (2013). A Study of Transition from Primary to Post-Primary School for Pupils with SEN: *NCSE Research Report No. 12*. Trim: NCSE.

Benner, A. D., & Graham, S. (2009). The transition to high school as a developmental process among multi-ethnic urban youth. *Child Development, 80*, 356–376.

Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education, 43*(1), 34-74.

Browne, J. R., II (2012). *Walking the equity talk: A guide for culturally courageous leadership in school communities.* California: Corwin Press.

Foley, T., Foley, S., & Curtin, A. (2016). Primary to Post-Primary Transition for Students with Special Educational Needs from an Irish Context. *International Journal of Special Education, 31*(2), 1-27.

Hargreaves, A., Earl, L., & Ryan, J. (1996). *Schooling for change: Reinventing education for early adolescents*. London: The Falmer Press.

Howard, S., & Johnson, B. (2004). Transition from primary to secondary school: Possibilities and paradoxes. Paper presented at *AARE International Education Research Conference*, Melbourne.

Jindal-Snape, D., Cantali, D., MacGillivray, S., & Hannah, E. (2019). Primary-secondary transitions: A systematic literature review. *Social Research Series*. Edinburgh: Scottish Government.

Martel, M. (2013). Sexual selection and sex differences in the prevalence of childhood externalizing and adolescent internalizing disorders. *Psychological Bulletin*, April 19.

Perkins, R., Clerkin, A., & Chubb, E. (2020). *Students' perspectives on learning mathematics and science: Results from TIMSS 2015 in Ireland*. Dublin: Educational Research Centre.

Pratt, S., & George, R. (2005) Transferring friendships: Girls' and boys' friendships in the transition from primary to secondary school. *Children & Society, 19*(1): 16–26.

Rice, F., Frederickson, N., Shelton, K. H., McManus, I. C., Riglin, L., & Ng-Knight, T. (2015). *Identifying factors that predict successful and difficult transitions to secondary school.* London: The Nuffield Foundation.

Ryan, V. (2018). *Making the transition: A student's mathematical journey from primary to post-primary school in Ireland.* Doctoral dissertation. University of Limerick. Retrieved from https://ulir.ul.ie/bitstream/handle/10344/7060/Ryan_2018_Making.pdf?sequence=6

Smyth, E. (2016). Social Relationships and the Transition to Secondary Education. *The Economic and Social Review, 47*(4), 451–476.

Smyth, E. (2017). *Growing up in Ireland, Report no.5: Off to a good start? Primary school experiences and the transition to second-level education.* Dublin: Stationery Office.

Smyth, E., McCoy, S., & Darmody, M. (2004). *Moving up: The experiences of first-year students in post-primary education.* Dublin: Liffey Press.

Symonds, J. E., & Galton, M. (2014). Moving to the next school at age 10–14 years: An international review of psychological development at school transition. *Review of Education, 2*(1), 1-27.

Tilleczek, K. (2007). Fresh starts/false starts: a review of literature on the transition from elementary to secondary school. Paper presented at the *Ontario Education Research Symposium*, Toronto: Ontario Ministry of Education.

Wang, M. T., & Eccles, J. S. (2012). Social support matters: Longitudinal effects of social support on three dimensions of school engagement from middle to high school. *Child Development, 83*, 877–895.

Waters, S., Lester, L., Wenden, E., & Cross, D. (2012). A theoretical grounded exploration of the social and emotional outcomes of transition to secondary school. *Australian Journal of Guidance and Counselling, 22(*2), 190–205.

West, P., Sweeting, H., & Young, R. (2010). Transition matters: Pupils' experiences of the primary– secondary school transition in the West of Scotland and consequences for well-being and attainment. *Research Papers in Education, 25*(1), 21–50.

White, J. (2020). *Supporting children's mental health and wellbeing at transition from primary to secondary school: Evidence review.* Edinburgh: NHS Health Scotland.

Access to and Engagement with Mathematics Across the Transition from Primary to Post-Primary School: Teacher Knowledge and Perceptions

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Research surrounding the transition has frequently cited the quality of students' interaction with teachers as having a significant association with engagement in mathematics. Thus in this cross-border study we employed a questionnaire with the aim of capturing teachers' knowledge and perceptions that underpin their instructional practices and ultimately impact on student achievement. A representative sample of 428 primary and 248 post-primary mathematics teachers from across both jurisdictions responded. The findings show that most primary teachers consider mathematics easier than other subjects to teach with only around 10% of post-primary teachers perceiving this to be the case. With the exception of post-primary teachers in Northern Ireland, perceived levels of content knowledge of the preceding/ensuing stage are low. All teachers reported a high sense of confidence in relation to teaching all strands across the mathematics curriculum and answering students' questions in class. Primary teachers were more likely to consider students well prepared in all strands of the curriculum upon exiting primary school than post-primary teachers. This disagreement was more pronounced when considering Algebra than any other strand. In this paper, we consider how these findings shape instructional practice and thus influence students access to and engagement with mathematics.

Introduction

Teachers play a central role in students' education and in their transition experience with positive interactions being associated with a greater interest in, and reduced difficulty with, mathematics (Smyth, 2017). The quality of these interactions are mediated by a teacher's impressions of mathematics and their own self-assurance in their pedagogical competency while interacting within particular environmental contexts (Cantley et al., 2021).

Following an evaluation of primary to post-primary transition arrangements for mathematics in NI, the Education and Training Inspectorate (2010) reported that teachers at either side of the boundary crossing had insufficient awareness of the curriculum and pedagogy employed in the other phase. To address this perceived knowledge deficit, a continuing professional development (CPD) project for teachers at either side of the transition, which specifically targeted mathematics (and also literacy) provision, was funded by the Department of Education in NI and commenced in January 2015. The CPD project sought to furnish teachers of mathematics in both phases with high-quality professional learning experiences, delivered via face-to-face training and supported by a virtual learning environment hosting a range of relevant resources, to promote the development of cross-phase curricular and pedagogical knowledge and skills. Between January 2015 and June 2016, a two-day training programme was offered to post-primary mathematics teachers, and two half-day training sessions were offered to primary school teachers. Funding was also provided to release primary and post-primary teachers of mathematics from normal teaching duties for two days to facilitate cross-phase collaboration (involving, for example, joint planning, joint staff development, and joint classroom observation). However, this was a once-off initiative and there was no similar transition-related CPD provision in RoI when the research was undertaken.

Thus, in this research, we investigated the four key elements that relate to *Teacher Knowledge and Perceptions* as detailed by Campbell et al. (2014). Specifically, we sought to ascertain the knowledge and perceptions of primary and post-primary teachers who teach in the year prior or subsequent to the transition in both the Republic of Ireland (RoI) and Northern Ireland (NI). Therefore, the questions that informed our study were:

1. What are teachers' self-perceptions of their knowledge of mathematical content in the preceding or ensuing stage?
2. How do they rate their own confidence in teaching mathematics?
3. What are their beliefs regarding mathematics teaching and learning?
4. Do teachers consider that they are familiar with students' dispositions and previous or forthcoming mathematical experiences?

These elements are particularly important as such *Knowledge and Perceptions* shape teachers' *Instructional Practice* and thus have an impact on *Student Achievement* (Campbell et al., 2014).

Methods

A teacher research advisory group was established to guide the development of two data collection instruments, a 6th Class/Year 7 Primary Teacher Questionnaire and a 1st Year/Year 8 Post-Primary Mathematics Teacher Questionnaire, and inform subsequent distribution to research participants. The sampling frame consisted of 3,300 primary and 723 post-primary schools in RoI as well as 827 primary and 202 post-primary schools in NI. On the basis of feedback from the teacher research advisory group, a random sample of 700 (RoI) and 450 (NI) primary schools as well as 400 (RoI) and 300 (NI) post-primary schools were selected. To allow a comparison to be drawn, the questionnaires employed across both jurisdictions and levels differed only with regards to specific terminology utilised in these contexts. Questions were designed to mirror the four key elements of *Teacher Knowledge and Perceptions* namely: knowledge of mathematical content in the preceding or ensuing stage; confidence in teaching mathematics; beliefs regarding mathematics teaching and learning; and awareness of students' prior or subsequent mathematical experiences and dispositions (Campbell et al., 2014, p. 423).

Results and Analysis

Participants

In total, 428 primary teachers returned completed questionnaires, which included 298 from RoI and 130 from NI, representing response rates of 42.6% and 28.9% respectively. In addition, 248 post-primary teachers responded to the questionnaire, including 173 from RoI and 75 from NI, which represented response rates of 43.3% and 25% respectively.

Mathematical Content Knowledge

Teachers were asked to indicate their perceptions of familiarity with the mathematical curriculum content and teaching methodologies in the proceeding/ensuing phase, a term commonly known as horizon content knowledge. Familiarity with such horizon knowledge is deemed to have an impact on teachers' decisions regarding "appropriate pedagogical approaches to use when teaching particular topics in the curriculum, so as to align with both prior and future learning" (Cantley et al., 2021, p. 43). Primary teachers in RoI indicated slightly higher levels of knowledge of post-primary curricula than their NI counterparts. However, despite the more positive views expressed by RoI teachers, the difference in perceived mathematical content knowledge between RoI and NI primary teachers was not statistically significant. Primary teachers in both jurisdictions revealed similar levels of familiarity with the recommended teaching methods for post-primary mathematics. On the other hand, post-primary teachers in NI indicated statistically significant higher levels of familiarity with both the curriculum ($p < .001$) and recommended teaching methods ($p < .001$) for final year primary mathematics than their counterparts in RoI (Table 1).

Table 1

Primary and Post-Primary Teachers' Perceptions of Mathematical Content Knowledge

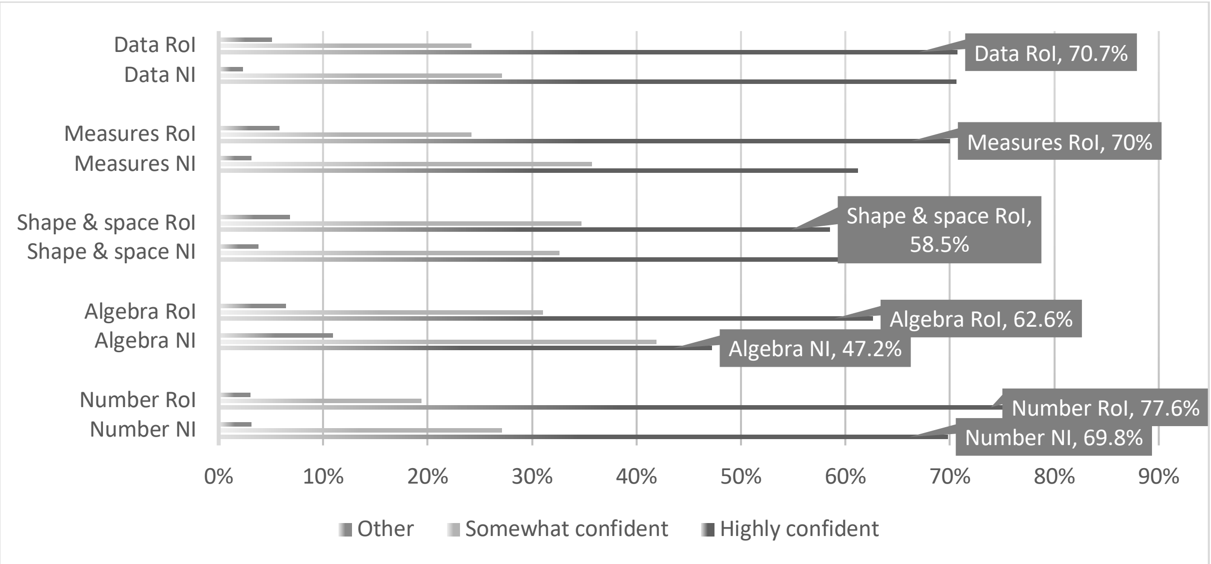
	Jurisdiction	Highly unfamiliar	Somewhat unfamiliar	Neither familiar nor unfamiliar	Somewhat familiar	Highly familiar
Familiarity with curriculum for first year post-primary mathematics	RoI (n=296)	29.7%	26.4%	6.8%	27.4%	9.8%
	NI (n=130)	31.5%	32.3%	11.5%	22.3%	2.3%
Familiarity with teaching methods for first year post-primary mathematics	RoI (n=294)	45.9%	26.5%	13.6%	9.9%	4.1%
	NI (n=128)	42.2%	28.1%	14.8%	14.1%	0.8%
Familiarity with curriculum for final year primary mathematics	RoI (n=173)	24.3%	28.3%	7.5%	31.8%	8.1%
	NI (n=75)	4.0%	21.3%	1.3%	57.3%	16.0%
Familiarity with teaching methods for final year primary mathematics	RoI (n=173)	43.4%	32.9%	8.1%	12.7%	2.9%
	NI (n=75)	17.3%	33.3%	16.0%	26.7%	6.7%

Pedagogical Content Knowledge for Mathematics

Teachers were asked to rate their confidence in teaching all areas of the mathematics curriculum. Primary teachers reported being most confident in teaching Number (Figure 1). Thus we compared responses from those who perceived they were *Highly Confident* in Number to each of the other strands. In both jurisdictions, primary teachers were statistically not as *Highly Confident* in teaching Algebra as Number ($p < .001$). Furthermore, primary teachers in RoI were statistically not as *Highly Confident* in teaching Shape and space ($p < .001$), and Measures ($p < .05$), as Number, whereas a comparison of Number with Data was nearly significant ($p = .06$).

Figure 1

Primary Teachers Pedagogical Content Knowledge for Mathematics



The majority of post-primary teachers in both jurisdictions perceived that they had high pedagogical knowledge across all strands of the mathematics curriculum (Table 2).

Table 2

Post-Primary Teachers Pedagogical Content Knowledge for Mathematics

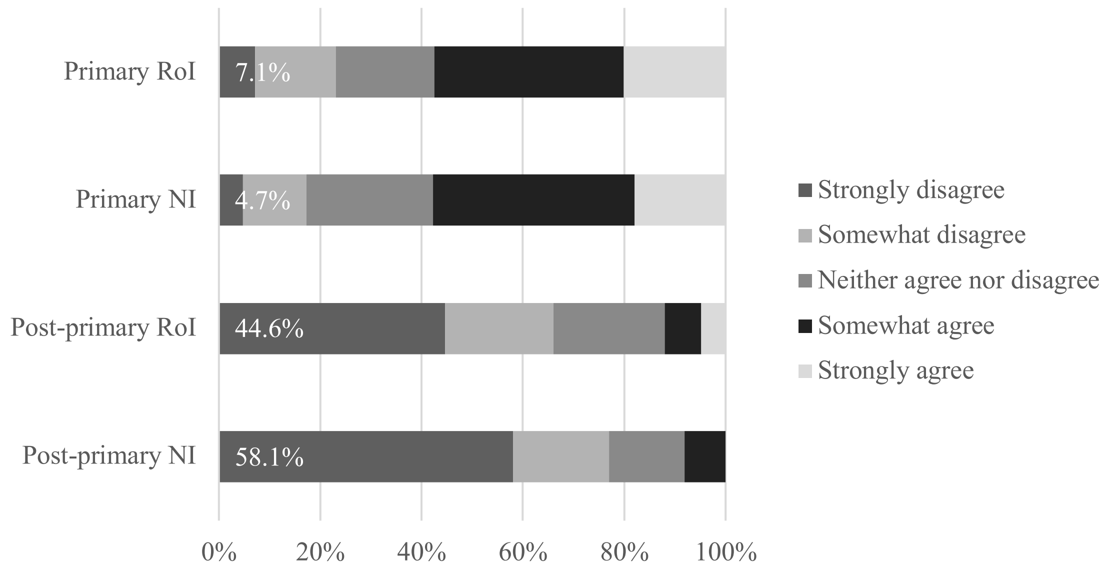
Strand	Number	Number	Algebra	Algebra	Functions	Geometry	Geometry	Data	Data
Jurisdiction	NI	RoI	NI	RoI	RoI	NI	RoI	NI	RoI
Highly confident	96%	85.3%	94.6%	90.6%	81.2%	93.3%	77.6%	92%	60.6%
Somewhat confident	1.3%	14.1%	2.7%	7.6%	16.5%	4%	20.6%	5.3%	30%
Other	2.7%	0.6%	2.7%	1.8%	2.3%	2.7%	1.8%	2.7%	9.4%

Beliefs Regarding Mathematics Teaching and Learning

The third element was to examine teachers' perceptions of mathematics teaching and learning more broadly. Primary teachers (57.5% RoI, 57.8% NI) consider mathematics easier than most subjects to teach with only around 10% of post-primary teachers agreeing (12% RoI, 8.1% NI; chi sq. p < .001) (Figure 2).

Figure 2

It is Easier to Teach Mathematics Than Other Subjects



Note. Data labels indicate the percentage of teachers who strongly disagreed.

However, teachers at both levels and in both jurisdictions believed that they are well able to answer students' questions in class (Table 3).

Table 3

I am generally well able to answer students' questions about mathematics in class.

	Primary teachers	Post-primary teachers
Strongly or somewhat agree	RoI (96.3%) NI (97%)	RoI (95.3%) NI (98.7%)

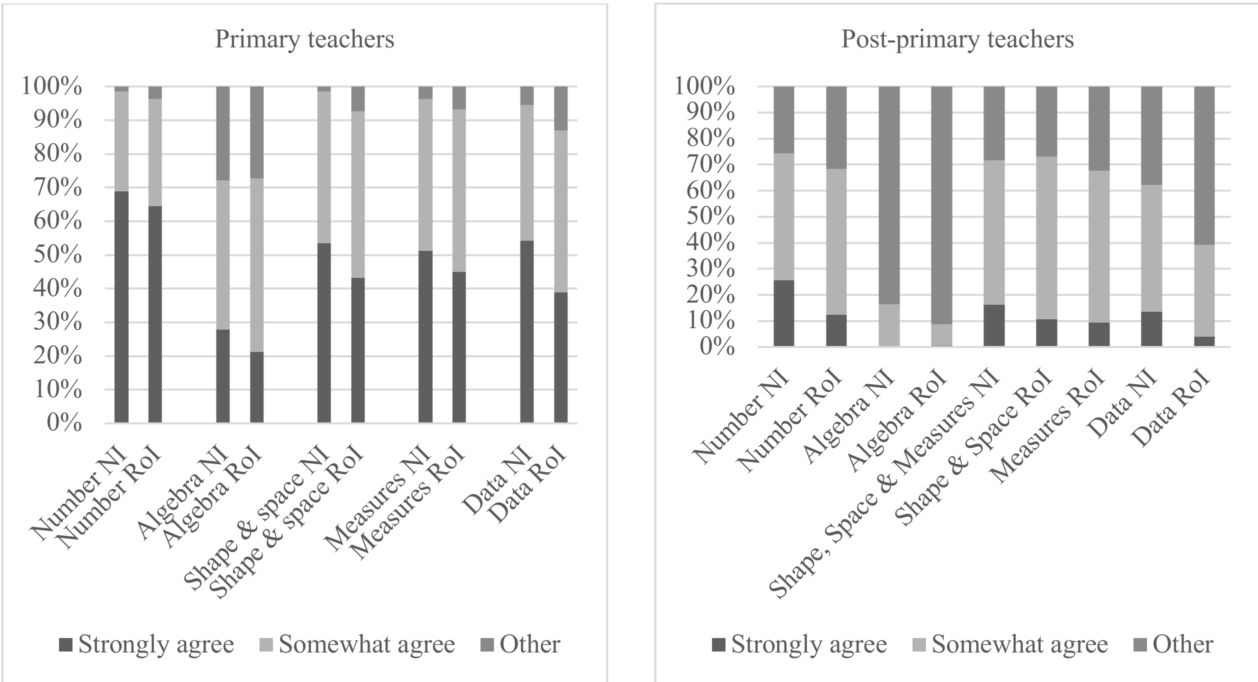
Awareness of Students' Prior Mathematical Experiences and Dispositions

Regarding students' prior mathematical experiences, teachers were asked if they perceived that students were well prepared in all curriculum areas upon exiting from primary school or entering post-primary. As can be seen from Figure 3, primary teachers perceived that students had strong mathematical experiences from which to build upon in post-primary school. This is in sharp contrast to post-primary teachers' perceptions. Particular concern was expressed about students' lack of foundational skills in Algebra. Over 70% of primary teachers in both jurisdictions agreed or strongly agreed that students were well prepared in

Algebra as opposed to just 8.8% and 16.2% of post-primary teachers in RoI and NI respectively (Figure 3).

Figure 3

Comparison of Post-Primary Teachers' Perceptions of Students' Prior Mathematical Experiences with that of Primary Teachers



Perceptions of students' dispositions towards mathematics can further influence the types of pedagogical practices teachers employ. Less than half of primary teachers in RoI (44%) and slightly more than half of primary teachers in NI (51.5%) believe that it is difficult to change students' attitudes towards mathematics. In comparison to their counterparts, 72% of teachers in NI consider that students' dispositions towards mathematics are already firmly established before they start in post-primary school ($p = .006$). Whereas just half of post-primary teachers in RoI perceive that students' dispositions towards mathematics are already firmly established, the majority of teachers (68%) favoured a fresh start approach to teaching students on entry to post-primary school over garnering information about students' mathematical dispositions from the primary school or using assessments to build up a profile of a student's prior learning.

Implications

The findings from this research call for the development of transition-related mathematics professional learning opportunities for teachers. The potential benefits of a cross-phase CPD would address issues of teachers' knowledge and perceptions of mathematics at transition and therefore support students' access to and engagement with mathematics.

References

Campbell, P.F., Nishio, M., Smith, T.M., Clark, L.M., Conant, D.L., Rust, A.H., DePiper, J.M., Frank, T.J., Griffin, M.J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

Cantley I., O'Meara, N., Prendergast, M., Harbison, L. & O'Hara, C. (2021). Framework for analysing continuity in students' learning experiences during primary to secondary transition in mathematics. *Irish Educational Studies, 40*(1), 37-49.

Education and Training Inspectorate. (2010). *Transition in mathematics: Primary to post-primary.* Belfast: Author.

Smyth, E. (2017). *Growing up in Ireland, Report no.5: Off to a good start? Primary school experiences and the transition to second-level education*. Dublin: Stationery Office.

Supporting Transitions in Mathematics: Initial Design Principles for Teacher Professional Learning

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This study presents initial design principles for a programme of professional learning for teachers focussed on supporting their student's transition from primary to post-primary mathematics. The three key principles proposed to support the design of this programme are: (1) Facilitating primary and post-primary teachers to collaborate as part of a professional learning community; (2) Providing opportunities for teachers to inquire into their own practice through Practitioner Inquiry; (3) Supporting teachers in co-designing rich mathematical tasks that can support and develop students' mathematical achievement.

Introduction

Teachers' Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008) can have an effect on students' experiences and achievement in mathematics. Knowledge of the mathematical horizon (a teacher's awareness of how mathematics is connected over the entire curriculum (Ball et al., 2008)) is one domain of MKT that can be extremely important at the transition between primary and post-primary school.

Transitions in education can provide several challenges for both students and teachers (Anderson et al., 2000; Hopwood et al., 2016). These challenges can be particularly prevalent in mathematics, as from the age of 10 upwards, students experience curricula in new educational environments, including experiencing with subject specialist teachers for the first time in the transition from primary to post-primary level. A wide range of factors that influence transitions in mathematics have been reported and these can affect both students' and teachers' experiences (the other papers in this symposium outline these factors in more detail).

This study proposes the initial design principles for the development of a programme of professional learning to support teachers of mathematics (primary and post-primary) focusing on the transition from primary to post-primary school. An Educational Design Research (EDR) approach is used to support both students and teachers across transitions in mathematics. This study will inform the initial design principles of the Erasmus+ funded Supporting Transitions Across Mathematics and Physics Education project (STAMPEd).

Design Principles for Teacher Professional Learning

Educational Design Research (EDR) addresses educational problems in real world settings and there are several descriptions or definitions of what is EDR. Plomp (2013) synthesises and describes EDR as

> "...the systematic study of designing, developing and evaluating educational interventions as solutions for complex problems in educational practice, which also aims at advancing our knowledge about the characteristics of these interventions and the processes of designing and developing them" (p.11)

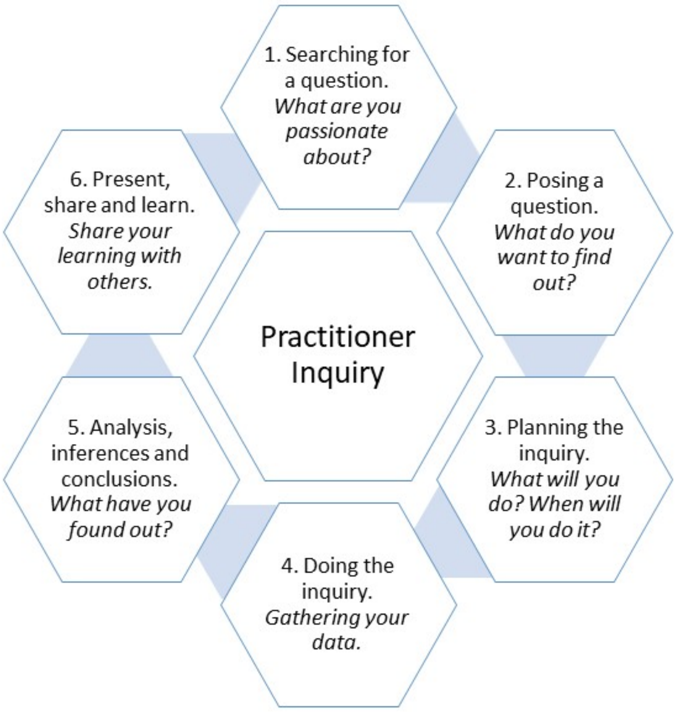
Lovatt et al. (2020) describe how studies that use an EDR approach develop interventions through continuous cycles of design, implementation and refinement, allowing researchers to reflect on the process with intention of identifying design principles. These design principles shape the development and implementation of the intervention, and can inform future design research studies. Their study designed an EDR framework that facilitated teachers working as a professional learning community to reflect and inquire into their own practice of using inquiry based learning in the science classroom (Lovatt et al., 2020). In this study, we use an EDR approach to propose initial design principles for a programme to facilitate teachers working as a professional learning community. The teachers will inquire into their own practice and collaborate to design rich tasks in mathematics that support student learning in the transition from primary to post-primary school.

Practitioner Inquiry

Practitioner Inquiry facilitates teacher professional learning by supporting teachers to inquire into their own practice and is defined as the "systematic, intentional study of one's own professional practice" (Dana & Yendol-Hoppey, 2014, p.12). The inquiry can lead to evidence informed changes and recommendations for the teacher's own practice. Teachers systematically work through an inquiry cycle, based on an initial inquiry question (Figure 1) and this cycle is iterative. Practitioner Inquiry is a powerful form of teacher professional learning (de Lange, 2020). It is intentional: that is, its purpose is to improve classroom practice with a focus on student learning. It is an inherent part of professional practice for teachers. Teachers inquire into something they are passionate about, and have ownership in the process. It is about collecting data. Teachers examine the student learning to help them to address their central question. And it is systematic. It is a continuous, ongoing process of learning with the collaboration of others (teachers, students, other stakeholders).

Figure 1

Practitioner Inquiry Cycle



Professional Learning Communities

Professional Learning Communities provide an opportunity for groups of teachers or educators to work together in a supportive, collaborative and positive environment. They are characterised by members having a shared vision, responsibility and values, and equitable participation. Effective Professional Learning Communities promote a culture of inquiry among their participants, and have a common interest and curiosity about student learning (Cochran-Smith & Lytle, 2015).

While practitioner inquiry has a focus on a teacher's own practice, it is important that there are opportunities for teachers to collaborate, share ideas and learn from the practice of other teachers. This is especially relevant when designing professional learning for teachers working in transitions, as this setting involves multiple teachers from different schools. Thompson et al. (2019) suggest that when PLCs collectively study a problem or element of practice that it can have the effect of improving teaching practices. Therefore, providing opportunities for teachers to form a professional learning community to inquire into their practice (through practitioner inquiry) can be an important element in teacher professional learning.

Rich Tasks

The design of mathematics tasks should include a range of dimensions that provide opportunities for learners to meet different needs at different times (Johnstone-Wilder and Mason, 2004). These dimensions can be considered as a spectrum, ranging from routine or closed to more open or rich tasks (as indicated in moving from left to right in Table 1). Teachers are encouraged to use a variety of these dimensions of mathematics tasks in designing student learning experiences.

Table 1

Dimensions of Mathematical Tasks. Adapted from Johnstone-Wilder and Mason (2004)

Tangential	*Fits into the core of the curriculum; represents a 'big idea'*	Essential
Contrived	*Uses processes appropriate to the discipline; learners value the outcomes of the process.*	Authentic
Superficial	*Leads to other problems; raises other questions; has multiple possibilities*	Rich
Uninteresting	*Thought provoking; fosters persistence*	Engaging
Passive	*Learner is a worker and decision maker; learners interact with other learners; learners construct meaning and deepen understanding*	Active
Infeasible	*Can be done within school and homework time; developmentally appropriate for learners; safe*	Feasible
Inequitable	*Develops thinking in a variety of styles; contributes to positive attitudes*	Equitable
Closed	*Has more than one right answer; has multiple avenues of approach making it accessible to all learners*	Open

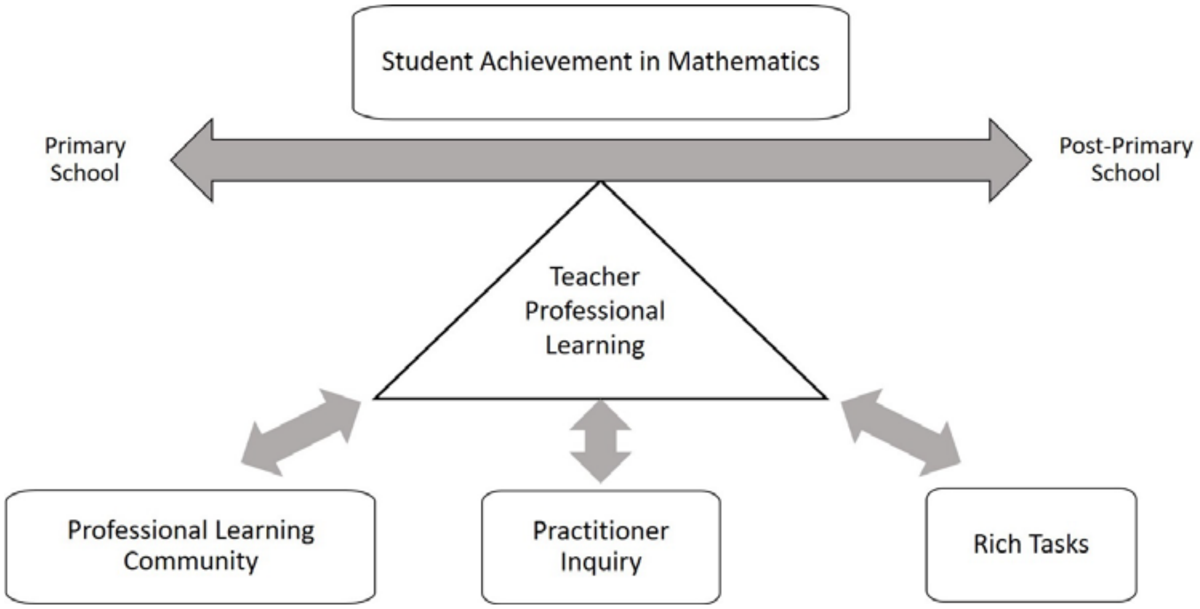
The term rich tasks is often used in mathematics as a general way to refer to tasks that include one or more of the dimensions outlined in Table 1 above. Knot et al. (2013) define a mathematics rich task as one that is *"complex, non-algorithmic, and non-routine, allowing for multiple strategies and representations and no single pathway to a solution."* (p. 600). The authors also emphasise that it is not just the description of a strategy or reasoning that is used to solve the task that is important. Students should be able to generalize and justify the strategy or reasoning used to arrive at an answer. Several characteristics and learner outcomes of mathematics rich tasks have been identified, such as a focus on inquiry, improving questioning, multiple methods, low threshold and high ceiling, promote reasoning and problem solving and encourage collaboration and discussion (NRICH). Such rich tasks can support students in developing mathematical thinking and reasoning and so it is important that teachers use these constructs in designing mathematics tasks for primary and post-primary students.

Initial Design Principles for Teacher Professional Learning

Our study considers the conceptual model proposed by Campbell et al. (2014) to support the development of teacher knowledge and perceptions of mathematics. The focus of our study is to facilitate teacher professional learning to enable student achievement in mathematics as they move from primary to post-primary school. The initial design principles for a model of teacher professional learning are presented in Figure 2 and are based on facilitating teachers to inquire into their own practice, work as part of a professional learning community and design rich mathematical tasks. By providing opportunities for teachers to engage in these activities, they will collaborate with others and investigate their own practice so that they are able to provide further support for students transitioning form primary to post-primary mathematics. This will also provide opportunities for teachers to broaden their own MKT, in the domain of horizon knowledge and other domains. Future studies will report on the use and implementation of these initial design principles; an EDR approach is currently being used to develop a programme of professional learning for primary and post-primary teachers as part of the Erasmus+ funded Supporting Transitions Across Mathematics and Physics Education (STAMPEd) project. The findings from this pan-European implementation will inform models for STEM teacher professional learning.

Figure 2

Design Principles for Teacher Professional Learning



References

Anderson, L. W., Jacobs, J., Schramm, S., & Splittgerber, F. (2000). School transitions: Beginning of the end or a new beginning? *International Journal of Educational Research*, *33*(4), 325-339.

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Campbell, P.F., Nishio, M., Smith, T.M., Clark, L.M., Conant, D.L., Rust, A.H., DePiper, J.M., Frank, T.J., Griffin, M.J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

Cochran-Smith, M., & Lytle, S. L. (2015). Inquiry as stance: Practitioner research for the next generation. *Teachers College Press*.

Dana, N. & Yendol-Hoppey, D., (2014). *The reflective educator's guide to classroom research: Learning to teach and teaching to learn through practitioner inquiry* (2nd ed.). Corwin.

Hopwood, B., Hay, I., & Dyment, J. (2016). The transition from primary to secondary school: Teachers' perspectives. *The Australian Educational Researcher*, *43*(3), 289-307.

Knott, L., Olson, J., Adams, A., & Ely, R. (2013). Task design: Supporting teachers to independently create rich tasks. In C. Margolinas (Ed.), *Task design in mathematics education (Proceedings of the International Commission on Mathematical Instruction Study 22)* (pp. 599– 608), Oxford, UK.

de Lange, L., (2020). Practitioner Inquiry in The Context of Inquiry Based Learning. Slovenia: University of Ljubljana. Retrieved April 28, 2021 http://archive3diphe.splet.arnes.si/files/2021/01/3D_VOLUME2.pdf

Lovatt, J., Grimes, P., McLoughlin, E., (2020) Educational Design Research for Teacher Professional Learning, Part A: Education Design Research. Slovenia: University of Ljubljana. Retrieved April 28, 2021 http://archive3diphe.splet.arnes.si/files/2021/01/3D_VOLUME4.pdf

Johnston-Wilder, S., & Mason, J. (Eds.). (2004). *Fundamental constructs in mathematics education.* Routledge.

NRICH (n.d.). *What are Rich Tasks?* https://nrich.maths.org/11249

Plomp, T. (2013). Educational Design Research: An Introduction. In Plomp, T., & Nieveen, N. (Eds.), *An Introduction to Educational Design Research*. Enschede, The Netherlands: SLO Netherlands Institute for Curriculum Development

STAMPEd (n.d.). *Supporting Transitions Across Mathematics and Physics Education.* https://www.stampedproject.eu

Thompson, J. J., Hagenah, S., McDonald, S., & Barchenger, C. (2019). Toward a practice-based theory for how professional learning communities engage in the improvement of tools and practices for scientific modeling. *Science Education*, *103*(6), 1423-1455.

Reflection on Project Maths after Ten Years: To What Extent Have Teaching Methods Changed?

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Ten years on from the introduction of the "Project Maths" initiative, this study aims to explore how it may have changed "common practice" in terms of pedagogical approaches within the Irish Leaving Certificate Higher level classroom, and to identify the main factors that influence teachers' choice in this regard. A mixed-methods approach to data collection was implemented using an online survey distributed to teachers using voluntary response and snowball sampling methods. Responses from 111 teachers indicated that direct instruction remains the predominant pedagogy in their classrooms, with perceived time constraints, teachers' level of comfort with an approach and their beliefs about its effectiveness being the predominant drivers of their choices. Results suggest a need for more professional development to increase teachers' level of comfort with the student-led pedagogies promoted by the reformed curriculum, as well as pointing to a need for research into their effectiveness, to achieve the objectives of Project Maths with respect to mathematical proficiency.

Introduction

Mathematics education in Ireland has undergone significant reform in the past dozen years, with a major initiative, "Project Maths", introducing a reformed curriculum initially to 24 schools from 2008 and then nationally from 2010. In both cases, the changes were phased in over three years concurrently at Junior and Senior Cycle. This was the first comprehensive curriculum reform in post-primary mathematics for over 50 years, aiming to change not only the content, which had evolved incrementally over the period (Oldham, 2019), but also the pedagogies and students' approaches to mathematics. It emphasised developing competences in solving problems in both familiar and unfamiliar contexts, via a more student-centred approach that would enhance engagement by focusing on investigative learning pedagogies (Byrne et al., 2021; National Council for Curriculum and Assessment [NCCA], 2013).

Just over 10 years since the national rollout began, a point has been reached where the students in the final-year cohort have experienced only the new curriculum. Teachers should also be familiar with it. The aims of the present study are *to establish what is "common practice" in terms of pedagogical approaches within the Leaving Certificate Higher level (LC HL) classroom*, and *to identify the factors that influence the use of those pedagogical approaches*. The decision to focus on Senior Cycle is due to:

- The introduction of a new Junior Cycle specification for Mathematics in 2018, with greater emphasis on learning through problem solving (which could confound the research, as current Junior Cycle practice may reflect this rather than Project Maths)
- The possibility of forthcoming changes to the Senior Cycle, for which feedback on the present situation could be relevant
- A study by O'Meara and Prendergast (2018), highlighting a lower level of satisfaction with time allocation for mathematics at Senior Cycle than at Junior Cycle

This paper outlines factors that led to introduction of the Project Maths initiative, setting them in the context of international trends in mathematics education. Key aspects of intentions and implementation of the initiative are highlighted. The methodology and findings of the study are presented; conclusions are drawn and possibilities for further work are set out.

Background and Context

Consideration of outcomes appropriate for students learning mathematics, and of the classroom practices that might best achieve these outcomes, produced lively international debate in the 1980s and 1990s. An important definition of "mathematical proficiency" emerged in 2001, specifying five intertwined "strands": conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). Thus, it became relevant to look for classroom practices that are especially powerful in promoting the different elements of proficiency (Groves, 2012).

Another international movement focused more broadly on curriculum. Throughout the 1990s and 2000s, there was pressure for mathematics curriculum "reform". A range of factors contributed: disillusionment with the level of abstraction in curricula; concern about students' limited capacity to apply taught material to new contexts; research from the learning sciences redefining best practice; and pressure generated by studies such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) (Conway & Sloane, 2006). Emphasis on "realistic" problem-solving gained popularity. Curricula in the so-called "reform" tradition are designed to focus on solving problems set in contexts, especially those relevant to the learners and their experiences (van den Heuvel-Panhuizen & Drijvers, 2014).

Many of the factors giving rise to pressure for change were pertinent for Ireland (Conway & Sloane, 2006). While much of the "modern" material adopted enthusiastically in the 1960s had been removed via successive revisions, curriculum content, especially for Higher level courses, remained rather "pure", as indeed it had been before the "modern" innovations (Oldham, 1980, 2019). Curriculum documents prior to 1970 did not specify learning intentions, but all later ones – albeit using differing vocabulary and formulated in different styles – focused on conceptual understanding and procedural fluency; some heed was paid also to strategic competence (solving problems) and adaptive reasoning. However, questions can be asked about the extent to which the intentions were implemented.

For years, there was a strong perception that a narrow form of procedural fluency remained the focus in many Irish classrooms, with (in Skemp's (1976) terminology) rules given without reasons, and with rote learning encouraged particularly by out-of-field teachers and those teaching lower-attaining students (Oldham, 1980, 2001, 2019). Hard evidence about classroom practice was elusive until 1996. Then, the results of the initial round of TIMSS ("TIMSS 1995") showed that expository whole-class teaching was the norm, group work was rare, and many Irish teachers prioritised the lower-order objective of remembering formulae and procedures over higher-order ones involving logical reasoning, creativity and application to the real world. The findings are nuanced, but – taken in the context of other factors – they suggest that this reflected teachers' beliefs, not only about the nature of mathematics as a subject, but also about the ways in which it is best taught and learned (Oldham, 2001). Supporting evidence came from an in-depth video study of twenty Junior Cycle mathematics lessons. In these classes, "[learning] appeared to be defined as a matter of memorising procedures and facts…. The objective was to ensure that students perfected their procedural skills" (Lyons et al., 2003, p. 143). Altogether, therefore, in the early 2000s the moment arrived for Irish post-primary mathematics education to be reconceived in the "reform" tradition, rather than incrementally revised. This led to the Project Maths (PM) initiative.

The PM Reform: Aspects and Implementation

The PM curricula were intended to build competences in solving problems in both familiar and unfamiliar contexts, thereby increasing learners' aptitude for logical thought and mathematical communication (NCCA, 2013). The approach was underpinned by the belief that emphasis on solving context-driven or applied problems would encourage higher-order thinking, promoting development of students' abilities to deal flexibly with problems and see connections between concepts (Johnson et al., 2019; Pegg, 2010). Eventually, the strands of mathematical proficiency were adopted as objectives (Kirkpatrick et al., 2001; NCCA, 2013).

To achieve the goals of the reform, changes were required, especially in pedagogy, textbooks and examinations (Conway & Sloane, 2006). A more student-centred pedagogical approach was advocated, with an emphasis on investigative learning in realistic or applied contexts: hopefully shifting teachers' focus towards conceptual understanding and strategic competence (cf. Groves, 2012). The move was supported by significant changes in the style of the examinations and by substantial professional development (Byrne et al., 2021; Johnson et al., 2019; O'Meara et al., 2017). Some content was removed to allow more time for active methodologies (Johnson et al., 2019). Overall, it was hoped to bring about more faithful implementation of the PM curricula than had been the case with earlier reform efforts. Separately, encouragement for greater uptake of LC HL was offered by the award of "bonus points" for university entry to students achieving suitable grades (Cosgrove et al., 2012).

Research to date has pointed to less than full adoption of recommended pedagogies. Early studies found that, although there was evidence of support for the constructivist and "reform" approaches encouraged, "traditional" teaching was still widespread (Cosgrove et al., 2012; Jeffes et al., 2013). More recent work by Johnson et al. (2019) indicated that the situation had not greatly changed; teachers were unconvinced that the altered pedagogies would improve learning. A major factor emerging from many studies is that of *time*; student-centred pedagogies were seen as more time-consuming, and teachers felt pressure to use direct instructional methods to ensure content coverage (Cosgrove et al., 2012; Irish Mathematics Teachers Association [IMTA], 2012; Johnson et al., 2019; O'Meara & Prendergast, 2018). The present study draws especially on the work of Johnson et al. (2019).

Research Questions

1. To what extent are the methods of instruction promoted by the aims and objectives of the PM curriculum used within the LC HL classroom?
2. What factors have most influence on the pedagogies used in the LC HL classroom?

Methodology

A web-based survey instrument was used to collect quantitative and qualitative data through closed and open questions, using the Qualtrics survey platform. The mixed-method approach was taken since the introduction of qualitative questions provided an opportunity to supply context and to allow for further explanation and enhancement of answers in the quantitative aspect of the survey (Bryman, 2012). The instrument consisted of 13 questions (11 closed and two open). Initial items addressed the level of teaching experience of respondents, both in general and specifically in relation to teaching the LC HL curriculum. The other items were of Likert type with five or more scale points. Respondents were asked to indicate their frequency of usage, in their LC HL classrooms, of each item in a list of pedagogies (see Table 1, below), chosen because they were advocated for PM or reportedly (still) used in classrooms. An option to add other pedagogies was provided. Respondents were also asked to rank their comfort level with each approach and their perception of its effectiveness. Two further, related, questions asked them to a) provide a rationale for their pedagogic choices (items such as "I use methods I have always used" or "I feel restricted and/or am afraid to try new methods"), and b) rank the influence of a list of factors on their choices (Table 2).

The survey was distributed in February 2020 to teachers of LC HL mathematics through professional networks, such as the Irish Mathematics Teachers Association (IMTA), and social media (Twitter), using voluntary and snowball sampling methods. The total of 111 responses was broadly in line with similar studies; the work by Johnson et al. (2019) received 147 responses. For the quantitative data, the software package SPSS was used to conduct descriptive and inferential analysis including Pearson's Correlation tests to identify potential relationships between variables. According to Norman (2010), tests such as Pearson's Correlation are robust enough to handle data from Likert-type items with five or more scale points. For qualitative responses, thematic analysis was used to identify key themes.

Limitations of the work should be noted. Survey items were designed in-house; despite piloting and discussion with colleagues, some element of bias may exist. While the wide reach of the IMTA may have helped to access participants across a range of experience and geographical locations, sampling did not use probabilistic methods, so the sample may not be representative (Bryman, 2012). Finally, data have the limitation of being self-reported.

Results

Analysis revealed that while the median level of teaching experience across all subjects was 11-15 years, the median number of years teaching LC HL mathematics was lower, at 6-10 years. However, teaching experience did not appear to influence other variables in the study.

Pedagogies Used and Influence on Choices

With regard to *pedagogical approaches* used, direct instruction remains predominant for the majority of the teachers; fewer than a quarter of teachers identified problem-solving, inquiry-based learning (IBL), group work or flipped approaches among their frequently-used strategies (Table 1). The more teacher-led methodologies ranked higher in terms of comfort and perceived effectiveness. While causal links between perceived effectiveness and comfort cannot be inferred, it is noteworthy that three of the seven methods have the same median for comfort and for effectiveness, while only one method differs by more than one scale point.

Table 1

Percentage of Teachers (n=111) Most Frequently Using Listed Pedagogies, with Median Comfort and Effectiveness Scores

Pedagogy	% of teachers using frequently	Median comfort level	Median perceived effectiveness
Direct instruction (chalk and talk)	61	7	7
Open-ended questioning	34	5	5
Discussion and debate	26	5	4.5
Independent problem solving	24	4	4.5
Group work	23	4	3
Inquiry-based learning (IBL)	14	3	3
Flipped classroom	7	3	1

Note. 1 = least comfortable / effective; 7 = most comfortable / effective.

The factor most influencing the teachers' choice of approach is *time*, with one teacher explicitly remarking that "Time pressure affects methods used to teach." Indeed, further examination of respondents' perceptions in relation to time constraints revealed that 65% do not believe there is adequate time to complete the course, and only 19% consider their pace as one that "suits" the students. Teachers' comfort and experience, and the group being taught, are other factors receiving considerable endorsement (Table 2).

Table 2

Percentage of Teachers (n=111) Ranking Factors Influencing their Choice of Approach

Factor	% of teachers ranking top influence	Median ranking
Time constraints	39	5
Comfort and experience with method	26	4
The group of students in the class	25	5
Facilities and resources available in school	8	3
Whole-school approaches to teaching and learning	1	2
Best practice guidelines from NCCA, research, etc.	0	2

Note. 1 = least influential, 6 = most influential.

Correlation Analysis

Correlation analysis revealed some interesting potential relationships between the level of comfort, perceived effectiveness, influencing factors, and types of pedagogies used. There was a statistically significant positive relationship between the use of *direct instruction* and the *level of comfort* with the method (r = 0.437, n = 74, p = 0.03). This connection was further illustrated by the positive relationship between a perception of being *restricted/afraid to try new methods* and *direct instruction* (r = 0.244, n = 73, p = 0.04). Qualitative statements indicating that teachers "cannot afford to waste time if experimental methods do not work," and that "on balance it simply isn't rewarded in the exam," support this connection. Exploration of factors positively associated with the student-centred approaches of *IBL* and *independent problem-solving* suggests the importance of a *whole-school approach to teaching and learning* (r = 0.276, n = 67, p = 0.02 and r = 0.272, n = 67, p = 0.03 respectively).

Discussion and Conclusion

This study aimed to identify the extent to which the methods of instruction promoted for the "Project Maths" curriculum are used in Leaving Certificate Higher level (LC HL) classrooms, and to identify factors associated with uptake of the different pedagogies. The results show that for the 111 teachers (not necessarily a representative sample) who responded to the purpose-designed survey, "direct instruction (chalk and talk)" – a method that was intended to be de-emphasised for Project Maths – is still the most frequently used pedagogy, with respondents being most comfortable in using it and most convinced of its effectiveness. Respondents were less comfortable in using the more "reform-type" methods promoted for Project Maths and also less convinced of their effectiveness. These findings are consistent with some ongoing resistance to methods still viewed, as stated by a respondent quoted above, as "experimental"; it also echoes an earlier point from the findings of TIMSS 1995, highlighting the importance of taking teachers' beliefs into account when trying to implement curriculum change. Factors reported as affecting choice of pedagogy are dominated by time constraints and to a lesser extent by comfort and familiarity, together with perceived relevance for the student group being taught. Perceptions that the high-stakes Leaving Certificate examination does not reward student-centred approaches are relevant also.

The results with regard to *frequently-used pedagogies* and *time* are similar to those from previous research (Johnson et al., 2019), in this case specifically for LC HL teaching. The qualitative data illustrate potential reasons why respondents may favour certain methods for the *group of students* that they are teaching. Specifically, there is a sense that the cohorts of students attempting LC HL are less academically strong than for previous HL curricula, given the allure of extra points: "Bonus 25 points encourages students who are not able for HL to hang on… ultimately increasing time constraints on teacher." One response provides a summary: "the system makes me teach the way I teach. Nothing significant will change until time, amount of content, or university selection processes change."

Kärkkäinen (2012) recognises that for teachers to engage fully with a reform; it is essential that they understand and agree with the reasoning behind it, its implications for their practice, and its consequences for their students. Findings reported above suggest that, if intentions for use of more active, student-led pedagogies are to be implemented, professional development addressing both rationale and hands-on practice is still required, for teachers across all ranges of experience. The need for even more professional development than accompanied the rollout of the Project Maths initiative was identified by Byrne et al. (2021).

Other points arise here. The present study does not address whether, or how, the "reform" pedagogies used in the Irish context actually lead to improvement with regard to the objectives of the reformed curricula and specifically each of the five strands of mathematical proficiency (Kilpatrick et al., 2001; NCCA, 2013). The work of Hiebert and Grouws (2007) indicates that effective teaching can occur with various forms of classroom organisation. Further work with regard to pedagogy might examine the effectiveness of different instructional methods in achieving mathematical proficiency through the present curriculum. Implications for eventual curriculum change include addressing the time factor – the major reported constraint to implementation of intentions – and the apparent mismatch between the pedagogies encouraged and those rewarded in the high-stakes examinations.

References

Bryman, A. (2012). *Social research methods* (4th ed.). Oxford University Press.

Byrne, C., Prendergast, M., & Oldham, E. (2021). Reforming Junior Cycle: Lessons from Project Maths. In D. Murchan & K. Johnston, *Curriculum change within policy and practice: Reforming second-level education in Ireland* (pp. 125–142). Palgrave Macmillan.

Conway, P. F., & Sloane, F. C. (2006). *International trends in post-primary mathematics education: Perspectives on learning, teaching and assessment*. National Council for Curriculum and Assessment.

Cosgrove, J., Perkins, R., Shiel, G., Fish, R., & McGuinness, L. (2012). *Teaching and learning in Project Maths: Insights from teachers who participated in PISA 2012*. Educational Research Centre.

Groves, S. (2012). Developing mathematical proficiency. *Journal of Science and Mathematics Education in Southeast Asia*, *35*(2), 119–145.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age Publishing.

Irish Mathematics Teachers Association. (2012). *Project Maths and the Irish Maths Teachers Association*. http://www.imta.ie/wp-content/uploads/2016/02/PM-IMTA-doc.pdf

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R., & Dawson, A. (2013). *Research into the impact of Project Maths on student achievement, learning and motivation: Final report*. NFER.

Johnson, P., Freemyer, J. V., & Fitzmaurice, O. (2019). The perceptions of Irish mathematics teachers toward a curriculum reform 5 years after its implementation. *Frontiers in Education, 4*(13). doi:10.3389/feduc.2019.00013

Kärkkäinen, K. (2012). Bringing about curriculum innovations *OECD Education Working Papers, No. 82*. OECD Publishing (NJ1).

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy of Sciences – National Research Council.

Lyons, M., Lynch, K., Close, S., Sheerin, E., & Boland, P. (2003). *Inside classrooms: The teaching and learning of mathematics in social context*. Institute of Public Administration.

National Council for Curriculum and Assessment (NCCA). (2013). Leaving Certificate Mathematics syllabus: Foundation, Ordinary and Higher level. Retrieved from https://ncca.ie/media/2122/lc-mathematics-syllabus.pdf

Norman, G. (2010). Likert scales, levels of measurement and the "Laws" of statistics. *Advances in Health Sciences Education*, 15(5), 625-–632.

Oldham, E. (1980). Post-primary mathematics in Ireland: An analysis of patterns in curriculum change. In D. J. Murphy & V. Rice (Eds.), *Mathematics, science and the gifted child: Conference proceedings and occasional papers* (pp. 49–59). Department of Higher Education and Research, Trinity College Dublin.

Oldham, E. (2001). The culture of mathematics education in the Republic of Ireland: Keeping the faith? *Irish Educational Studies*, *20*, 266–277.

Oldham, E. (2019). “Modern Maths” and “Project Maths”: Polar opposites or mirror images? In L. Harbison, & A. Twohill (Eds.), *Proceedings of the Seventh Conference on Research in Mathematics Education in Ireland (MEI7*) (pp.211–218). Institute of Education, Dublin City University.

O'Meara, N., Fitzmaurice, O., & Johnson, P. (2017). Old habits die hard: An uphill struggle against rules without reason in mathematics teacher education. *European Journal of Science and Mathematics Education, 5*(1), 91–109.

O'Meara, N., & Prendergast, M. (2018). Time allocated to Mathematics in post-primary schools in Ireland: Are we in double trouble? *Mathematical Education in Science & Technology*, 49(4), 501–516.

Pegg, J. (2010). Promoting the acquisition of higher order skills and understandings in primary and secondary mathematics. *[Proceedings of] Research Conference 2010* (pp. 35–38). Crown Conference Centre.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.

van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 521–525). Springer.

Ability and Learner Identity in Irish Primary Mathematics Classrooms

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Children are constructed as learners of mathematics by virtue of their participation within the classroom. Through its modes of measurement, discipline, sanctions and rewards, the discourse of mathematics classrooms make children visible as mathematically (dis)abled subjects. From this discourse "ability" emerges. Teachers transmit their own values and definitions of ability to their class. Children come to view themselves as learners of mathematics through the messages communicated by their teacher and through comparison with their peers. This study employs a mixed methods design approach (Teddlie & Tashakkori, 2009) across eight primary school classrooms at second and fifth class level in DEIS and non-DEIS schools. A total of twenty-four mathematics lessons were observed, while eighteen mathematics lessons were video recorded. Focus group interviews with children, child questionnaires and drawings were also employed in the data collection. Eight teachers and eight school principals were interviewed. This paper will examine the discourse around ability by drawing on the perspectives of teachers and children.

Ability and Learner Identity

According to social practice theory, students come to the classroom as part of many diverse communities in which they have formed their identities and they have to reshape their identities as they participate in the community of the classroom. It is in this reshaping of identity that learning resides (Lee, 2006). "Identities are constructed within a context of activity, pupils build an identity, that is a way that they explain themselves, within each community in which they participate" (Holland et al., 1998 p.270). Similarly, "Mathematics in practice becomes an issue of identity as well as cognitive process" (Barta & Bremner, 2009 p.91). Enabling students to build an identity as someone who is able to do mathematics is an important aim for a mathematics classroom. Teachers are the most important resource for developing student mathematical identities (Cobb & Hodge, 2002; Hayes, Mills et al., 2006). They influence the ways in which student's think of themselves as learners (Walshaw, 2004). While learning mathematical skills and knowledge, students are also developing beliefs and attitudes about the subject, and themselves as mathematical learners and practitioners (Grootenboer, 2013). Teachers of mathematics are in a powerful position because they can significantly impact on the mathematical identity and the futures of learners through the nature of the relational pedagogy they practice in their classrooms. This is evident throughout the everyday, routine mathematics classes that teachers and students experience. Feedback is a crucial feature of the teaching-learning process. Bloom (1976) identifies feedback, correctives and reinforcements as important elements of the instructional process. Feedback is considered to be one of the structuring conditions for learning, and is included alongside such variables as task presentation, sequencing, level and pacing of content and teacher expectations (Gipps & Tunstall, 1998). A major determinant of self-esteem is feedback from others, therefore children's self-evaluations are very often a reflection of significant others' evaluations, such as parents, teachers and peers. As far as academic self-esteem is concerned, teachers' evaluations are the most important, particularly in the early years of schooling. Children develop their 'self-image' in school through observing and feeling not only how the teacher interacts with them, but also how the teacher interacts with the rest of the class (Crocker & Cheeseman, 1988). The development of a positive self-concept in children is dependent upon perceiving themselves as successful, this in turn may depend on the way the child interprets the teachers' reaction to his/her performances. Often teachers, students and parents speak about success in mathematics education with reference to the concept of talent and of an inborn capability of mathematical thinking (Gellert et al., 2001). According to Boaler (2013), mathematics is a subject area that communicates the strongest fixed ability messages and thinking. Repeated teacher references to the "difficulty" of a particular mathematics text can serve as regulative discursive moves that position students and teachers in relation to cultural norms regarding ability and achievement (De Freitas, 2010). Through classroom practices messages are constantly communicated to children by teachers and schools regarding their ability and learner identity and this is succinctly summarised by Meighan and Siraj-Blatchford (1998) who argue that:

> …pupils tend to perform as well, or as badly, as their teachers expect. The teacher's prediction of a pupil's or group of pupils' behaviour is held to be communicated to them, frequently in unintended ways; thus influencing the actual behaviour that follows. (p. 309).

This is particularly demonstrated in the grouping of students for mathematical learning where students of similar 'ability' are placed in different ability groups and whose later differing achievement has been attributed to the grouping effect. A Foucaultian reading of this situation sees students as subjects taking up top-middle-bottom positions that the discourses of classroom, school and home create and maintain in practice, normalising the structure that establishes and then perpetuates the inequality. Schooling more openly acts as a discursive domain productive of the mathematically able being such as the 'numerate child', created and maintained through a framework of practices of assessment (standardised testing and teacher designed tests). Dowling (1998) challenged the notion of "ability" as fixed and viewed schools as endeavouring to categorise and separate students, especially through the teaching of mathematics. According to Walls (2009) the right/wrong nature of mathematics as presented by teachers, textbooks, families and peers through social interactions, significantly contribute to students' mathematical identities and construct themselves as a learner of mathematics. Rowland (1995a) argues that a child's level of mathematical competence cannot and should not be judged by the child's offering of a "correct answer". Rowland (1995a) suggests that when a child volunteers an answer that is not the "expected" teacher answer, it is important to investigate and explicate the child's thinking and reasoning behind it. With reference to the linguist Lakoff, Rowland (1995b) demonstrates how in oral explanations, students use "hedges" as "a shield against being wrong" (p. 350). The rewards and privileges that come with being correct are great. Rowland (1995b) observes that there is a regrettable absence of regard for the role 'uncertainty' plays in the mathematics classroom. Teachers and in turn students fail to recognise that being in a state of "uncertainty" is a necessary precondition to learning and that in "the making and learning of mathematics, uncertainty is to be expected, acknowledged and explicit" (Rowland, 1995b, p. 328). Recent research carried out by Boaler (2013) into ability and mindset in the mathematics classroom reveals that the types of tasks chosen by teachers communicate powerful messages regarding what mathematics and knowledge is important. Tasks convey what doing mathematics is all about. By engaging in tasks, students develop ideas about the nature of mathematics and mathematics learning (Anthony & Walshaw, 2009; Hodge et al., 2007). If children are assigned short, closed mathematics questions that have right or wrong answers and children are regularly getting them incorrect, it is very difficult to sustain the opinion that high achievement is possible with effort. In contrast, when tasks are open, with opportunities for learning, children can see the possibility of greater achievement and respond to these opportunities to improve (Boaler, 2013).

Methodology

In Irish primary schools, children are seldom consulted or given the opportunity to formally express or document their experience of learning in school. The pervasive social lens through which children's learning is examined is an adult one. This study employs a mixed methods design approach (Teddlie & Tashakkori, 2009) across eight primary school classrooms at second (7- and 8-year olds) and fifth class level (10- and 11- year olds). To allow for comparative analysis of social contexts, it was necessary to enlist two DEIS schools and two non DEIS schools at both second and fifth class level. All eight research schools were co-educational. Pseudonyms were used for the research schools and participants.

Table 1

Overview of Research Schedule and Research Methodology

September	Piloting of Children Questionnaire & Visiting the research schools			
October	**Schools**	***Visit One***	***Visit Two***	***Visit Three***
	Roadstown Quarryfield Bridge St Brookwood	Observation of Mathematics Lessons	Observation of Mathematics Lessons	Observation of Mathematics Lessons
	Abbeyside Knockbrack Summerville	Children's Drawings	Children's Questionnaire	Focus Group Interviews
	Mount Eagle	Focus Group Interviews	Focus Group Interviews	
November	Teacher Interviews (6 female, 2 male)			
February	School Leader Interviews (4 male, 4 female)			
Quantitative Instrument Design	***Child Questionnaire n =164***	Quantitative Analysis SPSS		
Qualitative Instrument Design	***Classroom observations n =24***	Thematic Analysis -Nvivo		
	Child Drawings n = 144	Thematic Analysis - Nvivo		
	Interviews with Children n = 40	Thematic Analysis - Nvivo		
	Interviews with Teachers n = 8	Thematic Analysis - Nvivo		
	Interviews with School Leaders n = 8	Thematic Analysis - Nvivo		

Mathematical Ability: The Perspective of Teachers

> Maths is a switch off thing…it develops as children go through school, being shown these mathematical things, seem complicated to them, try them and they get them wrong and feel like they have failed. So the next time they don't listen for as long…they say 'I'm not going to get this', or 'someone else is better at this than me', 'maths isn't my thing'…so they are being set up for failure (Tom, Principal, Roadstown, DEIS).

Teaching mathematics has "always been hard but in recent years with the addition of children with special needs in the class that's just making it certainly difficult" (Ms Rice, Brookwood, 2nd Class, non-DEIS). Schools in designated disadvantaged contexts embraced new initiatives and acknowledged that children "have the best of resources and excellent teachers" (David, Principal, Abbeyside, DEIS). However, in spite of this "it has all been a conundrum as to why our numeracy scores were so below the average" and "if they don't improve after all this [initiatives] there is no hope" (David, Principal, Abbeyside, DEIS). The dialogue in these schools revolve around performativity where educational success is that which can be measured and quantified. School leaders and teachers share clear views on mathematics which are both overtly and covertly transmitted daily across classrooms "it's hard enough to make maths interesting at the best of times. You either find it easy or you can find it very difficult" (Denis, Principal, Knockbrack, DEIS). Schools perpetuate the notion that people are "good" or "bad" at mathematics. The demarcation of people into "can" and "cannot" do mathematics suggests that very little can be achieved with children who are not innately adept at mathematics. Teachers shared a common discourse around ability. Ability was defined both positively and negatively. Positive attributes included being "high flyers", "very very good", "bright" and "stronger". The negative form of ability encompassed the innate "their own overall weakness", "low intelligence", being "less able", "weaker", "struggling with the curriculum" "lower achieving" and "not grasping the topics". Ability was also something that was measured in standardised tests "like you get from maybe the 7 or 8 Sten right down to not even registering on the scale you know" (Ms Cooper, Abbeyside, 5th Class, DEIS). For many teachers speed was synonymous with high ability "very quick you know they would be very quick working out answers" (Ms Keane, Knockbrack, 5th Class, DEIS). Teacher descriptions of children's progress in mathematics revealed much about how they viewed children's mathematical ability, the criteria for success or failure in mathematics and the labels and names used to classify children of differing ability. For some teachers, knowledge and competence revealed itself through the visible form of "hands up" and giving the "right answer". Firmly established across classrooms was the belief that in mathematics something was either right or wrong "Someone like Cody would be forever with his hand up to give you the answer and he'll always have the right answer" (Ms Bosworth, Bridge St, 2nd Class, non-DEIS). It was not uncommon for second-class children to have experienced anxiety when it came to mathematics. This was evident through classroom observations, children interviews and their drawings. The teacher recognised that anxiety and fear was something experienced by some children but in her accounts did not consider a possible connection between mathematics anxiety and teacher expectations:

> Julie is the little girl I had in at the door today crying that she didn't do well but she'll sit back because she knows this is very hard. She won't want to come up and try the whiteboard, she would be the type of child who doesn't want to get things wrong… she is a perfectionist and so she doesn't want to be seen to get anything wrong. (Ms Rice, Brookwood, 2nd Class, non-DEIS).

In the senior classes teachers attributed ability and "being good" at mathematics to be the by-product of attentive listening and children's adherence to the classroom norms and rules:

> Sarah...you wouldn't think that she was looking or listening. (Mr Keating, Summerville, 5th Class, non-DEIS).

> Tom and Patrick I say the fact that they are able to listen very carefully and watch and maybe being able to translate that into their work that they can follow the method and then apply it themselves on their own individually. (Mr Keating, Summerville, 5th Class, non-DEIS).

For some fifth class teachers, a child's attitude towards mathematics was a significant factor in determining performance in the subject "Stephen would be quite weak... he would be quite negative towards maths…he would give up quite easily and he needs a lot of support just to keep him interested in it" (Ms Keane, Knockbrack, 5th Class, DEIS).

Throughout teachers' accounts was the implication that a lack of mathematical ability was solely the fault of the child and success in mathematics was the reward that came with "listening" and "paying attention". The association between teachers' own pedagogical practices and how it impacts upon children's learning was ostensibly absent from their accounts.

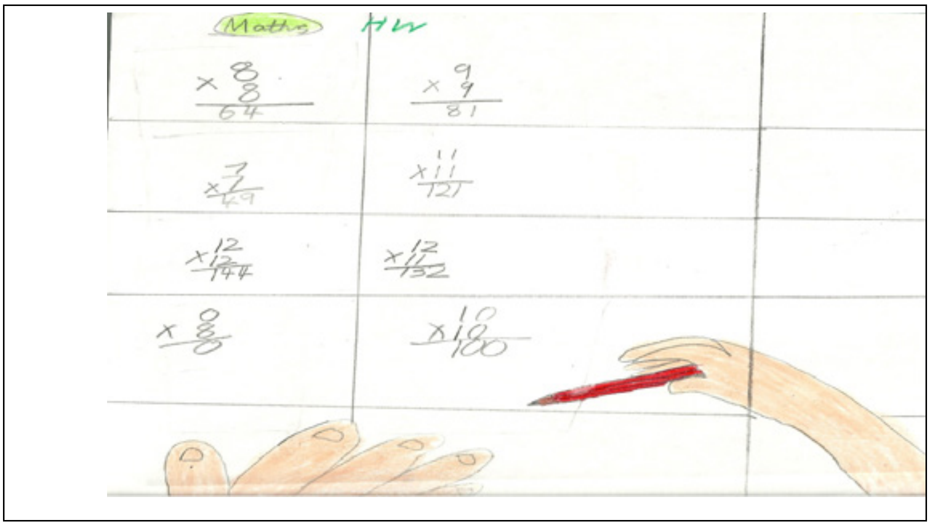
Children's Perspectives on Ability

> You can pay attention and just not 'get' what is going on (Diana, 5th Class, Mount Eagle, Middle Class).

According to one child being good at mathematics was attributed to the innate ability of having "a good brain" (Tony, Roadstown, 2nd Class, DEIS). For many children at second and fifth class level their identity as learners of mathematics is determined by speed activities, "being right" and "never getting maths wrong". This is reflected in child accounts and drawings "I don't normally get things wrong" (Aidan, Brookwood, 2nd Class, non-DEIS) and "I don't think I am very good at maths because I don't think I'm very good at understanding it" (Julie, Summerville, 5th class, non-DEIS).

Figure 1

Getting Maths Right (Summerville, 5^{th} Class, non-DEIS)



Comparisons with peers emerge as children rank themselves often unfavourably against their peers. As early as second class, children reveal a fixed mind-set regarding their ability where people are categorised into those who "can" and "cannot" do mathematics. The day-to-day experiences of the mathematics classroom affirms children in their personal held views of their ability:

Researcher: Would you say you are good at maths Keith?

Keith: I'd say alright, wouldn't be the best.

Cora: Well we have this real smart boy…Billy McCann and if you say real fast "what's 12 multiplied by 2?", he'd say the answer straight away.

Keith: Yeah he is smart. (Knockbrack, 5^{th} Class, DEIS).

and

Researcher: How can you tell if somebody is very good at maths?

Amelia: The first person ready. The last person ready, they need help with maths most.

Jacob: With the two lads…it's basically because they don't listen.

Researcher: Okay, so listening is important.

Zain: I am finished like fifth.

Jacob: I am finished like sixth. (Abbeyside, 5^{th} Class, DEIS).

Children could identify peers for whom mathematics was a challenge "Sometimes they find it very hard to do maths. It was never their favourite subject and they're not good at it" (Zain, Abbeyside, 5^{th} Class, DEIS). At the end of the third visit a girl spoke about a boy named Jeffrey and other children who sat at the same table whom she identified as "not good at maths" and explained that Eoin who is a good student "[Eoin] sits at their table to make them look smarter" (Abbeyside, 5^{th} Class, DEIS).

Conclusion

Children in this study constructed an image of themselves as learners of mathematics through the messages communicated by their teachers and through comparison with their peers. As early as second class children in this study displayed fixed mindsets regarding their ability in mathematics and the day to day experiences in the classroom affirmed them in their personal held views of their ability. The challenge for schools is to examine the fixed mindset culture that exists and to move beyond practices that label or define some learners as deficit and encourage teaching practices that value thinking, struggles and varied learning pathways of all children.

References

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the west. *Journal of Mathematics Education, 2*(2), 147-164.

Barta, J., & Bremner, M. (2009). Seeing with many eyes: Connections between anthropology and mathematics. In B. Greer, S. Mukhopadhyay, A. Powell & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 85-109). Routledge.

Bloom, B.S. (1976). *Human characteristics and school learning*. McGraw-Hill.

Boaler, J. (2013). Ability and mathematics: the mindset revolution that is reshaping education. *FORUM, 55*(1), 143-152.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*, 249-284.

Crocker, A.C., & Cheeseman, R.G. (1988). The ability of young children to rank themselves for academic ability. *Educational Studies, 14*(1), 105-110.

De Freitas, E. (2010). Regulating mathematics classroom discourse: Text, context and intertextuality. In M. Walshaw (Ed.), *Unpacking Pedagogy: New perspectives for mathematics classrooms* (pp. 129-151). Information Age Publishing Inc.

DES. (2011). *Literacy and numeracy for learning and life: The national strategy to improve literacy and numeracy among children and young people 2011-2020*. Stationery Office.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. Falmer Press.

Gellert, U, Jablonka, E., & Keitel, C. (2001). Mathematical literacy and common sense in mathematics education. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 85-102). Lawrence Erlbaum Associates.

Gipps, C., & Tunstall, P. (1998). Effort, ability and the teacher: Young children's explanations for success and failure. *Oxford Review of Education, 24*(2), 149-165.

Grootenboer, P. (2013). The praxis of mathematics teaching: developing mathematical identities. *Pedagogy, Culture and Society, 21*(2), 321-342.

Hayes, D., Mills, M., Christie, P., & Lingard, B. (2006). *Teachers and schooling making a difference. Productive pedagogies, assessment and performance*. Allen & Unwin.

Hodge, L., Zhao, Q., Visnovska, J., & Cobb, P. (2007). What does it mean for an instructional task to be effective? In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Vol. 1, pp. 329-401). MERGA.

Holland, D., Skinner, D., Lachicotte, W., & Cain, C. (1998). *Identity and agency in cultural worlds*. Harvard University Press.

Lee, C. (2006). *Language for learning mathematics: Assessment for learning in practice*. Open University Press.

Meighan, R., & Siraj-Blatchford, I. (1998). *A sociology of educating* (3rd ed.). Cassell.

Rowland, T. (1995a). Between the lines: The language of mathematics. In J. Anghileri (Ed.), *Children's mathematical thinking in the primary years: Perspectives on children's learning.* Continuum.

Rowland, T. (1995b). Hedges in mathematics talk: Linguistic pointers to uncertainty *Educational Studies in Mathematics, 29*, 327-353.

Teddlie, C., & Tashakkori, A. (2009). *Foundations of mixed methods research*. Sage Publications.

Walshaw, M. (2004). A powerful theory of active engagement. *For the Learning of Mathematics, 24*(3), 4-10.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematics lives.* Springer.

Resources, Representations and Reasoning: Young Children's Analytic Approaches in Solving Number Tasks

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This paper presents and discusses some findings of an intervention with first class boys investigating their use of mathematical resources and representations to aid and highlight mathematical reasoning. Across a series of reasoning tasks in the area of number, the boys were encouraged to solve, represent and discuss their solution attempts. Some interesting features of reason arose in this study. Children drew on mathematical resources and representations made available to them to aid and present further mathematical reasoning. In some cases, they extended this reasoning to evaluate the usefulness of particular resources and justify their adoption or rejection of them as well as to justify their choice of representations in line with their need for support as they moved toward mental methods. Some examples of the children's use of representations and reasoning are presented, with a focus on evaluation of representations as to the most mathematically appropriate and useful representations.

Reasoning and Representations

Mathematical reasoning is a complex cognitive process, which depends on knowledge and understanding. It involves the use of mathematical evidence and facts to support conclusions (Sarama & Clements, 2009). It stems from careful consideration of alternatives, and includes knowledge of how to justify conclusions (Kilpatrick, et al., 2001). A process of "noting patterns, generalizing relationships, making conjectures, questioning, and evaluating or constructing arguments and ideas" is necessary for the development of reasoning skills. Mathematical reasoning is an integral part of doing mathematics (Chapin et al., 2009, p. 78). It includes not only informal explanations and justifications but also intuitive and inductive reasoning based on pattern and analogy (Kilpatrick et al., 2001). The legitimacy of reasoning can be supported by discussing concepts and procedures; by representing problems, solutions and their understanding of mathematics in multiple ways; and by offering good reasons for the procedures and strategies they employ (NCCA, 2017).

Many forms of representations exist to convey mathematical ideas and develop reasoning skills: pictures, concrete materials, tables, graphs, diagrams, equations and symbols are just some of the forms used by mathematicians (National Council of Teachers of Mathematics, 2000; Lesh, Post and Behr, 1987). The process of organizing, recording and communicating mathematical ideas is aided by manipulation of objects or diagrams (Turner, 2013). An aid to seeing relationships in mathematics, materials are 'picturable' in that they can be remembered through mental imagery rather than through a word or symbol, which helps children to internalise their experience (Resnick & Ford, 1981). Representations can be used as tools to help record mathematical ideas, communicate thoughts and clarify understandings (Chapin et al., 2009). It is important for children to manipulate ideas to make connections between different representations as interacting with resources supports children in the process of discovery and in constructing meaningful understanding. These experiences can be enriched by teacher support eliciting and supporting children's mathematical connections (Turner, 2013).

Representational systems can be considered as vehicles of thought in mathematics (Nickerson, 2009). Representations highlight specific aspects of a mathematical concept which can support the process of explanation and develop understanding (Kaput, 1991; Ainsworth, 1990). Where representations appropriately capture mathematical concepts, then they can be used as tools to model and describe mathematical meaning and understanding and therefore promote the development of reasoning skills.

Mathematical Discussion

Just as representations do not contain but display mathematics, language does not transport knowledge but is a very powerful tool to orient children's conceptual construction by engaging them in discussion and explanation (Von Glasersfeld, 1991). According to Walshaw and Anthony (2008), mathematical discourse makes students' reasoning visible and open for reflection. Teachers play a role in engaging students in thoughtful mathematical conversations to develop explanations, make predictions, debate alternative approaches, clarify, or justify their thinking" (Brophy, 2001, p. 13). When learners engage in mathematical conversations they construct and improve their understanding through the exploration of ideas. Walshaw and Anthony (2008) state that environments where thoughts are shared enable students' own ideas to become resources for their learning. This makes it possible for them to communicate, reflect on, and evaluate their own and others ideas. The interplay between engaging with and evaluating mathematical reasoning and representing, through expertly guided discussion, creates an environment for powerful mathematical learning.

Format of the Study

This study was conducted in an all-boys school and a first class setting. The aim was to investigate the ways in which the children used representations to support reasoning, and to examine the roles of collaboration, and of the teacher in fostering reasoning. The work of two groups comprising ten children, was selected from whom to gather data as they engaged in a series of high-quality number tasks. Working in small groups, the boys were encouraged to record their solution attempts using the familiar representations of grouped and ungrouped base ten materials, counters, the empty number line, as well as pencil and paper and mental methods. Data sources included audio-recordings of children's interactions among their groups and with their teacher, work samples, researcher field notes and a reflective journal. The intention was to gather data across a period of five weeks, however the government-mandated closure of schools in response to the emerging COVID 19 first wave meant that data collection abruptly ceased after two weeks. Notwithstanding this, analysis of the available data sources allowed for an appropriate treatment of the research questions and for the emergence of some not entirely expected features.

Discussion

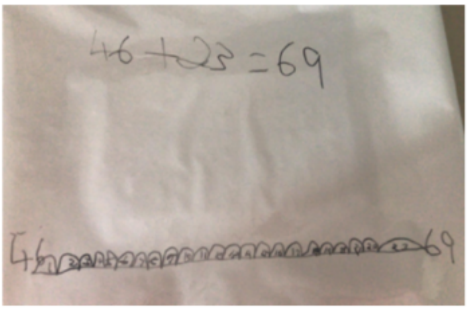
The examples presented below are chosen to demonstrate instances where children, engaging in quality collaborative tasks supported by access to representations and careful teacher questions, were able to move among representations to explore and express mathematical ideas and to evaluate their choices in connection to others children's representations, and in some cases to their own limitations. In addition, some examples are presented in which children evaluated the available representations in terms of their suitability for the task at hand. In all cases, pseudonyms are used.

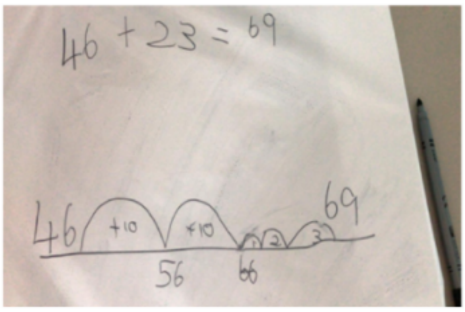
Evaluating Strategies in Peer Discussion

An indication of meaningful interaction both with the representation and the classroom discourse to refine thinking (Chapin et al, 2009) is exemplified in the task "True or false?" where children were asked to demonstrate whether double digit computations were correct. Different approaches using the same representation – the empty number line provided opportunities for children to display their mathematical thinking. Henry initially began by using an empty number line in single jumps (units) for the calculation 46+ 23= 69. In figure 1 below Henry adopted a 'count by ones' approach, while Andrew partitioned into tens to create more efficient jumps and modelled his approach to the group. In whole-class discussion, swift consensus was reached that Henry's approach was 'effective'. Now aware of the disadvantages of his chosen method, Henry altered his approach to incorporate his knowledge of place value to make his number line more efficient. His comment "Yeah, I'm doing jumps of tens. I forgot them, that's why I was taking so long. That's why Andrew finished before me!" shows his adoption of a more sophisticated way to manipulate the representation and his reliance on a peer to make this apparent. Interacting with the familiar representation allowed him to explore the mathematics, but the refining of the strategy was the result of seeing, discussing and evaluating his peer's approach in comparison to his own (Walshaw and Anthony, 2008).

Figure 1

Henry and Andrew's Approach to Proving an Addition Calculation





Flexibility in Approaches

Children proved very capable of using and rationalising varying strategies based on the numbers in a calculation. Although Andrew had initially used a number line to add, when working on a subtraction calculation in a similar numerical range, he chose to calculate in his head using mental representations of place value. He explained, "The units are the same so I can just get rid of them and count back the tens." Here he demonstrates he is able to reason his choice of selected method based on its effectiveness in the context of the problem. In addition to evaluating their reasoning in the context of the representations, the children also showed insight into their own inclinations and needs for support from representations (Walshaw and Anthony, 2008). For example, Roger used partitioning as part of an informal approach to addition. He partitioned the 2 digit numbers into tens and units. Adding the tens, then the units and then recombining, claiming “I don’t need the number line to keep track.” Andrew explained, I did a number line so I don’t get confused.” Varying degrees of reasoning reflected the children’s stage of understanding, “I used a number line because that’s the one I like. I like it ‘cause you can see where you are” (Henry). Henry chose to rely on a number line as it was the most familiar and enabled him to keep track of his work whereas Terry considered the efficiency of the approach, “I used place value because that’s the quickest for me. I did it in my head” as did Andrew: “I used place value for my jumps because it’s quicker when you have bigger numbers”. Methods were chosen based on their perceived usefulness whether it was the support of a number line or the efficiency of a mental approach (Fuson et al., 2005). The range of approaches adopted by the children and the flexibility throughout the reasoning activities made it clear that they reasoned with strategies or approaches selected to suit at some times the context and at others their aptitudes and abilities.

Children were able to evaluate strategies and resources based on their relevance to the task at hand, using different approaches to solve and check the calculations provided with the efficiency of methods and resources also considered. Boaler et al, (2017) suggests that fluency with known facts and ideas can assist children’s ability to reason numerically. The examples above provide insight into children with varying levels of acquired facts, which is exposed in the differing levels of support sought from the representations as opposed to reasoning mentally. The different and changing approaches used demonstrate the children show insight into the personal *and* mathematical factors influencing their choice of method or representation and can understood that there are multiple approaches that can be used to solve the calculations (Fuson et al., 2005). The speed and efficiency advantage to some children of using flexible number sense approaches was stated alongside frank acknowledgement by others of the benefit of using an empty number line in terms of providing clarity and guarding against confusion.

Evaluating the Utility of Representations

A particularly interesting aspect of reasoning which arose was the children’s evaluation of representations in the context of the task, and their awareness of the mathematical limitations and affordances of various representations. The role of the teacher was key here in sparking at least some of these discussions. Two examples are presented below:

Teacher:	Is the 20 frame helpful here?
Kelvin:	Not really.
Teacher:	Why not? What would be helpful?
Jim:	Cause it’s a 20 frame and that’s 2 ten frames but we need 3 groups.
Kelvin:	You could use groups of 10 (Dienes blocks).
Jim:	Yeah, you can’t stack that many blocks.

In the above brief excerpt, the teacher's suggestion is rejected on valid grounds, and the children can volunteer both a suitable and unsuitable alternative. The children are not merely evaluating the mathematics in terms of the representations (Kaput, 1991), but they are evaluating the representations in terms of the mathematics. Once again, just as they displayed flexibility in approaches above, they display flexibility in choosing useful tools for representation, choosing and rejecting them as the task demanded. Below we see that Ed is just as able to spot the limitations of Dienes blocks for this task as Kelvin was to see their benefit in the previous example.

Teacher:	How could we use them then to find all our combinations?
Jim:	You can split them and turn them into different ones.
Ed:	That's why you don't use blocks of ten. You can't split them. You need to use counters.
Jim:	Yeah, you can make more numbers if you use the counters.

In these examples, where the children evaluated and refined their representation methods in peer discussion; responding to and building on each other's statements provided them with opportunities to construct and refine their understanding (Brophy 2001). The conversations here are not influenced by personal preference for efficiency or stability offered by the one or the other method, but are squarely on the suitability of the tool to adequately represent the mathematics. The only distinction was on how the representations lent themselves to the task

Conclusions

The above features of the children's reasoning exemplify the complexity of developing the practice. The range of approaches adopted by the children and the flexibility throughout the reasoning activities made it clear that working on the tasks required more of them than consideration of the mathematics, but necessitated reflection on the problem and the selection of an appropriate way to represent it. Fuson (1988) suggests that children's number sense develops as they acquire the ability to be flexible with numbers. It had been anticipated that the children would engage with the resources, choosing and appropriately using them as tools to think with. However, the insight into the connections between their choice of representation and the level of support they required from it was a level of metacognition that was not foreseen. In addition, considering and evaluating the usefulness of the resources within the contexts of the tasks had not been anticipated, nor flagged in literature sources consulted. The children demonstrated they were capable of reflecting on their own and others thinking by evaluating and critiquing the strategies and resources being used. Beyond that, they showed insight into how these strategies were appropriate to their own preference and perhaps state of learning; and how the available resources aligned with the mathematical demands of the task. In this process that appears to have been supported by the teacher who questioned the children's approaches and provided with opportunities to describe and justify thoughts and actions (Katz, 2014). Moreover, attention was frequently drawn to the ways students were thinking about their approach and solution to generate shared understandings (Boaler et al., 2018).

References

Ahlberg, A. (2011). Communicating mathematics in primary school. In Emanuelsson, J., Fainsilber, L., Häggström, J., Kullberg, A., Lindström, B., & Löwing, M (Eds.), *Voices on learning and instruction in mathematics* (pp.79–95). National Centre for Mathematics Education

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education,* 33(2-3), 131–152. https://doi.org/10.1016/s0360-1315(99)00029-9

Chapin, S. H., O'Connor, C., & Canavan Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn*. Math Solutions.

Boaler, J., Munson, J., & Williams, C. (2017). *Mindset Math, Grade 4*. Newark: John Wiley & Sons, Incorporated.

Bills, C. (2000). Metaphors and other linguistic pointers to children's mental representations. In C. Morgan & K. Jones (Eds.), *Research in Mathematics Education Volume 3,* (pp. 141-154). British Society for Research into Learning Mathematics

Cotton, T. (2010). *Understanding and teaching primary mathematics*. Pearson.

Dijk, E. F., van Oers, B., & Terwel, J. (2004). Schematising in early childhood mathematics education: Why, when and how*?. European Early Childhood Education Research Journal*, 12(1), 71-83. https://doi.org/10.1080/13502930485209321

Kaput, J. J. (1991). Notations and representations as mediators of constructive processes. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 53-74). Kluwer.

Kilpatrick, J., Swafford, J., Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* National Academy Press. Retrieved from https://doi.org/10.17226/9822

Lesh, R., Post, T. R., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janiver (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum.

National Council for Curriculum and Assessment. (2017). *Primary mathematics curriculum: Draft specification - junior infants to second class. For consultation*. NCCA.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.

Nickerson, R. (2009). *Mathematical reasoning: Patterns, problems, conjectures, and proofs*. Taylor & Francis. Retrieved from https://ebookcentral-proquest.com.dcu.idm.oclc.org

Resnick, L.B. & Omanson, S.F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in institutional psychology* (pp. 41-96). Erlbaum.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Routledge.

Sarama, J. & Clements, D. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. https://doi.org/10.4324/9780203883785

Threlfall, J. (1996). The role of practical apparatus in the teaching and learning of arithmetic. *Educational Review*, 48(1), 3-12.

Turner, S. (2013). *Teaching primary mathematics*. SAGE

Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of applied developmental psychology*, *18*(1), 37-54. https://doi.org/10.1016/s01933973(97)90013-7

Van Oers, B., & Poland, M. (2007). Schematising activities as a means for encouraging young children to think abstractly. *Mathematics Education Research Journal*, 19(2), 10-22.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121-131. https://doi.org/10.1111/1540-5826.00068.

Private Lives: The Work of Mathematics Leaders in Irish Primary Schools

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Little is known of the enactment of subject-specific leadership across our education system. This national deficiency is aptly exemplified by our collective ignorance of mathematics leadership in the primary school sector. This research sought to address this gap by focusing upon ten individuals who self-identified as local mathematics leaders. Specific strands of inquiry included the nature of their duties, their generalised working habits, the supports they accessed and the skillset that they called upon in their work. Three research instruments were utilised to gather data: an initial participant questionnaire/profiler, a twenty-day participant activity log and a semi-structured interview format at the conclusion of the logging period. Following the merger of qualitative and quantitative data bases, a set of five cross-participant themes were identified for elucidation. Primarily, the themes addressed key findings encapsulating the critical influence of context upon the working emphases of the mathematics leader, the ever-growing complexity of the role, seeming contradictions within such leadership work, the universal absence of adequate time for mathematics leaders to lead, and, the apparent dearth of bespoke professional development and networking opportunities available to such personnel. This paper summarises the findings of an EdD. dissertation.

The Context

The critical influence of school leadership upon teacher efficacy and subsequent pupil outcomes is now an uncontested truth of modern education (Heck and Hallinger, 2014; Vale et al., 2010, and, Leithwood et al., 2008). Indeed, the latter authors emphatically note that "leadership acts as a catalyst without which other good things are quite unlikely to happen" (2008, p.28). This correlation between leadership efficacy and broader educational effectiveness has led to an understandable explosion in the literature dedicated to school leadership, particularly over the last two decades, both nationally and internationally. However, despite this enhanced examination of school leadership in Ireland particularly, little is known of the enactment of subject-specific leadership in the Irish context. Nowhere is this malaise more evident than in our unawareness of mathematics leadership in the primary sector. As Mathematics remains a core curricular area in our primary education syllabus, this lacunae is particularly unsettling. Coincidentally, this recognition also comes at a time of growing expectation being placed upon the mathematics teaching and learning provision in all schools (Department of Education and Skills (DES), 2017). Mathematics is not unique in this regard – across Ireland's primary and second-level sectors, middle and senior school leaders are being challenged to lead diverse curricular areas, each with their own intricacies and specialised demands. Policy makers are hungry for improvement in each discipline but are leaders adequately supported to realise these lofty ambitions? Are current leadership constructs enabling or undermining this drive? The practical leadership of mathematics as enacted in our primary schools, and as captured in this paper, provides an illustrative context to attempt to answer these questions. It is uncontested that school leaders cannot lead without assistance. Therefore, the research also strives to identify, and amplify the clamour for, practical supports and structures that mathematics leaders distinguish as being crucial to sustain their important work.

Learnings from the Literature

The research illustrates that distributed leadership patterns retain a strong foothold in most international school systems (Liu, 2020). Ireland's recently rebooted middle-management structures, still cling to the coattails of this international movement (Lárusdóttir and O'Connor, 2017). However, even the distributed model comes with the caveat that an authentic form of shared leadership requires the ceding of real power and influence to other competent, supported co-leaders (Shava and Tlou, 2018). Whilst alternative leadership styles tend to prioritise the people within the school organisation (such as transformational or servant leadership approaches), it is perhaps the instructional approach that best responds to the demands and specialisation of curricular leadership. Its emphasis on enhancing classroom pedagogy is as alluring as it is simplistic. Katterfeld teases out the practical implication of this style – the elucidation of academic purpose and high expectations, coupled with the creation of a "schoolwide focus on instruction through monitoring the progress of teaching and learning" (2014, p.1127). The Department of Education and Skills updated *Looking at Our Schools* leadership framework (2016) does begin to address the practical steps towards such an instructionally-focused form of school leadership in an Irish context. However, it's generalised, non-subject specific context is a missed opportunity to prioritise effective curriculum leadership. Similarly, the most recent DES circular (2018) aimed at reforming school leadership structures makes a small number of vague references to responsibility for curriculum development and implementation, but without any explicit subject-specific application. Indeed, the initially promising *Numeracy Link Teacher* role, first mooted during the early implementation of school self-evaluation in Ireland a decade ago, appears to have disappeared without trace within the primary school system. In this troubling context, it is particularly crucial to keep sight of the extensive international literature which links hands-on mathematics leadership and an improved instructional environment for pupils (Heck and Hallinger, 2014).

In the absence of relevant Irish research, the international literature reinforces the real phenomenon of localised mathematics leadership. Such activity is spearheaded by an assortment of constructs drawing from principal teachers, formally appointed middle management staff, volunteers and collaborative multi-member pods. The unique credibility of teacher leaders leading curriculum innovation and improvement among colleagues is a standout feature of the research (Jorgensen, 2016). Supports for these assorted leadership constructs varies from territory to territory, but Irish equivalents must enviously regard the dedicated release time, ongoing state-funded professional development and networking opportunities that are the norm elsewhere. In reality, a vast miscellany of curricular, pedagogical and organisational duties form the bedrock of these leaders' work (See Grootenboer et al., 2015; Sexton and Downton, 2014; Firestone and Martinez, 2007; Millett et al., 2004). Indeed, this specific literature directly contributed to the authors' isolating twelve commonly-accepted key mathematics leadership duties (See Table 1). It should be noted that this list is not exhaustive, but does convey the sheer breadth of the role. This survey formed an essential building block of this study. Furthermore, it should be acknowledged that with the guiding influence of subject-specific leadership charters, there currently exists a concerted effort in many North American school districts, as just one international example, to cover all the bases of mathematics leadership (See Balka et al., 2010; National Council of Supervisors of Mathematics, 2008 as exemplars of such guidance).

Table 1

Identified Duties of School Mathematics Leaders

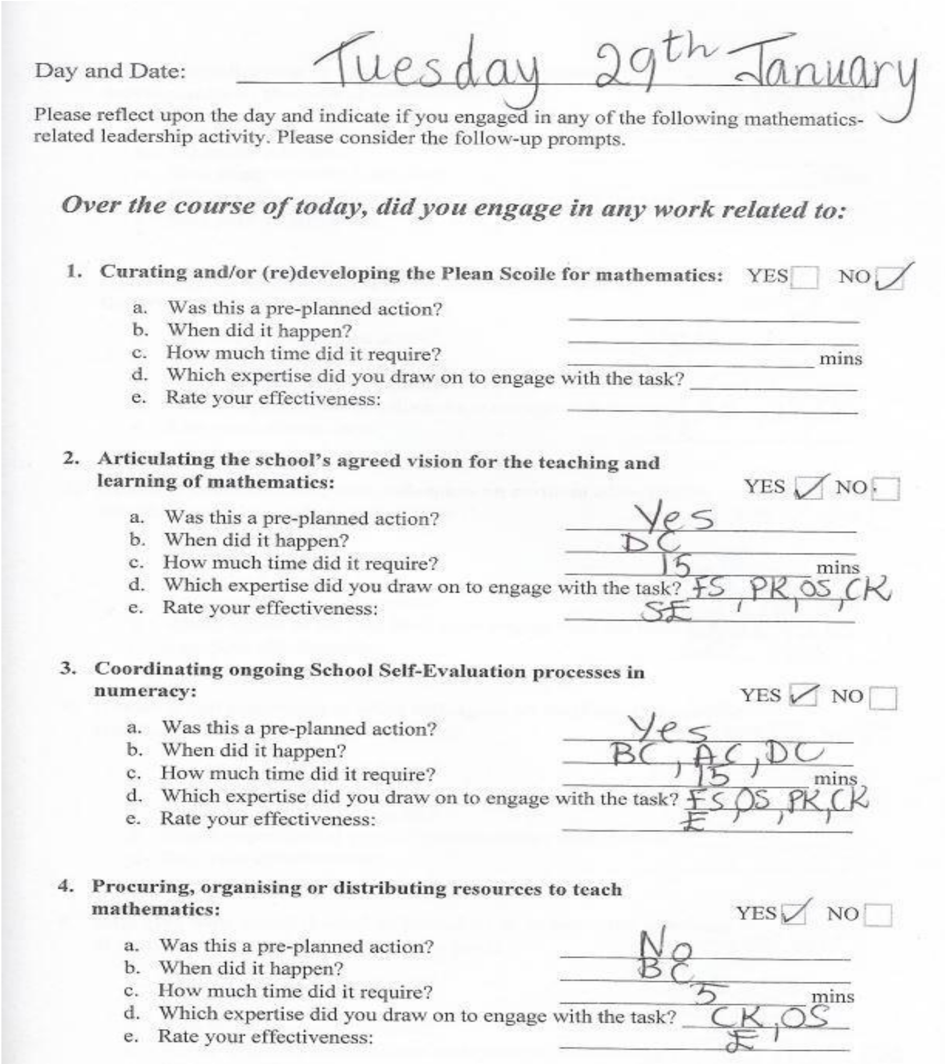
1. Curating and/or (re)developing the school plan for Mathematics.
2. Articulating the school's agreed vision for the teaching and learning of Mathematics.
3. Coordinating ongoing school self-evaluation processes in Numeracy.
4. Procuring, organising or distributing resources to teach Mathematics.
5. Informing colleagues of CPD opportunities and other new developments in the area of Mathematics.
6. Promoting the status and importance of Mathematics in the broader school community.
7. Advising and mentoring new colleagues on mathematics-specific teaching, learning and planning issues.
8. Advising and mentoring existing colleagues on mathematics-specific teaching, learning and planning issues.
9. Engaging with external services/providers to enhance the provision of mathematics teaching within the school.
10. Preparing materials for, and/or involvement in the administration of, student mathematics testing/other assessment.
11. Monitoring the standards of mathematics teaching and learning within the school.
12. Seeking and/or utilising the support of parents to enhance the teaching and learning capacity of Mathematics in school and/or at home.

This Research Project

This research focused upon ten individuals who self-identified as mathematics leaders. The representative cohort were drawn from both the principal (administrative and teaching) and teacher-leader communities. Leaders from DEIS, non-DEIS, rural, urban, developing and Irish language medium schools were represented in the sample. Some of the teacher leaders were unpaid volunteers who held a particular affinity for the subject. Specific strands of inquiry included the nature of the duties these mathematics leaders undertook, their generalised working habits, the supports they accessed and the precise skillset they exploited in their work. Data was gathered through an initial participant questionnaire/profiler, a subsequent twenty-day participant activity log and finally, a semi-structured interview format which built upon the insights gleaned from the logging data. Of the three instruments, the questionnaire/profiler provided the majority of qualitative data for the study. Inspired by Spillane and Zuberi's (2009, p.375) *Leadership Daily Practice Log*, each mathematics-leader participant was asked to complete a log of their mathematics leadership-related activities for a staggered four-week period during the 2018/19 school year. Additional detail (whether spontaneous or pre-planned, its scheduling and duration, the expertise it required and its overall effectiveness) was sought per logged action. For context, a screenshot of a completed page from one anonymised participant log is provided (See Figure 1).

Figure 1

Extract from Participant Activity Log



Day and Date: Tuesday 29th January

Please reflect upon the day and indicate if you engaged in any of the following mathematics-related leadership activity. Please consider the follow-up prompts.

Over the course of today, did you engage in any work related to:

1. **Curating and/or (re)developing the Plean Scoile for mathematics:** YES ☐ NO ☑
   a. Was this a pre-planned action? ______
   b. When did it happen? ______
   c. How much time did it require? ______ mins
   d. Which expertise did you draw on to engage with the task? ______
   e. Rate your effectiveness: ______

2. **Articulating the school's agreed vision for the teaching and learning of mathematics:** YES ☑ NO ☐
   a. Was this a pre-planned action? Yes
   b. When did it happen? DC
   c. How much time did it require? 15 mins
   d. Which expertise did you draw on to engage with the task? FS, PK, OS, CK
   e. Rate your effectiveness: SE

3. **Coordinating ongoing School Self-Evaluation processes in numeracy:** YES ☑ NO ☐
   a. Was this a pre-planned action? Yes
   b. When did it happen? BC, AC, DC
   c. How much time did it require? 15 mins
   d. Which expertise did you draw on to engage with the task? FS, OS, PK, CK
   e. Rate your effectiveness: E

4. **Procuring, organising or distributing resources to teach mathematics:** YES ☑ NO ☐
   a. Was this a pre-planned action? No
   b. When did it happen? BC
   c. How much time did it require? 5 mins
   d. Which expertise did you draw on to engage with the task? CK, OS
   e. Rate your effectiveness: E

Note. Abbreviations: **BC:** Before Contact/Teaching time, **DC:** During Contact/Teaching time, **AC:** After Contact/Teaching time, **FS:** Facilitation Skills, **PK:** Pedagogical Knowledge, **OS:** Organisational Skills, **CK:** Content Knowledge, **E:** Effective, and, **SE:** Somewhat Effective.

The Research Findings

The data-analysis process led to the generation of five overarching thematic findings:

Differing leaders, different activity emphases

Unsurprisingly, mathematics leaders were revealed as an industrious cohort across the twelve leadership activity domains identified within this study. Different types of leaders appeared to have more favoured aspects of their role. Principals and assistant principals tended to prioritise school development planning and numeracy-focused school self-evaluation requirements. They also made greater efforts to promote mathematics-specific professional development in their schools. Volunteer leaders were less enamoured with such administration, and were most prolific in offering informal mentoring to colleagues and in ensuring that mathematics equipment was readily accessible to co-teachers. These insights reinforce the international literature's assertion that the nature of mathematics leadership is mainly context sensitive - local leaders attempt to respond in a manner that is simultaneously cognisant of their school's particular needs, and their professional duty to prioritise what is most important therein.

PD Please, But Not As We Know It!

There exists a palpable hunger for a previously unexperienced form of professional development for mathematics leaders. This appetite is matched by an equally strong frustration with a current professional development offering that is solely focused on enhancing pedagogical know-how. Participants expressed a need for multi-faceted upskilling which encapsulates strengthening personal mathematics competency, enhancing interpersonal skills, building data-analysis and strategic planning nous, and cultivating a broader appreciation of the STEM configuration. The desire to interact with other mathematics leaders, and to build a community in order to support and share good practice was a recurring exhortation.

Mathematics Leadership and its Skill Set – Expert or Not?

Across eight mathematics leadership skills identified in the literature, participants revealed themselves as specialised and accomplished professional leaders. Simultaneously drawing on mathematical (both pedagogical and subject-matter based), analytical, interpersonal and logistical skillsets, this multi-disciplinary demand was evident in almost all of the captured leadership acts. This does not discount the more menial, logistical aspects of mathematics leadership which were ubiquitous in all settings. Despite a humble dismissal of the suggested expert tag by all participants, it is clear that mathematics leadership requires a skillset and a personal sense of mission which goes significantly beyond the norm.

The 'Do As I Say, Not As I do' Paradox.

Occasionally, the profiled leaders were unable or unwilling to make good on their intentions to lead in specific activity domains. Conversely, duties dismissed as being of lesser value sometimes featured significantly in the logged actions of many leaders. Two domains of activity stood out in this anomalous context: the infrequent instances of meaningful monitoring of the mathematics teaching and learning standards on a whole-school level, and, the preponderance of leadership acts and cumulative time devoted to the management of mathematics teaching resources and equipment.

Leaders shunned opportunities to oversee teaching and learning effectiveness, despite consistently referencing its criticality. Understandable local factors, cultural sensitivities and logistical concerns were all cited as obstacles. Some teaching leaders lamented the lack of release time to visit the class of colleagues during the working day; another remarked upon past experiences in her school where peer evaluation had ended unsatisfactorily for all parties; one leader cited the lack of written, explicit authority from the DES to visit the class of a peer. She was representative of a number of participants who felt that unless this leadership function was mandated by officialdom, supported by clear guidance, and, formally endorsed by the teaching unions, it remained unworkable. In a similar vein, some participants openly doubted the adequacy of their own mathematics knowledge base to pass judgement on the teaching of others.

On the other hand, despite downplaying its importance, many leaders devoted a considerable amount of their discretionary leadership time towards procuring, classifying, arranging and distributing mathematics resources and equipment. Interestingly, this kind of activity did serve a function as a gateway for volunteer or novice leaders to cut their teeth in school management. It should also be considered that in the case of more experiences leaders, their preoccupation with logistical duties can validly be rationalised by the human propensity "to gravitate towards doing what they know how to do" (Fink and Resnick, 2001 p.599), which may be further interpreted as remaining amicable and undemanding of colleagues.

Leading While Teaching – Mission Impossible?

It is undeniable that teacher leaders specifically are all the more effective because of their dual teaching and leading roles. They retain a foot in both camps and accrue added credibility with teaching colleagues for walking in their professional shoes. However, the primacy of their classroom function is most likely emasculating their leadership role. Participant after participant expressed profound frustration at the dearth of available time to lead. In parallel, their activity logs revealed snatched moments of leadership, shoehorned into a busy working day, sometimes occurring before and after contact time, and even during mandated breaks. This has inevitable consequences for the depth and quality of the hurried work they undertook. Significantly, participants who formed part of a mathematics leadership collective in their schools reported higher levels of role satisfaction and self-efficacy.

Recommendations

The study generated a set of recommendations which speak to an intended audience of mathematics leaders themselves, in-school management teams, boards of management, teacher support agencies, teacher unions and principal representative bodies, and the DES:

- Each school, irrespective of size, would hugely benefit from a formally appointed mathematics leader/leadership structure. Where relevant, this could entail a revitalization of the *Numeracy Link Teacher* role first envisaged by school self-evaluation guidance a decade ago.
- The DES should strongly consider the preparation of a mathematics-specific leadership framework to accompany its more generalised quality framework for school leadership and management.
- A broadly-based preparation and in-service support programme, devised and made mandatory for all aspiring and serving mathematics leaders, would provide unquestionable benefit to participants.

- Teacher unions, principal-representative bodies and other supporting agencies could collaborate to facilitate the creation of mathematics-specific leadership cells. In time, these localised collectives could become self-sustaining.
- Mathematics leaders can be encouraged to re-evaluate and prioritise the core aspects of their work. Within reason, leaders can be enabled to delegate the more clerical and logistical domains of activity that traditionally fall within their remit.
- Collaborative leadership structures, along the in-school management team (ISMT) model, provide a more sustainable form of mathematics coordination in schools. Schools, once adequately resourced by the DES, could explore the capacity of such structures.
- Dedicated release time must be made available to school leaders who, either individually or collectively, lead Mathematics in their school

All of these recommendations arising from the research are fundamentally contingent upon an acknowledgement of the local importance of the mathematics leadership role, and the potential it has to tangibly benefit the teaching and learning agenda at primary school level. It now falls upon all actors in this sphere to acknowledge these obvious truths. It is the writers' sincere hope that this study makes a small contribution to enhancing these painstakingly slow realisations.

References

Balka, D., Hull, T. & Harbin-Miles, R. (2010). A guide to mathematics leadership - sequencing instructional change. Corwin.

Department of Education and Skills. (2018). *Circular letter 0070/2018: Leadership and management in primary schools*. Retrieved from https://www.education.ie/en/Circulars-and-Forms/Archived-Circulars/cl0070_2018.pdf

Department of Education and Skills. (2017). National strategy: Literacy and numeracy for learning and life 2011-2020. Interim Review: 2011 – 2016. New Targets: 2017 – 2020. Retrieved from https://www.education.ie/en/Publications/Education-Reports/pub_ed_interim_review_literacy_numeracy_2011_2020.PDF

Department of Education and Skills. (2016). *Looking at our school 2016: A quality framework for primary schools*. Retrieved from https://www.education.ie/en/Publications/Inspection-Reports-Publications/Evaluation-Reports-Guidelines/Looking-at-Our-School-2016-A-Quality-Framework-for-Primary-schools.pdf

Fink, E. & Resnick, L. (2001). Developing principals as instructional leaders. *The Phi Delta Kappan,* 82(8), 598-606.

Firestone, W. & Martinez, M. (2007). Districts, teacher leaders, and distributed leadership: Changing instructional practice. *Leadership and Policy in Schools,* 6(1), 3-35.

Grootenboer, P., Edwards-Groves & Rönnerman, K. (2015). The practice of 'Middle leading' in mathematics education. *Proceedings of the 38th Annual Conference of the*

Mathematics Education Research Group of Australasia (MEGRA), (pp. 277-284). MERGA. Retrieved from https://files.eric.ed.gov/fulltext/ED572460.pdf

Heck, R. & Hallinger, P. (2014). Modeling the longitudinal effects of school leadership on teaching and learning. *Journal of* Educational *Administration,* 52(5), 653-681.

Jorgensen, R. (2016). Middle leadership: A key role of numeracy reform. *Australian Primary Mathematics Classroom,* 21(3), 32-37.

Katterfeld, K. (2014). Measuring leadership of math instruction: Investigating the validity of a survey scale for principals' leadership of middle school mathematics. *Journal of School Leadership, 24*, 1125-1154.

Lárusdóttir, S. & O'Connor, E. (2017). Distributed leadership and middle leadership practice in schools: A disconnect. *Irish Educational Studies,* 36(4), 423-438.

Leithwood, K., Harris, A. & Hopkins, D. (2008). Seven strong claims about successful school leadership. *School Leadership and Management, 28*(1), 27-42.

Liu, Y. (2020). Focusing on the practice of distributed leadership: The international evidence from 2013 TALIS. *Educational Administration Quarterly, 56*(5), 779-818.

Millett, A., Brown, M. & Askew, M. (Eds.). (2004). *Primary mathematics and the developing professional*. Kluwer Academic Publishers.

National Council of Supervisors of Mathematics. (2008). *The PRIME leadership framework*. Solution Tree Press.

Sexton, M. & Downton, A. (2014). School mathematics leaders' beliefs about their role when participating in a school mathematics project. *Australian Primary Mathematics Classroom,* 19(3), 3-5.

Shava, G. & Tlou, N. (2018). Distributed leadership in education, contemporary issues in educational leadership. *African Educational Research Journal, 6*(1), 279-287.

Spillane, J. & Zuberi, A. (2009). Designing and piloting a leadership daily practice log: Using logs to study the practice of leadership. *Educational Administration Quarterly, 45*(3), 375-423.

Vale, C., Davies, A., Weaven, M., Hooley, N., Davidson, K. & Loton, D. (2010). Leadership to improve mathematics outcomes in low SES schools and school networks. *Mathematics Teacher Education and Development, 12*(2), 47-71.

Accessing High Quality Mathematics Instruction in Measures: Reporting on a Constructivist and Project-based Approach in Primary Schools

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International and national assessments reveal that Irish primary pupils perform relatively poorly in measurement. This study explores the impact of a constructivist teaching intervention on pupil's understanding and experiences with measures. Two groups of sixth class pupils, control and intervention, from similar class sizes and school settings participated in the study. Pre-intervention assessments evaluated proficiency in Measures. The intervention group participated in a six-week intervention where pupils engaged in constructivist, project-based activities. They also participated in focus group interviews and surveys investigating their experiences and dispositions towards measurement. During the same period, the control group engaged with measures using a traditional textbook teaching approach. Following the intervention, both groups engaged in post-test assessments and a maths trail practical test to assess their hands-on measurement skills. The intervention group again participated in surveys and focus group interviews. Findings reveal that the intervention had little impact on content performance and minimal improvements in problem solving skills. The practical tests highlighted significant improvement for the intervention group in the 'hands on' application of skills such as physically measuring items, conservation, and estimation and more positive attitudes toward mathematics.

Introduction

Certain areas of the Mathematics Curriculum align themselves more readily to real life maths, where connections are made between the abstract world of the subject and everyday contexts. Nowhere is this more evident than the strand of measurement and its cognate areas of Length, Area, Weight, Capacity, Time and Money. Perplexingly, these are the same areas where Irish pupils perform below other content domains in international comparisons such as the Trends in International Mathematics and Science Study (TIMSS), Programme for International Student Assessment (PISA), and are also mirrored in our national studies. In the TIMSS 2015 study Irish pupils outperformed thirty-seven Organisation for Economic Development (OECD) countries. However, performance in geometric shapes and measurement was our lowest in the assessment and described as a 'relative weakness' when scrutinised against our mean score in maths (Clerkin et al., 2016). Similarly, our subscale score for the cognitive domains of 'Reasoning' was significantly lower than our overall maths scale score (compared to 'knowing', which was significantly higher). Similar patterns are evident in international and national assessments (Eivers et al., 2007; Surgenor et al., 2006), which recount Irish pupil's limited ability on items requiring higher order thinking skills particularly in real world contexts, and in the most recent TIMSS 2019 assessment (Perkins & Clerkin, 2019). Our more traditional instructional methodologies and reliance on textbook guidance may, we argue, contribute towards improving operation skills and procedural fluency while overlooking the higher order skills of Applying, Problem Solving, Integrating and Connecting.

Review of the Literature

Constructivism and Mathematics Education

A fundamental tenet of maths curricula in Ireland is child centred learning which places the child's personal experience at the heart of classroom pedagogy as active agents within an environment of discovery. Exploration, as opposed to teacher-led instruction, are seen as the scaffolds upon where incremental learning develops. The Mathematics Primary School Curriculum (NCCA, 1999) states 'a constructivist approach to maths learning involves the child as an active participant in the learning process' and that 'the importance of providing the child with structured opportunities to engage in exploratory activity cannot be over-emphasised' (p.5). Drafted plans for the prospective curriculum are equally congruous to this idea. Situating the child and their exploration at the centre of maths classroom methodologies traces its roots to the theoretical framework of constructivism. Fosnot (2005) aptly describes constructivism as what 'knowing' is and how one comes to 'know'. Indeed, Glasersfeld (1996) emphasises that we do not and cannot 'share' meaning; knowledge is acquired through involvement with content instead of imitation or repetition. Teachers who base their practice on constructivism reject the notion that meaning can be passed on to learners via symbols and transmission; that learners can incorporate exact copies of teacher's understanding for their own use; that whole concepts can be broken into discrete sub-skills and that concepts can be taught out of context. Indeed, constructivist teachers reject such pedagogies of control or telling (Larochelle et al., 1998) and embrace constructivist-informed teaching styles that mark a conscious effort to move from traditional, didactic and memory-oriented transmission models (Cannella & Reiff, 1994) to more student-centred approaches.

The Role of Textbooks

It is difficult to envision how mathematical understanding can be deepened and developed without incorporating opportunities for mathematical discussion and a forum where students' collaborative efforts can be communicated and visualised. Textbooks tend to portion learning into bite-sized pieces where little thinking is needed, and mathematical complexity is minimised. Memorising facts and formulas, Delaney (2016) contends, is often valued over presenting tasks that require thoughtful and creative responses. Problems are instead presented to pupils in a predefined way, an approach Noddings (1985) argues that removes the opportunity to wrestle with the problematic situation. Consequently, Delaney (2012) argues that traditional reliance on textbooks in Ireland is not conducive to developing children's problem-solving abilities, since, as he highlights, many of the problems in Irish maths textbooks are of poor quality. Indeed, the 2014 National Assessments of English Reading and Mathematics reports that instructional textbooks and teacher-led maths instruction acted as the main method of current classroom practice and report that 'a lower allocation of time to teaching the mathematical processes of reasoning and applying … compared with the more basic process of Knowing' (Shiel et al., 2014, p.4). Similarly, evaluation of the implementation of the revised curriculum in maths (DES, 2005a) found that in a significant number of classrooms, there was an overemphasis on didactic methodologies, teacher talk and the use of a single textbook. These findings were supported by Surgenor et al. (2006) who report that 83% of time in maths classes was allocated to instruction, with 95.5% of pupils reporting that the class textbook was afforded daily usage. Conversely, only 8.7% of over 4,000 pupils surveyed experienced daily contact with concrete materials. That said, more recent studies provide welcome evidence of a decrease in the reliance on textbooks and increase in the use of manipulatives in teaching of mathematics at primary level (O'Meara et al., 2019; Johnson et al., 2020).

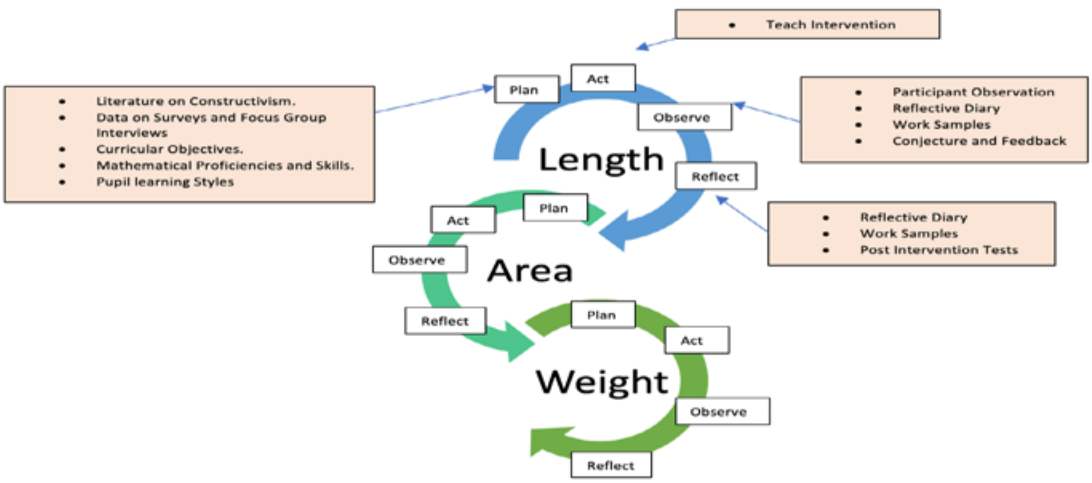
Methodology

This action research study (see Figure 1) was a pre-test, post-test control-group design. Two intact classes were selected as research participants. Both were from similar rural co-educational schools with comparable ability and class sizes. The intervention group consisted of 30 pupils (8 boys, 22 girls) and the control group had 31 pupils (16 boys, 15 girls). Ethical approval was gained from the college ethics committee; pupil' assent, parental and school consent were also received. This paper reports on the research question:

In what ways do constructivist and project based approaches to teaching support children's understandings of and experiences with measurement?

Figure 1

Action Research Design



The Intervention

Both classes covered three Measures strand units (length, area, weight), allocating two weeks of teaching to each. The intervention group, taught by one of the researchers, engaged in a constructivist informed teaching and learning intervention focusing on collaborative project-based learning, hands-on activities and opportunities to share strategies and engage in mathematical discourse. The class textbook was not used as a resource during the intervention. The control group enacted no changes to their regular classroom practice. They experienced a traditional classroom teaching approach where the textbook was the primary instructional guide.

Data Collection

This mixed methods approach collected both qualitative and quantitative data (table 1). Prior to the intervention, both groups were administered (1) the Drumcondra Primary Mathematics Test and (2) a measures content test to determine their mathematical competency in measures. A survey (modified from Fennema & Sherman (1978) *Mathematics Attitude Scales* and focus group interviews were also carried out at this stage with the intervention group to ascertain their experiences of and attitudes towards maths. Following the 6-week instructional period, both groups engaged in a maths trail to assess their kinaesthetic skills of measurement and repeated assessments (1) and (2) above. The intervention group engaged again with the survey and focus groups to investigate how they perceived and experienced the intervention. Participant observation and the use of a reflective diary were also employed. Observations gave rich insights into the challenges presented by both content and social dynamics for maths discussions and collaborative tasks. A reflective diary documented ongoing observations as pupils participated in each of the cycles of intervention. These substantially aided planning for subsequent lesson design and acted as a source of data and aide memoir when findings from the study were collated.

Data Analysis

Following the completion of the intervention cycles a hybrid process of inductive and deductive analysis was used to interpret the research data. Quantitative data from the pre and post intervention assessments was collated using Microsoft Excel. The programme was then used to create graphs to represent the data pictorially. Inductive analysis was used to investigate the study's qualitative data from focus group interviews and pupil surveys. Semi-structured focus group interviews were transcribed verbatim with codes assigned to responses that prompted common themes. These codes and similar findings from the pupil surveys were amalgamated to form categories, which were used to create themes. The deductive approach aimed to test theory and analyse data based on assertions used to design the research question at the outset. The analysis then assessed whether the collected data supported those hypotheses. Similarities between the inductively coded data were observed and triangulated with the numerical evidence from the quantitative research.

Table 1

Data Collection Method, Stage of Research and Group Researched

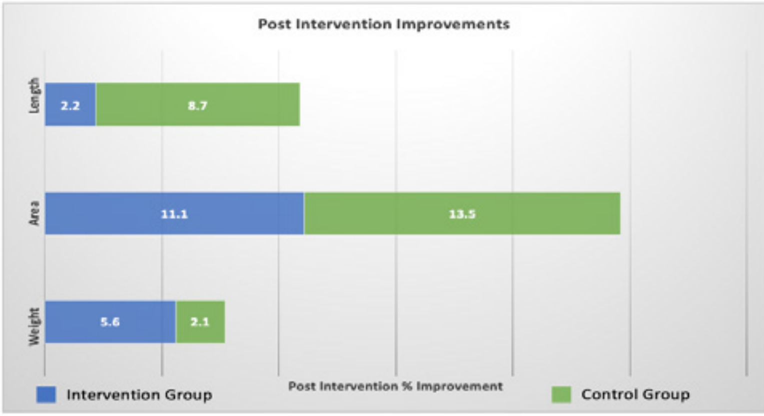
Data Collection	***Research Stage***	***Research Group***
Drumcondra Primary Maths Test	Pre and Post Intervention	Intervention, Control
Measures Content Test	Pre and Post Intervention	Intervention, Control
Measurement Practical Test	Post Intervention	Intervention, Control
Focus Group Interviews	Pre and Post Intervention	Intervention Group
'Measurement and Me' Survey	Pre and Post Intervention	Intervention Group
Teacher Observation	Pre and Post Intervention	Intervention Group
Reflective Journaling	Pre and Post Intervention	Intervention Group

Results

In pre-tests, both groups performed best on the weight strand unit and poorly on area. Post-test findings for the Drumcondra and Measures content tests revealed improvements in all three strand units for both the intervention and control groups, with the most comprehensive increment reported in area (see Figure 1). There was no reduction in the difference between both groups in mathematical knowledge and ability due to the intervention. In fact, the control group showed higher average gains in two of the strand units of length, weight and area (8.7%, 2.1%, 13.5%) compared to the intervention group (2.2%, 5.6%, 11.1%). In contrast, results from the maths trail practical assessment showed an advantage for the intervention group. Employing skills not evaluated in the pen and paper tests, the intervention group performed better in most tasks. They measured or estimated more accurately on six of the seven length tasks and four of the five area and weight tasks.

Figure 2

Post Intervention Improvements for Strand Areas (Percentage Increase)



Prior to the intervention, 14% of pupils stated they ‘don’t like Maths’ and cited a lack of stimulation and boredom as the reasons. When asked, ‘If you had a choice, what part of Maths would you get rid of? their responses indicated a clear distaste for strand units within ‘Number’ and an overreliance on the textbook. There was a change in pupil attitude towards maths as a result of the intervention. They reported enjoying the teaching approaches, in particular the collaborative and contextual nature of the activities. They indicated high satisfaction levels for using equipment and manipulatives (89%) and working with their hands (94%) during the intervention. Project work emerged from focus groups as a valued approach. When asked if they preferred it to book work, three quarters of the group replied that they did, citing collaboration with others and it being more ‘fun’ as the contributing factors.

In contrast, when asked if they enjoyed using the textbook, only 29% reported favourably. However, there was widespread acknowledgment that a textbook remains an important resource for teaching with the majority responding that they appreciated a mix of textbook and non-textbook work and saw the need for both with respect to activities like homework. It was observed that these sixth class pupils could discern that ‘fun’ and effective optimal learning experiences are not always accordant. This was evident in responses to the question of whether they *learned* more from project work and which they *preferred*. As one student stated, *"I prefer doing a mix of both because for homework, you learn more then as well as-- so if you can do the hands-on projects in school maybe and then the homework off the book, then you're getting a bit of both. And so you're learning, you're getting the best of both worlds"*. There was a considerable endorsement for collaborative lessons with peers alongside the enquiry-based learning and open-ended tasks, which were both elementary focal points in the intervention. Concerning the guidance level from their teacher in such task-work, most pupils felt the teacher's role was still important but perhaps different from what has been the case traditionally. 79% of the class pupils liked when the teacher *showed* them how to do something, but subsequent to the intervention, pupils who liked *listening to the teacher talk and explain* methods decreased from 57% to 39%. This was described by another student as *"I think a mix of both because there are some things you can't explain but the book might explain it better"*.

Discussion and Conclusions

Analysis of quantitative and qualitative data from this research leads us to arrive at three conclusions. First, *when the method of assessment matches instruction, results for both groups improve.* The post intervention results demonstrated a difference in the performances of both groups in the written and practical evaluations. These differences can be attributed to the match between the instruction and assessment methods. The control group did best on written assessment because this form of assessment matched the nature of instruction provided through the predominant traditional textbook approach they encountered. Similarly, the intervention group performed better in the practical assessments because this was the nature of instruction that they encountered. Second, *a mix of instruction methods, both constructivist and didactic approaches, constitutes best practice in supporting a diversity of learning styles.* Neither of the two approaches, constructivist or didactic, suited all pupils. The post intervention evaluations established that many pupils from both groups recorded a decrease in their scores. The focus group interviews and surveys demonstrated that the pupils discounted no single method of learning. Furthermore, the intervention itself only marginally altered the children's views. The pupils enjoyed the hands-on nature of constructivist maths but also had a considerable propensity for listening to instruction from their teacher. While not popular with the majority of the class, textbook exercises were still valued by many, and this opinion amplified after the intervention. Finally*, while indicating greater stimulation from engagement with constructivist approaches, pupil feedback also reveals their belief that an integration of both constructivist and didactic approaches is most beneficial for their learning.* The intervention brought about improvements in attitudes toward maths. Pupils enjoyed the collaborative and tactile nature of the lessons and 'going outside' the classroom to see learned skills performed in context. Although the feedback suggested pupils were unsure which method was ultimately more beneficial academically, there was an appreciation for the use of didactic approaches in certain situations, albeit a discernible preference for project-based maths over book centred tuition.

In conclusion, in a pupil-centred curriculum, there is a compelling need for educators to use a mixture of pedagogies to address and complement pupils' learning needs and preferences. Concurrent with this is the requirement that practices of assessment align with the pedagogies being used and a standardised means of evaluating hands on or practical learning be developed and implemented. As shown in the constructivist intervention of this study, practical hands-on maths has a central role in learning and has added value for pupils in terms of promoting the higher order maths skills of reasoning and problem solving. Measures, in particular, is an area of maths that allows for a transferability from what happens in the classroom to the real world. There is a need to be consistently mindful of this in the way it is taught. Although much of the strand's focus on computation is necessary, educators and policymakers need to ask what measurement applications and skills are most commonly used beyond the classroom and determine whether these are reflected in teaching methods? Ireland's STEM Policy Statement (DES, 2017) reports that learning experiences, such as those utilised in this study, support the development of dispositions that are essential in promoting STEM subjects and attracting pupils to studying STEM subjects. That continuum begins at the primary level and is influenced very much by how educators present mathematics to pupils in Irish classrooms.

References

Cannella, G., & Reiff, J. (1994). Individual constructivist teacher education: Teachers as empowered learners. *Teacher Education Quarterly*, *21*(3), 27-38.

Clerkin, A., Perkins, R., & Cunningham. R. (2016). *TIMSS 2015 in Ireland: Mathematics and science in primary and post-primary schools.* Educational Research Centre.

Department of Education and Science (DES) (2005a). *An evaluation of curriculum implementation in primary schools*. The Stationery Office.

Delaney, S. (2012). *A validation study of the use of mathematical knowledge for teaching measures in Ireland. ZDM, 44*(3), 427-441.

Delaney, S. (2016). How to be the teacher everyone wants. News: Education, *Irish Independent*, 01.12.16, Available: https://www.independent.ie/irish-news/education/how-to-be-the-teacher-everyone-wants-35254506.html

Eivers, E., & Clerkin, A. (2012) PIRLS & TIMSS 2011: *Reading, mathematics and science outcomes for Ireland*. Educational Research Centre. Available: http://www.erc.ie/documents/pt_2011_main_report.pdf. [Accessed 12 May 2019]

Eivers, E., Shiel, G., & Cunningham, R. (2007) *Ready for tomorrow's world? The competencies of Irish 15-year-olds in PISA 2006 summary report*. Available online at: http://www.erc.ie/documents/tomorrows_world.pdf

Fosnot, C.T. (2005). Constructivism revisited: Implications and reflections. *The Constructivist*, *16*(1).

Glasersfeld, E. (1996). Introduction: Aspects of constructivism. In C. T. Fosnot (Ed.), *Constructivism: Theory, perspectives and practice* (pp. 3-7). Teachers College Press.

Johnson, P., O'Meara, N. & Leavy, A.M. (2020). Factors supporting and inhibiting teachers' use of manipulatives around the primary to post-primary education transition. *International Journal of Mathematical Education in Science and Technology,* 1-23.

Larochelle, M., Bednarz, N., & Garrison, J. (Eds.) (1998). *Constructivism and education*. Cambridge Press.

O'Meara, N., Johnson, P. & Leavy, A. M. (2019). A comparative study investigating the use of manipulatives at the transition from primary to post-primary education. *International Journal of Mathematical Education in Science and Technology, 51*(6), 835-857. https://doi.org/10.1080/0020739X.2019.1634842

National Council for Curriculum Assessment (1999). *Primary mathematics curriculum teacher guidelines*. NCCA.

National Council for Curriculum and Assessment (2016). *Background paper and brief for the development of a new primary maths curriculum*. NCCA

Noddings, N., (1985). Small groups as a setting for research on mathematical problem solving. In E. A. Silver (ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives*. LEA.

Perkins, R., & Clerkin, A. (2020). *TIMSS 2019: Ireland's results in mathematics and science.* Educational Research Centre.

Shiel, G., Kavanagh, L., & Millar, D. (2014). *The 2014 national assessments of English reading and mathematics Volume 1: Performance report.* ERC Centre.

Surgenor, P., G. Shiel, S. Close and D. Millar, (2006) *Counting on success: Mathematics achievement in Irish primary schools*. Department of Education and Science Inspectorate.

Exploring One Teacher's Attempts to Differentiate Instruction in Remote Learning Environments

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Although prior research, documents, and teachers' experiences have focused on differentiating a single or a sequence of lessons, the study reported here examined teachers' attempts to differentiate every class, every day, over an entire semester. The study focused on the ideas of differentiation presented by Tomlinson to examine how mathematics teachers and math intervention specialists address differentiated instruction in remote learning environments. The study coordinated the documentational approach to didactics and Thompson and Harel's theory of meanings to examine teachers' (schemes of) meanings as they engaged in and discussed their attempts to differentiate instruction in remote learning environments. Analyses involved teachers' Reflective Mappings of Resource System and remote interviews. The findings reported here focus on the case of one mathematics teacher and highlight the importance of teachers' understandings and meanings in their attempts to differentiate instruction and the role digital resources play in supporting or hindering such practices.

Introduction

A number of international treaties and documents have affirmed the right of all children to education (e.g. United Nations General Assembly, 2006). Preparing all students to participate in life within a diverse society is a major educational challenge. According to Suprayogi et al. (2017), "Meeting student differences is challenging since these differences can be related to a large variety of student characteristics such as learner interests . . . cultural background, language level, [and] attitudes" (p. 298). Differentiated instruction aims to deal with the inherent differences between students by providing them with the best possible opportunities to learn and thrive. According to Tomlinson (2017), differentiated instruction "provides avenues to acquiring content, to processing or making sense of ideas, and to developing products so that each student can learn effectively" (p. 1). A long line of research supports the idea that differentiating instruction is a challenging and complex practice (e.g. Moosa & Shareefa, 2019). A variety of reports detail the challenges to teachers' implementation of differentiated instruction, including a lack of teacher preparation time and resources and a disconnect between teachers' understandings and their implementations of differentiated instruction. Furthermore, research has yielded mixed evidence of teachers' actual use of differentiated instruction, from teachers reporting they rarely or occasionally use differentiated instruction practices in their teaching to moderate or high rates of such practices (Roy et al., 2013).

As a consequence of the Covid-19 pandemic, schools have had to adapt to fulfil their many functions, challenging teachers to rethink ways to support their teaching and their students' learning. According to the OECD (2020), an almost universal response to the pandemic has been the use of digital technologies to support teachers, students and their families. Digital technology allows for new solutions to "what people learn, how people learn, where people learn and when they learn. Unfortunately, not all students have the same access to digital devices and online resources, and access varies greatly across countries (OECD, 2020). In Ireland, students from "socioeconomically disadvantaged backgrounds and students . . . in rural areas were . . . disproportionately affected by the digital divide, particularly in relation to broadband access" (Mohan et al., 2020, p. 67). The pandemic's restructuring of classrooms to remote or hybrid teaching and learning environments, along with teachers' motivations to find new and improved ways to support all of their students, makes examination of teachers' attempts to differentiate instruction in such environments an area prime for research. Therefore, the study presented here focuses on ideas of differentiation presented by Tomlinson (2017) and addresses the following research questions: 1) How do grades 6-12 mathematics teachers understand differentiated instruction? 2) How do grades 6-12 mathematics teachers understand the resources they utilize to support differentiated instruction?

Theoretical Framework

The study "networks" the documentational approach to didactics (Gueudet & Trouche, 2009) and Thompson and Harel's (Thompson et al., 2014) theory of meanings. The documentational approach to didactics analyses "teachers' work through the lens of 'resources' for and in teaching: what they prepare for supporting their classroom practices, and what is continuously renewed by/in these practices" (Trouche et al., 2018, pp. 1-2). In the documentational approach, resource is grounded in Adler's (2000) work, which defines a resource as anything likely to 're-source', or "to source again or differently" (p. 207), the teacher's work. That is, all the "resources that are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom" (Pepin & Gueudet, 2020, pp. 172-173). Such resources include text (e.g. textbooks, worksheets, tests) and other material resources (e.g. calculators); digital-/ICT-based resources (e.g. online textbooks, GeoGebra); discussions between teachers, orally or online; students' written work; teachers discussions with mathematics teacher educators; and so forth (Pepin & Gueudet, 2020). This conception of resource is particularly germane, since the study's main focus involved developing models of teachers' understandings of differentiated instruction as they attempted to differentiate instruction in remote learning environments; that is, as teachers attempted to differentiate instruction using one or more digital resources. The process of documentational genesis results in the development of a document and can be represented by the equation: Document = Resource(s) + Utilization Scheme. According to Gueudet and Pepin (2020), utilization schemes include both procedural schemes (e.g. how to use particular resources) and cognitive schemes (e.g. knowledge about the means that the resource offers).

Thompson and Harel's (Thompson et al., 2014) theory of meanings is based on Piaget's notion of assimilation to a scheme and focuses on teachers' (schemes of) meanings, where a scheme is defined as "an organization of actions, operations, images, or schemes—which can have many entry points that trigger action—and anticipations of outcomes of the organization's activity" (Thompson et al., 2014, p. 11). Such a focus enables researchers to identify and examine the assimilation of schemes, and the means for scheme transformation, via accommodation and reflective abstraction. According to Piaget, to understand is to assimilate to a scheme (Thompson et al., 2014). Therefore, attaching meaning (i.e. understanding) is constituted by assimilating to a scheme and the phrase "a person attached a meaning to a word, symbol, expression, statement, or action" means that the person assimilated the word, symbol, expression, statement, or action to a scheme (Thompson et al., 2014). Thompson and Harel's system address issues of understanding, meaning, and ways of thinking. Such a system allows for discussions of and investigations into in-the-moment and stable understandings (see Table 1). Such a system is productive for the study presented here, which focuses on teachers' understandings of differentiated instruction and digital resources not only in-the-moment, but also as these understandings potentially become stable; where an understanding becomes stable by repeatedly constructing it anew (Thompson et al., 2014).

Table 1

Definitions of Understanding, Meaning, and Ways of Thinking (Thompson et al., 2014)

Construct	Definition
Understanding (in-the-moment)	Cognitive state resulting from an assimilation.
Meaning (in-the-moment)	The space of implications existing at the moment of understanding.
Understanding (stable)	Cognitive state resulting from an assimilation to a scheme.
Meaning (stable)	The space of implications that results from having assimilated to a scheme. The scheme is the meaning.
Way of Thinking	Habitual anticipation of specific meanings or ways of thinking in reasoning.

As characterized by Thompson and Harel (see Table 1), an understanding is an in-the-moment state of equilibrium. Such a state of equilibrium may occur from assimilation to a scheme (i.e. stable understanding). According to Thompson et al. (2014), "A scheme, being stable, then constitutes the space of implications resulting from the person's assimilation of anything to it. The scheme is the meaning of the understanding that the person constructs in the moment" (p. 13). Alternatively, an in-the-moment state of equilibrium might be a state the "person has struggled to attain at that moment through functional accommodations to existing schemes . . . and is easily lost once the person's attention moves on" (Thompson et al., 2014, p. 13). Such understandings are specific to that moment in time and are "typical when a person is making sense of an idea for the first time" (Thompson et al., 2014, p. 446).

Study Participants

In this report, I present the case of one mathematics teacher (Monique), one of 18 teachers to participate in the study. Participating teachers were comprised of 15 teachers and three intervention specialists (i.e. teachers who assist students in inclusive mathematics classrooms with special education and social adjustment needs), were self-selected, and met the following criteria: a) mathematics teacher or math intervention specialist in any of grades 6 to 12 (i.e. teach students of ages 11-18 years); b) teach in a rural school district in the U.S.

state of Ohio; c) have an interest in investigating ways to differentiate instruction; d) have an interest in exploring how grade 6 to 12 mathematics teachers, math intervention specialists, and university mathematics education researchers can work collaboratively online to support mathematics teaching and learning; and e) have the availability and desire to spend approximately 10 hours of online collaboration over the course of 12 weeks. Teachers were asked to choose one element of differentiation to focus on (i.e. content, process, product, environment, affect) and attempt to a utilize digital resource (or resources) and implement strategies around this focus for three to five weeks. At the end of three to five weeks, teachers were asked to evaluate the "success" of this implementation (in terms of their students' learning) and modify their chosen element, introduce a new element altogether, or add a second element to the first for another three to five weeks. By the end of the semester (12 weeks), each teacher completed roughly two to three of these iterations. In addition, teachers were asked to describe their experiences through text (i.e. *Google Docs*), video (e.g. *Flipgrid*), or audio (e.g. *Audacity*) on a weekly basis. Finally, teachers participated in monthly online sessions designed for teachers to share and discuss their experiences with colleagues.

Data Collection

The study employed the reflective investigation methodology (Trouche et al., 2018) for data collection. According to Trouche et al. (2018), the reflective investigative methodology is naturally associated with case studies and grounded by the following five main principles: (1) broad collection of resources; (2) long-term follow up; (3) in- and out-of-class follow-up; (4) reflective follow-up; and (5) confronting a teacher's views on her documentation work. Although reflective investigation asserts the need for long-term follow-up because "[g]eneses are ongoing processes and schemes develop over long periods of time" (Trouche et al., 2018, p. 6), the study was designed to examine teachers' initial (or early) engagements with a resource (or sets of resources) or their engagements with a resource (or sets of resources) in novel ways. As such, follow-ups comprised discussions, interviews, and examinations of teachers' lessons and their descriptions of instruction over the course of 12 weeks. In the end, the study's data corpus consisted of reflective mappings of each teacher's resource system, inferred mapping of each teacher's resource system, video recordings of all online group meetings and individual teacher interviews (online), copies of and internet links to all materials teachers utilized during their lessons, teachers' weekly descriptions of their experiences, and teachers' responses to a pre- and post-survey designed to help make their meanings of differentiated instruction and digital resources to support such instruction explicit. The reflective mapping of a teacher's resource system (RMRS) is a methodological tool created by a teacher where the teacher is asked to draw a map to present their resources in a structured way based on her own reflection (Trouche et al., 2019). Similarly, the inferred mapping of the teacher's resource system (IMRS), is a methodological tool created by the researcher based on the observations of and interviews with the teachers about their resource work (Trouche et al., 2019).

Data Analysis

To make sense of teachers' attempts to differentiate instruction in remote learning environments required me to develop models of teachers' ways of operating—models that represented my observations of teachers' actions and interactions with their students, colleagues, and other resources (e.g. digital technology). Analyses involved developing models of teachers' understandings in order to explain how these understandings persisted or changed throughout the study. Using data generated from reflective investigation, these models endured or required modifications dependent on the data corpus supporting or contradicting teachers' hypothesized understandings. Such analyses allowed for teachers' understandings to be compared and contrasted and documentation of potential shifts in teachers' ways and means of operating.

Results

Results focus on the case of Monique, a high school mathematics teacher (teaching students of ages 14-16 years). At the start of the study, Monique had taught mathematics for 11-15 years (in the same school), spent between 31-60% of her professional time using the Internet when preparing for instruction, taught in a district where 31-60% of her students come from socio-economically disadvantaged homes, and classified 31-60% of her students as "low achievers in mathematics" and 1-10% of her students as "gifted in mathematics". Monique was one of eight mathematics teachers (along with two math intervention specialists) who initially operated with a stable understanding of "to demonstrate and allow for a variety of methods and strategies for solving problems." Teacher responses to a survey provided at the start of the study (pre-survey) illustrating such an understanding included: "Sometimes I present material in different ways with different strategies" and "I show them different ways to solve problems and let them choose what works best for them." In addition, these ten teachers believed that differentiated instruction was more important, and that their students believed it was more important to student learning than was the case for the remaining eight teachers. Finally, these ten teachers believed providing their students with choices (regarding classroom activities, assignments, and assessment) was important and that "one size" did not fit all students.

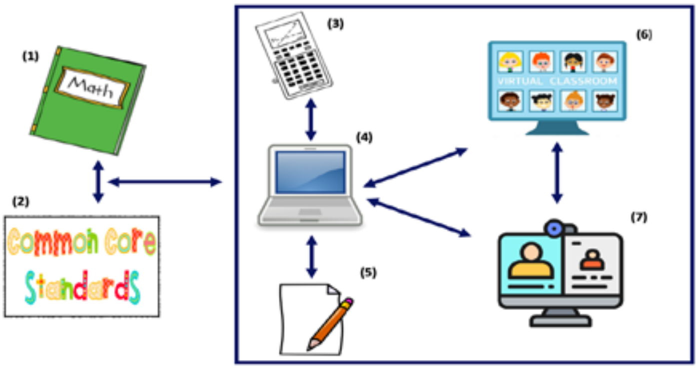
During her second individual interview, I confronted Monique—where "confront" is used in the sense of Brousseau to mean a focused comparison; bringing together for careful comparison—with her initial RMRS, resulting in a map illustrating a connected system of resources (i.e. initial Inferred Mapping of Monique's Resource System, IMRS) involving content and practice standards (grade- or course-specific statements of what students should understand and be able to do in their study of mathematics); the course textbook; resources for solving, graphing, and working on problems (e.g. *TI-84 Plus* graphing calculator, paper and pencil); and resources for virtual instruction (i.e. *Zoom*). Monique selected process as her initial focus element. When asked about her use of the TI-84 Plus graphing calculator and paper and pencil, Monique indicated that these resources provided her and her students with two different ways "to solve problems graphically and algebraically"—Monique's stable understandings for these resources. Such understandings fit Monique's "space of implications," which were limited to finding ways to provide her students with resources that "allow for a variety of methods or strategies to solve problems." Although Monique (and her students) utilized variety, when applicable, by solving problems graphically and algebraically, she never varied student working conditions (e.g. individual, pair, group), nor provided students with levels of support, challenge, or complexity. Considering Monique's experience and confidence with using digital technologies, she was asked to incorporate (over three weeks) Zoom breakout sessions, where students either worked in pairs or groups of three to discuss and solve problems. Monique was asked to move from group to group (i.e. breakout room) to manage discussions and provide support. In addition, Monique was asked to provide pairs or groups with problems of different levels of complexity as needed. The intent of these interventions was to encourage Monique to promote student collaboration and variety in problem complexity and how students demonstrated their understandings. At the end of the three weeks, Monique was interviewed to determine her level of comfort at using and managing the breakout rooms. Monique indicated that she felt students were getting more targeted support and desired to do more.

Next, Monique was asked to implement "tiered" activities and assignments over the next four weeks, where each pair or group of students were given targeted support, challenge, or complexity which Monique designed in advance. At the end of the seventh week, Monique indicated that the time required to design and manage these "tiered" breakout rooms was a bit overwhelming. Online group meetings and individual interviews (with Monique) indicated that she was spending time designing individualized instruction and pairing or grouping students by level of past achievement. As described by Tomlinson (2017), "While it is true that differentiated instruction . . . advocates attending to students as individuals, it does not assume a separate assignment for each learner" (p. 2). Furthermore, the goal of differentiated instruction is to "have students work consistently with a wide variety of peers and with tasks thoughtfully designed not only to draw on the strengths of all members of a group but also to shore up those students' areas of need" (Tomlinson, 2017, p. 4). These points were addressed with Monique, ideas that she stated "liberated" her from feeling overwhelmed. During the remaining weeks of the semester, Monique rotated pairs and group members and focused on meeting the needs of each group. The last three weeks of the study, Monique introduced and experimented with online collaborative white boards (e.g. *Jamboard*, *AWW* app) for small group collaborative problem solving. The intent of these interventions was to focus Monique's thinking on providing students with variation in how they learn, collaborate, and demonstrate their understandings.

By the end of the study, Monique's differentiation of process, as described by Monique during online group meetings and individual interviews, involved allowing students to make sense of the content – by thinking through, grappling with, and using important understandings and skills – at their own pace. Monique's final RMRS, online group meetings and individual interviews (involving Monique) resulted in a map illustrating a connected system of resources (i.e. final Inferred Mapping of Monique's Resource System, IMRS) displayed in Figure 1. Monique determines what to teach based on the (1) state standards and (2) course textbook, in-concert with feedback (i.e. data) she receives from her online interactions with and observations of students (6 and 7). Furthermore, each lesson involves Monique deciding how much she intends her students to use their (3) graphing calculators, (4) laptop (e.g. Jamboard, AWW app), and (5) paper-and-pencil.

Figure 1

Final Mapping of Monique's Resource System (IMRS)



Finally, due to the data corpus, it uncertain whether Monique had developed a stable understanding for differentiated instruction as "allowing students to make sense of the content at their own pace" through generalized assimilation, or if such an understanding was only a functional accommodation and will be assimilated to understandings that exhibit a lack of sense making (and focus on "allowing for a variety of methods and strategies for solving problems") once the study concluded (e.g. long term) or when classes return to face-to-face instruction.

Discussion and Conclusions

As demonstrated here with the case of Monique, coordinating the documentational approach and a theory of meanings not only supported the development of viable models of teachers' understandings of differentiated instruction, but also the design of interventions with the potential to support transformation to more inclusive understandings. Participating teachers indicated the Covid-19 pandemic forced them "to try new things to provide engaging lessons that incorporate exciting technology and resources for [their] students." Furthermore, attempts to incorporate new technologies pushed many teachers out of their comfort zones, making them learners alongside their students. Study limitations included the small sample size, the lack of student interviews, and the inability to observe instruction (either in-person or online), observe teachers as they prepare for instruction, interview teachers face-to-face in their own environments, and observe in-person meetings with colleagues. The study's findings highlight the importance of teachers' meanings in their attempts to differentiate instruction and the role digital resources play in supporting or hindering such practices. Finally, the study is relevant to the examination of teacher's work with and on digital resources as a means to support differentiation and builds on and expands existing research on differentiated instruction (e.g. Moosa & Shareefa, 2019) and research employing the documentational approach (e.g. Trouche et al., 2019), particularly as remote and hybrid teaching and learning continues throughout the U.S. and across the globe.

References

Adler, J. (2000). Conceptualizing resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, *3*(3), 205–224.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199–218.

Mohan, G., McCoy, S., Carroll, E., Mihut, G., Lyons, S., & Mac Domhnaill, C. (2020). *Learning for all? Second-Level education in Ireland during COVID-19*. Economic and Social Research Institute (ESRI) Research Series

Moosa, V., & Shareefa, M. (2019). The impact of teachers' experience and qualification on efficacy, knowledge and implementation of differentiated instruction. *International Journal of Instruction*, *12*(2), 587-604.

OECD. (2020, April 3). *Learning remotely when schools close: How well are students and schools prepared?* Insights from PISA (OECD Policy Responses to Coronavirus, COVID-19). OECD.

Pepin, B., & Gueudet, G. (2020) Curriculum resources and textbooks in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp.172-176). Springer.

Roy, A., Guay, F., & Valois, P. (2013). Teaching to address diverse learning needs: development and validation of a differentiated instruction scale. *International Journal of Inclusive Education*, *17*(11), 1186–1204.

Suprayogi, M. N., Vakcke, M., & Godwin, R. (2017). Teachers and their implementation of differentiated instruction in the classroom. *Teaching and Teacher Education*, *67*, 291-3- 1.

Thompson, P. W., Carlson, M. P., Byerley, C., & Hatfield, N. (2014). Schemes for thinking with magnitudes: An hypothesis about foundational reasoning abilities in algebra. In K. C. Moore, L. P. Steffe & L. L. Hatfield (Eds.), *Epistemic algebra students: Emerging models of students' algebraic knowing*, *WISDOMe Monographs* (Vol. 4, pp. 1–24). University of Wyoming.

Tomlinson, C. A. (2017). *How to differentiate instruction in academically diverse classrooms*. ASCD.

Trouche, L., Gueudet, G., & Pepin, B. (2020). The documentational approach to didactics. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer.

Trouche, L., Gueudet, G., & Pepin, B. (Eds). (2019). *The 'resource' approach to mathematics education*. Springer.

United Nations General Assembly. (2006). *Convention on the rights of persons with disabilities* (Resolution No. A/RES/61/106, Annex I), December 13, 2006. United Nations.

Self-Efficacy and Self-Belief in Mathematics and Access Student Progression

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This study examined the relationship between Access students' general self-efficacy, their beliefs about their mathematics abilities, the level of mathematics module they choose and their progression to higher education. An explanatory, sequential mixed methods approach was adopted for the study, which took place over three academic years, 2017/18, 2018/19 and 2019/20. During the study, questionnaires were completed by 184 students in the Access programme at Technological University Dublin and 24 students participated in interviews. All Access students must complete a module in advanced, intermediate or fundamental mathematics. Results revealed that students with higher belief in their mathematics ability were more likely to study advanced mathematics and more likely to pass mathematics modules. Students studying intermediate mathematics were most likely to progress to higher education, and non-Irish nationals, who had higher belief in their mathematics abilities than Irish nationals, were less likely to progress in advanced mathematics than their peers. Recommendations for improving students' belief in their mathematics ability and progression are provided.

Introduction

Access programmes provide students from backgrounds with little tradition of participating in higher education with an opportunity to access and progress in undergraduate studies (Technological University Dublin, 2020). However, there is little research on Access student progression. The goal of this study is to examine the relationship, if any, between Access students' general self-efficacy (GSE), their beliefs about their mathematics abilities (BMA), the level of mathematics module they choose and their progression to higher education.

Mathematics in Higher Education

Research has highlighted the importance of mathematics skills in education and in society (Wismath & Warrell, 2015). Mathematics is relevant in "high-status careers," particularly those in the high-technology sector (Ma & Johnson, 2008). Additionally, teaching students to use logic and reasoning enables them to develop critical thinking skills and to extend mathematical methods to other disciplines (Hodge, 2003). However, mathematics education can discriminate against and exclude some students due to lack of resources, racism, sexism, language deficiencies and a form of elitism where students are differentiated based on 'ability' (Skovsmose, 2004).

Irish students' performance in mathematics in the Leaving Certificate, the terminal examination for Irish secondary school students, can predict a student's performance in all academic disciplines (Hyland, 2011). Additionally, there is a strong link between performance in mathematics in the Leaving Certificate and successful progression to second year in higher education (Bergin & Reilly, 2005; Mooney et al., 2010; O'Shea, 2021).

Although mathematics skills have been shown to benefit students, over time, there has been a decline in higher education students' basic mathematics competencies that has become known as the "mathematics problem" (Hawkes & Savage, 1999; Faulkner, 2012). Additionally, higher education students commonly admit that they do not like mathematics, or they are "no good at mathematics" (Ryan & Fitzmaurice, 2017, p. 49). According to Marshall et al. (2017), students' negative experiences with mathematics in the past can result in avoidance and procrastination behaviours that affect students' course choices, their self-efficacy and their progression in higher education. Lopez and Lent (1992) found that prior performance was the most efficient predictor of high school students' self-efficacy in mathematics, and Hall and Panton (2005) found that students with more self-belief about their ability to succeed in higher education mathematics classes also had better mathematical skills.

There is a lack of research on Access student progression. However, the findings related to higher education students outlined above may also be relevant to Access students.

The Access Programme at Technological University Dublin

The Access programme at Technological University Dublin (TU Dublin) is a one-year programme offering an alternative route to higher education for mature students, and for young adults who are socio-economically disadvantaged (Technological University Dublin, 2020). In Ireland, mature students are 23 years or older on January 1 of the year they enter higher education and young adults are aged 22 years and under. All Access students must study one mathematics module each semester at fundamental, intermediate or advanced level.

This study is part of a larger study on the factors affecting the progression of Access students to undergraduate studies. A subset of the data from the larger study is provided here. The larger study revealed that demographic, psychosocial, educational, environmental and institutional factors affect Access student progression. The current study focuses on the effect of GSE and BMA on students' mathematical experiences in Access education and their progression to undergraduate studies. It aimed to answer the following research question:

> Is there a relationship between Access students' GSE and BMA and (a) the level of mathematics they study and (b) their progression to higher education?

Method

Ethical approval for this study was obtained from the ethics committee at TU Dublin. An explanatory, sequential mixed methods approach was adopted for the study, which took place over three academic years, 2017/18, 2018/19 and 2019/20. During the quantitative phase of the research, Access students completed a 29-item questionnaire at the start of the academic year. This questionnaire included the General Self-Efficacy Scale (Schwarzer & Jerusalem, 1995), which assesses ability to deal with unusual or difficult situations and was used to measure Access students' GSE. The scale includes 10-items with a 4-point Likert scale ranging from 'not at all true' to 'exactly true'. Additionally, students rated their BMA using a five-point Likert scale, where 1 represented 'excellent' and 5 represented 'poor'.

Although the questionnaire also included questions related to personality, motivation and relatedness, the focus of the current paper is self-efficacy, self-belief and progression.

Progression was measured based on whether students were offered a place at a higher education institution or not. The data was analysed using SPSS. Spearman's correlation (ρ), Mann-Whitney *U* tests (*U*) and Chi square tests (χ^2) were conducted. For all Chi-square tests, dichotomous variables were used. For example, to determine whether there was an association between gender and GSE score, female was coded as 1, male was coded as 2, GSE score below the median was coded as 1 and GSE score above the median was coded as 2.

During the qualitative phase of the research, 24 students who had completed a questionnaire at the start of the academic year participated in a one-to-one, semi-structured interview with the researcher before completing their final examinations. Interview participants included male, female, young adult, mature, Irish national and non-Irish national students. Interviews were approximately 30 minutes in duration and were transcribed and coded using the grounded theory approach outlined by Strauss and Corbin (1990).

Results

Questionnaires were completed by 184 students over the three years of the study. In all, 85% of students who studied foundation mathematics, 84% of those who studied intermediate mathematics and 89% of students who studied advanced mathematics progressed to higher education. Participant demographics are outlined in Table 1.

Table 1

Percentage of Access Students Studying Fundamental, Intermediate and Advanced Mathematics by Age, Gender and Nationality

	Fundamental Mathematics	Intermediate Mathematics	Advanced Mathematics
% enrolled	25%	64%	11%
Mature	30%	62%	8%
Young Adult	19%	66%	15%
Male	26%	59%	15%
Female	24%	70%	6%
Irish National	29%	61%	10%
Non-Irish National	18%	70%	12%

Results revealed that there was a weak positive correlation between Access students' GSE and their BMA ($\rho(169) = .215$, $p = .005$). Students with higher GSE scores (scores above the median) had higher BMA than those with lower GSE scores (scores below the median). Moreover, students with higher GSE scores were significantly more likely to rate their BMA as above average or excellent compared to those with low GSE ($\chi^2 = 8.00$, df = 1, $p = .018$).

Findings showed that there was a statistically significant relationship between students' GSE and the level of mathematics they studied. Students with lower GSE scores were significantly more likely to study fundamental mathematics than their peers ($\chi^2 = 6.25$, df = 1, $p = .012$). There was also a statistically significant relationship between students' BMA and the level of mathematics they studied. Students who rated their BMA as above average or excellent were significantly more likely to study advanced mathematics than students who rated their BMA as below average or poor (χ^2 = 10.06, df = 1, p = .007). Moreover, students who rated their BMA as excellent or above average were significantly more likely to pass their mathematics module (receive a total of 40% or more) than those who rated their BMA as average, below average or poor (χ^2 = 12.41, df = 1, p = .002).

Male and female students did not differ in their GSE scores (χ^2 = 1.71, df = 1, p = .19) or their BMA (χ^2 = 2.34, df = 1, p = .310) but males were significantly more likely to study advanced mathematics (χ^2 = 3.75, df = 1, p = .053). Additionally, non-Irish nationals had significantly higher BMA scores than their Irish peers (χ^2 = 12.78, df = 1, p = .002) and significantly higher GSE scores (χ^2 = 8.18, df = 1, p = .004). They were also more likely to study advanced mathematics or intermediate mathematics than Irish nationals (χ^2 = 3.58, df = 1, p = .059). There was no significant difference in GSE (χ^2 = .174, df = 1, p = .677) or BMA (χ^2 = 1.88, df = 1, p = .390) scores between young adults and mature students.

Mathematics and Progression

Overall, there was no statistically significant difference in progression based on Access students' mean rank for GSE (U = 2815.5, p = .425) or their BMA (χ^2 = 3.84, df = 1, p = .146). However, students studying intermediate mathematics were more likely to progress to higher education (χ^2 = 7.57, df = 1, p = .006) than those studying fundamental or advanced mathematics. They were also significantly more likely to have higher GSE scores (χ^2 = 4.38, df = 1, p = .036) and higher BMA scores (χ^2 = 4.70, df = 1, p = .096) than their peers. Mature students studying intermediate mathematics were more likely to progress than their young adult peers (χ^2 = 8.39, df = 1, p = .004). Non-Irish nationals who studied advanced mathematics had lower progression rates than Irish nationals (χ^2 = 2.92, df = 1, p = .087).

Qualitative Findings

In interviews, students contended that 'confidence' was important in progression. Lack of 'confidence' was mentioned by 11 interviewees (46%) as a problem for students who failed to progress. One interviewee, Dylan (male/mature/Irish national), recalled that a student who lacked BMA failed to progress. A second interviewee, Oscar (male/mature/Irish national), explained that he sometimes questioned whether he should continue with his studies.

'Overconfidence' was seen as a barrier to the progression of Access students also. Dylan explained that one student set his targets too high and failed to progress. He felt that this student chose the most demanding modules and could not keep up, leading to the student's departure from the programme.

Interviewees noted that Access students chose the level of their mathematics module depending on their BMA. Two interviewees, Oscar and Olena (female/Irish/young adult), noted that they chose to study fundamental mathematics because they lacked BMA. Oscar wanted to study media in higher education but changed his mind when he realised that he would be required to study intermediate or advanced mathematics. Alan (male/young adult/non-Irish national) noted that some Access students did not like mathematics.

Interviewees contended that mature students were more focused on their studies than young adults. However, they noted that both mature students and young adults engaged in informal learning communities that provided mathematics support.

Kassie (female/mature/non-Irish national), Tammy (female/young adult/non-Irish national) and James (male/young adult/non-Irish national) felt that English language difficulties affected their performance in some modules. Although non-Irish nationals may not have experienced language difficulties in mathematics, other modules may have been more difficult, particularly those that involved multiple choice questions.

Discussion

This study aimed to determine whether Access students' BMA and GSE affected the level of mathematics they study and their progression to undergraduate studies. Findings revealed that students who rated their BMA as above average or excellent were significantly more likely to study advanced mathematics than those who rated their BMA as average or below average. This may be because Access students with higher BMA have better mathematical skills or because their past performance in mathematics affected their BMA as indicated by Lopez and Lent (1992). Interviewees concurred that most Access students chose the level of their mathematics module depending on their BMA, particularly students studying fundamental mathematics. They contended that Access students commonly admit that they do not like or are "not good" at mathematics.

Students who rated their BMA as above average or excellent were significantly more likely to pass their mathematics module than students who rated their BMA as average, below average or poor. Wesson and Derrer-Rendall (2011) found that students generally have a good awareness of their academic abilities and House (2000) found that academic background and students' BMA are significantly related to the grades they achieve in mathematics courses in higher education. However, both quantitative and qualitative findings in the current study suggest that some students may have overrated their ability in mathematics.

Non-Irish nationals had higher BMA than Irish nationals. However, non-Irish nationals who studied advanced mathematics were significantly less likely to progress to higher education than their peers. Although non-native speakers of English in the advanced mathematics programme did not fail their mathematics module, interviewees noted that they may have failed to progress because they experienced difficulties in modules that required advanced English language skills. English competency can affect higher education students' academic achievement (Harris & Ní Chonaill, 2016; Paton, 2007).

Quantitative findings also revealed that mature students who studied intermediate mathematics were significantly more likely to progress to higher education than young adults. Interviewees explained this, noting that mature students may be more likely to progress because they put more effort into their studies than young adults. Mature students do better than their younger counterparts in higher education also (Faulkner et al. 2016; Hoskins et al., 1997).

According to interviewees, low BMA may have affected the progression of some students to undergraduate studies as students felt that they were not good at mathematics or could not engage in the mathematics module and left the Access programme as a result.

Conclusion

The findings from the current study reveal the importance of having positive BMA and higher GSE scores. Access students with higher BMA and higher GSE were more likely to study advanced mathematics and more likely to pass mathematics overall. However, it is important that students are not overconfident as this may result in failure to progress.

These findings indicate the importance of improving students' BMA and their GSE. Access students can be encouraged to improve their BMA if lecturers follow Heslin and Kelhe's (2006) recommendations for improving self-belief. This would involve engaging students in enactive self-mastery, which is achieved when people master at least part of a task; role-modelling, which involves watching someone else perform a task and forming ideas about how the task can be performed; and verbal persuasion, which involves positive self-talk or encouragement and praise from an educator. Moreover, Access students should be encouraged to monitor their understanding and progress in learning by adopting effective study strategies and see intelligence as a malleable quality. Ehrlinger and Shain (2014) found that these factors encourage accuracy in self-belief. Similar results have been found with higher education students, however, this is not something which has been examined objectively in longitudinal research where Access students are concerned so it is important to build up this knowledge base in a scientific way. This study goes some way to doing that.

References

Bergin, S., & Reilly, R. G. (2005). Programming: Factors that influence success. *Proceedings of the 36th SIGCSE Technical Symposium on Computer Science Education, SIGCSE 2005.* St. Louis, Missouri, USA.

Ehrlinger, J., & Shain, E. A. (2014). How accuracy in students' self perceptions relates to success in learning. In V. A. Benassi, C. E. Overson, & C. M. Hakala (Eds.), *Applying science of learning in education: Infusing psychological science into the curriculum* (pp. 142–151). Society for the Teaching of Psychology.

Faulkner, F. (2012). *An analysis of performance in mathematics for technology undergraduates and an investigation of teaching interventions for these students.* Doctoral thesis. University of Limerick.

Faulkner, F., Fitzmaurice, O., & Hannigan, A. (2016). A comparison of the mathematical performance of mature students and traditional students over a 10-year period. *Irish Educational Studies, 35*(*4*), 337–359.

Hall, M. J., & Ponton, M. K. (2005). Mathematics self-efficacy of college freshman. *Journal of Developmental Education, 28*(*3*), 26–28.

Harris, R., & Ni Chonaill, B. (2016). Inequality in the Irish higher education system: a case study of the views of migrant students and their lecturers on how English language proficiency impacts their academic achievement in an Institute of Technology. *Irish Journal of Applied Social Studies, 16*(*2*), 78–90.

Hawkes, T., & Savage, M. (1999). *Measuring the mathematics problem.* The Engineering Council.

Heslin, P.A., & Klehe, U.C. (2006). Self-efficacy. In S. G. Rogelberg (Ed.), *Encyclopedia of industrial/organizational psychology* (Vol. 2, pp. 705–708). Sage.

Hodge, B. (2003). Logical Thinking in Mathematics: From Oz to Awe! *Research and Teaching in Developmental Education, 19*(*2*), 41–46.

Hoskins, S. L., Newstead, S. E., & Dennis, I. (1997). Degree performance as a function of age, gender, prior qualifications and discipline studied. *Assessment and Evaluation, 22*, 317–328.

House, J. (2000). Student self-beliefs and science achievement in Ireland: Findings from the third international mathematics and science study (TIMMS). *International Journal of Instructional media, 27*(*1*), 107–115.

Hyland, A. (2011). *Entry to higher education in Ireland in the 21st Century*. Higher Education Authority.

Lopez, F. G., & Lent, R. W. (1992). Sources of mathematics self-efficacy in high school students. *The Career Development Quarterly, 41*(*1*), 3–12.

Ma, X., & Johnson, W. (2008). Mathematics as the critical filter: Curricular effects on gendered career choices. In H. M. G. Watt & J. S. Eccles (Eds.), *Gender and occupational outcomes: Longitudinal assessments of individual, social, and cultural influences* (55–83). American Psychological Association.

Marshall, E. M., Staddon, R. V., Wilson, D. A., & Mann, V. E. (2017). Addressing maths anxiety within the curriculum. *MSOR Connections 15*(*3*), 28–35.

Mooney, O., Patterson, V., O'Connor, M., & Chantler, A. (2010). *A study of progression in Irish higher education.* Higher Education Authority.

O'Shea, M. (2021). New Completion data shows that three out of every four undergraduate entrants completed and graduated from higher education. Higher Education Authority. https://hea.ie/2021/03/29/new-completion-data-shows-that-three-out-of-every-four-undergraduate-entrants-completed-and-graduated-from-higher-education/

Paton, M. J. (2007). Why international students are at greater risk of failure. *International Journal of Diversity in Organisations, Communities & Nations, 6*(*6*), 101–111.

Ryan, M., & Fitzmaurice, O. (2017) Behind the numbers. The preliminary findings of a mixed methods study investigating the existence of mathematics anxiety among mature students. *Adults Learning Mathematics: An International Journal, 12*(*1*), 49–58.

Schwarzer, R., & Jerusalem, M. (1995). Generalized self-Efficacy scale. In J. Weinman, S. Wright, & M. Johnston, *Measures in health psychology: A user's portfolio. Causal and control beliefs* (35–37). NFER-NELSON.

Skovsmose, O. (2004). *Critical mathematics education for the future.* International Congress on Mathematical Education.

Strauss, A., & Corbin, J. M. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* Sage Publications Incorporated.

Technological University Dublin. (2020). *HEAR scheme*. Technological University Dublin. https://www.tudublin.ie/study/undergraduate/how-to-apply/entry-pathways/hear-scheme/

Wesson, D., & Derrer-Rendall, N. (2011). Self-beliefs and student goal achievement. *Psychology Teaching Review, 17*(*1*), 3–12.

Wismath, S. L., & Worrall, A. (2015). Improving university students' perception of mathematics and mathematics ability. *Numeracy, 8*(*1*), Article 9.

Teachers' Use of Mathematical Picturebooks to Engage Children in the Upper Primary Years in Mathematics

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The use of picturebooks to engage children in developing mathematical proficiency is well documented. However, there is limited evidence as to their use with children in the upper primary years. Therefore, we developed a novel initiative in which we would source picturebooks that contained mathematical topics appropriate to this age cohort and trial them in the classroom. In this paper, we present the findings from the seven teachers who took part. The thematic analysis is aligned with the three core principles of Universal Design for Learning. Results demonstrate that use of picturebooks offer learners multiple means of accessing and engaging with mathematical concepts, multiple means of building conceptual representations of mathematics, and lend themselves to multiple means of action and expression in extending, internalising and showcasing learning.

Background

The Primary School Mathematics Curriculum is currently under redevelopment with the new specification due to be launched in 2022. Informed by commissioned research reports on children's mathematical learning in primary school, a draft specification was released for consultation, heralding a shift away from content objectives aligned to class level, to learning outcomes aligned along progression continua. Stemming from an ideology of what it means to be proficient in mathematics, teachers are encouraged to "use appropriate and evidence-based pedagogical approaches and strategies to foster engagement, ownership and challenge while connecting with children's life experiences and their interests" (National Council for Curriculum and Assessment [NCCA], 2020, p. 6). One such approach, and the one of focus in this research, is for teachers to plan "Experiences with mathematically-related stories [as these] have the potential to promote aspects of mathematical proficiency, including procedural fluency, adaptive reasoning and a productive disposition" (Dooley et al., 2014, p. 53).

In a recent study in Ireland, pre- and in-service teachers (n=154) reported perceived benefits to children's mathematical learning such as promoting engagement and inclusion, allowing for integrated learning across the curriculum, embedding mathematics in real-life problem-solving situations, supporting revision of concepts, and helping children visualise concepts (Prendergast et al., 2019). Despite this, the use of mathematically-related stories seems somewhat limited to the lower primary years. This appears to be quite consistent with research in the field and points to a dearth in the literature about suitability for use with older children. This was further confounded by a reported lack of availability of stories that would work for a specific mathematical topic and exacerbated by a lack of confidence and pedagogical knowledge (Prendergast et al., 2019). This motivated us to develop an initiative in which we would source and trial suitable mathematically-related stories in classrooms. In particular, we wanted to get feedback from teachers on how these stories could best be leveraged to support all learners to engage in challenging and meaningful ways of thinking and working mathematically.

Theoretical Framework

The focus of this study was to examine teachers' experiences of using mathematically-related stories in the classroom so as to engage all learners in mathematics. Thus the researchers drew on an *appropriate and evidence-based* teaching and learning framework, Universal Design for Learning (UDL). The three core principles of UDL comprise multiple means of engagement, representation, and action and expression. These principles are each expanded into guidelines and checkpoints which offer a set of concrete suggestions to help ensure that all learners can access and participate in meaningful, challenging learning opportunities. They further help teachers to identify different methods of engagement, representation, and expression that can be used in a particular lesson to allow learners to *access* the learning outcomes, remain engaged and *build* on their learning, and begin to *internalise* the approaches to learning in order to become *proficient* learners of mathematics (Center for Applied Special Technology [CAST], 2018). In the context of this initiative, the UDL framework was to act as an organiser, rather than a checklist, to highlight how mathematically-related stories can be used to meet the needs of all learners in the upper primary school mathematics classroom and to intentionally reflect on how quality learning interactions can be designed from the outset. In particular, we wished to consider:

1. If mathematically-related stories can engage all children in learning?
2. How representations of mathematical content in stories can support all learners to access and build understanding?
3. What opportunities mathematically-related stories afford all learners to demonstrate mathematical proficiency?

Methods

Collection Development

To create a list of titles for consideration, searches were performed within DCU Library's Juvenile Literature Collections, on Amazon.co.uk and Amazon.com and on educational publisher websites from the UK, US and Australia. The reference management tool *Zotero* was used to create a shared list of titles discovered during the initial search and to facilitate evaluation of the list. Titles considered had to meet the following selection criteria; (a) be in print; (b) have literary and aesthetic merit; (c) reflect numeracy themes contained in the curriculum; (d) have illustrations; (e) have an explicit emphasis on mathematical content through engaging in a narrative. Typically, the story would focus on a protagonist or group of characters that face some form of crisis which requires them to draw on understanding of mathematical concepts to solve the problem that they face (Trakulphadetkrai et al., 2019). We considered visuals necessary to inform, inspire and captivate learners. Therefore, from now on we are working with the definition of picturebook(s), written as one word, where the story or narrative is both implied and assumed. Furthermore, we narrowed our selection to titles that addressed mathematical concepts in the fourth to sixth class curriculum from the

Charlesbridge series comprising; Data: *Sir Cumference and the Off-the-Charts Dessert* (Neuschwander, 2013); Chance: *A Very Improbable Story* (Einhorn, 2008); Fractions: *Fractions in Disguise* (Einhorn, 2014); Ratio: *Pythagoras and the Ratios* (Ellis, 2010); Multiplication: *Multiplying Menace. The Revenge of Rumpelstiltskin* (Calvert, 2006).

Participants

A convenience sample of three teachers, one from 6th (T6), one from 5th (T5) and one from 4th (T4) class were invited to take part in the research. We also invited teachers from one senior national school. This case study comprised one 6th class (CS6), two 5th (CS5a), (CS5b), and two 4th (CS4a), (CS4b), class teachers. Packs were collated and distributed to classes in time for Maths Week 2020. Thus, a pack for a class of 30 pupils would contain six picturebooks of each of the five titles. This would allow flexibility in use from individual, pair-work, group-work or whole class activity. A Google Form was designed to capture teachers' initial response to the packs which formed the basis for the semi-structured interviews that followed.

Semi-structured Interviews

The interviews took place individually via Zoom and were audio-recorded and autotranscibed using Otter. A learning support teacher in the case study school collated feedback from colleagues and reported back to the research team. In summary, the interviews comprised three key question prompts which aligned with the principles of UDL: What book did you feel was most engaging for children and why? [multiple means of engagement]; Choose a book you consider represents mathematics best. Tell me about it. [multiple means of representation]; What activities did the children engage in to demonstrate and extend their learning? How did this book link with the engagement/representation/action and expression on the UDL framework? [multiple means of action and expression].

Coding

A deductive approach to data coding and analysis was adopted in this study. This technique enables the codes and themes to derive more from concepts and ideas the researcher brings to the data (Braun & Clarke, 2012). This top-down approach allowed us to code the data using the UDL principles of multiple means of engagement, multiple means of representation, and multiple means of action and expression. It should be noted that although the three UDL principles and associated guidelines are presented separately, they are interconnected with each other. Changes in practice, such as designing new tasks for children to demonstrate their understanding, may impact on the child's engagement with the mathematics. Thus, we have employed a 'best fit' model in coding the data comprising teachers' initial appraisal of the books and subsequent utility for the particular class level.

Findings and Analysis

Multiple Means of Engagement

In considering the 'why' of learning, we sought to ascertain teachers' perspectives on using the picturebooks to increase engagement, stimulate motivation and sustain enthusiasm for learning mathematics (CAST, 2018).

Engagement of the Learner. An initial consideration in the collection development was the literary and aesthetic appeal of the picturebooks which research has shown has the potential to cognitively engage the children in the mathematics (Elia et al., 2010). The response from teachers when the packs arrived was overwhelmingly positive. Teachers commented on the potential to "excite" and "motivate" the children (T5) as well as "increasing engagement with learning" (CS5b). "There was a lovely moment when the children saw the books for the first time, just after they submitted their assent forms which sparked curiosity and wonder around mathematics. As the children had their snacks, informal maths talk took place between them" (T4). Beyond the initial impressions of the picturebooks themselves, the teachers had to consider how they would use these in the classroom. Pivoting from an approach that tended to involve individual tasks with frequent opportunities for practice and consolidation to the use of picturebooks as a "teaching tool", was perceived as quite overwhelming and "would be planning times one hundred" (CS5A). For the most part, teachers decided to implement the picturebooks to coincide with planning of particular topics as this would "allow me to integrate the books more meaningfully into my teaching" (T6). "I didn't want to just throw the picturebooks in there as a one off lesson. I kind of wanted to build it into my maybe weekly or fortnightly theme for that month, or for that period of time when it came to maths" (T5). "Content-related storybooks can be used as an introduction to build interest and create a feeling of anticipation and focus for the lesson or as a culminating activity" (Capraro & Capraro, 2006, p.35). This was similar to how the teachers used the picturebooks, likening it to the teaching of English, "Children were asked to predict from the front cover, what the story was about and after one reading of the story (or having the story read to them), they were asked to make a summary" (T6). Teachers also concurred that "The problem presented in each story will be used to stimulate problem solving and talk and discussion in the lesson" (T5). Such practices in integrative contexts is supported by research which highlights the multiple opportunities that picturebooks offer in providing rich contexts for "learning activities that arise from children's interests, concerns, and questions and the educators links these to learning goals" in developing mathematical proficiency (Dooley, et al., 2014, p. 45).

Engagement with the Wider Community. It was hoped that engagement with the picturebooks would help to extend the learning of mathematics beyond the confines of the classroom as "Primary mathematics education should provide children with opportunities to engage with deep, meaningful and challenging mathematics in educational settings, including social and familial settings. Such engagement will result in children co-constructing knowledge and skills as they interact and collaborate" (NCCA, 2018, p.18). There were mixed feelings with regards to upscaling the use of mathematical picturebooks. “As a school community, the teaching and learning of mathematics is under review, with a push to move away from traditional "textbook" teaching, particularly in senior classes" (T4). This was reiterated by other teachers whose “colleagues are generally open to trying out new ways of teaching and learning in their classrooms" (T6) and would see this as “a different yet engaging way to teach maths” (CS4a). Although the potential of using picturebooks as part of Maths Week was highlighted (T5), there was caution expressed about use at home. “Parents may be interested but I think, would need a lot of hand holding/explicit direction” (CS5b). Others were more positive “Many of the consent forms were accompanied with thank you notes from parents" (CS6), and considered engagement with picturebooks at home “might give them [parents] also a better understanding of how to explain certain concepts in maths” (CS6).

Multiple Means of Representation

In this section we consider the ‘what’ of learning, with an explicit focus on how mathematics is represented in the picturebooks. “One key strength of that format is the way mathematical concepts can be represented in different ways, be it visually (through page illustrations), symbolically (through mathematical models and notations), and contextually (through meaningful contexts in which mathematical concepts are found)” (Trakulphadetkrai et al., 2019, p. 204). It has further been shown that strengthening children’s ability to move between and among representations; pictures, oral/written language, real-world situations, written symbols and manipulative models; improves the growth in mathematical proficiency (Lesh et al., 2003, cited in Chigeza, 2013, p. 179).

Representations of Mathematics through Story. Picturebooks can be a powerful vehicle for providing a meaningful context for representing mathematics and demonstrates that many mathematical concepts develop out of human experiences and interactions (Chigeza, 2013). This was particularly evident in *Pythagoras and* (his musical *rock* group) *the Ratios*. Whereas Pythagoras is infamous for his theorem on right-angled triangles, the underlying mathematical concept of this story is the particular mathematical relationship between musical notes. This picturebook, however, was used less frequently than others. One teacher commented that the children couldn’t even attempt to sound out the word *Pythagoras*. She considered pre-teaching the language involved but felt “it would take some of the magic out of the story” (T4). However, ratio is only introduced in the 6th Class curriculum and therefore this concept might be slightly beyond the reach of most children whereas an understanding of fractions is a key concept of the fourth and fifth class curriculum. *Fractions in Disguise* follows the story of a brand-new fraction, stolen at auction, and the subsequent invention of a *Reducer* machine which could be used to simplify fractions to their lowest terms, and thus reveal their true identity; “Quickly I spotted my first fraud. “That 3/21,” I said. “It’s really a 1/7, isn’t it?” I pointed my reducer at the fraction and dialled a 3. Both the numerator and the denominator were divided, and now I had a 1/7 before me, as I had suspected” (Einhorn, 2014, p. 17). It was seen that quite an abstract mathematical concept, when embedded in a meaningful context that children can relate to, gave children the opportunity to make sense of the mathematics involved (Elia et al., 2010). “This character created a really child-friendly, fun representation of a concept that helps simplify fractions efficiently and requires children to extend their use of multiples in a natural way" (CS4b).

Representations of Mathematics through Pictures and Text. The relationship between the pictures and the text in exemplifying the mathematical concepts was another important consideration. Predominantly the pictures were related to representing the story-element with the main exception, *Sir Cumference and the Off-the Charts Dessert*. The two protagonists, *Lady Di of Amater* and *Bart Graf*, engage in a bake-off with the pictures clearly illustrating the complete data cycle culminating in a representation of the data using pie and bar charts. Elia et al. (2010) contend that pictures which illustrate mathematical concepts support children's learning of mathematics and elicit mathematical thinking; "As they worked in pods there was good opportunity for orally explaining their maths findings related to the concepts, simplifying fractions, collecting and analysing data" (CS4a). The clever use of language and naming of the characters enabled the children to develop an appreciation of mathematics that was new to them, such as *Octavius* (Pythagoras is attributed with discovering that a string exactly half the length of another will play a pitch that is exactly an octave higher) and *Odds* the cat, where in one class, children assumed incorrectly that the story was going to be about odd and even numbers and led to a rich exploration of the story on probability (T4). Thus from a UDL perspective, we can see the potential use of picturebooks to enable all learners to "access powerful mathematical ideas relevant to their current lives and also learn the language of mathematics vital to future progression" (Chigeza, 2013, p. 182).

Multiple Means of Action and Expression

This principle focuses on how the teachers used the picturebooks in their classrooms and the options offered to the children to demonstrate their learning as it is particularly important that picturebooks offer participation opportunities for children (Elia et al., 2010). In this section, we examine how the teachers planned tasks in which the children could engage and develop new understandings. We also explore the various methods by which the teachers organised the learning environment to facilitate the children in expressing and sharing ideas.

Task Development. Tasks were developed by each of the teachers to ensure the active participation of the children and to provide opportunities for them to take action in the lessons. Many of the teachers utilised the tasks that were explicitly presented in the picturebooks. This often involved pausing the reading of the picturebook and encouraging the children to solve the problems faced by the characters. The teachers praised these tasks due to their cognitive demand and the richness of the learning opportunities they provided. One teacher stated that he "was amazed at the amount of learning" that emerged from bringing these tasks to life in the classroom (T5). From a UDL perspective, a number of the teachers identified that the tasks in the picturebooks challenged the thinking of all learners in the classroom. For example, one teacher reported that the tasks "had a low entry point but also had high challenge, [providing] optimal learning for all" (CS4b). A number of the teachers also designed their own follow-on tasks based on the events depicted in the picturebooks. For example, in *Multiplying Menace*, Peter is faced with saving the kingdom using a magic multiplier stick. In one classroom the teacher presented the children with her own imaginary activity involving the multiplier stick: "Imagine you are on a deserted island and when you look around you see the following: sand, the sea, a shark, a wooden box filled with treasure, a coconut palm tree, a pile of rocks and a multiplier stick" (T6). The children were required to describe how they would use the multiplier stick to escape from the island. A range of responses were provided by the children including "I'll multiply the shark by zero to get rid of the shark", "I'll multiply the tree by 1,000 and make a bridge" and "I'll multiply the sea by zero and then I can just walk back". In each of the classrooms, children were encouraged to use their own approaches to solve the tasks which allowed for multiple means of action. This was particularly evident in one classroom where the children were provided with access to a range of resources that they could physically manipulate to solve the problems and the children presented their work in various ways including through the use of drawings, symbols, numbers and letters (T5).

Classroom Organisation. The participating teachers organised their classrooms in a variety of ways to provide the children with multiple opportunities to take action in the lessons and to express their mathematical ideas. Firstly, in every classroom, the children engaged in whole class discourse around the mathematics and the picturebooks acted as a stimulus for these discussions. The opportunities presented by the picturebooks for engaging children in whole-class discussion were highlighted by a number of the teachers as having a positive impact on the children's learning. This was echoed in a study by Capraro and Capraro (2006) pertaining to the use of children's mathematics literature in geometry lessons. The researchers contended that children make meaningful connections through engaging in classroom discourse and responding to purposeful questioning. Questioning and open discussion was identified as a key tool in meeting the varied needs of children in a classroom. This was highlighted by one teacher who acknowledged that she was able to guide the discussion and her questioning to both support and challenge all learners (CS4a). Every teacher reported that they encouraged the children to work collaboratively on the tasks that emerged from the picturebooks. This afforded the children multiple opportunities to express their thinking and to discuss ideas with their peers. Working in pairs and small groups also provided regular opportunities for the children to hear the mathematical ideas of their classmates. Additionally, one teacher stated that children were encouraged to question and build upon each other's ideas (CS4b). In an online learning space, one teacher shared an audio recording of *A Very Improbable Story* with her class. Although it is assumed that children at this class level could read the story for themselves, the teacher wished to provide the children with the opportunity to sit back and enjoy the story (T6). Research has shown that "children can be mathematically engaged by being read a picture book when the role of the reader is restricted to the reading of the book (without any prompting)" (Van den Heuvel-Panhuizen & Van den Boogaard, 2008, as cited in Elia et al., 2010, p. 278). However, this teacher also noted the limitations of the online environment, in particular with respect to use of maths talk. This finding reflects that of other research which states that mathematically-related stories "used without a teacher who is asking questions, may not always be as effective as expected in evoking mathematics-related thinking" (Elia et al., 2010, p. 289).

Conclusion and Next Steps

It can be deduced from the findings that the teachers involved in this research found the picturebooks to be a valuable resource in supporting children to engage with rich meaningful mathematical experiences. Whereas we did not ask the teachers to identify titles for use in the upper primary school classroom themselves, the results show that there is merit in using such a pedagogical approach and thus we are developing guidance for teachers in the identification and evaluation of titles. The aim of this work is to build the confidence and competence of teachers in selecting picturebooks that will align with best practice as outlined in the draft mathematics specification. This information will be made available via DCU Libguides.

References

Braun, V. & Clarke, V. (2012). Thematic analysis. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology. Vol. 2, research designs.* American Psychological Association.

Capraro, R. M., & Capraro, M. M. (2006). Are you really going to read us a story? Learning geometry through children's mathematics literature. *Reading Psychology, 27,* 21-36.

Center for Applied Special Technology. (2018). *Universal design for learning guidelines version 2.2*. Retrieved from http://udlguidelines.cast.org.

Chigeza, P. (2013). Translating between and within representations: Mathematics as lived experiences and interactions. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (pp. 178-184). MERGA.

Dooley, T., Dunphy, E., & Shiel, G. (2014). *Mathematics in early childhood and primary education (3-8 years). Teaching and learning.* NCCA.

Edelman, J., Green, K., & Jett, C. (2019). Children's literature to inform mathematics teaching and learning: A systematic review of the research literature from 1999-2016. *International Journal of Science, Mathematics and Technology Learning, 26*(1), 49-60.

Elia, I., Van den Heuvel-Panhuizen, M., & Georgiou, A. (2010). The role of pictures in picture books on children's cognitive engagement with mathematics. *European Early Childhood Research Journal, 18*(3), 275-297.

National Council for Curriculum and Assessment. (2018). *Primary mathematics curriculum. draft specification junior infants to second class*. NCCA.

National Council for Curriculum and Assessment. (2020). *Draft primary curriculum framework.* NCCA.

Prendergast, M., Harbison, L., Miller, S. & Trakulphadetkrai, N. (2019). Pre-service and in-service teachers' perceptions on the integration of children's literature in mathematics teaching and learning in Ireland. *Irish Educational Studies*, *38*(2), 157-175.

Trakulphadetkrai, N. V., Aerial, J., & Yrjänäinen, S. (2019). Bringing mathematics alive through stories. In K. J. Kerry-Moran & J. Aerial (Eds.), *Story in children's lives: Contributions of the narrative mode to early childhood development, literacy, and learning* (pp. 199-225). Springer.

Exploring Mathematical Reasoning and Math-Talk Through Izak9

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This paper focuses on the use of Izak9 in promoting the development of mathematical reasoning and Math-Talk in an Irish classroom. Recent national and international assessments have shown many positive results in relation to the mathematical learning of Irish pupils (Shiel et al., 2014; Perkins & Clerkin, 2020). However, the skill of reasoning is a relative weakness among this cohort (Perkins & Clerkin, 2020). Reasoning is highlighted as one of the four 'elements' of the proposed new Primary Mathematics Curriculum (PMC), which is currently under development (National Council for Curriculum and Assessment (NCCA), 2016; NCCA, 2017). This study was qualitative in nature. The research methods utilised were focus group interviews and observations of mathematics lessons. Findings of this study demonstrated that mathematical reasoning skills were promoted in Izak9 lessons, specifically the exploration of patterns and the justification of mathematical processes and solutions. In addition, a Math-Talk environment was also supported through the use of the resource.

Introduction

The aim of this research was to explore the development of mathematical reasoning skills and the use of Math-Talk through the use of Izak9 in an Irish sixth-class classroom. The research has been drawn from a larger study in which the overarching research question was: What are the learning experiences created through the use of Izak9? This paper will focus on two of the emerging sub-questions: Can mathematical reasoning skills be developed through the use of Izak9?; To what extent can a Math-Talk environment be promoted through the use of Izak9?

The use of Izak9 was central to this research. Izak9 consists of a set of 27 cubes which can be separated into three sets of nine cubes to be used as sets in small groups in primary and early post-primary mathematics education. There are colours, numbers and other mathematical symbols and notations on the faces of the cubes (see Figure 1). Mathematical tasks to be used alongside the cubes are available through user login (Izak9, n.d.). The online user area contains a selection of tasks based on the number, algebra, shape and space and data handling.

Figure 1

Illustration of Izak9 cubes

This research comes at a time of curricular change in mathematics, with the development of the new PMC underway (NCCA, 2016). This time of transition offers an opportunity to research the mathematical learning experiences of Irish primary school pupils under the current Primary School Mathematics Curriculum (PSMC) (Department of Education and Skills (DES), 1999a), prior to the implementation of a new curriculum.

In recent years, there have been many positive results relating to Irish pupils' mathematical performance in both national and international assessments. The National Assessment of Mathematics and English Reading (NAMER) assesses Irish second class and sixth class pupils in four content areas (number and algebra, shape and space, measures, data) and five processes (understand and recall, implement, reason, integrate and connect, apply and problem solve). In the 2014 NAMER, significant improvement was noted in the performance of sixth class pupils in all content areas and processes when compared with 2009 results (Shiel et al., 2014). Irish pupils also performed significantly above the scale centrepoint for mathematics in the 2019 Trends in International Mathematics and Science Study (TIMSS) (Perkins & Clerkin, 2020). However, the TIMSS assessment outlined that the skill of mathematical reasoning is a relative weakness at both sixth class primary and second year post-primary levels. This research, therefore, sought to investigate pupils' use of mathematical reasoning through the use of Izak9. It also sought to explore how the use of Math-Talk may contribute to the development of this skill.

Literature Review

Mathematical Reasoning

Mathematical reasoning is listed as one of six skills that should underpin the five strands of the PSMC (DES, 1999a). The following features of mathematical reasoning resonate with this current research and will be explored: search for and investigate mathematical patterns and relationships; justify processes/results of mathematical activities, problems and projects (DES, 1999b, p. 69). Reasoning is also highlighted as one of the key elements of mathematical learning included in the proposed new mathematics curriculum (NCCA, 2017). In parallel with the PSMC, emphasis is placed on justification of mathematical processes, as well as the use of logic and generalisation of mathematical ideas.

Stein et al. (2008) describe an approach which enables mathematical reasoning through the use of collaborative learning and discussion. This model encourages learners to develop reasoning skills. Pupils work on a mathematical problem in a small group, before re-engaging with the whole class to discuss their work. The group work enables discussion and negotiation of the problem at hand in whatever way pupils see fit, in the knowledge that they may later share their solution with the group. On return to the whole class group there is still scope for further reasoning to occur. Different groups may bring different perspectives to the problem. Groups should be invited to outline their solutions or strategies, but may also be subjected to counter statements by teachers or peers. In these instances, groups must use logic to justify their case. This reasoning process enables the development of deep mathematical understanding through the use of negotiation, justification and generalisation. The success of a lesson such as that outlined by Stein et al. (2008) is dependent upon the engagement of the learners involved. The use of discussion, or Math-Talk, is central to this engagement.

Math-Talk

Math-Talk describes the use of discussion in explaining mathematical thinking, in an effort to support and extend learning (Chapin et al., 2009; Hufferd-Ackles et al., 2004; NCCA, 2016). The PSMC, although not referring specifically to Math-Talk, highlights the importance of talk and discussion in mathematics (DES, 1999a). A key aim of the PSMC is "to enable the child to use mathematical language effectively and accurately" (DES, 1999a, p. 12). Mathematical language includes content-specific mathematical vocabulary, but also incorporates the communication and expression of "mathematical ideas, processes and results in oral and written form" as well as the ability to "reason, investigate and hypothesise with patterns and relationships in mathematics" (DES, 1999a, p. 12). Further emphasis on the strategy of talk and discussion has been recommended in an Irish context (Dooley et al.,2014; NicMhuirí, 2012).

Through the effective development of a Math-Talk environment in the classroom, mathematical discourse can stretch beyond a tool simply for teacher evaluation. It can become exploratory territory whereby students can create shared understanding through consideration of the opinions of others and the revision of their own opinions (Wells and Arauz, 2006; NicMhuirí, 2012). With practice and teacher interaction, pupils should become more adept at refining their mathematical language (Wells & Arauz, 2006; Davis et al., 2010). Through the modelling and promotion of effective Math-Talk, teachers can facilitate learning activities which encourage students to reason mathematically. In particular, to explain, reflect and justify their thinking in mathematics.

Izak9

As a relatively new resource, there is limited research available into the use of Izak9 in mathematical teaching and learning. However, the research and reports that exist outline benefits associated with its use (Education Authority of Northern Ireland (EANI), 2016; Cantley et al., 2017). In Izak9 tasks, students participate in activities as part of a group, exploring and investigating possible responses to questions posed. An EANI report (2016) outlined the benefits of this in terms of collaboration and the students' engagement in learning. Following an intervention in using Izak9 as part of mathematics, the report showed a significant increase in "thinking skills and personal capabilities through collaborative/ peer learning" (p.3). Izak9 cubes aim to promote shared learning experiences (EANI, 2016; Cantley et al., 2017). Communication between learners in groups and in the whole class also improved significantly in the EANI study. The learning that is intended to take place through the use of Izak9 lends itself to the promotion of Math-Talk in the classroom. Students are given the opportunity to discuss the learning tasks in their groups and with the class, while teachers may act as facilitators to this (Cantley et al., 2017). Teachers are encouraged to use questioning to promote mathematical reasoning and discussion around the tasks. The significant development of the skill of communicating mathematically through the use of

Izak9 (EANI, 2016) lends itself to current and proposed Irish mathematics curricula, as communication is noted as a mathematical skill as well as an element in the PMC (DES, 1999a; NCCA, 2017). For the purpose of this study, Izak9 will therefore be explored in relation to the promotion of Math-Talk and the development of mathematical reasoning.

Methodology

The participants involved in this qualitative research were sixth class pupils attending an all-girls DEIS school in an Irish town. For this research, non-probability purposive sampling was used. This form of sampling is advantageous to small-scale research as it is less complicated and less costly to carry out (Cohen et al., 2011). There were 25 girls in the class, aged from 11 to 13. Ethical procedures were followed in line with Dublin City University (DCU) guidelines. The DCU Institute of Education Ethics application form was completed and approved prior to beginning research. Following this, informed consent was acquired by the school principal, board of management and parents. Children who had received parental/guardian consent to participate in the research were given a plain language statement in the form of a letter and an assent form. These outlined in child-friendly language what the research entailed, assuring that participation was voluntary.

The research methods utilised were focus group interviews, as well as the observation of six mathematics lessons in which Izak9 was the main resource in use. The lessons took place once a week over a six week period. Focus group transcripts and observational field notes from lessons were used as the primary data for the research.

For the Izak9 lessons, the class was split into six groups with four to five pupils in each. Two of these groups also took part in focus group interviews. These groups were interviewed twice in a semi-structured format - once before the Izak9 lessons commenced, and again after all six Izak9 lessons were completed. Questions were included in the interview schedule under the headings: experience of learning maths; group work; Math-Talk and reasoning. The class were observed in six mathematics lessons using Izak9. Each lesson lasted approximately one hour and the format of the lessons reflected the structure of that outlined by Stein et al. (2008). During each lesson, observational notes were recorded, detailing the events and interactions taking place. These notes served as a basis for more detailed field notes to be developed and analysed. Following each focus group interview and Izak9 lesson, the resulting data was reviewed and emerging themes were noted. After the lessons and focus groups were completed, all of the data was read thoroughly. Codes were assigned to data which was relevant to the area of research and which recurred throughout the data. The codes which emerged were then categorised in order to develop themes. These themes were: mathematical reasoning through Izak9; constructivist and social constructivist learning facilitated by Izak9; pupil engagement with Izak9; the use of Math-Talk in the classroom environment. All themes were explored in the resulting thesis. For the purpose of this paper, the themes of mathematical reasoning and Math-Talk are explored.

While great consideration was given to the design of this research, it is not without limitations. When using non-probability purposive sampling, it is important to highlight that data and results arising cannot claim to be representative of the wider population (Cohen et al., 2011).

Findings and Discussion

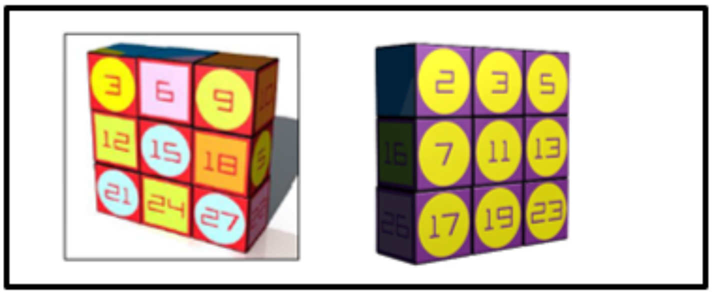
For the purpose of this paper, three of the six Izak9 lessons carried out in the larger study will be explored in addressing the research questions: Can mathematical reasoning skills be developed through the use of Izak9?; To what extent can a Math-Talk environment be promoted through the use of Izak9? The following subsections will outline mathematical reasoning and Math-Talk that took place during Lesson 1 and Lesson 2 (‘3x3 Demo’ and ‘Odd One Out’) and Lesson 4 (‘Magic Square’).

3x3 Demo’ and ‘Odd One Out’

In ‘3x3 Demo’ and ‘Odd One Out’ the pupils were instructed to build walls based on the first nine multiples of three and the first nine prime numbers (see Figure 2).

Figure 2

Multiples of three and prime numbers using Izak9 cubes



After the walls were built by the pupils, they were prompted by the teacher to discuss the patterns they could see on the walls. There was a heavy focus on simple patterns of colour and number. For example, in ‘3x3 Demo’, it was observed that the even numbers form a ‘diamond’ shape and the odd numbers form an ‘X’. The background shapes were also observed to form a pattern – circle, square, circle, square.

Some observations made in these lessons were inaccurate examples of patterns, as can occur in the initial stages of a Math-Talk environment (Davis et al., 2010; Wells & Arauz, 2008). For example, in the lesson ‘Odd One Out’, the top row had the numbers 2, 3 and 5. It was suggested that this was a pattern as 5 is the sum of 2 and 3. However, upon questioning it was agreed that as this does not continue on the other rows it is not considered a pattern. One participant looked at the difference between consecutive numbers on the wall in an attempt to discover a pattern. In this instance, through prompted continuation of her method, she discovered that this also did not seem to be a pattern. Prompts for justification by the teacher in such instances were an important component of the Math-Talk taking place, as referenced by Hufferd-Ackles et al. (2004).

It was observed that the majority of productive Math-Talk in both of these lessons occurred in the whole class segment at the end of each activity. At this point, refinement and further exploration of strategies was encouraged. This promotes the development of Math-

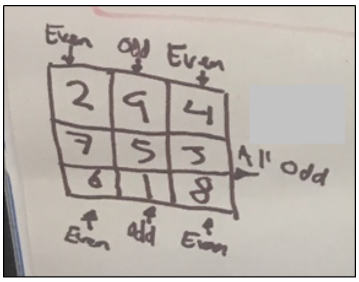
Talk skills from exploratory and imprecise to more coherent and accurate (Davis et al., 2010; Wells & Arauz, 2008). Lucy felt that this was a beneficial step in the learning process. "Afterwards the whole class had a big discussion on like what we did… I feel like that helped the most". The lesson 'Odd One Out', provided an example of such discussion. The class was asked to make the number 20 with two numbers, using subtraction only. The numbers available were 2, 3, 5, 7, 11, 13, 17, 19, 23. This problem was solved with relative ease by all groups (23 – 3 = 20). However, in the whole class discussion, a student questioned if this was the only solution. This was put to the rest of the group who proceeded to discuss their thoughts. It was concluded that as 23 is the highest available number and you must subtract, it is the only number you can begin with. For this reason 23 – 3 = 20 is the only possible solution. Pupil questioning here promoted the Math-Talk environment outlined by Hufferd-Ackles et al. (2004), and represented a move away from teacher-led learning to an environment in which pupils took responsibility for their own learning experiences. While the initial question was perceived as 'easy', through the use of effective Math-Talk, reasoning skills were further developed by facilitating a whole-class discussion after the task.

'Magic Square'

In the lesson 'Magic Square', the pupils were asked to build a 3x3 wall in which all of the rows, columns and diagonals added to the same number. After the task was completed, the class looked at the solutions of each group and reflected on these. Many observations were recorded with regards to the patterns of odd and even numbers in rows, columns and diagonals (see Figure 3). This led to speculation surrounding combinations of numbers that will give odd and even numbers. Again, in this instance, the pupils began to guide the direction of their learning. One participant wondered if there was a way of making 15 with just even numbers.

Figure 3

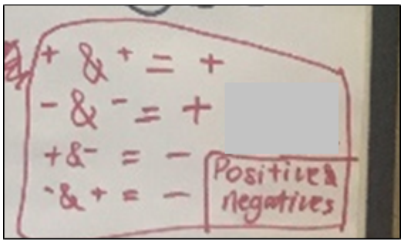
Sample of pupil observations in 'Magic Square'



This statement prompted an investigation by the class, in which the groups sought to prove or disprove the theory that it was impossible to make 15 using three even numbers. As part of this investigation, the class agreed that this theory was in fact correct. Through their work, they also discovered a generalised pattern regarding the sum of odd and even numbers. They deduced that the sum of two even numbers is always even, the sum of two odd numbers is always even, and the sum of an odd and even number is always odd. One pupil drew a comparison between this generalised pattern and something she had learned in maths earlier in the week. She wondered whether addition patterns of odd and even numbers were similar to the multiplication pattern of integers. Through examination of these general patterns, she observed similarities in their structure (see Figure 4 and Table 1).

Figure 4

Sample pattern exploration by participant



As the Izak9 lessons progressed, observations of patterns became a part of the reasoning process, furthering the pupils' ability to complete tasks and create opportunities for further learning. In '3x3 Demo' and 'Odd One Out' participants made simplistic observations based on the Izak9 walls. In 'Magic Square', the pupils began to seek out, explore and justify more complex patterns, at times leading to generalisations of mathematical ideas.

Conclusion

The research explored in this paper utilised qualitative research methods to explore the development of mathematical reasoning and Math-Talk skills through the use of Izak9. This research was particularly appropriate given the current development of the new PMC (NCCA, 2016; NCCA, 2017). The findings of this research demonstrated how Izak9 may be used to promote a Math-Talk environment in which the discussion of mathematical thinking can support and extend learning. In addition, the nature of this Math-Talk became more refined as the lessons progressed. The findings also demonstrated pupils' mathematical reasoning through the investigation of mathematical patterns. This observation began in an exploratory manner with simplistic patterns relating to colour and number observed. However, throughout the course of the Izak9 lessons, this developed into the formation of generalised patterns and relationships. This research was carried out on a small scale and therefore cannot claim to be representative of the wider population (Cohen et al., 2011). While there was not the scope in this paper to explore alternative mathematical resources to promote reasoning, this is an area that may warrant further research. In addition, educators should be provided with suitable continuous professional development (CPD) in the transition from the current PSMC to the new PMC, so as to address the shortcomings in current practice as highlighted in relevant studies (Perkins & Clerkin, 2020).

References

Cantley, I., Prendergast, M., & Schlindwein, F. (2017). Collaborative cognitive-activation strategies as an emancipatory force. *International Journal of Educational Research, 81*, 38-51.

Chapin, S. H., O'Connor, C., & Canavan Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn.* Math Solutions.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education.* Routledge.

Davis, A., Goulding, M., & Suggate, J. (2010). *Mathematical knowledge for primary teachers.* Routledge.

DES. (1999a). *Primary school curriculum: Mathematics.* The Stationery Office.

DES. (1999b). *Primary school curriculum: Mathematics teacher guidelines.* The Stationery Office.

Dooley, T., Dunphy, E., & Shiel, G. (2014). *Research report no. 18: Mathematics in early childhood and primary education (3-8 years) - teaching and learning.* NCCA.

EANI. (2016). *EANI Izak9 pilot evaluation report.* EANI.

Hufferd-Ackles, K., Fuson, K., & Sherin, M. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Izak9. (n.d.). *3x3 Demo*. Retrieved April 11, 2021, from www.izak9.com: https://www.izak9.com/teach/task/view?id=4&language=4

Izak9. (n.d.). *Odd One Out*. Retrieved April 11, 2021, from www.izak9.com: https://www.izak9.com/teach/task/view?id=6&language=4

NCCA. (2016). *Background paper and brief for the development of a new primary mathematics curriculum.* NCCA.

NCCA. (2017). *Primary mathematics curriculum: Draft specification - junior infants to second class. For consultation.* NCCA.

NicMhuirí, S. (2012). *Does it go without saying?: Mathematical thinking in whole class discourse*. (Doctoral thesis, Dublin City University).

Perkins, R., & Clerkin, A. (2020). *TIMSS 2019: Ireland's results in mathematics and science.* Educational Research Centre.

Shiel, G., Kavanagh, L., & Millar, D. (2014). *The 2014 national assessments of English reading and mathematics. Volume 1: performance report.* Educational Research Centre.

Stein, M., Engle, R., Smith, M., & Hughes, E. (2008). Orchestrating productive mathematical discussions: five practices for helping teachers move beyond show and tell. *Mathematical thinking and learning, 10*(4), 313-340.

Wells, G., & Arauz, R. (2006). Dialogue in the classroom. *The Journal of the Learning Sciences, 15*(3), 379-428.

Lesson Study as a Vehicle to Foster Teacher Agency

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The evidence in support of Lesson Study (LS) as a powerful approach to Teacher Professional Development (PD) continues to grow at a rapid rate. However, despite its widespread popularity, the mechanisms by which LS fosters teacher agency remain under-theorised. In order to address this theoretical gap, this paper proposes an emergent theoretical framework which seeks to explain how LS may foster teachers' achievement of agency. The paper describes how this framework derived from a systematic review of literature, which drew from a total of 32 empirical studies across a range of contexts. Guided by an ecological conceptualisation of teacher agency, a process of thematic analysis on data from studies allowed identification of a number of agency enablers, linked to LS activities. These links are used to generate an emergent theoretical framework, which holds relevance for researchers, LS facilitators, practitioners and policymakers. The timeliness of such a framework is notable given the recent publication of the draft Primary Curriculum Framework (NCCA, 2020), which espouses the importance of teacher agency in order to realise its aim of supporting children to achieve their full potential.

Introduction

The teaching profession in Ireland, like those in many jurisdictions, currently finds itself in a swiftly flowing river of educational change across both primary and post-primary sectors. The STEM subject disciplines have experienced an especially rapid flow of change across policy and curricula (DES, 2016; NCCA, 2020). STEM, which refers not simply to Science, Technology, Engineering and Maths, but a cross-curricular approach focusing on activities relevant to all four areas (Rosicka, 2016) does not exist as a standalone subject within the current Irish Primary Curriculum. This presents a challenge for generalist Irish primary teachers, who are expected to deliver effective integrated STEM teaching and learning experiences within mathematics and science (DES, 2016). Preparations are underway for a redeveloped primary curriculum, which aims to support a more authentic integrated approach to teaching and learning in all subjects, including the STEM disciplines (NCCA, 2020). In order to operationalise such changes in a way which most benefits outcomes for learners, Irish primary teachers require support in the form of professional learning opportunities which foster their achievement of agency (Darling-Hammond, 2005; NCCA, 2020). One school-based Professional Development (PD) approach which appears to offer promise in this regard is Lesson Study (LS) (Dudley et al., 2019; Ní Shuilleabháin & Seery, 2018). However, the means by which LS operates to support teachers' potential achievement of agency remains insufficiently theorised (Schipper et al., 2020). In order to address this theoretical gap, the researcher sought to answer the question: How and in what circumstances can Lesson Study enhance teacher agency in STEM and in the primary setting? This paper describes the process of how that question was addressed by a systematic review of existing LS literature, focusing on findings relating to agency enablers. The review process used an ecological conceptualisation of agency (Priestly et al., 2015) to identify and analyse possible connections between LS and teachers' achievement of agency in previous empirical studies, particularly in the area of primary STEM. Following from this, the paper explains how findings of the review led to the generation of an emergent theoretical framework and concludes by offering suggestions for how this framework may address the needs of practitioners and policy makers.

Lesson Study

LS, which originated in Japan over one hundred years ago, involves an action cycle whereby a group of teachers collaboratively plan, teach, observe and reflect on a research lesson taught with a group of pupils (Lewis, et al., 2006). Previous research has suggested that LS is effective in enhancing teachers' pedagogical skills and knowledge as part of curriculum reform (e.g. Ní Shúilleabháin & Seery, 2018); in fostering teachers' collaborative practice (e.g. Dudley et al., 2019; Lewanowski-Breen et al., 2020); and in supporting the development of positive teacher efficacy beliefs (e.g. Chong & Kong, 2012; Schipper et al., 2020). Seleznyov (2018) examined the fidelity of various adapted LS interventions in different jurisdictions. Their study identified seven critical components which are required in order for an LS intervention to be successful in enhancing teachers' learning:

1. The identification of a broad goal for pupil learning;
2. Teacher planning in collaborative groups drawing on relevant research and resources to create a research lesson;
3. A research lesson taught by one group member and observed by the others;
4. A post-lesson discussion using conversation protocols;
5. Repeated cycles of research using the findings from the post-lesson discussion;
6. The support of an outside expert or Knowledgeable Other (KO) throughout the process;
7. Opportunities for sharing new knowledge outside the LS group, for example, with other colleagues in their own or in other schools.

For the purpose of this paper, these seven critical components of LS as described by Seleznyov (2018) are adopted as a conceptual framework in order to examine existing studies related to LS.

Teacher Agency

Within the context of the draft Primary Curriculum Framework, an agentic teacher is described as "reflective, competent and capable of exercising professional judgement in response to individual learning needs in a variety of contexts" (NCCA, 2020, p. 6). Biesta and Tedder (2006, p.137) suggest that agency is not something teachers have, rather it is something which is achieved "by means of their environment". Similarly, Priestly et al. (2015) describe agency as an "emergent phenomenon of the ecological conditions through which it is enacted" (p.22). Furthermore, as Scanlon et al. (2020) contend, the achievement of agency cannot be considered static, but rather occurs on a processual basis, fluctuating from minute-to-minute, and is contingent on a complex myriad of factors. Thus, this paper adopts an ecological conceptualisation of teacher agency, according to which, achievement of agency derives from the complex interplay of individual efforts, available resources, contextual and structural factors as they converge in particular and unique ways (Priestly et al., 2015). In line with this conceptualisation of agency as ecologically constructed, this paper is underpinned by the view that agency is temporal, i.e. constructed based on the past (iterational), enacted in the present (practical-evaluative) and oriented towards the future (projective) (Emirbayer & Mische, 1998; Priestly et al., 2015). Agency-enabling factors include school cultures which feature strong horizontal relationships between colleagues, characterised by collegiality and sharing of practice (Poulton, 2020). Such cultures which emphasise teacher autonomy and professional judgement more broadly, rather than an overemphasis on accountability can also support teachers' achievement of agency.

Research Design and Methodological Approach

Given the focus on teacher agency, the review design was underpinned by a pragmatic epistemological orientation, which sought to ensure that the voices, views and lived experiences of teachers were represented in the included studies. A search protocol was prepared prior to commencing the search process, which included broad search terms, arranged according to three strands pertaining to LS: "Lesson study" and "agency"; "Lesson study" and "primary" or "elementary"; and "Lesson study" and "mathematics", "science" or "STEM". These terms were used to conduct a preliminary search of existing literature which returned over 3000 studies. Basic search limits were then set to include studies which focused on practicing teachers rather than preservice teachers; academic articles with full text accessible via electronic databases and articles which were published in English. Limits were also set to include studies from 2000 onwards, in order to focus on the most up-to-date research in relation to LS (Hennessy et al., 2019). The terms outlined in the search protocol were used to create search strings for each area of focus which were input to electronic databases Scopus, Education Source and Web of Science. Manual searches were then conducted of relevant conference proceedings as well as recent issues of International Journal of Lesson and Learning Studies (IJLS) towards the end of the literature retrieval process to ensure that the most current studies had been included. Reference lists from prior reviews were also checked in order to identify older seminal studies (Booth, 2016). Titles of returned studies were screened to remove duplicates. Abstracts of the remaining studies were then screened to retain only those which were relevant to the research question.

As the review sought to include studies from qualitative and quantitative domains, a quantitative critical appraisal checklist and qualitative critical appraisal checklist were designed and the relevant checklist was applied to returned studies (Hannes, 2011; Moher et al., 2009).These checklists were used to systematically examine research evidence to assess validity and relevance of their findings. The main reasons for exclusion of studies were that they were theoretical in nature, provided insufficient detail regarding the nature of the activities conducted during the LS or were not from the perspective of teachers. Following appraisal, data from included studies were then extracted. This data included the context, research design and findings from the study, as we all as direct quotes from teachers involved in studies. These data were then thematically analysed (Braun & Clarke, 2006) by deductive and inductive coding of instances where agency was constrained or enabled, as reported by participants.

Findings and Discussion

Following a process of identification, screening and critical appraisal, a total of 32 studies were deemed eligible for inclusion in the review. Thematic analysis of data from these studies were recorded as codes which were arranged under two categories: agency enablers and agency constrainers. For the purpose of the present paper, the focus will be placed on agency enablers derived from LS activities. While the research question sought to identify instances of agency enablement in the separate context of primary STEM, what emerged from the examination of findings from the included studies was that the factors which contributed to agency were not subject specific, rather these factors appeared to be common across multiple contexts in both primary and STEM. For the purpose of this paper, a summary of findings pertaining to agency enablers will be reported using illustrative quotes from some of the selected studies.

Agency enablers which were identified during LS activities were categorised as access to Pedagogical Content Knowledge (PCK), professional community membership and collaborative expertise. PCK describes the unique knowledge of curriculum, pupils and pedagogical strategies which are required for effective teaching (Shulman, 1987). Regarding PCK, for example, findings from one study (Coenders & Verhoef, 2019, p.228) noted that "going through complete Lesson Study cycles results in teachers realizing and internalizing new PCK and beliefs". Professional community membership describes the way in which the structured protocols of LS helps to create a sociocultural learning space for teachers, where they learn through engaging in critical reflective dialogue (Dudley et al., 2019). An example of this was evident in findings in another study (Brosnan, 2014, p.241), which speak of the "insulation and isolation" experienced by teachers which is ameliorated through engaging with other teachers and KOs in LS. In the case a further study (Chong & Kong, 2012), teacher learning in LS was attributed to "the constant collegial collaborative interactions between participants and KOs" (Baricaua Gutierez, 2018, p.813), which suggests such interactions foster agency under the category of collaborative expertise (Dudley et al., 2019).

Towards an Emergent Framework

Taking the enabling factors of PCK, professional community membership and collaborative expertise, Figure One aims to make explicit how the specific activities carried out as part of LS might foster each of the temporal dimensions of agency (Emirbayer & Mische, 1998; Priestly et al., 2015). While each enabling factor has been separated here for clarity, it is important to note that there is a degree of overlap, i.e. certain LS activities contribute to both PCK and professional community membership, for example, post-lesson reflective discussions. Within the framework, an overlap across temporal dimensions can also be seen, for example, the uncovering of tacit knowledge and beliefs spans both iterational, practical-evaluative and projective dimensions.

Figure 1

Emergent theoretical framework linking LS activities to temporal dimensions of teacher agency.

Agency enabler	***Access to Pedagogical Content Knowledge***		
Dimension of agency →	**Iterational**	**Practical-evaluative**	**Projective**
Phase of LS cycle↓			
Formulate goal	Tacit prior knowledge and beliefs about pedagogy and classroom practices uncovered		
Research & plan	Tacit prior knowledge, skills and experiences pertaining to overarching goal are drawn from participants by the K.O.	K.O. facilitates critical reflection on relevant research linked to area of focus of research lesson	Research lesson is planned with pupil responses and outcomes anticipated
Teach & Observe	Focused teaching and observation on pupil interaction with planned tasks which are documented in order to inform post-lesson discussion		
Reflect & Evaluate	Critically reflective dialogue facilitated by K.O., focused on taught research lesson, using the new knowledge generated to inform future teaching/ next LS cycle		

Agency enabler	***Access to Collaborative Expertise***		
Dimension of agency →	**Iterational**	**Practical-evaluative**	**Projective**
Phase of LS cycle↓			
Formulate goal	Tacit prior knowledge and beliefs about pedagogy and classroom practices uncovered, in order to identify learning outcomes		
Research & plan	Tacit prior knowledge, skills and experiences pertaining to overarching goal are drawn from participants by the K.O		
Teach & Observe	Opportunity to observe a colleague teaching, with focus on taught lesson and pupils' learning rather than on the person		
Reflect & Evaluate	Critically reflective dialogue facilitated by K.O., focused on taught research lesson, using the new knowledge generated to inform future teaching/ next LS cycle		

Agency enabler	***Access to Professional Community Membership***		
Dimension of agency →	**Iterational**	**Practical-evaluative**	**Projective**
Phase of LS cycle↓			
Formulate goal **Research & plan** **Teach & Observe**	Safe environment created by use of structured LS protocols-encourages uncovering of tacit beliefs ,	Establishment and strengthening of horizontal relationships	Discussion and sharing of variety of perspectives on purposes of

Reflect & Evaluate	knowledge and experiences in relation to pedagogy and classroom practices	between teachers within the LS group	education and broad vision for pupil outcomes

Limitations

A limitation deriving from features described in the framework is that they are relatively broad in nature, and may not be applicable to LS in every context. However, the emergent framework serves an important purpose, in that it has contributed to establishing a theoretical connection between agency and LS, which did not previously exist.

Implications

While this review sought to examine how and in what circumstances LS can enable teacher agency in the context of primary STEM, what emerged was that insufficient literature was available to examine LS and agency in this specific context. This highlights that despite LS receiving much scholarly attention, further empirical research is required in order to examine how these LS may contribute to, or indeed constrain, teacher agency.

The findings of this review have enabled the development of an emergent theoretical framework which seeks to make explicit how LS can contribute to teachers' achievement of agency. Due to its purely theoretical nature, the emergent framework would benefit from further application and testing in the field, for example, by teacher practitioners using it to support critical professional reflection on their own achievement of agency during LS. The framework may also be useful for LS facilitators who wish to foster teacher agency as part of their practice. It may also be useful for policy makers engaged in curricular reform, who may find the framework useful in guiding LS as a PD approach in the context of such reform.

References

Baricaua Gutierez, S. (2016). Building a classroom-based professional learning community through lesson study: Insights from elementary school science teachers. *Professional Development in Education, 42*(5), 801-817.

Biesta, G., & Tedder, M. (2006). *How is agency possible? Towards an ecological understanding of agency-as-achievement. Learning lives: Learning, identity, and agency in the life course*. Working paper. University of Exeter.

Booth, A. (2016). Searching for qualitative research for inclusion in systematic reviews: A structured methodological review. *Systematic Reviews, 5*(1), 74-97.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in Psychology, 3*(2), 77-101.

Brosnan, A. (2014). Introducing lesson study in promoting a new mathematics curriculum in Irish post-primary schools. *International Journal for Lesson and Learning Studies, 3*(3), 236-251.

Chong, W. H., & Kong, C. A. (2012). Teacher collaborative learning and teacher self-efficacy: The case of lesson study. *Journal of Experimental Education, 80*(3), 263-283.

Coenders, F., & Verhoef, N. (2019). Lesson study: Professional development (PD) for beginning and experienced teachers. *Professional Development in Education, 45*(2), 217-230.

Darling-Hammond, L. (2005). Teaching as a profession: Lessons in teacher preparation and professional development. *Phi Delta Kappan, 87*(3), 237-240.

Department of Education and Skills (DES). (2016). *STEM Education Policy Statement 2017-2026*. Retrieved 10th May 2019 https://www.education.ie/en/The-Education-System/STEM-Education-Policy/stem-education-policy-statement-2017-2026-.pdf

Dudley, P., Xu, H., Vermunt, J. D., & Lang, J. (2019). Empirical evidence of the impact of lesson study on students' achievement, teachers' professional learning and on institutional and system evolution. *European Journal of Education, 54*(2), 202-217.

Emirbayer, M., & Mische, A. (1998). What is agency? *American Journal of Sociology, 103*(4), 962-1023.

Eteläpelto, A., Vähäsantanen, K., Hökkä, P., & Paloniemi, S. (2013). What is agency? Conceptualizing professional agency at work. *Educational Research Review, 10,* 45-65.

Godfrey, D., Seleznyov, S., Anders, J., Wollaston, N., & Barrera-Pedemonte, F. (2019). A developmental evaluation approach to lesson study: Exploring the impact of lesson study in London schools. *Professional Development in Education, 45*(2), 325-340.

Groves, S., Doig, B., Vale, C., & Widjaja, W. (2016). Critical factors in the adaptation and implementation of Japanese lesson study in the Australian context. *ZDM, 48*(4), 501-512.

Hadfield, M., & Jopling, M. (2016). Problematizing lesson study and its impacts: Studying a highly contextualised approach to professional learning. *Teaching and Teacher Education, 60*, 203-214.

Hannes, K. (2011). Critical appraisal of qualitative research. In Noyes, J., Booth, A., Hannes, K., Harden, A., Harris, J., Lewin, S., & Lockwood, C. (Eds.). *Supplementary Guidance for Inclusion of Qualitative Research in Cochrane Systematic Reviews of Interventions*. Version 1. Retrieved 6th May 2020 http://cqrmg.cochrane.org/supplemental-handbook-guidance

Hennessy, E. A., Johnson, B. T., & Keenan, C. (2019). Best practice guidelines and essential methodological steps to conduct rigorous and systematic meta-reviews. *Applied Psychology: Health and Well-Being, 11*(3), 353-381.

Lewanowski-Breen, E., Shuilleabhain, A. N., & Meehan, M. (2020). Lesson study and the long-term impact on teacher professional community development. *International Journal for Lesson & Learning Studies, 10*(1), 89-101.

Lewis, C., Perry, R. & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35*(3), 3-14.

Moher, D., Liberati, A., Tetzlaff, J. & Altman, D. (2009). Preferred reporting items for systematic reviews and meta-analyses: the PRISMA statement. *PloS Medicine, 6*(7), 1-6.

National Council for Curriculum and Assessment (NCCA). (2020). *Draft primary curriculum framework.* Retrieved 6th July 2020 https://ncca.ie/media/4456/ncca-primary-curriculum-framework-2020.pdf

Ní Shuilleabháin, A., & Seery, A. (2018). Enacting curriculum reform through lesson study: A case study of mathematics teacher learning. *Professional Development in Education, 44*(2), 222-236.

Poulton, P. (2020). Teacher agency in curriculum reform: the role of assessment in enabling and constraining primary teachers' agency. *Curriculum Perspectives, 40*(1), 35-48.

Priestly, M., Biesta, G. & Robinson, S. (2015). *Teacher agency: An ecological approach.* Bloomsbury.

Rosicka, C. (2016). *From concept to classroom: Translating STEM education research into practice.* Australian Council for Educational Research (ACER).

Scanlon, D., Calderon, A., & MacPhail, A. (2020). Teacher agency in enacting physical education in a period of curriculum change and reform in Ireland. *The Curriculum Journal, 32*(1), 48-66. Retrieved from https://bera-journals.onlinelibrary.wiley.com/doi/full/10.1002/curj.80

Schipper, T. M., de Vries, S., Goei, S. L., & van Veen, K. (2020). Promoting a professional school culture through lesson study? An examination of school culture, school conditions, and teacher self-efficacy. *Professional Development in Education, 46*(1), 112-129.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-23.

Seleznyov, S. (2018). Lesson study: An exploration of its translation beyond Japan. *International Journal for Lesson and Learning Studies, 7*(3), 217-229.

An Exploration of the Impact that Continuous, Summative Assessment has on how University Mathematics Students Spend their Study Time

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A move to modularisation has seen an increase in the amount of continuous, summative assessments undergraduate students must complete. Students advocate for these assessments as it encourages them to remain engaged, and lecturers feel compelled to give them to ensure students do not neglect their modules. This phenomenon has been dubbed "the assessment arms race". In this study we examine the impact that continuous, summative assessment has on how Stage 3 mathematics undergraduate students in University College Dublin choose to spend their study time. Students completed weekly timesheets reporting how much time they spent on activities in their six modules in trimester 2, 2019-20. They were asked to describe the factors that influenced how they spent their study time, and if there were other activities they would have liked to complete. Although data collection was cut short owing to COVID-19, an analysis of the data up to that point provides clear evidence of the assessment arms race in action and highlights the behavioural impact that it has on how students approach their learning. We examine recommendations, and discuss possible solutions, to address this.

Introduction

> In all arms races, all players can be harmed because they need to take actions they would not otherwise choose. If this holds for the assessment arms race, then there are no winners, only losers (Harland, 2020, p. 18).

As Head of Teaching and Learning in the School of Mathematics and Statistics, the second author received feedback from undergraduate mathematics students about an excessive workload caused by in-trimester summative assessments. With a usual registration of six, 5-credit European Credit Transfer and Accumulation System (ECTS) credits per trimester, students claimed to experience weeks when the number of assignments due, and tests scheduled, resulted in an unmanageable workload. Anecdotally, students were not advocating for a great reduction in assessments and did not want 100% final examinations. Along with many lecturers, students felt that what was needed was better coordination of the timings of tests and assignments. This task fell to Programme Directors whose attempts to co-ordinate the scheduling were mainly unsuccessful. The flexibility of the modular system meant that students enrolled on a programme were frequently not taking the same six modules. In turn, module coordinators could argue that there were pedagogical reasons for the frequency and timing of their assessments, and if they reduced the number of assessments dramatically, students might end up ignoring their module.

This phenomenon University College Dublin (UCD) is experiencing is not unique. Studies conducted in Ireland (National Forum, 2016; O'Neill, 2019), the UK (Tomas & Jessop, 2019), and New Zealand (Harland et al., 2015) describe high levels of continuous, summative assessment in university undergraduate programmes, caused or exacerbated by a modular system – aptly called "the assessment arms race" by Harland et al. (2015). The authors of these studies call for universities to take a programmatic approach to assessment in order to promote deep learning and “slow scholarship” (Harland et al., 2015, p. 536). However, to do this requires a re-think of our programmes, and we wanted more than anecdotal evidence before embarking on such a change. Our aim in conducting this study was to obtain an evidenced-based, programmatic view of continuous, summative assessment in our school, and its impact on student study behaviour, in order to start a conversation amongst faculty. Specifically, we aimed to address the following research questions: How much time do students self-report spending on each module activity each week of the trimester? What factors influence students’ choices of how much time to spend on each module activity each week of the trimester? What module activities do students report wanting to spend more time on each week of the trimester?

To this end, in Trimester 2 of 2019-20, we recruited students from one of our undergraduate programmes to complete a weekly timesheet for the duration of the trimester. Unfortunately, the pandemic forced us to pivot online after seven weeks. Ultimately, student participation in the study was negatively affected and we did not collect data after week 11. In this paper we present the findings from the analysis of the data gathered until UCD pivoted online and examine implications for mathematics programme outcomes and design.

Literature Review

Harland et al. (2015) conducted in-depth interviews probing assessment practices at a university in New Zealand, with 62 lecturers and students from the Science, Humanities, and Professional disciplines. Irrespective of the discipline, they found that students preferred small continuous, summative assessments as opposed to high-stakes final examinations. The students reported that they had no time to work on non-graded work, and sometimes missed lectures and/or worked in groups to complete assignments or study. High-achieving students felt they were always working at sub-optimal levels, which created stress. Overall students exhibited a “love-hate relationship” (p. 535) towards assessment as they were stressed by the frequency of it and frustrated by the lack of coordination amongst lecturers. Lecturers were unhappy about assessing frequently, but the flexibility afforded by the modular system meant that if they did not, students may ignore their modules. This left them competing for students’ attention and exercising what Harland et al. call a “pedagogy of control” (p. 538).

Tomas and Jessop (2018) compared programme assessment data from 73 programmes across 35 research-intensive and 38 teaching-intensive universities in the UK. They found that the research-intensive universities had significantly higher summative assessment loads, and significantly more assessment by examination than the teaching-intensive institutions. They cite an example of a 3-year mathematics programme in a research-intensive university that had 227 summative assessments. They conjecture that with this level of frequency, the assessments must be relatively brief and question whether such tasks can encourage deep learning. The 16 Science students in the Harland et al. (2015) study reported having on average 1.44 graded assessments per week, with a module load of up to four modules per semester. A study of assessment practices in third-level programmes in Ireland (National Forum, 2016) found that in single-trimester modules, the average number of assessments was

2.6 and 4.5 for five and ten ECTS credits module respectively. This averages to 1.04 and 0.9 weekly assessments respectively. They found the most common module size in Ireland was 5 ECTS credits meaning that students may be taking up to six modules per semester.

All learning takes time, and when students are constantly being assessed, it is questionable whether they have the time and space needed to engage in deep learning and develop higher order skills (Harland et al., 2015; Tomas & Jessop, 2018) and have the opportunity to become self-directed and independent learners (Harland et al., 2018). The authors of the UK and New Zealand studies, along with O'Neill (2019) in Ireland, all advocate taking a programmatic approach to assessment – for example by assessing across the curriculum and focusing on programme outcomes. Harland and Wald (2020) provide further suggestions for "de-escalating" the arms-race. To address the scheduling issue, they observe that it may be necessary to reduce student choice in modules, and note that if workload is still an issue, then lecturers need to assess only that which they deem important. They acknowledge that this "will require careful planning and systemic change over time" (p. 10), while O'Neill (2019) observes that lecturers seem unwilling to move from the status-quo.

Method

For this study, an undergraduate mathematics student cohort was required that was enrolled to a suite of core modules in trimester 2, ensuring a level of homogeneity among participants. We also required that the group was sufficiently large to recruit participants from. Finally, in UCD, Science students choose their degree pathway at the end of their second year, therefore we were looking for students in Stages 3 or 4. Subsequently, the participants chosen for this study were Stage 3 undergraduate students (n=20) pursuing a single major in Applied and Computational Mathematics (ACM). This cohort was chosen as they have five core modules in trimester 2 of Stage 3. The sixth module is an elective. The third author was Programme Director for this group. While he was responsible for recruiting participants initially, the first author was responsible for contact with them after recruitment, and anonymising data as she was based outside of the School of Mathematics and Statistics and had no academic relationship, or otherwise, with the students. This allowed participants to remain anonymous to the other authors who held the previously mentioned support roles in the school. In total, nine students agreed to participate (45% response rate).

As we were keen for participants to remain engaged for a trimester, we had to strike a balance between the level of detail required of them, and the time it would take them to record it. Consequently, we asked participants to complete a spreadsheet provided on Google Drive at the end of each week, starting in week 2, indicating the hours spent on a list of activities across each of their six modules. The list of module activities was devised by the authors and was adapted over the first two weeks of the study as students described additional activities engaged in. The final list is as follows: lectures; tutorials; assignment; revising for test; reviewing lecture notes; and revising for final exam. We note that marks were not awarded for lecture attendance. For each module, participants were asked to provide open responses on the spreadsheet, describing the main factors influencing their study time and stating any activities they would have liked to spend more time on. While this is a quantitative study, the open responses allowed students to describe their decisions for the time they reported spending on each weekly activity. To incentivise consistent participation, any participant who completed the timesheet for a given week, was entered into a weekly draw for a €20 *One4All* voucher. If participants did not complete the sheet for a week, the draw was intended as an incentive to re-engage with the study, and a participant could win the draw multiple times. We planned to seek more in-depth explanations from students through interviews at the end of the trimester. However, this part of the study was impacted by the disruption caused by COVID-19.

The response rate varied each week from between one and eight students. The decision was taken to suspend data collection after week 11 of the trimester – the penultimate week of lectures. Only one student completed the spreadsheet in Weeks 10 and 11, therefore we will only include data collected up to Week 9 in what follows. Descriptive statistics were performed on the weekly time data and the average weekly time spent on each activity was calculated (RQ1). This was then compared with an assessment calendar, the students' comments on what influenced how they spent their time each week (RQ2) and what they would have liked to spend more time on (RQ3). UCD entered lockdown during the field-work break (two weeks when classes are not scheduled) when seven weeks of the trimester had been completed. Although the data collection was cut short, the results up to this first field-work break shed light on how students spend their study time, and their motivations for doing so. This period also includes the traditional midterm assessments. We will present our findings in the next section.

Results

The weekly participation rates in the study and the average time students self-reported studying each week are presented in Table 1. The study commenced in Week 2 and FW1 and FW2 denote two field-work break weeks when none of the core modules had scheduled lectures or tutorials. Of the nine participants, eight completed the spreadsheet in Weeks 4-6, with seven taking part in Week 7. The average time spent per week from Weeks 2-9 (including FW1 and FW2) ranged from 29 hours to 46 hours.

Table 1

Number of participants and average time spent studying each week

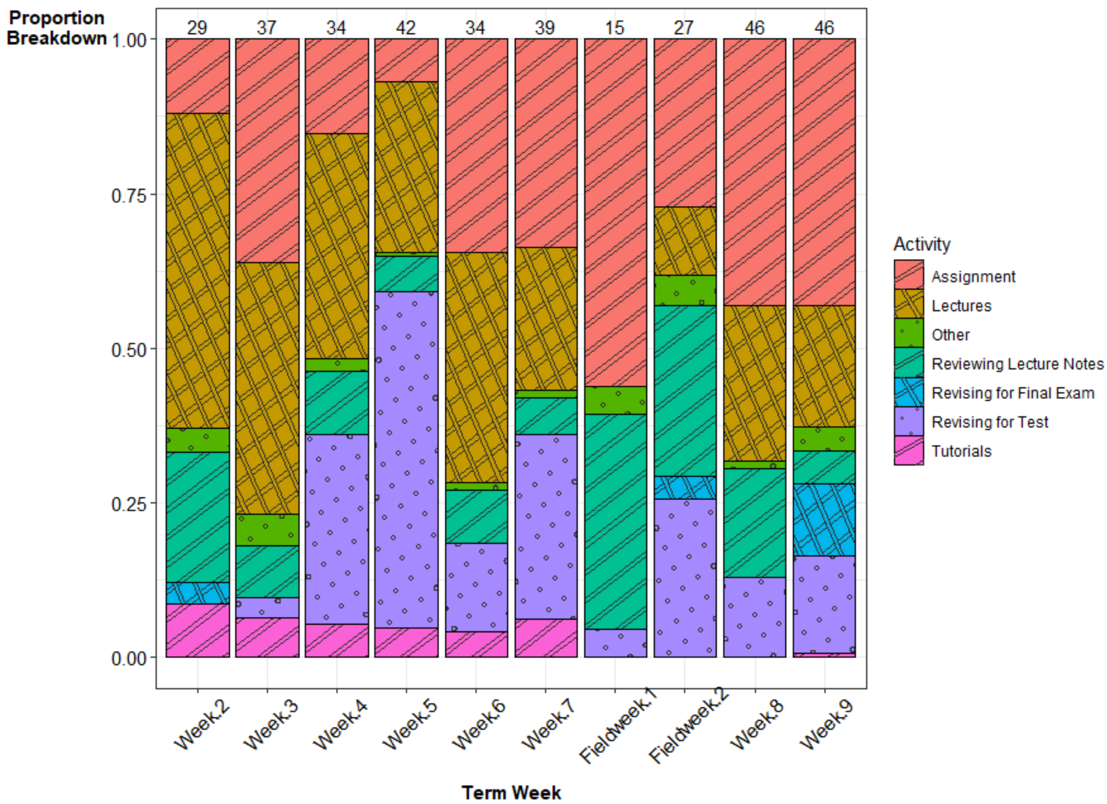
	Wk2	**Wk3**	**Wk4**	**Wk5**	**Wk6**	**Wk7**	**FW1**	**FW2**	**Wk8**	**Wk9**
Students	4	5	8	8	8	7	6	4	4	3
Av. Time	29	37	34	42	34	39	15	27	46	46

In terms of the first research question, the average time students self-report spending weekly on each module activity is presented in Figure 1. We address the second and third research questions by examining these results in conjunction with the assessment calendar for the five core modules (Table 2), and the student comments on the factors that impacted how they chose to spend their time and the activities they would like to have spent more time on.

There were no assignments due, or tests scheduled in Week 2, and the three activities that students spent most time on were attending lectures, doing assignments, and reviewing lecture notes. In Week 3, Module D had an assignment worth 7% due and all four participants that week stated that they spent time on this assignment. Two students noted that they worked on other assignments also - one due in week 4, and to prepare for the tests in Weeks 4 and 5. In terms of what they would have liked to spend more time on, two students in each of Weeks 2 and 3 mentioned that they would have liked to spend a little more time (1-2 hours) on some of the "ungraded" worksheets and reviewing or pre-viewing lecture notes.

Figure 1

The average proportion of time spent by students on each academic activity over the trimester and the average number of hours for each week are displayed at the top of the bar.



Attending lectures remains the activity with the highest proportion of time spent on it in Weeks 3 and 4 at 40.5% and 36.4% respectively. In Week 4, revising for a test is the next most popular activity at 30.8%. All participants said that the test in Module D influenced how they spent their time. Two commented that they spent time revising for upcoming tests in Week 5 also. Indeed Week 5 saw tests scheduled in three modules and the proportion of time students reported spending revising for tests increased to 54.5%. All seven participants commented on how the tests dominated how they spent their time, with two noting that it impacted on lecture attendance.

> Multiple in class tests meant I didn't have time to catch up on lecture notes and also affected my attendance in lectures. [Student 8, Week 5]
>
> The midterms influenced basically what I was doing during the week. As soon as one was over, I was preparing for the next one. […] I skipped a few lectures this week so that I could study for midterms during the week. Wish I could have went. I don't think it's valuable to go to lectures when midterms are on. [Student 5, Week 5]

The two core modules that did not have summative assessments in Week 5 were Modules D and E. Three of the participants stated that they would have liked to have spent time revising notes and doing "ungraded" assignments for Module E. One student skipped lectures in Module D to work on the assessments due, with another stating they would have liked to have had time to review the notes for this module in order not to fall behind.

Table 2

Timings of assessments due and tests scheduled with corresponding weightings

	Module A	Module B	Module C	Module D	Module E
Week 3				Assign. (7%)	
Week 4		Assign. (5%)		Test (10%)	
Week 5	Test (10%)	Test (7.5%)	Test (15%)		
Week 6					
Week 7		Assign. (5%)		Assign. (7%)	Test (20%)
FW1					Assign. (5%)
FW2					
Week 8	Test (20%)		MCQ (15%)		
Week 9		Assign. (10%)	Assign. (10%)		

There were no assessments due or scheduled in Week 6, but five of the six participants stated that the upcoming assessments in Week 7 influenced how they spent their time. The other student was sick for the week. The choice of which of the three assessments to concentrate on in Week 6 was an issue for some of the participants.

> [Module E] midterm worth 20% next week so most of my time was spent on that. That meant I had to neglect the [Module B] and [Module C] assignments which are due next week because they are worth more [sic]. [Student 9, Week 6]

Student 1 also spent "a lot of time" studying Module E, whereas Student 5 concentrated on the assignments but then stated that they would have liked to have spent more time revising Module E. Students 6 and 9 stated they would have liked to have spent 11 and 13.5 hours more respectively on the three assessments. Modules A and C did not have assessments scheduled for Week 7. Student 9 would have liked to work on "non-graded problem sheets" for Module C, while another student would have liked to pre-read lecture notes in it. Student 8 would have liked to have had time to review the lecture notes for Module A.

Unsurprisingly, Week 7 was dominated by the three assessments and students reported spending on average 24.7 hours, or 63.5% of their time, revising for the test or working on assignments.

> No time for going to lectures or studying modules other than the ones with all of the assignments and midterms due. [Student 1, Week 7].

The first field-work break was welcomed as a time to take a break, although one student spent it applying for internships and doing interview preparation. Indeed, while summative assessment deadlines heavily impacted how students spent their time during the trimester, students also reported that their schedules were impacted by being sick, completing short-courses, undertaking competitions, writing internship applications, preparing for and attending interviews, and of course later in the trimester by COVID-19.

Finally, some general comments about the module activities listed in the study. One student stated that the activity 'Reviewing lecture notes' is often part of completing an assignment. And revising for a test may include completing non-graded assignments:

> Primarily [worked on] graded assignments ensuring all answers were correct but I also spent notable amounts of time on nongraded assignments in preparation for the numerous midterm exams I will have over week 4 and 5. [Student 4, Week 3]

This comment was made in the third week of the trimester when students had more time to plan. From comments in later weeks we can surmise that as assessments became more frequent, notes were reviewed and assignments completed on a need-to-do basis.

Discussion and Conclusion

The findings show that as early as Week 4, summative assessments are playing a role in students' decisions on how to spend their study time. In Weeks 5-7, revising for tests or completing graded assignments dominates, and comes at the cost of missing lectures and ignoring other modules. This resonates with findings on one of the student assessment experiences reported by Harland et al. (2015). Students reported not having time to work on study activities outside those that were graded and reported missing lectures to cope with the workload. High-achieving students always felt like they were under-achieving and were experiencing stress as a result. In the open comments, some of our students did speak about an assessment being "very stressful because it was worth so much" or noting that "not much due this week so I wasn't so stressed". Given that the amount of extra time students reported wanting to spend on assessment activities increased during the trimester, it may indicate that they felt they were not performing at their best.

A weakness of this study is not having a complete set of data for the trimester nor having student interviews to expand on the data more. Nonetheless, we can see the assessment arms race in action from the student perspective. We are missing the lecturer perspective, although we suspect it would be similar to that of the lecturers in the Harland et al. (2015) study. We can see that lecturers' concerns about students ignoring their module if it is not assessed, are well-founded. We also did not seek evidence of whether the students were engaging in deep learning and developing higher order skills (Harland et al., 2015; Tomas & Jessop, 2018) given their assessment loads.

We are at the start of the journey to address what we believe is an "assessment arms race" in our undergraduate programmes, and we want to proceed quickly, but with caution. There are often many off-the-cuff suggestions when this problem is raised: only allow final exams; decree that all modules should have only one continuous summative assessment item, along with a final examination, and that the assessments be coordinated at the programme level (if this was even possible to arrange in a modular system) et cetera. However, the consequences of these decisions need to be considered. Like the authors of the studies mentioned above, we see possible solutions including some, or all, of the following: an elimination, or reduction, of student module choice; a reduction in the number of 5-credit modules; and/or, take a programmatic approach to assessment that ensures students meet high-level programme outcomes. An example of such a programmatic approach in an undergraduate ecology degree is provided by Harland (2020) where a research-based approach is taken. We suspect we are not unique across the Higher Education system in Ireland and look forward to conversations with colleagues at MEI to discuss this further.

References

Harland, T., McLean, A., Wass, R., Miller, E., & Nui Sim, K. (2015). An assessment arms race and its fallout: High-stakes grading and the case for slow scholarship. *Assessment and Evaluation in Higher Education, 40*(4), 528-541. https://doi.org/10.1080/02602938.2014.931927

Harland, T. (2020). *University challenge: Critical issues for teaching and learning.* Routledge.

Harland, T., & Wald, N. (2020). The assessment arms race and the evolution of a university's assessment practices. *Assessment and Evaluation in Higher Education*, *46*(1), 105-117. https://doi.org/10.1080/02602938.2020.1745753

National Forum for the Enhancement of Teaching and Learning. (2016). *Profile of assessment practices in Irish Higher Education*. https://www.teachingandlearning.ie/publication/profile-of-assessment-practices-in-irish-higher-education/

O'Neill, G. (2019). Why don't we want to reduce assessment? *AISHE-J: The All-Ireland Journal of Teaching and Learning in Higher Education, 41*(7), 1-7.

Tomas, C., & Jessop, T. (2019). Struggling and juggling: A comparative analysis of assessment loads across different universities. *Assessment and Evaluation in Higher Education, 44*(1), 43-53. https://doi.org/10.1080/02602938.2018.1463355

Storying Mathematical Identity Narratives

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This paper presents an analytical framework called storying stories. It was used to analyse narrative data that was collected as part of a PhD study into the mathematical identity of science and engineering students (MISE). This framework was selected because it relies on the co-construction of meaning between researcher and participant. Mathematical identity is commonly assumed to be a co-constructed phenomenon and thus, narrative methods of data collection and analysis have regularly been employed by mathematical identity researchers. However, the storying stories framework presented in this paper appears to have been overlooked, or perhaps discounted, by these researchers.

This stage of the PhD study involved one participant who took part in a pilot narrative interview concerning their mathematical identity. Although thematic analysis allows the development of themes that are common across participants, a new method of analysis was required to use narrative inquiry as a descriptive tool for an individual participant. This paper will present how this method for analysing and presenting such data was applied to this interview.

Introduction

The focus of this paper is the PhD research of the first author, conducted under the supervision of the second and third authors. The purpose of the study is to investigate the mathematical identity of science (including science education) and engineering students (MISE). This work builds on previous mathematical identity research conducted in Ireland (Eaton et al., 2013; see also Howard, 2019, p. 140). The overall aim of this research is to characterise and compare participants' mathematical identities and investigate how these identities change over time. To explore this, I conducted an online open-ended questionnaire, which I analysed using thematic analysis (Howard et al., 2019). I followed-up with focus groups to clarify these themes, explore any additional aspects and examine the changes in their mathematical identities during this time.

The focus of this paper is a pilot interview with one science education student who had previously participated in the open-ended questionnaire. The participant had experience teaching mathematics and physics at post-primary level through placement as part of their course. The purpose of the interview was to investigate how the participant's mathematical identity changed over time. Through the reflective process of a narrative interview, I hoped that reconstructing their mathematical identities would endow the participants with "a new sense of meaning and significance" (Clandinin & Connelly, 2000, p. 42) about their relationship with mathematics. Below, I will describe three concepts: mathematical identity, narrative inquiry and the *storying stories* framework for analysis. Secondly, I will demonstrate how this framework was applied to narrative interview data in MISE. Finally, I will present the results of the analysis and some conclusions.

Literature Review

In their seminal paper, Sfard and Prusak (2005) defined identity as "collections of stories ... that are reifying, endorsable and significant" (p. 16). Thus, they reimagined identity as a "communicational practice" centred on narratives rather than relying on static and extra-discursive notions of who one is (p. 16). Since that important reimagining, it has become more common to consider identity and narrative as related rather than synonymous (Howard, 2019, p. 140; Machalow et al., 2020). Following this viewpoint, I define mathematical identity to be the multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others (Eaton & OReilly, 2009, p. 228). In this study, I consider mathematical identity to be "storied ... through narratives" (Radovic et al., 2018, p. 28, Table 2). This implies that mathematical identity is fluid and that narratives are "enactments of identity, constructed in the moment" (p. 29). I also embrace the view of Kaasila (2007, p. 206), who explains that "one's mathematical identity is manifested when telling stories about one's relationship to mathematics, its learning and teaching."

Methodology

Narrative Inquiry

This study is situated in the area of narrative inquiry, which has been recommended for understanding experiences (Clandinin & Connelly, 2000, p. 20). It is appropriate for longitudinal studies such as MISE, because it is "a collaboration between researcher and participants, over time" (p. 20). This collaborative property aided me in positioning the participants as co-researchers who "shape the research process" (Cohen et al., 2007, p. 37) and whose agenda can predominate my own (p. 376). The telling of narratives "allows for growth and change" (p. 71) since narratives, like mathematical identity, are constructed and reconstructed as they are told, and over time (Howard et al., 2019, p. 1454; McCormack, 2000a, p. 286, 2004, p. 220).

In general, narrative researchers are concerned with "everyday or natural linguistic expressions, not with decontextualized short phrases or with abstracted counts" (Polkinghorne, 1995, p. 6). Such researchers acknowledge that context is important for sense-making (Clandinin & Connelly, 2000, p. 32) and aim to position the participant as the expert in order to represent and understand *their* experiences from *their* point of view. Narrative inquiries frequently feature semi-structured interview protocols and open-ended questions that "allow the speaker to 'hold the floor' beyond the limits of a usual [speaking] turn" (Mishler, 1991, p. 74).

I designed my interview protocol in a manner consistent with narrative inquiry and with Mishler's *interview as conversation* (Mishler, 1991). Narrative interviewing is a method of data collection that facilitates co-construction of meaning between interviewer and participant (p. 52). This is important because we are complicit in the creation of the world that we study (Clandinin & Connelly, 2000, p. 61). I was drawn to narrative interviewing because "[t]he goal of the narrative interview is to get the interviewee to tell stories about things that are important to him or her" (Kaasila, 2007, p. 207 emphasis added). To empower participants to do this, one must change the traditional interviewer-interviewee relationship to one of listener-narrator, where control of the conversation does not rest only with the interviewer (Mishler, 1991, p. 117).

Storying Stories

The *storying stories* framework is rooted in research concerning *life story* (Rosenthal, 1993) and *life history* (Schütze, 2008), both of which can be studied in terms of specific relationships (Szczepanik & Siebert, 2016, p. 287), i.e. relationship with mathematics. It seeks stories as data and generates stories through emplotment, which means it represents an analysis of narrative and a narrative analysis respectively (McCormack, 2004, p. 220; Polkinghorne, 1995). Previously, I used thematic analysis to focus on elements of mathematical identity that were common across participants in the questionnaire data (Howard et al., 2019). This constituted an analysis of narrative, since it allowed the development of themes that hold across the narrative data. To use narrative inquiry as a descriptive tool for an individual participant required a new method of analysis.

I used this framework for several reasons. Although stories/narratives occupy a central position in narrative interviewing, McCormack (2004, p. 219) points out that the task of analysing narrative data is "daunting" and that narrative research literature has been "largely silent" about how to do this. Furthermore, the "ordering principles" (Svašek & Domecka, 2012, p. 108) of stories have great potential for illuminating mathematical identity through the "dynamics of self-perception, self-projection, personal experience and transformation" (p. 108), all of which are present in the definition of mathematical identity given earlier.

This framework relies on the work of Schütze in the 1970s, who claimed that the three constraints of narration (to condense, to be detailed, to close the narrative at the end) significantly limit what a person says when telling stories and how they say it (Schütze, 2008, pp. 14–16; Svašek & Domecka, 2012, p. 110). By drawing from several other authors (Labov, 1972, 1982; McCormack, 2000a, 2004; Rosenthal, 1993), I compiled the following definitions of the five *narrative processes* that are used when telling stories about oneself:

- **Story**: Identified by "recognizable boundaries – a beginning and an end" (McCormack, 2000a, p. 288). A story is required to have a sequence of linked events/actions.
- **Description**: Details about static structures (Rosenthal, 1993, p. 8), such as people, places and routines (Schütze, 2008, p. 15), which reduce the "information gap" (p. 61) between interviewer and participant. They help the listener to get a more complete picture of the other narrative processes (McCormack, 2004, p. 224).
- **Argumentation**: Abstracted elements outside the story which present the perspective of the present (Rosenthal, 1993, p. 8). They add meaning to the other processes.
- **Theorising:** The narrator's general orientation at the moment (Rosenthal, 1993). Reflecting or trying to work something out (McCormack, 2000a, p. 290).
- **Augmentation**: Adding to, or expanding on, previous stories (McCormack, 2004).

The story is the main unit of this analysis. The other four processes may elaborate on these stories or they may include other elements from outside the stories. McCormack (2000a, pp. 288–289), taking inspiration from Rosenthal (1993), defines a story as having a recognisable beginning and end, along with a sequence of linked events/actions which together explain why the story was told (the point of the story). These events/actions can be organised chronologically or thematically (Mishler, 1991, p. 87; Rosenthal, 1993, p. 8). As such, a story consists of five distinct elements, two of which are optional:

- **Orientation (beginning):** Describes the general situation before, or at, the time of the first action. *Who, what, when, where?*
- **(Optional) Abstract:** Summarises the point of the story. Substance of the story as viewed by the narrator. *What was this about? Why is it being told?*
- **Linked events/actions:** *Then what happened?*
- **(Optional) Evaluation (the point of the story):** The narrator steps out of the story to explain what was in their mind at the time and how they felt about what was happening. Conveys the teller's emotions and attitudes to the narration. They may compare things that occurred and what might have occurred. This is the title of the story. *So what?*
- **Coda (end):** Finishes the story and brings the listener back to the present. *Then what happened? Nothing, I just told you what happened.*

Labov (1972, p. 370) proposed each of the guiding questions in italics above. Evaluation is an almost universal feature of narratives given by adults, since stories limited to events/actions don't always make a *point* to the listener (Labov, 1982, p. 226). Some authors consider narratives limited to events/actions to be *unreportable* (p. 227) or *unnarratable* (Georgakopoulou, 2007, p. 62), such is the proven importance of the evaluative element (Mishler, 1991, p. 83).

Methods

The first task was to identify the stories in the data. A story has three required features: a beginning (*orientation*), a middle (*linked events/actions*) and an end (*coda*); it may also contain *abstracts* or *evaluations*. When locating a story, the coda is easiest to pinpoint because it brings the listener away from the narrative and back to the present (Labov, 1972, p. 365). It signals that the participant's turn to speak has finished and the interviewer should speak next. Labov's definition of a narrative in terms of *narrative clauses* (p. 375, Table 9.1) was useful for distinguishing the other narrative processes from the linked events/actions, which indicate that a story is present.

With reference to Labov's definition, I identified the other narrative processes: *argumentation*, *description*, *theorising* and *augmentation*, which helped narrow the list of twenty-one potential stories down to ten. These were sufficient to categorise the narrative data since all parts of the interview were matched with one of the five narrative processes. To determine how these processes "enrich these stories ... to help the listener get the point" (McCormack, 2000a, p. 286), I examined the relationship between them by creating a spreadsheet to track which processes referred to a topic discussed earlier in the interview;

augmentations always do, but theorising and argumentation may also be connected to previous parts of the interview.

In the previous section, I emphasised the importance of evaluation in endowing stories with meaning. Most of the ten stories included an evaluative element without prompt from the interviewer. On one occasion, an evaluation arose when the participant noted that I might be getting the wrong message from their story about the maths learning centre (MLC):

ID069	Yeah so we worked together ... in smaller groups.
FH	Ok, so the collaboration.
ID069	Yeah, but like, we didn't go [to the MLC].

In four of the stories, the coda included a general evaluation. For example:

ID069	[W]e'd never done like, college exams and stuff, so it was just kind of all, *it was just a very different experience I think than secondary school.*

Often, this merging of coda and evaluation served to answer a question posed by me to the participant, or by the participant to themselves. For example, I asked "which modules influence your teaching practice?" and one minute later, the participant ends a story with the following coda:

ID069	[I]t was really helpful to have the reference to different things as we were doing them like.

As expressed by the literature, the coda sometimes explains the effects of a narrative (Labov, 1972, p. 365) or is merged with the final linked event/action (Labov, 1982, p. 226), both of which can be demonstrated by the following example from this interview:

ID069	So yeah ... I feel like a lot, not a lot like some of them understood it and some of them didn't.

This extract demonstrates how narratives processes of theorising and description can be used mid-story.

Results

This section presents the results of applying the storying stories framework to a narrative interview with a science education student. Ten stories were located in the data, the titles of which are shown below, along with the contributions from the four other narrative processes in italics. Note that some entries in italics added to a story, while some entries offered entirely new information. The titles are:

1. How you study maths in college and how you study maths for the Leaving Cert I think are different. (*I've learned how to learn maths now.)*
2. I know I was good at algebra before, but I'm really good at teaching it now. (*Which modules influenced me more?)*
3. The module just clicked in with lots of other mathematics that we came across.
4. If my student gets it one way, I'll go back and do it another way maybe. (*Teaching has made me more confident. I was inspired by my teacher in secondary school.)*
5. I worked really hard to stay in LCH maths and did better than I expected.

6. TY maths seemed like a waste of time, (*but there were some positives)*.
7. I wasn't getting back like “do you understand this” from my students.
8. I wouldn't personally use the area model, I don't really like it, but the male students did. *(Girls get stressed out about maths, whereas guys find it easier).*
9. In first year, I stopped going to the MLC (maths learning centre) because it was intimidating *but it has changed a lot.*
10. I started going to the MLC again because I needed to learn how to do certain things so I could keep doing them on my own until I kind of got it.

Since the evaluation and abstract explain why the story was told (McCormack, 2000a), the title for each story was drawn from these elements using the participant’s own words. Even though the sections of text matched with each process varied in length, it became clear that this interview was dominated by descriptions and theorising (57 of the 91 extracts from the interview). The process of argumentation, although used sparingly by this participant, contributed an important contextual framing of their experiences so they could be better understood by the interviewer.

Conclusions

The aim of this paper was to apply the *storying stories* framework to narrative interview data concerning mathematical identity. Since I conducted this interview using themes developed from a mathematical identity questionnaire, I wondered whether this type of narrative analysis would be applicable. Although Rosenthal (1993) proposed *life story* as the subject of the narrative biographical interview, other authors have noted that it can be presented in the context of specific relationships (Szczepanik & Siebert, 2016, p. 287). I further noted that the narrative processes were sufficient for categorising the interview transcript in this study, which indicates that discussing mathematical identity could be thought of as discussing one’s *mathematical life story.* Thus, the framework presented in this paper was successfully applied to data arising from such discussions.

In the interview, while they were mostly free to talk about anything they wished, the participant periodically checked whether I understood their words or whether they had answered my question. This emphasises the co-constructive nature of the interview setting, i.e. that the participant endeavoured to craft responses that were relevant to the interviewer and topic of the interview. Some details of the interview and stories have been omitted to preserve the confidentiality of the participant.

It is notable that although the story is the unit of analysis, narrative inquirers do not confine themselves to gathering stories alone (Clandinin & Connelly, 2000, p. 78). Rather, the stories facilitate the co-construction of meaning from the participant’s words through an *interpretive story*. The next step in the analysis is to construct an interpretive story for this interview (McCormack, 2000b) after the story titles are presented to the participant for review. This step allows the participant to amend or augment their stories and allows them the opportunity to discuss how their privacy can be ensured in the analysis that I publish (using pseudonyms for people and places, using analogies, selecting which [if any] elements of the interview they would prefer not to reveal in writing).

In conclusion, the storying stories framework is applicable to interview data concerning mathematical identity and was effective in distilling meaning from the participant's mathematical life story.

References

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research.* Jossey-Bass.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. Routledge.

Eaton, P., Horn, C., Liston, M., Oldham, E., & OReilly, M. (2013). Developing an instrument to explore mathematical identity: A study of students from several third level institutions in Ireland. In E. Arntzen (Ed.), *Proceedings of the Association for Teacher Education in Europe 38th Annual Conference (ATEE)* (pp. 280–296).

Eaton, P., & OReilly, M. (2009). Who am I and how did I get here? Exploring the mathematical identity of student teachers. In D. Corcoran, T. Dooley, S. Close, & R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education (MEI3)* (pp. 228–236). St. Patrick's College.

Georgakopoulou, A. (2007). *Small stories, interaction and identities.* John Benjamins Publishing Company.

Howard, F. (2019). Mathematical identity of science and engineering students in an Irish university. In L. Harbison & A. Twohill (Eds.), *Proceedings of the Seventh Conference on Research in Mathematics Education in Ireland (MEI7)* (pp. 139–146). https://doi.org/https://doi.org/10.5281/zenodo.3529748

Howard, F., Nic Mhuirí, S., & OReilly, M. (2019). " I don't like Maths as a subject but I like doing it ": A methodology for understanding mathematical identity. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 1453–1460). CERME. https://doi.org/https://hal.archives-ouvertes.fr/hal-02410134

Kaasila, R. (2007). Using narrative inquiry for investigating the becoming of a mathematics teacher. *ZDM - International Journal on Mathematics Education, 39*(3), 205–213. https://doi.org/10.1007/s11858-007-0023-6

Labov, W. (1972). The transformation of experience in narrative syntax. In *Language in the Inner City: Studies in the Black English Vernacular.* University of Pennsylvania Press.

Labov, W. (1982). Speech actions and reactions in personal narrative. In D. Tannen (Ed.), *Analyzing discourse: Text and talk* (pp. 219–247). Georgetown University Press.

Machalow, R., Goldsmith-Markey, L. T., & Remillard, J. T. (2020). Critical moments: pre-service mathematics teachers' narrative arcs and mathematical orientations over 20 years. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-020-09479-9

McCormack, C. (2000a). From interview transcript to interpretive story: Part 1 - Viewing the transcript through multiple lenses. *Field Methods, 12*(4), 282–297. https://doi.org/10.1177/1525822X0001200403

McCormack, C. (2000b). From interview transcript to interpretive story: Part 2 - Developing an interpretive story. *Field Methods, 12*(4), 298–315.

McCormack, C. (2004). Storying stories: A narrative approach to in-depth interview conversations. *International Journal of Social Research Methodology: Theory and Practice, 7*(3), 219–236. https://doi.org/10.1080/13645570210166382

Mishler, E. G. (1991). *Research interviewing: Context and narrative*. Harvard University Press.

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. International Journal of Qualitative Studies in Education, 8(1), 5–23. https://doi.org/10.1080/0951839950080103

Radovic, D., Black, L., Williams, J., & Salas, C. E. (2018). Towards conceptual coherence in the research on mathematics learner identity: a systematic review of the literature. *Educational Studies in Mathematics, 99*(1), 21–42. https://doi.org/10.1007/s10649-018-9819-2

Rosenthal, G. (1993). Reconstruction of life stories : Principles of selection in generating stories for narrative biographical interviews. *The Narrative Study of Lives, 1*(1), 59–91. http://nbn-resolving.de/ urn:nbn:de:0168-ssoar-59294%5CnNutzungsbedingungen:

Schütze, F. (2008). *Biography analysis on the empirical base of autobiographical narratives: How to analyse autobiographical narrative interviews - Part I.* https://doi.org/10.2307/j.ctvdf09cn.7

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22. https://doi.org/10.3102/0013189X034004014

Svašek, M., & Domecka, M. (2012). The autobiographical narrative interview: A potential arena of emotional remembering, performance and reflection. In J. Skinner (Ed.), *The Interview: An Ethnographic Approach* (pp. 107–126). Berg. https://doi.org/10.5040/9781474214230.ch-004

Szczepanik, R., & Siebert, S. (2016). The triple bind of narration: Fritz Schütze's biographical interview in prison research and beyond. *Sociology, 50*(2), 285–300. https://doi.org/10.1177/0038038515570145

The Student Perspective on COVID-19 Related Closures at Irish Universities with a Focus on Accessibility and Engagement

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This paper describes the impact of the COVID-19 closures on accessibility and student engagement. A survey was designed and administered to students who were enrolled in mathematics modules in an Irish university at the time of the closures. A total of 263 students from six universities responded to the survey. The survey comprised three sections: Teaching and Learning, Assessment, and Personal Experience, and centred on how the changes in teaching and assessment were viewed by students. This paper examines a subset of the survey that focuses on students' access to and engagement with online learning. The data shows that the abrupt changes had a significant impact on students' motivation in addition to other factors which impacted engagement. The responses indicate that most (but not all) students had access to the appropriate technology and infrastructure to engage with online learning.

Introduction

The COVID-19 crisis has had a major impact on education. The Irish Government announced the closure of schools and higher education institutions (HEIs) on the 13th of March 2020. HEIs remained closed for the 2019-2020 academic year, with all teaching and assessment done remotely. The situation remains largely unaltered in the 2020/21 academic year. In this paper we will consider the effects of the move to online learning on mathematics students.

There had been an increase in the use of online learning in universities before the pandemic, and in the previous decade, many researchers have studied online instruction in mathematics. Trenholm and Peschke (2020) describe the differences between face-to-face (F2F) and fully online (FO) instruction in mathematics communities of practice at university. They explain that the content being learned is the same as before, however significant changes in communication, interaction, and assessment have to take place when transitioning from F2F to FO learning (Trenholm & Peschke, 2020). We note, however, that research on massive open online courses, blended learning, or other technologically progressive methods in use prior to COVID-19 might not be as applicable to current teaching as one might expect. A more accurate characterization of what occurred following the closures is *emergency remote teaching* (ERT), described by Hodges et al. (2020) as:

> a temporary shift of instructional delivery to an alternate delivery mode due to crisis circumstances. It involves the use of fully remote teaching solutions for instruction or education that would otherwise be delivered face-to-face or as blended or hybrid courses and that will return to that format once the crisis or emergency has abated. (Hodges et al., 2020, p.6)

Researchers at University College Dublin carried out a similar survey to ours (Meehan & Howard, 2020). Their work reported on the positives (commuting, self-pacing) and negatives (internet connection, lack of peer interaction) students associated with distance learning. Hill and Fitzgerald (2020, p.3) also captured the student perspective, and they reported a 'feeling of disconnectedness and isolation' and decreased motivation from a reduction in engagement between students and lecturers.

We consider students to be equal stakeholders in mathematics education, and that their experiences offer valuable insights into how the closures were handled and how best to proceed. In this paper, we describe a subset of the results of a survey given to students who were enrolled in mathematics modules in Irish universities during the COVID-19 related closures. The research question is as follows:

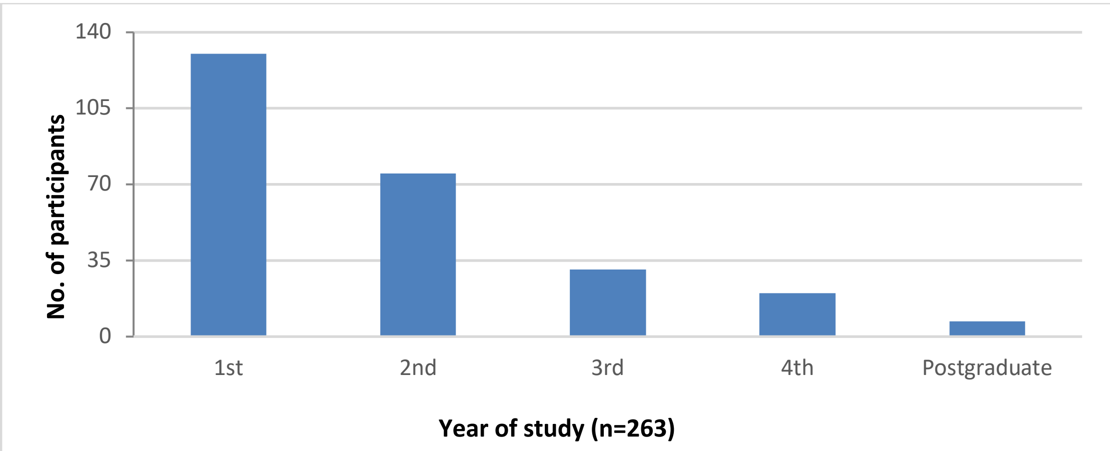
> How did the initial COVID-19 closure impact students' access to and engagement with learning?

Method

We designed a survey to inform the research question comprising 16 questions, divided into three sections: Teaching and Learning, Assessment, and Personal Experience. The subset of the survey that is discussed in this paper is Q5, and Q12-16, which relates to students' access to and engagement with FO learning. The survey received ethical approval by Maynooth University Ethics Committee, and was open from July 9th to August 9th, 2020, and as such only pertained to the students' experience from March 13th until the end of semester. The survey was shared with mathematics lecturers through mailing lists, many of whom forwarded the survey to their students. In total, 263 students from six universities responded to the survey (available at Hyland & O'Shea, 2021). The year of study for respondents is contained in Figure 1.

Figure 1

Year of study of participants



The majority of respondents were enrolled at Maynooth University (62%), with the remainder spread across five of the seven other Irish universities. The most frequent degree programmes represented in our sample were specialist mathematics courses (21%), teacher education (20%), and general science (19%). Students studying a single science subject (e.g., physics, chemistry, or biology) made up 13% of the respondents, while 11% of the group were studying mathematics as part of a BA programme, and 7% were enrolled in computing degrees. The survey responses generated a combination of qualitative and quantitative data. We analysed the qualitative data by following the general inductive approach to qualitative data analysis outlined by Thomas (2006). The approach allows "research findings to emerge from the frequent, dominant or significant themes inherent in raw data, without the restraints imposed by structured methodologies" (Thomas, 2006, p.283). The process, which uses coding to develop categories to condense the data, allows links between the research questions and the findings to be established (Thomas, 2006). Procedures to assess the trustworthiness of the category system, (independent coding, coding consistency check, and stakeholder checks) which Thomas (2006) describes, were also practiced during our data analysis. The quantitative data gathered in response to the closed response questions were tallied and are presented in tables in the Results Section.

Results

In this section, we present a subset of the results from the larger project. Though data was gathered on a wide range of questions in the survey, here we focus on data that describes students 'access to and engagement with FO learning. We begin by looking at the data on students' access to technology and suitable workspaces which might impact their ability to work remotely.

Accessibility of Remote Learning

Five closed questions were designed to learn about the equipment, infrastructure, and facilities students had access to while learning from home. The results (Table 1) show that most students had access to the appropriate equipment to facilitate their learning. Access was not universal however, with several geographical and economic barriers restricting access to equipment (desk, computer, printer/scanner), infrastructure (fast, reliable broadband), and facilities (quiet space). Almost one third of students did not have access to a printer/scanner and a similar proportion did not have reliable broadband. It is also notable that students often use apps on their phone to scan their work which we believe impacted the responses to Q4.

Table 1

Students' access to equipment, infrastructure, and facilities for distance learning

	Question	Yes	No	Prefer not to say
1	Did you have access to a quiet place to study?	200 (76%)	62 (23.6%)	1 (0.4%)
2	Did you have access to a table/desk?	249 (94.7%)	13 (4.9%)	1 (0.4%)
3	Did you have access to a PC or laptop?	256 (97.3%)	6 (2.3%)	1 (0.4%)
4	Did you have access to a printer/scanner?	186 (70.7%)	75 (28.5%)	2 (0.8%)
5	Did you have access to fast and reliable broadband?	166 (63.1%)	90 (36.5%)	1 (0.4%)

Students' Experience of the Transition to Distance Learning

Several open response questions were included in the survey to investigate students' personal experiences of the transition to distance learning. We focus on a subset of responses on how students accessed and engaged with FO learning. We begin with expressions of motivation, anxiety, and isolation (Table 2), which had a significant impact on student engagement.

Table 2

Student data on anxiety, isolation, and motivation

	Strongly agree	*Somewhat agree*	*Neutral*	*Somewhat disagree*	*Strongly disagree*	*Prefer not to say*
It was easier to motivate myself to learn during the lockdown than before	8% (*n*=21)	8.7% (*n*=23)	14.8% (*n*=39)	25.1% (*n=66*)	39.2% (*n*=103)	4.2% (*n*=11)
I felt more anxious about my learning during the lockdown than before	32.7% (*n*=86)	29.7% (*n*=78)	15.6% (*n*=41)	11.8% (*n*=31)	9.1% (*n*=24)	1.1% (*n*=3)
I felt isolated from my lecturer/class group	29.7% (*n*=78)	26.6% (*n*=70)	22.8% (*n*=60)	10.3% (*n*=27)	10.3% (*n*=27)	0.4% (*n*=1)

Students reported feeling increased anxiety and isolation during the COVID-19 closures. Though expected with such a sudden switch from F2F to FO learning, it is still concerning. Of the 263 respondents, 164 (62%) reported increased anxiety, 148 (56%) felt isolated from their peers and lecturers, and 169 students (64%) believed it was more difficult to motivate themselves during lockdown. An open response question asking for strategies to avoid isolation was also included on the survey; many responses described ways of increasing contact among students while acknowledging the complexity of such an issue in the current environment:

Student 152: I think it is just a consequence of isolation that can't really be avoided. It's difficult to replicate how it feels to go to a lecture in person just using online resources.

The increased isolation is also seen in responses to other parts of Q12, where students reported negative effects on communication with lecturers, peers, and support services, and a 'very negative' impact on their ability to study with peers which recurs throughout the data.

To finish the survey, three open response questions were asked about the challenging (Q14) and positive (Q15) aspects of teaching, learning, and assessment during the closures, and an opportunity for further comment (Q16) was provided. The responses relating to engagement are detailed below, with a focus on interaction and communication, which was the most frequently reported category. These responses are described below (in Tables 3, 4 and 5 respectively), where categories with five tallies or fewer are omitted.

The challenges students reported fell into five overarching categories: *interaction and communication* (*n*=97), *delivery of teaching* (*n*=90), *motivation* (*n*=75), *learning environment* (*n*=21), and *assessment* (*n*=20) (see Table 3). The category of *delivery of teaching* consisted of the *loss of in-person delivery* (*n*=28), *access to support* (*n*=54), and *access to resources* (*n*=8). *Access to support* was understandably the most frequently occurring category and is indicative of a change in interaction pattern brought about by the move online, with one student describing the challenges as:

Student 92: The 1 on 1 support from tutorials and maths support centre is very hard to recreate online. Lots of work can be self-directed however an hour or 2 of in person teaching can really help.

The category *interaction and communication* concerned the reduction of *peer interaction* (*n*=46), the difficulty of *asking questions in real time* during online lectures (*n*=30), students having *less communication* with their lecturer (*n*=16), and *lack of feedback* on work and assignments (*n*=5). Through *peer interaction*, we can see another example of how students' customary studying habits were uprooted:

Student 181: I found it difficult not having the library, Maths learning centre, and mostly my peers to study with.

We consider *interaction and communication* to be closely linked to *motivation*, which includes *loss of motivation* (*n*=52), issues with *self-pacing* (*n*=17), and feelings of *isolation* (*n*=3). Though student motivation can be impacted by many factors, it is unsurprising that the toll of the pandemic was felt by students:

Student 44: It's harder to motivate yourself to study at home, it feels more like a chore to have no peers to bounce ideas off, or to take small breaks between lectures.

Finally, challenges with the *learning environment* were mentioned by students, consisting of having access to an appropriate *study space* (*n*=13) and *issues with technology or internet* (*n*=8), relating back to the data on accessibility in Table 1.

Table 3

Challenging aspects of changes mentioned by students (Q14)

Category	Label	Tally
Interaction and communication	Peer Interaction	46
	Asking questions in real time	30
	Less communication with lecturer	16
Delivery of teaching	Access to support (MSC)	54
	Loss of in-person delivery	28
	Access to resources (e.g., books)	8

Motivation	Motivation	52
	Self-pacing	17
Learning environment	Home space	13
	Technology and internet	8
Assessment	Assessment format	13
Other		15

The social aspect of learning is a theme uniting many of the most frequently occurring labels in Table 3 (*in-person delivery, access to support, peer interaction, less communication with lecturer*, and *asking questions in real time*). The disconnect between this and FO learning led to issues with motivation and isolation which combine for 73% of the responses to this question.

Many positive aspects of the transition were also reported by students (Table 4), though benefits relating to access and engagement are few and far between. The absence of a fixed timetable for many students allowed them to work at their own pace, and at a time that best suited their circumstances. We note however, that self-pacing was also perceived as a negative for some students.

Table 4

Positive aspects of changes mentioned by students (Q15)

Category	Label	Tally
Learning	Self-pacing	44
	Individual study skills	26
Time	No Commute	35
	No set timetable	38
Resources	More internal resources (e.g., module-specific notes, solutions)	55
	Found external resources (e.g., third-party websites)	7
Assessment	Open book	8
	Online assessment	6
Other		14

Finally, the space for further comment which was provided at the end of the survey returned many of the previous findings (Table 5). In a sense, it caused respondents to prioritise their most salient opinion, which, in this case, relates to the delivery of mathematics. The most frequent response was that the learning of mathematics is a communal activity, that is to say, the way we engage our students when they are learning mathematics matters.

Table 5

All remaining comments mentioned by students (Q16)

Category	Label	Tally
Teaching and learning	Maths is a social subject/needs to be delivered in person	11
	All resources should be uploaded even when in-person returns	7
	Clarify standards across departments/institution	6
Personal experience	Negative experience	8
	Resilient and pragmatic viewpoint	8
	Positive experience	6
Assessment	Assessments need to be improved and standardized	6

Discussion

In this study, we describe the results of a survey designed to investigate the impact of the COVID-19 university closures have had on students studying mathematics with a focus on accessibility and engagement. Responses to Q12 (Table 1) showed that most respondents had access to the appropriate equipment and facilities to engage with lectures remotely, although many students were relying on their smartphones or tablets to scan and access resources. It is concerning that nearly a quarter of the respondents did not have a quiet place to study or to take their final examinations. Some improvements have been made in this area, with universities making quiet rooms available on campus so that certain students have a suitable location to take their examinations. The access to reliable and fast broadband is a more difficult issue to solve in the short term, but has been flagged previously (Becker et al., 2017).

The students' experience of the transition was investigated by asking them how their motivation, levels of anxiety, and feelings of isolation have been impacted by the closures. The results portray a drastic reduction in motivation along with increased anxiety and feelings of isolation. Unger and Meiran (2020) have reported this with respect to COVID-19 specifically and have called for students' mental health to be monitored during epidemics. Even though these students are digital natives, it seems many had negative experiences with remote learning and missed personal interactions with staff and peers.

Trenholm and Peschke's (2020) advise that significant changes in communication, interaction, and assessment are crucial when transitioning from F2F to FO learning even though the mathematical content remains unchanged. Our results suggest that these changes did not take place to the necessary level during the COVID-19 closures. This is not surprising since the move to ERT (Hodges et al., 2020) was so sudden and unplanned. At the time of writing, Irish universities are in their third semester of remote teaching and there is no doubt that the community have become more accustomed to the changes. An end to COVID-19 related closures is in sight but the lessons we have learned as a community should not be discarded (Hodges et al., 2020). Future events (e.g. weather and public health) may require ERT to be used again.

This research was undertaken to give a voice to students who were studying mathematics at the time of the COVID-19 related closures. Staff and students were faced with many difficulties, and our analysis has highlighted the areas of concern for students. In particular, our data has revealed the importance of personal contact with instructors and peers.

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References

Becker, S. A., Cummins, M., Davis, A., Freeman, A., Hall, C. G., & Ananthanarayanan, V. (2017). *NMC horizon report: 2017 higher education edition* (pp. 1-60). The New Media Consortium.

Hill, K., & Fitzgerald, R. (2020). Student perspectives of the impact of COVID-19 on learning. *All Ire. J. of Higher Educ.*, *12*(2).

Hodges, C., Moore, S., Lockee, B., Trust, T., & Bond, A. (2020). The difference between emergency remote teaching and online learning. *Educause review*, *27*, 1-12.

Hyland, D., & O'Shea, A. (2021). The student perspective on teaching and assessment during initial COVID-19 related closures at Irish universities: implications for the future. *Teach. Math. Appl.,* https://doi.org/10.1093/teamat/hrab017

Marinoni, G., Van't Land, H., & Jensen, T. (2020). The impact of Covid-19 on higher education around the world. *IAU Global Survey Report*.

Meehan, M., & Howard, E. (2020). *Undergraduate Mathematics Students' Perceptions of the Affordances and Constraints of Online Learning – Implications for Practice*. University College Dublin.

Thomas, D. R. (2006). A general inductive approach for analyzing qualitative evaluation data. *Am. J. of Eval., 27*(2), 237-246.

Trenholm, S., & Peschke, J. (2020). Teaching undergraduate mathematics fully online: a review from the perspective of communities of practice. *Int. J. of Educ. Technol. in Higher Educ.*, *17*(1), 1-18.

Unger, S., & Meiran, W. R. (2020). Student attitudes towards online education during the COVID-19 viral outbreak of 2020: Distance learning in a time of social distance. *Int. J. of Technol. in Educ. and Sci. (IJTES)*, *4*(4), 256-266.

The Design of an Algebra Concept Inventory

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In this paper, we report on the development of an Algebra Concept Inventory (ACI) intended for students transitioning from second- to third-level education. We begin by outlining the work done on concept inventories to date before describing some of the guiding principles of interdisciplinary design teams. Our methodology for developing the ACI is detailed in a step-by-step manner including the formation of the design team, defining the parameters of the ACI, shortlisting items, and piloting. The iterative design process resulted in a 31-item preliminary ACI. We conclude by highlighting aspects of how the interdisciplinary team functioned throughout the process before outlining some potential uses of the ACI which was rolled out during Semester 2 of the 2020-2021 academic year to all higher education institutions (HEIs) in Ireland. The ACI is available for interested practitioners upon request.

Introduction and Background

In this paper, we outline the creation of an Algebra Concept Inventory (ACI) with a focus on how the interdisciplinary design team was formed and collaborated throughout the process.

What is a Concept Inventory?

A concept inventory (CI) is a set of questions designed to assess students' conceptual understanding of a given topic. They originated in Physics Education Research in the early 1990s with the Force Concept Inventory (FCI) (Hestenes et al., 1992) which was based on the Mechanics Diagnostic Test (Halloun & Hestenes, 1985). The success of the FCI in physics has seen CIs designed in other subjects. Within mathematics, CIs have been designed and implemented in the areas of function (O'Shea et al., 2016), calculus (Epstein, 2013), precalculus (Carlson et al., 2010), and statistics (Stone et al., 2003). We believe that the work done on building CIs in mathematics is of huge value to the community. CIs have multiple applications in research and instruction. Though our immediate motivation for developing an ACI is as a diagnostic tool, CIs can be used for evaluating instruction, and as a placement examination (Hestenes et al., 1992). Thus, we seek to build on what has already been done by developing a concept inventory for algebra, which is fundamental to virtually all tertiary level mathematics and is a prerequisite to the aforementioned topics.

Interdisciplinary Teamwork

A key aspect of this work is creating an interdisciplinary team that will work together to develop the ACI. It is common for interdisciplinary teams to be formed to undertake projects such as this, where a certain topic is relevant to several adjacent areas of expertise (Claus & Wiese, 2019). In our case, we are developing a research tool with relevance to members of the mathematics and education communities, as well as educators in other adjacent subjects. Barriers to interdisciplinary team-based research are well described (e.g., Morse et al., 2007) and are often logistical; COVID-19 has exacerbated these problems. For this project to progress as effectively as possible, research on interdisciplinary teamwork was examined for guiding principles and best practice.

Nancarrow et al. (2013) combined a large survey with an extensive literature review to identify the attributes of a good interdisciplinary team using qualitative content analysis. They identified 10 themes that support effective interdisciplinary teamwork. We have adopted eight of these: leadership and management; communication; appropriate resources and procedures; appropriate skill mix; climate; individual characteristics; clarity of vision; and respecting and understanding roles. The other two, personal rewards, training, and development; and quality and outcomes of care, are not relevant to our CI.

Many studies have identified competencies relating to teamwork and collaboration. Communication within the team (Molyneux, 2001), team structure (Xyriches & Lowton, 2008), and knowledge integration (Claus & Wiese, 2019) were the most frequently occurring in our review, which provided excellent guidance for our work with the ACI design team.

Stages of Design

In this section, we break down the creation of the ACI into five distinct stages, detailing the formative decisions and obstacles encountered during the design process.

Recruiting the Design Team

Algebra plays a central role in second level mathematics education and continues into a variety of pure and service mathematics undergraduate courses. The wide range of uses of algebra, coupled with the different applications each course may focus on, can require the emphasis of different facets of the content area. Given this, it was important that the ACI design team included members from as many different stakeholders as possible. The expertise of the following groups was acknowledged: Lecturers in Mathematics, Engineering, Physics, Business, other service subjects, and Teacher Education; Members of the Mathematics, Science, and General Education communities; Members of the Mathematics Support community; Second level Mathematics teachers; and Students.

While a representative from each stakeholder group would be optimal, logistic reasons would make it impractical. This is in line with the thoughts of Xyriches and Lowton (2008) that team size and composition are key themes in interprofessional teamwork. It was decided that a team of seven members[1] would begin the project. Each of the team members possess expertise relating to more than one of the above roles, but members of the general education community, a second level mathematics teacher, and a student voice were omitted from the

[1] Dr Eabhnat Ní Fhloinn (DCU) and Dr Micheal Carr (TU Dublin) were recruited to the design team and attended the first meeting of the team where significant progress was made on defining the parameter of the ACI. Their involvement beyond this stage was curtailed by workload and we would like to thank them for their work on the project and their continued support.

design team. This decision was made to keep the team at a manageable size and with the knowledge that their perspectives would be heard and incorporated later in the process.

Defining the Parameters of the ACI

Once the design team was formed, the priority was to establish the meaning and intention of the ACI before advancing. Algebra is an extensive content area within mathematics that begins at primary level and extends beyond undergraduate level. This can cause uncertainty with respect to the facets of algebra that should be included in the inventory. To ensure that the design team and future administrators understood the scope of the ACI, the following definition of this scope was agreed: *The elements of algebra a student should understand having completed second level education.* There were several factors that influenced our definition, most notably previous work on CIs and an algebra decomposition (Figure 1), discussed below. It is common for CIs to discuss what their threshold for inclusion is, for example, Epstein (2013, p.1018) discussed in relation to the Calculus Concept Inventory (CCI) "… a set of very basic concepts that all sides agree students should – must – be expected to master in, for example, first semester calculus". The team agreed that the wording used by Epstein provided clarity (for the CCI) and a scaffold from which we could work. Though Epstein elected to use the first semester of tertiary level as a time stamp, we preferred the end of second level. At this stage in their education, students will have studied the same mathematics syllabi (at primary and second level) albeit to different standards. All higher and further education programmes will have different approaches to mathematics that will lead to an increasingly heterogeneous population as they continue through their education. Second, including this time stamp also benefitted our algebra decomposition because we could use the senior cycle syllabus (NCCA, 2021) to inform our work.

The idea of building a decomposition for the subject in focus is common to CI studies. It began with Hestenes et al. (1992, p.142), who developed a table that separated the force concept into six 'conceptual dimensions', all of which are necessary for a complete conception of force. Other CI designers have proceeded in a similar way (e.g., O'Shea et al. 2016;). Our decomposition was developed through multiple meetings of the design team. Initially, each member shared a preliminary decomposition with the group to form an exhaustive list of content for inclusion. This was examined by a team member, who removed redundant entries and gave structure to the decomposition. This version of the decomposition was returned to team members who made further suggestions on content and structure. Multiple rounds of feedback and revision took place until the group reached a consensus about the decomposition. Figure 1 details our deconstruction of the concept of algebra.

Figure 1

Algebra Decomposition

Equality	Solution	MER
Meaning of '=' • Evaluate vs equivalent to • Balance model • Symmetry • Transitivity • Misuse (≠, =>) • Identity **Equations** • Rules for solving equations • Properties of operations (commutativity, associativity, etc.)[2] **Inequalities** • Rules for solving inequalities	**Meaning of solution(s)** • Equations vs inequality **MER** • Algebraic • Graphical (x-intercept, POI,) **What's valid vs invalid**	**Fractions as:** • Parts of whole • Size of portion • Quotient • Ratio • Decimal • Percentages • Graph/chart **Word problems to equations and vice versa**
Expressions	**Variables**	**Operations**
Fractions **Polynomials of order ≤2**	**Variable as:** • Constant • Placeholder • Multiple values simultaneously **Zero** **Proportional reasoning**	**Manipulations with all operations** • +, -, x, ÷, (), exp., log. **Expanding, factorizing, transposing, simplifying** **PEMDAS**

Our decomposition divides algebra into six sections: Equality, Expressions, Solution, Variables, Multiple External Representations (MER)[2], and Operations. Each comprises smaller subsections which detail the intricate aspects of each section that is included in our decomposition. Some subsections apply to more than one section (e.g., Fractions, MER) and appear as often as required for the reader to interpret our algebra decomposition. In this sense, we consider the decomposition to be exhaustive, with the exception of properties of operations. Though they are in the decomposition under Equality, including each property for each operation would be impractical. Ultimately, the team decided to acknowledge their importance through inclusion in the decomposition, consider items assessing them in our shortlist, and then prioritize the most important ones during the creation of the ACI.

Shortlisting Potential Items

Shortlisting questions for the ACI involved gathering many items that, collectively, assess each aspect of the decomposition. The algebra decomposition provided tremendous clarity to the task and was a constant point of reference that aided communication and fast-tracked many aspects of this phase of the project. Using an online space for collecting and developing items was necessitated by COVID restrictions but was also helpful when refining tasks and is recommended by Carlson et al. (2010).

The shortlisted items came from multiple sources. Research articles, algebra tests, and textbooks were consulted, and several items were adapted for the ACI. Algebra tests included a sample of international assessments (e.g., TIMSS), national assessments, and mathematics

[2]We use MER (e.g., text, pictures, equations) in the same manner as Ainsworth (1999), who explored the different ways MERs can be used to support learning and detailed a functional taxonomy of multiple representations. An example of multiple representations in mathematics would be the use of graphs, tables, and equations when teaching functions.

diagnostic tests at third level. These are indicative of what is valued by assessors and provide a rich pool of items. Previous research on algebra and task design were also studied for items, and a significant number of items were developed by members of the design team. All methods contributed to our shortlist, which began with over 100 items that mapped to at least one facet of the decomposition. Team-developed items were more frequent, primarily in responses to specific facets of the decomposition that are less common in testing or textbooks.

Once collected, each team member provided feedback on all items. The feedback was collated and reconciled which resulted in a refined list of fewer items. This list was returned to each team member for further feedback, beginning an iterative process which occurred several times. Sometimes new items were added as they were discovered, or items were created in response to discussions around shortlisted items. As each round of revisions occurred attention to detail increased, and the standard for inclusion became higher, with later rounds being stricter wording, ordering, and general presentation, in addition to the mathematical merit of each item. Factors considered at this stage are the students' familiarity with notation, and ability to interpret the question as intended. Wage et al. (2005) highlighted issues such as how the questions are presented to students (linked to MER) and whether questions focused a single concept from the decomposition or required students to use knowledge of multiple concepts. Similarly, Halloun and Hestenes (1985) were forced to remove two problems from their assessment because, despite being well-posed, they were misinterpreted by students during interview more often than not.

Each question was developed and administered as a multiple-choice question (MCQ). We chose MCQs because they are simpler to complete and to analyze than open response items. Halloun and Hestenes (1985, p. 1044) concluded that the MCQ version of their assessment 'measures the same thing as the written version but more efficiently'. Interviews with students will be conducted later in the project. This will inform distractor – an incorrect option on an MCQ – choice, and even item inclusion. We planned to administer a written version of the ACI to students to select distractors, as done by Halloun and Hestenes (1985) and others. However, lack of access to students made this impossible. We chose three options per question for the ACI because we were keen to minimize the word count of the ACI as much as possible. Vyas and Supe (2008) include reading time among the numerous practical advantages of including three options over four or five. More importantly, they claim that there was "no significant change in the psychometric properties of the 3 options test when compared with 4 and 5 options" (p.130).

Another motivation for reducing the word count of the ACI was to accommodate a Certainty of Response Index (CRI) for each question. The CRI method is used to distinguish between lack of knowledge and misconceptions at both individual and group level. The approach was used by Hasan et al. (1999) on Halloun & Hestenes' (1985) Mechanics Diagnostic Test. In essence, the authors (Hasan et al., 1999) used a CRI with each question to associate how confident the student was in their response. Responding with low confidence (independent of the student's correctness), indicates a lack of knowledge. High confidence with a correct answer is a justification of the student's confidence in their answer, but high confidence attached to an incorrect answer indicates a misconception. In total, over 100 items were reduced to 31 using the iterative design process.

Piloting the ACI

Piloting the ACI began as soon as the design team had agreed on the items to include. This allowed individuals outside of the design team to offer insights beyond those of the design team. Piloting the ACI also allowed for the online format of the ACI to be tested before rollout began[3].

The piloting and refinement of CIs is well described in the literature. Hestenes et al. (1992) recruited lecturers and students to provide feedback on their tests prior to rollout. O'Shea et al. (2016) included second level teachers in their study. Interviewing students who have taken the CI in question (often called cognitive laboratories) is also commonly done (Hestenes et al., 1992) to learn more about the answers provided and associated reasoning..

During piloting, the ACI was shared with representatives of the following groups: a lecturer in Mathematics and Teacher Education, a member of the Mathematics Support community, a second level Mathematics Teacher, and a student. We were particularly interested in their opinion on the language and symbols used. As outlined above, well-posed questions can often be read in an unintended manner by students or may contain terminology that is beyond the level of the test. The feedback we received during piloting was shared with the design team which led to minor changes (all of which pertained to the foreword preceding the ACI). This resulted in a 31-item preliminary ACI, advertised to students in February 2021.

Data Collection and Validation

The collection of data is a vital stage in the validation of a CI. Questions of reliability and validity are significant for CIs and require many participants to engage with each item. The FCI's reliability was based primarily on the Mechanics Diagnostic Test (MDT), a precursor to the FCI, from which over half of its items are taken. Halloun and Hestenes (1985) used four mechanisms 'to establish the face and content validity' of the MDT: feedback from experienced lecturers; testing of graduate students; interviewing undergraduate students; in-depth analysis of high achievers' answers.

The most detailed analysis outside of physics was carried out by Steif and Dantzler (2005) on their Statics CI. They ascertain their CI's reliability, content validity, criterion-related validity, and construct validity through numerous calculations (Cronbach's alpha, Spearman's rho, confirmatory factor analysis, etc.). O'Shea et al. (2016) used Rasch Analysis, as well as Cronbach's alpha, to investigate the validity and reliability of their test. Carlson et al. (2010) discuss internal and external content validity and report going through an iterative process of administering the Precalculus Assessment (PCA) and carrying out follow-up

[3] CIs are traditionally pen and paper tests. Administering a pen and paper test currently is not possible and so the decision was made to convert the ACI into an online exam.

interviews with students. They also 'examined external measures to further establish PCA's validity as a tool for determining students' preparedness for beginning calculus … by correlating students' post-course PCA scores with their course grades (p.124)'.

Data collection relies on the success of its rollout strategy. The ACI is no exception, and the effort made by the extended mathematics education community and other departmental colleagues was of paramount importance, especially considering that in-person testing was not possible. Our rollout strategy comprised two parts: personal emails to lecturers with whom we had a previous working relationship, and general emails sent to mailing lists. The ACI went live in February 2021 and received 330 responses as of April 2021.

Discussion

In this paper, we detail the design of an Algebra Concept Inventory (ACI) by an interdisciplinary research team. We found Nancarrow et al.'s (2013) 10 themes to support interdisciplinary teamwork very useful to our project. Within this, team structure, communication, and knowledge integration were the most applicable.

The results will be used to design teaching materials that will be used with subsequent cohorts of students nationwide. The initial data from the ACI has already highlighted specific aspects of algebra about which students possess misconceptions. These misconceptions certainly occur in all HEIs, and we intend to make our resources freely available. We hope to continue to increase the number of responses to validate our ACI instrument.

A validated CI can be used as a diagnostic tool, for evaluating instruction, and placement exams (Hestenes et al., 1992) which offer exciting avenues to extend the research. The rapid, pandemic-induced transition to online teaching in HEIs has accelerated work that focuses on the affordances of online and blended instruction (Hyland & O'Shea, 2021). One avenue to pursue is that of a personalized, adaptive approach to teaching and learning (Walkington, 2013). The ACI could have a role in 'triaging' students for entry onto such a flexible learning pathway, and for determining appropriate progression. Hestenes et al. (1992) talk about threshold scores in the FCI near 60% and 80%. Failure to achieve 60% means "a student's grasp of Newtonian concepts is insufficient for effective problem solving" whereas achieving 80% is indicative of a true Newtonian thinker. Corresponding thresholds could be identified in relation to the ACI to inform decisions about entry points and pathways.

References

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*(2-3), 131-152.

Carlson, M., Oehrtman, M., & Engelke, N. (2010). The precalculus concept assessment: A tool for assessing students' reasoning abilities and understandings. *Cognition and Instruction, 28*(2), 113-145.

Claus, A., & Wiese, B. (2019). Development and test of a model of interdisciplinary competencies. *European Journal of Work and Organizational Psychology, 28*(2), 191-205.

Epstein, J. (2013). The calculus concept inventory-measurement of the effect of teaching methodology in mathematics. *Notices of the American Mathematical Society, 60*(8), 1018-1027.

Halloun, I. A., & Hestenes, D. (1985). The initial knowledge state of college physics students. *American Journal of Physics, 53*(11), 1043-1055.

Hasan, S., Bagayoko, D., & Kelley, E. L. (1999). Misconceptions and the certainty of response index (CRI). *Physics Education, 34*(5), 294-299.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher, 30*(3), 141-158.

Hyland, D. & O'Shea, A. (2021). *The student perspective on COVID-19 related closures at Irish universities.* In review.

Molyneux, J. (2001). Interprofessional teamworking: what makes teams work well?. *Journal of Interprofessional Care, 15*(1), 29-35.

Morse, W. C., Nielsen-Pincus, M., Force, J. E., & Wulfhorst, J. D. (2007). Bridges and barriers to developing and conducting interdisciplinary graduate-student team research. *Ecology and Society, 12*(2).

Nancarrow, S., Booth, A., Ariss, S., Smith, T., Enderby, P., & Roots, A. (2013). Ten principles of good interdisciplinary teamwork. *Human Resources for Health, 11*(1), 1-11.

National Council for Curriculum and Assessment. (2021). Curriculum online - Mathematics [online]. Available from: https://www.curriculumonline.ie/Senior-cycle/Senior-Cycle-Subjects/Mathematics/ [accessed 1 April 2021]

O'Shea, A., Breen, S., & Jaworski, B. (2016). The development of a function concept inventory. *International Journal of Research in Undergraduate Mathematics Education, 2*(3), 279-296.

Steif, P. S., & Dantzler, J. A. (2005). A statics concept inventory: Development and psychometric analysis. *Journal of Engineering Education, 94*(4), 363-371.

Stone, A., Allen, K., Rhoads, T. R., Murphy, T. J., Shehab, R.L., & Saha, C. (2003). The statistics concept inventory: A pilot study. In *33rd Annual Frontiers in Education, 2003. FiE 2003.* (Vol. 1, p. T3D-1). IEEE

Vyas, R., & Supe, A. (2008). Multiple choice questions: a literature review on the optimal number of options. *National Medical Journal of India, 21*(3), 130-3.

Wage, K. E., Buck, J. R., Wright, C. H., & Welch, T. B. (2005). The signals and systems concept inventory. *IEEE Transactions on Education, 48*(3), 448-461.

Walkington, C. A. (2013). Using adaptive learning technologies to personalize instruction to student interests: The impact of relevant contexts on performance and learning outcomes. *Journal of Educational Psychology, 105*(4), 932.

Xyrichis, A., & Lowton, K. (2008). What fosters or prevents interprofessional teamworking in primary and community care? A literature review. *International Journal of Nursing Studies, 45*(1), 140-153.

Young Children's Identifications of the Most and Least Likely Outcomes of Experiments

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The aim of this study was to investigate the probabilistic thinking of young children, focusing in particular on the judgements that influence their identifications of the most and least likely outcomes of experiments. Research studies present conflicting results pertaining to young children's potential to engage in probabilistic thinking and a wide variance exists across international mathematics curricula regarding the age at which children receive formal probabilistic instruction. At present, young children in Ireland are not formally introduced to probability until Third class when they are approximately 8 or 9 years old. In this study, the probabilistic thinking of 16 children aged 5-6 years was examined using task-based group interviews. The results suggest that young children are capable of engaging in sophisticated probabilistic thinking and highlights that the current practice of formally introducing children to probability in Third class warrants further investigation.

Introduction

Assessing the probability of an event is an everyday occurrence and both adults and children encounter regular opportunities to construct probabilistic understandings and to develop probabilistic thinking skills in their daily lives. However, research into young children's probabilistic thinking has produced inconclusive and conflicting results regarding the potential of children to understand probabilistic situations and, consequently, further research is required to identify the strengths and limitations of young children's probabilistic thinking (Bryant & Nunes, 2012).

The study described here sought to examine the probabilistic thinking of young children through investigating their responses to probabilistic tasks. The children were asked to identify the most and least likely outcome of a variety of experiments and to justify their thinking. In this paper we examine the literature pertaining to young children's probabilistic thinking. Drawing upon this literature, we present findings from our examination of children's probabilistic thinking, as evidenced through their engagement in four tasks.

Literature Review

Probability is not a new mathematical concept. As such, this literature review includes recent research studies along with literature that gives a historical perspective. We draw largely on the work of international researchers due to the absence of research that has been undertaken into probabilistic thinking in Ireland, particularly relating to young children. Throughout this paper, the term *young children* refers to children aged between 3 and 8 years.

Piaget and Inhelder are widely recognised as the first researchers to study the development of probabilistic thinking in children and their research paved the way for further research into this area (Ben-Zvi et al., 2018). Through conducting clinical interviews with children aged between 4 and 12 years, they concluded that children's ability to engage in probabilistic thinking is linked to their cognitive development and that the systematic understanding of probability commences between the ages of 9 and 12 years (Piaget & Inhelder, 1975). However, Piaget and Inhelder's findings have also been contested by various researchers. For example, Bryant and Nunes (2012) argued that it involved the use of an unfamiliar context while others have contended that the questions were based on children's verbal abilities and, as a result, may not reflect the children's probabilistic thinking because the verbal abilities of children develop later (e.g. Fischbein & Gazit, 1984).

In contrast to Piaget and Inhelder, several researchers have suggested that children possess basic notions of probability from a young age. Fischbein (1975) systematically studied the literature relating to children's probabilistic thinking and was among the first researchers to contend that even preschool children can possess an intuitive understanding of probability. For example, a study by Nikiforidou et al. (2013) found that children aged between 4 and 6 years express stable understandings of probability and can identify the most likely outcome of events.

Theory about early probability learning remains relatively new and further research is required into how young children's probabilistic thinking develops over time (Supply, 2020). However, a framework designed by Jones et al. (1997) almost twenty-five years ago may be useful for describing and predicting young children's responses to probabilistic scenarios. The role of this framework in guiding the task design and data analysis processes in this study are discussed in the methodology section of this paper.

Methodology

Participants

The participants were selected from a Senior Infant class in the school where one of the authors of this paper was teaching. Within this convenience sample, a smaller sample of 16 children were chosen to participate through the use of stratified sampling. This allowed for an equal number of boys and girls to be chosen at random and led to the creation of groups comprising of two boys and two girls. This gender balance was sought as unequal numbers of boys and girls have been found to disadvantage certain group members (Swann, 1992).

Data Collection

Task Design. The task design process was guided by the probabilistic thinking framework designed by Jones et al. (1997). The tasks used related to a single construct to allow for a fine-grained analysis of the children's thinking. This study focused on the probability of an event construct because a number of researchers have investigated young children's probabilistic thinking in relation to this construct and their findings differ regarding the types of reasoning demonstrated by young children when identifying the most/least likely outcome (e.g. Nikiforidou & Pange, 2010; Piaget & Inhelder, 1975). Jones et al. (1997) presented learning descriptors at each level of the probability of an event construct which acted as a guide when designing the tasks. In order for the children's responses to be mapped onto the framework, it was necessary to provide the children with opportunities to identify the most/ least likely outcome of an experiment and to examine if the children's justifications involved subjective, quantitative, or numerical judgements, or a combination of these judgements. The tasks that were completed during the original study in which this probabilistic thinking framework was formulated typically involved the children making a prediction, carrying out an experiment, and comparing the results to their predictions (Jones et al., 1997). The tasks in the current study were modelled on a similar format.

The Interview. Task-based group interviews were utilised as the primary method of data collection in this study. Interviewing children about their mathematical thinking enables researchers to look beneath the surface and can reveal insights into a child's learning that otherwise may go undetected (Ginsburg, 1997). This research tool involves the interviewer and participants interacting in relation to tasks which are introduced in a pre-planned manner (Goldin, 2000). The interviews were conducted in small groups. Group interviews have been shown to generate richer responses than individual interviews, providing opportunities for children to share ideas, hear opposing views, and challenge each other's thinking (Littleton & Mercer, 2013). The limitations of group interviews were also recognised throughout the study. In a group interview it can be difficult to ascertain if children are sharing their own thoughts or if they are agreeing with the views of others, repeating these ideas with little understanding. Thus, the children's comments were not analysed in isolation. Their thinking was tracked throughout each task and the potential impact of the group on their thinking was examined.

Data Sources. Video-recording was the primary method of data collection utilised in this study as it captured the children's behaviour in audio and visual form. The children's utterances, gestures, pauses, intonations, and expressions provided insights into their thinking and assisted the researchers in elucidating the meaning of their spoken words. Photographs were taken of the children's use of resources and copies of their drawings were collected. These artefacts supported inferences made from the children's spoken ideas and enabled a more rigorous analysis than could be afforded by examining the transcripts in isolation.

Data Analysis

Data from the interviews were drawn upon to generate the most accurate interpretation of the children's thinking. A deductive approach to data analysis was chosen to allow the children's probabilistic thinking to be examined in relation to the aforementioned framework designed by Jones et al. (1997). The chosen codes of 'subjective', 'transitional', 'informal quantitative', and 'numerical' related to the four levels of probabilistic thinking identified by Jones and his colleagues.

Findings and Analysis

The children were presented with four tasks and each task was broken into three smaller tasks. For example, Task 1 was broken into Task 1.1, 1.2, and 1.3. The tasks varied in complexity and a range of resources were utilised to explore the children's thinking. For example, spinners were introduced in Task 2.2. The children were asked to identify the most and least likely outcome from a spinner that had two possible outcomes. As this was the children's first use of a spinner during the interview, a spinner that displayed three cats and one dog was used, thereby creating a discrete scenario in which the children could count the number of animals to express the probability quantitatively or numerically. However, in Task 4.2, the children were presented with a spinner for the final time and the segments were not of equal size to explore how the children would respond to a continuous situation in which the events were not equally likely. The children's responses were analysed using the probabilistic thinking framework designed by Jones et al. (1997). The focus was on gathering examples of the children's thinking under each level rather than on identifying a dominant level of thinking for each child because children's thinking is fluid and these levels represent an approximation of their thinking at a particular moment, in response to a particular set of tasks.

Evidence of Subjective Thinking (Level 1)

The children's comments were classified as representing subjective thinking when their probabilistic judgements were based on personal beliefs and preferences (Jones et al., 1997). The children expressed subjective thinking 50 times during the interviews which represented 9% of the children's probabilistic judgements. The children expressed a variety of subjective beliefs when justifying their choice of the most/least likely outcome. The most common form of subjective reasoning used related to the position of a particular object within a bag or its location on a dice or spinner:

Sarah: Yellow because, probably all the yellows will probably be in the corners and all the blues will be in the inside *(bag of bears).*

Jane: I think four because, emm, I actually think two because two is normally at the bottom *(dice)*.

Mark: Purple because it mostly starts at purple at the top and then goes back around and goes at the top again *(spinner)*.

This form of subjective reasoning was most prevalent during the tasks that involved identifying which colour bear was most/least likely to be chosen from a bag. However, in Task 3.3 the children were asked to identify the colour counter that was most/least likely to be chosen from a bag of counters and none of the children solely justified their thinking by referring to the position of the counters in the bag. Consequently, the increased use of subjective thinking during the tasks involving drawing a bear from a bag cannot be directly attributed to the context of drawing items from a bag.

The children's use of subjective thinking was also evidenced when their choice of the most/least likely outcome was influenced by their favourite colour. For example, when asked which colour on a spinner was the most likely outcome, Alex justified his choice of colour by stating that it was his favourite colour on the spinner (Task 4.2). During a task involving drawing a bear from a bag, Jane claimed that she didn't know which colour was most likely because blue and yellow were her favourite colours (Task 1.2).

The subjective judgements used by the children in this study were not restricted to the position or colour of a particular outcome. The children's thinking was also influenced by other factors such as the power they used when rolling a dice, the potential impact of previous outcomes, the size of the numerals on the dice, and external factors such as the impact the wind could have on the spinner. For example, three of the children indicated that they held a belief that previous outcomes could have an impact on future events:

Conor: Because it won last time.

Ben: I think I'm, well, going to lose because I landed on a six last time and that means I might land on a six again.

Mark: Emm, lose because I already won.

From the above extracts, it appears that Ben and Conor exhibited a *positive recency bias* because they held the belief that a previous outcome is more likely to occur again. In contrast, it appears that Leah exhibited a *negative recency tendency* as she believed that because she won previously, she was less likely to win again. It is surprising that only three children in the study based their probabilistic reasoning on previous outcomes because research has found that children are often influenced by previous experiences (e.g. Kazak & Leavy, 2018).

Piaget and Inhelder (1975) claimed that young children have subjective tendencies because they lack a grouped organisation of thought which does not develop until later. However, the fact that only 9% of the children's probabilistic judgements reflected subjective thinking appears to indicate that the children in this study have developed deeper levels of thinking than Piaget and Inhelder perceived as possible for their age.

Evidence of Transitional Thinking (Level 2)

The children's comments were classified as representing transitional thinking when they exhibited a readiness to recognise the significance of quantitative measures while also reverting to subjective reasoning (Jones et al., 1997). Comments that referred to uncertainty without quantification were also considered to represent transitional thinking, as recommended by Polaki et al. (2005). The children expressed transitional thinking 58 times during the interviews which represented 10% of the children's probabilistic judgements.

Six of the children referred to informal quantitative reasoning while also expressing subjective reasoning, as demonstrated by the following comments during Task 1.1 in which the children were asked which colour bear was most likely to be drawn from a bag:

Daniel: There's more green and there's only one red and it might be buried.

Emma: The red because there's only one and it might be at the bottom.

Mark: 'Cause there's more green and the red could be at the bottom.

Jane: And there's more red and probably everyone likes greens.

These comments reflect that the children were in a period of transition, beginning to recognise that the quantity of each colour bear influences its chance of being chosen, while continuing to be bound by subjective reasoning.

The majority of the children's comments that were classified as transitional thinking referred to uncertainty without quantification. On 45 occasions, when asked to identify the most/least likely outcome, the children acknowledged that the outcome was uncertain, as evidenced in the following extracts:

Conor: Because there's any number that you could get.

Grace: Because you don't know what you're going to land on.

Hannah: We don't know, because like you could get any colour.

The children most commonly referred to uncertainty without quantification during tasks involving equally likely outcomes. For example, during Task 3.2, the children were asked for the most/least likely outcome when a traditional six-sided dice is rolled. Most of the children recognised that that a most/least likely outcome did not exist. While some children stated that there was just one of each number on the dice or that each number had the same amount, in many cases the children did not refer to quantities, instead discussing the unpredictability of the outcome. For example, Hannah referred to the uncertainty associated with rolling a dice, stating that "no one knows what they're going to get". This aligns with previous research findings that children often equate equal likelihood with uncertainty (Watson, 2005).

Evidence of Informal Quantitative Thinking (Level 3)

The children's comments were classified as representing informal quantitative thinking when they used quantitative reasoning to justify their choice of the most/least likely outcome (Jones et al., 1997). The children expressed informal quantitative thinking 463 times during the interviews which represented 80% of the children's probabilistic judgements. In tasks involving discrete situations, the children made regular references to part-part relationships in justifying their choice of the most/least likely outcome. For example, in Task 2.2, Emma identified the cat as the least likely outcome, stating that "there's only two cats and there's four dogs on the dice". On several occasions, the children also made explicit comparisons between quantities, using words such as more, most, less, and least:

Sarah: Because there's *more* dogs than cats.

Ben: Because there's four reds, so reds are the *most* so I think that.

Tom: There's *less* cats than dogs.

Mark: You've *least* purple so everyone knows you're least likely to get purple.

The above extracts appear to indicate that the children recognised that the quantity of each part impacts its chance of occurrence in discrete situations.

Task 4.2 involved a continuous situation as the children were presented with a spinner that was shaded one-half orange, one-third blue, and one-sixth green. The children's informal quantitative justifications pertaining to this task differed to those shared during discrete situations in which the children referred to specific quantities to justify their thinking. Contrastingly, the children's informal quantitative judgements in response to the continuous situation presented in Task 4.2 involved general references to the comparable sizes of the segments, as can be seen from the following extracts in which Tom and Sarah, during two separate interviews, were justifying their choice of green as the least likely outcome:

Tom: Because green is smaller than blue and orange.

Sarah: Green because green is tiny and the orange and blue are much bigger.

While the children appeared to recognise that the size of each segment should be considered when identifying the most/least likely outcome, their limited knowledge of fractions appeared to prohibit them from making explicit references to the quantity represented by each segment.

Evidence of Numerical Thinking (Level 4)

The children's comments were classified as representing numerical thinking when they assigned valid numerical measures to describe the probability of an event occurring (Jones et al., 1997). The children exhibited numerical thinking on five occasions which represented just one percent of the children's probabilistic judgements. This type of thinking was only used by the children in response to Task 4.2 and 4.3 in which the children were discussing spinners that were shaded one-half orange. For example, during Task 4.2, Daniel identified orange as the most likely outcome "because orange is half.", while during Task 4.3, Shane stated that "half is orange and half is blue so they have an equal chance". These children recognised that the orange segment represented half of the spinner and, through doing so, identified the part-whole relationship between the orange segment and the entire spinner. These comments represented the only times when the children referred to part-whole relationships. Throughout the interviews, most of the children's judgements were based on comparisons between each part rather than comparing the quantity of one part to the overall quantity. This echoes findings of several studies that children understand proportions as part-part relations before they understand part-whole relations or fractions (e.g. Nunes & Bryant, 1996). Consequently, it appears that the children's use of numerical thinking may have been limited by their previous mathematical experiences, in particular in relation to fractions.

Conclusion

The children's engagement in probabilistic tasks revealed detailed information pertaining to their probabilistic judgements. Many of the children demonstrated robust probabilistic thinking despite their limited experiences of probability. The children referred to uncertainty throughout the interviews, suggesting an awareness of the unpredictability associated with random phenomena. The children's dominant level of thinking was identified as informal quantitative reasoning. However, some of the children appeared to be in a period of transition between subjective and informal quantitative reasoning because although they predominantly attempted to quantify probabilities, at times they exhibited unpredictable tendencies to regress to subjective judgements. Numerical judgements were used infrequently by the children due to their limited knowledge of fractions which restricted their use of part-whole reasoning.

This study demonstrated the potential of Senior Infant children to exhibit robust reasoning in response to probabilistic tasks. The children appeared motivated by, and interested in, the probabilistic tasks. This raises questions regarding the age at which children are introduced to formal probabilistic instruction. However, further research is required into how children's probabilistic thinking develops in order to design instruction that is appropriately challenging and that will have positive implications for the children's everyday lives and their probabilistic understandings.

References

Ben-Zvi, D., Makar, K., & Garfield, J. (2018). Introduction. In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.), *International handbook of research in statistics education* (pp. 193-228). Springer.

Bryant, P., & Nunes, T. (2012). *Children's understanding of probability: A literature review (full report).* Nuffield Foundation.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children.* Reidel.

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics, 15*(1), 1-24.

Ginsburg, H. P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* Cambridge University Press.

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-545). Lawrence Erlbaum Associates.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics, 32*(2), 101-125.

Kazak, S., & Leavy, A. M. (2018). Emergent reasoning about uncertainty in primary school children with a focus on subjective probability. In A. Leavy, M. Meletiou-Mavrotheris, & E. Paparistodemou (Eds.), *Statistics in early childhood and primary education: Supporting early statistical and probabilistic thinking* (pp. 37-54). Springer.

Littleton, K., & Mercer, N. (2013). Educational dialogues. In K. Hall, T. Cremin, B. Comber, & L. C. Moll (Eds.) *International handbook of research on children's literacy, learning and culture* (pp. 291-303). Wiley-Blackwell.

Nikiforidou, Z., & Pange, J. (2010). The notions of chance and probabilities in preschoolers. *Early Childhood Journal, 38*(4), 305-311.

Nikiforidou, Z., Pange, J., & Chadjipadelis, T. (2013). Intuitive and informal knowledge in preschoolers' development of probabilistic thinking. *International Journal of Early Childhood, 45*(3), 347-357.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics.* Blackwell.

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children.* W. W. Norton.

Polaki, M. V., Lefoka, P. J., & Jones, G. A. (2005). Dealing with compound events. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 191-214). Springer.

Supply, A. S., Van Dooren, W., Lem, S., & Onghena, P. (2020). Assessing young children's ability to compare probabilities. *Educational Studies in Mathematics, 10*(3), 27-42.

Swann, J. (1992). *Girls, boys and language.* Blackwell.

Watson, J. M. (2005). The probabilistic reasoning of middle school students. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 145-168). Springer.

Impediments to Adult Learner Engagement in Higher Education Mathematics Learning: Obstacles to Creating a Classroom Culture of Enquiry

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Learner engagement with mathematics can oftentimes be sporadic and reactive, but in the context of non-traditional, (part-time, undergraduate) adult learners, perhaps more so. This paper outlines some key areas for investigation and research, begins to outline some of the key literature and theoretical perspectives whilst at all times considering impediments that hinder adult engagement with mathematical learning – both individually and corporately. Moreover, it discusses the hypothesis that learners who disengage do so over time and occupy a series of three zones or so-called 'coping states', which appear to exist within a continuum characterised as reticent, retreating and withdrawn. It is further hypothesised specific, situational triggers exist, which impede learning and precipitate disengagement, and if identified, may be mitigated or ameliorated. The paper's function is to provide the reader with an overview of the study thus far undertaken, the key hypotheses identified and subsequent research questions in advance of undertaking the review of theoretical perspectives and key literature.

Creating a Classroom Culture: Dialogic Teaching and Learning

The genesis for this research derives from many years' experience of teaching and observing adult learners' difficulties learning mathematics in higher education. Struggles with articulating their mathematical thinking, in addition to much greater difficulties communicating with each other and their instructor were most obvious and disheartening. Learners appeared oftentimes incapable of being understood as they intended, whilst acting as if mathematical language usage was the preserve only of the teacher and not germane to them. Apparently, they did not see the need to appropriate the exemplars of the teacher. Equally, when in small group or whole class discussions, the obvious nervousness or awkwardness felt by some self-conscious adults speaking corporately seemed to exacerbate their rejection of apposite mathematical language. Some reverted to common, everyday colloquialisms, including slang words instead of clear, concise, communication utilising appropriate jargon. Often, this resulted in learners becoming frustrated – sometimes with themselves, other times with their peers and occasionally with their instructor – worryingly, some learners disengaged from the learning subsequently, and decoupled from their groups when collaborating. Re-engaging such learners I found to be quite difficult; and once I noticed some learners criticising and blaming one another for their lack of progress, I resolved to investigate potential solutions for mitigation.

There are instances within the literature e.g. see (Goos, 2002) and (Mason and Johnston-Wilder, 2006) whereby teacher-researchers have attempted to discern the kernel of establishing an environment where mathematical thinking is articulated corporately within classrooms and amongst peers. The focus of my research will involve overlaps with these researchers as well as others such as Schoenfeld's seminal research from 1983 onwards.

Furthermore, (Alexander, 2020, p.1) argues that language is at the heart of learning, an assertion supported by emancipatory constructivists such as (Freire, 1970) and (Mezirow, 1991). Alexander describes it this way:

> "In its pursuit of the metalinguistic alongside the communicative, dialogic teaching is **more** than just 'classroom talk'. It is as distinct from the question-answer and listen-repeat routines which most of us experienced as school as it is from everyday conversation, aiming to be more **consistently searching and reciprocal** than both" (emphasis added).

Social classroom interactions in a reciprocal, dialogic fashion mediates socially constructed learning. If mathematical articulation such as self-talk, peer-to-peer talk, and classroom talk are being discussed then the topic of dialogic teaching should be considered. Therefore, the literature so far has assisted me in deriving the following hypotheses, which have in turn led to the research question(s).

One hypothesis is that poor or reduced learner articulation negatively affects classroom communication, mathematical thinking, and collaboration, impeding self-regulation and metacognition, and opposes the formation of a metacognitive and dialogic classroom culture. Moreover, improved learner articulation mediates improved communication, which in turn, leads to improved mathematical thinking, or at least mediates foundations for establishing a metacognitive classroom culture of enquiry. (Mason and Johnston-Wilder, 2006, p.37) have referred to this phenomenon in another way, namely the so-called 'conjecturing atmosphere'. They describe it as an atmosphere

> "…[where] anybody can be asked to explain their thinking so as to try to convince others. Thus when two people disagree, each can try to persuade the other, …, thereby initiating mathematical thinking."

A further hypothesis is that learners who disengage from learning mathematics do so in a nuanced way and it appears such disengagement is a process, not unlike a sliding scale or a continuum, with degrees of disengagement. One working hypothesis is that this continuum may be categorised into at least three distinct zones or so-called 'coping states': reticent, retreating, and withdrawn, i.e. the *reticent-retreating-withdrawn (RRW) continuum*, through which disengagement is mediated.

Moreover, classroom culture, teacher role-modelling of mathematical articulation and language, are also relevant. How can dialogic teaching improve engagement for, or at the very least, remove impediments preventing anxious learners engaging in mathematical learning? It seems to me that a *culture of enquiry* within the classroom, championed by the teacher, is one obvious solution - see (Goos, 2002 and 2004). It seems both interesting and fitting to consider carefully therefore, any effect of a reciprocal, dialogical teaching and learning paradigm established within a classroom culture of enquiry, interfacing with adult learners on the *reticent-retreating-withdrawn* (*RRW)* continuum. Therefore, the main research question currently is stated as:

What impediments exist which diminish adult learner engagement in mathematics and corrupt communication between learners and instructor, and interfere with collaborative learning of mathematics?

Subsequent, related research questions include:

a) What might hinder, impede or be antagonistic to the learning of mathematics collaboratively? Are there specific settings in, or specific triggers with which learners are hindered in their collaboration and/or learning? Is mathephobia a factor?
b) How might these impediments be ameliorated or mitigated? What role might metacognitive control play in learners being facilitated towards metacognitive collaboration?
c) With reference to the establishing of a collaborative metacognitive community of enquiry, how might learners mediate, preserve, and ultimately perpetuate such a paradigm? What provides an impetus that spurs on learning in this way? Are there specific catalysts for impetus?

Understanding the Adult Learner of Mathematics - Affective and Other Characteristics

Learner Cohort

The learner cohort in consideration comprised mainly self-funded adult learners i.e. over 18 years of age, and who have concluded their traditional school obligations. Some will have completed secondary schooling to Leaving Certificate standard, whilst others will have varying levels of schooling, terminating at various stages. Levels of attainment will vary, accordingly.

All are engaged in a one-semester Engineering Mathematics module, taken as part of an overall programme which ultimately provides a level 6 Higher Certificate in Engineering. It is worth noting this cohort differs to so-called ‘mature students’ within the Irish educational system; these learners are over-18, studying part-time, usually evening-attending, rather than full-time students over the age of 23 years, pursuing level 6-8 programmes.

Learner Mathematical Background

Learners present with varying levels of mathematical ability and past success. Over the years, much of the cohort presents with similar characteristics. For example, they are usually in full or part-time employment, perhaps working shifts and may be absent for one or more sessions as a result and are predominantly male. Mathematical anxiety is present throughout this cohort and typically, depends on age, surprisingly. It appears that the length of time since traditional schooling ended is a determining factor in whether mathematical anxiety is likely to manifest. Research by (Multon et al, 1991) and (Lanigan, 2007) found that self-efficacy and performance were affected to a greater extent for older adult learners and low achieving adult learners.

Moving forward, it will be instructive to highlight key general characteristics of adult learners, their motivations for learning as adults, and differences in comparison to traditional students. Participation is, in the main, voluntary and for an intended purpose. Pressure may exist in those situations where learners may be pursuing programmes for CPD reasons and participation may be less than voluntary. In many cases the learner has an agenda to achieve some learning goal that may be linked to employment or career prospects, where employers may have some vested interest, whilst others simply have social reasons, or for self-actualisation. A majority of adult learners will have had some experience of traditional schooling. Continuing Education is understood by (Rogers, 2007, p.35) as "relatively advanced professionally oriented programmes for adults who have already been educated". Continuing education therefore includes those adult learners whose experiences are other than traditional, and undoubtedly, the evening classrooms and lecture theatres of higher education institutes are a nexus predominated by such cohorts.

Additionally, key characteristic differences are acknowledged which help to separate the adult learner from the learner as a child - it is remarkable that not all characteristics need to be present for an adult to be defined. Adults are self-recognising and recognise other adults; self-determined, fully grown, developed, and moving towards greater maturity. If children are 'growing up', then according to (Rogers, 2007, pp.40-41), adults are 'grown ups' and have 'arrived'.

In terms of perspective, adults have a more balanced approach to life, having some sense of far-sightedness, and tend not to act childishly, however adults can act like children, notably in education settings. Adults have responsibilities, managing their own lives and perhaps the lives of others such as dependents. They usually possess autonomy and are relatively secure. In terms of diversity, individuals are different and come to learning with different motivations and life experiences, affiliations and expectations. Children generally tend to flock together and stand out less in the crowd.

Adult learners have different motivations for entering academia to traditional school-leavers. Generally, adults are more self-directed and focussed and quite determined and somewhat resilient due to life-experience. Maturity is an obvious difference between these cohorts and (Perry, 1970), illustrates the journey of development and maturation (intellectually at least) of traditional learners through their college years. It is worth noting that most students traditionally attain so-called 'adulthood' whilst attending college, however, it does not mean they are considered 'adult learners', as the term applies.

(Rogers, 2007, p.15), argues adult learners in general, engage in (to varying degrees) a so-called 'learning contract' with the learning provider, the terms of which may be made explicit or not, but an implicit 'bargain has been struck' nonetheless between learner and provider. In contemporary times we are seeing more and more often the student as customer and the customer has found the complaints department especially when they feel their expectations have not been met.

Nonetheless, despite the much lauded and oft reported 'experience' adults bring to academia, it remains a two-edged sword: along with life experience, adults also bring forms of 'baggage' with them into the classroom, mainly relating to their previous negative experiences. Often they have been conditioned into an unrealistic expectation of a didactic pedagogy that leaves them feeling lost, confused and disappointed. Research by (Meltzer,

2002) and (Zopp, 1999) amongst others, found that previous mathematical experience as well as 'trigger events' related to mathematics education contributed towards mathematics anxiety in adult learners.

Particularly in mathematics learning, mistake-making is disliked intensely because it is perceived as a form of 'stupidity' by many adult learners. Mistakes are to be avoided, and if made, are to remain hidden and undivulged; such dysfunctionality, it is hypothesised, impedes learning significantly in adults. Moreover, (Lanigan , 2007, pp.22-23) found older adults particularly held beliefs that their 'school' mathematics was now superseded by 'new' mathematics – essentially, they viewed their own mathematical capital as near to worthless in the current setting. The research of both (Skemp, 1971) – *understanding mathematics relationally versus instrumentally* – and (Ernest, 2004) – *absolutist versus fallibilist philosophies in mathematics* – seem to be germane and have consonance with the overall research

There is, consequently, a number of resulting hypotheses; the first is mathephobia manifests in adult learners of mathematics and is mediated through inability or reticence to coherently communicate with other learners and instructor. Several potential causes are identifiable: anecdotally, resorting to negative talk (self, peer-to-peer, or with instructor) is a clear indication of poor or reduced mathematical self-efficacy and self-image, potentially indicating moderate to high levels of mathematical anxiety.

It is further hypothesised that undiagnosed or hidden levels of mathematical anxiety in adult learners mediates disengagement. Such anxiety may reveal itself in a number of ways, including the manner in which adult learners deal with mistake-making, for example.

Adult Learner Resistance to Learning Mathematics

Learner resistance to learning in higher education is a deceptively complex and multi-faceted phenomenon that has remained relatively misdiagnosed or ill-defined within the literature, according to (Tolman and Kremling, 2017, p.3), who researched cohorts of mature nursing students in higher education and described the aetiology of student resistance with their *IMSR* (*integrated model of student resistance*) see Figure 1. They define student resistance as *"…an outcome, a motivational state in which students reject learning opportunities due to systemic factors"*. They go on to emphasise the point that resistance is a motivational state, an outcome of multiple interacting factors, as opposed to a trait that endures over time or exists as part of a student's personality.

Furthermore, they go on to describe the difficulties instructors encounter when introducing a much more active learning paradigm within their classrooms. Clearly, there is potential for this current research to propose one or more solutions which mitigate learner resistance in spite of the typically didactic pedagogical expectations of many adult learners undertaking higher education. It is noteworthy (Tolman and Kremling, 2017) have identified within their research a form of passive resistance with learner characteristics not too dissimilar to those of a disengaged learner who could be situated on the *RRW-continuum*, most likely *withdrawn*.

The subsequent hypothesis is, that adult learner resistance to learning mathematics may in fact be 'signal' rather than 'noise', as (Tolman and Kremling, 2017) found, and just as their findings on student resistance appear to indicate a fluid, reactive nature to the phenomenon, it is further hypothesised there exist similarities with the coping states within the *RRW-continuum,* most notably *withdrawn*, with mathematical anxiety being involved or at least interacting in some form, perhaps from past negative experiences, instigating learners to ultimately withdraw from learning. It is hypothesised that the *RRW-continuum* as a phenomenon is equally fluid, dynamic and most certainly reactive in nature. It is hypothesised the *RRW-continuum* is a more nuanced interpretation of student resistance. The resulting research question is as follows:

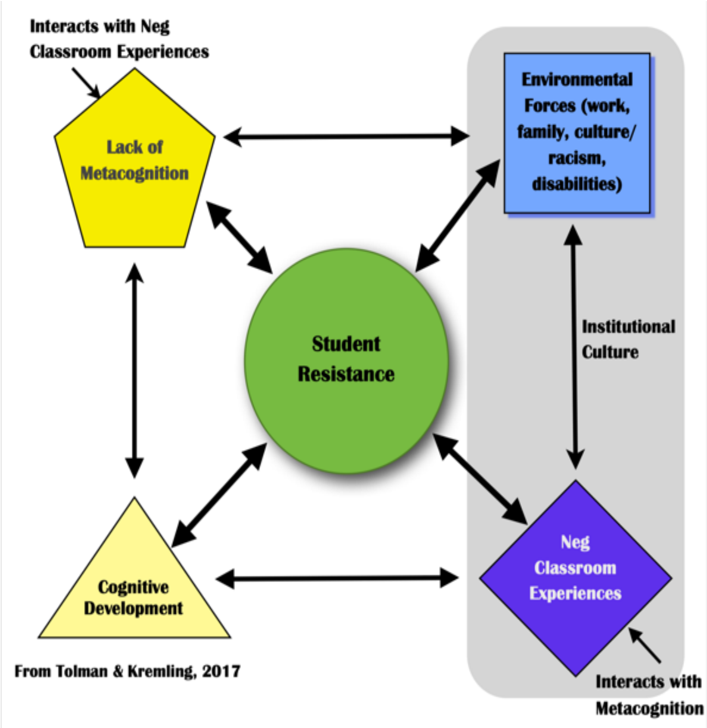
> In terms of adult learner motivation, what impact might learner self-efficacy, conation, learner resistance and mathematical anxiety have on engagement, collaboration, and mathematical thinking, and in mitigating impediments?

Subsequent, related research questions include:

a) What evidence might exist for adult learner engagement with (some or all of) negative self-talk, negative peer-to-peer talk and/or negative talk towards or from the instructor? What role, if any, does learner self-efficacy play?
b) If so, what type of negative talk exists, and might it be triggered by certain cues or events? Might it be possible to categorise such triggering events, and might they be typified in some way?
c) What relationship, if any, exists between adult learner self-efficacy and negative self-talk? What effect might this have on collaborative learning in adults? Might there be some effect on adult learner participation (especially learners on the reticent-retreating-withdrawn continuum) in a collaborative learning context?
d) What are the effects of mathematical anxiety (also known as mathephobia) on adult learners and in a collaborative context? If anxiety manifests, what is the nature of the manifestation? Is learner resistance, for example, a key indicator of undiagnosed or hidden mathematical anxiety in adult learners?
e) If learners resist learning or engaging, what might be the root cause(s) of such behaviour?

Figure 1

Integrated Model of Student Resistance (IMSR)



References

Alexander, R. (2020). *A dialogic teaching companion* (1st ed.). Routledge.

Ernest, P. (2004). *Philosophy of mathematics education journal edited by Paul Ernest*. http://Www.People.Ex.Ac.Uk/PErnest/Pome18/PhoM_%20for_ICME_04.Htm. http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/

Goos, M. (2002). Understanding metacognitive failure. *The Journal of Mathematical Behavior*, *21*(3), 283–302. https://doi.org/10.1016/s0732-3123(02)00130-x

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, *35*(4), 258. https://doi.org/10.2307/30034810

Lanigan, M. (2007). *The effects of learner self-confidence on competence in mathematics* (Master's dissertation). Waterford Institute of Technology. https://repository.wit.ie/3400/1/Dissertation%20FINAL

Mason, J., & Johnston-Wilder, S. (2006). *Designing and using mathematical tasks* (None ed.). Tarquin Group.

Meltzer, D. E. (2002). The relationship between mathematics preparation and conceptual learning gains in physics: A possible "hidden variable" in diagnostic pretest scores. *American Journal of Physics*, *70*(12), 1259–1268. https://doi.org/10.1119/1.1514215

Mezirow, J. (1991). *Transformative dimensions of adult learning* (1st ed.). Jossey-Bass.

Multon, K. D., Brown, S. D., & Lent, R. W. (1991). Relation of self-efficacy beliefs to academic outcomes: A meta-analytic investigation. *Journal of Counseling Psychology*, *38*(1), 30–38. https://doi.org/10.1037/0022-0167.38.1.30

Perry, W. (1970). *Forms of ethical and intellectual development in the college years: A scheme* (1st ed.). Holt, Rinehart & Winston.

Rogers, A. (2007). *Teaching adults* (3rd ed.). Open University Press.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, *196*(2), 1–38. https://doi.org/10.1177/002205741619600202

Skemp, R. R. (1971). *The psychology of learning mathematics* (1st ed.). Harmondsworth.

Tolman, A. O., Kremling, J., & Tagg, J. (2016). *Why students resist learning: A practical model for understanding and helping students* (Reprint ed.). Stylus Publishing.

Zopp, M. A. (1999). *Math anxiety, the adult student and the community college* (Ed. D. Dissertation). Northern Illinois University. https://www.proquest.com/openview/7a5e951615662118a0145b6803a43814/1?pq-origsite=gscholar&cbl=18750&diss=y

Outdoor Activities to Discover Medians and Centroid of Triangles

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The article describes the steps that led students to the discovery of properties of triangles and to the construction of medians, perpendicular bisectors (axes), and notable points. The approach used is that of Inquiry and Embodied cognition in outdoor context. The activity involved 15 students from a middle school in Trieste, Italy. The results were tested by proposing to the same students five types of exercises on axes, medians, heights and bisectors and the notable points connected to them. Success rates were averaged and divided between those that were subjected to outdoor activities (axis and median) compared to those that were carried out in the classroom (heights and bisectors). A further comparison was made between the success rates in drawing the objects under investigation. Finally, we investigated through Mentimeter the students' appreciation of the outdoor activity. 100% of students found the activity fun and helpful and the 85% of them considered group work to be useful.

Introduction

We are in a pandemic era, the need to move is even more important, like that of being together, collaborating, asking, and solving problems, breathing outdoors. Italy has had an extraordinary history of open-air public schools that began in the early 1900s for weak, fragile children who risked contracting TB (D'Ascenzo, 2018). After some experience it was noticed that these students learned more and better than the others, so they were also extended to "normal" children. The experience continued even during fascism. After the war it resumed strongly and ended in 1977 in Imola (Bo), when the last outdoor school closed, under the pressure of full-time and declining enrolments. In 2016, the "National Network of Outdoor Schools" was established in Italy, which groups and harmonizes the different experiences of a structured outdoor school system, where lessons outside the classroom can be taught, promoting students to rediscover a relationship with nature. More than 53 institutes currently belong to it. In the early 1900s, the outdoor school movement was widespread in many countries (Germany, England, France, Spain, United States, etc.) too. Recently there has been an increased interest in the development of outdoor and adventure education programmes (Fägerstam & Samuelsson, 2012). It would be an opportunity to use the virus catastrophe to change schools using the good practices of the past that have a lot to say even in the "digital" era. Never like now adolescents and young people need a true relationship with the teacher, to take responsibility, to learn from experimentation, to do manual and artistic work.

The purpose of this article is twofold: it is intended to show how an outdoor activity should be presented with a view to the Embodiment, the Inquiry and with the aim of facilitating peer collaboration; to test whether outdoor activities have been effective for understanding concepts and whether they have been appreciated by students. The activity presented involved 15 students from a middle school in Trieste, Italy.

Theoretical Background

Learning outside the classroom essentially can be defined as use of resources out of the classroom to achieve the goals and objectives of learning (Knapp, 2010; Smith & Walkington, 2020). The constant focus on textbooks and formal mathematical practice might invoke a view among students that mathematics is abstract, distanced and only useful in a classroom context. Existing research on outdoor learning in mathematics indicates positive affective outcomes and possible academic benefits from learning mathematics in an out-of-school context (Daher & Baya'a, 2012; Moffett, 2011). Moreover, outdoor environments are real-life contexts enabling children to internalise, transfer and apply mathematical ideas and provide direct experience, the students need to be active in the learning process (Moffett, 2011). It lends itself to the Inquiry-based mathematics education, a student-centered form of teaching whose guiding principle is that the students are supposed to work in ways like how professional mathematicians work (Artigue & Blomhøj, 2013; Dorier & Maass, 2014): they must observe phenomena, ask questions, look for mathematical and scientific ways of answer these questions, interpret, and evaluate their solutions, and communicate and discuss their solutions effectively. Cooperative learning gives the opportunity to discuss and reason with others and justify one's mathematical thoughts on how to solve different mathematical problems. Cooperative outdoor learning in mathematics gives the possibility to observe that a task at hand can be solved in more than one way and that more than one "right" solution to the problem may exist. The sensorimotor experiences arising from the environment also play a paramount role in learning (Wilson, 2002).

Embodied cognition is described as a bodily sense of knowing, expressed through physical movement and sensory exploration with environments (Kim et al., 2010; Merleau-Ponty, 2002; Smith & Gasser, 2005; Varela et al., 1991). There is complexity in the processes that may be involved in the development of embodied cognition as "knowledge depends on being in a world that is inseparable from our bodies, our language, our social history" (Varela et al., 1991, p. 173). According to Glenberg (2010) perception and how memory works is affected by how people move their bodies. To that vein, Hu, Ginns et al. (2015) suggested that pointing and tracing gestures might enhance geometry learning by activating an "increased working memory channel". The role of gestures as semiotic tools, contributing to deeper understanding of mathematical concepts (Arzarello et al., 2009).

Learning geometry can foster the ability to think logically, develop problem solving ability and reasoning, and can support many other topics in mathematics (Duval, 1995; Fischbein, 1993; Mariotti, 1995). According to Carden & Cline (2015) visualization refers to mental processes that describe visual or spatial information. Furthermore, for Arcavi (2003), visualization is a process that is the result of the creation, interpretation, reflection of images, diagrams with the aim of describing and communicating information on the development of an idea that is not known in advance to obtain a higher understanding.

The Methodology

In the first part we describe those steps that led students to the discovery of properties regarding triangles, to the construction of median, perpendicular bisectors (axis) and notable points. The approach used is that of Inquiry and Embodied cognition in an outdoor context. The activity takes place in the "Classroom under the sky" (for other activities see https://www.youtube.com/watch?v=lGJbz_d7OUs&t=80s). The environment is already welcoming in itself: a small pond right on the edge of a laurel grove, an open lawn that converges to the maple tree in the centre of the space, under which a blackboard and seats for students are placed. The students can also make use of portable shelves, to support books and notebooks. The activity involved 15, six grade students, 4 boys and 11 girls, from a middle school in Trieste, Italy. Students were randomly divided into 4 groups of 4/5. The results were tested by proposing, to the same 15 students, 5 types of exercises, one of these about drawing, on axes, medians (which were taught outside the classroom), heights and bisectors (which were taught in the classroom), and the notable points connected to them. Success rates were averaged and divided those that were subjected to outdoor activities compared to those that were carried out in the classroom. Finally, we investigated through Mentimeter the students' appreciation of the outdoor activity.

The Activities

The activities regard the discovery of the rules of drawing, those rules that lead hand and body to trace elements that take position on the surface of the lawn, with established criteria and simple tools, ancestors of the modern squares and compasses that crowd our school desks. Ropes, broomsticks, wooden stakes are our tools and some plastic caps. The square is the same one used by the ancient Egyptians 5 thousand years ago: 1 piece of rope divided, by means of knots, into 12 equal pieces. A short video of the activity carried out can be found at: tinyurl.com/outdoortriangles.

The Perpendicular Line and the Axis

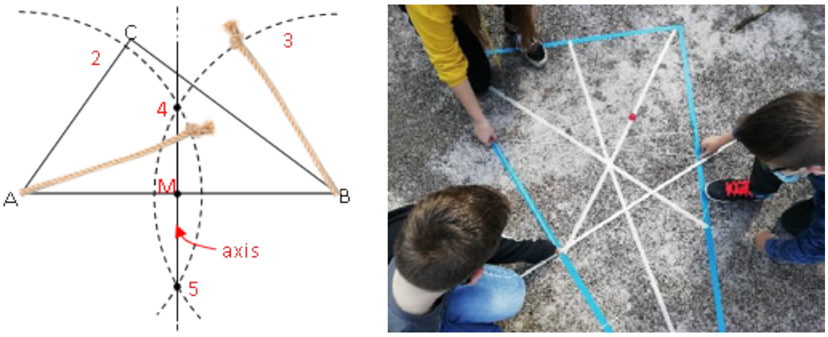
The activity starts by asking the kids to build a triangle by giving them 3 pieces of wooden planks deliberately of inappropriate lengths: they must find out why it is not possible. They discover, after several attempts, the fundamental property: The sum of the lengths of any two sides of a triangle is always greater than the length of the third one.

After building whatever triangles they wish, each group is asked to identify the midpoint of each side of the triangle. One group uses a piece of rope as a unit to measure one side and then calculates the half, another group uses a rope to measure one side, it joins the two extremes obtained to have the half.

At this point some teacher's suggestion is needed, students don't know yet certain techniques (Figure 1). By fixing one end of the string to the vertex of the triangle (A), a sufficiently wide arc is drawn (2), so it is possible to identify all the points that are at the same distance from that vertex: we call it an arc of circumference. We move to vertex (B), belonging to the same side, repeating operation (3) thus identifying two crossing points of the arcs which are both at the same distance from the ends of 4 and 5. By joining these two points, a straight line is drawn, which marks its midpoint (M) on the side. Now it is their turn: the students understand that they must position themselves on the vertices of the triangle, so crouched on the ground, bent on the knees, they hold down the vertex of competence with their fingers on the ground. Another member of the group hands one end of the rope to the partner who holds it on the vertex. After leaving the vertex, as there are no other classmates to help, a student identifies another point on the rope so that the distance from the first extreme is greater than half of the side, which is done by estimating the position of the midpoint. He lowers himself, grasps this end of the string between index, thumb, and middle finger and, with some difficulty, also binds the plaster to proceed with the formation of the bow. At the suggestion of the teacher, he forms a loop on the string and inserts the chalk into it to be freer in movement. At this point, with the back bent but in an upright position, the pupil draws an arc: his task is to keep the rope always in tension, so that the radius remains constant. Some students suspect that these axes form right angles to the side. Three students take the knots of our piece of rope divided into 12 equal pieces every 3, 4 and 5 segments and stretch it to obtain a right triangle. The impression is confirmed by positioning our large right-angled triangle of rope with the right angle placed on the identified midpoint (M). It is really true! The traced segment passes through the midpoint of the side perpendicularly. It is now up to the teacher to propose a name for this remarkable line: the axis.

Figure 1

The Axis



The Circumcenter and the Circumference Circumscribed

We note that in all the triangles of the groups the three axes cross in a single point. But “obtuse triangle group” has a strange design: did they do something wrong? Their point lies outside the triangle, on the longest side, the one opposite the obtuse angle. They explain how they proceeded, and the “acute triangle groups” confirm that the classmates followed the correct procedure. The peer review is a fundamental aspect in mathematics: it favors collaboration, the exchange of ideas and critical observation. It is precisely by leveraging on collaboration that students can build theorems themselves and then have the satisfaction that the theorem will not take the name of some important mathematician but will be reported in the notebook with the acronym of their names. Let us go back to the obtuse triangle and the axis. The only thing that has changed compared to the other “experiments” is precisely the shape of the triangle, so we can argue that in the obtuse triangles the point of intersection of the axis ends outside the triangle. It is time to give a name to this center-point: we call it Circumcenter. At this point the students are invited to look for another characteristic. In the right triangle, the crossing point is neither “inside” nor “outside”, but it is “up”: it is in fact on one side; the longer one, facing the right angle.

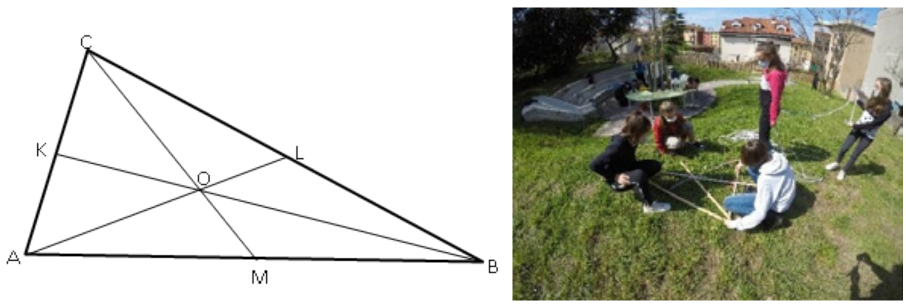
They are asked to discover other particularities: some suspect that the distances from the vertices to the circumcenter may matter. Here the circle comes into play, which although not treated in the 6^{th} grade, always arouses its charm. The end of the rope is now fixed on the circumcenter while another student stretches the rope until it reaches a vertex. He holds the end between his forefinger and thumb and checks that by rotating around the triangle by 360° it touches all the vertices. The circumference is circumscribed and therefore the point is called circumcenter.

The Centroid

We continue to take advantage of the fact that we have already identified the midpoints of the sides to discover another notable point: the centroid. What could be done with the midpoints of the three sides of the triangle? Some propose to unite them among themselves, others to unite them at opposite vertices. Both hypotheses are verified. Finally, the students join each vertex of the triangle to the midpoint of the opposite side: all these segments cross in a single point, O (Figure 2). The name of a geometric part is almost never accidental: since it arrives at the midpoint of the side, these segments are called medians. The meeting point is called centroid and it is the centre of gravity.

Figure 2

The Centroid



Once again, the challenge of identifying properties starts. The students soon discover that this point divides each median into two parts such that the one containing the vertex is longer than the other. Invited to measure them, the students discover that the first part is exactly double the other.

The Test

The next day a test containing 5 exercises was administered to attest the knowledge learned on axis and medians experienced outdoors as well as in classroom, but also height and bisectors learned only in class. Exercise 1. “Keywords table”: Write an “X” in the cell where there is a characteristic that serves to define the notable segment of the triangle (height, median, bisector or axis). Exercise 2. “Definitions”: Define: The bisector is ... The height is ... The median is ... The axis is ... Exercise 3. “Characteristics”: Report at least one characteristic of each segment and each notable point of the triangle: Medians ... Bisectors ... Axis ... Exercise 4. “Notable points”: Fill in the table indicating in which triangles the notable points are internal (write “int”), external (write “ext”) or lie on one side or a vertex (write “up”). Exercise 5. “Drawing”: Draw the required parts on the triangles, identify the crossing point; when you can, draw the inscribed or circumscribed circumference.

Results

Table 1. shows the averages of the success rates on all 5 exercises that concerned axis and medians (first row), topics addressed outdoors as well as in the classroom, with the same ones on heights and bisectors (second row). The differences of the previous percentages have been added in the third row.

Table 1

Success rate

	Success rate														
Axis & Median	73	61	39	54	35	80	93	73	61	79	68	73	13	35	70
Height & Bisector	63	45	25	31	25	82	92	19	43	41	47	44	0	30	65
Differences	10	16	13	23	10	-1	1	54	18	37	21	29	13	6	5

We immediately notice a significantly higher score of the exercises on the topics covered outdoors, axis and medians, with respect to height and bisectors treated in the classroom. Only for a student with an excellent success rate, the results are comparable: correct answers “outdoors” 80%, “indoors” 82%. For five of the other 15 students, the outdoor activity resulted in an advantage of more than 20%. In outdoor activities, the results are also more homogeneous: a percentage change coefficient of 50% for axis and 60% for medians with respect to 103% for heights and 99% for bisectors. The difference between outdoor and indoor activities (see Table 2) is even more evident for Exercise 5 on drawing: the average success rate is for outdoor activity equal to 75% (axis) and 70% (median) while for indoor activity 38% (heights) and 41% (bisectors).

Table 2

Exercise 5: Success rate

	Success rate of Exercise 5 on Drawing														
Axis & Median	100	60	35	23	50	100	100	100	100	100	100	100	40	55	100
Height & Bisector	90	53	23	20	0	100	100	13	23	23	53	23	0	13	100
Differences	10	7	12	3	50	0	0	87	77	77	47	77	40	42	0

Finally, we wanted to investigate through Mentimeter the students’ appreciation of the outdoor activity. The students were asked to write the first 5 words that came to mind when they think about the outdoor activity. The results are summarized in Figure 3 below (on the left) together with the same request made the next day in the light of the previous answers (on the right). In both cases the expression "funny / fun" was the most voted followed by "collaboration / working with and in group / group / being together".

Figure 3

Mentimeter

POSITIVE MOOD	
Fun	15
Beautiful	8
Joy / Happiness	4
Relaxing	16
STATE OF NEGATIVE MOOD	
Difficulty	2
SOCIALITY	
Team work	10
Cooperation	7
Stay together	3
LEARNING	
Learn	5
Interesting	7
Instructive	4
Comprehension	3
Definitions	2

Conclusions

Covid-19 launches a challenge to schools today in a strong crisis and that of outdoor schools is a real way that connects students with reality, nature, dexterity, art and a new responsibility towards creation, others, themselves. The reduction of opportunities for socialization has led to various psychological disorders in adolescents: panic attacks and anxiety. It should therefore come as no surprise that fun and socializing activities are of interest and approval. The survey carried out anonymously at the end of the outdoor lesson shows that 100% of students found the activity fun. Several used terms such as "beautiful, joy, happiness". 85% of students considered group work to be formative and useful, and the words "interesting, formative, learning" were reported by almost all students. Test results have also always been to the advantage of outdoor activity.

References

Arcavi, A. (2003). The role visual representation. *The Learning of Mathematics Educational Studies in Mathematics*, 52, 215-241.

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM*, 45(6), 797-810.

Arzarello, F., Paola, D., Robutti, O., Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics* 70, 97-109.

Carden, J. & Cline, T. (2015). Problem solving in mathematics: the significance of visualisation and related working memory. *Journal Educational Psychology in Practice*, 1-12.

Daher, W., & Baya'a, N. (2012). Characteristics of middle school students learning actions in outdoor mathematical activities with the cellular phone. *Teaching Mathematics and its Applications*, 31(3), 133-152.

D'Ascenzo, M. (2018). *Per una storia delle scuole all'aperto in Italia*, Pisa, Edizioni ETS.

Dorier, J.L., & Maass, K. (2014). Inquiry-based mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp.300-304). Springer.

Duval, R. (1995). Geometrical pictures: Kinds of representation and specific processings. In R. Sutherland & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education* (pp. 142-157). Berlin: Springer.

Fägerstam, E., & Samuelsson, J. (2012). Learning arithmetic outdoors in junior high school-influence on performance and self-regulating skills. *Education 3-13, 42*(4), 1-13.

Fischbcin, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*, 139-162.

Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *WIREs Cognitive Science*, 1, 586-596.

Hu, F., Ginns, P., & Bobis, J. (2015). Getting the point: Tracing worked examples enhances learning. *Learning and Instruction, 35*, 85-93.

Kim, M., Roth, W. M. & Thom, J. (2010). Children's gesture and the embodied knowledge of geometry. *International Journal of Science and Mathematical Education*, 9(1), 207-238.

Knapp, C. E. (2010). Teaching for experiential learning: Five approaches that work. *Journal of Experiential Education*, 33, 288-290.

Mariotti, M. A. (1995). Images and concepts in geometrical reasoning. In R. Sutherland & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education*. Springer.

Merleau-Ponty, M. (2012). *The phenomenology of perception*. Translated ed D. A. Landes. Routledge.

Milrad, M. (2010). Collaborative design of mathematical activities. *Proceedings of CERME 6*, Lyon France, 1101-1110.

Moffett, P. V. (2011). Outdoor mathematics trails: an evaluation of one training partnership. *Education 3-13, 39*(3), 277-287.

Moss, M. (2009). Outdoor mathematical experiences: Constructivism, connections, and health. *Mathematics Teacher Education*, *4*, 263-273.

Smith C., Walkington C. (2020). Four principles for designing embodied mathematics activities. *Australian Mathematics Education Journal, 1*(4), 16-20.

Varela, F. J., Thompson, E., & Rosch, ER. (1991). *The embodied mind: Cognitive science and human experience*. MIT Press.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, 9, 625-636.

Primary Mathematics Technological Anxiety: A Mixed Methods Exploratory Case Study of Primary Pre-Service Teachers

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Using the Technological Pedagogical Content Knowledge (TPACK) survey (Koehler & Mishra, 2008) and an Abbreviated Mathematics Rating Scale (AMARS, Richardson & Suinn, 1972) to measure the Technological Knowledge and Mathematics Anxiety of 30 Postgraduate (PGCE) Primary pre-service teachers, the exploratory case study highlighted several indicators of Mathematics Technological Anxiety. Although data was collected using a variety of quantitative and qualitative measures, only the quantitative findings are initially discussed in this paper. Pre and posttest statistical analyses revealed that although pre-service teachers' mathematics anxiety levels significantly decreased by the end of the course, technological knowledge did not significantly improve. Additionally, the TPACK survey revealed that technological anxiety ensued when student teachers felt pressurized to utilize technology to improve their personal mathematics content knowledge.

Introduction

Where an abundance of studies exist on the mathematics anxiety of primary preservice primary teachers (Boyd et al., 2014), limited research is available on the technological or computer anxiety of preservice teachers (Eko Setyarini, 2018; Tatar et al., 2015). Nevertheless, although research has shown strategies to reduce mathematics anxiety (Finlayson, 2014; Sidiqi, 2017) very few of these studies refer to the use of technological interventions as methods to reduce mathematics anxiety with primary preservice teachers specifically. This case study was conducted to contribute to this body of knowledge by highlighting the impact of using iPads as tools to improve Maths Content knowledge (CK) and changing levels of pre-service teachers' Maths anxiety. This paper will discuss the quantitative findings of TPACK and Maths anxiety levels of 30 PGCE primary preservice teachers studying in Northern Ireland (NI).

Review of Literature

NI Context

In 2008, the GTCNI (General Teaching Council for Northern Ireland) investigated the numbers of teachers on its register that held one or more Science, Technology, Engineering and Mathematics (STEM) teaching qualifications, or other academic STEM qualifications. Stewart (2014, p. 4) reported that the investigation found that, "10% of all teachers registered in Northern Ireland had a science or mathematics background and only 23% of STEM specialists worked in primary schools." This was then followed by the Report of the STEM Review (2009), where Perry (2012, p. 2) noted "a continual decline in interest in STEM subjects beginning in the latter years of primary education." Nevertheless, Hilton (2016) found that integrating technology into the mathematics classroom helps reduce Mathematical Anxiety (MA) in students and positively influences student engagement.

Mathematical Anxiety (MA)

Introduced as *number anxiety* by Dreger and Aiken (1957), MA is defined by Dowker (2016, p. 508) as "severely disrupts…mathematical learning and performance, both by causing avoidance of mathematical activities and by overloading and disrupting working memory during mathematical tasks". Although the study of mathematics anxiety has spanned almost sixty years, minimal research exists regarding the impact of school initiatives upon teachers' mathematics anxiety and more specifically preservice teachers' MA.

Barry (2017, p. 25) comments, "Technology helps take away the pressure and anxiety associated with worksheets and the traditional teaching practices in math and provide an avenue to explore and enjoy doing mathematics". Both Heinrich (2012) and Henderson and Yeow (2012) identified Initial Teacher Education (ITE) courses as a necessary support for effective integration of tablet devices. Several studies (Hourigan & Leavy, 2017; Tatar et al., 2015) have also shown that integrating mobile technology in the mathematics classroom can have a positive effect towards reducing MA among preservice teachers. Nevertheless, Shamoon (2014, p. 2) highlighted that as "the level of mathematical anxiety is considerably higher among students within ITE programmes compared to other university students". Therefore, the challenge for teacher educators have been to selectively utilise the most appropriate technologies which aim to enhance mathematical learning instead of hindering it.

Boyd et al. (2014) and Harper and Danne (1998) conclude that if preservice teachers are highlighted to their individual levels of maths anxiety and are taught methods to avoid transmitting their own negative dispositions within their mathematical pedagogy then this has proven to reduce mathematics anxiety. Rayner et al. (2009) more specifically highlighted the importance of preservice teachers demonstrating a proficiency in both mathematical procedures and concepts to decrease maths anxiety levels. Sloan, (2010) & Furner & Berman, (2003) agreed that conceptual understanding, should be explored by creating environments which are student-centred and encourage the use of mathematical manipulatives which place an emphasis on sharing multiple processes and methods to help decrease anxiety. Flipped classrooms have been used to promote more student-centred opportunities during university teacher education classes.

iPad Use Within I.T.E Mathematics Teaching and Learning

Clarke and Luckin (2013, p. 11) described the iPad as '*a powerful, portable, personal learning partner*,' which was fast becoming the essential toolbox for the 21st century classroom. Mango (2015) after surveying pre-service teachers use of iPads reported overwhelmingly that learning was more enjoyable, and they were able to remain more focused on classroom tasks when using iPads. Maich & Hall (2016, p. 23) concur stating, '*iPads can improve classroom learning*.' Walter (2011) reported many advantages of iPads including; allowing a smooth transition from software-specific projects with a steep learning curve to smaller scale apps-based learning tasks, ease of use with apps instead of software training, as well as portability and kinaesthetic interactions that traditional desktop or laptop computers could not offer. Indeed, Clarke and Luckin (2013) have highlighted various models and strategies for iPad implementation used throughout schools both nationwide and internationally and it is clear from practitioner blogs (Andrews, 2013; Page Burdick, 2013; Swanson, 2013) that although overall teachers' perceptions on iPad use enhanced their learners' experience and transformed their pedagogical style. Cheu-Jay (2015) indicated that mathematical classroom success was more attributable to the fact that teachers who integrated iPads into their lessons tended to do more Project Based Learning (PBL), which had a profound improvement to student learning across grade levels. Baran (2014, p. 17) lists six key research findings in her review of mobile learning in Teacher Education, concluding with the finding that several pedagogical affordances support mobile learning integration into teacher education settings. Nonetheless, although some UK iPad research exists, many studies (Burden et. al, 2012; Heinrich, 2012; Hopkins & Burden, 2014) indicate that there remains "the need for further research in the area of educational use of iPads for both pre- and in-service teachers in addition to how attitudes towards technology affect classroom integration" (Tohill, 2014, p. 113).

Ulster University's PGCE Primary use of Technology

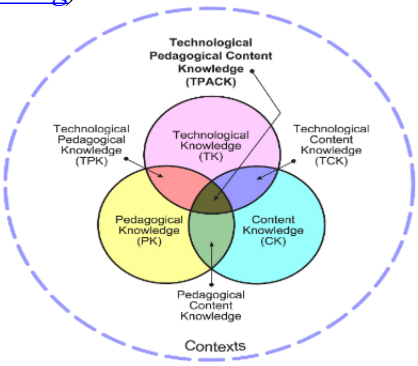
The use of technology within the teaching content of the PGCE Primary provision is correlated directly to the emerging technologies utilized in NI's Primary schools. A wide range of mobile and static devices are utilized, and student teachers effectively and selectively integrate pedagogical software and hardware within the NI Curriculum. With the introduction of iPads, an evaluation of how the devices were being implemented was prompted and the use of a mathematical TPACK survey initiated the findings.

Theoretical Framework

The TPACK framework (Koehler & Mishra, 2008) focused upon the interdependence of Technological Knowledge (TK), Pedagogical Knowledge (PK) and Content Knowledge (CK) to '*integrate the use of digital tools and resources effectively in curriculum-based teaching.*' (Harris et al., 2017, p. i). Benton-Borghi (2013) highlight that the intricate linkage between TK, PK and CK is evidently embedded in most Initial Teacher Educational thinking.

Figure 1

TPACK framework (http://tpack.org)



Methodology

Research Questions

1. Did the effective use of iPad technology decrease mathematics anxiety among primary pre-service teachers?
2. Does iPad technological knowledge (TK) increase student teachers' mathematical content knowledge (CK)?
3. Is there a correlation between mathematical anxiety and mathematical content knowledge of PGCE preservice teachers?

30 PGCE Primary pre-service teachers: ten male and twenty females studying in Ulster University. Although the case study initially began with 33 participants, the attrition rate resulted in three students withdrawing before the end of the study.

Instruments

Table 1

Data Collection Tools

Quantitative Collection Tools	*Data Collection Date*
TPACK survey (Koehler &Mishra, 2008).	*August 2016*
A seven-point Likert-type scale	*June 2017*
AMARS (Richardson and Suinn, 2012).	*August 2016*
25 statements - 5-point Likert scale	*June 2017*

Following the approval of Ulster University's Ethics Committee, the study began with the distribution of Mathematics TPACK surveys and Mathematics Anxiety Tests (AMARS) to all participants at the beginning of the PGCE Primary course (pre-test) and at the end (post-test). All steps to conceal the identity of the participants were taken. Voluntary informed consent was obtained from the participants. The researchers took all the necessary steps to ensure all participants are aware of the confidentiality of any information provided. Each participant completed a biographical profile survey which was complied with the legal requirements in relation to the storage and use of personal data as set down by the Data Protection Act (1998). To ensure anonymity of responses, each student was randomly allocated an identifier number to match up to the biographical profile survey.

Results

Research question one: Did the effective use of iPad technology decrease mathematics anxiety among primary pre-service teachers?

The quantitative analyses of the Abbreviated Mathematics Rating Scale (AMARS, 2012) revealed a highly significantly reduction to PGCE Preservice teachers' Mathematics Anxiety (MA) levels by the end of the PGCE primary course. Table 2 illustrates the Paired Sample T-Test statistical significance with gender split. MA levels were highly significantly reduced for females by the end of the PGCE course although both genders showed a significant decrease.

Table 2

Paired Samples T-Test of AMARS

Gender		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig (2-tailed)
					Lower	Upper			
Male	AMARSpre TOT - AMARSpost TOT	14.33333	14.96663	4.98888	2.82896	25.83770	2.873	9	.021
Female	AMARSpre TOT AMARSpost TOT	15.80000	19.45467	4.35020	6.69493	24.90507	3.632	19	.002

Research question two: Does iPad technological knowledge (TK) increase student teachers' mathematical content knowledge (CK)?

Within the TPACK survey explored student teachers' perceptions of Maths CK. Table 3 demonstrates the t-test conducted between the pre and post CK means and it is also split by gender. The results in Table 3 indicate that male student teachers perceived their Maths CK to slightly improve with P=0.164. Whereas for female student teachers P=0.002 highlighting that a highly significant improvement was made to their Mathematics CK.

Table 3

Paired Samples T-Test on Maths Content Knowledge

Gender		Mean	Std. Deviation	t	df	Sig. (2-tailed)
Male	preCKTOT - postCKTOT	-2.20000	4.58984	-1.516	9	.164
Female	preCKTOT - postCKTOT	-2.95000	3.77631	-3.494	19	.002

The crosstabulation results of the pre and post CK variable descriptives are outlined in Table 4 below. This was highlighted to identify the specific variables of Maths CK that held the highest significance. A significance difference was highlighted between the Pre-CK Question 1 (PVM=4.40, SD=1.102) and Post C K Question 1 (PTVM=5.73, SD=0.944); t (29) = -5.884, P=0.000. This drew the conclusion that PGCE student teachers felt that they had a wider and deeper understanding of various Maths concepts they planned to teach by the end of the course. Also, there was a higher significance difference between the Pre-CK Question 5 (PVM=5.27, SD=0.785) and Post C K Question 5 (PTVM=5.80, SD=0.714); t (29) = -3.764, P=0.00. This concluded that student teachers felt better equipped by the end of the course to have various methods and strategies which developed their mathematical understanding. Therefore, when the overall means of the CK totals were calculated P=0.001, this showed a highly significant improvement to CK between pre and post TPACK surveys.

Table 4

Descriptive Variables of Maths CK within the TPACK survey

Descriptive Variables	Pre Var Mean	Post Var Mean	N	Pre Var SD	Post Var SD	Sig. (2-tailed)
1. I have a wide and deep understanding of the subjects I plan to teach.	4.40	5.73	30	1.102	0.944	0.000
2. I know about various examples of how mathematics applies in the real world.	5.43	5.83	30	0.898	0.34	0.056
3. I have sufficient knowledge about mathematics.	5.60	5.70	30	0.675	0.915	0.610
4. I can use a mathematical way of thinking.	5.37	5.70	30	0.999	0.877	0.057
5. I have various ways and strategies of developing my understanding of mathematics.	5.27	5.80	30	0.785	0.714	0.001

When analyzing TK exclusively within the TPACK survey, no significant difference was found in the paired sample T-Test even when the means were split by gender. T (29) = -1.601, P= 0.120. Therefore, in order to gain a further insight into why there was no significant TK improvement, a crosstabulation between the descriptive variables showed that only two TK statements held some significance. As Table 5 illustrates, these findings indicated that although PGCE student teachers learned technology more easily by the end of the course and knew more about different technologies, no overall significant difference was found to their TK.

Table 5

Descriptive Variables of Technological Knowledge within the TPACK survey

Technological Knowledge	Pre Var Mean	Post Var Mean	N	Pre Var SD	Post Var SD	Sig. (2-tailed)
I know how to solve my own technical problems.	5.23	5.60	30	1.135	1.003	.094
I can learn technology easily.	5.73	6.07	30	0.521	0.640	.005
I keep up with important new technologies.	5.27	5.47	30	0.868	0.973	.339
I frequently play around with the technology.	5.10	5.30	30	0.995	1.368	.476
I know about a lot of different technologies.	4.20	5.00	30	1.126	1.203	.006
I have the technical skills I need to use technology.	5.57	5.57	30	1.203	0.774	1.000
I have sufficient opportunities to work with different technologies.	5.47	5.43	30	1.008	1.194	.897
When I encounter a problem using technology, I seek outside help.	4.87	4.77	30	1.507	1.478	.775

Therefore, it was non-conclusive that the improvements to CK could have been be attributed to TK. As the improvement to TPACK was highly significant P=0.000, the T-Tests listed in table 6 show that TK was the only element of TPACK that didn't improve significantly. Table 6 highlights the need for qualitative data to explore in greater depth the qualitative reasons why student teachers' TK did not significantly improve. Hence, the quantitative data was non-conclusive if the improvements to Mathematics CK was attributed to TK.

Table 6

Paired Samples T-Tests for all seven TPACK domains.

TPACK VARIABLES	P- VALUES
TKTOT	P = 0.120
CKTOT	P = 0.001
PKTOT	P = 0.000
PCKTOT	P = 0.000
TCKTOT	P = 0.018
TPKTOT	P = 0.000
TPACKTOT	P = 0.000

Research Question Three: Is there a correlation between MA levels and Maths CK?

Linear regression was used to ascertain if there was a correlation MA levels and Maths CK. The dependent variable in this case was MA with CK from the TPACK survey as the independent variable. The overall idea of using regression was to examine whether there was a correlation between MA levels and doing a maths test. The result showed a positive correlation with $Y = 0.675$.

Discussion and Recommendations

To conclude, quantitative results showed that PGCE student teachers' Maths CK significantly improve and MA levels also significantly decreased by the end of the course, however, TK did not improve. The findings showed that the use of iPads specifically, was not a medium by which student teachers felt comfortable implementing to improve their Maths CK. The TPACK survey did highlight that if iPads were implemented in mathematical teaching and learning in a scaffolded constructivist way, incorporating peer collaborative shared learning, then this pedagogic approach could be more impactful within Initial Teacher Education (ITE) programmes.

Despite the TPACK survey indicating that TK remained unimproved, qualitative data is required to explore why this was so. It is likely however, that due to the changing nature of our digital world and the evolution of TK, it is unlikely that PGCE student teachers would acquire all the TK necessary and the need for Continuous Professional Development (CPD) of new technologies is essential in order to have a fuller understanding of the affordances of technologies effectively implemented within mathematical teaching and learning. Currently, the GTCNI has no stipulated CPD requirement for qualified teachers, whereas in Scotland, the GTC requires a minimum of 35 hours per year of CPD. Although the monitoring of this CPD requirement has been questioned (Kennedy et al., 2008), the benefit to the effectiveness of ICT pedagogical practices in Scottish primary mathematical education is outweighed (Burden et al., 2012).

Limitations

The limitations to this study was the small sample size of the PGCE cohort involved and it would be an interesting further study to make comparisons of mathematical and technological knowledge of other PGCE primary cohorts in the UK mainland. Additionally, as this paper explored quantitative findings exclusively, it would be interesting to follow this with some qualitative findings which would substantiate the differential statistics found.

References

Boyd, W., Foster, A., Smith, J., & Boyd, W. E. (2014). Feeling good about teaching mathematics: Addressing anxiety amongst pre-service teachers. *Creative Education, 5*, 207-217.

Brown, A., Westenskow, A., & Moyer-Packenham, P. (2011). Elementary pre-service teachers: Can they experience mathematics teaching anxiety without having mathematics anxiety? *Issues in the Undergraduate Mathematics Preparation of School Teacher: The Journal, 5*, 1-14.

Burden, K., Hopkins, P., Male, T., Martin, S., & Trala, C. (2012). iPad Scotland evaluation. The University of Hull.

Eko Setyarini, M. C. (2015) Understanding teachers' computer anxiety. *ETERNAL (English Teaching Journal), 6*(1). https://doi.org/10.26877/eternal.v6i1.2297

Kennedy, A., Christie, D., Fraser, C., Reid, L., McKinney, S., Welsh, M., Wilson, A., & Griffiths, M. (2008). Key informants' perspectives on teacher learning in Scotland. *British Journal of Educational Studies, 56*(4), 400-419.

Richardson, F., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric Data. *Journal of Counseling Psychology, 19*(6), 138-149.

Sidiqi, N. (2017). Effective strategies to reduce math anxiety in teachers and students; A research paper submitted in conformity with the requirements for the degree of Master of Teaching Department of Curriculum, Teaching and Learning Ontario Institute for Studies in Education of the University of Toronto.

Sloan, Daane, & Giesen (2002). Mathematics anxiety & learning styles: What is the relationship in elementary preservice teachers? *School Science & Mathematics Journal, 102*(2). https://doi.org/10.1111/j.1949-8594.2002.tb17897.x

Stewart, J. (2014). Research & Information Service Briefing Paper: Paper 38/14, *Science in the Revised Curriculum*. Retrieved from http://www.niassembly.gov.uk/globalassets/documents/raise/publications/2014/education/3814.pdf

Tatar, E., Zengin, Y., & Kağızmanlı, T. B. (2015). What is the Relationship between Technology & Mathematics Teaching Anxiety? *Educational Technology & Society, 18*(1), 67–76.

A Study of the Experience of Able Mathematicians in Secondary Schools in Ireland

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In 1993, the Report of the Special Education Review Committee in Ireland outlined best practise for students who were 'exceptionally able and talented' in a number of areas including 'specific academic aptitude' and 'creative productive thinking'. However, to date, there has been limited research focus on how to challenge able mathematicians. In 2010, the Project Maths curriculum reform was introduced into Irish schools at both Junior and Senior cycle levels. There is much emphasis in the new syllabus on teaching for conceptual understanding, reasoning, justification and problem solving (NCCA, 2015) which is in line with the recommendations above. My study aims to examine whether able mathematicians feel challenged in post-primary schools in Ireland and what can be done, within a diverse classroom, to give them a more engaging learning experience. As part of the project, I designed a series of workshops through which I could examine the attitudes of students to creative problem solving. This paper is primarily a review of international research into the importance of creative tasks for identifying and challenging able mathematicians and a description of the design of my study. Data is still being collected but a preliminary analysis has been carried out, the findings of which will be discussed.

Introduction

My research project aims to examine how challenged second level mathematics students feel, what they enjoy about their mathematics classes and which aspects they believe will help them reach their mathematical potential. In this paper I will describe the design of my project and give some preliminary results based on data gathered from 95 students in 5 different Irish schools. I have designed and trialled workshops to push the students into areas of uncertainty and encourage them to question, explore and discuss mathematics. In Barbeau & Taylor (2009) mathematical challenge is seen as an essential aspect of education in that it provides students with opportunities to experience enjoyment and satisfaction whilst also enhancing valuable life skills such as patience, persistence and flexibility. Providing students with an engaging experience that enhances their understanding of the mathematics of real life is a key feature in the current Irish syllabus (NCCA, 2015). However, early research into the impact of Project Maths showed that students are still "being presented with tasks that do not require them to engage widely with the mathematical processes promoted through the revised syllabuses", (Jeffes, Jones, Wilson, Lamont, Straw, Wheater, & Dawson, 2013, p. 45). There is evidence that the level of cognitive demand in Leaving Certificate examinations has increased since the introduction of the Project Maths syllabus (O'Connor, Ní Shúilleabháin, & Meehan, 2019) but the extent to which the aims and objectives of the syllabus are being implemented in the classroom needs further investigation. Textbooks still play a key role in classroom instruction (Jeffes et al., 2013) yet research has shown that those used in Ireland offer a low level of high cognitive demand or creative thinking tasks (O'Sullivan, 2017). In light of this my study will examine whether able mathematicians, who may have an excellent capacity for creative reasoning and transfer of conceptual knowledge, are given sufficient opportunities to do so in the classroom.

Defining an Able Mathematician

An able mathematician is seen in this study as a student who has the potential to achieve highly in school and thereafter. In light of research which has shown that a lack of high achievement in school mathematics assessments does not prevent mathematical accomplishment (Pehkonen, 1997), an able mathematician also includes those who have a great interest in mathematics yet may not be high achievers in exams. To allow all students to have the opportunity to maximize their creativity, standard assessments of their ability should not be the only determining factor for their selection as an able mathematician. Intelligence, personality and perseverance need consideration as well, given the evidence that there are many high achievers who do not excel in traditional assessments (Mellroth, 2018; Nolte & Pamerien, 2017; Sheffield, 2003).

Literature Review

Why Able Mathematicians Need Challenge

The literature on education for mathematically able students stresses the need for challenge (Mann, 2006; Nolte & Pamerien, 2017; Sheffield, 2003) but defining exactly what we mean by challenge is more complex. The Cambridge online dictionary describes challenge as being faced with ‘something that needs great mental or physical effort in order to be done successfully and therefore tests a person’s ability’. By such a definition all students deserve to be challenged, the difficulty comes with knowing how to challenge all students and balancing this with other priorities as a teacher. Nolte & Pamperien (2017) believe that challenging problems are essential for the development of cognitive and emotional skills in high ability students. Similarly, Sheffield (2003) suggests that challenge is instrumental in the development of the brain and that mathematics is the ideal discipline in which to do this. Creativity is identified as one of the key characteristics of mathematically able students (Leikin & Lev, 2013) and as such should be an essential element of classroom instruction.

The Role of Creativity in Recognising and Fostering Challenge

The last two decades have seen an increased interest in mathematical research on giftedness and creativity and, in particular, in the crucial role creativity has on mathematical cognition. This heightened interest can be seen by the introduction of thematic groups on giftedness and creativity in international conferences such as the International Congress on Mathematics Education (ICME) and The International Group for Mathematical Creativity and Giftedness (MCG). Research papers for these conferences have focused on the importance of creativity and on specific areas such as defining, recognising and fostering creativity.

Defining creativity has proved a complex task and there is a lack of a widely accepted definition amongst researchers. Up until recently it was felt that this “lack of an accepted definition for mathematical creativity hindered research efforts” (Mann, 2006). Torrance, the acclaimed “Father of Creativity”, based his assessment of creativity on a measure of the evidence of fluency, flexibility, novelty and elaboration in a mathematician's work (Leikin & Lev, 2013). Building on Torrance's concepts, multi-solution tasks have been shown to be a means by which the 'relative creativity' of students can be identified and fostered (Leikin & Lev, 2013). The definition that best describes my study is that creativity at school level is "the process that results in unusual (novel) and/or insightful solution(s) to a given problem" that are new relative to the student's mathematical experiences and those of his peers (Liljedahl & Sriraman, 2006, p.19).

Inhibiting Factors to Pursuing Creative Tasks in Schools

One of the major obstacles in regular classrooms is the 'myth' that high ability students can teach themselves (Sheffield, 2017). Research in Finland and Britain on the role teachers can have has shown that in some cases up to 40% of high ability students are underestimated by their teachers (Freeman, 1998). Other surveys quote teachers have felt "hindered by constraints on time and material resources in teaching bright pupils, and that any available extra provision was targeted towards the least able" (Freeman, 1998, p.11). A teacher's ability to see the creative potential of students can also be obscured by an over emphasis on classroom instruction based on teacher examples and rote learning of algorithms. This is substantiated by research on the theory of functional asymmetry in the human brain which has highlighted the danger of placing too much focus on routine algorithms, which exercise the left hemisphere, at the expense of the creativity and spatial awareness, which exercise the right (Pehkonen, 1997). However, there has been a move towards support for Pehkonen's theory and more recently it has been acknowledged that mathematical creativity can, and must, be developed in all students (Mellroth, 2018; Sheffield, 2003).

The Irish Context

Ireland has catered for 'gifted' students through summer schools and extra-curricular courses such as the Olympiad, Maths Circles and Irish Centre for Talented Youths (CTYI) since the 1990's. My study sees creativity as a characteristic of a wider section of students than those who have been classified as 'gifted' and aims to investigate opportunities to expand creative reasoning skills within the school curriculum.

The motivation behind the Project Maths curriculum reform, in 2010, was largely to address the findings in reports on international assessments such as The Programme for International Assessment (PISA) and Trends in International Mathematics and Science study (TIMSS). In 2015, Ireland was found to be below the OECD average in the higher order level 5/6 problem solving tests in PISA (OECD, 2016). As a country, Ireland performs well on 'knowing' procedures but higher order thinking and 'reasoning' have been neglected in favour of drilling students on procedural fluency. In order to enhance the recognition of the need to foster creative reasoning, both teachers and students must believe in its merits.

Theoretical Framework

My research is guided by Vygotsky's Sociocultural Theory of Cognitive Development and Sriraman's Five Principles to Maximize Creativity (Figure 1). These frameworks form the structure upon which the study was designed. By doing so I hope to investigate how challenged able mathematicians feel in Irish classrooms and what impact creating an environment based on Sriraman's five principles can have on their motivation, enjoyment and perseverance when solving unfamiliar tasks.

Vygotsky's Sociocultural Theory of Cognitive Development

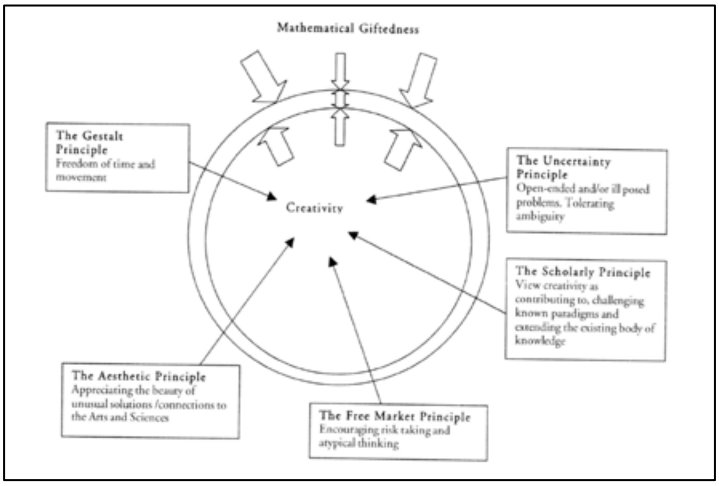
A fundamental construct of Vygotsky's theory is his belief that creativity is the key to cognitive development by enabling students to construct their own knowledge through collaboration with others. He suggested that "development processes do not coincide with learning processes. Rather, the development process lags behind the learning process" (Vygotsky, 1978, p.90). For this reason, student learning should be in what he called 'zones of proximal development' (ZPD) where students work on more advanced tasks than those they currently feel comfortable with. Collaboration and exposure to tasks in the ZPD are considered critical in enabling students to reach their mathematical potential.

Sriraman's Five Principles to Maximize Creativity

More recently, Sriraman (2005) outlined five principles to maximize creativity for second level students (Figure 1). The first of these requires students to be given the opportunity to engage in the four-stage creativity process of the Gestalt psychology principle: initiation-incubation-illumination-verification. Of these four stages, the most important stage with regard to creativity is the incubation stage when the mind unconsciously reaches a solution. Sriraman also emphasised the appreciation of unusual student solutions, exposing students to uncertainty and creating an environment that encourages student discussion.

Figure 1

Harmonizing creativity and giftedness at upper second level.



Note. This is reproduced from Sriraman (2005).

Methodology

I designed 2 workshops to allow students to work in small groups on multi-solution tasks. The workshops were held a week apart to enable the students to incubate the problems posed and some of the techniques that had worked for them. Before the workshops, the students were given a Likert scale survey to gather data on their current experience of mathematics in school and their level of self-confidence with regards to mathematics. After participating in the workshops, the students were re-surveyed to assess the impact, if any, of the workshops on their thinking, problem-solving strategies, self-efficacy and motivation. Voluntary focus group interviews, of 3 or 4 students, approximately 40 minutes long, were held a week after the workshops to give the students an opportunity to answer more open-ended questions and to discuss in more detail their experience of the tasks in the workshops.

Structure of workshops

The workshops were run face to face, in three different schools in early 2020, but the pandemic necessitated that those for the final two schools were run via an online platform using a similar group set-up via breakout rooms. The schools selected included single sex and mixed gender schools from a variety of locations. The students were selected from Transition Year and the teachers were asked to offer the workshops to students who were either high achievers in school, or those who showed a particular interest in mathematics but may not have been in the top 15% in traditional assessments. They were then placed into randomly selected groups of three or four where they were given one or two tasks per workshop that were designed to be unfamiliar and suitable for diverse classrooms. I was guided by the definition of 'an unfamiliar task' as "one for which students have no algorithm, well-rehearsed procedure or previously demonstrated process to follow", (Breen & O' Shea, 2015).

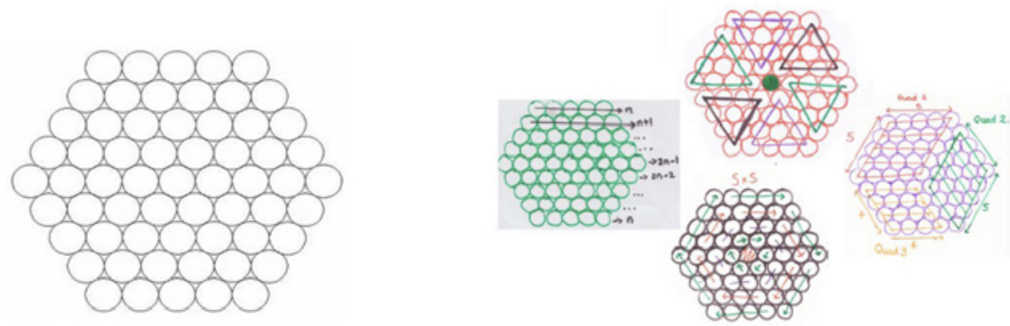
Other key features, based on my chosen frameworks of Vygotsky (1986) and Sriraman (2005), were that the tasks selected were challenging, multi-solution tasks intended to encourage creative thinking, peer collaboration and perseverance whilst avoiding putting the students under time pressure to solve them. The students were given no guidance as to the method to employ and were required to make connections across various strands of the Leaving Certificate syllabus to solve the tasks. They were encouraged to discuss the problem-solving strategies they employed as a group and the merits of a number of different solution methods presented to the class. My role was that of a facilitator where I only intervened to ask probing questions or to encourage a line of thinking that the students had suggested but lacked the confidence to pursue.

Sample Tasks

The Nrich Steel Cables task (see Figure 2 below) is an example of one of tasks selected because of its suitability to challenge all students simultaneously in diverse classrooms. The students were given the template on the left, a 'size 5' cable with 61 strands, and asked to find how many strands would be in a 'size 10' cable in as many ways as possible. They were then asked to find how many strands would be in a 'size *n*' cable and to explain their reasoning to the group. The next phase of the task was carried out in the second workshop to give students time to incubate the problem. The brief was to discuss the solutions presented to them (diagram on the right of Figure 2 below) and to suggest how the methods employed could be used to find the strands in a 'size 10' and 'size *n*' cable.

Figure 2

Steel Cables Task.



Note. Reproduced from www.nrich.maths.org/7760

Data Collected

In addition to the student surveys, the discussions in the working groups and breakout rooms were audio recorded and the written workings of the students were collected as data. I have so far conducted nine focus group interviews based on questions aimed at assessing the students' experience of school mathematics and the workshops. Of the 95 students who have agreed to participate I have collected surveys from 80 students and 26 have taken part in the group interviews. I have one remaining set of workshops and interviews to complete.

Preliminary Results

A preliminary analysis of the responses to both surveys has been carried out and the percentages of responses to a sample of questions in the surveys are given in the table below:

Table 1

Sample of results from 80 student surveys.

Survey Question	**Strongly agree /Agree %**
I am often bored in class while the teacher explains the solution to other students.	71.1%
Most of the time we work on mathematics problems from the textbook.	86.8%
Most problems I do in school can be answered by recall of examples and formulae.	93.4%
Having to think about the method in the workshops was different to what usually happens when we use the textbook in class.	91.4%

I have only carried out a preliminary analysis of the focus group interviews for 3 schools but can give a flavour of their answers to questions about their school experience and their impressions of the workshops. Some recurring themes have emerged from the interviews analysed, such as student descriptions of class as "very repetitive", "a lot of waiting around", "we don't ever discuss maths" and "we just stick with the method we are shown in the example, we don't have to think about it". Discussion and peer collaboration, as advocated by Vygotsky and Sriraman, do not seem to be key features of mathematics class for these students. Similarly, there is little evidence to support the integration of Sriraman's 'five principles to maximize creativity' in their normal classrooms. In addition to the above recurrent themes, key words that have been reiterated in these particular interviews were feelings of "boredom" in school classes in contrast to feelings of "freedom" in the workshops.

Discussion

The preliminary analysis highlights that able mathematicians feel that classroom instruction is overly focused on examples rather than concepts. In contrast, the main aspects of the workshops they enjoyed were having to use trial and error to think of methods for themselves and discussing these with their peers. This study is grounded in the belief that creativity is an essential aspect of providing challenge to able mathematicians but as evidence suggests (Mellroth, 2018; Nolte & Pamerien, 2017 and Sheffield, 2003) it can also be accomplished in a diverse classroom, such as is common in Ireland. The anticipated dilemma for teachers is how to ensure that the students are challenged without being overwhelmed by the cognitive leap from the traditional algorithmic textbook questions. It is hoped that the combination of data in this study will help facilitate the exploration of creative problem-solving methodologies for diverse classrooms in Ireland.

References

Barbeau, E., & Taylor, P. (Eds.). (2009). *ICMI Study—16 Volume: Mathematical challenge in and beyond the classroom.* Kluwer.

Breen, S., & O'Shea, A. (2015). Adams. G. (Ed.) *Proceedings of the British Society into Learning Mathematics 35*(1) February.

Report of the Special Education Review Committee (SERC) (1993). Retrieved January 12, 2021 from https://www.sess.ie/sites/default/files/Categories/Exceptionally_Able/SERC_Exceptionally_Able_Talented.pdf

Freeman, J. (1998). *Educating the very able: Current international research.* The Stationary Office.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw., Wheater, R., & Dawson, A. (2013). *Research into the impact of Project Maths on student achievement, learning and motivation: Final report.* NFER.

Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematical excelling adolescents: What makes the difference? *ZDM – The International Journal on Mathematics Education,* 45(2), 183–197.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the Learning of Mathematics, 26,* 17-19.

Mann, E. (2006). Creativity: the essence of mathematics. *Journal for the educated of the gifted, 30*(2), 236-260.

Mellroth, E. (2018). *Harnessing teachers' perspectives: Recognizing mathematically highly able pupils and orchestrating teaching for them in a diverse ability classroom.* Karlstad University Press.

National Council for Curriculum and Assessment (NCCA). (2015). Leaving certificate mathematics syllabus: Foundation, ordinary and higher level. Stationary Office.

Nolte, M., & Pamperien, K. (2017). Challenging problems in a regular classroom setting and in a special foster programme. ZDM – *The International Journal on Mathematics Education, 49*(1), 121-136.

O' Connor, R., Ní Shúilleabháin, A. & Meehan, M. (2019). Investigating cognitive demand of higher-level leaving certificate mathematics examination tasks pre- and post-curricular reform. In L. Harbison and A. Twohill (Eds.), *Proceedings of the seventh conference on research in mathematics education in Ireland*, (pp. 203-210). MEI.

Organisation for Economic Co-operation and Development (OECD). (2016). *Pisa 2015: Results in focus*. Paris.

O' Sullivan, B. (2017). *An analysis of mathematical tasks used at second-level in Ireland.* PhD thesis. DCU, Ireland.

Pehkonen, E. (1997). The state-of-art in mathematical creativity. ZDM – *The International Journal on Mathematics Education*, *29*(3), 63-67.

Sheffield, L. (2003). *Extending the challenge in mathematics. Developing mathematical promise in K-8 students.* Corwin Press.

Sheffield, L.J. (2017). Dangerous myths about "gifted" mathematics students. ZDM – *The International Journal on Mathematics Education, 49*(1), 13-23.

Sriraman, B. (2005). Are mathematical giftedness and mathematical creativity synonyms? A theoretical analysis of constructs. *Journal of Secondary Gifted Education*, *17*(1), 20-36.

Vygotsky, L.S. (1978). *Mind in society. The development of higher psychological processes.* Cambridge.

Evaluating the Impact of Mathematics Support Using Moderation

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Mathematics and Statistics Support has existed formally within Irish higher education for twenty years. Evaluations of the effectiveness of engaging with such student support suggest improvements in students' grades, confidence, retention, progression, completion and employability, among other factors. Distinguishing student success due to mathematics support engagement from students' other practices and use of academic resources such as lectures, tutorials, peer support and online materials is difficult. In this paper we present findings from a quantitative and longitudinal analysis of visitors and non-visitors of the UCD mathematics support centre over six years. We employed a technique from social psychology research literature known as moderation to address two research questions relating to the university mathematics module grades of students who use, and do not use the institution's mathematics support centre. Moderation analysis revealed that visiting the centre more often has a significant impact on the relationship between Leaving Certificate mathematics grades and university mathematics grades. Findings indicated that using mathematics support bridges the gap between lower and higher achieving Leaving Certificate mathematics students in terms of their university mathematics results.

Introduction

Mathematics and statistics support (MSS) is an optional, non-timetabled service often in the form of a dedicated physical space where students can drop in or pre-book an appointment to gain assistance with their mathematical or statistical learning. MSS was first established in Ireland at the University of Limerick in 2001 with a similar initiative established at University College Dublin (UCD) from 2003. The latest survey of MSS provision on the island of Ireland (Cronin et al., 2016) revealed that 25 of 30 (83%) higher and further education institutions surveyed offered MSS in some form with 16 such institutions providing a dedicated centre for their support. Various attempts, both qualitative and quantitative, to evaluate MSS have been conducted throughout Ireland and internationally over the past twenty years (Matthews et al., 2013). In this paper we report on a quantitative analysis, via moderation, of longitudinal UCD Mathematics Support Centre (MSC) usage data to answer the following two research questions: (1) Does visiting the MSC accentuate the positive relationship between students' prior school mathematics results and their university mathematics module results? (2) If so, for which students is it most beneficial and does the number of visits matter? The hypothesis was that visits to the MSC did accentuate the positive relationship, with more visits meaning greater accentuation of the relationship between students' second-level school mathematics results and their university mathematics results.

Literature Review

Previous evaluative studies of the impact of MSS on students' success include both quantitative and qualitative methodologies. Dzator and Dzator (2020), utilised student surveys including open-ended questions to evidence student satisfaction and retention due to the service. Rickard and Mills (2018), and Jacob and Ni Fhloinn (2019) conducted quantitative studies linking visits to the MSS centre with improved university results while controlling for prior academic achievement. Matthews et al. (2013), and Lawson et al. (2020) have synthesised evaluative studies on the impact of MSS in their respective literature reviews. These studies show the wide ranging positive impact of MSS on learners, staff and institutions. As MSS has become a more permanent and embedded student resource within higher education there has been an evolution in scholarship from justifying centres' existence via usage figures and positive student feedback to more sophisticated evaluative techniques such as regression analysis. However, such positive student engagement with MSS and correlations with student success measured via final grades for example, do not imply a causal relationship. As Lawson et al. (2020) state 'robust evaluation of the effectiveness of mathematics support alongside effective ways of engaging the disengaged remain the most important research areas in mathematics support.' (p.1248). In the national context, an all-Ireland survey of MSS provision (Cronin et al., 2016) asked MSS coordinators to list their most difficult challenges in providing MSS of which 'reaching the non-engaging students' and 'getting students to engage earlier [in their university life]' were the top two difficulties prioritised by 19 of 22 respondents. The issue of MSS student engagement has deteriorated further with the advent of wholly online MSS brought on by the COVID-19 pandemic. In UCD attendance figures have decreased by 59% from 4,283 to 1,762 student visits for the corresponding periods of April to December in 2019 and 2020 (Mullen et al., 2021a; Mullen et al., 2021b). This pattern of decreased MSS engagement due to COVID-19 is replicated internationally (Hodds, 2020). Thus when the return of on-campus MSS provision resumes it will be more important than ever to evidence the effectiveness of MSS on student success for a new generation of students.

Methodology

Data was gathered over six academic years involving ten semesters between Spring Semester of 2015 and Autumn Semester of 2019. This data came from three sources, namely: (a) MSC visit data recording the number of visits, time of visit and the module code for each student visitor over the study period; (b) Assessment results in letter grade form for all students enrolled in the 27 modules in this study; and (c) students' prior mathematics learning achievement as measured by the Irish Leaving Certificate (LC) mathematics results. We note that all three data sources emanate from official sources ((a) and (b) from UCD Registry and Assessment respectively and (c) from the Central Applications Office via UCD Student Records), and thus are not student self-report data.

To comply with General Data Protection Regulations (GDPR) and the university's Office of Research Ethics the data was aggregated in the form of 227 'bins'. A bin represents a group of (not necessarily distinct) students with four traits in common. These traits are: (1) mathematical module type, (2) the year group of student enrolment, (3) the university letter-grade module result achieved by the student, and finally (4) the number of MSC visits the student made for that module (including non-visitors).

The 27 modules in the study fell into six types of university mathematics module which were MATH1, MATH2, ACM1, ACM2, MST and STAT. MATH1 denotes a mathematics module taken in stage one of a student's undergraduate degree programme, ACM2 denotes an Applied and Computational Mathematics subject taken in stage two, STAT denotes statistics modules taken in either stage one or two and MST denotes another type of mathematical module again taken in stage one or two. The two year-groups category from which the students first completed the module were 2015-2016/17 (five semesters) and 2017/18-2019 (five semesters). The final letter-grade result these students received in their respective module(s) are A, B, C, D, or F, where F denotes a failing grade of less than 40%. The passing grades A-D are commensurate with how UCD defines these grades numerically[1]. Finally, the number of times the students visited the MSC for each module fall into four distinct categories, 0, 1, 2-4 or 5+ visits in the relevant time period.

The reasons for these category choices were to maximise the number of data observations subject to preserving student anonymity in compliance with GDPR and ethical guidelines. There were 12,163 unique students in the study but 25,768 bin entries. Thus, each bin had between 3 and 1,766 entries, with a bin entry representing one module taken by one student. Hence a student can be in a bin multiple times if the student was enrolled to more than one mathematics module of this study and received the same final grade and used the MSC the same amount of times for those modules. A student can also be in multiple bins if the student was enrolled to more than one mathematics module of this study and received a different final grade and/or used the MSC a different number of times for those modules.

The LC mathematics level (Higher or Ordinary) and grade for each entry was provided by UCD Student Records. These grades were converted to a 12-point ordinal scale shown in Table 1. An average of these converted results was taken to create the average LC result for each bin. For example, the average LC result for bin 1 was 10.03, a H3 grade. The final university mathematics module grade was also converted from 'A to F' to '5 to 1' where A=5, B=4, C=3, D=2 and F=1. The average LC result for each bin, the final university mathematics module result for each bin (fixed for each bin e.g. bins 1 to 4 all received an A), and the number of MSC visits category for each bin, were used to create 227 observations.

[1] A: 70-100%, B: 60-69.99%, C: 50-59.99%, D:40-49.99%, F:<40% (https://maths.ucd.ie/tl/grading/en02)

Table 1

Conversion of Leaving Certificate Grades to a 12-point Scale

Scale	Leaving Certificate grade and percentage
12	H1: Higher Level, 90-100%
11	H2: Higher Level, 80-89.99%
10	H3: Higher Level, 70-79.99%
9	H4: Higher Level, 60-69.99%
8	H5: Higher Level, 50-59.99%; O1: Ordinary Level, 90-100%
7	H6: Higher Level, 40-49.99%; O2: Ordinary Level, 80-89.99%
6	H7: Higher Level, 30-39.99%; O3: Ordinary Level, 70-79.99%
5	H8: Higher Level, 0-29.99%; O4: Ordinary Level, 60-69.99%
4	O5: Ordinary Level, 50-59.99%
3	O6: Ordinary Level, 40-49.99%
2	O7: Ordinary Level, 30-39.99%
1	O8: Ordinary Level, 0-29.99%

Note. Leaving Certificate grades and percentages sourced from https://www.theleavingcert.com/points-calculator/

Data Analysis

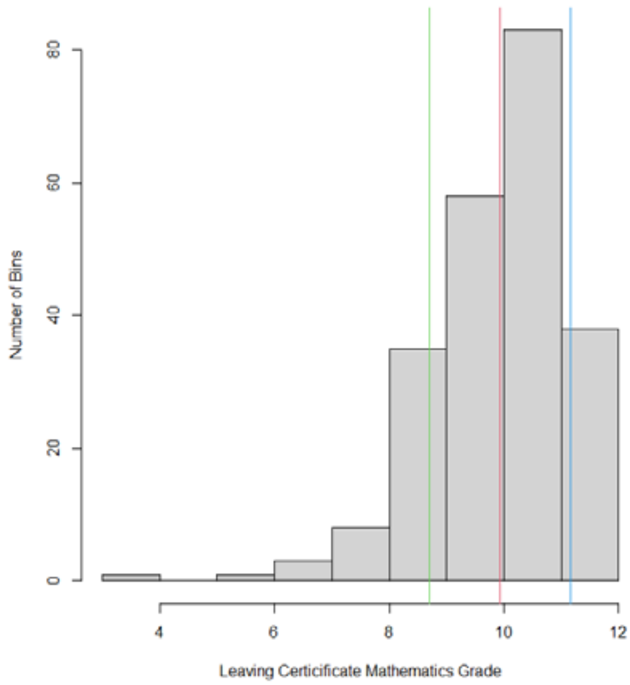
Data was analysed using moderation, a statistical method which studies the effect of a moderator variable (in this study the number of visits to the MSC) on the relationship between an independent or predictor variable (LC mathematics result) and a dependent variable (university mathematics module result). A moderator variable can change the direction and/or the strength of the relationship between an independent and dependent variable (Baron & Kenny, 1986). Moderation can be tested using hierarchical multiple regression, looking at the interaction effect between the moderator and predictor variables and whether this interaction is significant in predicting the dependent variable.

Results

Moderation analysis was used to answer the research questions: (1) Does visiting the MSC accentuate the positive relationship between students' prior school mathematics results and their university mathematics module results? (2) If so, for which students is it most beneficial and does the number of visits matter? The significance of the interaction effect between visiting the MSC and LC grades in predicting university mathematics grades was investigated. Figure 1 shows the spread of the average LC mathematics results of the bins with the mean and plus/minus one standard deviation highlighted. Note that the minimum UCD entry requirement mathematics grade is O6/H7 and many of the 27 modules included in the analysis require at least O2/H6 so the histogram is left-skewed. Table 2 presents the bivariate correlations for the three variables.

Figure 1

Average Leaving Certificate Mathematics Results Histogram



Note. The red, blue and green lines show the mean, and one standard deviation above and below the mean.

Table 2

Correlation between Leaving Certificate Mathematics results, Final University Mathematics results, and MSC visits

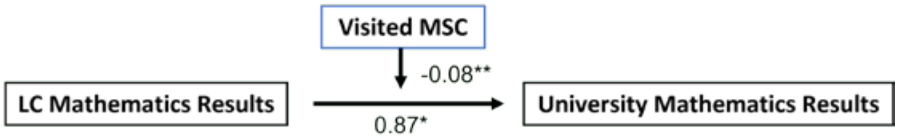
	LC Mathematics Result	Final University Result	MSC Visits
LC Mathematics Result	1		
Final University Result	0.48*	1	
MSC Visits	-0.30*	0.072	1

Note. * indicates p<0.01.

Using Hayes' (2017) PROCESS model 1 in SPSS, the moderating effect of visiting the MSC was investigated, as shown in Figure 2.

Figure 2

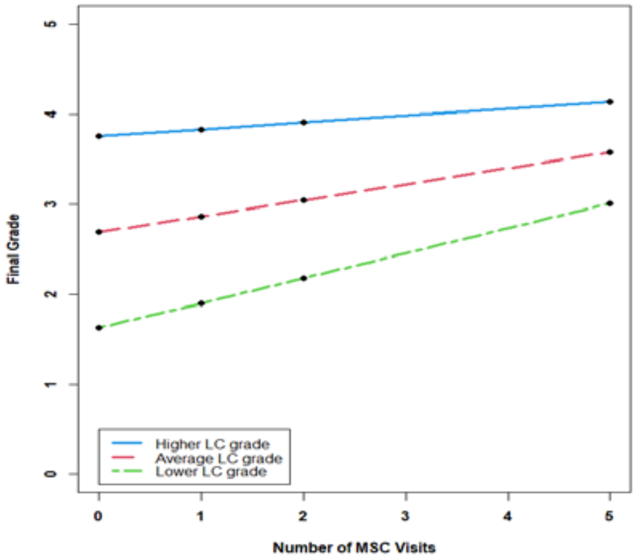
Visiting the MSC moderating the relationship between Leaving Certificate mathematics results and university mathematics module results.



Note. *unstandardised coefficient, s.e = 0.12, p = 0.00. **unstandardised coefficient, s.e. = 0.03, p = 0.01.

Figure 3

The number of visits to the MSC moderates the relationship from Leaving Certificate mathematics results to final university mathematics results.



Note. Regression of the university final mathematics result on the number of MSC Visits at specific values of LC mathematics grade is shown. Results compare for lower (1 standard deviation below the mean), average (mean), and higher (1 standard deviation above the mean).

Visiting the MSC does influence the strength of the relationship between LC results and university mathematics results as a significant interaction effect was found with an unstandardised coefficient of -0.08 (s.e. = 0.03, $p = 0.01$). Simple slopes analysis, (Preacher et al., 2006), shown in Figure 3, reveals that the more visits a student makes to the MSC, the higher their final university mathematics grade is; this effect is more pronounced for students with lower LC results. In other words, Figure 3 compares the final university mathematics results of lower (one standard deviation below the mean), average (mean), and higher (one standard deviation above the mean) LC mathematics students. The positive effect of visiting the MSC is strongest for the lower achieving students (the steep green line) compared to the higher achieving students (flatter blue line). Notably there is a positive difference after just one visit to the MSC, and with an increasing number of visits, for all three groups. In summary, a greater number of visits to the MSC is related to higher university mathematics results, particularly for the lower achieving students.

Discussion and Conclusion

Determining the impact of MSS engagement on student success in subsequent examination performance is a difficult task. Simple analyses can be prone to a post hoc fallacy, whereby improvements in students' performance can be ascribed to a single intervention – or to a range of them – and do not necessarily take into account the wide variety of other potential influences on students' academic performance. Simply put, correlation is mistaken for causation. Thus rigorous and careful analyses are required to ensure the efforts of such student academic support provision is not undermined. This is achieved in the present paper by undertaking a robust statistical analysis (moderation) of a very large cohort consisting of 12,163 students over a considerable time period of six years.

We have provided evidence that students from lower second-level school mathematical backgrounds experience a greater benefit from engaging with their institution's mathematics support centre than their higher-achieving peers. Students from higher school mathematics backgrounds experience a ceiling effect but still benefit from greater interaction with mathematics support. As hypothesised, students who had used mathematics support five times or more experienced the greatest accentuation in the relationship between their LC mathematics results and final university mathematics results but it is clear that even those who visited the MSC only once still benefited in comparison to non-users. This aligns with previous research indicating that just one visit to a MSS centre can benefit students (Jacob and Ni Fhloinn, 2019). These findings also build on existing Irish and international research demonstrating second-level mathematics performance as a predictor of third-level mathematics performance. The advancement made in the current study however distinguishes student success due to mathematics support engagement from students' other practices. While this may be unsurprising it is important to document nonetheless.

Short-term effect analyses and/or small sample size studies claiming positive effects of MSS on student mathematics performance are strengthened by such longitudinal studies as carried out here. Such studies bolster claims that MSS provision works for those students who avail of it, and can be used as evidence to encourage those who have yet to utilise its services. In addition, utilising a large data set involving dozens of university modules and thousands of students allows for generalisations that MSS works for academic modules of varying mathematical rigour (e.g. service versus specialist courses), and students of varying mathematical aptitudes, to be made. Thus this paper sets a baseline for examining trends among different student cohorts' engagement (or non-engagement) with their institution's MSS offering.

Student engagement with MSS, especially from those with lower mathematical attainment backgrounds, must continue to be encouraged so that all such students can gain these benefits. We intend to build on this work to examine whether university students from non-traditional entry routes (e.g. mature, international, HEAR[2], DARE[3] and QQI-FET[4]) benefit (or lose out) disproportionally from MSS engagement (non-engagement) than their peers who enter university from more traditional routes.

[2] HEAR stands for Higher Education Access Route (https://accesscollege.ie/hear/).

[3] DARE stands for Disability Access Route to Education (https://accesscollege.ie/dare/)

[4] QQI-FET stands for Quality and Qualifications Ireland Further Education and Training (https://www.qqi.ie/Articles/Pages/FET-Awards-Standards.aspx).

References

Baron, R. M., & Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of personality and social psychology*, *51*(6), 1173.

Cronin, A., Cole, J., Clancy, M., Breen, C., & Ó'Sé, D. (2016). *An audit of Mathematics Learning Support provision on the island of Ireland in 2015*. National Forum for the Enhancement of Teaching and Learning in Higher Education.

Hayes, A. F. (2017). *Introduction to Mediation, Moderation and Conditional Process Analysis, Second Edition: A Regression-Based Approach*. Guilford Publications.

Hodds, M. (2020). *A report into the changes in Mathematics and Statistics support practices due to Covid-19*, The Sigma Network, 2020. Jacob, M., & Ni Fhloinn, E. (2018). A quantitative, longitudinal analysis of the impact of mathematics support in an Irish university. *Teaching Mathematics and its Applications, 38*, 216–229.

Lawson, D., Grove, M., & Croft, T. (2020). The evolution of mathematics support: a literature review, *International Journal of Mathematical Education in Science and Technology*, *51*(8), 1224-1254.

Matthews, J., Croft, T., & Lawson, D. (2013). Evaluation of mathematics support centres: A review of the literature. *Teaching Mathematics and its Applications*, 32, 173–190.

Mullen, C., Pettigrew, J., Cronin, A., Rylands, L., & Shearman, D. (2021a). Mathematics is different: student and tutor perspectives from Ireland and Australia on online support during COVID-19. *Teaching Mathematics and its Applications.* [In Press]

Mullen, C., Pettigrew, J., Cronin, A., Rylands, L., & Shearman, D. (2021b). The rapid move to online mathematics support: changes in pedagogy and social interaction. *International Journal of Mathematical Education in Science and Technology.* https://doi.org/10.1080/0020739X.2021.1962555

Preacher, K. J., Curran, P. J., & Bauer, D. J. (2006). Computational tools for probing interaction effects in multiple linear regression, multilevel modeling, and latent curve analysis. *Journal of Educational and Behavioral Statistics*, *31*, 437-448.

Rickard, B, & Mills, M. (2018). The effect of attending tutoring on course grades in Calculus I, *International Journal of Mathematical Education in Science and* Technology, 49(3), 341-354.

Modelling Division: Towards a Local Instructional Theory for the Teaching of Multi-Digit Division

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This paper reports on a teaching experiment in which a hypothetical learning trajectory was enacted. The aim of this learning trajectory was to support children to develop efficient strategies for calculating multi-digit division computations in ways that made sense to them. This paper analyses the strategies that they used and how these changed over the course of the teaching experiment. Findings indicate that the teaching approach, which emphasised trialling multiple solution strategies and selecting and justifying computation methods, allowed children to develop efficient, meaningful solution strategies.

Introduction

This paper investigates a teaching experiment focused on long division. This took place in the first author's classroom as part of the Maths4All project, a SFI funded project which aims to develop resources for, and with, teachers. In this paper, we analyse the strategies that children developed and how these changed over the course of the experiment.

Literature Review

The draft specification of the Irish primary mathematics curriculum proposes mathematical proficiency as the central goal for mathematics teaching (National Council for Curriculum and Assessment [NCCA], 2017). Mathematical proficiency is conceived as consisting of the intertwined strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). Strategic competence is understood to mean the ability to formulate, represent and solve problems. Adaptive reasoning involves the capacity for logical thought, reflection, explanation, and justification. This vision of desirable learner outcomes has implications for teaching. In particular, five meta-practices are advocated in the research reports underpinning the draft specification (Dooley et al., 2014). While all meta-practices are relevant to the teaching of division, we highlight the proposed emphasis on mathematical modelling. Distinct from traditional understandings of teacher modelling, where a teacher might use concrete materials or other resources to model mathematical ideas, in mathematical modeling the focus is on supporting children's own modelling of problems- their use of mathematics to describe a context and develop meaningful solutions (c.f., Suh & Seshaiyer, 2017). As children develop their own models of situations, there will be conceptual and procedural components (Lesh & Harel, 2003). On a conceptual level, a model describes how elements of a system relate to each other but it may also have accompanying procedures for accomplishing goals.

There are strong implications for the teaching of mathematical operations if mathematical proficency is accepted as the ultimate goal of teaching. Traditionally, procedural fluency is a priority and many teachers understand mathematics learning as concerned primarily with memorising number facts and facility with conventional digit-based algorithms (Schulz, 2018). A contrasting approach is followed in the Netherlands, where informed by Realistic Mathematics Education, there is an early focus on supporting children's informal mental calculations. Over time, focused efforts are made to guide development from informal methods to formal algorithms using progressive schematization of informal strategies (van Putten et al., 2005). Informal methods may involve partitioning of number but generally maintain place value. This contrasts with the digit-based strategies, used in the conventional division algorithm, which operate on individual digits in a procedural way. Algorithms can minimise the demands on working memory and on reasoning processes but once introduced, may inhibit the use of number-based, informal calculation strategies (Schulz, 2018) and thereby inhibit children reasoning adaptively about the task. Amongst the relatively few studies of multi-digit division, Schulz (2018) has presented a theoretical and empirical analysis outlining the ways in which division strategies rely on two types of reasoning abilities: reasoning about relations between numbers, and reasoning about relations between operations. Repeated addition and subtraction are generally the first intuitive strategies for division. More developed strategies recognise and use the multiplicative relationships between the dividend and the divisor. Advanced strategies decompose or adapt the dividend and/or divisor to create easier calculations from which the final solution can be derived. It is also possible to categorise division strategies according to the ways in which students create multiples of the divisor (chunking) to be subtracted from the dividend (van Putten et al., 2005). *Low-level chunking* refers to using doubling or small multiples while *high-level chunking* refers to subtracting higher multiples or chunks, such as ten times the divisor.

Methodology

Hypothetical learning trajectories (HLT) are understood to involve teachers designing sequences of instructional activities that they imagine will support children in moving from their current levels of thinking to the desired goals (Simon, 1995). These trajectories are considered to be hypothetical because, until tasks are enacted, the teacher can only imagine how children might engage. The teacher-researcher, first author of this paper, developed and iteratively refined the instructional sequence described in this paper over four years of teaching at this class level, though previous iterations were not formally researched. Gravemeijer (2004) recommends the use of HLT to describe the planning of instructional activities in a classroom on a day-to-day basis. He notes that in developing HLTs teachers may draw on local instructional theory. For Gravemeijer, local instructional theories include a clear description of (i) learning goals, (ii) planned instructional activities and (iii) an empirically grounded theory of how the instructional activities might develop students' thinking. This paper presents the first empirical analysis of data related to these activities and thus is moving toward meeting part (iii) of Gravemijer's conditions.

All teaching is underpinned by understandings of the overarching purpose. For both authors, this involves a commitment to developing children's agency, authority and identity through a pedagogical approach which involves enactment of the five meta-practices (Dooley et al., 2014). In practice, this involved lessons where a small number of problems were explored in great depth, with children encouraged to discuss, analyse and trial methods proposed by others as well as give justifications for their choice of strategy, i.e., numerous opportunities were created for children to engage in adaptive reasoning. The ultimate goal of the hypothetical learning trajectory was that children develop efficient strategies for calculating multi-digit division computations that make sense to them. As outlined in the overview on Table 1, the first two lessons were exploratory in nature, with no strategies presented by the teacher. The long division algorithm was introduced for the first time during Lesson 3 but it was presented as an alternative method rather than a superior method. In the final lessons, children solved division problems in a variety of ways and justified the reasonableness of their chosen approaches. Across these lessons, digital records of board work were collected as well as children's written solution strategies. At the end of each lesson, children were invited to provide a short, written reflection in response to a question posed by the teacher. These questions generally prompted children to reflect on strategies used or to select and justify their preference of strategy. All relevant ethical procedures were followed in the collection of this data and in total eighteen children participated in this study.

Table 1

Unit Overview

Lesson	*Extension*
Lesson 1 - 'Punnets of Strawberries' A punnet holds 23 strawberries. How many punnets can be filled from a basket holding 115 strawberries?	228 ÷ 38; 176 ÷ 36; 279 ÷ 54; 375 ÷ 17
Lesson 2 - 'School Library' A school library has 719 books. How many shelves, each holding 24 books, will be needed to display the books?	416 ÷ 15; 786 ÷ 19; 805 ÷ 22; 751 ÷ 45
Lesson 3 - 'The Car Transporter' A car transporter delivered 216 cars over 18 trips. How many cars were carried on each trip?	Explore 'Mandeep's Method' – The Long Division Algorithm.
Lesson 4 - 'Long Division' Solve 389 ÷ 17 using 'long division' and/or in other ways.	638 ÷ 25; 736 ÷ 34; 716 ÷ 18; 417 ÷ 16
Lesson 5 - Going Around in Circles (nrich) A railway line has 27 stations on a circular loop. If I fall asleep and travel through 312 stations, where will I end up in relation to where I started? Which station will I end up at?	If it is midday now, will it be light or dark in 539 hours? What time will it be?

We are interested in understanding how children's models of division, their conceptual and procedural understandings of division situations (c.f., Lesh & Harel, 2003), developed across the teaching sequence. We are guided by the research questions: (i) what strategies did children use as they engaged in this sequence of activities? (ii) how did these change over the course of the teaching experiment? Our focus on 'strategies' gives explicit attention to the procedural aspects of children's solutions. Some inferences about conceptual understandings are also possible. Given the constraints of this paper, only a subset of the data was analysed. Seven students were selected from the larger group and their reflections and recordings of their strategies across all lessons were analysed. Descriptors for the division strategy used in each problem were assigned. These strategies, described in more detail below, align with those identified in the literature (c.f., Schulz, 2018; van Putten et al., 2005).

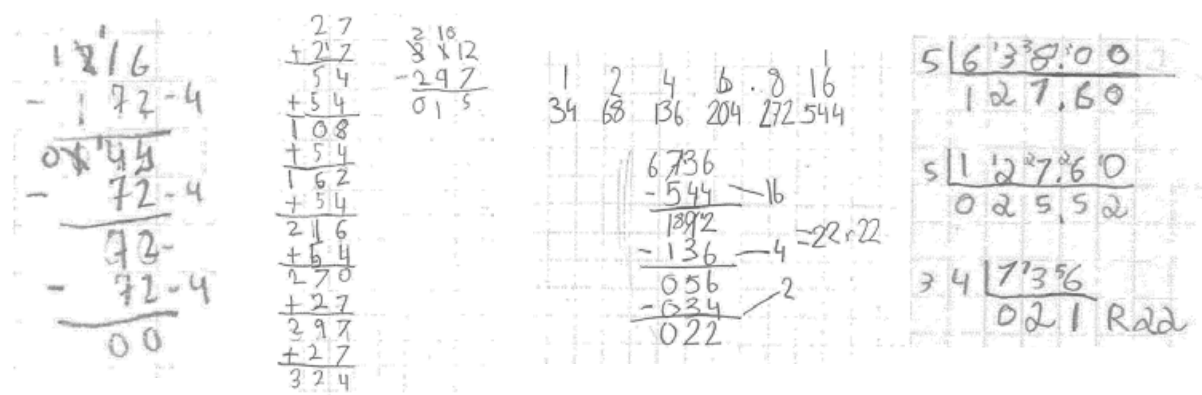
Findings

These findings offer insight into the division strategies that were developed by seven, fifth class participants as they engaged with the tasks of the HLT. The approaches emerged as the children engaged in lessons which aimed to foster adaptive reasoning (Kilpatrick et al., 2001), where the teacher used the meta-practices advocated in Dooley et al. (2014). Analysis of the strategies that different children employed, at different points of the HLT, offers some insight into their trajectories of learning and the reasoning that they were engaged in. These findings share the variety of division strategies that emerged from the participants and provide some insight to their learning trajectories. Findings are of relevance given the pedagogical alignment with the advocated emphasis on children's mathematical modeling in the draft curriculum specification. The children's strategies are discussed under six categories below.

Figure 1

Division Strategies Developed by Fifth Class Children

(i) (ii) (iii) (iv)



Repeated Addition and Repeated Subtraction

Strategies involving repeated addition and repeated subtraction were used repeatedly by the participants in this study. During Lesson One 'the strawberry problem' was presented for the children to solve collaboratively, using an approach that they could justify. All participants used repeated addition or subtraction, as one of their approaches to solving this problem. Strategies, such as this, are indicative of reasoning about relations between operations (Schulz, 2018). In the case of 'the strawberry problem' the children applied their understanding of subtraction to the unfamiliar division context of *115 ÷ 23.* Formatting the task as a word problem offered flexibility for the development of various approaches and the numbers were chosen to accommodate repeated addition and subtraction. The frequent use of repeated addition and subtraction during initial lessons correlates with Schulz's (2018) suggestion that these are often the first intuitive strategies for division. The numbers chosen, as part of the HLT, were organised with the quotient increasing as the children progressed through the tasks. As the quotient increased, repeated addition and subtraction strategies became cumbersome and all participants adapted their initial approaches to solve the tasks more efficiently, through low-level chunking. Some of the participants developed this strategy independently while others adopted it following the sharing of strategies. Repeated addition, repeated subtraction and low-level chunking remained the preferred solution strategies for a number of the participants throughout this study.

Low-Level Chunking

Figure 1 (i) and Figure 1 (ii) illustrate how two child adapted their initial strategies to improve efficiency. All of the participants adapted their repeated addition and subtraction strategies but some were more methodical in their approach than others. Some children appeared to make decisions before beginning their calculation, such as deciding to quadruple the divisor, as shown in Figure 1 (i). The strategies adopted by others appeared to emerge as they worked on a problem. Figure 1 (ii) depicts a child's low-level chunking, taken from Lesson Five. This child's strategy appeared to emerge as they worked on the task, using a combination of adding 27, and adding double 27, to reach 312. We can see that when the child surpassed 312, they returned to the previous step of the calculation and used that sum, 297, to calculate the remainder for their solution. The fact that the strategy emerges as the child enacts it, indicates engagement in reasoning and decision making throughout the process.

In developing low-level chunking, which involves using basic multiples, e.g., doubles, of the divisor in conjunction with repeated addition or subtraction, the participants demonstrated both reasoning about the relations between operations and reasoning about the relations between numbers (Schulz, 2018). The children made connections between the division problem context and the operations of addition and subtraction. They also engaged in reasoning about number through their comparison of the dividend and the divisor and their endeavour to manipulate the divisor to expediate their calculations.

High-Level Chunking

A number of participants adapted and extended low-level chunking to invent a new, more efficient strategy, which we have coded as high-level chunking. High-level chunking involves a deeper engagement with the multiples of the divisor and the comparison of these multiples to the dividend. Figure 1 (iii) depicts the use of this strategy to solve, *736 ÷ 34*. While there is an error in this child's recording, it appears that they were involved in considerable reasoning about the relationship between the divisor and the dividend. The participant created a list of multiples, of 34, and appeared to take cognisance of the dividend, scaling up their multiples until they found one that was sufficiently close to the dividend to make their strategy efficient. Some children used a calculator to help them establish the multiples of the divisor as part of this strategy.

The work in Figure 1 (iii) was completed during Lesson Four, after the children were introduced to the formal, long division algorithm. At this point, some students were using the formal algorithm and some were using their preferred invented strategies. The child's work in Figure 1 (iii) demonstrates deeper reasoning about the relations between numbers and operations (Schulz, 2018), compared to that demonstrated with low-level chunking. The adoption of a high-level chunking approach is indicative of reasoning associated with number sense, multiplication, subtraction and division and of the connections that can be utilised to solve problems in novel contexts. While all the participants in the study utilised low-level chunking, only some adopted a high-level chunking approach. In respecting the different trajectories of learning of the children, a broad variety of student invented strategies were shared, discussed and praised during whole-class discussions at all stages of the unit of work.

Missing Factor

The Missing Factor approach emerged during Lesson One and many of the participating children utilised it repeatedly in subsequent lessons. The strategy involved estimation and trial-and-improvement as a child aimed to determine the missing factor in multiplication sentences to solve division problems e.g., determine the missing factor in *24 x __ = 720* to identify the solution to *720 ÷ 24.* This strategy suggests reasoning about relations between operations (Schulz, 2018), as the children utilise the inverse relationship between multiplication and division. Some participants appeared to experience greater success with this strategy than others. Those who engaged in deeper reasoning about the relations between numbers (Schulz, 2018) and who were able to make accurate estimates could use this strategy efficiently whereas those who found estimation more difficult engaged in a longer series of trial-and-improvement cycles and tended to prefer other strategies. Schulz (2018) identifies strategies that use the multiplicative relationships between the dividend and the divisor, such as this, as being more advanced than chunking strategies.

Decompose the Divisor and Divide Stepwise

Figure 1 (iv) shows two different solution strategies that one participant employed during Lesson Four. The first involved decomposing the divisor and dividing stepwise, to solve *638 ÷ 25*. This child was the first participant to propose the decomposition of the divisor. They utilised many different approaches throughout the unit but they employed this strategy during each of the five lessons. When they first developed the approach they recorded, “I came up with my own way. An example of my way is, instead of dividing 657 by 9 I divide 657 by 3 and that equals 219. Then I divide 219 by 3” (Child C, Reflections). The child appeared to take ownership of this solution strategy, using it repeatedly and sharing it with their classmates during whole-class discussions. Schulz (2018) views strategies that involve the decomposition of the divisor, in this way, as advanced strategies of division.

This participant encountered a difficulty, in employing their strategy, when the dividend was not divisible by the decomposed divisor, as occurred in the case of Figure 1 (iv). However, they appeared eager to learn about the relationship between remainders and decimals and continued to choose this division strategy. During the unit of work there were many opportunities for the children to share strategies with their classmates and to use each other's ideas. After this child shared their approach with the class a number of students adopted it, perhaps because of its contrast to the strategies discussed thus far. This strategy demonstrates reasoning about relations between operations (Schulz, 2018) as the child manipulated the divisor to apply the short division algorithm that was familiar to them. It also demonstrates reasoning about the relations between numbers (Schulz, 2018) as they explored divisibility and the relationship between remainders and decimals.

Standard Algorithms

During Lesson Three the standard long division algorithm was introduced to the children. Initially, they solved a problem using invented methods and subsequently the new strategy, which was dubbed 'Mandeep's Method,' was presented. The children were encouraged to make sense of 'Mandeep's Method,' to compare it to their own invented strategies and to use it themselves. In previous lessons they were asked to use one another's methods and, in a similar way, they were now being asked to try a new method. Memorisation of the procedure was not encouraged and there was an understanding that the children could choose to use their preferred, invented methods or the long division algorithm.

All participants utilised the long division algorithm during the final two lessons. Some moved towards using it exclusively, some used it in conjunction with an invented method as a way of self-correcting and some tried it but then reverted to invented strategies. One particular participant used the long division algorithm in conjunction with an invented strategy during Lesson Four but then move towards using the algorithm exclusively during the final lesson. In their reflection at the end of Lesson Four they noted, "I would use the long way/new way. I would use it because, to me, it's easier and quicker" (Child E, Reflections). This reflection appears to support the idea that the use of an algorithm can minimise demands on working memory, making finding a solution easier and quicker (Schulz, 2018). Cognisance was taken of the multiple trajectories of learning present in the classroom and emphasis was placed on progressing each child's understanding and on developing their conceptual understandings of the division of multi-digit numbers.

In another case during Lesson Four, a participant engaged in reasoning about the relations between operations by making connections between the long division and the short division algorithms, adapting the latter to accommodate multi-digit division. The second solution strategy, depicted in Figure 1 (iv) and solving $736 \div 34$, depicts how a participant adapted the short division algorithm, that they would have encountered during the previous school year, to take account of the multi-digit divisor. The child developed this solution strategy after being introduced to the long division algorithm. The solution strategy follows the same general procedure as the long division algorithm but the child demonstrated strong mental arithmetic skills as many steps were completed mentally rather than symbolically.

Conclusion

The findings discussed in this paper highlight one aspect of the collected data, with this paper presenting the first empirical analysis thereof. This analysis offers insight into the reasoning of the participants in this study, as they engaged with the division problems, and highlights the varying strategies that they developed for solving said problems, many of which stretched beyond the anticipated student responses predicted in the planning of the HLT. The findings highlight the participants' capacity to develop efficient and meaningful solution strategies and to engage meaningfully with the formal algorithm. The previous discussion focuses on the children's solutions and samples of their work but this is just one facet of the situation. The importance of the pedagogical approach, in developing adaptive reasoning, cannot be understated and merits further exploration. This paper aimed to offer insight into the broad variety of strategies that the participants utilised, but it is important to note that each child followed an individual trajectory of learning. The individual trajectories that the participants followed also merit further investigation and study.

References

Dooley, T., Dunphy, E., Shiel, G., Butler, D., Corcoran, D., Farrell, T., NicMhuirí, S., O'Connor, M., Travers, J., & Perry, B. (2014). *Mathematics in early childhood and primary education (3-8 years). Teaching and learning. National Council for Curriculum and Assessment (NCCA). Research report no. 18*. NCCA

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical thinking and learning*, *6*(2), 105-128.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical thinking and learning*, *5*(2-3), 157-189.

NCCA (2017). *Primary mathematics curriculum: Draft specification junior infants to second class*. NCCA.

Schulz, A. (2018). Relational reasoning about numbers and operations–Foundation for calculation strategy use in multi-digit multiplication and division. *Mathematical Thinking and Learning*, *20*(2), 108-141.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for research in mathematics education*, *26*(2), 114-145.

Suh, J. M., & Seshaiyer, P. (2017). *Modeling mathematical ideas: Developing strategic competence in elementary and middle school*. Rowman & Littlefield Publishers.

van Putten, C. M., van den Brom-Snijders, P. A., & Beishuizen, M. (2005). Progressive mathematization of long division strategies in Dutch primary schools. *Journal for Research in Mathematics Education*, *36*(1), 44-73.

Making Squares: Children's Responses to a Tangram Task

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This paper reports on children's responses to a tangram task. The task was designed, based on the draft specification of the primary mathematics curriculum, to facilitate children's exploration of shape properties and their engagement in mental and physical transformations of tangram pieces. The task was enacted with three cohorts of children from first and second class. Analysis shows some children's concept images of squares to be limited and their thinking dominated by prototypical geometric images. This research is pertinent in the context of the proposed changes to the Shape and Space strand of the primary curriculum.

Introduction

The lesson at the centre of this research was developed in collaboration with the Maths4All project team. Maths4All, funded by Science Foundation Ireland, develops resources for and with teachers to support high quality mathematics teaching. A lesson focusing on tangram activities was designed with reference to the draft specification of the primary mathematics curriculum (National Council for Curriculum and Assessment [NCCA], 2017). This lesson was trialled in three primary classes. This paper presents an analysis of children's responses to one of the lesson tasks. The expectations in the draft specification are quite different to existing curriculum objectives (Government of Ireland, 1999). For this reason, our analysis gives insight into the possibilities and challenges of working toward new curriculum expectations. The literature review below first presents an overview of research on children's thinking in Shape and Space then explores the Irish context.

Children's Geometrical Reasoning

The van Hiele framework, Table 1, describes progressive levels of geometric thinking, with initial levels dominated by visual imagery (Fuys et al., 1984). Increasingly sophisticated levels of description, analysis, abstraction and proof are understood to develop in response to appropriate opportunities for learning (Clements & Battista, 1992). The framework is recognised as having the potential to inform decisions around the appropriateness of tasks. This is important as primary students often experience teaching that emphasizes only the identification and naming of shapes with little offered that would develop their reasoning at higher levels (Sinclair & Bruce, 2015). van Hiele theory contends that the teacher has a crucial role in the development of children's geometric reasoning (Fuys et al., 1984). It is recommended that teaching must attend to supporting the development of rich and varied concept images of geometric shapes (Sinclair et al., 2016). *Concept image* is understood to mean the cognitive structure associated with the concept. This includes all mental images and associated properties and processes (Tall & Vinner, 1981). Children's exploration of non-prototypical examples (and non-examples) in different positions or orientations is recommended as a way to develop rich concept images (Nic Mhuirí, 2020).

Table 1

The earliest levels of the van Hiele model of geometric thinking

0. Pre-recognition	Children may attend to only a subset of a shape's visual characteristics and may be unable to identify many common shapes.
1. Visual	Children recognize shapes solely by their appearance, often by comparison with a known prototype. Limited/no awareness of shape properties.
2. Descriptive/ Analytic	Children characterise shapes by their properties but do not perceive relationships between properties. The child may be unable to identify which properties are necessary and/or sufficient to describe the object.
3. Abstract/ Relational	Children can perceive relationships between properties and between figures. They can form meaningful definitions, classify shapes and give informal justifications for their classifications.

Note. This overview draws on Clements and Battista (1992) where level 0 was added due to a perceived lack in the original model. The levels shown are those considered to be most pertinent to primary education. Reprinted from Nic Mhuirí (2020).

Composing and decomposing shapes is a key element in geometric reasoning (Clements et al., 2004). This type of reasoning can be connected to transformations and visuospatial reasoning. While different terminology and definitions are offered, at heart visuospatial reasoning is concerned with visualising objects and manipulating them mentally, for example, visualising a shape being rotated through a turn (Sinclair et al., 2016). Such reasoning is understood to be central to mathematical and other forms of thinking. The growth in attention to visuospatial reasoning in recent years is accompanied by a growing recognition that age-appropriate activities that involve explicit attention to transformations should be part of children's early learning experiences (Sinclair & Bruce, 2015).

The Irish Context

International assessments such as the Trends in International Mathematics and Science Study (TIMSS) facilitate comparison of Irish children's achievement relative to other populations. The TIMSS assessment takes place every four years and is administered at fourth class and second year level in Ireland. Measurement and Geometry form one domain of the TIMSS assessment at fourth class. Assessment tasks include solving problems involving length, mass, volume, time, perimeters of polygons, area of triangles and partial squares, lines and angles, and two- and three-dimensional shapes. For the second year TIMSS assessment, Geometry is a domain in its own right. The most recent data available is from TIMSS 2019. Though Ireland was one of the highest performing countries at both class levels, Irish students showed a relative weakness in fourth class on the Measurement and Geometry domain, and in second year Irish students showed a relative weakness in the Geometry domain (Perkins & Clerkin, 2020). Similar findings are reported in the Programme for International Student Assessment study (Perkins & Shiel, 2016). Thus, despite high achievement in most areas of mathematics, the findings of international assessments highlight the need for careful consideration of the teaching and learning of Shape and Space in this country.

Currently, the primary curriculum is undergoing significant reform. As mentioned above, a draft specification of the new primary mathematics curriculum (NCCA, 2017) from Junior Infants to Second Class, has been published. This draft is organised around a set of broad learning outcomes and the *strand units* of the 1999 curriculum (Government of Ireland, 1999) are reimagined as *learning outcome labels*. For the Shape and Space strand, it also suggests significant changes in terms of content. While teachers will recognise the learning outcome labels of *spatial awareness and location* and *shape* from their previous experience, it is likely that *transformation* will be more problematic. Learning outcomes for this label include, “Explore and describe the effects of shape movements” (stage 1) and “Visualise and show the effects of transformations on shapes” (stage 2) (NCCA, 2017, p. 35). The sample learning experiences described in the progression continua (p.66-67) give further insight into how it is envisaged that these outcomes might be achieved. These focus on physical and mental manipulation of shapes (visualisation) as a site for developing language to describe simple transformations, e.g., flip, turn, slide.

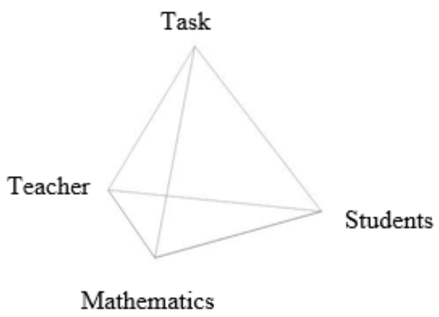
Methodology

This paper relates to a tangram lesson which was designed using the draft specification of the primary mathematics curriculum (NCCA, 2017). The goals of this lesson included that children would recognise the same shape in different orientations, that they would combine tangram pieces to form a variety of shapes and that they would name, compare and describe the properties of different tangram pieces. It was envisaged that children would also identify and discuss shape transformations in these activities, for example, physically or mentally rotate or flip tangram pieces and describe their actions or thinking. This lesson was taught on three occasions by the authors of this paper, twice at first class level and once at second class. Neither author was class teacher in any of these cases and we had limited insight into children’s previous experiences. Each lesson was recorded with three video-cameras. Two cameras were fixed, while a videographer operated the third camera. Videos were reviewed after each lesson. No changes were made to the focus task but reflection on, and refinement of, planned teacher questioning did occur. The research question which guides this paper is: What is the nature of children’s geometric thinking elicited by the focus task? Pseudonyms are used to report our findings.

We use the didactical tetrahedron (Rezat & Sträßer, 2012) as a theoretical framework. This framework, Figure 1, conceives of artefacts - alongside teachers, students, and mathematics - as fundamental constituents of any teaching situation. Drawing on Vygotsky’s notion of a psychological tool as one which impacts the mind, Rezat and Sträßer (2012) contend that all tools used in mathematics teaching can be considered as psychological tools or artefacts. Each face of the tetrahedron represents different perspectives on the teaching-learning situation. Various artefacts might be considered in relation to the research lessons, but for the purposes of this paper we focus on tasks as artefact. Notwithstanding the crucial role of the teacher in orchestrating learning opportunities, our focus in this paper is primarily on *task-mathematics-student* face as we attempt to investigate students' geometrical thinking elicited by the task. Given the constraints of the paper, we focus only on the fourth and final task in this lesson, Making Squares (details on Table 2 below).

Figure 1

Didactical Tetrahedron (Rezat & Sträßer, 2012) with Task as Artefact



First, the mathematical ideas that underpin the task were identified. While many of these ideas were discussed at the planning stage, planning for teaching tends to be focused on articulation of learning goals for children. For the purposes of analysis, we aimed to explicate the underpinning mathematics clearly. Secondly, we considered both the task as written in the planning documents and the task as implemented by the teachers of each lesson (c.f., Stein et al., 1996). Finally, each video was reviewed and relevant segments showing children's responses to the task were identified. These included occasions where children's responses were evident from visual appraisal of the video data alone, for example, evidence of a number of composed squares visible in front of an individual student. These also included occasions where video data captured extended conversations between the teacher and various children. All examples of student responses were listed and common responses to the task, including errors, were identified. Below we present an analysis of this data with reference to what we deem the most relevant or interesting examples of children's thinking.

Findings

Table 2 outlines details of the task and the underpinning mathematics. The task, sourced from *nrich.maths.org*, was selected as it has potential to develop the chosen learning outcomes. In enacting this task, we decided to make multiple tangram sets available to students. We did not want students to have to deconstruct their squares to make new ones and we intended that children would review the squares they had constructed and identify which ones were the same and different in terms of their component parts and/or the transformations needed to align orientations. The mathematical ideas underpinning this task are also listed on Table 2. It should be understood that it was not expected that students would understand all of these mathematical ideas, or indeed that all of them would become explicit through engagement with the task. That said, these details are vital as they form the background against which children's thinking is considered. Angle concepts run through all of the identified mathematical ideas and the combination of geometric and measurement reasoning involved in, c*omposing and decomposing shapes and angles* highlights the complexity and interwoven nature of these ideas. While formal measurement was not employed in this lesson, direct and visual comparison were used by students to check, for example, that the angle of a constructed square was the same as that of the single square piece. In addition, children also made judgements about whether the length of sides 'matched' or not (c.f., Clements et al., 2004). We note the gap between the mathematics described here and the expectations of the 1999 curriculum (Government of Ireland, 1999) where *transformation* does not feature at all and where the *Angles* strand unit is not introduced until second class.

Table 2

Overview of task and underpinning mathematics

Task presented on whiteboard	Orchestration of of task	Underpinning Mathematics
 *Tangram pieces are made from a square cut into seven pieces.* *Can you make other squares using some, not all, of the pieces?* *Can you make five different squares?*	The task was read to the children. Children were provided with multiple different sets of tangrams to experiment with. Teacher questions encouraged children to check their solutions and to try to make 'different' squares, for example, "I see you have lots of two-pieces squares, do you think you can make a three or four-piece square?" (Lesson 1, first class)	*Properties of the square* A square is a 2D-shape with four equal sides and four right angles. Opposite sides are parallel (and equal). *Composing and Decomposing* A square/angle/length can be composed of, or decomposed into, a number of smaller subunits. *Transformations* Shapes can be physically or mentally moved around in space by reflecting, translating and rotating.

Across all three lessons, many children's initial responses to the task involved the creation of two-piece squares using right-angled triangles of the same size, see Figure 2 (i). This sometimes evolved into larger squares made of multiple copies of a two-piece square as subunits. It appeared that, initially at least, more children were successful in combining repeated iterations of the same shape to form squares rather than combining different shapes, see figure 2 (ii). This was obvious in the relatively large number of two- and four-piece squares (made of repeated squares or triangles) compared to three- and five-piece squares. A small number of children made three- and five-piece squares relatively quickly, but the vast majority of the children continued to experiment with two- and four-piece squares until the teacher intervened to encourage experimentation with other variations. A number of children were also observed to make seven-piece squares relatively quickly after the activity was initially introduced. It seems likely that these creations were guided by the image of the complete seven-piece tangram that was shown on the interactive whiteboard.

While most children appeared to complete the initial compositions of two-piece squares with ease, one child, Síle, from first class engaged in an extended discussion with the teacher where she articulated some uncertainty. While Síle had aligned two right-angled triangles of the same size accurately to make a square, she was unconvinced that the resulting composition was actually a square. When questioned about why she did not think it was a square, the child appeared to struggle to articulate her thinking. When pressed, she stated, "Because it's not-" and pointed at the middle of the composed shape, where the edges of the two triangles met. She nodded when the teacher asked, "It's not one piece?" The same child had previously, with no observed difficulties, engaged in an activity where tangram pieces were combined to make animal shapes. It appears here that the particular concept image she had for 'square' did not include squares composed of subunits. Across all three lessons, children had repeatedly identified squares shown standing on a point, rather than sitting on a horizontal base as 'diamonds' and some children appeared to understand diamonds as quite distinct from squares. This is another example of children's limited concept images of squares and most likely arises as a result of exposure to largely prototypical representations.

A number of other different attempts to construct squares were made. For example, a number of children made quadrilaterals that were not squares. Some of these were approximations of squares, where despite small errors, attempts to construct equal sides, right angles and opposite sides parallel were obvious, for example, see Figure 2 (iii). In other cases, children constructed non-square rectangles or parallelograms. Seán, who had successfully constructed a square from four smaller square subunits, then went on to attempt to construct a square from four parallelograms, as shown in Figure 2 (ii). He claimed that this was a square and when asked why he thought it was a square, he said that, "it's got four sides but it's a bit slanted". He appeared to recognise the visual difference between the shape that he had created and 'other' squares but did not verbally identify any other properties of a square. In the lesson with second class, the teacher attempted to probe children's understanding about the properties of different quadrilaterals and the following conversation occurred.

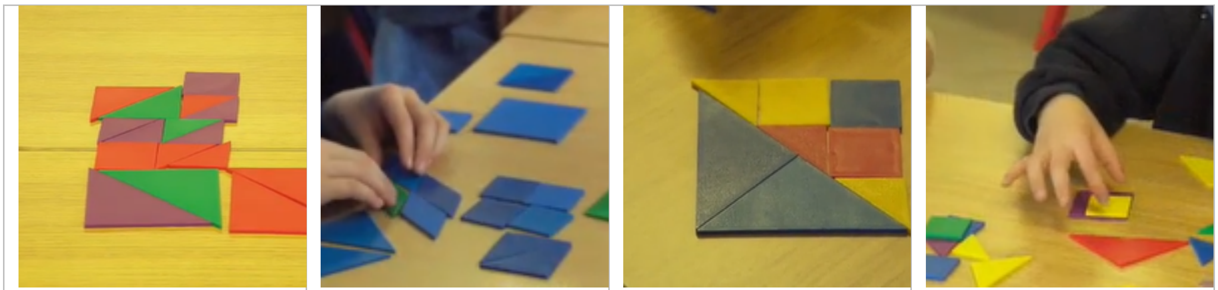
Teacher: What's the difference between squares and other types of rectangles? What's so special about squares?

Ciara : They're smaller. So like if you cut a rectangle in half, so like, you can make a square. You can make a rectangle by putting two squares.

Ciara demonstrated impressive levels of visuospatial reasoning particularly given that no singular (non-square) rectangular pieces were available to children at this time. Her response does seem to suggest though that she is drawing on the prototypical image of a 2 x 1 rectangle with width double its height. Other examples of strong visuospatial reasoning were evident in children who recognised, and described in informal language, the relationships between shapes in various different orientations and the movements necessary to change the appearance of shapes in different ways.

Figure 2

Samples of children's work



(i) Multiple iterations of 2-piece squares.	(ii) Iterations of single shapes. Child constructing non-square quadrilateral	(iii) Approximation of Square	(iv) Comparing corners of squares

Discussion

Across these lessons, we saw instances where children's thinking appeared to align with level 1 of the van Hiele framework- it was dominated by visual imagery and children had limited understanding of shape properties. While this might be expected for first and second class children, it is problematic when the imagery which guides their thinking is prototypical in nature, limiting their concept images for given geometric shapes. While much attention was given above to the limitations in children's thinking, we argue that this was a useful task for uncovering and extending that thinking. For example, teacher questions prompted students to count sides and to test and compare the size of corners on various composed shapes against the square-piece, as per Figure 2 (iv). This hands-on exploration of shape properties supported identification and naming of same and the multiple examples of composed squares that were created should enrich children's concept images. In addition, the task focused on composing shapes, a pillar of geometric reasoning, and opportunities were created for describing transformations and their effects on shapes. The fine-grained learning trajectories described by Clements et al. (2004) outline how understanding of angles is used (or not) in shape composition tasks at various stages of development depending on whether the child possesses a sense of angle as a quantitative entity. Our observations align with their research in that we observed a number of children engaging in trial and error approaches to the task, while others operated with greater intentionality and anticipation- they selected and combined shapes that they predicted would fit together to make a square based on visuospatial reasoning involving mental transformations of the selected shapes.

This paper gives some insight into Irish children's thinking about a tangram task. As per the didactical tetrahedron, we recognise the crucial role of the teacher in supporting children's mathematical exploration but did not have scope to address this here. Shape composition activities have the potential to address the proposed learning outcomes of the new primary curriculum (NCCA, 2017). The analysis of task and students' responses presented here offers insight into how the reformulated learning outcomes might be achieved.

References

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. *Handbook of research on mathematics teaching and learning,* 420-464.

Clements, D. H., Wilson, D. C., & Sarama, J. (2004). Young children's composition of geometric figures: A learning trajectory. *Mathematical Thinking and Learning, 6*(2), 163-184.

Fuys, D., Geddes, D., & Tischler, R. (1984). *English translation of selected writings of Dina van HieleGeldof and Pierre M. van Hiele.* Retrieved from https://files.eric.ed.gov/

Government of Ireland. (1999). *Primary school curriculum: Mathematics*. Stationery Office

NCCA. (2017). *Primary mathematics curriculum: Draft specification junior infants to second class.* NCCA.

Nic Mhuirí, S. (2020). *Shape and space in the senior primary classes.* NCCA.

Perkins, R., & Clerkin, A. (2020). *TIMSS 2019: Ireland's results in mathematics and science.* Educational Research Centre.

Perkins, R., & Shiel, G. (2016). *PISA in classrooms: Implications for the teaching and learning of mathematics in Ireland.* Educational Research Centre

Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: Artifacts as fundamental constituents of the didactical situation. *ZDM, 44*(5), 641-651.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Sinclair, N., & Bruce, C.D. (2015). New opportunities in geometry education at the primary school*.* *ZDM, 47*, 319–329.

Sinclair, N., Bussi, M. G. B., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM, 48*(5), 691- 719.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational studies in mathematics, 12*(2), 151-169.

Mathematical Content Knowledge of Pre-Service Teachers: Implications for Consecutive ITE Programmes

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This study examines the mathematical content knowledge (MCK) of pre-service post-primary mathematics teachers (N=85) on commencing their Professional Master of Education (PME) initial teacher education (ITE) programme. Participants' cognitive and conceptual proficiency with curriculum-aligned mathematical content was evaluated using a validated paper and pencil test, on commencement of their PME studies. Findings indicate that pre-service mathematics teachers exhibit stronger proficiency with Junior-Cycle curriculum-aligned content and weak proficiency with Senior-Cycle concepts, regardless of the curriculum strand. Given the lack of recent research examining MCK relating to pre-service teachers, and the focus of postgraduate ITE programmes on pedagogical aspects of teaching, this research identifies concerns with ITE programmes that may need to be addressed to support pre-service mathematics teachers' development and induction into the profession.

Introduction

The Professional Master of Education (PME), a Level 9 Masters qualification, was established following a review of initial teacher education provision in the Irish context in 2012 (Department of Education and Skills, 2012). Students entering the PME to qualify as post-primary mathematics teachers will have undertaken degree level studies in mathematics or related areas, as outlined by the subject requirements to register with the Teaching Council of Ireland (Teaching Council, 2013). Given the consecutive postgraduate nature of the PME programme, with entrants having completed degree studies in their subject area(s), the focus is primarily on the development of pedagogical practices and professional experiences in the classroom. It is fundamentally recognized that Initial Teacher Education (ITE) teachers' mathematical content knowledge (MCK) does not alter during their teacher qualification programme (Osborne 2013). Accordingly, the purpose of our study was to examine the MCK of pre-service mathematics teachers (N=85) commencing their PME programme[1]. Lowrie and Jorgensen (2016) stress that most significant studies relating to pre-service mathematics teachers' MCK are dated and have been overtaken by studies focused on PCK (pedagogical content knowledge). Given that research has indicated the essential function of a teacher's mathematical knowledge base for undertaking key teaching and learning roles such as lesson preparation, facilitating classroom discussion and creating purposeful learning opportunities (e.g. Baumert et al., 2010), it is important that we examine the MCK of pre-service

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mathematics teachers. Examining MCK can alert us to issues within ITE programmes that could be addressed prior to induction into the teaching profession.

Pre-Service Teachers and Mathematical Knowledge

Mathematical knowledge for teaching (MKT), which includes both subject matter (MCK) and pedagogical (PCK) considerations (Ball et al., 2008), is essential for mathematics teaching and supporting student learning (Baumert et al., 2010). MKT is utilised for example when using different approaches to explain mathematical concepts to students, interpreting students' answers, using various representations, and selecting examples that help develop students' understanding (Clivaz & Ní Shúilleabháin, 2019). However, despite such obvious importance, research studies have shown evidence of inadequate knowledge of mathematics for teaching amongst teachers (Ma, 1999; Slattery & Fitzmaurice, 2014). Similarly, Lowrie and Jorgensen (2016, p. 205) probe the "PCK fever" that has emerged in relation to researching teacher knowledge, at the detriment of examining content knowledge. Baumert et al.'s (2010) research demonstrates that limited subject matter knowledge can have detrimental effects on a teacher's PCK and, consequently, negative effects on instructional quality and student progress. Furthermore, they find that these differences in teachers' MCK persist across an entire teaching career.

There are various reasons cited for this in the literature but of concern to us is the role of ITE. Thanheiser et al. (2013) suggest that there are inadequacies in teachers' knowledge of mathematics when they graduate from their ITE programmes and many lack conceptual understanding of the mathematics they will be required to teach (O'Meara et al., 2017). For example, Slattery and Fitzmaurice (2014) carried out a study at an Irish university to measure pre-service post-primary mathematics teachers' conceptual understanding of fraction division. The results showed that participants – who were near the end of their degree programme – had a fragmented understanding of the fraction concept and were unable to explain the invert and multiply rule. They relied on a series of "rules without reason" to answer the questions posed. Thus, when teaching, these pre-service teachers would have to rely on a series of learned procedural steps, as they did not have the conceptual understanding necessary to teach for understanding (O'Meara et al., 2017).

Baumert et al. (2010) suggest that ITE programmes should increase the attention given to teachers' subject matter knowledge and, more specifically, achieve a balance between MCK and PCK. Mathematics teachers need to possess knowledge of the curriculum content at a much deeper level of understanding than their students (Krauss et al., 2008). Affording pre-service teachers the opportunity to re-examine post-primary school mathematics content from an advanced perspective may be an important element in preparing them to teach mathematics meaningfully (Artzt et al., 2012). A deeper understanding of mathematical concepts may also enable teachers to access a wider collection of strategies for explaining and illustrating mathematical content to their students (Ma, 1999). This present study looks to examine the MCK of pre-service post-primary mathematics teachers on a broad scale as they commence their ITE programmes and provides evidence of issues in relation to subject matter knowledge that may be addressed during the two-year PME programme. For the purposes of this study MCK is concerned with "an understanding of mathematics concepts anticipated to be taught" (Norton, 2019, p.530) and, accordingly, pre-service teachers' proficiency with curriculum-aligned content (Ní Ríordáin et al., 2017).

Methods

A paper-and-pencil test developed to examine Irish post-primary out-of-field mathematics teachers' knowledge of curriculum-aligned content (see Ní Ríordáin et al., 2017) was utilised in the study. The TEDS-M conceptual framework supported the development of item design for the paper-and-pencil test (Ní Ríordáin et al., 2017; Tatto et al., 2008) and items were developed that closely align with the Irish post-primary mathematics curriculum (Krauss et al., 2008). The original paper-and-pencil test had 24 MCK items, including 10 multiple-choice (MC) and 14 open-ended (OE) items. However, for the purpose of this study a reduced version, 17 MCK items (9 MC, 8 OE), was utilised due to time constraints in collecting the data. Three of the MC items consisted of two sub-items, therefore participants were asked to complete 20 items in total.

The paper-and-pencil test was administered to participants in their first mathematics pedagogy lecture of the term, on commencement of their PME programme. The test was of closed-book form and no calculators were allowed. All those in attendance at the first lecture consented to participate in the study. We acknowledge that this may be a limitation of the study as participants may be focused on other aspects of their studies and not focused on mathematical content in this first lecture. Participants (N=85) were recruited from four (of eight) institutions in Ireland offering the PME programme over the course of three intakes (2015 – 2017), allowing for a more considerable sample for data analysis and to make appropriate inferences. A convenience sampling approach was utilised in the study, whereby all four authors had access to the participants through their involvement in the delivery of the PME programme in their participating universities.

The scoring of the MCK items on the paper-and-pencil test involved two different scoring processes. Ní Ríordáin et al. (2021) provide a detailed description of the analysis of the data. The first scoring process involved the calculation of a *cognitive score*, with each item scored based on the correctness of the answer with an overall percentage score returned for each participant. MC items were scored either 0 (incorrect) or 1(correct); OE items were scored 0 (incorrect), 1(partially correct) or 3 (correct). In addition, a cognitive proficiency rate for each individual item was determined by calculating an overall mean score for each item as a percentage of the total possible score (1 or 3) of that item. The second scoring system employed examined conceptual errors made by participants on items answered. Each item was broken down into key concept(s) necessary for answering the question and linked to the post-primary mathematics curriculum. Based on a participant's solution to a given item, a *conceptual error* score was calculated, and an overall percentage error score returned.

Key Findings

The first stage of the cognitive scoring analysis examined the overall scores for each pre-service teacher. This overall cognitive score provides a gauge of a pre-service teacher's performance based on their responses to the test items. The cognitive score was calculated as a percentage out of a possible 42 marks. The first notable finding is the low mean score of only 40.2% with a standard deviation of 18.02. However, there is a range of 88.1% with a maximum score of 92.86% achieved by one pre-service teacher. The second stage of analysis examained the proficiency rates for each item on the test (Table 1). An item's mean score is divided by its total possible score (1 or 3) in order to calculate the proficiency rate. Only four items (of twenty) had a proficiency rate higher than 50%. It is important to note that all of these four items are linked to Junior Cycle mathematics and that no such strengths were recorded at Senior Cycle level. Also included in Table 1 are three items with a proficiency rate between 40%-50%. Only one of these items is at the Leaving Certificate level; all others are linked to the Junior Cycle curriculum. For the purpose of this paper, test items with proficiency rates below 40% fall below minimum standards expected in mathematical studies. Thirteen items out of twenty fell below the required 40% pass rate.

Table 1

Pre-service teacher cognitive proficiency with curriculum-aligned content

Strand & Concepts[2]	Level: JC/LC	Proficiency Rate (%)	*SD*
A&F: Making predictions about what comes next in a pattern	JC	91.0	27.5
N: Solving problems involving shopping: % discount; Performing calculations with percentages	JC	64.3	34.4
G&T: Properties and equations of a line – slope, x/y intercepts; Relationship between the slopes of parallel lines; Labelling axes w/ appropriate scales	JC	60.6	35.5
G&T: Properties of a Square; Applying Pythagoras's Theorem; Operations with Surds	JC	58.3	34.5
N: Relationships between the number systems	JC	46.3	37.8
S&P: Probability: Finding the probability of equally likely outcomes	JC	46.0	49.7
S&P: Representing Data Graphically: Mean vs. Median	LC	45.0	49.9
A&F: Writing an arithmetic expression for the terms in a sequence	JC	38.7	39.8
S&P: Standard Deviation	LC	35.0	47.8

[2] **A:** Algebra; **F:** Functions; **N:** Number; **G:** Geometry; **T:** Trigonometry; **S&P:** Statistics & Probability.

S&P: Mean vs. Median	LC	29.0	46.1
S&P: Count the number of ways to select *r* objects from *n* distinct objects	LC (HL)	28.0	45.6
F: Recognize a bijective function and find its inverse	LC (HL)	26.3	42.4
F: Differentiation	LC	25.3	32.5
A: Solve simultaneous equations with two variables and interpret results	LC	25.3	39.5
F: Graph of the function $f(x) = x^3$; Transformations $f(x) + a$; Graphs of inverse functions are reflections over $y = x$.	LC	25.3	23.8
S&P: Median	LC	20.0	39.5
S&P: Interquartile Range	LC	19.0	39.5
F: Associate derivatives with slopes and tangent lines	LC	19.3	39.5
F: Relationships between number systems	JC	18.0	38.6
A: Solve simultaneous equations with infinite number of solutions	LC	15.2	31.0

Overall, the pre-service mathematics teachers demonstrated poor proficiency with the Functions strand. All items had a proficiency rate of below 40%. Although these items relate to LC content, all but one are at the Ordinary level. The case is similar for the Statistics and Probability strand where participants demonstrated poor proficiency in five of the seven items connected to this strand, four of which are at the Ordinary level and relate to representing data graphically; the other item relates to Counting. Pre-service teachers demonstrated, on average, a pass standard proficiency with the remaining two items: one related to JC Probability content and the other LC content identifying data sets in which the Mean and Median were equal. The data relating to the Statistics and Probability content suggests a poor knowledge of concepts overall, particularly items involving Senior Cycle and Higher level.

In general, no distinct pattern emerged from the data in relation to strengths and weaknesses relating to the other strands. For example, Item 13a (finding what comes next in a pattern) with the highest proficiency rate (91%, with a standard deviation of 27.5) involved content from Junior Cycle Algebra. Yet its related item, 13b (writing arithmetic expressions for terms in a sequence), had a proficiency rate of only 38.7%, with a standard deviation of 39.8. The four items with the highest proficiency rates involve three different strands (Algebra, Number and Geometry), but all content is related to Junior Cycle. The items with the weakest proficiency rates involve Number and Algebra also, but at Senior Cycle. Therefore, overall, the data suggests that pre-service teachers demonstrate a stronger proficiency with Junior Cycle curriculum aligned content and poor proficiency with Senior Cycle content, irrespective of the curriculum strand.

An analysis of pre-service mathematics teachers' overall conceptual scores establishes a relatively high occurrence rate of errors amongst the 85 participants. Overall, of the questions answered, there was a 45.86% mean occurrence rate for conceptual errors or evidence of incomplete conceptual understanding. The implications of this is that more than two out of every five errors identified as possible were made by these pre-service teachers. Given that there is variation in the response rates for individual items, it is difficult to undertake a comparison of occurrence rates on individual concepts. However, there are several items with a high response rate, with a high conceptual error occurrence rate, that can extend our understanding of the cognitive scores and proficiency rates outlined in the previous paragraphs. For example, items 3-7 relating to Statistics and Probability had a high response rate with a high occurrence rate (50%-78%) for conceptual errors relating to the identified concepts. In general, items around or below the average occurrence rate for conceptual errors and with a high response rate are associated with Junior Cycle content relating to Geometry and Trigonometry, Number and Algebra. It is interesting to note variation in occurrence rates within items with high response rates. For example, Item 10 examines Number Systems. Pre-service teachers demonstrated low occurrence rates for conceptual errors relating to Junior Cycle concepts of Natural and Rational Numbers, but an increase in conceptual errors is evident with higher order concepts such as Prime and Complex Numbers relating to Leaving Certificate content. This is consistent with findings relating to pre-service teachers' cognitive proficiency rates with curricular aligned content. These insights help identify the areas of weakness that may have caused participants to answer an item incorrectly and by profiling pre-service mathematics teachers' knowledge of content related items it can help us address such misconceptions within our teacher education programmes.

Discussion & Conclusion

The findings of this research suggest that while these pre-service teachers may be well equipped to teach Junior Cycle mathematics content, they do not demonstrate the same proficiency with Senior Cycle content. Furthermore, there are strands of the curriculum (e.g. Statistics & Probability and Functions) where pre-service teachers demonstrate poor cognitive and conceptual knowledge. This research aligns with the findings of Slattery and Fitzmaurice (2014) that record shortcomings in pre-service post-primary teachers' knowledge of mathematics. Given the importance of teachers' adequate knowledge of content to develop learners' conceptual understanding and to positively impact on learner achievement (Baumert et al., 2010), these findings may have broad implications for consecutive post-primary ITE programmes in Ireland.

The results of this research suggest that, despite the rigorous content requirements of the Teaching Council (2013), pre-service teachers on consecutive programmes may require additional subject matter knowledge prior to their commencement of teaching. In line with recommendations from Baumert et al. (2010), ITE programmes should increase the attention given to pre-service teachers' content knowledge and explicitly align MCK and PCK in PME mathematics pedagogy modules. This is significant as, according to Baumert et al. (2010), high levels of teacher PCK is not achievable without a high level of subject matter knowledge and both have significant impact on student achievement and consideration needs to be given to LC content. For example, in pedagogy modules, MCK materials should not mirror that of programmes for mathematics majors, but rather explicitly link mathematical content at a deeper level of understanding to that of the school curriculum (Artzt et al., 2012). On the basis of this evidence, any reduction of the current minimum subject specific pedagogy from 5 ECTS credits (Teaching Council, 2013) should be avoided. It may be of interest to repeat this research with additional cohorts of pre-service teachers who have studied the revised post-primary mathematics curriculum as part of their own post-primary education and may therefore be more familiar with the revised curriculum content, particularly that of inferential statistics which was not compulsory content prior to 2012.

Given calls for a renewed focus on content knowledge in teacher education (Lowrie & Jorgensen, 2016), further research should be conducted on both pre-service and in-service mathematics teachers' knowledge on a larger and longitudinal scale. Such research would be of particular relevance in informing any future ITE and teacher education policies. In the context of various ITE programmes underway across the country, it may be of particular interest to compare and contrast the MCK of pre-service teachers from consecutive and concurrent programmes. Additional research should be conducted to investigate how best we might support the development of pre-service teachers' MCK over the duration of their PME programme. The strong association between MCK and classroom practices always needs to be taken into consideration and by assessing pre-service teachers' MCK it allows for the development of awareness of the challenges they may face (Norton, 2019). How we utilise this information thereafter needs to be considered in our PME programme delivery.

References

Artzt, A. F., Sultan, A., Curcio, F. R. & Gurl, T. (2012). A capstone mathematics course for prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education, 15*(3), 251-262.

Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133–180.

Clivaz, S. & Ní Shúilleabháin, A. (2019). What knowledge do teachers use in lesson study? A focus on mathematical knowledge for teaching and levels of teacher activity. In Huang, R., Takahashi, A., & Ponte, J. P. (Eds). *Theory and practice of lesson study in mathematics* (pp. 419-440). Springer.

Department of Education and Skills (2012). *Report of the international review panel on the structure of initial teacher education provision in Ireland: Review conducted on behalf of the Department of Education and Skills.*

Krauss, S., Baumert, J. & Blum, W. (2008). Secondary mathematics teachers’ pedagogical content knowledge and content knowledge: Validation of the COACTIV constructs. *ZDM: The International Journal on Mathematics Education, 40*(5), 873-892.

Lowrie, T. & Jorgensen, R. (2016). Pre-service teachers’ mathematical content knowledge: Implications for how mathematics is taught in higher education. Teaching *Mathematics and Its Applications, 35*, 202-215.

O'Meara, N., Fitzmaurice, O. & Johnson, P. (2017). Old habits die hard: An uphill struggle against rules without reason in mathematics teacher education. *European Journal of Science and Mathematics Education, 5*(1), 91-109.

Osborne, S. (2013). Preservice education: Room for improvement but no crisis. *Independent Education, 48*(2), 18–19.

Ní Ríordáin, M., Ní Shúilleabháin, A., Prendergast, M. & Johnson, P. (2021). Irish pre-service mathematics teachers’ knowledge of curriculum-aligned content. *Irish Educational Studies*. https://doi.org/10.1080/03323315.2021.1899030

Ní Ríordáin, M., Paolucci, C. & O’ Dwyer, L. (2017). An examination of the professional development needs of out-of-field mathematics teachers. *Teaching and Teacher Education, 64*, 162-174.

Norton, S. (2019). Middle school mathematics pre-service teachers’ content knowledge, confidence and self-efficacy. *Teacher Development, 23*(5), 529-548.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teacher's understanding of fundamental mathematics in China and the United States.* Lawrence Erlbaum Associates.

Slattery, J. & Fitzmaurice, O. (2014). “Ours is not to reason why, just invert and multiply”: An insight into Irish prospective secondary teachers' conceptual understanding of the division of fractions. *Irish Educational Studies, 33*(4), 467-488.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R. & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics: Conceptual framework.* Teacher Education and Development International Study Center, College of Education, Michigan State University.

Teaching Council (2013). *Teaching Council registration: Curricular subject requirements (post-primary) for persons applying for registration on or after 1 January 2017.*

Thanheiser, E., Philipp, R.A., Fasteen, J., Strand, K. & Mills, B. (2013). Preservice-teacher interviews: A tool for motivating mathematics learning. *Mathematics Teacher Educator, 1*(2), 137–147.

Validation of Task Items on a Screener for Initial Algebra

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A standardised criterion referenced assessment known as a screener has been developed for initial algebra. This screener is a formative assessment which has been designed to be of use to teachers in the second-year Irish post-primary mathematics classroom. This paper outlines the design of the screener as guided by the assessment triangle and the subsequent steps in selecting and validating task items for use on the screener. There are two types of items included - selected response (SR) and constructed response (CR). Classical Test Theory (CTT) is used to analyze the results of the SR task items in terms of item difficulty, discrimination, and functioning distractors. Important construct validity evidence arises from the statistical analyses of the item responses. The results of these analyses inform the presentation and layout of the SR task items on the final draft of the screener to ensure the construct of initial algebra is operationalised fairly.

Introduction

Initial algebra is the period when students transition from arithmetic to algebra, an area of mathematics education known for its difficulties (Kieran, 2007). Much research and effort has been focused on the area of algebraic thinking and initial algebra to improve students' engagement and attainment with the subject (Kieran et al., 2016). Formative assessment (FA) should now be a vital part of mathematics teaching and learning and it should form part of the "Assessment toolkit" for all classrooms (NCCA, 2019). However, it is noted in the literature that few adequate assessments are available to provide formative information on a student's progress with algebra, but they are essential to allow timely and informed instructional decisions for teachers (Ketterlin-Geller et al., 2019). The overall aim of this study was to profile second year post-primary students' (approx. age 14) knowledge of initial algebra and to do so an appropriate measurement instrument was required. This standardised screener developed for this study takes the form of a summative assessment (SA) to be used in a formative manner, as it can provide information to teachers about their students' current understanding of algebra and identify any gaps in their knowledge and understanding. The focus of this paper is on the development and validation of selected response (SR) task items for the standardised screener. First the design of the screener is outlined which explains how the task items were identified and selected for inclusion. Subsequently, the methodology employed to establish construct validity using item level analysis is given. Finally, an overview of the results of item analyses are presented and the resulting changes to the screener are documented.

Screener Design and Development

Nichols et al., (2017) state that assessment design is a "sequence of development actions aimed at accomplishing specific goals" and assessment development is "the execution of the planned course of action" (p. 15). The assessment triangle as described by Pellegrino et al. (2001) guided the many interconnected decisions in the design and development of the screener. The assessment triangle consists of three interconnected elements; (i) Cognition; a theory of how students gain competence in a domain, (ii) Observation; the task items or content used to evidence the learning and cognition, (iii) Interpretation; the methods used to interpret and analyse the evidence (Nichols et al., 2017).

The cognition vertex of the assessment triangle is addressed by establishing the construct of interest. This is done by aligning the knowledge, skills, and abilities (KSAs) required for initial algebra with an appropriate conceptual framework. An established framework for algebraic thinking by Kaput et al., (2008) was identified in the literature which aligns closely with the Junior Cycle specification. Research by Blanton et al. (2018) to develop a framework for algebra organises Kaput's two core aspects into the following three areas (i) generalised arithmetic, (ii) equivalence, expressions, equations, and inequalities, and (iii) functional thinking. This was adopted as the conceptual model for initial algebra in this study. Related content domains from the Irish syllabus were used as a framework to align the prerequisite and algebra content areas identified in the literature (O' Brien & Ní Ríordáin, 2017). These content areas framed the pertinent KSAs required for success in algebra as guided by the conceptual framework adopted for this study.

Cronbach and Meehl (1955) state that the construct to be measured consists of a universe of content, from which items that are domain relevant should be sampled, and from these a test that is representative of the domain can be developed. By defining the construct of interest "initial algebra" and using a conceptual model aligned with relevant content areas and the Irish syllabus a systematic search of the literature for studies that assessed students in each area was conducted. Task items contained in all the relevant studies were recorded in an item bank for possible inclusion in the assessment. This item bank together with a proposed experimental first draft was then forwarded to an expert panel for review. The revised draft was returned containing 21 task items assessing the pertinent content areas and utilised for piloting and further development of the screener. Two types of item format are utilised on the screener; multiple-choice known as selected-response (SR), and constructed-response, objective scoring (CROS) (Haladyna & Rodriguez, 2013). Of the SR items some used the Complex Multiple-Choice format where these items allow for answers that are incorrect, part correct, or fully correct (polytomous item). The remaining items used the Conventional Multiple-Choice format with four or five possible answers and only one of these is the absolute correct answer (dichotomous item). Part of the development of the screener was developing an SR format for existing constructed-response items identified in the literature. When creating an SR format, distractors are based on common errors to help inform teachers of the common errors and misconceptions made by their students. It is known that writing distractors is one of the most difficult aspects of item development and they often require revision from their first iteration (Haladyna & Rodriguez, 2013). The development of the initial distractors based on existing literature through to the final format based on the statistical analysis of the results is an important outcome of this study.

Finally, reporting of results to students must be an integrated part of the assessment design and development (Lane et al., 2016). An objective scoring system was established for the screener which then formed the basis for the statistical analysis conducted. Two forms of scores were developed for the items: first a cognitive score which awards a score when an item is correctly answered, and second, an error score which identifies a type of error associated with an item. A cognitive score (CS) is one which identifies a cognitive process, for example, knowledge, recall, interpretation, or synthesis (Anderson & Morgan, 2008). A CS was applied consistently to all items whereby a completely correct response was awarded 2, partly correct 1 and an incorrect response 0. Error scores have been developed for items to identify both potential procedural errors and misconceptions. An error score of '0' means that the student demonstrated evidence of understanding the concept and an error score of '1' reflects incomplete KSAs.

Methodology

Leedy and Ormrod (2010) succinctly define the validity of a measurement instrument as "the extent to which the instrument measures what it is intended to measure" (p. 31). The validity was established in terms of content, criterion, and construct validity, known as the trinitarian concept of validity (Foster, 2017). The focus in this paper is on the SR task items analyses which forms part of construct validity. According to Cohen et al., (2018, p. 257), there are two main stages in addressing construct validity; (i) ensuring the construct has been clearly and adequately defined and (ii) operationalising the constructs fairly. In defining the construct of initial algebra, the cognition vertex of the assessment triangle was used to establish the KSAs and a conceptual model as outlined above. Furthermore, the conceptual model was then aligned with the KSAs, which allowed for relevant items to be mapped to Junior Cycle specification to ensure the construct of interest was established clearly (O' Brien & Ní Ríordáin, 2017).

The aim of this study is to validate the items for use with Irish second-year students specifically, to ensure the construct has been operationalised fairly for the population of interest (Cohen et al., 2018). It is important to note that subject matter expertise outweighs statistical guidelines when deciding if items should be retained, if it is believed they are measuring something important (Haladyna & Rodriguez, 2013). The revised draft of the screener returned from the expert panel was administered using pen and paper in a pilot school (n = 67) with a short survey. Task items were revised based on the results of this administration and feedback from teachers and students in the pilot school. Subsequently, the revised screener consisted of seventeen SR items and four CROS items. It was administered using pen and paper to 576 second-year students in 19 post-primary schools (29 classes) across Ireland in October 2016 and again in April 2017. There were two reasons for administering the screener twice to the same group of students; first was to enable the test-retest reliability measure of stability for the screener results (Cohen et al., 2018), and second was to measure the change in the students' performance over the academic year.

Comprehensive item analyses utilising Classical Test Theory (CTT) were undertaken for the responses in the main study once each student had been assigned a complete set of CSs. First the facility index (FI) was calculated for each item to assess item difficulty. The FI equates to the proportion of correct responses for dichotomous items and ranges between 0 and 1, with an acceptable range from 0.3 up to 0.8 (Haladyna & Rodriguez, 2013). For polytomous items (a CS of 0, 1 or 2 in this study) the FI is calculated as the arithmetic mean for all respondents and accordingly ranges from 0 to 2 with an acceptable range of 0.6 up to 1.6 for this study (Finch, 2016). Subsequent analysis focused on item discrimination which is the relationship between the item response and performance on the screener overall (Haladyna & Rodriguez, 2013). In addition, multiple choice items were assessed for non-functioning distractors, that is a less than a 5% response rate, which should be removed (Ibid).

Findings

The discussion here focuses on the item analysis of SR items. Each item that had an FI outside the acceptable guidelines was identified, followed by the identification of items that did not discriminate well. Table 1 below details the results for the SR items. Each SR item and the content it assesses is listed with the associated FI and whether it discriminates well at each administration. Ten SR items were identified as requiring revision and/or further analysis. The analysis of distractors showed that six items had non-functioning distractors at both administrations. A final measure for item analysis is non-response and any item having greater than 15% of students not answering requires review (Anderson & Morgan, 2008). The items with non-functioning distractors are given together with the percentage of students who did not respond to the item on the screener at each administration. The cells highlighted in grey in Table 1 highlight the issues with the items. It is important to note at this point that evidence of students struggling with the subject of initial algebra had emerged from government reports and state examination results (O' Brien & Ní Ríordáin, 2017). Therefore, these results and the FI for each item should be viewed in light of this information.

Table 1

Results of Item analyses for SR Items

Item number and content (CS; polytomous 0, 1,2; dichotomous 0,2)	**Facility Index (FI)**		**Does the Item discriminate?**		**Non-functioning distractor**		**% Non-response**		**Revised Layout**
	Oct	Apr	Oct	Apr	Oct	Apr	Oct	Apr	
2. Procedural Fraction Knowledge (0, 1, 2)	0.72	0.92	Yes	Yes	Yes	Yes	12.1	7.1	Yes[1]
3. Procedural Fraction Knowledge (0, 2)	0.38	0.46	Yes	Yes	No	No	12.3	10.5	Yes[2]

4. Equivalent Fractions (0, 2)	0.42	0.49	Yes	Yes	Yes	Yes	8.3	5.7	Yes[2]
5. Relational Fraction Knowledge (0, 2)	0.20	0.30	No	Yes	No	No	13.1	8.0	No
6. Proportional Reasoning (0, 2)	0.72	0.72	No	No	No	No	2.7	0.8	No
7. Exponents (0, 2)	0.36	0.33	No	No	No	No	16.4	9.2	Yes[2]
8. Exponents and the Distributive Property (0, 2)	0.09	0.12	No	No	Yes	Yes	17.8	10.5	Yes[2]
9. Order of Operations (0, 2)	0.30	0.36	Yes	Yes	No	No	9.7	6.1	Yes[2]
10. Distributive Property (0, 1, 2)	0.71	0.79	Yes	Yes	Yes	Yes	12.6	8.0	Yes[1]
11. Comparing and Ordering Numbers (0, 2)	0.65	0.67	No	Yes	Yes	Yes	9.9	6.5	No
14. Variables (0, 2)	0.23	0.25	No	No	No	No	8.6	5.7	No
17. Expressions (0, 2)	0.47	0.56	Yes	Yes	No	Yes	18.9	12.4	Yes[2]
18. Equation solving (0, 1, 2)	0.43	0.54	No	No	No	No	18.9	10.5	Yes[2]
19. Equation solving (0, 1, 2)	0.88	1.10	Yes	Yes	Yes	Yes	17.3	8.2	Yes[3]
20. Equation forming (0, 2)	0.22	0.25	No	No	No	No	15.3	7.1	No
21. Patterns – situation (0, 1, 2)	0.59	0.80	Yes	Yes	No	No	16.6	7.5	Yes[3]
21. Patterns – form the equation (0, 1, 2)	0.34	0.51	Yes	Yes	No	No	18.9	9.2	Yes[3]

Note. [1] Items are polytomous, and their revised layout is discussed in the subsequent paragraph. [2] Items are dichotomous, and their revised layout is discussed in the subsequent paragraph. [3] Items require further revision discussed below.

As evidenced in Table 1 many items need revision for a final version of the screener based on the item analyses. The use of Complex Multiple-Choice format for the pen and paper administration of the screener in the main study was not ideal however necessary for data inputting and analysis together with ease of scoring and identification of student errors. Most items on the screener are selected response and the most appropriate format for these items is to present three options in a vertical list (one correct and two distractors). However, where there are more functioning distractors a four or five option vertical list is also appropriate (Haladyna & Rodriguez, 2013). All dichotomous SR items will be presented in this format with any non-functioning distractors removed[2]. Polytomous items (more than one correct response) will have answers offered vertically with a YES/NO option printed beside each answer, where students are asked to circle a response to each answer[1]. Based on the evidence from item analysis it is recommended that items should have their format and layout revised to ensure the construct of initial algebra is operationalised fairly.

Two items required more revision than others given non-functioning distractors, the layout of the answers, item difficulty and non-response particularly in the October administration[3]. The first of these items assessing equation solving was developed specifically for this study based on a study by Chung and Delacruz (2014). The item asks the student to identify the next correct step when solving the following equation; $7h - (3h - 2) = 38$. It is recommended this item is presented with only one correct answer $7h - 3h + 2 = 38$ as opposed to the two possible correct answers offered in the main study, $7h - 3h + 2 = 38$ and $7h + (-1)(3h - 2) = 38$. The first distractor, $7h - 3h - 2 = 38$, was the most chosen answer by students so this will remain as it identifies an important error in the application of the distributive property. A new distractor has been developed $-21h^2 - 14h = 38$ from observing the student workings which showed the attempt to multiply '7h' into the brackets.

The second item assessing patterns was a constructed response item from a study by Ayalon et al. (2015). The item was adapted for this study with the development of selected response answers. The issue of having more than one correct answer arises for this item, together with the amount of reading required. The student is asked how they would describe the process for finding the perimeter of one hundred hexagons, without knowing the perimeter of ninety-nine hexagons. Originally, responses were developed by the first author based on various student strategies identified by Ayalon et al. (2015). However, on consultation with the expert panel an additional correct option was added as it was in line with the methods of teaching patterns in Ireland. The item requires a lot of reading and therefore it cannot be said that knowledge of patterns alone is being assessed. It would be favourable to reduce the number of options to reduce word count as well as having the unfavourable two correct answers. The distractor analysis shows that the original correct answer as identified by Ayalon et al. (2015) in their study is preferable. Therefore, it is advisable to retain the item in its original format with 4 possible options. However, it is important to note that this item may need to be adjusted again after further administrations.

Discussion & Conclusion

This paper outlined the assessment triangle framework used to guide the development of a screener for initial algebra. The focus of this paper was on the validation of the SR items for use in the final version of the screener. The use of item analyses identified issues with several task items, and these have been amended accordingly. The distractors which are the incorrect answers to SR items are often the most challenging aspect of item development (Haladyna & Rodriguez, 2013). The development and testing of the distractors as part of this research has been an important contribution to international research in addition to the development and validation of the task items for use in the Irish classroom.

This research has produced insights into students' issues with learning initial algebra, many of which were identified in the literature and are common to students worldwide. It confirms what Irish post-primary teachers have highlighted, that knowledge of fractions, decimal number magnitude, exponents, integers, order of operations and variables are key content areas where there is a lack of understanding (Shiel & Kelleher, 2017). The overall aim of developing the screener for use in the Irish classroom is to assist teachers identify student errors and possible root causes of these errors. The careful development of distractors for the SR task items on this screener enables this is an efficient manner. Therefore, the use of this screener as a formative assessment to create a profile of students' knowledge in the Irish classroom will be efficient and useful for teachers. It will allow for timely instructional decisions to help prevent and rectify skill gaps for students and is therefore an important contribution of this research (Ketterlin-Geller et al., 2019).

References

Anderson, P., & Morgan, G. (2008). *Developing Tests and Questionnaires for a National Assessment of Educational Achievement.* The World Bank.

Ayalon, M., Watson, A., & Lerman, S. (2015). Functions represented as linear sequential data: relationships between presentation and student responses. *Educational Studies in Mathematics, 90*(3), 321-339.

Blanton, M., Brizuela, B. M., Stephens, A., Knuth, E., Isler, I., Gardiner, A. M., Stroud, R., Fonger, N., & Stylianou, D. (2018). Implementing a framework for early algebra. In Kieran C. (eds) *Teaching and learning algebraic thinking with 5-to 12-year-olds* (pp. 27-49). ICME-13 Monographs. Springer, Cham.

Bloom, B. S. (1971). *Handbook on formative and summative evaluation of student learning.* McGraw-Hill.

Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *The Journal of Mathematical Behavior, 32*(3), 613-632.

Chung, G. K., & Delacruz, G. C. (2014). Cognitive readiness for solving equations. In O'Neil, H., Perez, R., Baker, E. (eds.), *Teaching and Measuring Cognitive Readiness* (pp. 135-148). Springer.

Cohen, L., Mannion, L., & Morrison, K. (2018). *Research Methods in Education* (8th ed.). Routledge.

Cronbach, L. J., & Meehl, P. E. (1955). Construct validity in psychological tests. *Psychological Bulletin, 52*(4), 281-302.

Finch, W. H. (2016). *Applied psychometrics using SPSS and AMOS.* Information Age Publishing.

Foster, C. (2017). Validity in educational and psychological assessment. *Research in Mathematics Education, 19*(2) 108-111.

Haladyna, T. M., & Rodriguez, M. C. (2013). *Developing and validating test items.* Routledge.

Kaput, J. J., Carraher, D. W., & Blanton, M. (2008). *Algebra in the early grades.* Routledge.

Ketterlin-Geller, L. R., Shivraj, P., Basaraba, D., & Schielack, J. (2019). Universal screening for algebra readiness in middle school: Why, what, and does it work? *Investigations in Mathematics Learning, 11*(2), 120-133.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. K. Lester (Ed.), *Second handbook of research on mathematics Teaching and learning* (pp. 707-762). Information Age.

Kieran, C., Pang, J., Schifter, D., & Fong Ng, S. (2016). *Early algebra: Research into its nature, its learning, its teaching*. Springer International Publishing.

Lane, S., Raymond, M. R., & Haladyna, T. M. (2016). *Handbook of test development* (2nd ed.). Routledge.

Leedy, P. D., & Ormrod, J. E. (2010). *Practical research planning and design* (9th ed.). Merrill Prentice Hall.

NCCA. (2020). *Junior cycle mathematics guidelines for the classroom-based assessment and assessment task.* Retrieved from https://curriculumonline.ie/getmedia/f5af815d-5916-4dc9-bfda-4f3d73bc4787/Assessment_Guidelines_Mathematics.pdf

Nichols, P. D., Kobrin, J. L., Lai, E., & Koepfler, J. (2017). The role of theories of learning and cognition in assessment design and development. In A. Rupp & J. Leighton (Eds.), *The handbook of cognition and assessment: Frameworks, methodologies, and applications* (pp. 13-40). John Wiley & Sons, Inc.

O' Brien, A., & Ní Ríordáin, M. (2017). Examining difficulties in initial algebra: Pre-requisite and algebra content areas for Irish post-primary students.. In Dooley, T., & Gueudet, G. (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 1-8) (CERME10, February 1-5, 2017). DCU Institute of Education and ERME.

Pellegrino, J. W., Chudowsky, N., & Glaser, R. (2001). *Knowing what students know: The science and design of educational assessment.* National Academy Press.

Shiel, G., & Kelleher, C. (2017). *An evaluation of the impact of project maths on the performance of students in Junior Cycle mathematics.* Retrieved from https://www.erc.ie/wp-content/uploads/2018/07/PM_EvaluationStrand1.pdf

The Uptake of Higher Level Mathematics: An Analysis of Students' Reasons for Studying Mathematics in its Most Advance Form

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There is a large body of research which highlights the importance of students studying mathematics at an advanced level and many Governments and policy makers worldwide are constantly looking at ways to increase the uptake of advanced mathematics. In Ireland, the Bonus Points Initiative was introduced in 2011 in an attempt to increase the number of Senior Cycle students opting to study higher level mathematics and to improve Irish students' mathematical capabilities. Despite a rise in the number of students studying higher level mathematics at this time very little research has been conducted to determine if it was the initiative, the new curriculum or other factors that led to the surge in uptake. This study investigates Irish students' reasons for participating in higher level mathematics and seeks to determine if the reasons for studying higher level mathematics differs across gender. The findings suggest that the points system currently in place in Ireland is the main driving factor behind students' participation in higher level mathematics while parents are very influential actors in the decision-making process also. Differences in the reasons offered by males and females were also unearthed with higher levels of mathematical self-efficacy among male respondents influencing their decision to study higher level mathematics.

Introduction

There is a large body of growing research which highlights the importance of students studying mathematics at an advanced level (Attridge & Inglis, 2013). For example, Chinnappan et al. (2008) determined that higher-level mathematics facilitates the development of a variety of skills that underpin a scientifically literate workforce. Kennedy, Lyons and Quinn (2014) added that higher-level mathematics courses in high school are critical if we are to produce graduates who are capable and confident in making informed decisions about various real life issues. The literature also highlights the importance of advanced mathematics for developing students' logical thinking and reasoning abilities. For example, in a year-long study of students in the United Kingdom, Attridge and Inglis (2013) recorded differences in the conditional reasoning abilities of advanced mathematics students compared with non-mathematics students. The Irish senior cycle higher level mathematics syllabus places particular emphasis 'on the development of powers of abstraction and generalisation and on the idea of rigorous proof' (DES, 2013, p. 11). With this in mind, many researchers hypothesize that there is a correlation between participation rates in higher level mathematics and participation in other science subjects such as physics (Chinnappan et al., 2008; Kennedy et al., 2014) and chemistry (Donovan & Wheland, 2009). Furthermore, many science, engineering, and technology Level 8-degree courses in Ireland have minimum requirements for attainment in Leaving Certificate higher level mathematics, meaning that without studying mathematics in its most advanced form at upper secondary level many students are limiting the options available to them at third level and this will have a detrimental effect on the potential supply of graduate recruits for third-level science, technology, engineering, and mathematics (STEM) courses (EGFSN, 2008). This is particularly important in Ireland where the Government have set a goal that the country will become a leader in Europe with regards to developing and deploying STEM talent by 2026 (DES, 2017).

Despite the importance of higher-level mathematics many countries worldwide have reported low numbers of students opting to study mathematics in its most advanced form. For example, in Australia, Goodrum, Druhan and Abbs (2012) found that all high school science subjects, mathematics included, were experiencing dramatic declines. In addition to this, in the UK participation in higher level mathematics, that is mathematics post-GCSE level (age 16), has been a cause of concern for many years. According to Noyes (2013) only 10-15% of 16-year-old students choose to continue their study of mathematics and he reported that this figure is low when compared with other developed countries. Similar problems were also reported in the USA and India (National Commission on Mathematics and Science Teaching, 2000; Garg & Gupta, 2003). In Ireland, while the uptake of mathematics at Senior Cycle is not a cause for concern, due in no small part to the fact that mathematics acts as a gatekeeper to higher education, for many years there has been concern around the low numbers of students opting to study higher level mathematics. For example, in 2011 15.8% of Irish students opted to study higher-level mathematics for their Leaving Certificate compared with 63.7% for English; 73.4% for Physics; 81.7% for Chemistry; 74.7% for Biology and 32.2% for Irish. In order to address this problem, and to improve the standard of mathematics among second level graduates, the Irish government introduced the Bonus Points initiative. This initiative rewarded students with 25 additional CAO points[1] if they achieved 40% or higher in their Leaving Certificate mathematics exam. This initiative was introduced in 2012 and the proportion of students studying higher level mathematics jumped from 15.8% in 2011 to 22.8% in 2012 and there has been a steady increase year on year ever since, with 32.9% of students sitting the higher level mathematics exam in 2019. This suggests that the Bonus Points Initiative has been successful in achieving one of its aims, to increase the proportion of students studying higher level mathematics. However, in the same year that the Bonus Points Initiative was introduced a new mathematics curriculum, known locally as Project Maths, which placed a much stronger emphasis on teaching mathematics for understanding and through the use of real life applications was also introduced so this too may have contributed to the recent surge in the

[1] Students are awarded points based on the result they obtain in each subject in the Leaving Certificate and the number of points awarded varies depending on whether the subject is studied at higher or ordinary level. A student who achieves the top grade (H1 = 100% - 90%) at higher level is awarded 100 while the same score at ordinary level is awarded 56 points (see http://www2.cao.ie/downloads/documents/CommonPointsGradingSystem.pdf). The total number of points a student accumulates across six subjects dictates the third level college courses for which they are eligible.

uptake of higher level mathematics. This study seeks to determine which of these factors, if any, play a role in students' decision to pursue higher level mathematics.

In addition to this, in recent years, elsewhere around the globe, many researchers have invested a lot of time and effort into identifying possible reasons why students do not continue to study mathematics in upper secondary school or why they do not choose to study mathematics in its most advanced form. A range of different reasons have been identified including low levels of perceived competence in mathematics among students (Nagy et al., 2010); students' dissatisfaction with mathematics (Hine, 2019); perceived level of difficulty of the subject (Hine, 2019) and the excessive amount of time that the subject requires in order to succeed (Chen & Liu, 2009). However, very little research has been conducted internationally or in Ireland to determine the reasons behind students' decision to pursue higher level mathematics. Uncovering these reasons may help policy makers and educators to develop more strategies that will help to improve uptake of higher level mathematics, retention in higher level mathematics and performance in the subject.

Research Questions

In order to address the aforementioned gaps in the literature this study sought to determine Irish students' reasons for opting to study higher level mathematics. As a result, the research questions underpinning this study are:

1. What are the most influential factors in students' decision to pursue higher level mathematics in Ireland?
2. Do the reasons for studying higher level mathematics differ across gender, and if so, in what way?

Methodology

Research Design and Instrument

To address these research questions the authors chose to employ a survey research design. They utilised a survey which had been designed and validated for use in Queensland, Australia, another jurisdiction where incentives are in place for the uptake of advanced mathematics. Two versions of this survey were developed one for those who chose to study higher level and one for those who chose to study mathematics at ordinary level. For this paper, the authors will solely focus on the survey designed for students studying higher level.

The survey yielded both qualitative and quantitative data. Quantitative data was collected in Section A and Section B of the survey. In Section A student demographics were recorded while in Section B students were provided with eighteen potential reasons for opting to study higher level mathematics and asked to state, on a five-point Likert scale[2], if they agreed or disagreed that each of the reasons played a significant role in their decision. Qualitative data was collected in Section C of the survey when three open ended questions were posed to students. First students were asked to outline any other reasons that contributed to their decision

[2] The Likert Scale used was as follows: 1 = Strongly Agree; 2 = Agree; 3 = Neither Agree nor Disagree; 4 = Disagree; 5 = Strongly Disagree.

to study higher level mathematics and then they were asked of all the reasons listed, including any they provided themselves, which did they consider the most influential reason. Finally, students were then asked to describe how they came to the decision to study higher level mathematics and when they made this decision.

Sample

In total the authors sought to survey 2000 second level students and in order to achieve this number 12 schools were selected using convenience sampling. Five vocational schools and seven secondary schools were invited to participate, a school breakdown that aligned with the national breakdown. However, two schools withdrew and the final sample consisted of three vocational schools and seven secondary schools. In total 1706 senior cycle students responded to the survey and 53.4% (*n* = 911) of the sample were studying higher level mathematics at the time the survey was conducted. It is the responses of these 911 students that will be analysed and discussed in this paper. Table 1 outlines the gender and year group of the sample. 48.3% of the sample were male and 50.7% were female. 54.3% of the sample were in 5th year while the remaining 45.7% were in sixth year.

Table 1

	5th Year	**6th Year**
Male	236	204
Female	254	208
Other/Prefer Not to Say	5	4

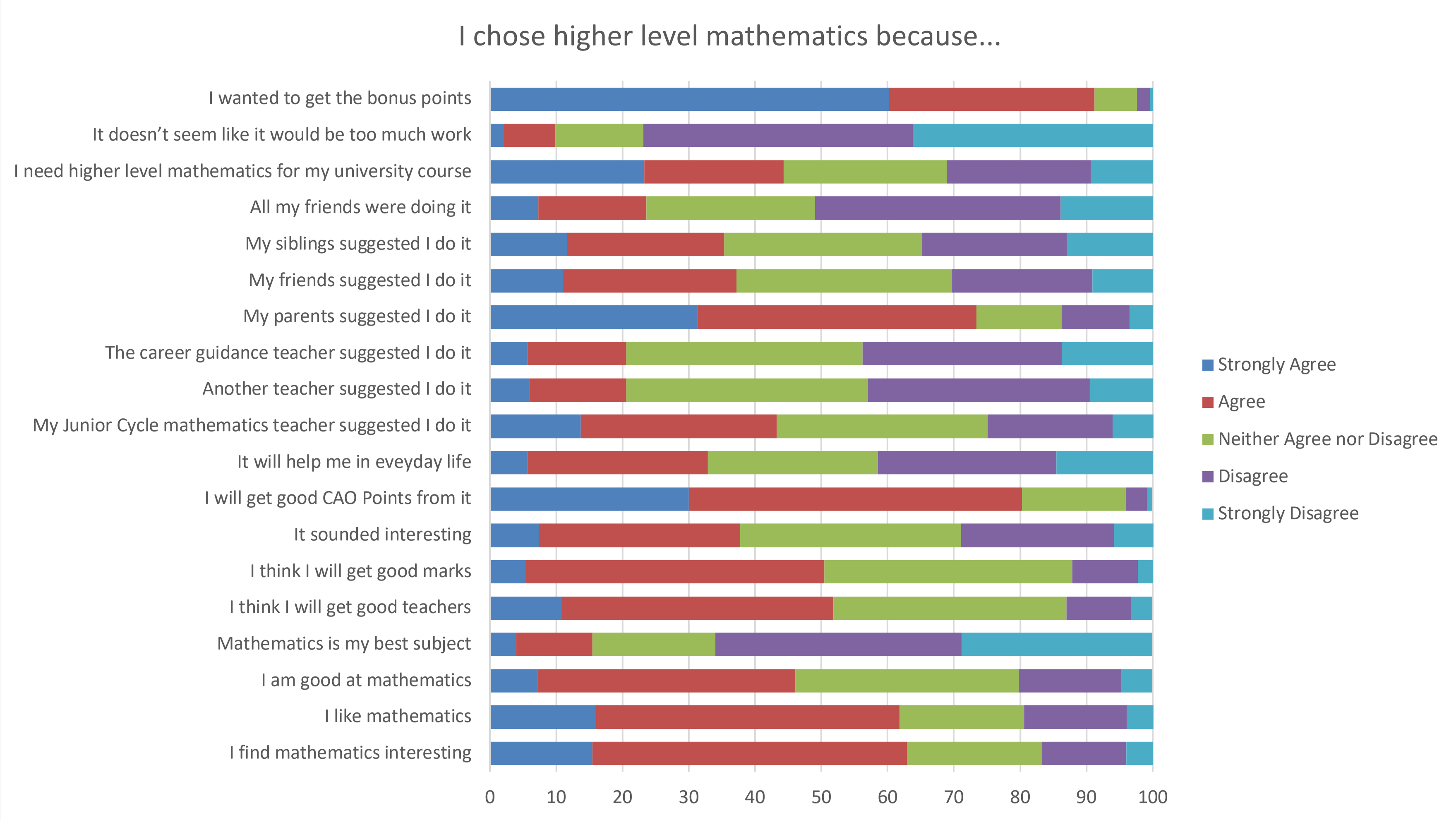
Results

The first research question sought to determine the reasons behind students' decision to study higher-level mathematics and to determine if it was in fact bonus points that led to the recent surge in the uptake of higher level mathematics in Ireland. Figure 1 shows students' level of agreement with the 18 reasons for studying higher level mathematics. It shows that the three reasons which had strongest levels of agreement were *I wanted to get bonus points* (91.2% of students agreed or strongly agreed with this statement); *I will get good CAO points from it* (80.3% of students agreed or strongly agreed with this statement) and *my parents suggested I do it* (73.4% of students agreed or strongly agreed with this statement). This shows that CAO points, and in particular the provision of bonus points, is one of the real driving factors in the uptake of higher level mathematics while parents are the most significant actors influencing a student's decision to study higher level mathematics.

Students were also asked to outline what they believed to be the most influential factor out of all of those listed. The results are presented in Figure 2. In total 893 students offered a response to this question and almost half of these students, 46.2% (*n* = 413) indicated that of all the possible reasons listed bonus points was considered the most influential factor in their decision. A further 7.2% (*n* = 64) stated that the CAO points on offer was the determining factor. This indicates that the majority of students were extrinsically motivated to study higher level mathematics and the main reason for over half of the students in this study opting for

Figure 1

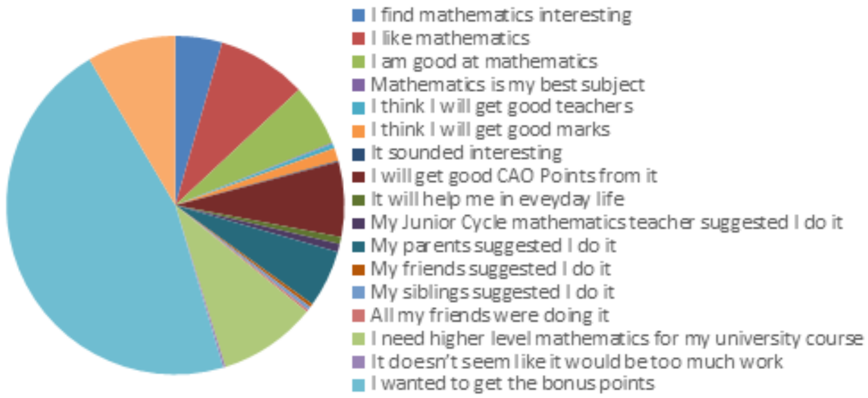
Students' level of agreement with the different reasons for studying higher level mathematics



higher level mathematics was as a result of the points system in place in Ireland. On the other hand, only 4.4% of respondents stated that they chose higher level mathematics because they found it interesting; 5.9% stated that they chose higher level because they are good at mathematics while a mere 0.7% of respondents said they opted for higher level mathematics because the skills developed will help them in their everyday life. In addition to this, a further 8.5% of students gave an answer of "other" or cited more than one reason when asked to outline the most influential factor in their decision. Students who gave such answers were asked to elaborate and analysis of this qualitative data showed that of the 76 students in this category, 58 were there because they offered more than one factor, and 93.1% (n = 54) of these responses included a reference to bonus points (n = 35) and CAO points (n = 19).

Figure 2

Predominant reason for studying higher level mathematics as reported by students



The authors also wished to investigate if the reasons for studying higher level mathematics differed across gender. Of the 18 reasons outlined in the questionnaire, significant differences between the levels of agreement offered by males and females were recorded in nine of the statements. The most notable differences were recorded for the statements *"I need higher level mathematics for my university course"* and *"I think I will get good marks"*. The average male score for *"I need higher level mathematics for my university course"* was 2.50 (s.d. = 1.27) while the median score among this cohort was 2. The corresponding mean among female students was 2.94 (s.d. = 1.27) while the median was 3. As the data for this response was not normally distributed and the data was ordinal a Mann Whitney U test was carried out to determine if the differences recorded were statistically significant. This test showed that the male score was significantly lower than the female score (U = 80627, $p < 0.001$), meaning that male students were more likely to agree with this statement. For the second statement, *"I think I will get good marks"* the mean score for males was 2.42 (s.d. = 0.76; median = 2) while the mean score among females was 2.73 (s.d. = 0.86; median = 3). In this instance the responses were normally distributed but because the dependent variable was ordinal a t-test was not appropriate and so a Mann Whitney U test was again conducted to determine if the differences recorded were statistically significant. This test showed that again the differences recorded were statistically significant (U = 80860.5, $p < 0.001$). This indicates that males were more likely to agree with this statement than their female counterparts. Another interesting finding, when results were compared across gender, was that for all the reasons relating to self-efficacy (e.g. *I study higher level mathematics because I am good at maths* or *I study higher level mathematics because maths is my best subject*) males recorded a significantly lower average score than females, thus suggesting that reasons of this nature are more likely to influence a male's decision to study higher level mathematics compared to females. This indicates that males' self-belief in relation to mathematical performance is higher than females and this played a role in their choice to pursue higher level mathematics.

Conclusion

The first research question sought to determine the influential factors in students' decision to pursue higher level mathematics. The findings have shown the true extent of the influence of the bonus points initiative on students' decision to study mathematics in its most advanced form. While recent studies have suggested that this was the case (Authors, 2019), this is the first study that has definitively shown the influence of this initiative and the authors hypothesise that without this initiative and the CAO points system in Ireland it is highly likely that the proportion of students studying higher level mathematics would be drastically lower. With the points system as the driving force behind students' decision-making process it is little wonder that many teachers believe that students are pursuing higher level mathematics despite struggling with the content (O'Meara et al., 2020). This presents many challenges to students and teachers alike and so the authors recommend that policy makers look at other initiatives to improve students' attitudes towards mathematics in lower secondary school in the hope that intrinsic reasons such as liking mathematics or having a deep-rooted interest in the subject will play a more influential role in students' decision to study higher level mathematics in the future.

This research study also sought to determine if gender played a role in students' reasoning for studying higher level mathematics. The results showed that males were more likely to suggest that their perceived capability in mathematics and their liking of the subject was a reason behind their decision to study higher level mathematics compared to females. While research has pointed to differences in attitudes towards mathematics across gender (Frenzel, Pekrun & Goetz, 2007) for many years this study shows that these differences are contributing to students' decision to study higher level mathematics and so it is important, going forward, that efforts are made to improve female students' attitudes towards mathematics in Ireland in the hope that this will have a knock on effect on female students' opting to study higher level mathematics.

References

Attridge, N., & Inglis, M. (2013). Advanced mathematical study and the development of conditional reasoning skills. *PloS one*, *8*(7), e69399.

O'Meara, N., Prendergast, M., & Treacy, P. (2020). What's the point?: Impact of Ireland's bonus points initiative on student profile in mathematics classrooms. *Issues in Educational Research*, 30(4), 1418-1441.

Chen, A., & Liu, X. (2009). Task values, cost, and choice decisions in college physical education. *Journal of Teaching in Physical Education*, *28*(2), 192-213.

Chinnappan, M., Dinham, S., Herrington, A. J., & Scott, D. (2008). *Year 12 students' and higher mathematics: emerging issues.* University of Wollongong.

Department of Education and Skills (DES). (2013). *Leaving certificate mathematics syllabus.* Department of Education and Skills.

Department of Education and Skills (DES). (2017). *STEM education policy statement 2017–2026.* Department of Education and Skills.

Donovan, W. J., & Wheland, E. R. (2009). Comparisons of success and retention in a general chemistry course before and after the adoption of a mathematics prerequisite. *School Science and Mathematics*, *109*(7), 371-382.

Expert Group on Future Skills and Needed (EGFSN). (2008). *Statement on raising national mathematical achievement.* EGFSN.

Frenzel, A.C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics - A "hopeless" issue? A control value approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education, 22*(4), 497-514.

Garg, K. C., & Gupta, B. M. (2003). Decline in science education in India–A case study at+ 2 and undergraduate level. *Current Science*, *84*(9), 1198-1201.

Goodrum, D., Druhan, A., & Abbs, J. (2012). The status and quality of year 11 and 12 science in Australian schools. *Australian Academy of Science*.

Hine, G. (2019). Reasons why I didn't enrol in a higher-level mathematics course: Listening to the voice of Australian senior secondary students. *Research in Mathematics Education*, *21*(3), 295-313.

Kennedy, J., Lyons, T., & Quinn, F. (2014). The continuing decline of science and mathematics enrolments in Australian high schools. *Teaching Science*, *60*(2), 3.

Nagy, G., Watt, H. M., Eccles, J. S., Trautwein, U., Lüdtke, O., & Baumert, J. (2010). The development of students' mathematics self-concept in relation to gender: Different countries, different trajectories? *Journal of Research on Adolescence*, *20*(2), 482-506.

National Commission on Mathematics and Science Teaching for the 21st Century. (2000). *Before it's too late: A report to the nation from the National Commission on Mathematics and Science Teaching for the 21st Century.* US Department of Education.

Noyes, A. (2013). The effective mathematics department: Adding value and increasing participation?. *School Effectiveness and School Improvement*, *24*(1), 1-17.

Mobile Technology Supporting Algebra and Numeracy skills in a Maths Support Learning Centre

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The mathematical ability of students entering third level education in Ireland has been a cause of concern for some time now. This presentation will introduce an action research project designed to identify the most common mathematical challenges students attending the Maths Support Centre (MSC) at an institute of higher education present with. The paper will explore the effectiveness of a mobile learning application on student engagement and learning. Mobile technology applications remain an underutilised resource within higher education and with an increasing number of students using both smartphones and tablets, this study explores the influence of a specific mathematical mobile application on student learning and engagement. The innovative study will present findings helpful to academic staff teaching mathematics and statistics modules, assisting their planning and teaching approaches. Insights from the research and the use of mobile technology will help create effective resources for the students, which in turn support their mathematical learning experience.

Introduction

The level of mathematical readiness for students entering third level education in Ireland has continued to generate widespread concern both in the academic community and the general public in recent times (e.g. O'Donoghue 2002, Gill et al. 2010). One popular approach to tackle these mathematical issues in third level institutions in Ireland is the setting up of maths support centres (MSC). Lawson et al. (2003) described mathematics support as

> "a facility offered to students (not necessarily of mathematics) which is in addition to their regular programme of teaching, lectures, tutorials, seminars, problem classes, personal tutorials, etc." (p.9).

This study aims to analyse the prevalence of mathematical topics students present with at the Maths Learning centre at an institute of higher education. In identifying the student's mathematics "trouble spots" the study will develop effective supports for a number of these. Initial data collected from the IT Sligo maths support centre indicate that the areas of basic algebra and arithmetic that are causing most difficulty with attending students. The research will make use of technology to help provide the services to assist the students.

The study explores the influence of a mobile application on student engagement and how mobile technologies might be used to support student learning. Mobile learning technologies have been recognised as emerging tools to improve teaching and learning (Traxler 2007). The use of textbooks in class are being challenged by the pace of development of mobile technologies which have increased at a significant pace over the last few years.

> "More and more young people are now deeply and permanently technologically enhanced, connected to their peers and the world in ways no generation has ever been before. […] More and more of what they need is available in their pocket on demand" (Prensky, 2010, p. 2).

In the US, the National Council of Teachers of Mathematics (2000) considered technology as

> "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (p. 3).

As documented by Olive et al (2010), it is envisaged that the technologies will also help support the students and improve the way they approach, learn and understand the problematic concepts. There are many potential benefits of using mobile technologies for learning such as facilitating learner-generated contexts, as well as personalising the learning for the student as detailed by Cochrane (2010). Calder et al (2016) describes how the features of mobile technologies enable alternative ways to explain, process, encounter and investigate mathematical concepts. The learner to control the flow of information and choose the amount of mathematical content that they wish to access at any one time. These potential benefits make mobile technology seem an extremely useful tool for the learning and teaching of mathematics. This study will explore the how mobile technologies might be used to support student learning in the centre utilising a highly adaptive mobile application tailored to a student's skill and maths level.

Maths Support Centre at IT Sligo

This study takes place in the Maths Support Learning Centre (MSC) at Institute of Technology Sligo - a free drop-in centre to support all IT Sligo student's mathematical needs. Cronin et al. (2015) highlight the extent to which the landscape of maths support centres in higher education institutions in Ireland has changed for the positive since 2008.

The Maths Support Centre at the Institute of Technology, Sligo was set up in 2012 as a special inter-school initiative of the institute with the purpose of supporting students' mathematics learning across all programmes by providing a dedicated area with supervised access and resources to support students in a relaxed environment. The centre delivers appropriate support services for students on service mathematics courses and addresses the mathematical needs of special groups. The findings of this study will help assist MSC and academic staff members with mathematics and statistics modules to plan, enhance and potentially change their teaching approaches of the highlighted topics.

Mathews et al. described the main aims of the maths support centre as

> "to address issues surrounding the transition to university mathematics and to support students' learning of mathematics and statistics across the wide variety of undergraduate courses that require an understanding of mathematical concepts and techniques." [15, p. 3]

This MSC opened initially for 2 hours during academic term in semester 1 and 3 hours in semester 2. The centre was proving to be a success and a valuable resource to both struggling and able students based on feedback received from students who used the centre. However, large increases in the number of students attending the MSC over the previous semesters led to a re-evaluation of opening hours. With the support of the newly appointed Educational Development Manager, Institute management agreed to hire a MSC manager and increase the number of contact hours to 24 hours per semester. This has allowed the MSC to extend the existing mathematics support it currently provides to both incoming and existing students in terms of the drop-in clinics, and expand MSC services to include structured mathematics sessions Monday -Thursday, provide revision sessions close to examination times, and deliver online support tutorials, group tutorials and one to one sessions.

The current restrictions and safety concerns around COVID-19 posed a challenge for the proposed study. IT Sligo's delivery plan for the 2020/21 academic year involves a blend of remote delivery and some on campus delivery but only for activities that require active face-to-face engagement with academic staff. As a result, to mitigate any effect of the COVID-19 pandemic on the MSC and on face to face meetings with students, the centre went fully online for the 2020-2021 academic year. Through the booking system on the MSC Moodle page, students can book one to one online sessions or group tutorials with the various maths support tutors. To reduce the impact of COVID-19 on student numbers to the centre, the marketing team at IT Sligo promote the centre through the different college social media platforms. Regular emails are also sent by staff and through the student's union to all students highlighting the opening hours for the MSC and the free online one to one mathematical service offered. All students have access to the MSC Moodle page where they get more information on who to contact, the opening days and times. The MSC is in weekly contact with the maths lecturers at IT Sligo and any student that the lecturer identifies as struggling in their class are also encouraged to seek help and get in contact with the math support centre tutors.

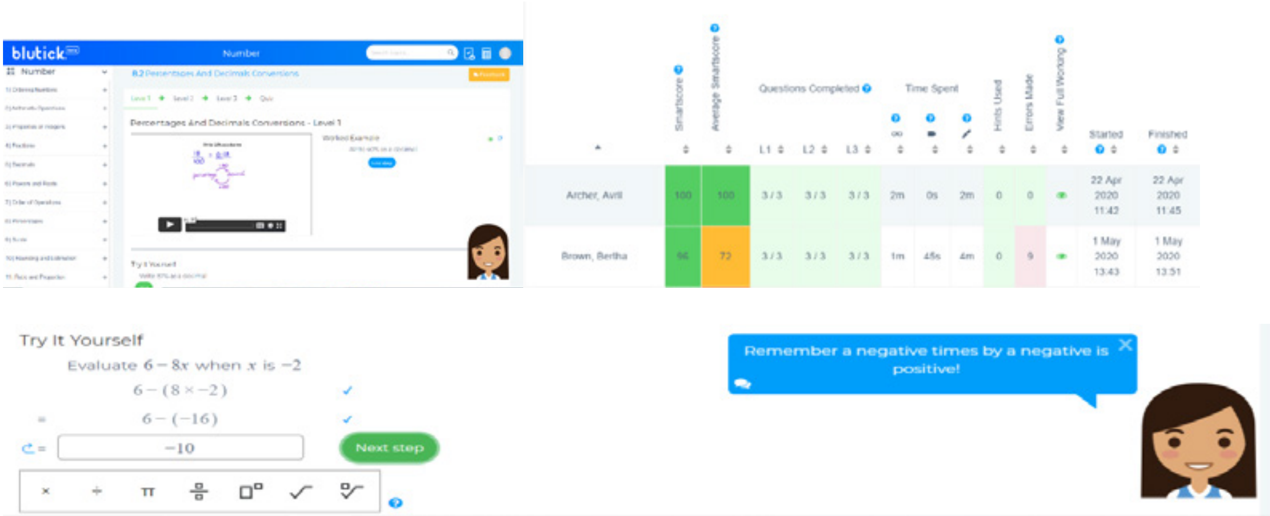
Blutick Maths Application

The maths application chosen for this study is the Blutick maths app (https://blutick.com/). It is an AI powered maths teaching platform and works as a reactive AI, by applying a large number of algorithmic tests to what the students input. Based on the student's response it then generates a probabilistic assessment of the best response to any mistake made by the student. The app takes into account a variety of different factors, including the stage of the problem the student is currently at, the question context and the mathematical working itself. Positive feedback is generated and designed to both encourage the student and point them in the right direction. There are many thousands of apps available to help students with their mathematical difficulties. Most however simply mark the students attempt or provide the answer to the problem. The Blutick app has the ability to indicate to students where they made their mistake giving the students a better understanding of the problem at hand. Calder & Campbell (2016) argue that apps that provide instantaneous feedback of this type allow the learner to both experiment and take risks with their learning and knowledge intake.

The app provides learning in small accessible chunks by guiding and teaching the students through the different mathematical concepts with a combination of short video content, fully worked out examples, interactive questions and quizzes. The app also processes and stores the lines of work that students enter into the system. It generates suggestions for these scenarios that initially do not offer the intelligent feedback. These scenarios are then examined, with relevant feedback built into the system, ensuring the app is learning over time. As the student works through the various tasks the app identifies what feedback works best, creating a more effective feedback and teaching tool. According to Kukulska-Hulme (2010), mobile technologies allow the learner to improve their knowledge whenever the need arises, with learning occurring in everyday and more varied environments. The task summary page provides more detailed information on each of the tasks undertaken. The instructor can view the amount of time spent viewing the various videos, the number of questions completed at the different levels, the number of mistakes made, and the number of hints used. Full workings of each question completed by the student is also available together with any feedback offered by the AI instructor. Task summaries also allow the instructor to keep track of the tasks set and the work that the students are completing. The app allows the learner to work at their own pace and repeat the necessary mathematical skills as many times as they wish in order to master the concepts undertaken. Rather than just guessing the solution, the app encourages the students to think about the various steps involved in arriving at the solution. It promotes active learning rather that passive learning.

Figure 1

Shows some screenshots from the Blutick app.



Study Methodology

This study stems from the researchers positionality as a mathematician with over six years' experience of teaching mathematics online. The researcher is part of the digital champion initiative, a collaboration between GMIT & LYIT (www.digitaled.ie), where he is a digital champion for the School of Business and Social Sciences mentoring staff on developing blended and online teaching and learning experiences.

IT Sligo students volunteer to be part of the study. Any student, 18 years and older who present themselves to the Maths Support Centre are deemed eligible to take part in the study.

At the beginning of the academic year, students across the college complete a maths diagnostic test to try to identify any weaknesses in their numeracy skills. Lawson et al. (2003) describes diagnostic testing as an effective way to highlighting widespread areas of mathematical weakness. The diagnostic test is accessed using the college virtual learning environment (VLE) platform Moodle in the form of an online quiz. The quiz consists of twenty questions across a number of basic numerical topics with the difficulty level of the questions roughly equivalent to Ordinary Level Junior Certificate Maths. Each student is given instant feedback on completion of the test through a raw mark. As a result, students obtaining less than fifteen out of twenty questions correct are encouraged to attend the MSC to improve the numeracy skills and mathematical competency. Any student that has performed poorly in the maths diagnostic test or have presented themselves to the MSC tutors in need of improving their mathematical skills are invited to participate in the study.

The researcher has reviewed current literature as well as engaging with colleagues running similar maths support centres in other Irish HEI's to help identify the most appropriate data to capture from the study group and aims to identify the different mathematical topics that the students at IT Sligo present with at the centre.

For this study data has been collected from students who have attended the MSC from AY2018/19 and AY2019/20. Course of study, college year, area of difficulty, how often the student attends the centre were some of the data that was collected from each student visit. This was primarily done to gauge the number of visits per semester, the percentage of visits per year and course of study and the area of difficulty. Data collected during this time period, shows there has been on average 350 students visits each semester, with 125 unique visits. Figure 2 gives an overview of the study and the steps involved in collecting both qualitative and quantitative data.

Figure 2

Mobile technology supporting learning in the MSC study overview

Towards the end of semester one of the 2020/2021 academic year a pilot study was conducted with a group of students to test the Blutick app and identify any difficulties or questions that the students may have in relation to operating the app. During semester two of AY 2020/21 25 students started to engage with the mobile maths app to complement their learning. Initial student data collected from MSC centre visits highlighted the areas of basic algebra and basic arithmetic that were causing most difficulty with attending students. Approximately 70 tasks in both the areas of algebra and basic arithmetic were set for the students to complete. The topics covered range from working with fractions, percentages, and basic arithmetic skills to simplifying expressions and solving both linear and quadratic equations. Students typically use the app in their own time for one to two hours a week over 12 weeks of semester working through various questions which are supported with short videos and worked examples. Reflective events are held throughout the study to capture student feedback. Detailed and comprehensive quantitative and qualitative data will then be collected which will help inform the way in which the use of a mobile maths app will assist in the students learning. The app allows many ways to view a student's progress and engagement with the app. The mark book element of the app allows detailed information on individual tasks to be collected and allow a deeper understanding on where each student is having difficulty. Each task set for the learner has three levels of difficulty followed by a quiz. The student must get six questions correct in the quiz before the task is marked complete. A smart score is assigned to each task as completed by the learner. The score represents the number of questions that are answered correctly but also considers the degree of understanding of the task by looking at other factors such as skipping questions, making mistakes, and using the hints available. Analysis of these results will help measure the effectiveness of this mathematical mobile application on student learning together with identifying the various trouble spots as the students work through the various tasks assigned. The Mathematics and Technology Attitudes Scale (MTAS) (Pierce et al. 2007) together with focus groups will be used to collect additional data, gauge student's engagement and progression with the app and to identify potential areas of improvement.

The MTAS is a questionnaire with five subscales: affective engagement, behavioural engagement, mathematical confidence, confidence with technology and attitude to using technology for learning mathematics. The focus groups for the first cycle will be held at the end of semester two in 2021 and again at the end of the summer for the group involved in the second cycle of the study. A focus group interview was deemed a most suitable qualitative technique for use in this situation. Denscombe (2007, p.115), states a "focus group consists of a small group of people, usually between six and nine in number, who are brought together by a trained moderator (the researcher) to explore attitudes and perceptions, feelings and ideas about a topic". Accurate records of student visits to the centre during the academic years 2020/21, 2021/22 will be collected and maintained, and the information gleaned will help identify the mathematical issues facing the students with a view of providing unique ways to alleviate them.

Results & Discussion

The initial result from the first 2 action research cycles will be presented. The first cycle or cohort of students are completing 70 tasks in the areas of algebra and basic arithmetic over a 12-week semester block, May 2021. The second action research cycle will consist of a group of students that partake in the college summer school over a 3-week period towards the end of August 2021.

The results will examine how the students engaged with the app and whether the app assisted the student to master the various concepts undertaken. Feedback from the students will look to identify the positive features of the app from their use of videos, worked examples and the ability to work at their own pace. Data will also help identify both the mathematical difficulties the students encountered and their comprehension of information in the areas of basic algebra and numeracy. Tasks summaries will identify the number of questions completed at the different levels, the number and types of mistakes made together with the number of hints used. The ability to view every line of the student's workings as they complete each task, together with the feedback the system generates, will allow a more detailed understanding of both the key problematic areas and how the students used the application to improve their mathematical ability over time.

Conclusion

This presentation will provide an overview of the action research project on mobile technology supporting algebra and arithmetic skills in a maths support centre at an Irish HEI. The worked examples provided by the mobile application, such as the line by line feedback and short videos, describe underlying concepts in mathematical tasks and help scaffold the students, enabling them work through problems on their own leading developing a deeper understanding. The mobile application programme students are engaging with assist their mathematical comprehension will be presented along with the results from two action research cycles.

One of the biggest challenges is both the recruitment of students to the study and keeping the students on track and engaging with the app during a busy academic semester. The current restrictions and safety concerns around COVID-19 has presented another challenge for the proposed study, even though the MSC was setup to provide online support. Despite the many challenges, mobile learning technologies offer enormous potential to enhance mathematical learning and remain an under used resource in third level education.

References

Calder, N. S., & Campbell, A. (2016). Using mathematical apps with reluctant learners. *Digital Experiences in Mathematics Education.* https://doi.org/10.1007/s40751-016-0011-y.

Cochrane, T. D. (2010). Exploring mobile learning success factors. *Research in Learning Technology, 18*(2). https://doi.org/10.1080/09687769.2010.494718.

Cresswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Sage.

Cronin, A., Cole, J., Breen, C., Clancy, M., & Ó Shea, D. (2015). *An audit of mathematics Learning support provision on the island of Ireland*. National Forum for the Enhancement of Teaching and Learning in Higher Education.

Denscombe, M. (2007). The good research guide for small-scale social research projects (3rd ed.). McGraw-Hill.

Gill, O., O'Donoghue, J., Faulkner, F., Hannigan, A. (2010). Trends in performance of science and technology students (1997–2008) in Ireland*, International Journal of Mathematical Education in Science and Technology, 41*(3), 323-339.

Kukulska-Hulme, A. (2010). *Mobile Learning for Quality Education and Social Inclusion.* Policy Brief published by UNESCO Institute for Information Technologies in Education. http://iite.unesco.org/policy_briefs/

Lawson, D., Croft, A., & Halpin, M. (2003). *Good practice in the provision of mathematics support centre.* LTSN Maths, Stats & OR Network.

Mathews, J., Croft, T., Lawson, D., &Waller, D. (2012). *Evaluation of mathematics support centres: A review of the literature.* Sigma Centre for Excellence in Mathematics and Statistics Support.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* NCTM.

O'Donoghue, J. (2002). *Mathematics: transition from second level to university.* Fr Ingram memorial lecture to the Irish Mathematics Teachers Association.

Olive, J., & Makar, K., with V. Hoyos, L. K. Kor, O. Kosheleve, & R. Straesser (2010). Mathematical knowledge and practices resulting from access to digital technologies. In C. Hoyles & J. Lagrange (Eds.), *Mathematics education and technology – Rethinking the terrain*. The 17th ICMI Study (pp. 133 -177). Springer.

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education, 48*(2), 285–300.

Presky, M., 2010. *Teaching digital natives. Partnering for real learning*. SAGE

Traxler, J. (2007). Defining, discussing and evaluating mobile learning: The moving finger writes and having writ…. *International Review of Research in Open and Distance Learning 8*(2).

An Investigation into Pre-Service Post-Primary Mathematics Teachers' Knowledge of Problem-Solving

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We discuss the mathematical problem-solving proficiency of pre-service post-primary mathematics teachers in an Irish University, where the participants were undertaking concurrent teacher education programmes. The conceptual framework of this study is based on the work of Chapman (2015) who outlines that problem-solving proficiency is a key component in the effective teaching of problem-solving. We describe the range of data collection exercises undertaken as part of this study, and report in detail on one whereby the participants undertook two problems following a 'Think Aloud' protocol in recorded interviews. The interviews were analysed using a general inductive approach and five main themes were identified in the participants' approaches to problem-solving. We report here on the analysis of the interviews and the role of problem-solving proficiency in the teaching of problem-solving.

Introduction and Background

In this paper, we report on a project that investigates the capacities of pre-service teachers to teach problem-solving. This project is being undertaken by the first author as her doctoral research. In this introduction, we provide the background to the project and motivate our research questions. We outline the key issue of characterising problem-solving in mathematics, outline its central role in school curricula (discussing the performance of Irish school students on international assessments of mathematical problem-solving), and link this role to the importance of teacher preparation for teaching problem solving.

Problem-Solving in Mathematics

Acknowledging Polya's (1945) efforts to put problem-solving at the centre of mathematical instruction, Schoenfeld (1992) attests that there is a wide variety of meanings for the terms "problems" and "problem solving": this has been highlighted more recently by Lester (2013). The variations in these definitions are further discussed in Owens and Nolan (2019). Recognising the need for a clear definition the following Three Key Characteristics were identified, effectively defining our perspective on problem-solving: (i) Problem-solving includes a goal; (ii) it is not immediately clear to the problem-solver how to achieve the goal; (iii) the problem-solver must organize prior knowledge to generate reasoning towards achieving the goal.

Problem-Solving in School Curricula

It is evident that problem-solving plays a key role in mathematics education nationally and internationally (Shiel & Kelleher, 2017). Mathematical problem-solving occupies a privileged position in the Irish post-primary mathematics syllabus, and is at the centre of both the Junior Cycle and Senior Cycle curricula.

Problem-Solving Capacities in Ireland and Internationally: PISA and TIMSS

In Shiel and Kelleher (2017), information from both the PISA 2012 and TIMSS 2015 reports regarding Irish students' problem-solving competencies was analysed. The PISA (2012) report showed that Irish students performed above the OECD average in applying mathematical concepts and in relating solutions back to the original problem. However, the report highlighted that Irish students were less capable in the process of translating real-world problems into mathematical representations that are productive in solving problems, relative to other problem-solving processes. The TIMSS test results showed that Irish students demonstrated most proficiency in tasks that required recall of memorized facts, carrying out learned procedures, and retrieval of information from representations such as tables or charts. Overall, this indicates a need to improve the problem-solving capacities of Irish students.

The Preparation of Mathematics Teachers

The role of the teacher plays a critical role in students' learning. According to Hattie (2012, p. 18), "teachers are among the most powerful influences in learning". Hattie suggests that while it is important what teachers do, it is most important that the teachers can effectively review the impact their actions have on their students' learning. Teacher education programmes are viewed as a critical stage in teachers' development (Teaching Council of Ireland, 2017). During which, prospective teachers' beliefs regarding teaching and learning should be considered and challenged: these beliefs will be brought forward into their professional practice (Teaching Council of Ireland, 2017).

Research Questions – A First Look

Our research questions are given in detail below, but we note at this point that in light of the discussion above, we are motivated to ask: do pre-service teachers hold the appropriate capacities to teach mathematical problem-solving in secondary schools in Ireland? How are these capacities to be developed? And what are these capacities? We now discuss the different aspects of the conceptual framework within which we ask our specific research questions.

Conceptual Framework- Scope of This Study

Our conceptual framework addresses several related aspects of the study. We revisit the concept of mathematical problem solving, and then discuss the concepts of problem-solving work and strategies; learning problem solving; teaching problem solving and teachers' capacities for teaching problem solving. As discussed above, and at more length in Owens and Nolan (2019), we have associated Three Key Characteristics with the concept of problem solving. This has implications for the selection of problems used in our study and for our interpretation of students' actions and words in their engagement with our study.

As stated above in the Three Key Characteristics, the problem-solver must organize prior knowledge to generate reasoning towards achieving the goal of the problem. Mason et al. (2011) highlight that the understanding of mathematical content is one factor in mathematical thinking. This is supported by Polya (1945) who states that when problem-solving, it is essential for the problem-solver to have some knowledge of the subject matter and have the ability to select the relevant items from this pre-existing knowledge. He notes that to reach a solution, the problem-solver must recall previously solved problems, definitions and other mathematical facts. Polya explains that heuristics, the study of procedures, are independent of this subject-matter. He states that since the aim of heuristics is generality, it is therefore applicable to a variety of problems (Polya, 1945). Since heuristics can be useful in producing successful problem-solving, it is essential for teachers to be aware of the different heuristics that are accessible to their students (Lester, 2013). However, as pointed out by Lester (1994) it is not enough to teach about heuristics but heuristics should be practiced through a variety of problems. This is supported by Mason et al. (2011) who states that there is a wide acceptance that it is essential to develop an understanding of what happens during the process of attempting a problem along with developing and becoming proficient in strategies.

In addition to teachers needing a knowledge of subject matter, Polya (1945) points out that it is essential for a teacher to have a positive disposition towards problem-solving if their students are to have a positive attitude. The teaching of problem solving does not simply rely on the techniques employed but it "comes from the identity and integrity of the teacher" (Palmer, 1998 p.149) meaning "we teach who we are" (p.2). Lester and Kroll (1993) declare that the affective domain is an important contributor to problem solving behaviour. The affective domain includes attitudes, feelings and emotions. Beliefs impact on problem solving performance since beliefs contain their subjective knowledge about self, mathematics and the topics dealt with in particular mathematical tasks (Lester & Kroll, 1993). Similarly, Mason et al. (2011) identify the affective domain as an influential factor in problem-solving. Mason demonstrates the importance for teachers to understand the role that teachers play in creating an environment which promotes confidence and elements of success for their students.

Lester (2013) highlights that it is widely agreed that the development of students' problem-solving capabilities is a main goal of mathematics instruction. The realisation of this goal involves multiple factors such as metacognition and beliefs along with factors associated with the teacher (Schoenfeld, 1992). Schoenfeld (1992) describes metacognition as one's own knowledge about one's own cognitive processes. He highlights that metacognitive ability plays an essential part in problem solving, and he notes that this is the structure that that allows problem solvers to dismantle more challenging problems into subtasks, prioritize and order the importance of each subtask and then complete each subtask in sequential order. Although Lester (1994) highlights the benefits of monitoring behaviours during problem-solving, he identifies that it is difficult to teach students monitoring behaviours. Mason et al. (2011) state that monitoring behaviours can be developed through practice of questions with particular focus on reflection. They note that it is success in overcoming situations of being stuck in a problem that promotes positivity in the problem-solver. Through reflection of feelings involved while problem-solving with actions, it can help the problem-solver relate these feelings when they arise again in new situations to productive actions (Mason et al., 2011). The Rubric writing approach allows the problem-solver to monitor their progress and give structure to the problem-solver through avoiding switching between different plans of attack (Mason et al., 2011).

Thus our conceptualisation of mathematical problem solving begins with the Three Key Characteristics above, and acknowledges the central role played by the employment of heuristics or other strategies, as well as the importance of metacognition and affective factors.

Our conceptualisation of teaching problem-solving draws heavily on the synthesis of research on this topic carried out by Chapman (2015). The role of the teacher is to support their students' development of problem-solving skills, and of the appropriate habits of mind (metacognitive skills) and affective factors (productive disposition) that underpin successful problem-solving. To identify what capacities teachers need to teach problem-solving effectively, Chapman (2015) conducted an extensive review of the literature with research articles dating from 1920 to 2015. Chapman identifies three main components that make up the mathematical problem-solving knowledge for teaching. These components are: 1) Problem-solving content knowledge (PSCK), 2) Pedagogical problem-solving knowledge (PPSK), and 3) Affective factors and beliefs. These three components are made up of six different capacities. PSCK is made up of the following three capacities; knowledge of problems, knowledge of problem-solving, and knowledge of problem- posing. The two capacities that make up PPSK are; the knowledge of students as problem-solvers, and the knowledge of instructional practices. Chapman's identification of these capacities align with frameworks offered by Lester (2013) and Guerin (2017).

This paper focuses on the capacity *knowledge of problem-solving*. This capacity entails teachers' proficiency in problem-solving and in understanding the nature of approaches to problem solving. Chapman (2015) outlines that teachers' own proficiency in problem-solving is essential for them to be able to understand students' approaches and predict the implications of these approaches. Problem-solving proficiency is defined as "what is necessary for one to learn and do genuine PS successfully" (Chapman, 2015, p.9). Kilpatrick et al. (2001) state that the components of mathematical proficiency are not one-dimensional and are interdependent. Chapman proposes that since mathematical proficiency is interwoven, then problem-solving proficiency is too. She suggests that to support students in developing their problem-solving proficiency, teachers must be able to solve the problems and also understand the elements associated with the development of problem-solving proficiency.

Research Questions

In the context described in our introduction and in the setting of the conceptual framework just described, we now state our full set of research questions. Question 1:What do pre-service teachers understand a mathematical problem to be? Question 2 (a):Are pre-service teachers proficient in problem-solving? Question 2(b): Are taught strategies implemented while problem-solving? Question 3:What are pre-service teachers' capacities in relation to problem posing? Question 4:What beliefs do pre-service teachers hold regarding problem-solving? The research question addressed in the present study is research question 2 a).

Methodology

Participants were recruited on a voluntary basis and were pre-service mathematics teachers (PSMTs) undertaking a concurrent initial teacher education programme. The participants were taking a module that includes the study (and practice) of mathematical problem-solving. This module adopted the Rubric Writing approach to problem-solving (Mason et al., 2011).

The PSMTs were interviewed on a one-to-one basis by one of the researchers (EO). The interview consisted of the PSMTs being given two mathematical problems and asked to solve them, following a ‘Think Aloud’ protocol. Working on problem solving often involves strategies and involves metacognitive and affective aspects. We used think-aloud to create a space for students to display these. Cowan (2019, p. 1) describes the ‘Think Aloud’ process as “a voluntary activity in which learners having been asked to tackle a relevant task, talk their thoughts out aloud, while engaging with the task”. The interviews came to an end when the participants had nothing further to add to their attempt.

All the problems used in the interviews were taken from the NRICH website (NRich, 2019). The problems dealt with the topics of probability, geometry, trigonometry, number, and proportion and ratio. To categorise the tasks, both researchers independently compared the task to the following two criteria of a problem: 1) there is a goal, 2) it is not clear how to reach the goal.

The interviews were recorded and transcribed. Cohort One completed one interview during the module while participants in Cohort Two and Cohort Three both conducted two interviews. These were conducted near the beginning of the module when problem-solving had been introduced in the course content and where limited instruction in the Rubric Writing method (Mason et al, 2011) had been received. The other interview was post-module. Nine participants in Cohort One completed the single interview for that cohort. Five participants in Cohort Two completed the first interview and three of these five completed the second interview. Five participants from Cohort Three completed both interviews.

The data were analysed using a general inductive approach in order to account for both the different strategies that participants may employ and affective utterances that would occur while problem-solving. The data analysis of the interview transcripts involved the repetitive process of coding, comparing, and grouping the data with similarities to construct categories (Jones and Alony, 2011).

Results

The analysis described above led to the identification of five main themes (or categories) in all three cohorts. These categories are; *Introduce, Productive reasoning, Unproductive reasoning, Resilience,* and *Identity.* Analysis of the interviews from Cohort Two and Cohort Three found that there was evidence of participants questioning themselves. This is referred to as *Productive Questioning* and is viewed as a sub-category of *Productive Reasoning.* Revision of the transcripts of Cohort One were done in order to identify if

Productive Questioning was evident, and it was not found to be so. Participants 1-9 were in Cohort One, participants 10-14 were in Cohort Two, and participant 15-19 were in Cohort Three. The excerpts below exemplify each category: in these, Px/Cy refers to Participant x of Cohort y. Table 1 shows the number of occurrences of each theme in the interviews. Columns 1 and 2 indicate the relevant cohort interview and problem respectively.

Table 1

Occurrence of themes count for every problem by the three different cohorts.

Cohort	Problem Number	Introduce	Productive reasoning	Unproductive reasoning	Resilience	Identity	Productive Questioning
1	1	0	23	8	2	8	0
[N=9]	2	34	38	19	25	22	0
2 Pre	3	2	21	4	3	2	7
[N=5]	2	24	26	1	5	14	31
2 Post	4	4	16	0	2	3	6
[N=3]	5	9	12	4	2	5	8
3 Pre	3	3	10	4	0	2	0
[N=5]	2	3	8	8	3	5	4
3 Post	4	5	9	12	5	6	1
[N=5]	5	9	12	8	4	3	5
	Total	93	175	68	51	70	62

Introduce refers to the introduction by the problem-solver of diagrams, constructions within given diagrams, and notation. Mason et al (2011) highlights that the introduction of diagrams and appropriate notation plays a key role in organizing information when problem-solving. Examples of participants' use of *Introduce* include:

P10/ C2: "ok if I set x as time, told travels, x +20 +y"
P11/C2: "So I am going to start by drawing a picture."

The *Productive Reasoning* category includes statements made or actions taken by the participants that promote progress towards a solution of the problem. This category includes the interpreting of information given in the question, use of prior knowledge, specializing and generalising.

P13/C2: "well we know that ¼ is more than ⅕ and less than ⅓."
P15/C3: ""So then I would use Pythagoras to look at the top triangle."

As stated above, *Productive Questioning* was evident in the interviews of both Cohort Two and Cohort Three. This category refers to the participant questioning themselves on their work towards a solution, their chosen strategy, or how to proceed. This *Productive* *questioning* is seen as a sub-category of *Productive reasoning* as the questioning helped participants towards achieving a solution.

> *P13/C2: "Can I find the distances from the courtyard that would be helpful? … Is there a way to make right angled triangles to help?"*
>
> *P12/C2: "So his average speed overall was 93.5km/h. How does that help?"*

Unproductive reasoning involves actions or statements which do not help (or even constrict) the problem-solver from progressing or being successful. This includes procedural errors, making assumptions, misconceptions, and persisting with a line of reasoning despite previously stating that it is incorrect.

> *P16/C3: "we assume that he's at his average speed for almost an hour".*
>
> *P18/C3: "I'm just going to have to guess 3.37 and I don't even know why."*

Resilience includes statements that reflect a participant learning from mistakes, demonstrating a willingness to restart or try a new strategy, and demonstrating a positive response when faced with difficulty.

> *P13/C2: "So what is some other ways?"*
>
> *P18/C3: ""I'm just writing down I'm stuck. I'm writing down where I'm stuck. I'm trying to, I don't know how to find a formula to find A the time after."*

Statements that indicate a participants' self-belief and confidence make up the *Identity* category. This involves the affective domain which is seen as an important influence on problem-solving behaviour (Lester & Kroll, 1993).

> *P12/C2: "I just hope I'm on the right path here. […] I'll see where it goes."*

Conclusion and Next Steps

From the analysis of the interviews, five main themes were identified. With the exception of Problem One, *Introduce* appeared to be a starting point for *Productive Reasoning*. This was evident through the introduction of diagrams, notation, and constructions within given diagrams. This use of *Introduction* as a starting point of a strategy indicates proficiency amongst the PSMTs in both problem-solving and the implementation of the taught strategy provided by Mason *et al.* (2011). However, the exception mentioned indicates that use of this strategy is tied to the problem under consideration. The use of different elements of *Introduction* signal towards proficiency as outlined in the conceptual framework through the use of heuristics. In Cohort Two and Cohort Three, *Productive Questioning*, as described above, was evident in the PSMTs' problem-solving attempts. This questioning was not evident in Cohort One. *Productive Questioning* statements were particularly prominent in both Problem 2 and Problem 5 which were both trigonometry problems. It is encouraging to note that statements categorised as *Productive Reasoning* were not only the most common categorisation but also outnumbered *Unproductive Reasoning* statements by a factor of four. This indicates problem-solving proficiency amongst the PSMTs through demonstrations of procedural fluency, strategic competence, and conceptual understanding. However, this view must be tempered by the fact that Unproductive Reasoning statements outnumber Resilience statements in all but two of the rows of Table 1. Resilience, including the ability to re-start, reflect, and identify misconceptions is vital in problem-solving as identified by Mason et al. (2011). While we have not coded Identity statements as indicating a positive or negative disposition towards problem-solving, it is noteworthy that this important element of our overall problem-solving framework emerges in the interviews. It may be of concern that there an increase in the *Unproductive Reasoning* statements of Cohort Three participants between the pre- and post-module interviews. There did not appear to be any other increase in the four other categories between the pre- and post-module interviews. Future work will involve the analysis of the interviews in terms of explicit implementation of taught strategies by the PSMTs when problem-solving.

References

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *LUMAT (2013–2015 Issues)*, *3*, 19–36.

Cowan, J. (2019). The potential of cognitive think-aloud protocols for educational action-research. *Active Learning in Higher Education*, *20*(3), 219–232.

Guerin, A. (2017). The beliefs of primary mathematics prospective teachers about mathematical problem solving [University of Limerick].

Hattie, J. (2012). Visible Learning for Teachers: Maximizing Impact on Learning Routledge.

Jones, M., & Alony, I. (2011). Guiding the use of grounded theory in doctoral studies – an example from the Australian film industry. *Faculty of Commerce - Papers*, 95–114.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, *10*(1 & 2), 245–278.

Mason, J., Stacey, K., & Burton, L. (2011). *Thinking mathematically* (2nd ed.) Prentice Hall.

NRich. (2019). *Nrich Mathematics*. Nrich. https://nrich.maths.org/

Owens, E., & Nolan, B. (2019). An investigation into the problem-solving capacities of preservice post-primary mathematics teachers. *Proceedings of the British Society for Research into Learning Mathematics*, *39*.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton University Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (Vol. 1, pp. 334–370). MacMillan.

Shiel, G., & Kelleher, C. (2017). *An evaluation of the impact of project maths on the performance of students in Junior Cycle mathematics*. 1–190.

Teaching Council of Ireland. (2017). *Initial teacher education: Criteria and guidelines for programme providers* (pp. 1–32).

Combining Student-Led Lab Activities with Computational Practices to Promote Sensemaking in Financial Mathematics

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Creating an engaging enquiry-based teaching environment in computational science modules is fundamental in the development of a deep understanding of the underlying principles, relations, and core concepts in the related subject. In this contribution we describe how we have implemented computational lab practices in a Computational Finance undergraduate module to foster students' computational thinking and overall learning. In particular, we discuss the use of computation jointly with groups activities as central elements to facilitate sensemaking in Financial Mathematics and we show how the modern definition of "inclusive computation" has been embedded harmonically in the design of tailored student-led lab practices.

Introduction

In the last few years, the need for well-prepared STEM graduates, equipped with data analysis and computational modelling skills, has increased in both modern academia and industry. There has been increased attention on computation in mathematical subjects (Lockwood et al., 2019), and consequently traditional undergraduate education has been adapted in different ways. One strategy has been to make use of tailored computational and problem-solving activities where students must work in groups to solve complex problems in realistic STEM contexts (Irving et al., 2017). In this setting, Computational Finance is a relatively new and highly interdisciplinary subject, fundamental to cover high-level roles in the financial sector and to master Financial Mathematics as well. Whilst some researchers refer to a body of literature in Finance Education (Diamond & Smith, 2011; Hoadley et al., 2015, 2016) only a limited portion of these specifically investigate the Computational Finance curriculum; thus, the area is under-researched. The focus of this paper is to show how we have implemented specific computational lab practices in ACM30070, Computational Finance, a core module in stage 3 of the BSc in Financial Mathematics, School of Mathematics and Statistics, UCD. We describe how the modern definition of "inclusive computation" (Caballero & Hjorth-Jensen, 2018) has been embedded within a student-led educational activities and how specific lab practices have been accordingly designed. In particular, we show how those practices contribute reciprocally to use computational thinking to enrich the mastery of financial mathematics and the theory of financial mathematics to enrich students' computational thinking.

Context and Learning Goals

The purpose of ACM30070 is to provide a practitioner-oriented education in implementing financial models and to embed computational thinking in mathematical and financial contexts, with the aim of bringing current educational efforts in line with the increasing demand for problem-solving and quantitative skills in the industry. The module is core for stage 3 students attending the BSc in Financial Mathematics (FM), and it is optional for stage 3 students attending the BSc in Applied and Computational Mathematics (ACM). In total, 50 students attend the module, with 35 FM and 15 ACM. In stage 1 and 2 both FM and ACM students attend modules on Pure and Applied Mathematics, Statistics and Finance (FM students only), but they do not have any prior exposure to computational modelling of real-world financial problems. They also attend an introductory coding module in Python. To set up the learning goals, a series of discussions with STEM education researchers and industry professionals has been ongoing since 2016. Starting from the outcomes of those conversations and referring to the principles of backward course design (Wiggins & McTighe, 2005), we agreed that upon successful completion of the module, students will be able to: apply financial mathematical theory and quantitative methodologies to real-world situations; understand industry practices; identify salient features of a financial system that can be translated into a model; judge the suitability of a model, critically understanding its limitations; write computer code to solve common problems in the financial sector; collect, create, manipulate and analyse financial datasets; understand basic numerical methods and use them to solve problems; synthesize and communicate outcomes of a scientific computing problem.

To design the overall course structure, we referred to the Seven Research-Based Principles for Smart Teaching described in Ambrose et al. (2010). Those principles focus on how learning begins with empirical evidence; for this reason, they can be easily applied in classes where students have a modest background in a subject, which is the case for students enrolling in ACM30070. A comprehensive description of the module design process can be found in Perrotta (2021). Keeping in mind the aforementioned learning outcomes and the fact that "engagement and motivations strongly influence what students learn" (Ambrose et al., 2010), one of the main drivers in any design component was making the study of computational finance as authentic and engaging as possible through practices set in a real-world financial context. The choice of student-led lab practices aims also to foster FM students' motivation, achievement, persistence and retention (Good et al., 2012; Irving et al., 2017). In particular, Funkhouser et al. (2018) have shown that the laboratory is the ideal place for students to feel part of an academic community, since they have the opportunity to engage in authentic practices and build knowledge through collaboration with peers.

In this paper we focus on a detailed description of the lab activities in ACM30070, Spring 2021 offering, and we show how the above considerations and the modern definition of "inclusive computation" have been essential to build group practices, to develop computational thinking and to foster sensemaking in financial mathematics.

Theoretical Background

In this section we offer an initial working definition of computing and its relationship to computational thinking and modelling. Then, we introduce the modern definition of "inclusive computation". Finally, we provide the general definition of sensemaking in science education and list the steps of sensemaking process.

Computing and Computational Thinking

The definition of *computing* is prone to several interpretations as it includes many different activities. The K-12 Computer Science Framework presents computing as features of computer literacy, education technology, digital citizenship, IT and computer science (K-12 Computer Science Framework, 2016, p.13-14). Weintrop et al. (2016) develop a Computational Practice Taxonomy, focusing on the application of computational thinking to mathematics and science (p.128). In our study we referred to the following definition of computation in mathematics: "the practice of using tools to perform mathematical calculations or to develop or implement algorithms in order to accomplish a mathematical goal" (Lockwood et al. 2019, p.3). By calculations, we include both numerical and symbolic ones, such as simplifying algebraic expressions, generating numerical structures with particular characteristics, or numerically estimating error. The tools used in calculation could range from pen and paper to a particular programming language depending on the complexity of the problem at hand. From an algorithmic perspective, computing involves developing, using, or implementing a logical sequence of steps known as an algorithm. Using computation in financial mathematics has several potential benefits: students can engage in the modelling process to make complex problems manageable, and they can use computation to explore the applicability and utility of underlying financial principles. Strictly connected to computation is the definition of computational thinking and computational modelling. The notion of computational thinking has historical roots that stretch back for decades (see Tedre and Denning (2016) for a historical account). Wing (2006), modernized the term describing it as "taking an approach to solving problems, designing systems and understanding human behaviour that draws on concepts fundamental to computer science" (p. 33). In subsequent years other researchers have refined the definition (e.g., Aho, 2012). In a 2014 blog post, Wing articulated the definition we currently adopt: "Computational thinking is the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer - human or machine - can effectively carry out" (Wing, 2014).

Inclusive Computation

Starting from the above definitions of computing and computational thinking, we referred to the "inclusive computation" framework introduced by Caballero & Hjorth-Jensen (2018). In this framework, computation practices are not restricted to writing programme statements from scratch, learn a coding language syntax and debugging a code. In fact, the "*inclusive computation*" definition includes a wide range of high-level coding activities like: having students working on their own or in groups with simulations and/or algorithms to understand the main characteristics of a mathematical/physical/financial model; giving students pieces of code to complete or modify on their own or in groups in order to adapt them to a different problem; critically inspecting and judging computational inputs and outputs; advising students on open-ended group projects where they write code from scratch.

Inclusive computational practices have been heavily used to design physics undergraduates' modules in Michigan State University and Georgia Tech, USA (e.g., Caballero et al., 2012; Caballero & Hjorth-Jensen, 2018). They have included computation and computational thinking as a central element, and not simply as a tool in the design process. We have adapted such definition to a financial mathematics context, proposing lab activities pertinent to students' future professions (Barrett & Moore, 2011; Schmidt et al. 2009). Students are constrained by the programming language to certain syntactic structures and must learn to contextualize problems in a way that produces a precise representation of the financial model. They learn how to use computation, computational thinking and financial mathematics, in harmony, to solve a real-world financial problem and to identify important features of a financial system that can be translated into a model. Finally, students understand how to judge the suitability of a model and critically grasp its limitations. Below, we include a detailed case study to show how inclusive computation has been implemented in practice.

Sensemaking in Science Education

The problem of investigating how students can "make sense" of science became relevant in recent years. However, even if many researchers agree on an intuitive definition of *sensemaking*, the related literature is fragmented. Odden & Russ (2017) propose the following definition, that we adopt as theoretical framework: "sensemaking is a dynamic process of building or revising an explanation in order to "figure something out"—to ascertain the mechanism underlying a phenomenon in order to resolve a gap or inconsistency in one's understanding." (p.3). We refer to this definition because it unifies the three primary approaches describing sensemaking: sensemaking as an epistemological frame, as a cognitive process and as a discourse practice. In summary, the process of sensemaking involves (a) realising that there is a gap or contradiction in one's knowledge, (b) iteratively proposing ideas and attempting to connect them to prior knowledge or other ideas, and (c) evaluating that these ideas are consistent and do not lead to additional contradictions. In this paper, we show how inclusive computational practices (defined above) may provide opportunities for sensemaking in computational finance in a lab setting.

Toward Lab Practices

Lab activities in ACM30070 are intended to build a teaching environment in which students can develop a deeper and more robust knowledge in FM. Sensemaking will grow as students are actively made accountable in the construction of their knowledge. The weekly schedule, the technology and logistics chosen, and the staff selection have strongly contributed in supporting these objectives and in preparing students to be independent learners. In terms of weekly schedule, students attend 4 slots of 50 minutes each per week, divided into two lectures, one tutorial, and one lab scheduled at the end of the week. To pre-activate learning and individual reflections, pre-class materials are uploaded on the course management system. Those materials typically include slides, notes, short videos as well as proper formative assignments like guided programming activities and/or working questions. The lectures are devoted to the financial modelling part, while tutorials are intended for computational practices and problem-solving activities. Lectures and tutorials are aimed to provide students with suitable skills and knowledge to make them as independent as possible in performing the weekly lab activities, given that they are fully student-led. In terms of technology and logistics, since "students' motivation determines, directs, and sustains what they do to learn" (Ambrose et al. 2010), we chose specific digital technologies and programming languages that are meaningful from an educational perspective and used in the financial industry. Students attending ACM30070 learn VBA for Excel, Python and Fincad (FAS). FAS is a financial software widely used in financial firms. It is very helpful for learning, since it is intuitive to use, and each workbook is equipped with extensive documentation on both the theory and computational side. VBA for Excel and Fincad are also frequently used to create complex financial spreadsheet models. Python programming is easy to learn, the code is compact and in general highly readable, as its syntax is close to that of mathematics. Python programs are developed in Spyder notebooks, allowing easy access to not only raw code but also the results of its execution throughout, plots and powerful data analysis. Beyond the technology, the "physical" classroom space has been crucial to support the student-led aspect. Before March 2020, all activities took place in Active Learning rooms, equipped with round tables and movable whiteboards. To adapt this class format to off-campus teaching, the classes have been live-streamed on Zoom in Spring 2021. The round tables and class discussions have been substituted with the "breakout rooms" and "poll" features in Zoom and practices have been redesigned to be delivery-mode independent.

Finally, problem-based learning environments and student-led activities require the facilitation of experts to ensure that students have a productive and engaging experience. As a result, we took great care in the selection of the tutor (T) and teaching assistant (TA). The current T attended the module in 2018-19 as a FM student, and was the TA in 2019-20. The TA attended the module in 2019-20. Both were among the best performing students in their year. In preparation for their roles, they were provided with the aforementioned literature on problem-based learning, inclusive computation and sensemaking. They also received a one-day training on module contents, module schedule, assessment components and how to guide and scaffold student learning before the start of term. Finally, 20-minute briefing and debriefing meetings with the lecturer (L) have been organized on a weekly basis for each lab during the term. Additionally, both the T and TA received funding to further develop learning materials for the module. Using their perspectives as students, they heavily tailored content to facilitate student-led discussions and attend to students' needs.

Lab Practices

As mentioned above, weekly lab activities are fully student-led and involve the participation of the L, the T and the TA as facilitators. The same lab structure is proposed each week, but it is applied to different kinds of practices. There is no pre-class assignment; all activities are entirely covered in the lab. In preparation for the specific day activity, students are required to review the contents of the weekly lectures and tutorial. During the first part of the lab, students work in groups on modelling, pseudocoding, data analysis and other related activities (see example below). To foster the student-led aspect of the labs and peers' collaboration, the L and the T observe the groups' discussions and, only if needed, will intervene to: give some hints on implementing the day's lab, guide students' brainstorming or pose relevant questions to address critical thinking. In the second part, each group chooses a representative to present the group outcomes to the whole class. The L and T guide groups in presenting their results and encourage dialogue between groups in order to come to a conclusion. The TA acts as a moderator in groups dynamics, answering easier questions and takes field notes during the whole lab.

The selection of the groups has been central to foster engagement and peer-learning. Groups were constructed by the L and stayed the same for the whole term. Students are grouped according to their ability, based on historical academic performance to date (the L has access to all past grades of any student); gender and possible minorities balance have also been taken into account. There are two concepts that are central to support motivation: the *subjective value of a goal* and the *expectancies*, or expectations for successful attainment of that goal (Ambrose et al., 2010). To positively set students' expectation to contribute to the group success, groups member had similar annual GPA to date and they were made aware of that. This way, they were all *expected to be able to contribute in the same way* within their group. Neither leader-follower dynamics nor pedagogical problems were observed during the group' activities. Both individual and group formative feedbacks have been provided weekly. Annotated slides, question solutions and full working codes are provided after each class/lab to give students the opportunity to self-reflect on the activities done. Finally, after each lab, students are invited to fill out a Google Form survey to critically reflect on the activities.

The following case study is introduced by way of example; it shows how topics have been presented in a scaffolded manner to lead students to connect prior and new knowledge, and to apply what they have already learned to new related contexts. In particular, this case study exemplifies how inclusive computational practices have been designed and integrated for sensemaking, and how student-led practices and group activities have been implemented.

The Implied Volatility Case Study

This case study has been developed to offset the difficulties encountered by the students in approaching the concept of implied volatility of a European Vanilla Option. Usually, students learn the definition of implied volatility and run a code to compute it, but they do not understand the mathematical assumptions and relation linking implied volatility and option price. This practice consists of three exercises, of increasing levels of difficulty. It has been designed following the steps of the sensemaking process and using some of the inclusive computational practices described above. The implied volatility model has been developed during the lectures. Since there is a non-linear relation between option price and volatility, a root-finder algorithm is needed for its calculation and the related code has been presented during the tutorial. Before attending the current lab, students are required to review those materials. The lab opens with the T doing a quick walk-through of the tutorial code, followed by the presentation of the first exercise, where students have to figure out why the code breaks down for a given set of input data (step 1 sensemaking). The key faulty element in this first exercise is not a syntax or coding error, but an incoherence between the provided dataset and the model assumptions. To enhance the student-led aspect of the practice, students do not receive any initial hint from the L or the T, instead they have been equipped with a debugging worksheet and a few questions to drive them in collaboration. Thanks to discussions between group members, facilitated at a later time by the L and the T, they learn how to critically inspect and judge computational inputs and outputs and how to integrate the mathematical and computational modelling (step 2 sensemaking). As a second exercise, they are provided with another (working) set of input data and they are required to do a sensitive analysis (step 3 sensemaking). As a third exercise, assigned as a homework, they are required to individually adapt the model, modify the code and perform a sensitive analysis for a different option type. Finally, they are required to bring their solution at the next lab to discuss the individual outcomes in each group and propose a unique class solution.

Conclusions

The over-arching purpose of the lab practices described in this paper is to help students to develop a robust knowledge allowing them to understand and/or create financial models, translate them into code and both quantitatively and qualitatively compare these models with real-world data. The use of an inclusive definition of computation jointly with suitable technology have been central to contribute to sensemaking. In this framework, computation is not simply a skill to learn but it represents one of the constitutive pillars to master financial mathematics within an enquiry-based and student-centred learning environment. In this paper we have described the design process, the theoretical framework and the implementation plan of ACM30070 lab activities, aimed to harmonize student-led practices with inclusive computational practices to promote sensemaking in financial mathematics. We have also explained how groups' activities coupled with moderators' interaction allowed for a free-flowing student-led experience. A case study has been presented as an example. The labs received very positive feedback from students. Two students quoted:

> "I think that the labs that require us to design some code in real-world scenarios help me to understand the computational part of the course best. It's one thing to read the other code and implement it but the opportunity to write your own forces you to understand the deeper intricacies in the code and you become more aware of the parts that you do and do not understand, which you can then fix."

> "The group work was very helpful today. It helped us understand the different thought processes that go into creating a specific macro and what the correct outcomes are. It's important to see that just because you did something differently to someone else it does not mean that you are incorrect. There are similar advantage and disadvantage to the method that you or the person whose code you are comparing against have with your respective code. I liked working alongside others."

Given the importance of computation in modern science and the lack of literature referring to computational finance, our next step is to contribute to educational research. In particular, we want to investigate and measure to what extent the enquiry-based and student-led practices, as well as the "inclusive" computational practices proposed are successful in effectively facilitating sensemaking in FM. In-class data collection for this research study ran in ACM30070 Spring 2021 offering. We collected quantitative and qualitative data including: weekly lab and tutorial attendance, continuous assessment grades, final grades, group project numerical outputs as quantitative data, labs field notes, weekly Google Form, a final Google Form and group project diaries, as qualitative data. Data analysis will start in July 2021 with outputs expected for a future work.

References

Aho, A. V. (2012). Computation and computational thinking. *The Computer Journal*, *55*(7), 832-835.

Ambrose, S. A., Bridges, M. W., DiPietro, M., Lovett, M. C., & Norman, M. K., (2010). *How learning works: Seven research-based principles for smart teaching*. Wiley Press.

Barrett, T., & Moore, S., (2011). *New approaches to problem-based learning. Revitalising your practice in higher education*. Routledge.

Caballero, M. D., Kohlmyer, M. A. & Schat, M. F. (2012). Implementing and assessing computational modeling in introductory mechanics, *Physics Education Research*, 8, 020106.

Caballero, M. D., & Hjorth-Jensen, M. (2018). Integrating a computational perspective in physics course. *New Trends in Physics Education Research*, 47-76, Nova Science.

Diamond, R., & Smith, H. (2011). Threshold concepts: A disciplinary enquiry in quantitative finance. *Social Science Research Network*, http://ssrn.com/abstract=1890837

Funkhouser, K., Caballero, M., Irving, P., & Sawtelle, V. (2018). What counts in laboratories: Toward a practice-based identity survey. *Proceedings of 2018 Physics Education Research Conference*, American Association of Physics Teachers.

Good, C., Rattan, A., & Dweck, C. S., (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. *Journal of Personality and Social Psychology*, *102*(4), 700.

Hoadley, S., Wood, L., Kyng, T., & Tickle, L. (2015). Threshold concepts in finance: Conceptualizing the curriculum. *International Journal of Mathematics Education in Science and Technology, 46*(6), 824-840.

Hoadley, S., Wood, L., Kyng, T., & Tickle, L. (2016). Applying Threshold Concepts to Finance Education. *Education + Training*, *58*(5), 476-491.

Irving, P., Obsniuk, M., &Caballero, M. D. (2017), P3: a practice focused learning environment. *European Journal of Physics, 38*(5), 055701.

K-12 Computer Science Framework. (2016). Retrieved from http://www.k12cs.org

Lockwood, E., DeJarnette, A. F. & Thomas, M. (2019). Computing as mathematical disciplinary practice. *Journal of Mathematical Behavior*, 54, 100688.

Perrotta, A. (2021). A learner-centered approach to design a Computational Finance module in higher education, *HEAd'21 - 7th International Conference on Higher Education Advances,* Editorial Universitat Politècnica de València, 405-412.

Schmidt, H., van der Molen, H., te Winkel, W., & Wijnen, W. (2009). Constructivist, problem-based learning does work: A meta-analysis of curricular comparisons involving a single medical school. *Education Phycologist*, *44*(4), 227-249.

Tedre, M., & Denning, P. J. (2016). The long quest for computational thinking. *Proceedings of the 16th Koli Calling International Conference on Computing Education Research,* 120-129.

Wing, J. M. (2006). Computational thinking. *Communications of the ACM, 49*(3), 33–35.

Wing, J. M. (2014). Computational thinking benefits society. Retrieved from http://socialissues.cs.toronto.edu/index.html%3Fp=279.html

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology, 25*, 127–147.

Content Analysis of Mathematics Textbooks and Adapted Lorentz Curves

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In this paper, we introduce an adaption of the Lorenz curve as a powerful tool for graphically presenting content analyses of mathematics textbooks. Three textbooks currently used with year-one children in Sweden, one Swedish-, one Finnish and one Singaporean-authored, were analysed against the eight categories of Foundational Number Sense (FoNS), which is a set of literature-derived and instruction-dependent competences that all children need to acquire if they are to become successful learners of mathematics. The adapted Lorenz curve highlighted, inter alia, major differences in the distribution of FoNS-related tasks across the three textbooks. In general, the Swedish-authored textbook offers repeated cycles of FoNS-related opportunities, the Finnish-authored textbook offers such opportunities continuously, and the Singaporean-authored textbook typically offers FoNS-related opportunities only within its earlier pages.

Introduction

Two years ago, at MEI7, we introduced moving averages as a tool for presenting textbook content analyses (Petersson et al., 2019). This paper extends that work by offering a second approach, which we label adapted Lorenz curves, which also shows in graphically transparent and reader-friendly ways how textbooks emphasise and distribute content.

Why Analyse Textbooks?

There are at least three reasons for analysing mathematics textbooks. First, particularly in cultures in which textbook production is effectively deregulated, it is important to evaluate a textbook's content against curricular specifications. In such circumstances, where textbooks are intended to make the curriculum visible (Park & Leung, 2006; Son & Senk, 2010), teachers should have confidence that they address adequately the system's expected outcomes. Importantly, dependent on national context, the textbook is afforded different responsibilities with respect to what children are expected to learn, with differing consequences for the teachers who use them. On the one hand, for example, are systems like Cyprus, where teachers are mandated to use the government-produced textbooks that, de facto, represent the intended curriculum (Travers & Weinzveig, 1999). Here, there is, in essence, no need for teachers to evaluate textbooks' curricular resonance because there are no permitted alternatives (Xenofontos, 2019). On the other hand, in a system like Ireland, where few teachers do not use textbooks as the basis for their teaching (Mullis et al., 2012) and publishers operate in a deregulated market, the responsibility for any evaluation of their resonance with the state-mandated learning outcomes lies with the teacher. Put another way, in Ireland, due to individual authors' interpretation of curricular expectations, textbooks there form part of the implemented curriculum (Travers & Weinzveig, 1999).

The second reason, also of particular relevance to Ireland, is that even if two textbooks are similarly adequate in their addressing of curricular expectations, they may do so in different ways. Topics and the subtopics within them may be sequenced differently or topics may be given different emphases. Generic competences like problem solving may be privileged more in some books than in others, with some offering such tasks in a continuous chain of opportunity and others locating them at the different chapters' ends. In other words, in systems in which teachers have multiple choices with regard to the textbooks they use, comparing textbooks' approaches to mathematics is important. For example, Neuman et al. (2014) found substantial variation in the support for teachers in textbooks written for use in Swedish primary schools. Thus, while this may not be the case for teachers in countries like South Korea, where textbook reviews occur centrally at ministry level (Son & Senk, 2010), teachers in systems like Ireland and Sweden are, de facto, expected to be able to undertake such reviews themselves.

The third reason concerns the importation of textbooks from one country to another. Over the last few years, publishers, particularly in countries with unregulated textbook markets, have imported textbooks from countries whose students have excelled on large-scale tests of achievement. For example, Singaporean textbook series have been adapted for use in England and Sweden (Petersson et al., 2019), the Netherlands (van Zanten & van den Heuvel-Panhuizen, 2018) and the United States (Hoven & Garelick, 2007). While the publishers of such imports typically claim that their books have been adapted to local curricular expectations, it is important for to acknowledge that teaching and learning are deeply culturally situated (Merttens, 2015).

In sum, while all three reasons invoke, in different ways, notions of curriculum matching, they also allude to three different but important perspectives governing how textbooks are produced and used (Rezat and Strässer, 2015). The first is the author perspective, which acknowledges both the addressed curriculum and individual authors' preferences. The second is the user perspective, which refers to how mathematics books are used by students and teachers and reflects, in varying degrees, what Johansson (2006) has described as room for manoeuvre. The third is the content perspective, which deals with, essentially, the didactical aspects of a book's content, including its distribution, presentational variation, and levels of difficulty. In this paper, acknowledging such matters, we offer an analytical tool hitherto unknown in research on school mathematics textbooks, which we believe facilitates all forms of textbook analysis and comparison in powerfully visual ways. By way of demonstration, we draw on three textbooks currently used with year-one children in Sweden. While each book has its roots in a different curriculum tradition, each, according to its publisher, has been adapted to address the Swedish national curriculum. These are Matte Eldorado, a Swedish-authored series, Favorit, an adapted Finnish-authored series, and Singma, an adapted Singaporean-authored series.

Methods

Analytical Framework

As with our earlier paper (Petersson et al., 2019), analyses drew on the eight categories of Foundational Number Sense, hereafter FoNS, which is a set of literature-derived and instruction-dependent competences that all children need to acquire if they are to become successful learners of mathematics (Andrews & Sayers, 2015). For each book, all tasks that expected some form of action on the part of the student were examined by at least two members of the project team. Each task addressing one of the FoNS categories, shown in Table 1, was coded '1' for that opportunity and '0' otherwise. In this manner, every task in each book attracted eight codes, according to the presence or absence of the eight FoNS categories. In this way, many tasks, particularly geometrical, attracted no codes, while many attracted several. For example, Figure 1 shows a task inviting children to "compare the number of dots" and then "write either = or ≠" in the box. This task occurred before the book introduced addition, and was viewed as encouraging solution by counting and coded for *systematic counting*. Expectations of equality or inequality led to its being coded for *quantity discrimination*, the dot patterns not only alluded to *different representations of number*, but also hinted at subitising and an *awareness of the relationship between number and quantity*. In other words, this task attracted four codes of '1' (codes 2–5 in table 1) and four of '0'.

Figure 1

A multiply coded textbook example

Table 1

Summary of the eight categories of foundational number sense

FoNS Category	Teachers encourage children, within the range 0-20, to
1. Number recognition	Identify, name and write particular number symbols
2. Systematic counting	Count systematically, forwards and backwards, from arbitrary starting points
3. Number and quantity	Understand the one-to-one correspondence between number and quantity
4. Quantity discrimination	Compare magnitudes and deploy language like 'bigger than' or 'smaller than'
5. Different representations	Recognise and make connections between different representations of number
6. Estimation	Estimate the magnitude of a set of objects or place a given number on empty number line
7. Simple arithmetic	Perform simple addition and subtraction operations
8. Number patterns	Recognise and extend number patterns, identify a missing number

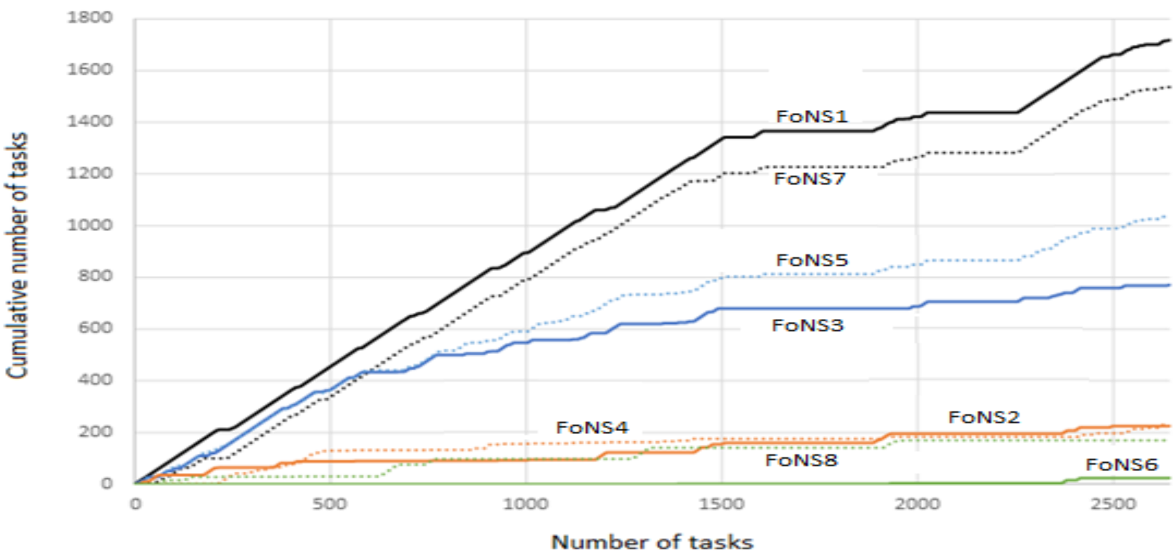
Analytical Tool: The Adapted Lorenz Curve

The process outlined above means that all the tasks in each book collectively create a sequence of ones and zeros where each '1' affirms that the task addresses a particular FoNS category. In this way, premised on all tasks in a book forming a time-delimited series of activities, different forms of analyses can be undertaken. In particular, an adapted Lorenz curve is simply a plot of the cumulative sum of the number of occurrences of a FoNS category. This means, for example, that if the first six tasks in a book were coded 0, 1, 1, 0, 0, 1… for, say, FoNS1, its cumulative sum becomes 0, 1, 2, 2, 2, 3…, a sequence amenable to a graphing process similar to that of a cumulative frequency curve. Indeed, as we show below, an adapted Lorenz curve shows how particular content, say FoNS 1, is distributed throughout the textbook. Of course, such a process does not account for teachers exploiting possible room for manoeuvre (Johansson, 2006) through choosing to alter the order in which they teach, or even omit, the material (Mesa, 2004), but it does offer a clear indication of the authors' intentions, emphases and timing.

Results

Figure 2

Adapted Lorenz curve for Eldorado

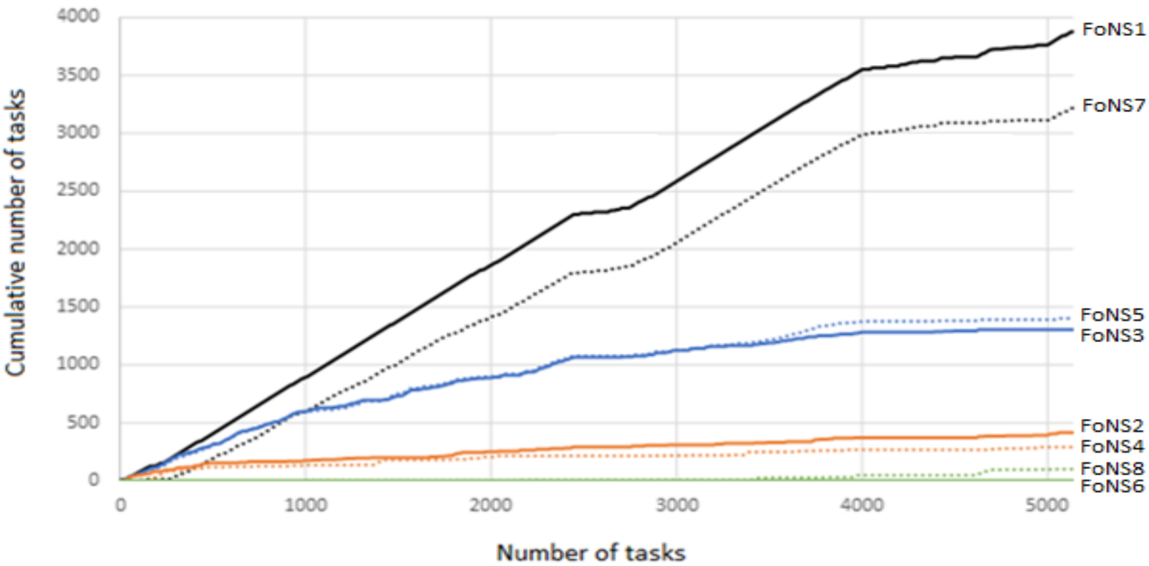


Figures 2-4 show the adapted Lorenz curves for each of the eight FoNS categories for each book respectively. It can be seen clearly that each time a task is coded for the presence of a FoNS category, the adapted Lorenz curve rises and when that category is missing, it contributes a horizontal component. While several similarities and differences can be inferred from the three figures, we turn first to the former. The most obvious similarity is that all three books place the greatest emphasis on FoNS1 and FoNS7 (number recognition and simple arithmetical operations respectively). Moreover, the vertical differences between the two graphs, indicate that FoNS7 develops at slower pace than FoNS1. Elsewhere, in each figure, they are effectively parallel, meaning that they tend either to occur simultaneously or not at all. The second is that all three books emphasis FoNS3 and FoNS5 (Awareness of the relationship between number and quantity, and awareness of different representations of number respectively), although this is stronger in Eldorado than either Favorit or Singma. Across the three figures, the graphs for FoNS3 and FoNS5 are essentially parallel, with the consequence that they occur either simultaneously or not at all. The third is that the remaining FoNS categories either receive low emphases or, as with FoNS6 (estimation) are essentially absent. Before discussing their differences, we now turn to the characteristics of each book.

With respect to Eldorado, it can be seen in Figure 2 that the majority of the different FoNS curves show a repeated pattern of periods of growth, indicative of tasks repeatedly addressing the category under scrutiny, followed by periods of constancy, indicating no tasks addressing the category under scrutiny. This pattern is especially evident in the tasks found in the second half of the book, highlighting, it seems to us, two important features. The first is that these patterns, which tend to occur simultaneously, are distinguished only by the differences in the gradients of the growth periods. In other words, Eldorado seems to address several FoNS categories at the same time. The second is a corollary of the first, namely, the periods of constancy, represented by the horizontal lines, emphasise sections in the book where topics other than number are presented.

Figure 3

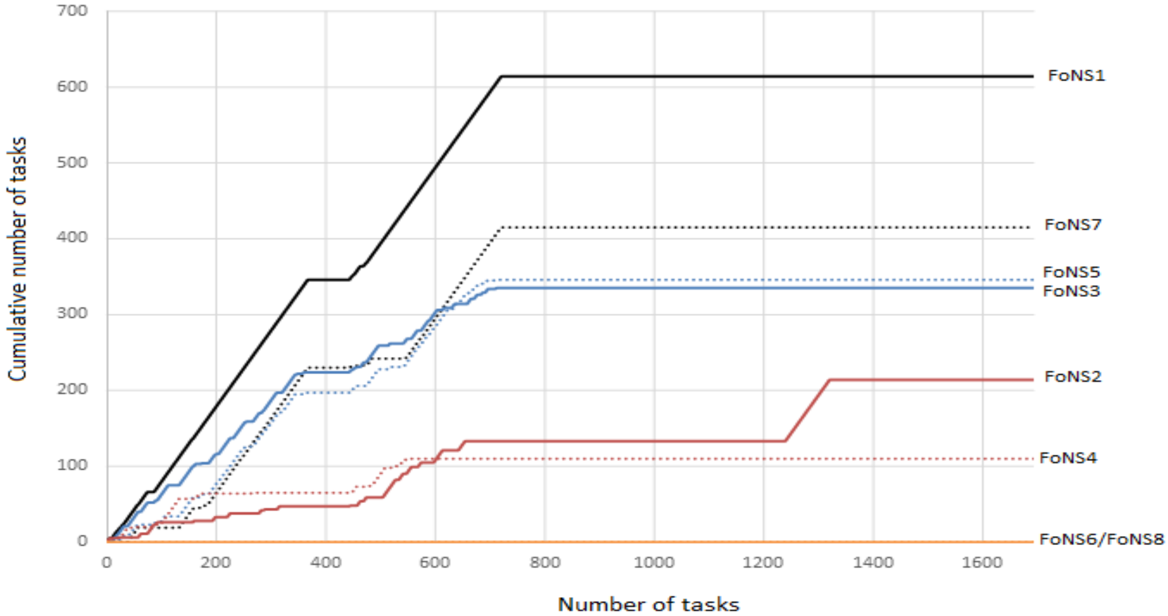
Adapted Lorenz curve for Favorit (the scale follows the number of tasks)



From the perspective of Favorit, the curve for each FoNS category shown in Figure 3 (except FoNS6) has hardly any plateaus. There are hints, around the middle of the book, of limited FoNS-related activity, but, overall, the graphs show that number-related opportunities are a constant presence. The only exceptions are FoNS6 (estimation) and FoNS8 (number patterns). The former is, de facto, absent and the latter, introduced in a very limited manner towards the end of the book. The different Singma graphs shown in Figure 4, present a very different story. With a single exception, no tasks were found to address any FoNS category after the first 40% of the books' tasks. In fact, our analyses found that later number-related tasks fell outside the FoNS categorisations, involving only numbers in the range 21-100. The solitary exception is FoNS2 (systematic counting), which is clearly subject to a second period of emphasis around three-quarters of the way through the book.

Figure 4

Adapted Lorenz curve for Singma



From the perspective of differences, the graphs shown in each figure highlight well very different didactical representations of the different FoNS categories. For example, irrespective of the FoNS category under scrutiny, the graphs for Swedish-authored Eldorado show a repeated pattern, whereby a category is present for a period followed by its being absent for a period. By way of contrast, and acknowledging the effective absence of two FoNS categories, when Finnish-authored Favorit addresses a FoNS category it does so continuously throughout the book. Finally, Singapore-authored Singma, with a single exception in FoNS2 and two FoNS categories absent, essentially offers continuous opportunities for children to engage with five FoNS categories during the early part of the book, after which FoNS disappears from a child's experience as more complex material is introduced.

Discussion

In an era in which textbooks written in countries deemed successful on international tests of achievement are imported into other countries, typically on the untested assumption that they must be of a higher quality than those produced by the importing countries, it is important for researchers to have efficient tools for evaluating the efficacy and, for teachers, the adaptability, of such imports. In this paper, we have analysed three textbooks currently used with year-one children in Sweden. One, Eldorado, is Swedish-authored, while the others, Favorit and Singma, are Finnish- and Singaporean authored respectively. Analyses were structured by the eight categories of Foundational Number Sense, a core set of literature-derived and instruction-dependent competences that all children need to acquire if they are to become successful learners of mathematics (Andrews & Sayers, 2015). In undertaking these analyses, we have extended an earlier study in which we introduced moving averages as a novel tool for analysing the content of school mathematics textbooks (Petersson et al., 2021), by introducing adapted Lorenz curves, to further extend an important and necessary toolkit. As we have shown, the adapted Lorenz curves have highlighted well how textbooks' authors emphasise and distribute different forms of mathematical knowledge.

The analyses identified important differences and similarities, confirming, we argue, the relevance and efficacy of the adapted Lorenz curve. The most obvious similarity was that all three books emphasised the same four FoNS categories concerning; number recognition; simple arithmetical operations; awareness of the relationship between number and quantity; and awareness of different representations of number respectively. A second was that all three books offered few opportunities for children to engage with the remaining four FoNS categories. A major difference, well exemplified by the adapted Lorenz curves, concerned the distribution of FoNS-related tasks across the three textbooks and the didactical implications inferred from them. In general, Swedish-authored Eldorado appears to present repeated cycles of opportunity for children to engage with the different FoNS categories of competence. Finnish-authored Favorit offers such opportunities continuously throughout the school year and Singaporean-authored Singma offers FoNS-related learning, with a single exception, only within its earlier pages. In other words, if a textbook's role is to make visible a system's curricular expectations (Park & Leung, 2006; Son & Senk, 2010; Travers & Weinzveig, 1999), then these three commonly-used textbooks not only present very different images of the Swedish curriculum but also require teachers to manage intelligently the room for manoeuvre they offer (Johansson, 2006).

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References

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal, 43*(4), 257–267.

Hoven, J., & Garelick, B. (2007). Singapore Math: Simple or complex? *Educational Leadership*, *65*(3), 28–31.

Johansson, M. (2006). *Teaching mathematics with textbooks: A classroom and curricular perspective* (PhD dissertation). Luleå University of Technology.

Merttens, R. (2015). Textbooks from Shanghai and Singapore A national debate. *The Mathematical Gazette, 99*(546), 391–401.

Mesa, V. (2004). Characterizing practices associated with functions in middle school textbooks: An empirical approach. *Educational Studies in Mathematics, 56*(2), 255–286.

Mullis, I.., Martin, M., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Neuman, J., Hemmi, K., Ryve, A., & Wiberg, M. (2014). Mathematics textbooks' impact on classroom instruction: examining the views of 278 Swedish teachers. In Silfverberg, Harry, Kärki, Tomi & Hannula, Markku S. (eds.), *Nordic research in mathematics education* (pp. 215–224). University of Turku, Department of Teacher Education.

Park, K., & Leung, F. (2006). Comparative study of the mathematics textbooks of China, England, Japan, Korea, and the United States. In F. K. S. Leung, K. Graf & F. J. Lopez-Real (red.), *Mathematics education in different cultural traditions - a comparative study of East Asia and the West: The 13th ICMI study* (pp. 227–238). Springer.

Petersson, J., Sayers, J., Rosenqvist, E., & Andrews, P. (2019). Opportunities for year-one children to acquire foundational number sense: Comparing English and Swedish adaptations of the same Singapore textbook. In L. Harbison & A. Twohill (Eds.), *Proceedings of the Seventh Conference on Research in Mathematics Education in Ireland* (pp. 251–258). Institute of Education, Dublin City University.

Petersson, J., Sayers, J., Rosenqvist, E. & Andrews, P. (2021). Two novel approaches to the content analysis of school mathematics textbooks. *International Journal of Research and Method in Education, 44*(2), 208–222. https://doi.org/10.1080/1743727X.2020.1766437

Rezat, S., & Straesser, R. (2015). Methodological issues and challenges in research on mathematics textbooks. *Nordic Studies in Mathematics Education, 20*(3-4), 247–266.

Son, J., & Senk, S. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74*(2), 117–142.

Travers, K., & Weinzveig, A. (1999). The second international mathematics study. In G. Kaiser, E. Luna & I. Huntly (Eds.), *International comparisons in mathematics education* (pp. 19-29). Falmer press.

van Zanten, M., & van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM: The International Journal on Mathematics Education*, *50*(5), 827–838.

Xenofontos, C. (2019). Primary teachers' perspectives on mathematics during curriculum reform: A collective case study from Cyprus. *Issues in Educational Research*, *29*(3), 979–996.

Teachers' Perceptions of the Impact of the Grinds Culture: A Focus on Post-Primary Mathematics

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Grinds can be defined as education outside the formal schooling system where a tutor teaches a particular subject(s) in exchange for a financial gain. Their provision has become a widespread phenomenon internationally in recent years, no more so than for the subject of mathematics. In this paper we sought to investigate mathematics teachers' perceptions of the impact of the grinds culture in the subject at post-primary level in Ireland. The data was gathered using an online survey designed by the authors and circulated to post-primary mathematics teachers in November 2020. The findings from responding teachers (n = 305) revealed mixed opinions, with both positive and negative impacts identified. Many teachers acknowledged the benefits of one-to-one support that grinds can provide and the resulting increase in students' confidence in the subject. However, teachers also noted that for some students', grinds can be a substitute for a lack of motivation and work ethic and their provision can often lead to disengagement in class.

Introduction

In this paper we sought to investigate mathematics teachers' perceptions of the impact of the grinds culture in the subject at post-primary level in Ireland. Grinds, often referred to as 'shadow education', are defined by Smyth (2009, p.2) as *"paid private tuition outside of, and additional to, the formal schooling system"*. In essence, they involve additional tutoring outside of the formal school day. Stevenson and Baker (1992) termed it 'shadow education' as all members of the tutoring triangle (tutors, students and parents) wish for it to remain low profile. Yung and Bray (2017) also noted that it 'shadows' and to some extent copies the regular school curriculum. For many decades shadow education has been popular in East Asian countries influenced by Confucian cultural traditions including China, South Korea, Hong Kong, and Japan (Bray, 2013). For example, in China the 2004 'Urban Household Education and Employment' survey indicated that tutoring was received by 74% of primary, 66% of lower secondary and 54% of upper secondary students (Bray, 2009). However, in recent years there has been a notable surge in the uptake of private tuition globally with Bray (2020) reporting that this phenomenon is now prevalent across the globe. With regard to Ireland, Smyth (2009) reported that 45% of students surveyed in her sample had received grinds in their final year of schooling in 2003. This was a significant increase from 32% of the same age-group a decade earlier (Smyth, 2009). More recently, using data from the 'Growing up in Ireland' study, McCoy and Byrne (2019) reported that 60% of Irish 17 years olds participated in 'shadow education'.

Due to the prevalence of private tuition, many researchers have sought to ascertain the factors which drive the uptake of grinds. In Ireland, the Leaving Certificate (LC) examination acts as a gatekeeper to third-level education as a student's entry relies almost entirely on their performance in this summative State examination. A study carried out by Smyth and Banks (2012, p. 302) determined that students' performance in the LC has 'very significant consequences for young people's future life chances'. There is no doubting that high stakes nature of such terminal examinations are a driving force in the uptake of grinds. This is not just an Irish occurrence. For example, similar findings were reported in many districts of China where examinations in upper and lower secondary school have become increasingly competitive and has led to parents enrolling their children in private tuition in order to enhance their chances of being accepted into prestigious schools or universities (Zhang & Bray, 2016). Interestingly, the Finnish education system does not place an emphasis on high stakes examinations, and this is one of the few countries where shadow education is "barely visible" (Bray, 2020, p.4).

While the provision of grinds can be found across almost every subject, a UK study by Ireson and Rushforth (2005) found that mathematics was the subject area where private tuition was most in demand. They found that 19% of students in Year 13 (students aged 17-18 years old) reported receiving private tuition in mathematics while only 8% of students received private tuition in the next most popular subject, English. Their study also found that less than 3% of students were availing of private tuition across other curricular subjects. More recently, these findings were supported by the work of Kim and Jung-Hoon (2019) and that of Bray (2013, p. 415) who found that "Mathematics and the national languages tend to be in especially high demand [for private tuition]". In Ireland, Smyth et al. (2007) found that a significantly higher proportion of students availed of mathematics grinds compared to other subjects.

There are many interlinking reasons why the demand for mathematics grinds may be high, compared to other subjects. Bray (2020) asserted that many avail of additional tutoring in order to compensate for shortcomings in the mainstream education system. In Ireland, Prendergast and O'Meara (2017) reported on shortcomings in relation to an overcrowded mathematics curriculum and a shortage of class time to complete this curriculum. Furthermore, as previously mentioned, the LC examination, acts as a gatekeeper to third-level education in Ireland. However, its high stakes nature is even more pronounced for the subject of mathematics. Firstly, the subject is considered a necessary entry requirement for many college courses. Secondly, since 2012, students are now awarded an extra 25 'bonus points' in their overall LC examination results if they achieve ≥ 40% in advanced mathematics. This initiative has resulted in record numbers now opting for the Higher Level (HL) paper at both Junior and Senior Cycle. For example, between the years 2011 – 2019 the numbers taking LC HL mathematics have increased from 15.8% to 32.9% (SEC, 2011 - 2019). While there are many merits in increasing the numbers studying mathematics at an advanced level, there have been some unwarranted consequences. For example, in a study by Prendergast, O'Meara, and Treacy (2020), many responding teachers voiced concerns about the mathematical standard of some students now continuing at HL. Some believed that the awarding of bonus points are promoting a 'grinds culture' in the subject; 'It promotes a grinds culture where if a parent throws enough money at the problem the problem will be solved…' (Teacher response in Prendergast et al., 2020). The research detailed in this paper aimed to explore mathematics teachers' perceptions of the impact of this 'grinds culture' in more detail. It sought to address the following research question:

- What are mathematics teachers' perceptions of the impact of the grinds culture at post-primary level in Ireland?

Methodology

As part of a larger study, the authors designed an online survey that sought to investigate mathematics teachers' perceptions of the scale, nature, driving forces, and impact of the grinds culture that currently exists in the subject at post-primary level in Ireland. Given that almost all of the studies on grinds have been conducted quantitatively (Hajar, 2018), the survey the authors designed enabled them to generate mixed data through the inclusion of dichotomous, multiple choice, Likert scales and open-ended questions. The finalised instrument, which was piloted with eight experienced mathematics teachers, comprised of six sections. One of these sections focused specifically on the impact of grinds. Here teachers were asked to indicate their level of agreement with a series of seven statements on the impact of grinds on various domains using a five-point Likert scale. Four of these statements related directly to the impact of grinds on students' knowledge, understanding, and performance in mathematics. For example, 'In general, receiving grinds increases students' conceptual understanding of mathematical concepts'. The remaining three statements were associated with the impact of grinds on the affective domain in the subject. For example, 'In general, receiving grinds improves students' attitudes towards mathematics'. These Likert scale statements were proceeded by an open-ended question where teachers were asked for further comment on their opinion regarding the impact of grinds.

The target sample for this study was all teachers of mathematics across the 723 post-primary schools in Ireland. The Qualtrics survey was distributed online, and the link was widely circulated on a variety of social media platforms and to a number of professional bodies, including the Irish Mathematics Teacher Association. In total, 305 teachers responded to the survey.

The quantitative data for this paper, which provided information on teachers' level of agreement on the impact of grinds on various domains, was recorded in SPSS and was analysed using descriptive statistics. The data from the subsequent open-ended question on impact was transcribed into a Microsoft Word document and an inductive 'bottom up' thematic content analysis was performed on the teachers' responses in relation to the positive and negative impacts of the grinds culture. The work of Braun and Clarke (2006) provided a framework for this analysis. It was a flexible and recursive process, with repeated movement back and forth as initial codes were generated, and themes were reviewed.

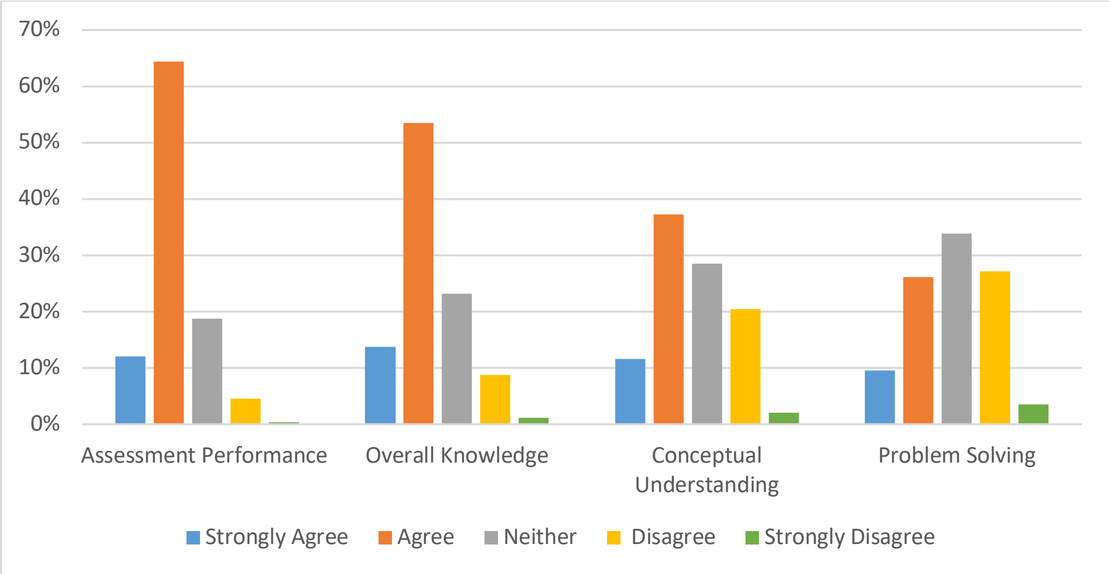
Findings

As noted, 305 mathematics teachers responded to the survey and 38% of these identified as currently giving grinds. The data in relation to teachers' levels of agreement (n = 284) with each of the seven statements are outlined in Figures 1 and 2. As evidenced from

Figure 1, 76% of responding teachers were in agreement that receiving grinds increases students' performance in mathematical assessments. However, less than half of this number (36%), agreed that receiving grinds increases students' problem-solving capabilities.

Figure 1

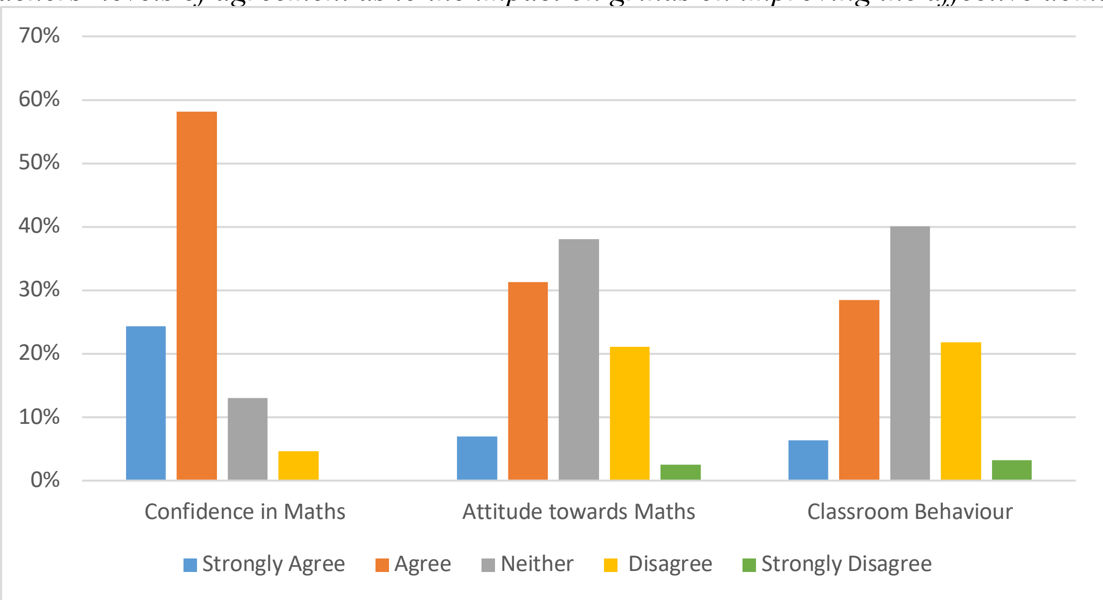
Teachers' levels of agreement as to the impact on grinds on improving various domains



Regarding the affective domain, the highest proportion of teachers (82%) were in agreement that receiving grinds increases students' confidence in mathematics. However, on the other hand, only 35% agreed that receiving grinds positively alters students' behaviour in the classroom.

Figure 2

Teachers' levels of agreement as to the impact on grinds on improving the affective domain



These findings were elaborated on through the open-ended question where teachers were asked for further comment on their opinion regarding the impact of grinds. Analysis of this data highlights the mixed views of teachers with both positive and negative impacts being noted.

From a positive perspective, the most common impact that teachers noted was the benefit of one-to-one support that grinds can provide for struggling students.

T114: Weaker students sometimes just need the one-to-one element to help them grasp a concept.

T212: One-on-one will always be a more effective way of communicating an idea to a student.

In line with the quantitative data, many teachers also mentioned increased student confidence as a positive impact.

T175: Some students need that reinforcement and it gives them the confidence to excel in the subject.

T220: I believe they can be highly effective in instilling confidence in a student.

Other positive impacts noted by some teachers were that grinds are an opportunity for students to ask questions and seek help (T84: Opportunity to ask the questions that they won't ask in class) and also that they can increase student understanding and improve grades.

From a negative perspective, there were a number of common impacts that responding teachers noted. The first was that grinds often lead to disengagement in class.

T8: Students often participate less in class, they won’t ask questions as they feel like it’s okay as they will do it in their grinds.

T157: Students become disengaged in the classroom because when they encounter something difficult they don't need to try and I quote ..." I'll do it in grinds"....

Secondly, it was noted that grinds are often a substitute for lack of student motivation and work ethic.

T20: Papering over the cracks of a culture of minimal effort.

T134: Many students think doing a grind a week is a substitute for hard work and study.

Following on from this, some teachers felt that grinds encourage rote learning and are exam focused.

T298: Students will frequently be shown shortcuts and tricks. Focus is very often on the answer rather than the process.

Furthermore, they can lead to students developing a negative attitude/ opinion towards the class teacher.

T94: Often it can alter a students opinion of the teacher negatively. They often think their grinds teacher is 'better' than their teacher. Maybe sometimes that is true. However, there is a huge difference in one-on-one help and sitting in a class with 30 other students which they don't consider.

Discussion and Conclusion

The findings of this study reveal the mixed views that mathematics teachers have in relation to the impact of the grinds culture that is permeating the subject at post-primary level in Ireland. There is no doubting the benefits of the one-to-on support that grinds can provide. As summed up by one responding teacher, 'there is a huge difference in one-on-one help and sitting in a class with 30 other students'. This positive impact of grinds was also noted by students in a UK study conducted by Hajar (2018). One student noted that "instead of the teacher talking to everyone they're just talking to you and giving you advice on what you should do in a specific task" (p. 523). This is best summed up by Kim (2016) who noted that grinds can focus on the needs, learning styles and academic goals of the individual student. Such personalised attention can undoubtedly have a positive impact on students' affective domain (Hajar, 2018). Responding teachers in this study were very cognisant of this positive impact, particularly in relation to students' confidence. For example, of all seven Likert scale statements, the highest proportion of teachers (82%) were in agreement that receiving grinds increases students' confidence in the subject. Given the well documented issues around the affective domain in mathematics, which are often associated with low confidence, low self-concept, and mathematics anxiety, particularly in relation to female students (O'Rourke & Prendergast, 2021) and students attending DEIS schools (Perkins et al., 2013), this is an important positive impact. However, not every family can pay for grinds and thus these potential positive impacts are not available to all and can exacerbate rather than improve social inequalities (Bray & Kwok, 2003). As one responding teacher in this study noted "You pay and you get the privilege and advantage that puts you up the pecking order".

While these impacts of grinds on creating further inequality and also covering up wider systemic issues (e.g. shortage of class time and overcrowded curriculum) were mentioned by some teachers, the main negative impacts that emerged were more classroom related. Although 76% of responding teachers were in agreement that receiving grinds increases students' performance in mathematical assessments, it was clear from the qualitative data that teachers felt that grinds are often a substitute for a lack of student motivation and work ethic in class. This is in line with some of the criticisms noted by teachers in a study by Wang and Bray (2016) investigating attitudes towards private supplementary tutoring in Hong Kong. One teacher in their study noted that "It also gives students a wrong perception that they don't have to work hard. They can just rely on tutoring" (p.879). In a related point, a strong theme to emerge from the qualitative data of this study was that grinds often lead to disengagement in class. In the quantitative data, one in four teachers also disagreed that receiving grinds improves students' classroom behaviour. A similar finding was noted in the work of Zhan et al. (2013) who determined that private tutoring may have reduced the students' respect for their teachers.

Given the dearth of research in this area, particularly from an Irish perspective, this paper offers important insights into Irish mathematics teachers perceptions of the grinds culture. There is no doubt that shadow education can be helpful and have positive effects on student learning and society (Kim & Jung-Hoon, 2019). On the other hand, it also has potential for distorting some educational processes and may have social repercussions such as greater inequalities (Baker, 2020). Despite this, there is an air of inevitability to its continued growth. As determined by Byun and Baker (2015), the provision of grinds is 'unlikely to be banned or fall into disuse as its connection to the main social institution of formal education has become too strong' (p.10). Thus, it is important that research such as this continues to investigate the impact of grinds from a variety of perspectives and hypothesize how it may shape the future of education.

References

Baker, D. P. (2020). An inevitable phenomenon: Reflections on the origins and future of worldwide shadow education. *European Journal of Education*, 55(3), 311-315.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in psychology*, *3*(2), 77-101.

Bray, M. (2009). *Confronting the shadow education system: What government policies for what private tutoring?* UNESCO International Institute for Educational Planning (IIEP).

Bray, M. (2013). "Shadow education: Comparative perspectives on the expansion and implications of private supplementary tutoring." *Procedia-social and behavioral sciences 77*, 412-420.

Bray, M. (2020). Shadow education in Europe: Growing prevalence, underlying forces, and policy implications. *ECNU Review of Education.* https://doi.org/10.1177/2096531119890142

Bray, M., & Kwok, P. (2003). Demand for private supplementary tutoring: conceptual considerations, and socio-economic patterns in Hong Kong. *Economics of education review*, *22*(6), 611-620.

Byun, S. Y., & Baker, D. P. (2015). Shadow education. *Emerging trends in the social and behavioral sciences: An interdisciplinary, searchable, and linkable resource*, 1-9.

Hajar, A. (2018). Exploring Year 6 pupils' perceptions of private tutoring: evidence from three mainstream schools in England. *Oxford Review of Education*, *44*(4), 514-531.

Ireson, J., & Rushforth, K. (2005). Mapping and evaluating shadow education. *ESRC Research Project.* Institute of Education, University of London.

Kim, Y. C. (2016). *Shadow education, curriculum, and culture of schooling in South Korea.* Palgrave Macmillan.

Kim, Y.C, & Jung-Hoon, J. (2019). "Conceptualizing shadow curriculum: definition, features and the changing landscapes of learning cultures." *Journal of Curriculum Studies*, *51*(2), 141-161.

O'Rourke, I., & Prendergast, M. (2021). Mathematics as a gendered subject: a deeper insight into students' attitudes in Irish post-primary schools. *Irish Educational Studies*, 1-20.

McCoy, S., & Byrne, D. (2019). Shadow Education among Irish secondary school students: evidence for system reform? Paper presented at *European Conference on Educational Research (ECER)*, Hamburg, Germany, 4th September.

Perkins, R., Shiel, G., Merriman, B., Cosgrove, J., & Moran, G. (2013). *Learning for life: The achievements of 15-year-olds in Ireland on mathematics, reading literacy and science in PISA 2012*. Educational Research Centre.

Prendergast, M., & O'Meara, N. (2017). A profile of mathematics instruction time in Irish second level schools. *Irish Educational Studies*, *36*(2), 133-150.

Prendergast, M., O'Meara, N., & Treacy, P. (2020). Is there a point? Teachers' perceptions of a policy incentivizing the study of advanced mathematics. *Journal of Curriculum Studies*, *52*(6), 752-769.

Smyth, E. (2009). Buying your way into college? Private tuition and the transition to higher education in Ireland. *Oxford review of education*, *35*(1), 1-22.

Smyth, E., & Banks, J. (2012). High stakes testing and student perspectives on teaching and learning in the Republic of Ireland. *Educational Assessment, Evaluation and Accountability*, *24*(4), 283-306.

Smyth, E., Dunne, A., Darmody, M., & McCoy, S. (2007). Gearing up for the exam?: The experience of junior certificate students. *Economic and Social Research Institute (ESRI) Research Series*.

State Examinations Commission (SEC). (2011 - 2019). *State examinations statistics*. Retrieved from https://www.examinations.ie/statistics/

Stevenson, D. L., & Baker, D. P. (1992). Shadow education and allocation in formal schooling: Transition to university in Japan. *American Journal of sociology*, *97*(6), 1639-1657.

Wang, D., & Bray, M. (2016). When whole-person development encounters social stratification: Teachers' ambivalent attitudes towards private supplementary tutoring in Hong Kong. *The Asia-Pacific Education Researcher*, *25*(5), 873-881.

Yung, K., & Bray, M. (2017). Shadow education: Features, expansion and implications. In T. Tse & M. H. Lee (Eds.), *Making sense of education in post-handover Hong Kong: Achievements and challenges* (pp. 95–111). Routledge.

Zhang, W., & Bray, M. (2016). Shadow education. In *Spotlight on China* (pp. 85-99). Sense Publishers.

Zhan, S., Bray, M., Wang, D., Lykins, C., & Kwo, O. (2013). The effectiveness of private tutoring: Students' perceptions in comparison with mainstream schooling in Hong Kong. *Asia Pacific Education Review*, *14*(4), 495-509.

Harnessing spatial thinking to support mathematics teaching and learning

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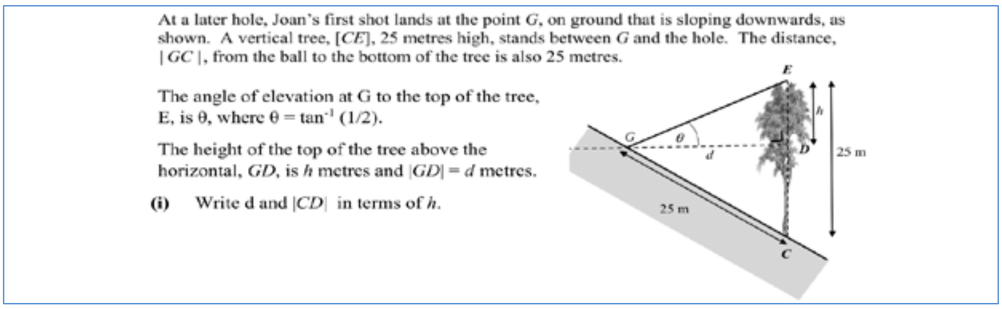
There is a growing consensus that spatial thinking is fundamental in how students conceive, express and perform mathematics. Decades of research show that building spatial skills yields measurable impacts on learning. This paper highlights however that the translation of research into explicit and systematic classroom practices to support spatial thinking in post-primary mathematics is not widespread. The aim of this paper is to: present a rationale for spatially enhancing mathematics curricula and pedagogy; to consider some existing tools and frameworks in the field; and to highlight the need for research that develops our understanding of effective practice that promotes spatial thinking in the mathematics classroom.

Irish Post-Primary mathematics: Should more focus be placed on spatial reasoning?

Spatial thinking or spatial reasoning (SR) is of growing importance in our technological world (Diezmann & Lowrie, 2012). SR involves the location and movement of objects and ourselves, either mentally or physically, in space. It concerns a considerable number of concepts, tools and processes (NRC, 2006). Three spatial skills that have been consistently studied in the literature are mental rotation, spatial orientation and spatial visualisation (Frick, 2019). These spatial abilities are malleable. Uttal et al. (2013) performed a meta-analysis examining 217 spatial training studies over a 25-year period and concluded that spatial training is an effective means for improving SR in people of all ages and across a variety of training techniques. There is also evidence that SR can also be developed through exposure to spatially rich learning experiences (Reilly et al., 2017). Developing SR has both moral and economic implications. Firstly, because it may mitigate against a spatial ability gender gap that exists to the disadvantage of females (Halpern, 2020), and secondly given that spatial ability is a strong predictor of future career choice (Wai et al., 2009) SR development is likely to improve gender equality p-STEM fields (Sorby et al., 2018).

Figure 1

Leaving Certificate Higher Level Mathematics Question 9, Paper 2, 2015



Researchers have long been aware that spatial ability and mathematics are connected (MacFarlane Smith, 1964). "The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related." (Mix & Cheng, 2012, p. 206). A study by Hawes et al. (2019) found that children's numerical and spatial skills collectively explained 84% of the variance in mathematics achievement. The emerging consensus is that spatial thinking plays a fundamental role in how people conceive, express, and perform mathematics (Hawes & Ansari, 2020). For example, the National Council of Teachers of Mathematics (2000) recommends the use of visual representations as an instructional tool and diagrams are extensively used in mathematic textbooks and examination questions (figure 1). Taking advantage of visual representations makes good sense because students answer more word problems correctly when the problem is accompanied with a diagram (Hembree, 1992). However, it is also likely that students would benefit if they developed their representation comprehension skills given that they are a strong predictor of student learning in mathematics (Pantzaria et al. 2009) and because without support students often struggle to correctly interpret diagrams (Kozhevnikov et al., 2007). Giofrè et al. (2012) showed that the academic achievement of post-primary school geometry students was strongly related to their visuospatial working memory. Harris et al. (2020) connect differences in mathematical problem solving as a function of SR and mathematics content, in this study spatial factors accounted for 32% of the variance in grade 8 mathematics scores, and again this was particularly evident in the area of geometry. While both Stieff (2013) and Mix et al. (2016) argue that it may be particularly important to provide students with spatial scaffolding when students are learning a novel mathematical concept. Other research indicates that SR may even be required for mathematical reasoning (Cheng & Mix, 2014).

Aware of the strong connection between SR and mathematics Bishop (1980, p. 267) asks: "How much responsibility should mathematics teachers take for the training and teaching of spatial abilities?" This question is deserving of due consideration. In the US "learning to think spatially" is a key goal of education across school curricula (NRC, 2006) but in Ireland research supporting SR development has yet to meaningfully impact on our post-primary mathematics curricula. While "learning to think and communicate numerically and spatially" is a goal of the primary level curriculum it is omitted from post-primary mathematics curricula (NCCA, 2017, p.12) and it is notable there isn't any reference to "spatial" in the Key Skills of Junior Cycle curricula reform and rationale document (NCCA, 2015).

Two strategies for harnessing spatial thinking to support mathematical learning

If we accept the strong evidence linking spatial thinking skills and mathematical success, the key question for researchers and educators naturally arises: How can spatial thinking research findings be translated into classroom practice to improve learning? Newcombe's (2017) OECD working paper responds with two strategies for harnessing spatial thinking to support STEM learning. Strategy 1 involves direct training of spatial skills and Strategy 2 involves spatialising the curriculum, using tools suited to spatial thinking.

Strategy 1: Train spatial skills

For most people who hear about the link between spatial skills and mathematical learning, the obvious implication might be that we should train spatial skills. Diezmann and Lowrie (2012) advocate that spatial thinking requires explicit instruction. There is merit in this approach. Sorby et al. (2018) have shown that spatial skills instruction improves spatial cognition and boosts STEM related performance while others such as Cheng and Mix (2014) have demonstrated that training effects in spatial tasks are transferable to mathematics and calculation skills. However, a challenge for Strategy 1 remains that it requires adding more components to an already crowded curriculum or finding time for a new programme in an already-packed timetable. It may be difficult to find teachers or departments willing to take responsibility for or have the expertise to deliver this new material in schools.

Strategy 2: Spatialising the curriculum

Although it may still require curriculum development and professional development support for teachers, Strategy 2 has the great advantage that it offers the potential to support effective teaching and learning while developing SR at the same time. Spatial thinking is not an add-on to curricula, but rather an approach to thinking fundamental to the interpretation of graphics and complementary to mathematical and logical thinking (Barwise & Etchemendy, 1991). Though there are not many research examples specifically relating to spatialising post-primary mathematics curricula, a recent Australian project that demonstrated positive outcomes engaged middle school children in spatially enhanced mathematical learning activities and found that they outperformed the control classes in spatial reasoning (Lowrie et al., 2018). Syahputra (2013) also demonstrated that students' spatial ability can be improved by learning mathematics using a realistic mathematics education (RME) approach characterised by: the use of real or imagined context; the use of models; the connection between and between mathematical topics; the use of interactive methods; and appreciation of variations in answers and student contributions. Woolcott et al., (2020) recognise that SR is a potentially powerful but under-utilised bridging mechanism between real-world experiences and mathematics teaching and learning. This is because mathematical concept formation is connected to our interaction with the three-dimensional world in both a mathematical and non-mathematical way.

Newcombe et al. (2019) suggest that specific spatial skills may support specific mathematical exercises, in which case intervention should focus on the relevant spatial skills, however mathematics learning may be facilitated overall by a general spatial way of thinking, what has been called a spatial turn of mind. There are already some powerful tools that might support this spatial turn of mind for mathematical learning. For Newcombe (2017) these include: the use of symbolic systems such as spatial language and visual systems for communicating information, analogical learning, and learning that is grounded in embodied experience of the world. Collectively, the various tools allow us to leverage the spatial-mathematics connection, in which we "strive to incorporate spatial skills into the curriculum efficiently and pragmatically" (Newcombe, 2017, p. 24). A practical guide for mathematics teachers to make more effective use of spatial teaching techniques in the classroom is Paying Attention to Spatial Reasoning (PATSR) (Ministry of Education in Ontario, 2014). It recognises that spatial reasoning is not a separate strand of mathematics, nor is it confined to geometry but rather that it is a process that can support learning and communicating across all strands. It asks a question that is of central importance here: How do we get started bringing awareness and development of spatial reasoning into teaching practise? It identifies nine specific ways to promote and scaffold SR in mathematics lessons using what this paper will refer to as "spatial enhancements" (SEs): (i) Teachers need to understand what spatial thinking is, and think of ways to support it within the content that they are teaching, (ii) Emphasise the strand of geometry and spatial sense, (iii) Emphasise spatial language, (iv) Encourage visualisation strategies, (v) Emphasise and celebrate visual displays of data, (vi) Use gestures and encourage students to gesture, (vii) Provide meaningful opportunities to investigate mathematical concepts and problems using manipulatives, (viii) Provide playful opportunities for students to exercise their spatial reasoning, and (ix) Take advantage of technology to promote spatial reasoning. Though each of these SEs are supported with by research further work is required to understand their particular role in a cohesive strategic approach to spatialising teaching and learning. This short paper can't address all of these SEs but of particular importance in the current Irish is perhaps the recommendation to place emphasis on geometry.

Emphasise strands of geometry and spatial sense

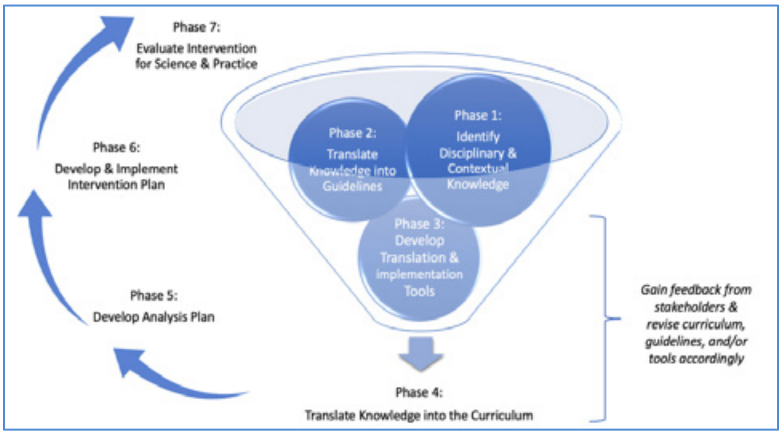
Though "much of the thinking that is required in higher mathematics is spatial in nature" (Jones 2001, p. 55) Newcombe et al. (2019) recognises that spatial reasoning is a particularly strong contributor to mathematics achievement in the strand of geometry. This may be of particular interest to Irish post-primary educators because geometry is a relative weakness among our students. In TIMSS 2019, as in previous TIMSS assessments, geometry was the subscale in which Second Year students from Ireland score significantly lower than our countries overall mathematics score with a gender gap in favour of males on this subscale (Perkins & Clerkin, 2020). This is consistent with the performance of 15-year-old Irish students in "Space and shape" PISA tests (Clerkin et al., 2016). This weakness has also been identified in the Chief Examiners report (2015) noting that at higher level candidates struggled with questions from the geometry strand "in particular co-ordinate geometry and the construction of a triangle" (DES, 2015, p.15). It has been noted in the past that geometry is getting less time and attention in the classroom than other mathematics topics and less than the international average (Clerkin, Perkins & Chubb, 2018). It is perhaps fair to say that geometry is deserving of renewed attention for a number of reasons outlined above.

Developing a framework for spatialising mathematics curricula:

Strategy 2 requires that tools suitable for spatialising the curriculum would be developed. There is a shortage of systematic guides for teachers who wish to optimise spatially rich approaches in their classrooms. Gagnier and Fisher (2020) present a seven step Knowledge Translation Framework (KTF) to guide the infusion of SR research into science curricula. This offers a framework that could potentially be adapted to mathematics education (figure 2).

Figure 2

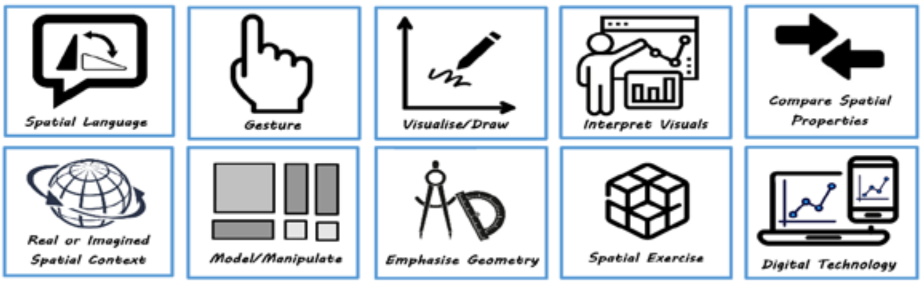
Knowledge Translation framework Gagnier and Fisher, 2020



The development of this framework is a substantial and complex task. Phase 3 of the KTF requires the creation of research translation, implementation and refinement tools that support the systematic translation of SEs into the curriculum, because this relates directly to classroom practice, resources and pedagogy it is of particular interest to teachers of mathematics. Gagnier and Fisher (2020) created instructional supports including example lesson plans, a spatial word bank poster and SE icons for science education. Similar supports are required for mathematics educators. Using the research outlined in this paper ten spatial tools or SE scaffolds for mathematics have been identified and icons representing these strategies have been created (figure 3).

Figure 3

Spatial enhancement icons for mathematics



These SE scaffolds include: being particular about spatial language; using physical gesture; encouraging sketching, constructions and graphing; promoting visualisation strategies; engaging meaningfully with the spatial properties of visual representations; incorporating spatial contexts into mathematical tasks; using manipulatives and models; emphasising geometry; promoting spatial activities; and using DS and technology to engage students with dynamic spatial representations. Embedding SEs that promote SR into lesson planning, practice and assessment would add focus and support a systematic approach to spatialising mathematics teaching and learning. While there is some research supporting each of these SEs more is required if we are to understand how to effectively combine them to shape mathematics curricula and pedagogy.

Conclusion

Mathematics teaching and learning would be enhanced if educators recognised the benefits of promoting SR in mathematics and found effective ways of doing it.

> "It is likely enough that spatial intelligence is an important element in STEM success that we should use this idea in designing curricula, training teachers, setting goals and developing assessments, while simultaneously evaluating the effectiveness of the efforts and continuing basic research on the mechanisms" (Newcombe 2017 p.37).

Newcombe's OECD paper (2017) proposes two worthy approaches for exploiting this finding Strategy 1 is to directly train students in spatial skills and Strategy 2 involves spatialising mathematics curricula using tools that support spatial thinking. Though it might seem ideal to pursue both strategies in parallel further research is required to understand how to effectively and efficiently do this and there exist challenges to implementing each strategy. To effectively realise Strategy 2 a systematic framework that translates research into practice (KTF) will need to be developed along with professional development that promotes teachers' understanding of the important role of SR in mathematic learning, up-skills teachers to effectively embed spatial enhancements (SEs) into teaching and learning and perhaps to develop teachers own SR abilities. Research has identified a number of practical SEs that could be used to scaffold the development of SR and embed spatially rich approaches into classroom practice. These offer potential for the development of curricula and effective pedagogy but require further research to help educators understand how to use and combine these tools collectively to shape the curriculum. There are many questions that remain to be answered as literature provides little practical advice on: How often SEs should be used? Is a combination of SEs more effective than using one in a lesson? How frequently SEs should be used in a lesson? Which spatial enhancements best support the learning of specific mathematical content? Will it take more time to cover the curriculum using spatially enhanced lessons? Does the inclusion of SEs facilitate conceptual understanding or procedural fluency? And, are there specific considerations for diverse learners?

Mechanisms by which spatial skills training can promote mathematical thinking are still not well understood (Young et al., 2018). If we knew more about these mechanisms, educators would be better positioned to appropriately use and develop spatially rich approaches and utilise students' spatial skills to aid mathematical learning and develop framework for spatialising mathematics. A promising avenue for future work is not just to support spatial thinking in general to show students can use this kind of thinking to solve particular kinds of mathematical problems (Casey, 2004). Also, little is known about the teachers' current competence and confidence in teaching using SEs and about the prevalence of spatially rich approaches currently being used in post-primary mathematics lessons in Ireland. The next steps in this research which focuses on Strategy 2 will be to develop a KTF for mathematics and then to collaborate with post-primary teachers in the development, teaching and assessment of spatially rich post-primary mathematics lessons to understand how to effectively use SEs and other supports to harness untapped potential of SR to improve the teaching and learning of mathematics.

References

Bishop, A. J. (1980). Spatial abilities and mathematics education—A review. *Educational Studies in Mathematics*, 11(3), 257-269.

Casey, B. (2004). Mathematics problem-solving adventures: A language-arts-based supplementary series for early childhood that focuses on spatial sense. Engaging young children in mathematics: *Standards for Early Childhood Mathematics Education*, 377-389.

Clerkin, A., Perkins, R and Cunningham, R. (2016). TIMSS 2015 Mathematics and Science In Ireland: In Primary And Post-Primary Schools. *Educational Research Centre*

Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, 15(1), 2–11.

Diezmann, C. M., & Lowrie, T. (2012). Learning to think spatially: What do students 'see'in numeracy test items? *International Journal of Science and Mathematics Education*, 10(6), 1469-1490.

Frick, A. (2019). Spatial transformation abilities and their relation to later mathematics performance. *Psychological Research*, 83, 1465–1484.

Gagnier, K. M., & Fisher, K. R. (2020). Unpacking the Black Box of Translation: A framework for infusing spatial thinking into curricula. *Cognitive Research: Principles and Implications*, 5(1), 29.

Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences*, 23, 114-122.

Halpern, D. F. (2000). *Sex differences in cognitive abilities* (3rd ed.). Mahwah: Erlbaum.

Harris, D., Lowrie, T., Logan, T., & Hegarty, M. (2021). Spatial reasoning, mathematics, and gender: Do spatial constructs differ in their contribution to performance? *British Journal of Educational Psychology*, 91(1),

Hawes, Z., Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin and Review* 27, 465–482 (2020).

Hembree, R. (1992), "Experiments and relational studies in problem solving: A meta-analysis", *Journal for Research in Mathematics Education*, Vol. 23, pp. 242-289

Jones, K. (2002), Issues in the Teaching and Learning of Geometry. *In*: Linda Haggarty (Ed), *Aspects of Teaching Secondary Mathematics: perspectives on practice*. London: Routledge Falmer. Chapter 8.

Lowrie T., Logan T. (2018) The Interaction Between Spatial Reasoning Constructs and Mathematics Understandings in Elementary Classrooms. In: Mix K., Battista M. (eds) *Visualizing Mathematics. Research in Mathematics Education*. Springer, Cham.

MacFarlane-Smith, I. (1964) *Spatial Ability: its educational and social significance.* London: University of London Press.

NRC (2006). Learning to think spatially: GIS as a support system in the K–12 curriculum. Washington, DC: National Academic Press.

NCCA (2015). Junior Key Skills. Department of Education and Skills

NCCA (2017). Junior Cycle Mathematics. Department of Education and Skills

National Council of Teachers of Mathematics (2000). *Principals and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA.

Newcombe, N. (2017). Harnessing spatial thinking to support stem learning. *OECD Education Working Paper* No. 161

Newcombe, N., Booth, J., & Gunderson, E. (2019). Spatial Skills, Reasoning, and Mathematics. In J. Dunlosky & K. Rawson (Eds.), *The Cambridge Handbook of Cognition and Education* (Cambridge Handbooks in Psychology, pp. 100-123). Cambridge: Cambridge University Press.

Ontario Ministry of Education. (2014). Paying Attention to Spatial Reasoning, K-12 Support Document for Paying Attention to Mathematics Education. Queen's Printer for Ontario.

Pantziara, M., A. Gagatsis and I. Elia (2009), "Using diagrams as tools for the solution of non-routine mathematical problems", *Educational Studies in Mathematics*, Vol. 72/1, pp. 39-60. Unclassified

PDST (2021). Retrieved online at https://www.projectmaths.ie/for-teachers/digital-learning/learn-to-use-geogebra/. Accessed July 5th 2021(Newcombe, 2017)

Perkins, R. & Clerkin A. (2020). TIMSS 2019 Ireland's results in mathematics and science. *Educational Research Centre* (ERC).

Reilly, D., Neumann, D. L., & Andrews, G. (2017). Gender differences in spatial ability: Implications for STEM education and approaches to reducing the gender gap for parents and educators. *In Visual-spatial Ability in STEM Education* (pp. 195-224). Springer, Cham.

Sorby, S., Veurink, N., & Streiner, S. (2018). Does spatial skills instruction improve STEM outcomes? The answer is 'yes'. *Learning and Individual Differences*, 67, 209-222.

Syahputra, E. (2013). Increasing Spatial Ability of Students through the Application of Realistic Mathematics Learning. Yogyakarta: *Cakrawala Journal*. November 2013.

Mathematics Self-Efficacy in PISA and Relevance to Teaching and Learning in Irish Classrooms

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The self-efficacy beliefs of students and their impact on school performance have been investigated in PISA studies across a range of areas, including mathematics. Self-efficacy refers to the beliefs one has in one's abilities and actions to produce desired outcomes. These beliefs influence students' behavioural, cognitive and motivational engagement in learning. They are a significant factor in dimensions of performance such as application, persistence and resilience in the face of challenges. Mathematics was a main focus of investigation in PISA 2012. Mathematics self-efficacy was strongly associated with mathematics performance at the country level. Countries with higher mean performance in mathematics were those where students are more likely to report feeling confident about being able to solve a range of pure and applied mathematics problems. The relevance of these findings to the teaching and learning of mathematics in the context of Irish classrooms is discussed.

Introduction

Students' beliefs about their abilities in specific domains of academic and school activities, their self-efficacy beliefs, are now widely recognised as a key factor in school performance (Bandura, 1986,1997; EACEA, 2011; OECD, 2003, 2013; Pajares, 1996; Usher & Pajares, 2008, Zimmerman, 1999). Since it was introduced by Bandura (1977), the concept of self-efficacy has received extensive attention from educational researchers interested in its role in students' academic and school performance. Perceived self-efficacy, the "beliefs one has in one's capabilities to organise and execute the courses of action required to produce given attainments" (Bandura, 1997, p.3) is posited as a key factor in the actions and efforts that people undertake across a range of areas, including health functioning and education.

As a fundamental part of his social cognitive theory, Bandura (1986) contended that unless people believe they can produce desired outcomes, they have little incentive to act or persist in the face of challenges. For this reason, how people behave can often be predicted more accurately by the beliefs they hold about their abilities, rather than by their actual abilities. This occurs because self-efficacy perceptions powerfully influence what individuals do with the knowledge and skills they have. This helps to explain why there is often a mismatch between people's behaviour and achievement and their levels of ability. Bandura (1977, 1986) advances a view of human functioning that gives a central role to cognitive, vicarious, self-regulatory and self-reflective processes. People are seen as self-organising, proactive, self-reflecting and self-regulating. How people interpret the results of their behaviour informs and alters their environment and the personal factors they possess. This is the basis of Bandura's concept of reciprocal determinism, the view that personal factors, behaviour and environmental influences interact in reciprocal fashion. In the classroom setting, for example, students' self-beliefs can be enhanced when students alter their thoughts and emotions (personal factors), when their teachers use effective classroom strategies (environmental factors) and when students themselves improve their own practices of self-management (behaviour). Social cognitive theory is rooted in a view of human agency in which people proactively engage in their own development. Fundamental to this sense of agency is the fact that individuals have self-beliefs that enable them to exercise a degree of control over their thoughts, feelings and actions.

In the context of education, self-efficacy beliefs influence students' behavioural, cognitive and motivational engagement in learning and are a significant factor in dimensions of performance such as application, effort, persistence and resilience in the face of challenges. Self-efficacy beliefs operate through the mediating role they play in how people make use of the knowledge and skills they possess. While students with a high sense of efficacy tend to pursue more challenging learning goals and are more resilient in resisting adverse academic influences, students with diminishing self-efficacy can become caught in a downward cycle of academic underachievement, leading to unhelpful attitudes towards school and learning (Bandura, 1986, 1997). The PISA 2000 report, *Learning for Life* (OECD, 2003, p.8) noted that "The degree to which students believe in their own efficacy is the strongest single predictor of whether they will adopt strategies that make learning effective". In mathematics, while performing can well lead to an increased sense of efficacy, students with low levels of self-efficacy are at risk of underperforming even though they may have the ability. If students do not believe they have the ability to accomplish specific tasks, they are less likely to employ the effort and strategies required to complete tasks successfully. Thus, the lack of adequate self-efficacy contributes to student underachievement and can become a self-fulfilling prophecy (OECD, 2013).

Student Self-Efficacy Findings PISA 2012

Mathematics was a main focus area in PISA 2012. The study investigated a range of students' self-beliefs including mathematics self-efficacy, mathematics self-concept and mathematics anxiety. The results confirmed previous evidence that, while different mathematics beliefs are related, they are conceptually distinct. Mathematics self-efficacy refers to the extent to which students believe in their own ability to manage mathematical tasks effectively and to overcome difficulties. Students' mathematics self-efficacy was found to be strongly associated with mathematics performance at the country level. Countries with higher mean performance in mathematics were those where students are more likely to report feeling confident about being able to solve a range of pure and applied mathematics problems. Mathematics self-efficacy has also been found to be a predictor of students' selection of mathematics-related areas of study and careers with a significant mathematics component (Hackett, 1995).

For PISA 2012 (OECD, 2013), along with the completion of the performance tasks, students were asked to report on their level of confidence in doing a range of pure and applied mathematical tasks involving some algebra. The study reported that "The relationship between students mathematics efficacy and mathematical performance was strong in 2003 and remained strong 2012 (a correlation of 0.5) on average, across OECD countries and for 23 countries and economies" (OECD, 2013, p.83). Across the countries, mathematic achievement is, on average, associated with an increase of 49 score points per standard deviation increase in self-efficacy– "the equivalent of an additional year of school" (OECD, 2013, p. 93).

Mathematics self-efficacy was also investigated in relation to students' gender and socio-economic status. The study reported that girls and socio-economically disadvantaged students are more likely to have low levels of self-efficacy than boys and socio-economically advantaged students. In relation to girls' self-efficacy, it was stated that "gender differences are striking when students are asked to report on their ability to solve applied mathematical tasks, particularly when the mathematics problem is presented in terms of tasks that are associated with stereotypical gender roles" (OECD, 2013, p.83). Disadvantaged students were also found to be generally less likely than advantaged students to feel confident about their ability to manage specific mathematics tasks. While these differences partly reflect differences in mathematics performance related to socio-economic status, they were large and statistically significant differences, even with comparing students who performed similarly in mathematics. Significantly also, mathematic self-efficacy tended to increase among countries where students had reduced levels of mathematics anxiety.

In Ireland, students' mean score on the PISA 2012 self-efficacy index was "not significantly different from the OECD average score" (ERC, 2013, p.12). Students in Ireland have similar levels of self-efficacy as the average student across the OECD. Also comparable with findings across OECD countries was that "male students (in Ireland), report significantly higher mean scores than females on self-efficacy [0.32 scale points higher]" (ERC, 2013, p.12). Students attending girls' secondary schools had significantly lower self-efficacy [by 0.34 points] than students attending boys' secondary schools (ERC, 2013, p.124). The study also reported that in Ireland "Students attending schools in the School Support Programme (SSP) under DEIS had a significantly lower mean score on self-efficacy [by 0.26 points] compared with students in non-SSP schools" (ERC, 2015, p.124)[1].

Student Self-Efficacy and Attainment

Recognising the role of self-efficacy beliefs in school and academic settings can contribute to our understanding of why there may be a gap between students' accomplishments and their actual capabilities. Efficacy beliefs impact on individuals' thought patterns and emotional reactions. The conversion of knowledge and abilities into proficient action is governed by self-referent thought, activated through an individual's cognitive, motivational and affective processes. Students with low self-efficacy may believe that the challenge is greater than it is in reality, a belief that limits their capacity to address the task demands in a successful manner. Individuals with high self-efficacy, in contrast, will tend to approach difficult tasks and activities with a higher degree of composure. While some

[1] The School Support Program under DEIS (Delivering Equality of Opportunity in Schools) is a national programme designed to ameliorate the effects of disadvantage in schools.

individuals can experience undermining self-doubt about capabilities they clearly possess, others may have confidence about what they can accomplish despite possessing modest skill. Thus, self-efficacy is concerned not primarily with the amount of skills an individual possess, but with what he or she believes can be done with these skills in different situations and contexts.

It is important to note that self-efficacy is not the only, or even the most important, influence on achievement outcomes. No amount of self-efficacy will produce a competent performance when the necessary skills are absent. Researchers have highlighted the issue of overconfidence and students' having a miscalculated sense of efficacy. Students who lack skills or understanding in a domain may also experience the additional challenge of not being aware of their limitations in the area in question. Moreover, researchers have drawn attention to the complex relationship between beliefs and achievement, suggesting a circularity, which may also be a cross-cultural phenomenon (Williams & Williams, 2010). Nonetheless research on self-efficacy, which has been widely investigated in education contexts for 40 years, continue to inform discussion on factors influencing students' academic performance.

In the learning context of the classroom, it is not solely the learner's experiences that influences his or her self-efficacy beliefs; rather, it is the interpretation and inferences that people make about experiences, situations and performances that cause efficacy beliefs to be altered. Bandura (1997) described four main sources of efficacy-enhancing experiences: mastery experience, vicarious experience, verbal persuasion, and physiological and affective states. Based on the work of Pajares (2008), Usher (2009) and Zimmerman (1999), teaching practices which attend to the sources of students' self-efficacy can be described as including:

- *Mastery experiences*: scaffolding the learning; breaking tasks into manageable steps and achieving incremental gains in learning; focussing on learning goals as distinct from performance goals; focussing on the process of learning and developing skills; viewing errors as a part of the learning; differentiated approaches responsive to individual needs; increasing student's capacity as an independent learner; fostering a sense of agency
- *Vicarious experience*: using peer learning and co-operative group work approaches; promoting collaboration and reducing the competitive orientation of the classroom; maximising the instructional function over the comparative function of peer models by focussing on skill development
- *Verbal persuasion*: encouraging students to develop their own internal standard for measuring progress; framing evaluative feedback as gains rather than shortfalls; persuading students that skills are acquired through effort and perseverance
- *Emotional and physiological states*: reassuring students when they are becoming overly anxious about challenges in learning mathematics; reducing time pressures and providing clear guidance in relation to learning tasks

Thus, in the mathematics lesson, while teachers cannot directly raise students' self-efficacy, they can provide opportunities for students to experience and interpret their learning in ways that facilitate the development of a sense of efficacy.

Self-Efficacy in Mathematics Classrooms in Ireland

Very few studies have examined students' self-efficacy in relation to mathematics in the Irish classroom context. Walsh (2013) investigated whether students' experiences of approaches to teaching and learning of mathematics in classrooms can facilitate the development of their self-efficacy beliefs in mathematics. Four schools, two primary and two post-primary, participated in the study that employed mixed methods to acquire both qualitative and quantitative data.

The study found that teachers endeavoured to support students in developing confidence in their abilities in mathematics. Teachers at both levels were conscious of the significant influence of students' level of confidence in the mathematics learning experience. Teachers reported using strategies to promote students' confidence, such as giving praise and encouragement and they provided individual and small group support to students experiencing difficulties. The study found, however, that a number of aspects of classroom practice inhibited opportunities for the promotion of self-efficacy (see below). Moreover, the concept of self-efficacy and its role in the learning and teaching experience were under-utilised and teachers referred to a limited range of strategies to enhance students' efficacy and confidence in mathematics.

Mastery experiences that serve as indicators of capability through the development of skills are the most influential source of efficacy information in the learning context (Bandura, 1997). The description of the mathematics lessons from the study data reflected processes that were, for the most part, teacher-directed. The students' voicing of dependence on the teacher in order to enhance their skills and gain confidence in mathematics was a recurring theme at both the primary and post-primary levels. Students' discussion about mathematics contained frequent references to "the book", "the page", "tests" "exams", "results" "grades", "right answers" and "wrong answers". Accordingly, students revealed a predominant focus on performance goals rather than learning goals (Elliot & Dweck, 1988), as evidenced in a persistent concern with getting "right" answers and doing well in tests and examinations. Self-efficacy theorists contend that to support students in developing a sense of efficacy it is helpful to focus on "learning" or "mastery" goals which identify the progress that has been made in gaining knowledge and skills. However, participating students and teachers in the study referred more frequently to results and levels of performance rather than progress and gains made from students' starting points. Consequently, there were limited opportunities for students to recognise and affirm progress in their own learning and to develop a sense of efficacy in relation to the acquisition of new knowledge and skills in mathematics.

Vicarious experience and verbal and social persuasion provide further potential as sources of self-efficacy in the learning environment. In this study students and teachers reported that groups were used in mathematics lessons, though co-operative and collaborative approaches were not a regular feature of classroom practice. Concerning students' self-efficacy in the school context, Bandura (1997, p.176) noted that the evidence indicated that both "performance attainments and favourable self-appraisals are best achieved through co-operative effort that is organised to work well". Several features of co-operative learning approaches overlap with approaches to support the development of students' self-efficacy. These include: establishing and working towards agreed goals; support for increased co-operative as against competitive behaviour; and support for the group dimension, contributing to a sense of collective efficacy. The influence of vicarious experience is also at play, as within the co-operative framework students have increased opportunities to observe the work of other group members and identify peer models from whom they can learn and so enhance their own sense of efficacy. Thus, the infrequent use of collaborative group approaches limited opportunities for efficacy-enhancing learning experiences during such activities.

A key finding of this study concerned the latent and powerful role of social comparative factors on students' behaviour in the classroom. This was evidenced in students' reluctance to engage in help-seeking practices such,as asking questions, where their own possible weaknesses in mathematics could be revealed to their classmates. Apprehension about getting the "wrong answer", appearing "stupid", or being perceived as-less able than peers was articulated. Thus, the potent and potentially inhibiting influence of social comparison factors on aspects of the students' participation in the learning experience may reduce opportunities for the development of their sense of agency and self-efficacy.

Assessment practices were a critical element of the students' experience of learning in mathematics. Students' interpretations of the results and feedback from summative and formative assessment played a key role in the development of their self-efficacy in mathematics. The study found that assessment, particularly in the form of tests and examinations, was a central element of the experience of the participating students. While assessment for learning was an ongoing element of classroom practice, the preponderance of focus was on summative assessment processes. Standardised test scores were a key concern of the teachers in the primary schools. In the post-primary schools, the students' experience of mathematics was largely concerned with preparation for state examinations.

While teachers identified "lack of achievement" and "failure" as crucial factors in students' confidence, such experiences are relative to the evaluation of task performance. Usher and Pajares (2006, p.137), observed, from their study of the sources of the efficacy beliefs of students, that "academic feedback must be crafted with particular care to how it might be interpreted". When individuals doubt their abilities, they require explicit and frequent feedback on progress that provides them with repeated affirmations of their abilities (Bandura,1997).

The study found that for the post-primary student there is a significant relationship between students' attitudes to mathematics and their perceptions of mastery experiences and feedback in the classroom. Students who perceived themselves as supported in making progress in mathematics and receiving encouraging feedback indicated more positive attitudes to mathematics. These findings underscore the relevance of providing high-quality feedback to students to enhance efficacy and facilitate the development of positive attitudes to mathematics (Hattie & Timperley, 2007, NCCA, 2007). Overall, the study drew attention to the role of self-efficacy beliefs in the student experience in mathematics. It also identified areas of convergence between the aims and approaches identified in self-efficacy research to support the implementation of learner-centred and co-operative approaches in mathematics education (DES, 2010; ERC, 2016; NCCA, 2007; Pajares, 2008; Stipek et al., 1998; Usher, 2009).

Conclusions

Raising students' attainment in mathematics has been a policy priority in Irish education for many years, particularly through the implementation of *Literacy and Numeracy for Learning and Life: The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (Department of Education and Skills, 2011). The STEM Education Policy Statement 2017-2026 (DES, 2017) has set out ambitious areas for action considered necessary to achieve an improved STEM (Science, Technology, Engineering and Mathematics) education experiences and outcomes for learners. The policy's vision is that students' learning experiences should nurture a range of qualities, including curiosity, problem solving and creativity, along with confidence and persistence. Substantial developments in the post-primary mathematics curriculum programme have been implemented over more than a decade. The revision of a new primary mathematics curriculum is well advanced (Dooley, 2019).

Students' performance in mathematics continues to be an area of attention in Irish education. National and international studies, including PISA, provide important sources of information in relation to progress. While in PISA 2018 the overall mean mathematics score of students in Ireland was 499.6, significantly above the OECD average of 489.3 (ERC, 2019), Ireland was not among the countries with the highest-performing students. Ireland's mean score ranked 16th of the 37 OECD countries. Ireland had significantly fewer lower-performing students than the OECD average. However, there were also significantly fewer students performing at the highest levels. The PISA reports (OECD, 2003; 2013) have provided substantial data on the influential relationship between students' self-efficacy beliefs and performance. Few research studies have been undertaken in this area in the Irish school context. Further investigation into how practices in Irish classrooms influence the development of students' sense of efficacy in mathematics would contribute to teachers' knowledge of this significant dimension of teaching and learning.

References

Bandura, A (1977). Toward a unifying theory of behavioural change *Psychological Review*, *84*, 191-215.

Bandura, A. (1986). *Social foundations of thought and action.* Prentice Hall.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. Freeman.

Department of Education and Skills. (DES). (2011). *Literacy and numeracy for learning and life: The national strategy to improve literacy and numeracy among children and young people.* Stationery Office.

Department of Education and Skills. (DES). (2017). *STEM education policy statement 2017-2026*. Stationery Office.

Dooley, T (2019). *Learning and teaching primary mathematics: An addendum to NCCA research reports 17 and 18.* NCCA.

EACEA (2011). *Mathematics education in Europe: Common challenges and national policies.* European Commission: (EURYDICE).

Educational Research Centre (2013). *PISA 2012, Learning for Life*: *The achievements of 15-year olds in Ireland on mathematics, reading literacy and science in PISA 2012.* ERC.

Educational Research Centre (2016). *PISA in classrooms: Implications for the teaching and learning of mathematics in Ireland.* ERC.

Educational Research Centre (2019). *Learning for the Future: The performance of 15 -Year-olds in Ireland on reading literacy, science and mathematics in PISA 2018*. ERC.

Elliot, A., & Dweck, C. (1988). Goals: An approach to motivation and achievement. *Journal of Personality and Social Psychology, 54*, 5-12.

Hackett, G. (1995) Self-efficacy in career choice and development. In A. Bandura (Ed.), *Self-efficacy in changing societies* (pp.232-258). Cambridge University Press.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research. 77*(1), 81-112.

National Council for Curriculum and Assessment (2007). *Assessment in the primary school classroom: Guidelines for schools.* NCCA.

OECD (2003).*Learning for life, student approaches to learning results from PISA 2000.* OECD.

OECD (2013). *PISA 2012 Results: Ready to learn, students' engagement, drive and self-beliefs,* Volume 111. OECD.

Pajares, F. (1996). Self-efficacy beliefs in achievement settings. *Review of Educational Research, 66*, 543-578.

Pajares, F. (2008). Motivational role of self-efficacy beliefs in self-regulated learning. In D. Schunk & B. Zimmerman (Eds), *Motivation and self-regulated learning: Theory, research and applications.* Routledge.

Stipek, D., Salmon, J., Givvin, K., Kazemi, E., Saxe, G., & MacGyvers, V. (1998). The value (and convergence) of practices promoted by motivation researchers and mathematics education reformers. *Journal for research in Mathematics Education*, *29*, 465-488.

Walsh, W. (2013). *Self-Efficacy and its role in the teaching and learning of mathematics*: *A case study*; Unpublished doctoral thesis, Trinity College, Dublin.

Williams T. & Williams K. (2010). Self-Efficacy and performance in mathematics: Reciprocal determinism in 33 nations: *Journal of Educational Psychology*, *102,* 453-466.

Usher, E. (2009) Sources of middle school students' self-efficacy in mathematics: A qualitative investigation. *American Education Research Journal, 49*(1), 275-314.

Usher, E & Pajares, F. (2008). Sources of self-efficacy in school: Critical review of the literature and future directions, *Review of Educational Research, 78*(4), 751-796.

Zimmerman, B. J. (1999). Self-Efficacy: An essential motive to learn. *Contemporary Educational Psychology, 25*, 82-91.

A Framework for Identifying Teacher Competencies of Mathematical Modeling

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With the proposed introduction of a new Primary Mathematics Curriculum in Ireland, mathematical modeling[1] is a pedagogical approach to teaching mathematics that Irish primary school teachers may not be familiar with. This article explores literature in the field that can support teachers in this role and identifies a gap in the educational field of mathematical modeling at primary school level. A proposed framework for identifying teacher competencies of mathematical modeling is presented as means of providing professional development in mathematical modeling.

Introduction

In the midst of reform at primary school level in Ireland and with the proposed introduction of a new Primary Mathematics Curriculum, it is essential that teacher competencies to implement these changes are identified and supported. There are a number of proposed changes from the *Primary Mathematics Curriculum* (PMC) (Department of Education[2] (DoE), 1999) that are highlighted in the research. One such change is the implementation of meta-practices that teachers should engage with, when teaching mathematics (Dooley et al., 2014, p. 36). These include maths-talk, development of a productive disposition, formative assessment, cognitively challenging tasks and mathematical modeling. While teacher competencies for these meta-practices will need to be established, there appears to be a gap in the literature regarding mathematical modeling at primary school level. This article will explore literature in the area of mathematical modeling at primary school level as well as identifying a proposed framework for identifying teacher competencies. This will be an essential prerequisite to professional development provision as part of the roll out of the new primary mathematics curriculum.

Literature Review

Mathematical Modeling

Mathematical modeling describes the process of developing a model and can begin with a real life problem. Models are simplified representations of reality that allow the application of mathematics (Greefrath & Vorholter, 2016, p.9). During the mathematical modeling process pupils mathematise real world problems in mathematical terms. Single mathematising in the modeling cycle only requires one step to transfer a real life problem to a model whereas complex mathematising requires more than one step. A model presented by

[1] Modelling and Modeling are found in the literature. For the purpose of this paper the later spelling will be used as in Dooley et al., (2014) and its addendum (Dooley, 2019).

[2] Formally the Department of Education and Science

Blum and Leib (2007) breaks down the modeling process into a number of steps; constructing, simplifying, mathematising, working mathematically, interpreting, validating and exposing. Mathematical modeling can promote the development of mathematical content, process-oriented skills and general life skills for example being critical with data represented in the media or social skills. The duality mathematical modeling plays also needs to be investigated where mathematical modeling itself can develop useful skills and concepts with pupils but can also be a means to learning mathematics.

Systematic Review of the Literature

Stohlmann and Abarracín (2016) conducted a systematic review of the literature, investigating what studies have been conducted at elementary grades (10 years and under) in mathematical modeling. Mathematical modeling is defined as “an iterative process that involves open-ended, real world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations” (Stohlmann & Abarracín, 2016, p. 1-2). Twenty-nine publications were included in the study where mathematical modeling content, assessment data, population, unit of analysis and effectiveness data was gathered. Data collected was generally qualitative in nature using audio and video recordings, student work and researcher field notes. Modeling eliciting activities were the most common approach utilised, however, the majority of the studies are by Lyn English who engaged in a three-year longitudinal study and published papers throughout, so this may not be representative of the greater research. One study investigated the learning of heuristics and the metacognitive processes for mathematical applications, emphasising the importance problem solving plays in the mathematical modeling process. Ten of the studies were conducted in Australia, six were in the US and Europe represented seven articles including one from the Irish context. Only four studies included professional development for participating teachers. Ultimately, it was found that young children are capable of mathematical modeling and that they benefit from it greatly.

Problem Solving

Problem solving is an essential part of the mathematical modeling process but also of the PMC (DoE, 1999) where it is described as a skill that needs to be developed with pupils as well as context for developing concepts and skills. Pupils need “to analyse mathematical situations; to plan, monitor and evaluate solutions; to apply strategies; and to demonstrate creativity and self-reliance in using mathematics” to develop higher order skills (DoE, 1999, p. 8). The implementation of mathematical modeling will build on current practices of problem solving in the primary school context.

International Assessments

Ireland’s performance in international assessments such as Trends in Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) portray the progression Ireland has made in mathematics in recent years for example in PISA 2018 the OECD average was 489.3 and Ireland scored 499.6, similar to TIMSS 2019 where Ireland scored 48 points above the centre point. While Ireland has performed particularly well, it is important to identify where more progress could be made. Sixteen countries significantly outperformed Ireland in PISA 2018 and seven countries in TIMSS 2019 such as China and Singapore. The Education Research Centre (ERC) have highlighted our under performance at higher benchmarks in these international assessments such as PISA 2018 where Ireland achieved 8.2% in the level 5/6 or advanced benchmarks in contrast to Korea who scored 21.4%. Similar findings were highlighted in TIMSS in 2019 where it depicts that only one in seven (15%) of pupils in fourth class achieved the advanced benchmark. Furthermore, McKeown et al., outline that "a number of countries with overall mean scores not significantly different from Ireland's had proportionately more students at Levels 5-6, including Sweden (12.6%) and the UK (12.9%). In Northern Ireland, 8.3% of students performed at Levels 5-6" (2019, p. 8). Comparable findings were portrayed by the ERC for TIMSS 2019 (Perkins & Clerkins, 2020). TIMSS 2019 reported a relative weakness in the cognitive domain of reasoning (Perkins & Clerkins, 2020).

Considering Irelands' strong position in international assessments, it is relevant to look at National Assessments of Mathematics and English Reading (NAMER) in Ireland. In a performance report published by the ERC in 2014, it was found that "there is scope for pupils in Second and Sixth classes to improve further on higher level mathematical processes, including Apply & Problem Solve" (Shiel et al., 2014, p. 15). A further context report was published in late 2015 and in an effort to explain possible reasons for challenges in developing higher benchmarks and raising standards to compete with countries that are significantly outperforming Ireland it identified a number of possible areas of concern; "While the nature and focus of mathematics instruction is likely to be a factor, other factors affecting curriculum implementation are also relevant, including support for teachers in the form of professional development, time allocated to teaching mathematics, the quality and appropriateness of support materials such as tests and text books, the support pupils receive at home and at school, and pupils' dispositions" (Kavanagh et al., 2015, p. 16). Furthermore, problem solving with emphasis on mathematical modeling, realistic contexts and collaboration is emphasised and recommended (p. 17). This forms the basis of a rationale to introduce mathematical modeling into the Irish primary classroom where more realistic mathematics problems may be explored. It will also investigate how professional development can support teachers in implementing this in their classrooms.

Research in the Irish context

In 2014, a research paper was published to support the development of a new Primary Mathematics Curriculum in Ireland (Dooley at al., 2014). Two approaches to mathematical modeling were identified as part of this process; Realistic Maths Education (RME) (van den Heuval-Panhuizen, 2003) and Lyn English's longitudinal study in Australia (eg. 2006, 2007, 2008). Dooley re-emphasised these approaches in a further report published in 2019. In the RME approach mathematical modeling can be identified as an organising activity from which a model emerges (Gravemeijer & Stephan, 2002) and deeper, more flexible understandings are generated. The emphasis is on the mathematisation process and the generalisation of models that can be used in a variety of situations. The literature describes the transition in the modeling process as a model of a situation to a model for thinking about mathematics (Gravemeijer & Stephan, 2002). Content related goals associated with this perspective include solving realistic problems, the development of modeling skills and developing an understanding of the real world. English's approach to mathematical modeling is identified as a means of addressing realistically complex situations where models or conceptual tools are developed. These are needed for a specific purpose or a goal. This could be considered an extension to problem solving currently experienced at primary school. The first stage of this modeling cycle begins with a modeling eliciting activity and progresses to model exploration and model application with related problems. Critical reflection is essential in this process.

While the RME approach to mathematical modeling can be categorised as an organising activity and English and her colleagues' approach can be identified as means of addressing realistically complex situations, both approaches focus on the mathematisation that occurs as part of that process. This will form the basis for the theoretical perspective adopted in this study.

Teacher Competencies in Mathematical Modeling

In the literature, competencies of mathematical modeling are presented, however, teacher competencies required to teach mathematical modeling are limited. One such article published by Ferri and Blum (2009), as part of Congress of the European Society for Research in Mathematics Education (CERME 6), explored teacher competencies for mathematical modeling in teacher education. A number of competencies were regarded as important in this research including; theoretical, task related, teaching and diagnostic. Assessment was also mentioned but was not expected from teachers with no experience of it. The seminar was broken down into five parts; theory, practice (tasks), theory and practice, presentations and lastly lesson-reflection (p. 2048). The intention of this research was that students would learn mathematical modeling as well as develop strategies to teach it. Cooperative learning strategies were applied to achieve this where activities such as jigsaw, think-pair-share, round robin brainstorming, silent writing conversations and inside-out-circle were utilised. For teachers engaging in mathematical modeling for the first time it would be essential to become familiar with mathematical modeling competencies through professional development before teacher competencies could be addressed as described above.

Professional Development

In 2020 a report was published from the Education Research Centre (ERC) which outlined "the importance of focusing an evaluation on the core features of effective Teacher Professional Learning (TPL), rather than the mode of delivery or type of activity" (Rawdon et al., 2020. p. 2). *Cosán,* The Natural Framework for Teacher Learning, outlines learning processes that teachers experience during TPL. These include; mentoring/ coaching, practice and collaboration, research, reading and professional contributions, immersive professional activities, and courses, programmes, workshops and other events.

Professional development is an essential part of improving school performance (Hargreaves, 1994) and transformative practices are key for effective change in practice for teachers (Kennedy, 2005). Collaborative Professional Inquiry models are deemed to be more transformative where teacher agency and autonomy are central. Considering this, a transformative model will be established where "all models and experiences that include an element of collaborative problem identification and subsequent activity, where the subsequent activity involves inquiring into one's own practice and understanding more about other practice, perhaps through engagement with existing research" (Kennedy, 2014, p. 693).

Guskey's (2000) five-level evaluation framework includes participants' reactions; participants' learning; organisational support and change; participants' use of new knowledge and skills; and, student learning outcomes. Evaluation at one level is not enough to see change. Guskey's framework does not include core features of effective TPL which is outlined as an effective means of evaluating and therefore, evaluation would have to focus on the model of CPD which is collaborative professional inquiry. However, conceptual frameworks from Desimone (2009), Merchie et al., (2018) and King (2014) include features of effective TPL. Desimone (2009), evaluation model suggests five main features of effective TPL including; content focus, active learning, coherence, duration and collective participation. This model assumes the material being covered is content related which would have to be adopted for the implementation of mathematical modeling, a meta-practice. Merchie et al., (2018) framework builds on Desimones' work where it outlines linear stages of professional development including features of the intervention, teacher quality, teacher behaviour and student behaviour. Alongside this, contextual factors as well as teacher and student personal characteristics are included. King (2014), emphasises diffusion of TPL where she speaks of the ripple effect within the system. Schoenfeld (2017), Teaching for Robust Understanding (TRU) Framework, includes five dimensions of powerful classrooms, including; the content, cognitive demand, equitable access to content, agency, ownership and identity, and formative assessment. These are divided into three proficiency levels to assist teachers to reflect on their teaching. The purpose is for professional development as opposed to evaluation of practice. The focus also shifts from the teacher to student learning.

Research Questions

It is intended that this research could support teacher educators and teachers in the implementation of mathematical modeling in Ireland. It is envisioned that this research will identify the goal mathematical modeling could play in the Irish context for example; will it be used as a tool to learn mathematics (content), to develop mathematical modeling competencies (processes) in their own right or both. The research will explore perspectives of mathematical modeling that would work best in the Irish context and it will investigate teacher competencies needed to implement mathematical modeling in the primary classroom. The three research questions include; what are the goals of mathematical modeling in the Irish context; what theoretical framework of mathematical modeling would best fit the Irish context; and what teacher competencies are needed to implement mathematical modeling in the Irish primary classroom?

Discussion

It is proposed to answer three research questions to identify areas of support Irish teachers will require in the implementation of mathematical modeling in the Irish primary classroom. In order to accomplish this, it is suggested that a Collaborative Professional Inquiry model of TPL be formed with at least six participants where they engage in a six part programme of professional development over six months. The six parts include; establishment of a Collaborative Professional Inquiry, theory of mathematical modeling, practice and design of mathematical modeling tasks, theory and practice- implementation of mathematical modeling in classroom, assessment to inform practice- implementation of mathematical modeling in the classroom with a specific emphasis on formative assessment, reflection on theory and practice (of the implementation of mathematical modeling). Reflection will form a key role in data collection, so principles of Brookfield (1995), Rolfe (2001) and Gibbs (1988) will be integrated throughout the research. Data will be collected from multiple perspectives of the teacher, the group, the researcher/ professional development facilitator.

Conclusion

Proposed Methodology

Constructivism will embody this research where ontological assumptions will allow for multiple meanings and perspectives within historical and social context. An interpretive stance will be reported where the investigation will be "interested in the ways in which individuals and groups construct their world through their actions, beliefs and values" (Hamilton & Whittier, 2013, p. 27). This research will provide a case study at primary level in Ireland, which will lead curriculum innovation in implementing mathematical modeling to inform policy. Key conditions will be necessary for participants to partake in the Professional Collaborative Inquiry including; a learning culture promoted by leadership, external expertise (researcher), time for collaboration, teacher agency and voluntary participation (Kennedy, 2014; King, 2014). The TRU Framework by Schoenfeld (2017) will be adapted throughout the research as a reflective tool whereas, Guskey framework including Desimone core features will be used as an evaluative component. Both the core features of professional development and the transformative model of Collaborative Professional Inquiry can be evaluated in this research.

References

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modeling problems? In C. R. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA–12): Education, engineering and economics,* 222–231. Horwood Publishing.

Brookfield, S. (1995). *Becoming a critically reflective teacher*. Jossey-Bass.

Department of Education. (1999). *Primary mathematics curriculum.* Department of Education.

Desimone, L.M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181-199.

Dooley, T., Dunphy, E., & Shiel, G. (2014). *Mathematics in early childhood and primary education (3-8 year olds): Teaching and learning. Research report 18.* NCCA.

Dooley, T. (2019). *Learning and teaching primary mathematics: An addendum to NCCA research reports 17 and 18*. NCCA.

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics, 63*(3), 303-323.

English, L. (2007). Cognitive psychology and mathematics education: Reflections on the past and the future. *The Montana Mathematics Enthusiast, Monograph 2*, 119–126.

English, L. (2008). Setting an agenda for international research in mathematics education. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 3–19). Routledge.

Ferri, B., & Blum, W. (2009). Mathematical modelling in teacher education- Experiences from a modelling seminar. *CERME 6* (pp. 2046-2055). INRP.

Gibbs, G. (1988). *Learning by doing: A guide to teaching and learning methods*. Oxford Further Education Unit.

Gravemeijer, K., & Stephan, M. (2002). Emergent models as an instructional design heuristic. In K. Gravemeijer, R. Lehrer, B. Van Oers, & L. Verschaffel (eds.) Symbolizing, modeling and tool use in mathematics education. *Mathematics Education Library*, *30*, 145-169. Springer.

Greefrath, G. & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: Approaches and developments from German speaking countries.* Springer International Publishing.

Guskey, T. R. (2000). *Evaluating professional development.* Corwin Press.

Hamilton, L., & Whittier, C. (2013). *Using case study in education research*. Sage.

Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture in the postmodern age*. Cassell.

Kavanagh, L., Shiel, G., Gilleece, L., & Kiniry, J. (2015). *The 2014 national assessments of English reading and mathematics. Volume 2: Context report*. Educational Research Centre.

Kennedy, A. (2005). Models of continuing professional development: A framework for analysis. *Journal of In-service Education, 31*(2), 235- 250.

Kennedy, A. (2014). Understanding continuing professional development: the need for theory to impact on policy and practice. *Professional Development in Education*. *40*(5), 688-697. https://doi.org/10.1080/19415257.2014.955122

King, F. (2014). Evaluating the impact of teacher professional development: an evidence-based framework. *Professional Development in Education, 40*(1), 89-111.

Merchie, E., Tuytens, M., Devos, G., & Vanderlinde, R. (2018). Evaluating teachers' professional development initiatives: towards an extended evaluative framework. *Research Papers in Education, 33*(2), 143-168.

McKeown, C., Denner, S., McAteer, S., & Shiel, G., with O'Keefe, L. (2019). *Learning for the future: The performance of 15-year-olds in Ireland on reading literacy, science and mathematics in PISA 2018*. Educational Research Centre

Micah, S. S, & Lluís, A. (2016). What is known about elementary grades mathematical modelling. *Education Research International*, 2016(2), 1-9. https://doi.org/10.1155/2016/5240683

Perkins, R., & Clerkins, A. (2020). *TIMMS 2019: Ireland's results in mathematics and science.* Retrieved from https://www.erc.ie/wp-content/uploads/2020/12/03-ERC-TIMSS-2019-Report_A4_Online.pdf

Rawdon, C., Sampson, K., Gilleece, L., & Cosgrove, J. (2020). *Developing an evaluation framework for teachers' professional learning in Ireland: Phase 1 Desk-based research.* Educational Research Centre.

Rawdon, C., Sampson, K., Gilleece, L., & Cosgrove, J. (2020). *Developing an evaluation framework for teachers' professional learning in Ireland: Phase 1 Desk-based research (Executive summary).* Educational Research Centre.

Rolfe, G., Freshwater, D., & Jasper, M. (2001). *Critical reflection for nursing and the helping professions*: *A user's guide*. Palgrave.

Schoenfeld, H. (2016). *An introduction to the teaching for robust understanding (TRU) framework*. Berkeley, California.

Stohlmann, M. S., & Albarracín, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International, 2016*(2), *1-9.* https://doi.org/10.1155/2016/5240683

Shiel, G., Kavanagh, L., & Millar, D. (2014). *The 2014 national assessments of English reading and mathematics. Volume 1: Performance report.* Educational Research Centre.

van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35.

Wenger, E., McDermott, A., & Snyder, W. (2002). *Cultivating communities of practice: A guide to managing knowledge.* Harvard Business School Press.

"It's a Bit Like Going to McDonald's – In the Moment You Feel Satisfied, You Feel Great, But an Hour Later You are Hungry": Mathematics Teachers' Views of Professional Development in Ireland

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This paper details the results of a study that aims to reimagine the professional development offered to Irish secondary school mathematics teachers. This reimagined design will be based on four perspectives, one of which is that of current mathematics teachers in the Irish context. It is this perspective that is the subject of this paper. Through a series of semi-structured interviews and a follow-up national survey, teachers expressed their opinions on what professional development should and shouldn't entail. These opinions can be grouped into the following themes: the role of the internet in professional development; the provision and sharing of teaching and learning resources; having opportunities to discuss teaching approaches and pedagogy styles with colleagues; engagement with and access to professional development; professional development activities; the facilitator; and the role of professional conversations in teachers' continued professional development.

The Role of Teacher Professional Development

The endeavour to ensure that mathematics is accessible for all communities of learners is a noble one. It is linked, however, with the ongoing professional development of those in the role of "teacher" within these communities to ensure they remain equipped with the skills and knowledge necessary to meet the needs of their mathematics learners and create an environment that allows for meaningful learning for all. After graduation from their initial teacher education programmes, professional development opportunities are often the only form of formal education teachers experience in relation to how they teach. As such, Desimone et al. (2002) credit professional development as a crucial mechanism to increase and enhance teacher knowledge and teacher classroom practices. Additionally, Flecknoe (2000) found that teachers developed constructive attitudes towards both their subject and the act of teaching after engaging in professional development, and the students of those teachers who experimented with new strategies and practices introduced at a professional development programme increased their desire to learn. In the subject area of mathematics, Smith (2004) witnessed increased levels of motivation and enthusiasm among teachers who had engaged in particular professional development programmes.

When conducted effectively, professional development has the potential to challenge teachers and assist them to develop new skills. In an ever-changing world, teachers need to adapt how they teach to adequately prepare their students for further education, work and life in general. This requires teachers to understand both teaching and learning, address the particular needs of students, and connect students' experiences with the goals of the curriculum (Darling-Hammond, 2005). Developing, changing, and improving one's instructional practice is therefore a life-long (or career-long) endeavour and requires the assistance of various, effective professional development opportunities. The research project discussed in this paper sought to investigate the professional development available to Irish mathematics teachers from four perspectives – the theoretical and experimental literature perspective, the international experience perspective, the policy perspective, and the perspective of current Irish mathematics teachers. The aim of the project was to design a reimagined model of professional development that incorporates and aligns with each of these four perspectives. It is the understanding gained from the fourth perspective that will be the subject of this paper. First, however, a brief overview of the definition of effective professional development will be provided.

Defining Effective Professional Development for Teachers

While research suggests that teacher professional development is crucial for the improvement of systems, schools, and student achievement, some argue that the term *professional development* is conceptually vague (Coffield, 2000) and the concept ambiguous and contested (Friedman & Phillips, 2004). Hoban (2002) encourages separation of the terms *professional development* and *professional learning* while Wei et al. (2009) describe professional development as a subset of professional learning. Wei and colleagues conceptualise professional learning as the product of any activity that results in an increase in teacher knowledge or a change in teacher practice, while they refer to professional development as the activities that are formal, planned, and aim to impact a teacher's professional learning when designed. It is this distinction and definition that will be used throughout this paper. This definition of professional development does not include unplanned experiences that happen to result in professional learning, although the authors do recognise the importance of these instances.

There is a broad consensus in the field of education research on the characteristics of effective professional development. Knapp (2003) summarises the work of many by providing the following list of six essential characteristics that increase the probability that a professional development activity impacts the classroom practices of a participating teacher. Knapp (2003) argues that professional development opportunities should:

1. Concentrate on classroom teaching that emphasizes high learning standards and on evidence of students' learning to standard.
2. Focus on building teacher's pedagogical content knowledge.
3. Model "preferred" instructional practices (e.g. active learning), both in classrooms and in adult learning situations.
4. Locate professional learning in collaborative, collegial – and generally school-based – learning environments.
5. Offer rigorous and cumulative opportunities for professional learning over time.
6. Align with reform initiatives. (pp. 119-120)

Unfortunately, there is a lack of alignment between much of the professional development on offer to Irish mathematics teachers and what are deemed best practices in this regard. The professional development opportunities that are most widely available to Irish mathematics teachers are generic workshops, a model of professional development that has been shown to be often ineffective due the lack of connection to teachers' individual contexts and practices (Hawley & Valley, 1999). Therefore, Irish mathematics teachers are in urgent need of a restructured and reimagined model of professional development, a model which this wider research project aims to understand and develop. In order to create such a model, the opinions and experiences of current Irish mathematics teachers were sought to ensure that teacher voice was central to the redesigned model and that the model aligns with the needs and wants of current Irish mathematics teachers. As outlined earlier, this perspective is one of four perspectives that will be incorporated into the final design, but it is the results from this teacher perspective that will outlined in the remainder of this paper.

Gathering and Analysing the Perspectives of Current Irish Mathematics Teachers

In October 2020 an invitation was sent to all secondary school mathematics teachers in Ireland, via their school administrators' email, to partake in a semi-structured interview on the topic of professional development. In this email, teachers were provided with three questions that would form the core structure of the interview so that they would have time to consider their responses prior to the recorded phone interview. These three questions were:

1. In what area of mathematics teaching/curriculum reform would you like to receive support as a maths teacher?
2. What provisions/supports/forms/types of professional development would be most helpful for you in your role as a maths teacher?
3. What forms/types of professional development do you currently engage with in your role as a maths teacher? What has been your recent experience with professional development? How helpful has this been for you as a maths teacher?

Between October 2020 and January 2021, 24 teachers were interviewed. Each interview was audio recorded and transcribed. A thematic analysis was conducted on the interview transcripts using Braun and Clarke's (2006) phases of thematic analysis as follows:

1. Each interview was read and reread.
2. On the second reading the content of each interview was summarised into a series of "I" statements.
3. Related or similar statements from all teachers were placed together.
4. Related or similar statements were rewritten as a single statement along with a frequency count to show how many teachers this new statement linked with.
5. Similarly themed statements were then grouped and a theme name chosen.
6. Theme names and statements were read and reread.
7. Overly similar theme names and statements were combined until a final list of distinct themes and statements remained.

Following this thematic analysis, the resulting statements were collated into a survey. Surveyed teachers were asked to respond via a Likert scale of agreement/disagreement. The purpose of the survey was to determine what proportion of a broader number of teachers agreed or disagreed with the views of the 24 interviewed teachers. When the survey was piloted, pilot participants stated that the survey was too long. As a result, all statements that related to the views of only one interviewed teacher were removed, with 52 statements remaining in the final document. In late February 2021, the final survey was sent to all secondary school teachers of mathematics in the 730 secondary schools in Ireland via their school administrator's email. 160 responses were received. An overview of the results of these interviews and the follow up survey are provided in the next section.

Results: Professional Development from the Perspective of Irish Mathematics Teachers

Both the interviews and follow up survey sent to teachers had three main sections that aligned with the three interview questions – in summary, what teachers would like professional development to focus on, what teachers would like professional development to look like, and what teachers' recent experiences with professional development had been. The latter two categories were inherently linked as many interviewed teachers used their recent experiences with professional development to describe what they would and would not like to experience in future professional development opportunities. Due to the short nature of this paper, it is only these two categories that will be discussed. The following subsections outline the results of the interview process in the form of the statements that emerged from the thematic analysis. In addition, the results of the follow-up survey, in the form of the percentage of surveyed teachers that either agreed or strongly agreed with each statement, are provided in the brackets. Each subsection is a theme that emerged from the interview data.

Online Professional Development Opportunities

The theme of "online professional development" and the role the internet could, can, and does play in the professional development opportunities available to mathematics teachers was discussed by 17 of the 24 interviewed teachers. The common views expressed by two or more of these teachers were summarised into the following five statements:

- I would benefit from all professional development opportunities for maths teachers being advertised and/or available on a single website (91%).
- I would benefit from professional development that is offered online (85%).
- An increase in the amount of professional development opportunities offered online would make professional development more accessible to me (82%).
- I would benefit from having access to recorded online professional development that I can engage with at my own convenience (88%).
- I would like to have access to online "refresher" videos where Leaving Certificate higher level mathematical concepts are explained (78%).

Teaching and Learning Resources

The theme of "teaching and learning resources" was discussed by 18 of the 24 interviewed teachers. This theme encompasses concepts such as the sharing of resources among teachers, being provided with resources at professional development opportunities, and the lack of resources available in some schools. The common views expressed by two or more of these teachers were summarised into the following seven statements:

- I would benefit professionally from opportunities to share resources with other maths teachers (85%).
- I am more likely to use resources presented at professional development opportunities in my teaching if I am provided with the resource at the end of the professional development opportunity (89%).
- I am more likely to use resources presented at professional development opportunities if the presentation includes evidence of the resources being used in a school setting (84%).
- I would benefit from access to a bank of resources online that teachers could both download and add to and share their experiences of implementing the resources in a comments section attached to the particular resource (92%).
- I am more likely to use resources presented at professional development opportunities in my teaching if the resources are easily accessible online (94%).
- I am more likely to use resources/activities presented at professional development opportunities if they take less than 10 minutes to do in class and are easy to implement (78%).
- I feel that those facilitating professional development opportunities need to be more aware of the limited resources available in many schools (84%).

Teaching Approaches and Pedagogy Sharing

The theme of "teaching approaches and pedagogy sharing" was discussed by 15 of the 24 interviewed teachers. This theme most often appeared when teachers were discussing what they would like their professional development to consist of, rather than what they are currently engaging in. The common views expressed by two or more of the interviewed teachers were summarised into the following four groups of statements:

- I would like to engage in professional development where teachers gather to share how they currently teach a particular topic, with a variety of approaches being discussed (77%).
- I would be willing to share my own approaches with teachers from my own school (88%).
- I would be willing to share my own approaches with teachers from a number of schools (73%).
- Following the sharing of ideas, I would like time to trial these ideas in my classroom before gathering with this group of teachers again to discuss my experience (78%).
- I would benefit from in-school professional development consisting of regular department meetings where we share ideas, try out changes and then report back to one another on our experience (78%).
- Having access to a large number of sample lessons or resources related to a particular pedagogy/approach to teaching would help me better implement this pedagogy/approach into my classroom on an ongoing basis (85%).

Engagement and Access

The theme of "engagement and access" was discussed by 20 of the 24 interviewed teachers. This theme consists of topics relating to when, where, and with whom teachers would like to engage in professional development opportunities. The common views expressed by two or more of the interviewed teachers were summarised into the following six statements:

- I am willing to engage with professional development outside of the normal school day (68%).
- I would like professional development opportunities to be facilitated more regularly than they currently are (81%).
- I would like for my entire mathematics department to have the opportunity to engage in professional development as a group (83%).
- I would benefit from a broader range of professional development opportunities being offered (71%).
- I would be more motivated to engage in professional development opportunities if management in my school designated a specific time (during or after the school day) when all teachers were to engage in professional development of some sort (73%).
- I would be more likely to engage in professional development if time spent engaging counted towards my Croke Park hours (88%).

Professional Development Activities

The theme of "professional development activities" was discussed by 14 of the 24 interviewed teachers. This theme consists of topics relating to what teachers would like to do, and with whom, during professional development opportunities. The common views expressed by two or more of the interviewed teachers were summarised into the following four statements:

- I benefit more from professional development that allows me to engage with and trial the resources or ideas being presented throughout the opportunity (79%).
- If engaging in group work at a professional development opportunity, I would benefit most from being grouped with teachers from my own school or from schools that are similar to my own (71%).
- All content presented at professional development opportunities should be grounded in research and be deemed a best practice principle (75%).
- I would benefit from the opportunity to witness (either in person or online) a fellow teacher teaching (70%).

Facilitator of Professional Development Opportunities

The theme of "facilitator" was discussed by 14 of the 24 interviewed teachers. This theme consists of topics relating to who teachers would like to facilitate professional development opportunities, what experience they would like this person to have, and what this person could do to make the opportunity as beneficial as possible. The common views expressed by two or more of the interviewed teachers were summarised into the following six statements:

- I would prefer for professional development opportunities to be led by experienced maths teachers with an extensive knowledge of the content/idea being presented and/or discussed (89%).
- If something novel is being presented at a professional development opportunity the presenter must be an expert in that particular field (74%).
- The facilitator of the professional development opportunity must be knowledgeable about the realities of current classrooms and pitch their ideas at an appropriate level (95%).
- It is important to me that the facilitator of the professional development opportunity is open to questions and facilitating discussions (94%).
- I would benefit from the facilitator modelling the approaches being discussed with the teachers present acting as the students (63%).
- Professional development opportunities would be more beneficial if all providers (e.g. PDST, Education Centres, IMTA, universities) worked together and presented a common message (89%).

Professional Conversations

The theme of "professional conversations" was discussed by 14 of the 24 interviewed teachers. This theme consists of topics relating to how teachers benefit from the opportunity to converse with colleagues from their own school and from other schools and how they would appreciate more organised times to engage in these conversations. The common views expressed by two or more of the interviewed teachers were summarised into the following three statements:

- I benefit from conservations with teachers from different schools about the content being presented when attending professional development opportunities (83%).
- I would benefit professionally from having regular group meetings with the other maths teachers in my school to ask questions and collaborate on resources and ideas (84%).
- I would be more likely to transfer knowledge gained from a professional development opportunity back into my classroom if I was given time the next day to discuss this knowledge with my department colleagues and plan for the implementation given our school context (81%).

Conclusions & Next Steps

Throughout the interview and survey processes, Irish secondary school mathematics teachers have been very clear on what they want in relation to professional development. The high levels of agreeance among the surveyed teachers suggests that the views of the interviewed teachers are shared by a wider proportion of secondary school mathematics teachers in Ireland. These views are not dissimilar to the literature on best practice regarding teacher professional development, however these teachers have provided details regarding the opportunities, activities, characteristics, and supports that would allow these best practice principles to be translated into the Irish education system effectively. In order to progress the requests made by teachers, these results will be combined with what are deemed best practices by the theoretical and experimental literature in the field, the experiences of professional development developers internationally, and the Irish education policy relating to the professional development of secondary school mathematics teachers. Collaboration with the main teacher professional development stakeholders will then need to begin to ensure that meaningful, impactful professional development is accessible to all secondary school teachers of mathematics in Ireland.

References

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101.

Coffield, F. (2000). Introduction: A critical analysis of the concept of a learning society. In F. Coffield (Ed.), *Differing visions of a learning society.* Policy Press.

Darling-Hammond, L. (2005). Developing professional development schools: Early lessons, challenge, and promise. In L. Darling-Hammond (Ed.), *Professional development schools: Schools for developing a profession* (pp. 1-27). Teachers College Press.

Desimone, L., Porter, A., Garet, M., Yoon, K., & Birman, B. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis, 24*(2), 81-112.

Flecknoe, M. (2000). Can continuing professional development for teachers be shown to raise pupils' achievement? *Journal of In-Service Education, 26*(3), 436-457.

Friedman, A., & Phillips, M. (2004). Continuing professional development: Developing a vision. *Journal of Education and Work, 17*(3), 361-376.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development. In L. Darling-Hammond and G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127-150). Jossey-Bass.

Hoban, G. F. (2002). Teacher learning for educational change. Open University Press.

Knapp, M. S. (2003). Professional development as policy pathway. *Review of Research in Education, 27*(1), 109-157.

Smith, A. (2004). *Making mathematics count: The report of Professor Adrian Smith's inquiry into post-14 mathematics education.* Retrieved July 22, 2020 https://dera.ioe.ac.uk//4873/

Wei, R. C., Darling-Hammond, L., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the United States and abroad.* National Staff Development Council.

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