On the Potential of the Reynolds Stress Approach to Model Convective Overshooting in Grids of Stellar Evolution Models

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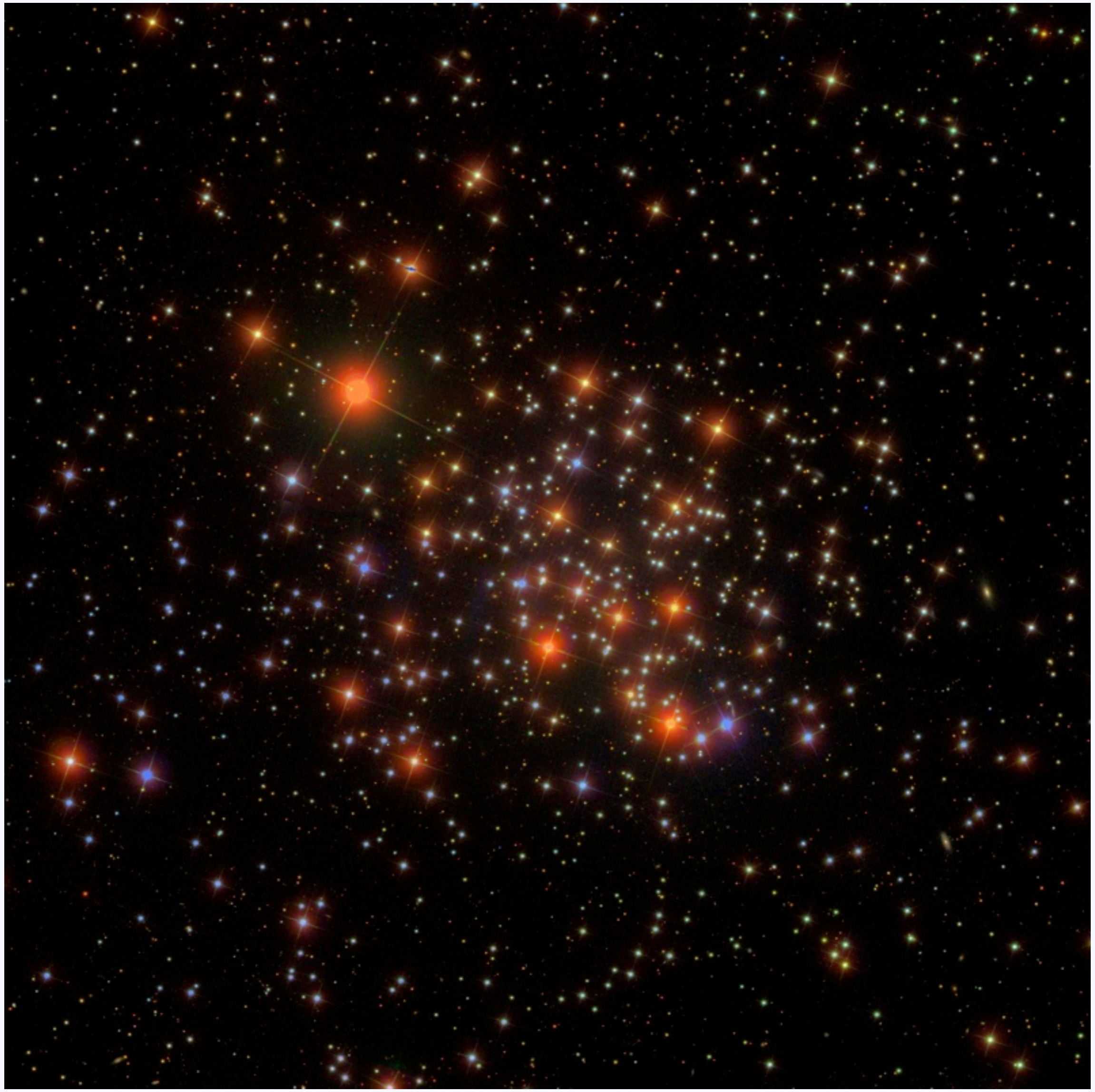
Evolution of Low Mass Stars I

Open Cluster M67

Simultaneous visualization of g-, r-, and -i bands of Sloan Digital Sky Survey DR14 observations of M67.



Processing: https://www.sdss.org/dr14/imaging/jpg-images-on-skyserver/

Evolution of Low Mass Stars II

Determination of the age of M67

Isochrone for 4 Gyrs

t = 4 Gyrs stellar evolution tracks for different masses match absolute brightness vs. $(B\text{-}V)_0$ colour index as derived from observations.

Models require overshooting to match its TO (main sequence turn off) morphology at $M_V < 3.5$.

VandenBerg et al. 2006, ApJS 162, 375

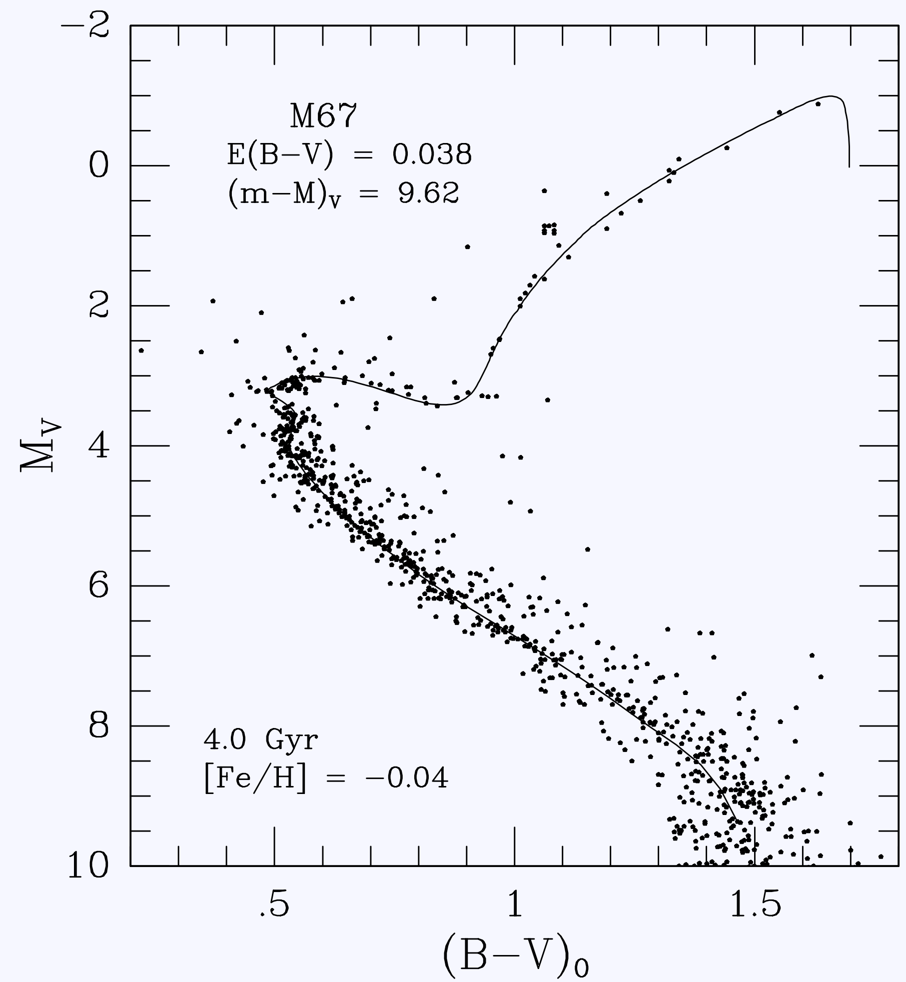


FIG. 3.—Main-sequence fit of a 4.0 Gyr isochrone for [Fe/H] = −0.04 to the *BV* photometry of M67 by Montgomery et al. (1993), on the assumption of the indicated reddening and apparent distance modulus.

Evolution of Low Mass Stars III

Choose microphysics, physical processes, numerics

- then calibrate initial helium content Y_0 and mixing length (MLT) parameter α as follows:

- change Y_0 & α for a 1 $M_\odot$ model at solar metallicity till $L_\odot$ and $R_\odot$ of the present Sun can be reproduced (t=4.567 Gyrs), use those for further work (D.O. Gough & N.O. Weiss (1976), MNRAS 175, 589)

- or use 3D simulations to calibrate α or construct a patched model atmosphere (calibrate Y_0 as usual)

- advances in 3D simulations and new calibration procedures (see, for example, the work of A.C.S. Jørgensen, Z. Magic, J.R. Mosumgaard, R. Trampedach; H.G. Ludwig; F. Spada; and their colleagues)

  → currently favoured approach for stellar model grids for preparing PLATO

Convection Models I

But how to deal with other situations ?

- Tachocline region in cool dwarfs?
  - realistic simulations for the deep stellar interior?
- Multiple convective layers?
  - A stars, massive stars interiors, with zones deeply inside the star
- Convective zones driven by nuclear burning ?
  - F-O type, RGB, AGB, He burning stages, ... (also deeply inside)
- Interaction with mechanisms of mixing
  - overshooting with / without compositional gradients
  - rotation
  - ...

→ requires a more flexible approach

Convection Models II

Ensemble averages

- Moment expansion of hydrodynamic equations
  - based on work by O. Reynolds (1894), Keller & Friedmann (1925)
  - equation for A(t,x,y,z) → split: A = <A> + A'
  - average equation for A → equation for <A>
  - non-linear terms: <A> depends on <A'A'>
  - subtract eq. for <A> from eq. for A → eq. for A' → eq. for <A'A'>
  - *non-linearity → infinite hierarchy of moments → closure problem*

    → examples: Xiong (1978, 1986, ...), Canuto (1992, 1993, ...)
- Reduce complexity: non-local turbulent convection models
  - many examples: Gough (1977), Kuhfuß (1986, 1987), ...

Convection Models III

One-point closure turbulence models

- Physical meaning of ensemble averages in physical space
  - mean values: <T>, <P>,… → thermal structure
  - fluctuations: $w = W - \langle W \rangle$, $\theta = T - \langle T \rangle$ → turbulent components
  - SOMs: 2nd order moments $\langle w\theta \rangle$, ... → energies, turb. pressure
  - TOMs: 3rd order moments $\langle w\theta^2 \rangle$, ... → non-local transport
  - usually also horizontally averaged
- closure assumptions
  - higher order moments → from means, SOMs & TOMs
  - 4th order moments: "Gaussian" (QN, quasi-normal), mass-flux models
  - "local models" → TOMs = 0
  - "non-local models": → TOMs: diffusion models, (damped) QNA, ...

→ physical completeness: number of differential equations ↑ , complexity ↑

Convective Overshooting I

Analysis for 1-equation non-local convection models

If a dynamical equation is used at all in current models of OV, it usually only deals with the turbulent kinetic energy K, e.g., models by J.-P. Zahn, I.W. Roxburgh, and scaled-down versions of more complete models (Kuhfuß & others).

Characteristic: $F_{conv} < 0$ coincides with $F_{rad} > F_{total}$, $\nabla < \nabla_{ad}$ (marginally so here in region B where convective penetration occurs, followed by ∇ approaching ∇_{rad} (region C, the *thermal boundary layer*).

Allows realistic TO in isochrones.

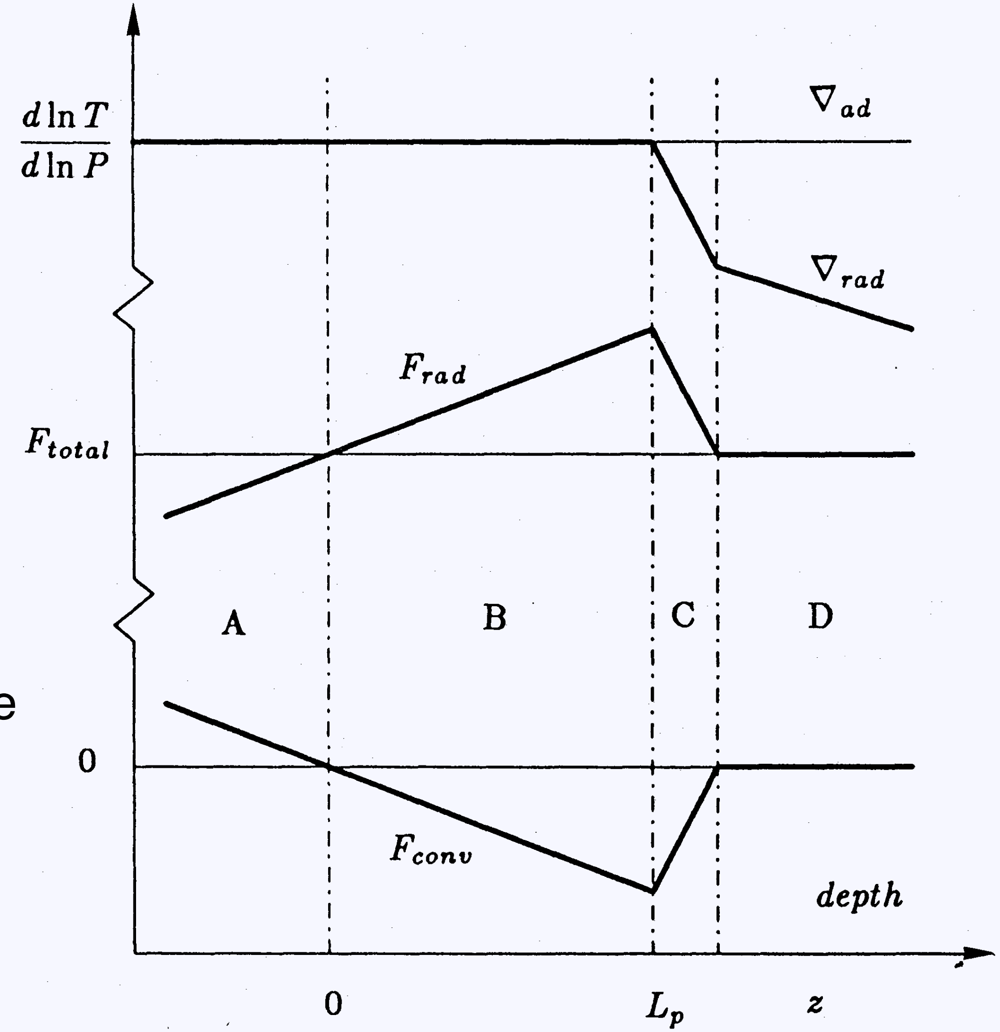


Fig. 1 from J.-P. Zahn (1991), A&A 252, 179 (no countergradient region here !).

Convective Overshooting II

Re-analysis based on Reynolds stress approach

In the stationary limit of the model by Canuto & Dubovikov (1998), ApJ 493, 834, considering dynamical equations for turbulent kinetic energy $K = \overline{q^2/2} = (\overline{u^2} + \overline{v^2} + \overline{w^2})$, convective flux $\overline{w\theta} = F_{\mathrm{conv}}/(\overline{c_p \rho})$, and temperature fluctuations $\overline{\theta^2}$, ignoring compressibility effects, and assuming a low Prandtl number while neglecting temperature fluctuations in radiative flux computations we obtain

$$\partial_z \left(\frac{1}{2}\overline{q^2 w} + \overline{pw} \right) = g\alpha_{\mathrm{v}}\overline{w\theta} - \varepsilon, \quad (1)$$

$$\partial_z \left(\frac{1}{2}\overline{w\theta^2} \right) = \beta\overline{w\theta} - \tau_\theta^{-1}\overline{\theta^2}, \quad (2)$$

$$\partial_z \left(\overline{w^2\theta} \right) = \beta\overline{w^2} + (1-\gamma_1)g\alpha_{\mathrm{v}}\overline{\theta^2} - \tau_{\mathrm{p}\theta}^{-1}\overline{w\theta}. \quad (3)$$

Note that β is the superadiabatic temperature gradient. A very similar system of differential equations is obtained for the Kuhfuß (1986), A&A 160, 116 model.

(adapted from Kupka 2020, edts. Rieutord et al., EDP Sci. Proc., 69-110)

Convective Overshooting III

Results of this re-analysis

Since Priestley & Swinbank (1947), Proc. Roy. Soc. Ser. A 189, 543 we know that the convective flux is *not coupled* to the entropy (superadiabatic) gradient.

A non-zero flux of temperature fluctuations forces the formation of a layer with $F_{conv} > 0$ despite being locally stable (Deardorff 1966, J. Atm. Sci. 12, 503).

For the same reason a non-zero flux of kinetic energy forces the formation of a layer (filled by plumes) further away from the convective zone, where $F_{conv} < 0$.

Capturing these effects → 3 differential equations for a self-consistent model.

If such a model (e.g., Kuhfuß 1986, A&A 160, 116) is coupled to a stellar evolution code, the whole star becomes fully mixed (Flaßkamp 2003, PhD thesis). This does not happen for commonly used models with 1 non-local equation.

➔ So what is wrong here ? Anisotropy ? Dissipation ?

Convective Overshooting IV

Conclusions from further analysis

Accounting for asymmetry between kinetic energy in horizontal and vertical flows is correct, but solves the problem only for unphysical values.

Analyze dynamical equation for the dissipation rate of turbulent kinetic energy, ε: key contribution left out if computed from mixing length $\varepsilon = C_\varepsilon K^{1.5} / \Lambda$, $\Lambda = \alpha H_p$.

Why use large scale lengths ($\Lambda \sim r_{core}$) to describe an overshooting layer which is much smaller than that in mid / late B type main sequence stars ?

A major physical phenomenon extracting energy from convective flow in stable stratification are g-modes ➔ ε depends on the Bruntt-Väisälä frequency, accounted for in the dynamical equation of Canuto & Dubovikov (1998), ApJ 493, 834.

➔ Idea: use this equation to construct a new scale length Λ

➔ Kupka, Ahlborn & Weiss, in prep. for A&A (2021) (Paper I)
Ahlborn, Kupka & Weiss, in prep. for A&A (2021) (Paper II)

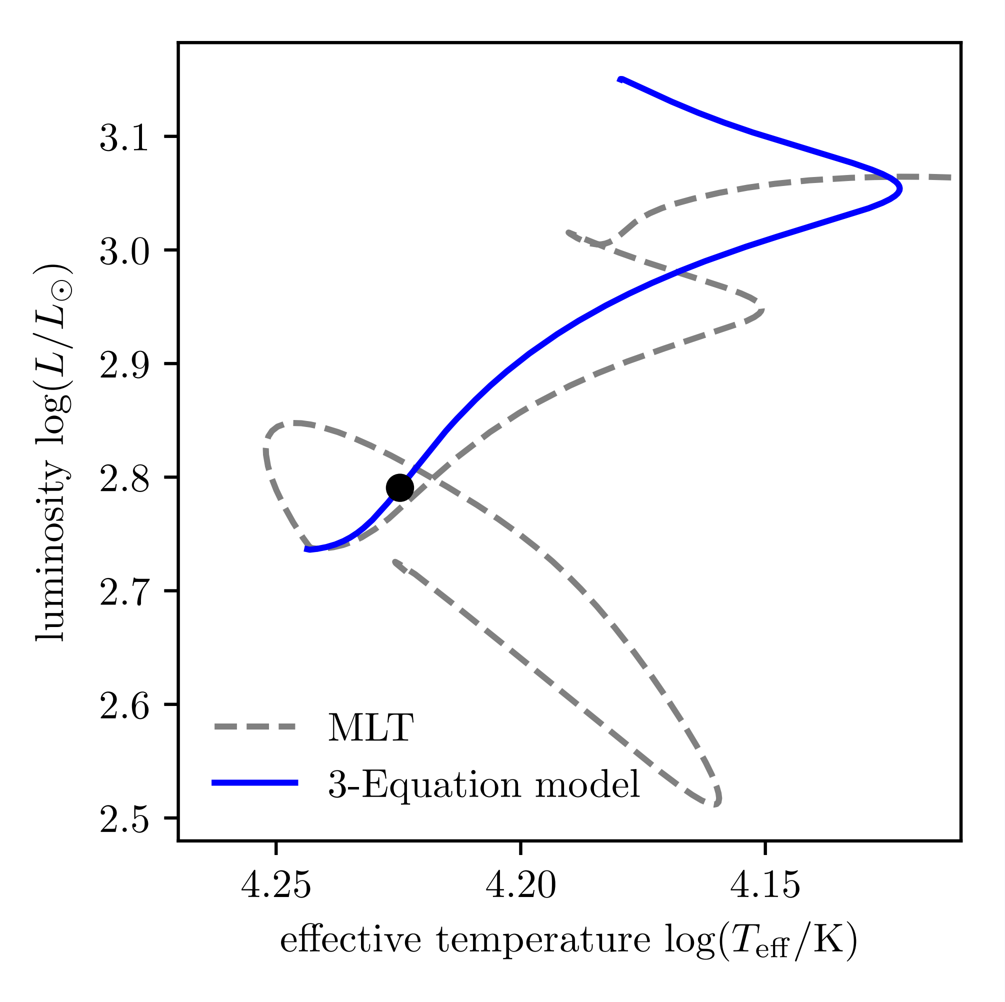
Convective Overshooting V

Evolution of a 5 $M_\odot$ star

Evolution with the new, modified 3-equation model from zero age at the main sequence till end of core hydrogen burning.

Pre-main sequence track: starting point for MLT & non-local model tracks.

→ largest difference near turn-off: L increase and MS widening, as required from observations, but now with finite extent of OV zone



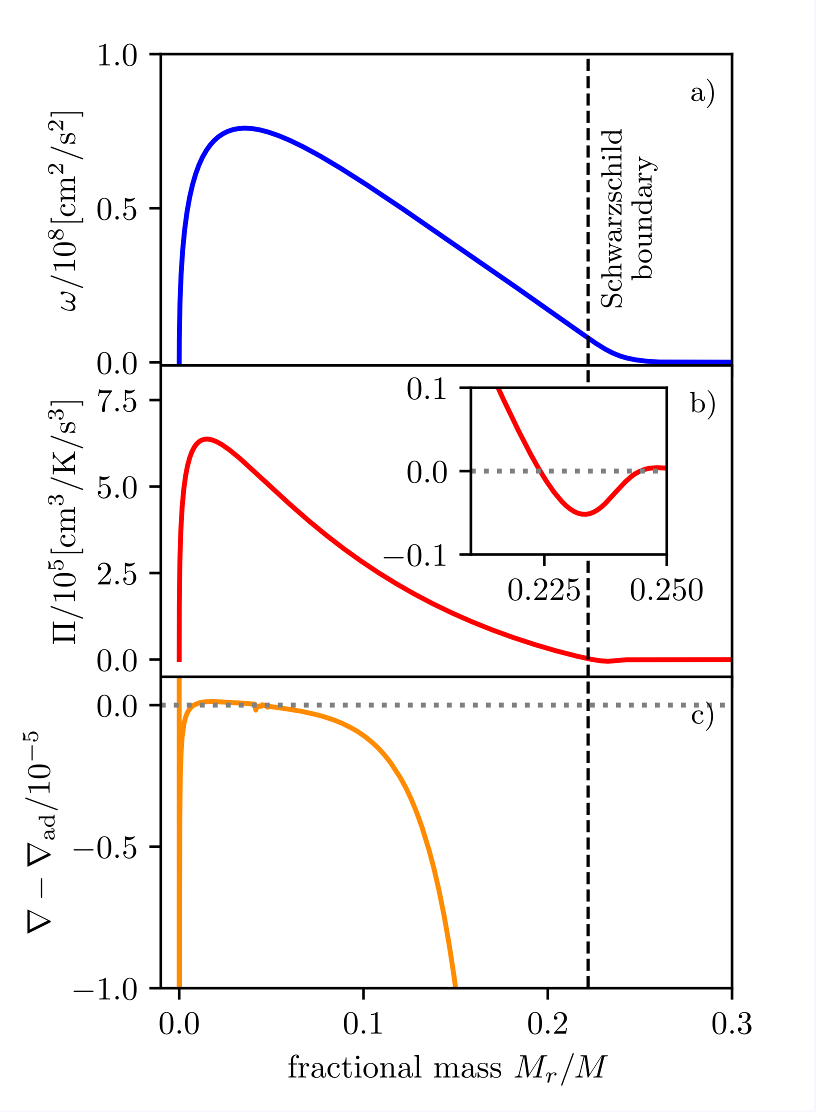
(Ahlborn et al. 2021, A&A in prep.)

Convective Overshooting VI

Evolution of a 5 $M_\odot$ star

Scaled kinetic energy ω, convective flux Π, and superadiabatic gradient $\nabla - \nabla_{ad}$ obtained from the new 3-equation model. Note the small region of negative convective flux outside ($M_r/M \gtrsim 0.225$) the Schwarzschild boundary of the local model (see inset).

The superadiabatic layer is followed by a large Deardorff (countergradient or subadiabatic) layer for $0.05 \lesssim M_r/M \lesssim 0.22$), as shown in the bottom panel.



(Ahlborn et al. 2021, A&A in prep.)

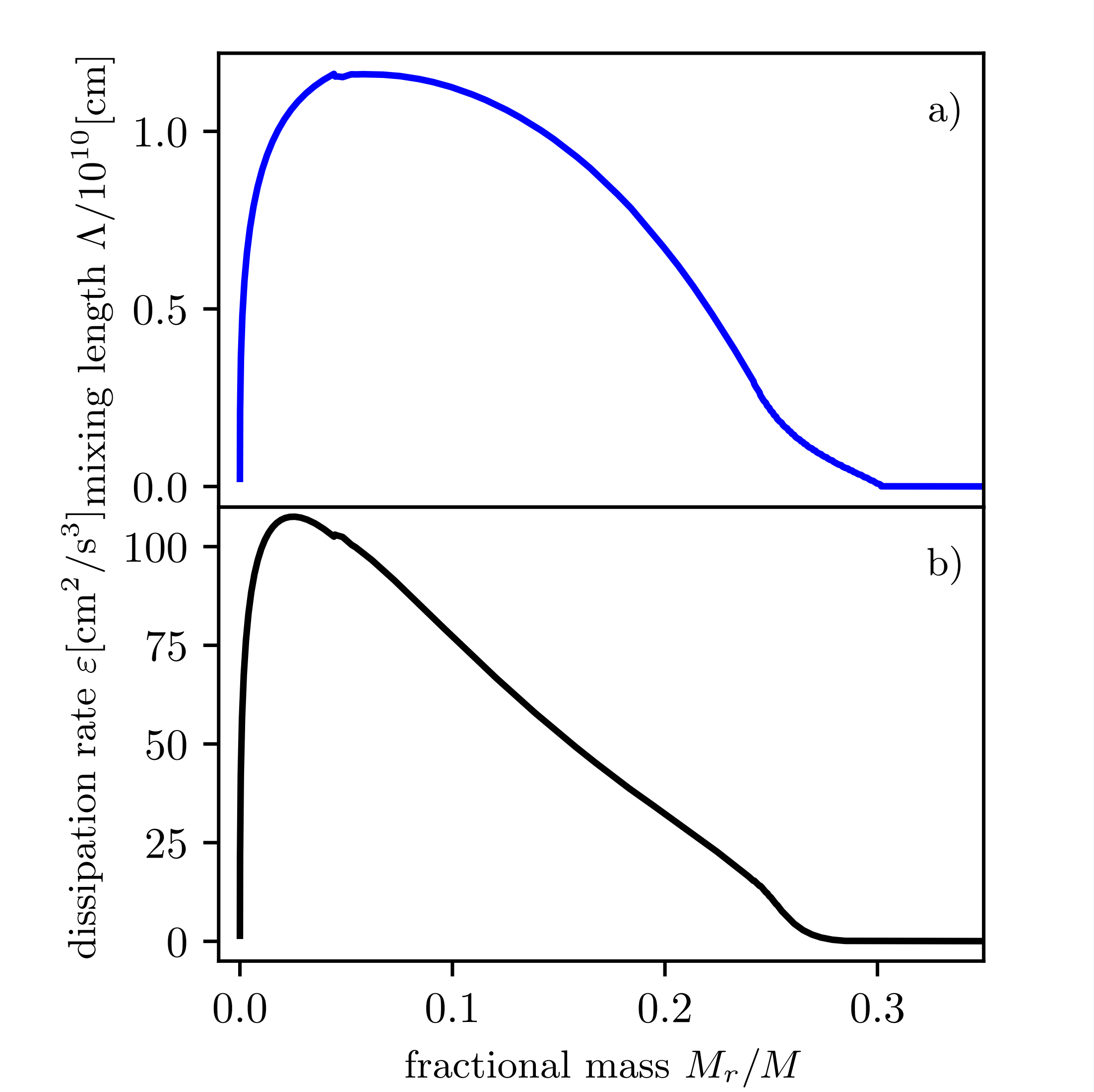
Convective Overshooting VII

Evolution of a 5 $M_\odot$ star

Dissipation length scale Λ (top panel) and *dissipation rate of turbulent kinetic energy ε* in the new 3-eq. model.

Note: computing Λ from a fixed fraction of H_p fails to let drop the characteristic dissipation length sufficiently fast both in the subadiabatic Deardorff layer and in the zone where $F_{conv} < 0$ which then leads to too much mixing.

The 1-eq. model is structurally different and less sensitive to an oversimplified model for Λ.



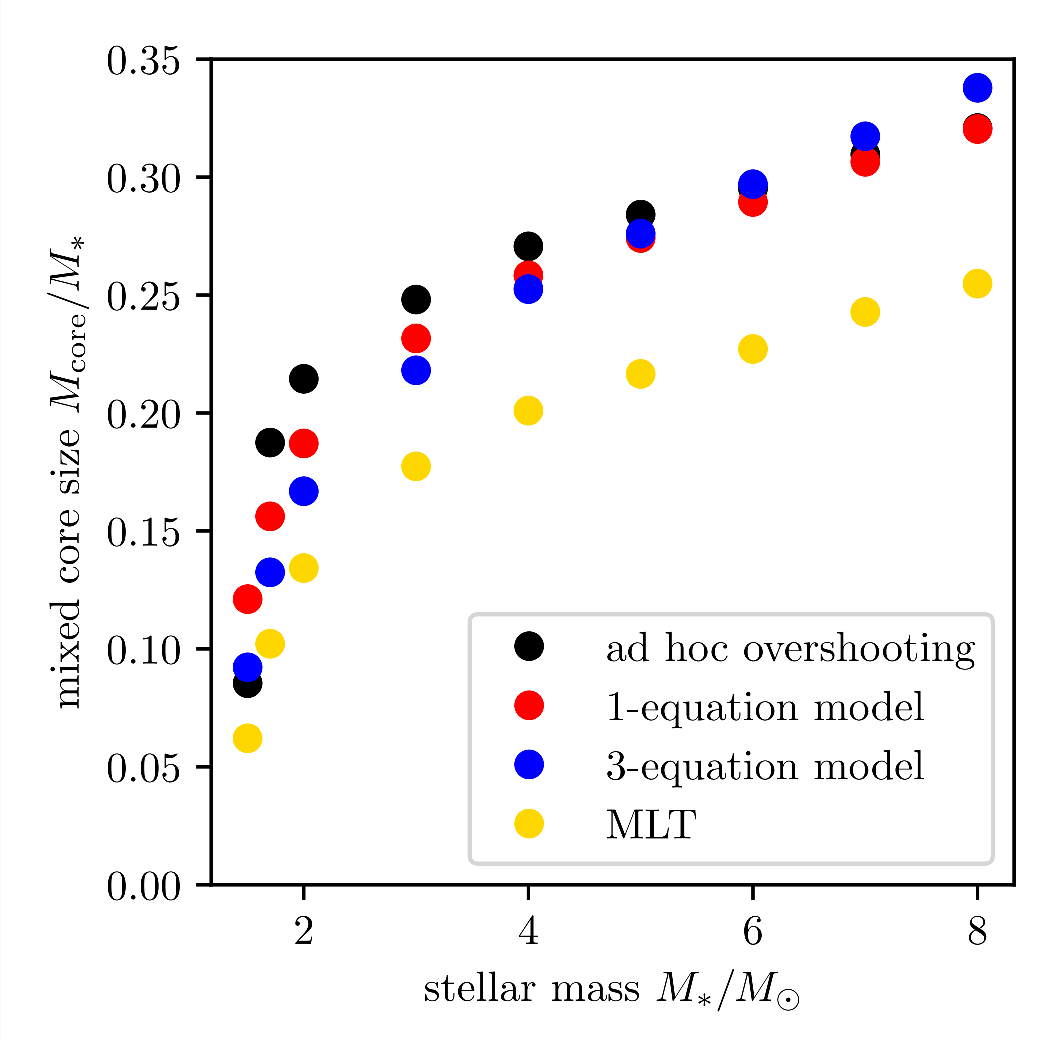
(Ahlborn et al. 2021, A&A in prep.)

Convective Overshooting VIII

Evolution for 1.5-8 $M_\odot$

Core sizes obtained for
1) standard MLT based models,
2) MLT models with *diffusive (exponential) overshooting and cut-off to limit the size of small cores*,
3) for the 1-equation non-local model,
4) for the new 3-equation non-local model.

Model 1) fails on the data (not shown here), *model 2)* requires adhoc optimization to match those data (black dots shown here). Standard parameters for models 3) and 4) used (no tuning to match observations).



(Ahlborn et al. 2021, A&A in prep.)

Convective Overshooting IX

Added values of new 3-eq model

– Formation of a subadiabatic (Deardorff) layer
  - as expected from most LES & DNS of convective overshooting
  - and from experiments and geophysical cases of overshooting
  - has different properties with respect to p-mode and g-mode physics
  - accessible to seismology in the not too distant future ?

– Restricted overshooting
  - extent roughly compatible with observational constraints for the entire range from 1.5 to 8 $M_\odot$, original parameters (dependencies checked)
  - underlying physical effect must operate also for models which achieve such restriction for other reasons (1-eq. model, model by Xiong 1986).

– Applicability
  - implemented & affordable: standard Henyey-based stellar evolution codes

...THANK YOU FOR YOUR TIME !