IGS Global Seminar, 2021.10.13, 20:00 UTC

Of cold ice, warm ice and water: thermodynamics of ice sheets and glaciers

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Thermodynamics

- From Greek θέρμη (therme), meaning "heat",
  and δύναμις (dynamis), meaning "power".
- Branch of physics that deals with heat, work and temperature, and their relation to energy, radiation and physical properties of matter.

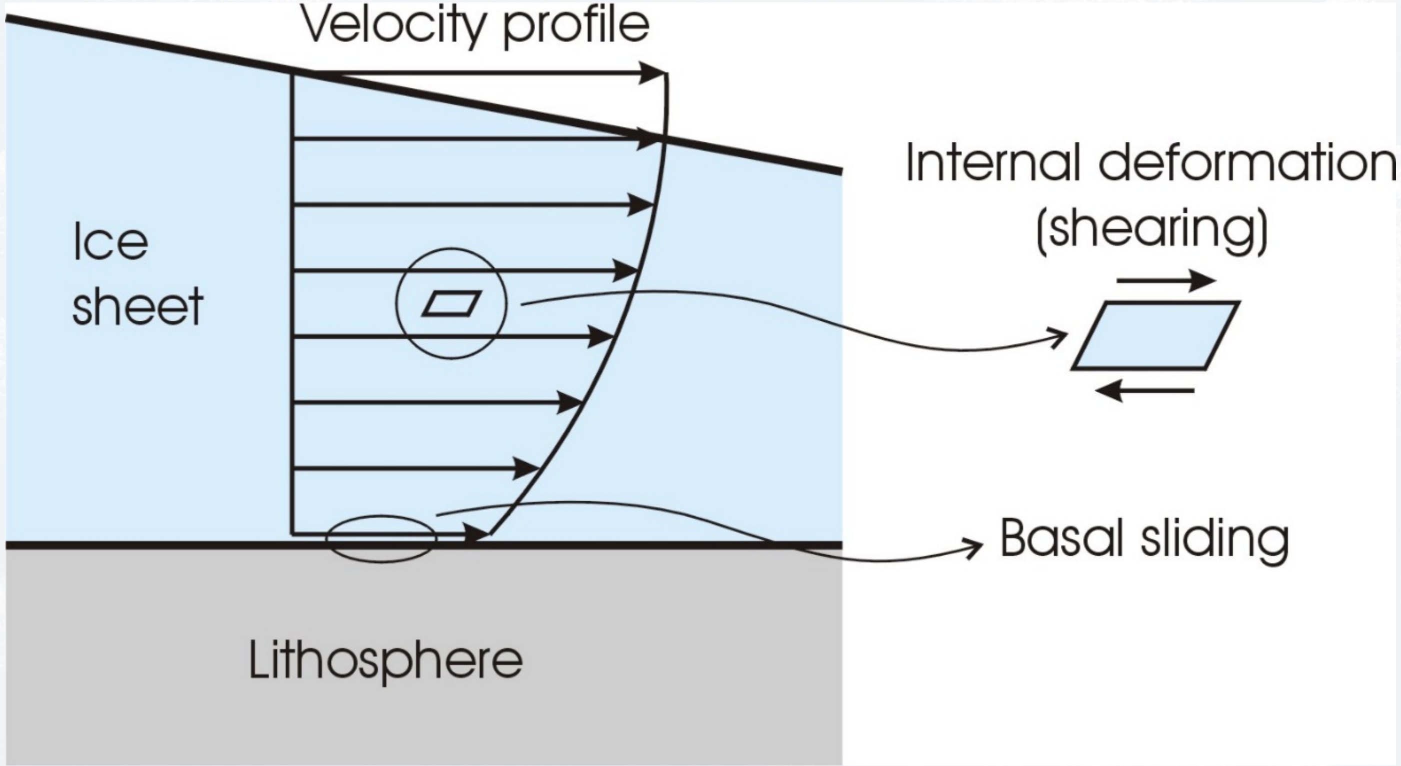
(https://en.wikipedia.org/wiki/Thermodynamics)

Why bother?

Two mechanisms contribute to ice flow

Internal deformation
(ice = viscous fluid)

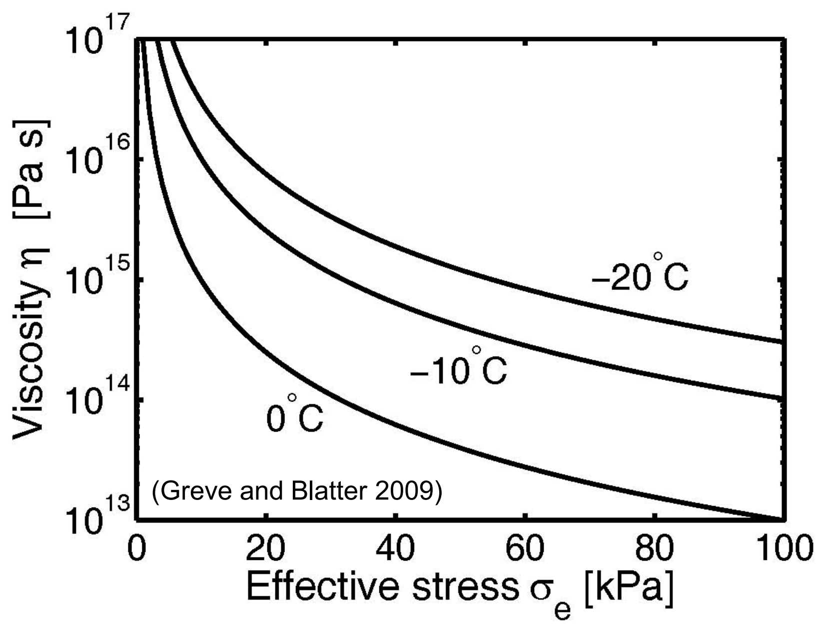
Basal sliding
(on hard rock or
soft sediment)



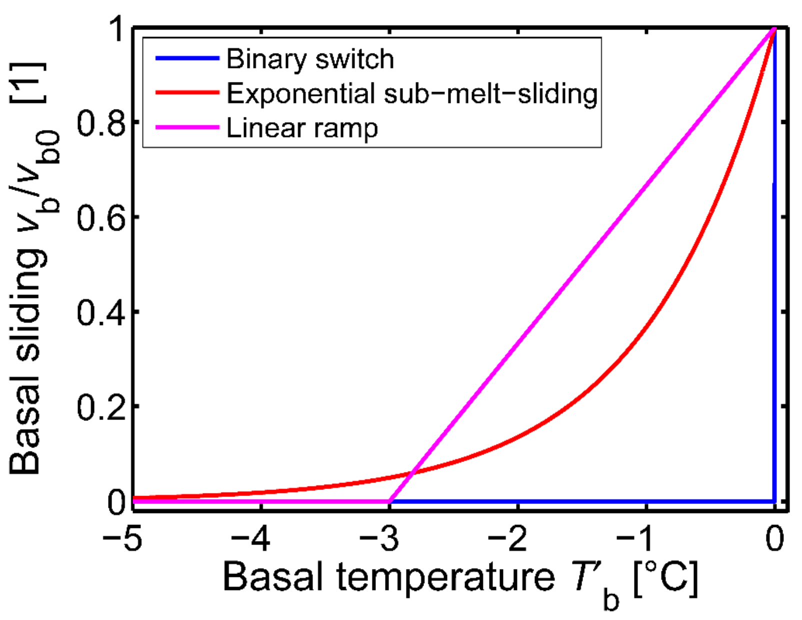
Why bother?

Both internal deformation and basal sliding depend strongly on temperature (and water content)

Viscosity of polycrystalline ice

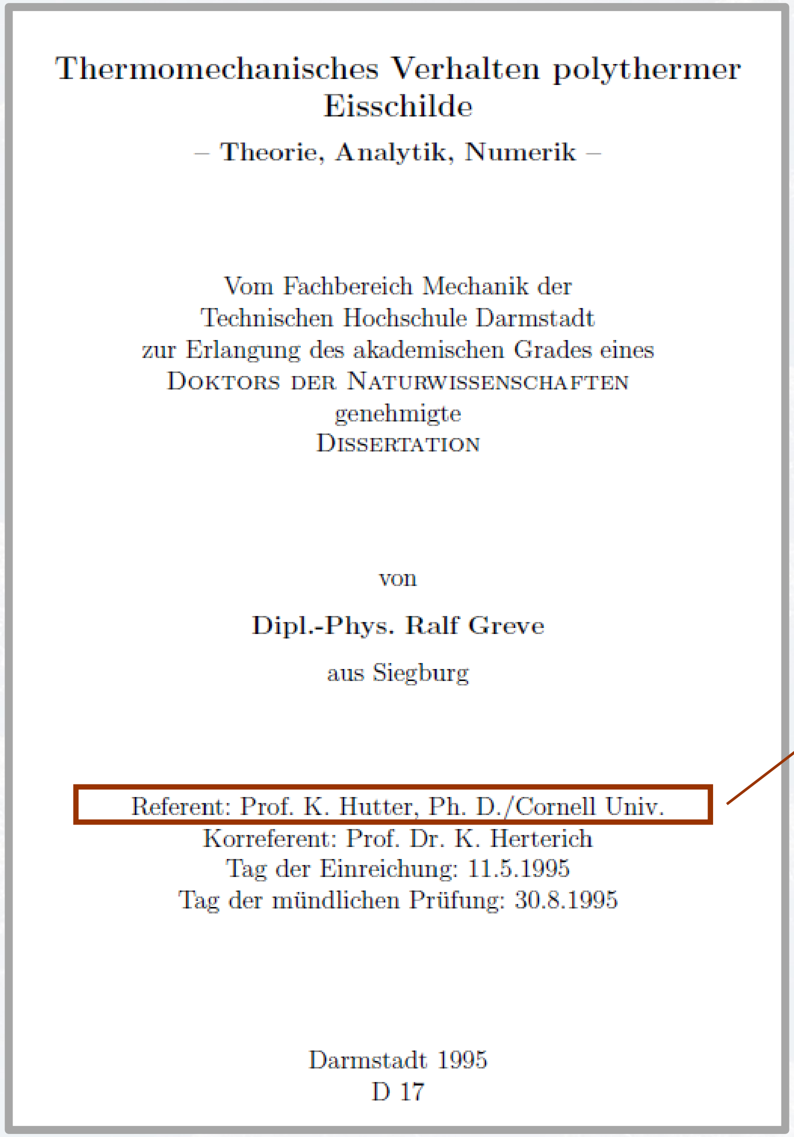


Basal sliding



→ Flow of ice sheets and glaciers: Thermo-mechanically coupled problem!

Why me?



Thermomechanisches Verhalten polythermer Eisschilde

– Theorie, Analytik, Numerik –

Vom Fachbereich Mechanik der
Technischen Hochschule Darmstadt
zur Erlangung des akademischen Grades eines
Doktors der Naturwissenschaften
genehmigte
Dissertation

von

Dipl.-Phys. Ralf Greve

aus Siegburg

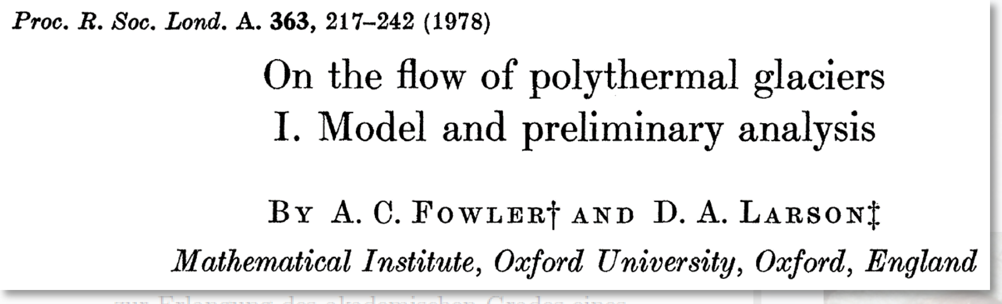
Referent: Prof. K. Hutter, Ph. D./Cornell Univ.
Korreferent: Prof. Dr. K. Herterich
Tag der Einreichung: 11.5.1995
Tag der mündlichen Prüfung: 30.8.1995

Darmstadt 1995
D 17

(https://doi.org/10.5281/zenodo.3815324)

Why me?

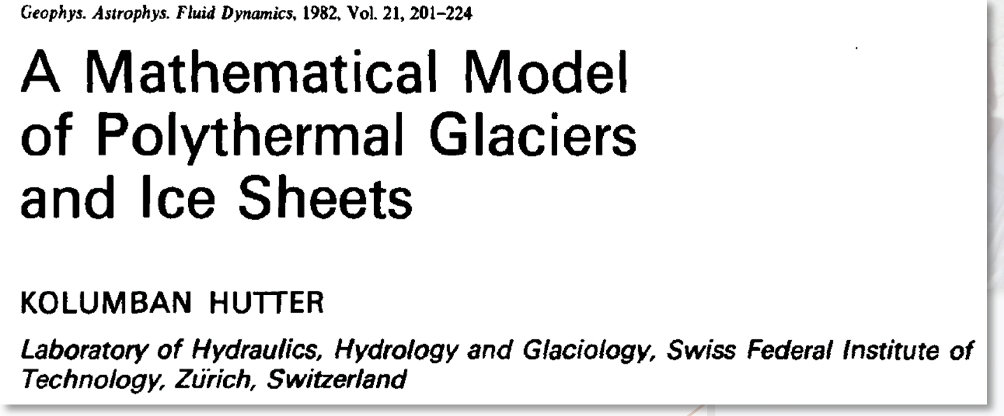
Based on…



Proc. R. Soc. Lond. A. **363**, 217–242 (1978)

On the flow of polythermal glaciers
I. Model and preliminary analysis

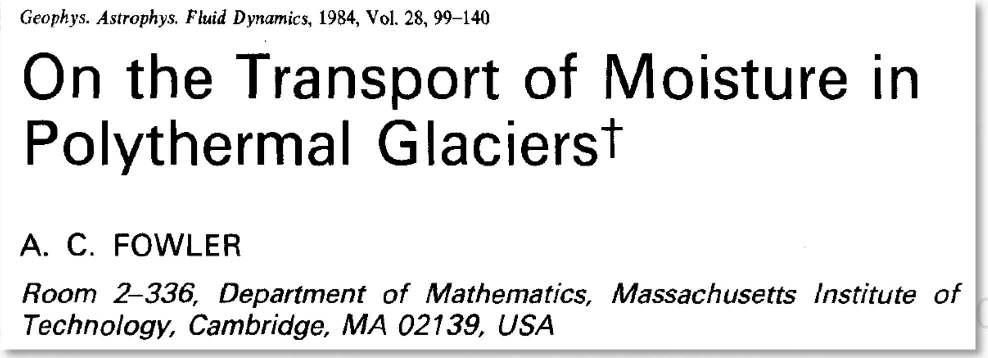
By A. C. Fowler† and D. A. Larson‡
Mathematical Institute, Oxford University, Oxford, England



Geophys. Astrophys. Fluid Dynamics, 1982, Vol. 21, 201–224

A Mathematical Model
of Polythermal Glaciers
and Ice Sheets

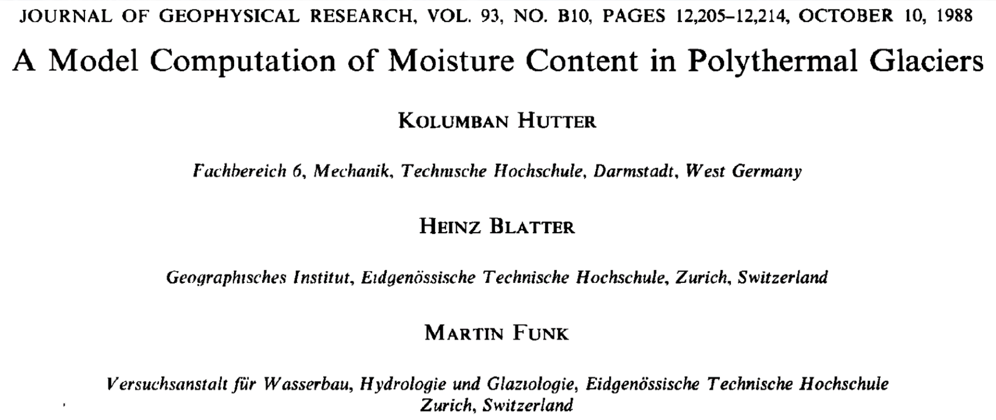
KOLUMBAN HUTTER
Laboratory of Hydraulics, Hydrology and Glaciology, Swiss Federal Institute of Technology, Zürich, Switzerland



Geophys. Astrophys. Fluid Dynamics, 1984, Vol. 28, 99–140

On the Transport of Moisture in Polythermal Glaciers†

A. C. FOWLER
Room 2–336, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 93, NO. B10, PAGES 12,205–12,214, OCTOBER 10, 1988

A Model Computation of Moisture Content in Polythermal Glaciers

Kolumban Hutter
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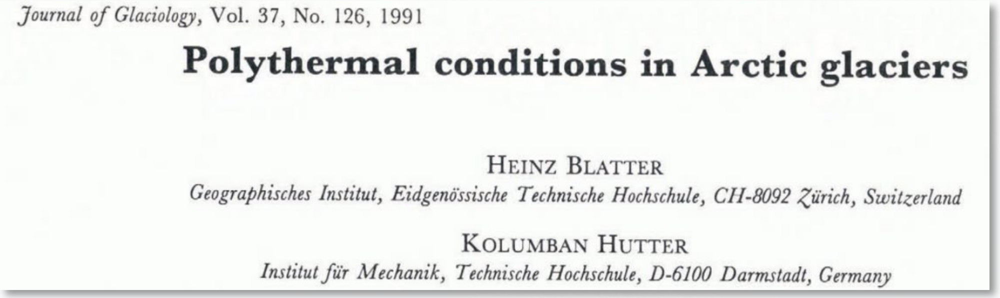
Heinz Blatter
Geographisches Institut, Eidgenössische Technische Hochschule, Zurich, Switzerland

Martin Funk
Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie, Eidgenössische Technische Hochschule Zurich, Switzerland

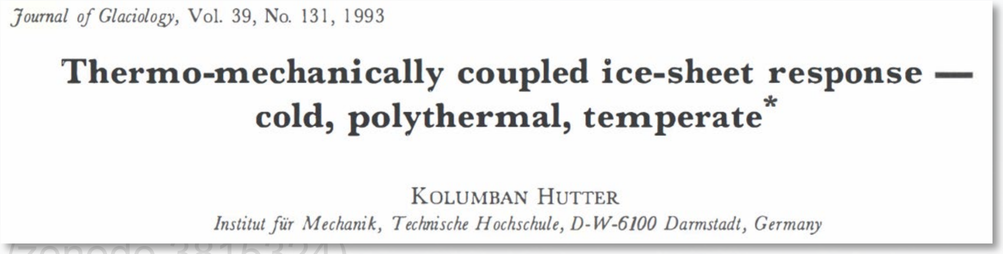


Journal of Glaciology, Vol. 37, No. 126, 1991

Polythermal conditions in Arctic glaciers

Heinz Blatter
Geographisches Institut, Eidgenössische Technische Hochschule, CH-8092 Zürich, Switzerland

Kolumban Hutter
Institut für Mechanik, Technische Hochschule, D-6100 Darmstadt, Germany



Journal of Glaciology, Vol. 39, No. 131, 1993

Thermo-mechanically coupled ice-sheet response — cold, polythermal, temperate*

Kolumban Hutter
Institut für Mechanik, Technische Hochschule, D-W-6100 Darmstadt, Germany

Temperature computation

Temperature equation:

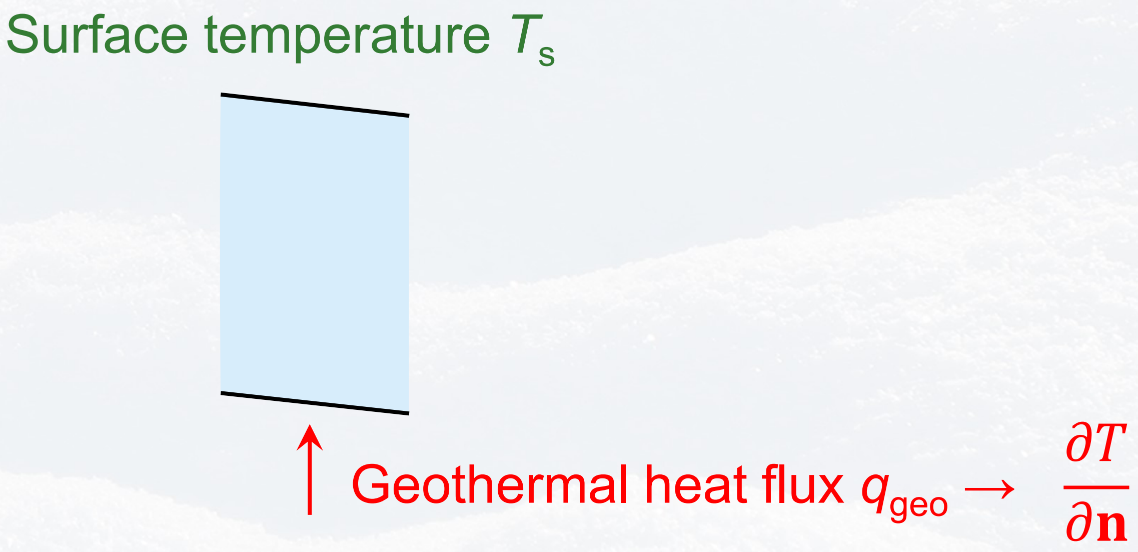
$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\rho c}\,\mathrm{div}(\kappa\ \mathrm{grad}\ T) + \frac{\Phi}{\rho c}$$

Material time derivative
(local derivative + 3D advection)

Heat conduction
(diffusion)

Strain heating
(dissipation)

Boundary conditions:

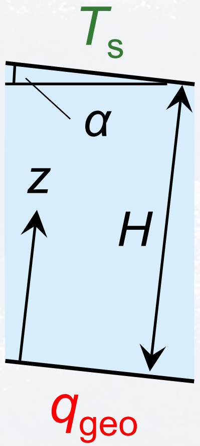


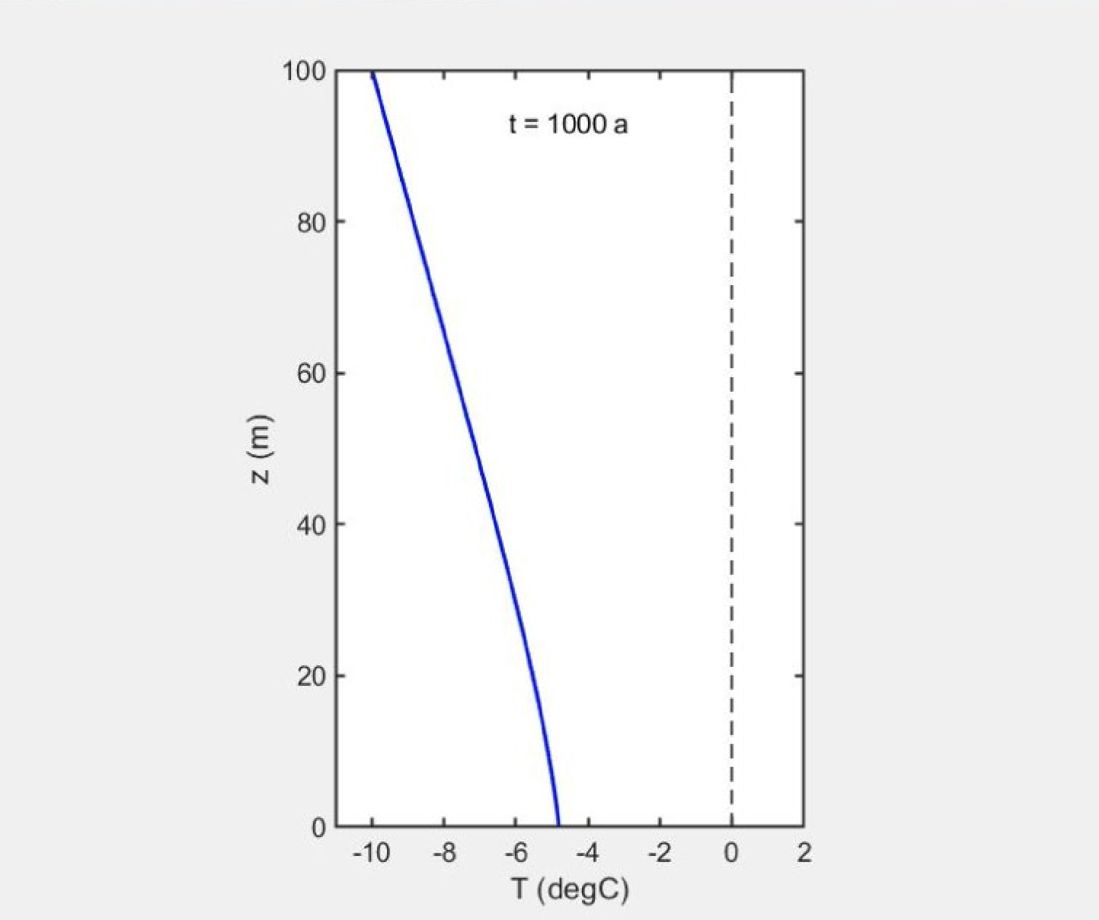
Surface temperature T_s

Geothermal heat flux $q_\mathrm{geo} \rightarrow \dfrac{\partial T}{\partial \mathbf{n}}$

Some MATLAB tests for an ice column…

$H = 100$ m, $\alpha = 10°$, $T_s = –10$°C, $q_{geo} = 50$ mW m^{-2}, $T_{init} = –10$°C, $t = 0…1000$ a

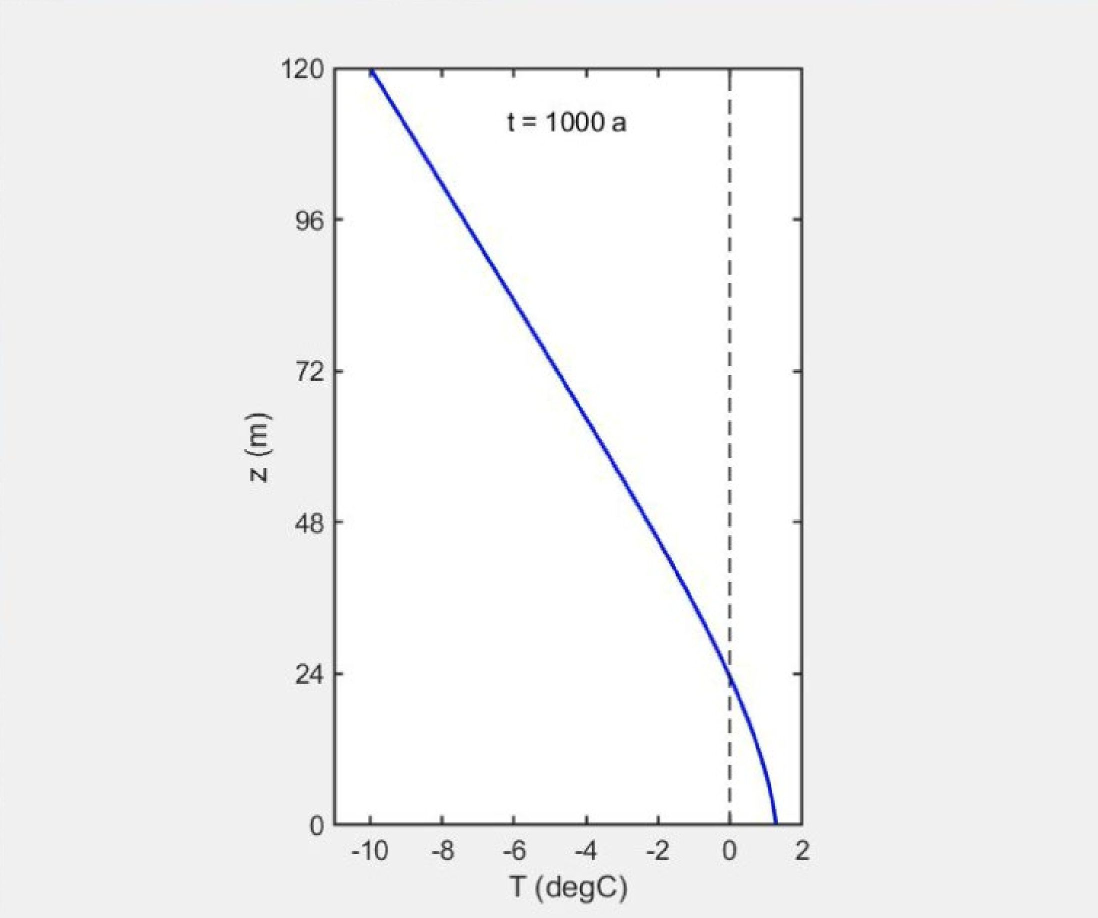




Good!

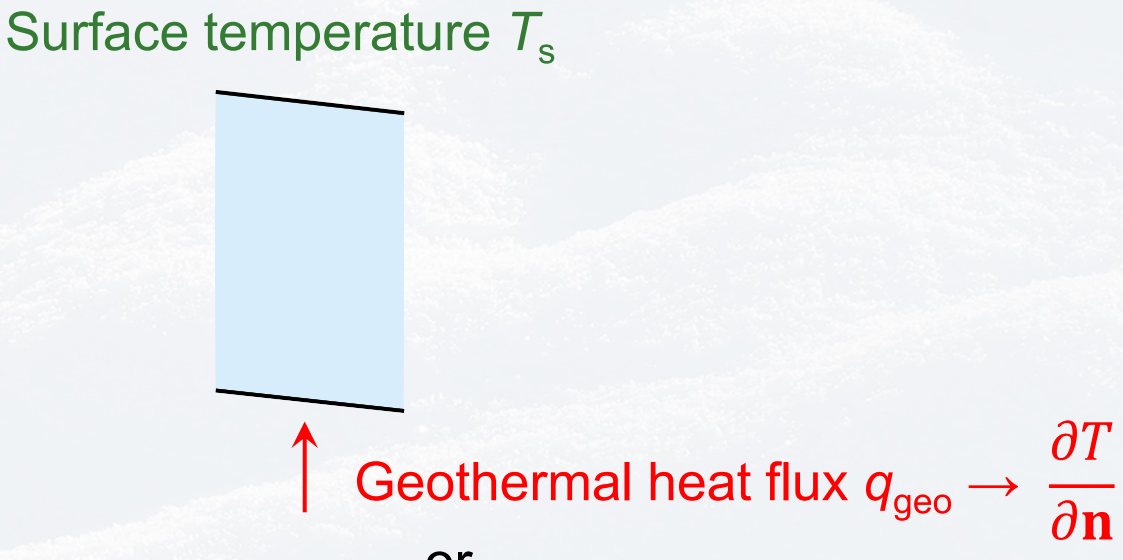
Some MATLAB tests for an ice column…

Let's make it a bit thicker: *H* = 120 m



Not good!

Correction of the boundary conditions required



Surface temperature T_s

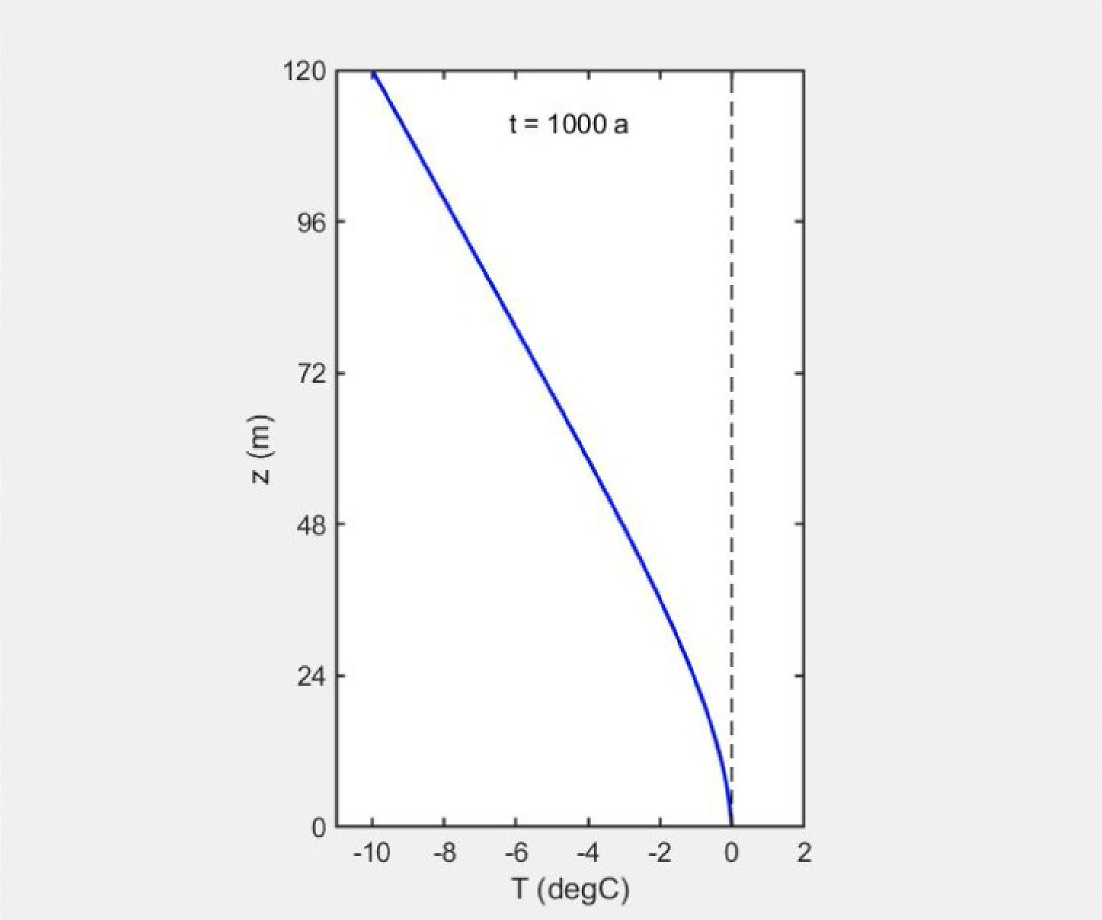
Geothermal heat flux $q_\mathrm{geo} \rightarrow \dfrac{\partial T}{\partial \mathbf{n}}$ (cold base)

or

Basal temperature T_pmp (temperate [warm] base)

Some MATLAB tests for an ice column…

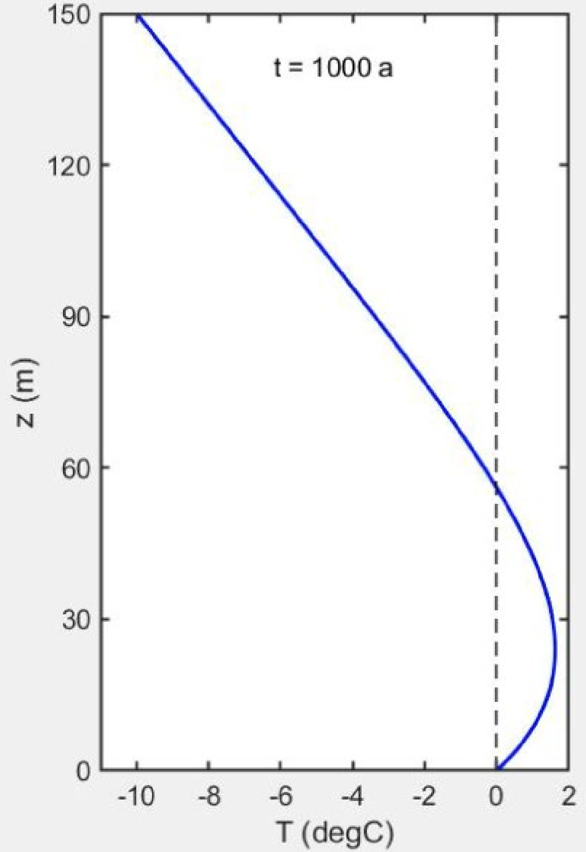
Second try with *H* = 120 m



Good!

Some MATLAB tests for an ice column…

Let's make it even thicker: $H = 150$ m



Not good!

Correction of the temperature required

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\rho c}\mathrm{div}(\kappa\,\mathrm{grad}\,T) + \frac{\Phi}{\rho c}$$

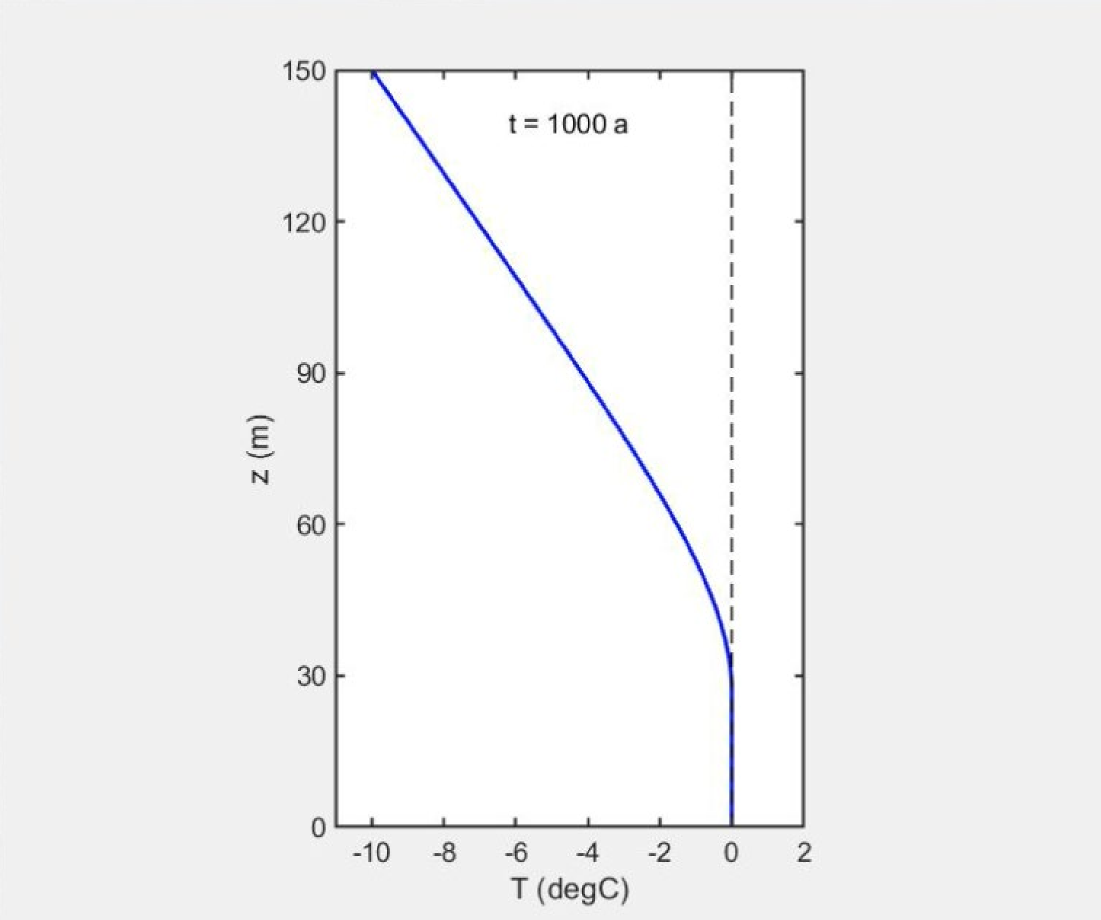
with the secondary condition

$T \le T_{\mathrm{pmp}}$ (reset to $T = T_{\mathrm{pmp}}$ if violated)

→ distinguish between cold ice ($T < T_{\mathrm{pmp}}$)
and temperate [warm] ice ($T = T_{\mathrm{pmp}}$)

Some MATLAB tests for an ice column…

Second try with *H* = 150 m

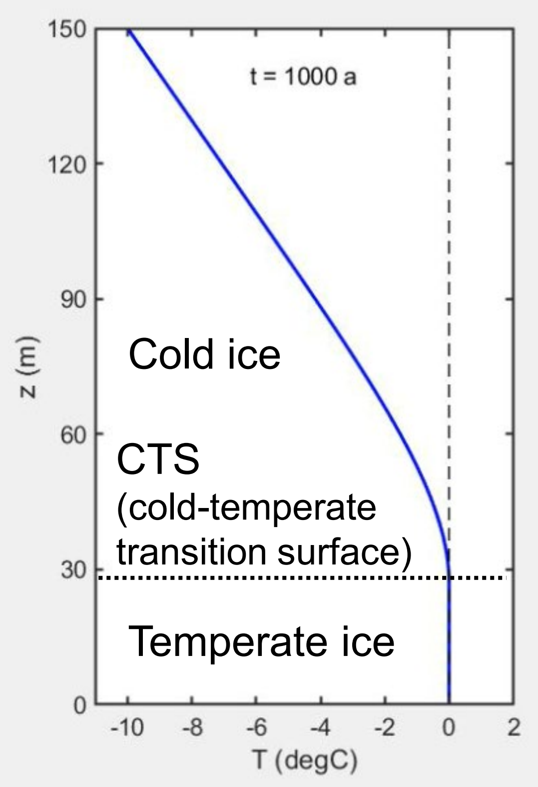


Good!

Cold-ice method

As described on the previous slides:

- Solve temperature equation.
- Basal boundary condition for either cold or temperate base.
- Reset temperatures to T_{pmp} if needed.



Polythermal conditions:
Both cold and temperate ice present.

Really good?

No!

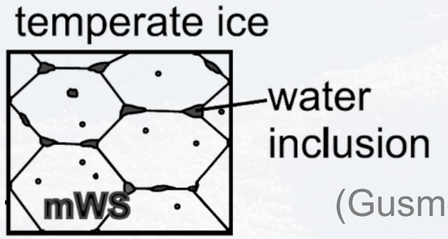
- ➢ Resetting does not conserve energy.
- ➢ Energy jump condition at the CTS not necessarily fulfilled.

Polythermal method

Temperature equation as before, but only solved in cold ice.

Water-content equation in temperate ice:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{1}{\rho}\,\mathrm{div}(\nu\,\mathrm{grad}\,W) + \frac{\Phi}{\rho L}$$

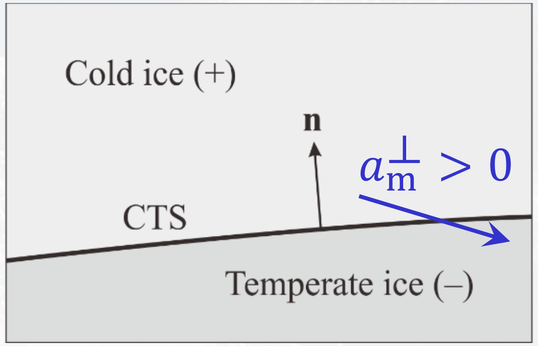


(Gusmeroli et al., 2010)

(microscopic water system)

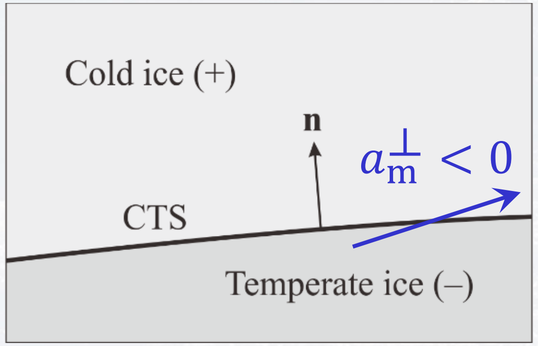
Energy jump condition at the CTS:

Melting conditions:



Ice flow from cold to temperate
→ ∂*T*/∂**n** and *W* continuous across the CTS.

Freezing conditions:

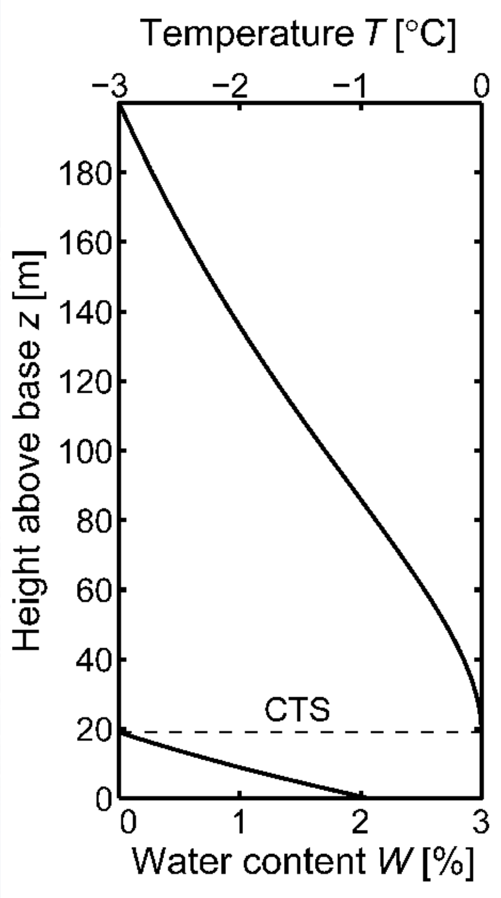


Ice flow from temperate to cold
→ ∂*T*/∂**n** and *W* jump across the CTS.

Steady-state solution for an ice column

$H = 200$ m, $\alpha = 4°$, $T_s = -3$°C / -10°C, $a_m^\perp = +0.2\ \mathrm{m\ a^{-1}}$ / $-0.2\ \mathrm{m\ a^{-1}}$

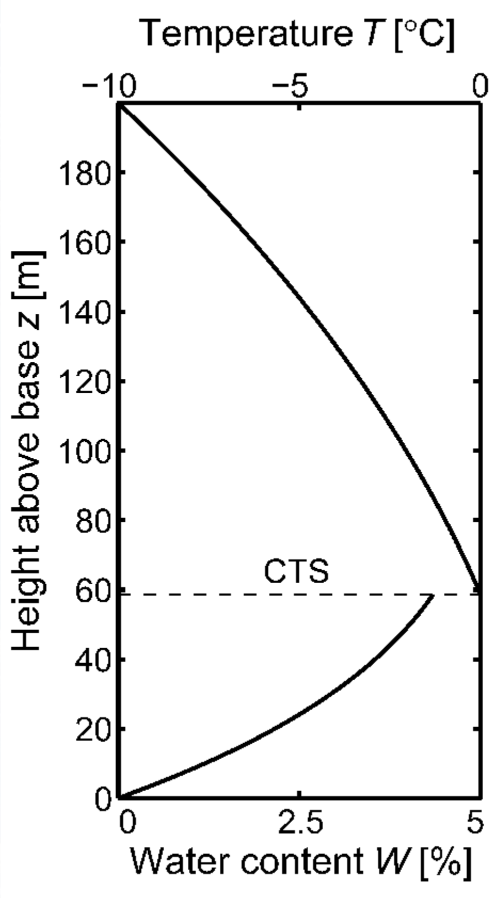
Melting conditions, $a_m^\perp = +0.2\ \mathrm{m\ a^{-1}}$



$$\left(\frac{\partial T}{\partial z}\right)^+ = \left(\frac{\partial T}{\partial z}\right)^- = 0$$

$$W^+ = W^- = 0$$

Freezing conditions, $a_m^\perp = -0.2\ \mathrm{m\ a^{-1}}$



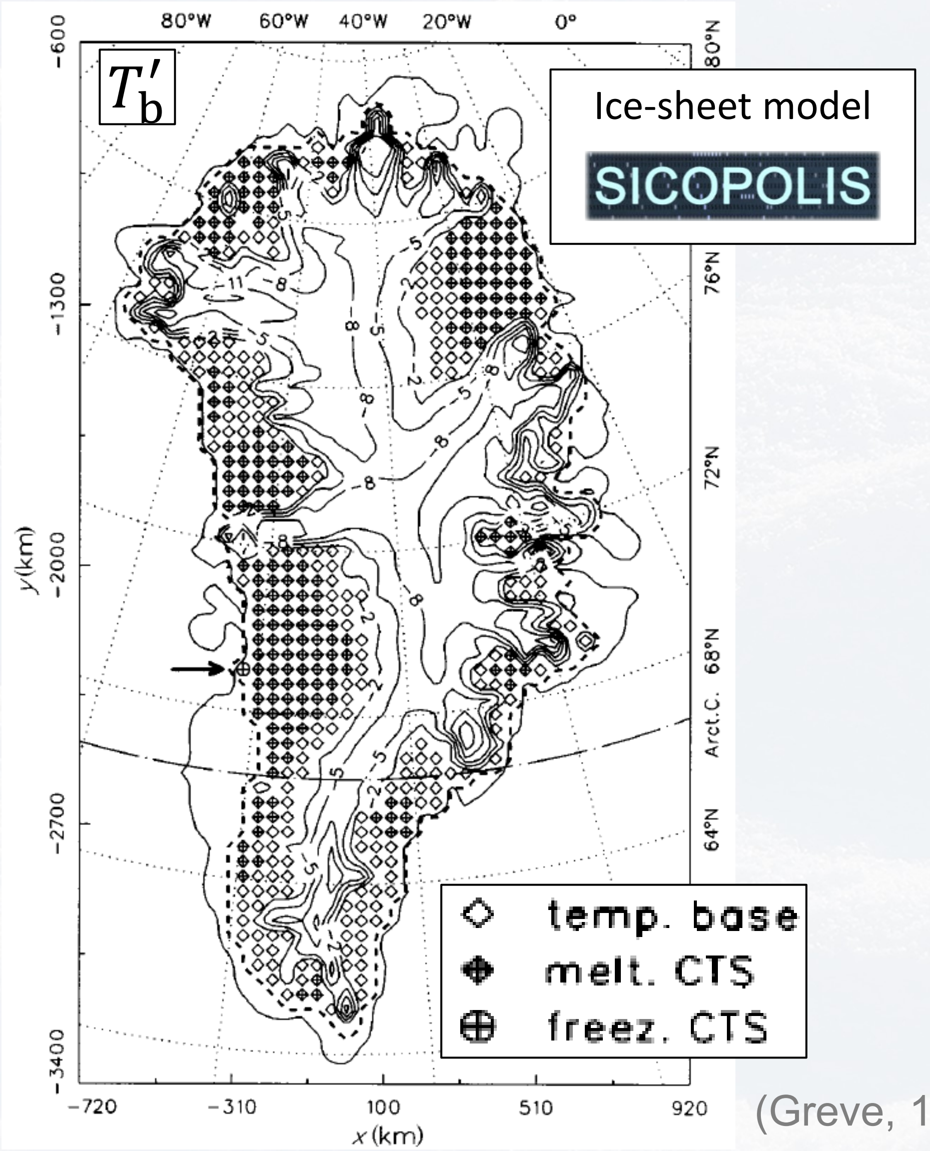
$$\left(\frac{\partial T}{\partial z}\right)^+ < 0$$

$$\left(\frac{\partial T}{\partial z}\right)^- = 0$$

$$W^+ = 0$$

$$W^- > 0$$

Steady-state solution for the Greenland ice sheet



Ice-sheet model

SICOPOLIS

(Greve, 1995, 1997)

Areas with

- cold base
- temperate base
- temperate layer

occur.

At 40 km resolution,
freezing conditions only detected
for a single grid point (→)

→ not that important.

Enthalpy method

One common thermodynamic field

$$\text{Enthalpy } h = \text{fct(Temperature } T,\ \text{water content } W)$$

for cold and temperate ice:

(Aschwanden et al., 2012)

$$h(T,W) = \int_{T_0}^{T} c(T')\,\mathrm{d}T' + LW$$

Enthalpy equation for cold and temperate ice:

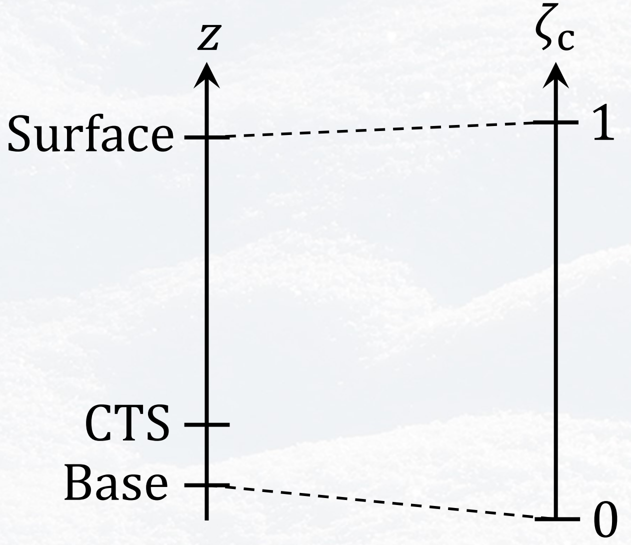
$$\frac{\mathrm{d}h}{\mathrm{d}t} = \mathrm{div}(k\ \mathrm{grad}\ h) + \frac{\Phi}{\rho}$$

$$\text{with } k = \begin{cases} \dfrac{\kappa}{\rho c} & \text{for cold ice} \\ \dfrac{\nu}{\rho} & \text{for temperate ice} \end{cases}$$

Thermodynamics solvers in SICOPOLIS

Cold-ice method (COLD)

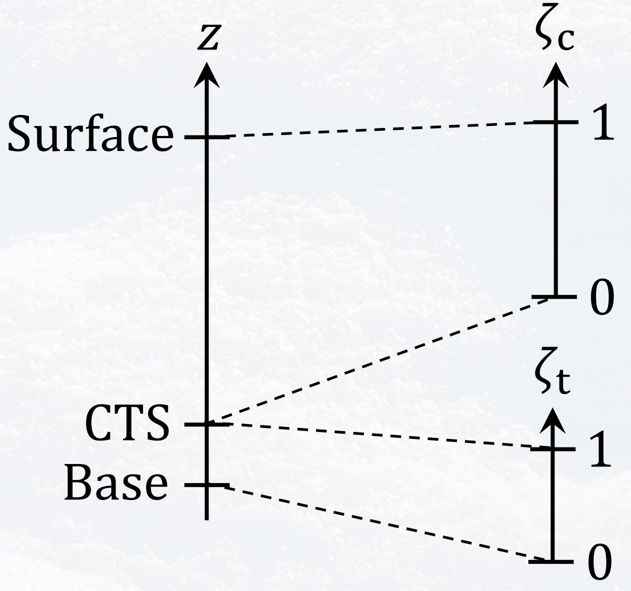
- Terrain-following coordinates (sigma transformation), one common domain $\zeta_c = 0 \ldots 1$ for cold and temperate ice.



Polythermal method

- Two separate domains $\zeta_c = 0 \ldots 1$, $\zeta_t = 0 \ldots 1$.
- Enforcement of the energy jump condition at the CTS:

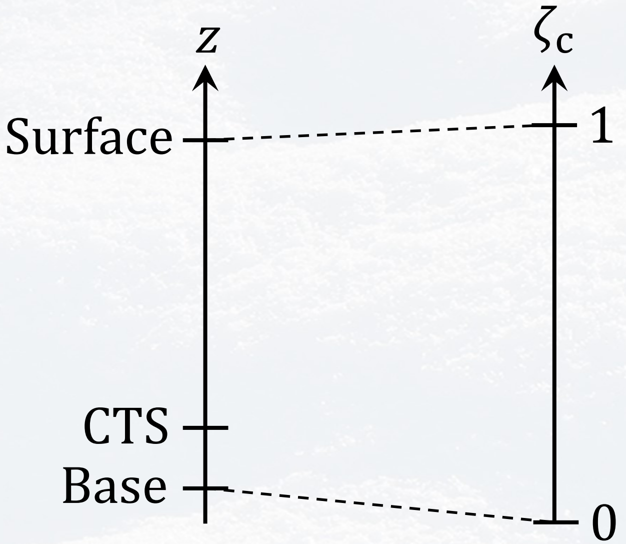
Melting and freezing conditions → POLY1.
Only melting conditions → POLY2.



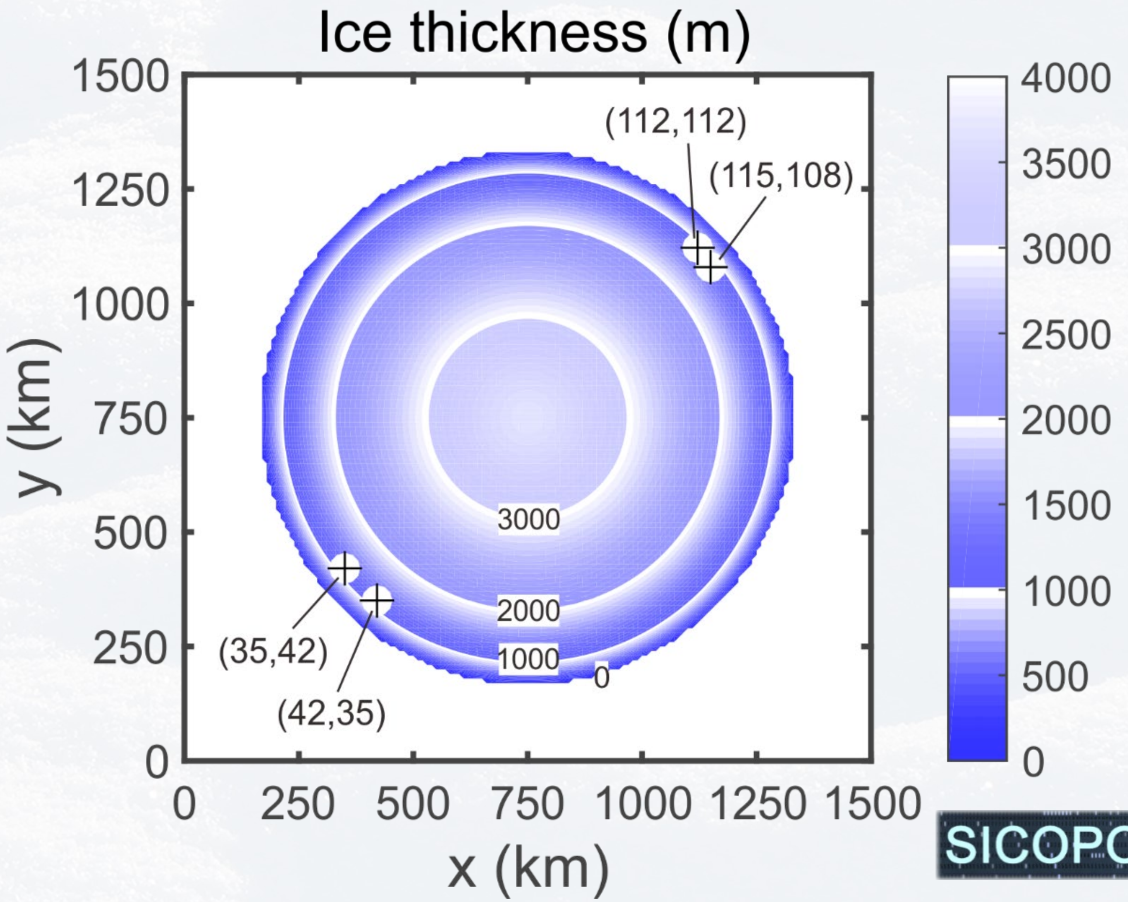
Thermodynamics solvers in SICOPOLIS

Enthalpy method

- One common domain $\zeta_c = 0\ldots1$ for cold and temperate ice.
- Enforcement of the continuity of the temperature gradient at the CTS:
  - No → conventional enthalpy scheme (ENTC).
  - Yes → melting-CTS enthalpy scheme (ENTM).



EISMINT Phase 2 SGE experiment A1 produces a Greenland-like ice sheet (Payne et al., 2000; Greve and Blatter, 2016)

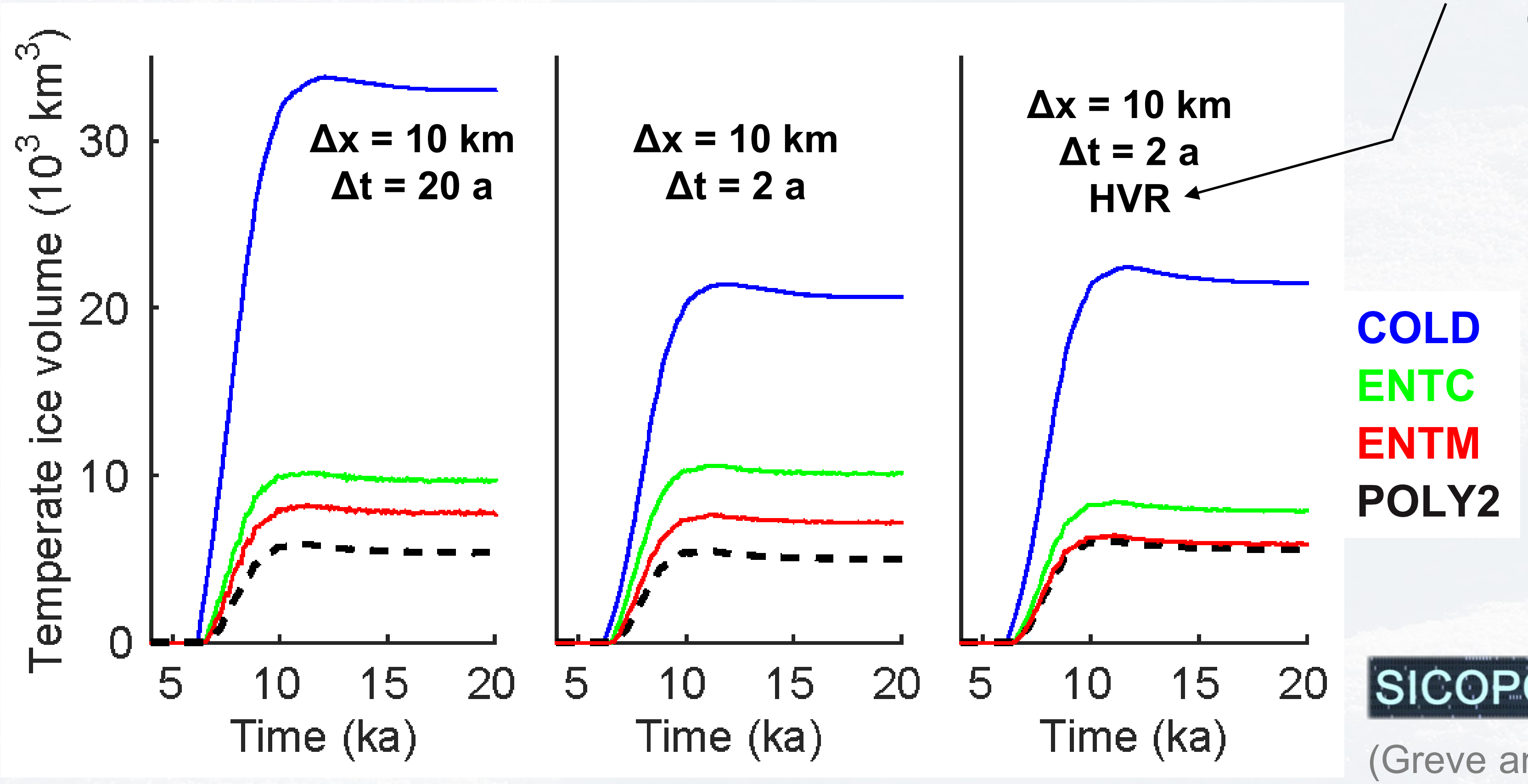


SICOPOLIS

$V_{tot} = 2.1 \times 10^6\ \mathrm{km}^3$

POLY2, Δx = 10 km, Δt = 2 a

Exp. A1: Evolution of the temperate ice volume



HVR: high vertical resolution
(5 × standard)

COLD:
Much too thick.

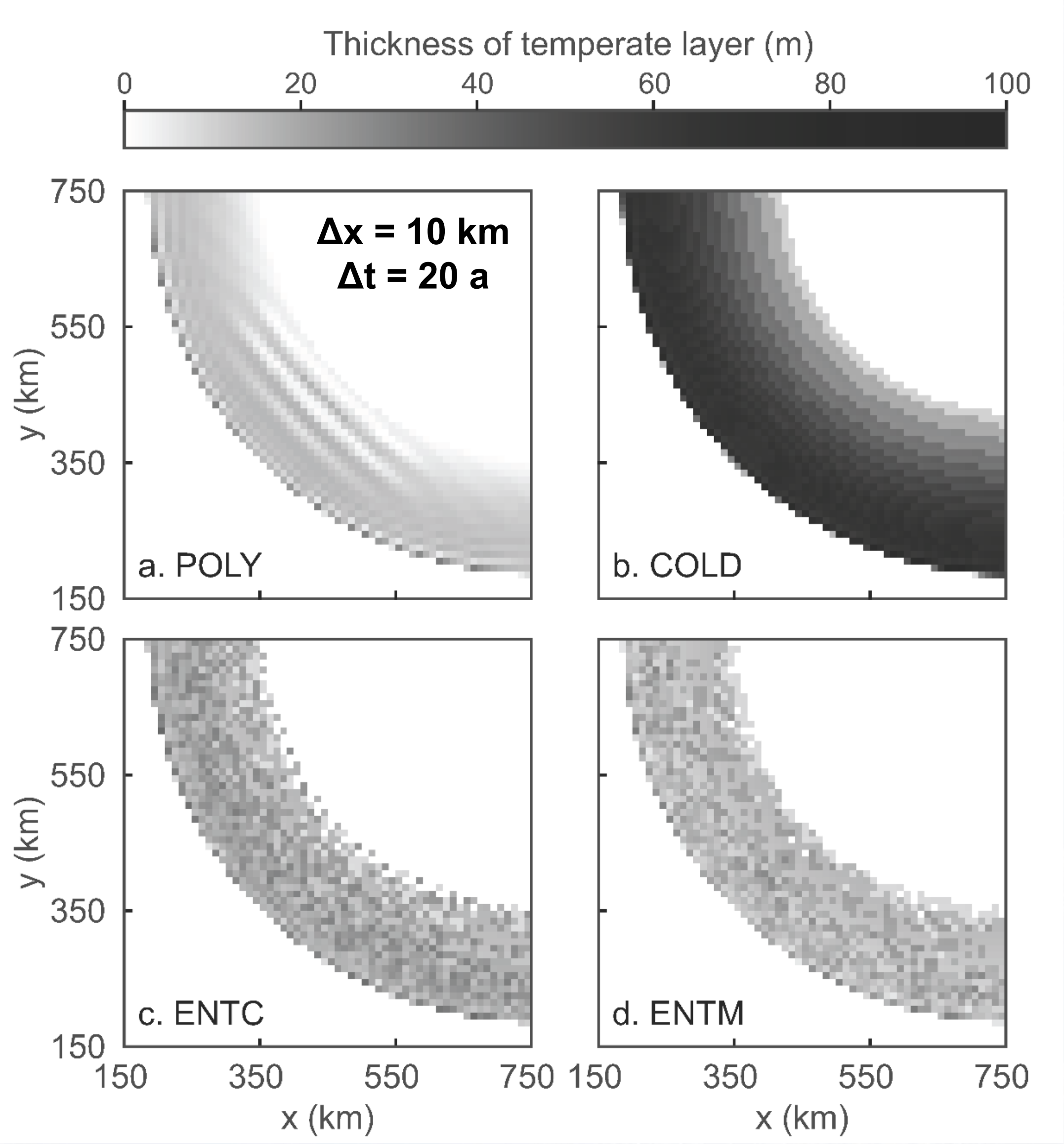
ENTC:
Somewhat too thick.

ENTM:
Converges to POLY2.

SICOPOLIS

(Greve and Blatter, 2016)

Exp. A1: Thickness of temperate ice layer



POLY2:
A bit wavy (instability).

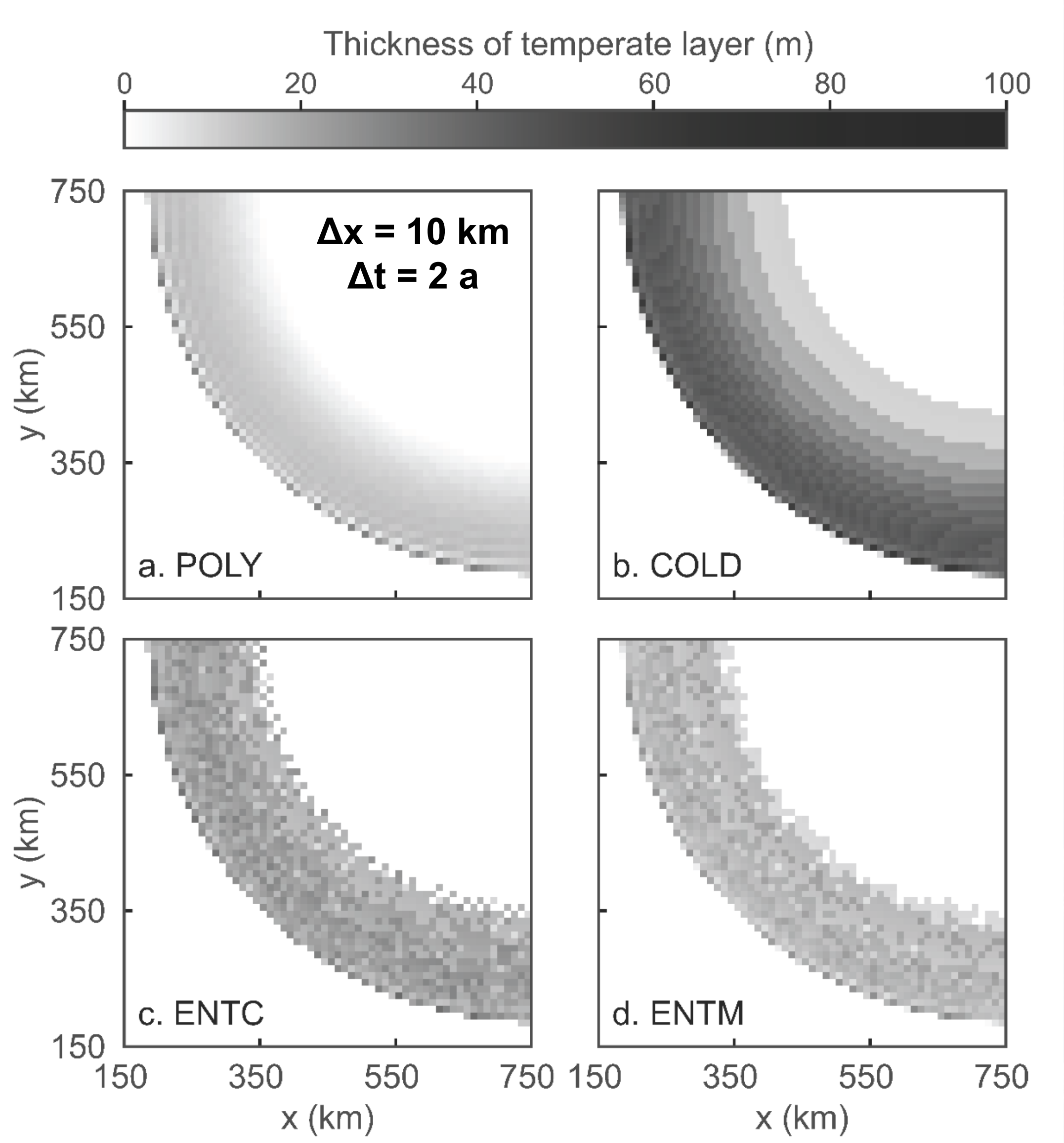
COLD:
Much too thick.

ENTC & ENTM:
Somewhat noisy.

SICOPOLIS

(Greve and Blatter, 2016)

Exp. A1: Thickness of temperate ice layer



POLY2:
Now fine (stable).

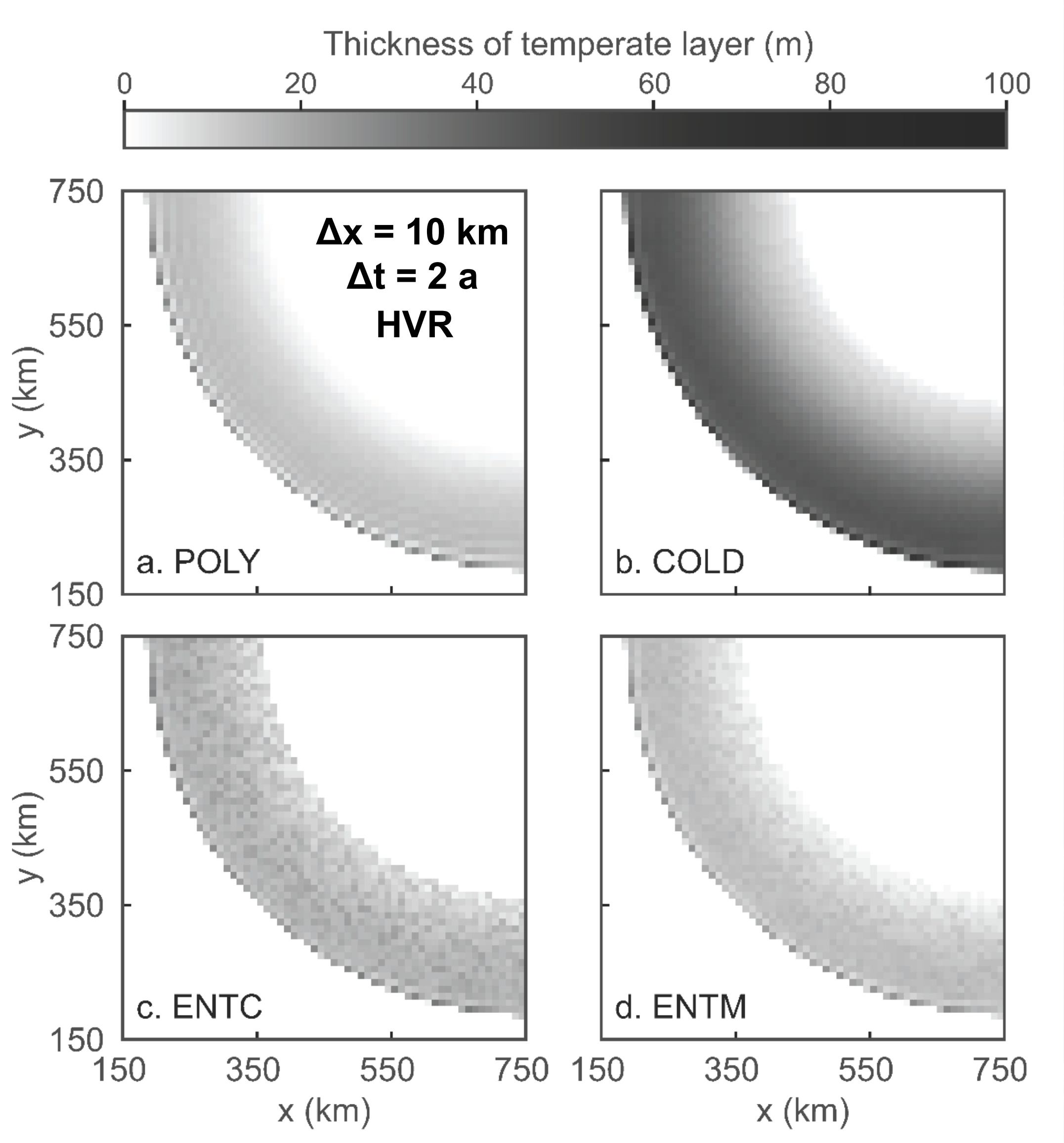
COLD:
Still much too thick.

ENTC & ENTM:
Still somewhat noisy.

SICOPOLIS

(Greve and Blatter, 2016)

Exp. A1: Thickness of temperate ice layer



POLY2:
Fine (stable).

COLD:
Much too thick.

ENTC & ENTM:
Still slightly noisy.

ENTM:
Close to POLY2.

SICOPOLIS

(Greve and Blatter, 2016)

Summary

- Ice sheet/glacier thermodynamics relevant for ice flow.

- Polythermal conditions must be considered in a model:

  COLD scheme is easiest, but physically inadequate.
  POLY1/2 schemes are best, but cumbersome implementation.
  ENTC and ENTM schemes are good compromises.

- Not covered in this talk:

  Drainage of excess water (> a few %) from temperate ice.
  Macroscopic water system: supraglacial, englacial & subglacial hydrology.

Next week's seminar

UC SANTA CRUZ

Subglacial Precipitates Record East Antarctic Ice Sheet Response to Ocean Forcing

Gavin Piccione (he,him)
PhD Candidate
UC Santa Cruz

IGS

IGS Global Seminar
October 20, 2021

Thank you!

😊

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German Research Foundation (DFG)

Federal State of Hesse, Germany

German National Academic Foundation

Japan Society for the Promotion of Science (JSPS)

Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT)

Institute of Low Temperature Science (ILTS), Hokkaido University

Appendix

References

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