

Option Valuation in the Presence of Market Sentiment: Application to Listed Companies in the CAC40 Index

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Abstract— Asset pricing theory based on rationality was widely criticized in literature. Indeed, the noninclusion of investor behavior and assuming market efficiency led to the weaknesses of option valuation through the traditional Black and Scholes model (1973). In this paper we examine the effect of the inclusion of investor behavior in the option pricing model. We test whether the Black and Scholes model in presence of sentiment behavior can lead to an improvement of the calculation of call price. Using daily data of 30 listed companies of France in the CAC40 index for the period June 18, 2009 to May 09, 2018, results showed that the introduction of sentiment effect in the Black and Scholes model provides better estimates of the call price than that obtained by the standard Black-Scholes model. In fact, we obtain an average gain of about 44% in terms of relative change in mean square error between both methods.

Keywords— Black and Scholes model, market sentiment, call price, implied volatility, heterogeneity of behavior.

INTRODUCTION

In 1973, Black and Scholes developed a seminal work for which they proposed a mathematical model to simulate the dynamics of financial market. This mathematical model covers derivative financial instruments. Since that date, applications of derivatives considered this model as fundamental and the famous one for estimating and forecasting the price of stock options. Most studies applying Black and Scholes model (B-S model) concluded that the right way to eliminate risk in buying and selling stocks for investing is summed up in hedging the options in an investment portfolio.

The B-S option-pricing model is a simple mechanism for valuing calls under certain assumptions for being relevant to real markets characteristics. It is a mathematical model that simulates the dynamics of the derivatives such as forwards, futures, options and swaps. According to its simple form, it is assumed that this model is based on partial differential equation (PDE) that governs the evolution of the stock's price in financial markets. Some assumptions are imposed to the original B-S model. Market efficiency was considered as the main assumption of the B-S model, from which it assumes that economic agents are perfectly rational, and on the other hand, it assumes the absence of imperfections in the financial markets. It is supposed that the financial market consists of one risky asset and one risk-free asset. The second assumption is to suppose that the rate of return on the risk-free asset is constant. The third assumption advocates that the instantaneous logarithmic return of the risky asset's price follows a geometric Brownian motion. The fourth one is to assume that the risky asset is not obliged to pay dividend. As we assume also that there are no risk-free profit opportunities (no arbitrage), in other way



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you can borrow/lend money with a rate that is equal to the interest rate of the risk-free asset, you can buy/sell any amount of stocks and finally there are no commission for buying/selling securities in the market.

Although the wide application of the Black and Scholes model and its impressive successes, it should be noticed that the above-mentioned assumptions are not empirically valid and market efficiency hypothesis cannot be achieved in most cases. However, market frictions and constraints can lead to biases in valuation. Multiple factors are responsible to obtain imperfections in the market. For example, we can cite the presence of transaction and information costs (Jensen (1978), Grossman and Stiglitz (1980)), the presence of asymmetry information (Allen and Gorton (1993)), short selling constructions (Harrisson and Kreps (1978), Duffie et al. (2002) and Scheinkman and Xiong (2003)). These imperfections lead to the weaknesses of the B-S model application to some real-life examples for financial markets in order to determine the price of a call or put option.

Many studies on behavioral finance showed that psychology factors can influence decisions made by investors, which may cause inefficiency in financial markets. According to behavioral finance and the unrealistic rationality of some investors, valuation errors can result when rational investors confront irrational investors. Investors qualified as noisemakers or noise traders (De Long et al. (1990) and Shleifer and Summers (1990)), or investors suffered from overconfidence (Daniel et al. (2002) and Abreu and Brunnermeier (2003)).

Another important common measure of market behavior is sentiment bias. Compared to other psychological biases, sentiment behavior affects rationality of investors and leads to unrealistic making decisions. Yang and al. (2016) proposed a model for the valuation of option in the presence of sentiment psychology factor relative to the option and other factors related to the underlying. They highlighted that option pricing can be affected by stock sentiment and option sentiment. By incorporating these sentiment factors in their proposed model, they concluded that both stock sentiment and option sentiment have a significant effect on the option price. Market sentiment can be measured either by sampling, such as confidence indexes or by using the information that is often found in market data, such as the put /call ratio, implied volatility is often used to measure the market sentiment. Yang and Gao (2014) studied the influence of stock index futures sentiment and the stock index sentiment on stock index futures returns. By using trading data for the Chinese stock index futures market, they constructed stock index futures sentiment and stock sentiment at daily, weekly, and monthly frequencies to incorporate sentiment factor in their study. They concluded in the empirical analysis that stock index futures sentiment and stock sentiment have significant impact on stock index futures returns. Zghal et al. (2020) studied the impact of market sentiment and information asymmetry on option pricing model. Based on S&P 500 options to the period ranging from March 17, 2000 to June 14, 2013 they found that the results obtained by B-S model in presence of imperfections are more effective than those obtained by traditional B-S model.

In this paper, we attempt to explain how market sentiment could influence the valuation of European options. Our contribution consists to integrate the sentiment of investor in the B-S model (1973) to consider the heterogeneity of participant behaviors in the options market. In other way, we are interested in this research to analyze the influence of the introduction of market sentiment to the B-S model of option valuation in order to better observe what happens in the options markets in terms of volatility. The purpose of this paper is to shed light on the robustness of B-S model when considering sentiment behavior applied to 30 listed companies of France in the CAC40 index for the period ranging from 18/06/2009 until 05/09/2018.

This paper is structured as follows. Section two presents the methodology employed in this research as it presents the introduction of sentiment factor in B-S model. Section three is devoted to the empirical analysis for which we present the results and make comparison between traditional B-S model and B-S sentiment model in computing price of a European call option. Section 5 concludes the paper.

II-BLACK AND SCHOLES MODEL IN PRESENCE OF SENTIMENT BEHAVIOR

According to B-S model (1973) and under a certain number of assumptions based on the market efficiency and the PDE, the evolution of the price stocks is written as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial s^2} + rS \frac{\partial V}{\partial s} - rV = 0 \quad (1)$$

Where,

V: represents the price of the option which is considered as a function of two variables: the stock price S: and the time t.

r: represents the risk-free interest rate

 σ : represents the volatility of the log returns of the underlying security. From equation (1) we can rewrite: $\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV - rS \frac{\partial V}{\partial S}$ (2)



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According to equation (2) we can notice that the left-hand side represents the change of the option V in value due to the time effect and the right-hand side represents the risk-free return from a long position in the option and a short position consisting of $\frac{\partial V}{\partial s}$ shares of the stock.

At maturity (date T), the value of a call or a put option can be obtained by the following equations:

Price of a European Call option: $C_{E,T} = \max(0, S_T - K)$ (3)

Price of a European Put option: $P_{E,T} = \max(0, K - S_T)$ (4)

These boundary conditions given in equations (3) and (4) included to the functional form of B-S model implies that the Black and Scholes formula for the price of a European call option for a non-dividend paying stock can be written as:

 $C_{RS}(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$

S: the price of a security

T: the date of expiration

t: the current date

X: the exercise price

r: the risk-free interest rate

 σ : the volatility measured by the standard deviation of the underlying asset

N(.): the cumulative distribution function of a normal distribution

d1 and d2 represent the inputs that are given by:

$$d_1 = \frac{\ln \frac{S}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\frac{\sigma\sqrt{T}}{\sigma}}$$

 $d_2 = d_1 - \sigma \sqrt{T}$

In order to calculate the call option price according to the previous formula obtained from the standard B-S model, we have developed a procedure on Visual Basic (VBA), named Call_BS to facilitate the calculation. The computation of call price is based on the recognition of the underlying price, exercise price, risk-free rate, time to maturity and volatility. Panel 1 presents the VBA program to compute the European call option.

Panel1: Visual Basic program for computing European call option by using standard B-S model

Function Call_BS(S, K, T, r, sigma)

d1 = ((Application.Ln(S / (K))) / (sigma * Sqr(T))) + (0.5 * sigma * Sqr(T))

d2 = d1 - sigma * Sqr(T)

nd1 = WorksheetFunction.NormSDist(d1)

nd2 = WorksheetFunction.NormSDist(d2)

 $Call_BS = (S * nd1) - (K * Exp(-r * T) * nd2)$

End Function

The volatility is considered as a non-observable variable that should be estimated. Two approaches are commonly used in practice to estimate volatility: the empirical or historical method and the implicit method. The calculation of implied volatility, for a given Call whose price is available on the market noted CMKT, from the B-S equation can be found by considering a function of one unknown variable (the implicit volatility $\hat{\sigma}$) that equalizes the theoretical Call C (r; T; K; S; $\hat{\sigma}$) to that of the market. We have then the following relation:

$$C(r; T; K; S; \widehat{\sigma}) = C_{MKT} = S.N\left(\frac{\ln\frac{S}{K} + \left(r + \frac{1}{2}\widehat{\sigma}^{2}\right)T}{\widehat{\sigma}\sqrt{T}}\right) - Ke^{-rT}N\left(\frac{\ln\frac{S}{K} + \left(r - \frac{1}{2}\widehat{\sigma}^{2}\right)T}{\widehat{\sigma}\sqrt{T}}\right) = C_{BS}(\widehat{\sigma})$$
(5)

To solve equation 5 and obtain the implied volatility an algorithm of interpolation on VBA software was employed. This can be considered as a useful indicator for investors as it corresponds to the volatility expected by the participants in the market over the life of the option and it is reflected in the option premium. A recursive procedure, based on Newton's algorithm, was considered for the computation of the implied volatility for which the calculated implied volatility related to date j will be used next day j+1 to re-compute the price of the purchase option.

The employed method for calculating implied volatility, from the option value of the market, is based on the B-S formula. This method consists to create two adjacent volatility sequences that converge to the real values of the volatility. Firstly, we calculate the value of the option $C_t(\sigma)$ for the maximum volatility, σ_{max} and the minimum volatility, σ_{min} . We have then a bounded interval for the volatility. These bounds are initialized with values close to 1 for σ_{max} and close to 0 for σ_{min} .



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The value of the option is respectively designated by $C_{max} > C$ for the maximum value of volatility σ_{max} and by $C_{min} < C$ for the minimum value of volatility σ_{min} (*C* represents the value of the option in the market).

After that, the value of the option $C_t(\sigma)$ is calculated for each mean value of the volatility interval bounds $(\sigma = \frac{\sigma_{max} + \sigma_{min}}{\sigma})$. For the iterative procedure, we do the following:

If $C_t(\sigma)^2 < C$ then $C_{min} = C_t(\sigma)$ and $\sigma_{min} = \sigma$ If $C_{max} = C_t(\sigma)$ and $\sigma_{max} = \sigma$

This iterative calculation stops when $|C_t(\sigma) - C|$ is less than a fixed predetermined value. Panel 2 presents the VBA program, named colimplcall, to compute the implied volatility.

Panel2: Visual Basic program for computing implied volatility

Function volimplcall(price As Variant, S As Variant, K As Variant, T As Variant, r As Variant) As Variant Dim sigmasup As Variant

Dim sigmainf As Variant Dim sigmamiddle As Variant Dim BSsup As Variant **Dim BSinf As Variant** Dim BSmiddle As Variant sigmasup = 0.8sigmainf = 0.1For i = 1 To 200 sigmamiddle = 1 / 2 * (sigmasup + sigmainf)BSmiddle = Call_BS(S, K, T, r, sigmamiddle) If price < BSmiddle Then sigmasup = sigmamiddle Else sigmainf = sigmamiddleEnd If Next i volimplcall = 1/2 * (sigmasup + sigmainf)End Function

Next, the calculated implied volatility obtained in panel2 will be used to compute the call price through B-S model in presence of sentiment behavior to consider irrationality property of some investors. To calculate the Call price of the European option price by the B-S model in the presence of sentiment, an algorithm was used and programmed on VBA. This method consists of calculating the value of the option based on the implied volatility $\hat{\sigma}$ which consider irrationality of investors caused by sentiment behavior in the market. Call price with sentiment will be calculated according to the following formula:

$$\begin{split} C_{BSS} &= SN(d1') - Ke^{-rT}N(d2') \\ d1' &= \frac{\ln \frac{S}{K} + \left(r + \frac{1}{2}\widehat{\sigma}^2\right)T}{\widehat{\sigma}\sqrt{T}} \\ d2' &= d1 - \widehat{\sigma}\sqrt{T} \end{split}$$

with $\widehat{\sigma} = \sigma_{rationnel} + \sigma_{irrationnel}$

Panel 3 presents the VBA program to compute the European call option by using B-S model in presence of sentiment behavior in the market, named Call_BSS.

Panel3: Visual Basic program for computing European call option by using B-S sentiment model

 $\begin{aligned} & Function \ Call_BSS(S, K, T, r, implvol) \\ & d1 = ((Application.Ln(S / (K))) / (implvol * Sqr(T))) + (0.5 * implvol * Sqr(T)) \\ & d2 = d1 - implvol * Sqr(T) \\ & nd1 = WorksheetFunction.NormSDist(d1) \\ & nd2 = WorksheetFunction.NormSDist(d2) \\ & CallBSS = (S * nd1) - (K * Exp(-r * T) * nd2) \\ & End \ Function \end{aligned}$



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III- EMPIRICAL ANALYSIS

A. Data Description

In this paper we selected 30 listed companies of France in the CAC40 index. We considered daily data ranging from 18/06/2009 until 05/09/2018 and that corresponds to 2406 observations. Data were collected from Thomson Reuters database. The selected companies and variables are presented in table 1.

Table 1 Companies and variables selection				
Companies	Variables			
Financial: BNP PARIBAS. Crédit Agricole. Société	C: price of a call option			
Générale. Axa. Automotive: Renault. Peugeot. Valeo.	P: price of a put option			
Commercial: Danone. Carrefour. L'Oréal. LVMH. Pernod.	S: price of the underlying asset			
Unibail. IT services: CAP Gemini. Atos.	X: strike price of the option			
Advertising: Publicis. Vivendi.	r: interest rate			
Construction: Lafarge. Saint Gobin. Vinci. Oil and gas: Total. Air liquid.	t: time to expiration			
Electric and equipment: Schneider. Micro-electronics	s: volatility of the underlying asset			
company. Water: Total. Air liquid. Medical: Essilor. Sanofi. Aerospace: Airbus. Pneumatic: Michelin.				

Table 1 Companies and variables selection

In this research, we use the stock option that is based on the fact that individuals in the market do not follow the same behavior and then they act differently in their choices. In other way, heterogeneity of investors affects the behavior in the market and the introduction of investor's irrationality into the B-S model (1973) will have a significant effect on call option calculation. Therefore, when evaluating option prices on daily data we assume that investor's strategy may depend on volatility to take into account the effect of sentiment behavior on call option valuation, i.e. when volatility is low in the market chartist investor acting more in the market than fundamentalist and vice versa.

In order to evaluate the performance of the option valuation B-S model in presence of sentiment behavior, we try to compute in the empirical analysis the price of a call using the B-S model with and without sentiment behavior and make comparison between both models by using the mean square error criteria (MSE).

B. Empirical results

In this section we calculate the price of a Call through Visual Basic programs developed in Panels 1 and 3. After that, a comparison between both methods are considered to empirically test the effect of introducing market sentiment into the model of Black and Scholes (1973) when evaluating options.

To compare the models, we defined the following variables that we must calculate:

- C_{BS} : is the price of a European call option calculated using the formula of B-S model (1973)
- C_{BSS}: is the price of a European call option calculated using the model in the presence of sentiment
- C_M: is the price of a European call option observed on the market.

The following graphs present C_{BS}, C_{BSS} and C_M of some selected companies.



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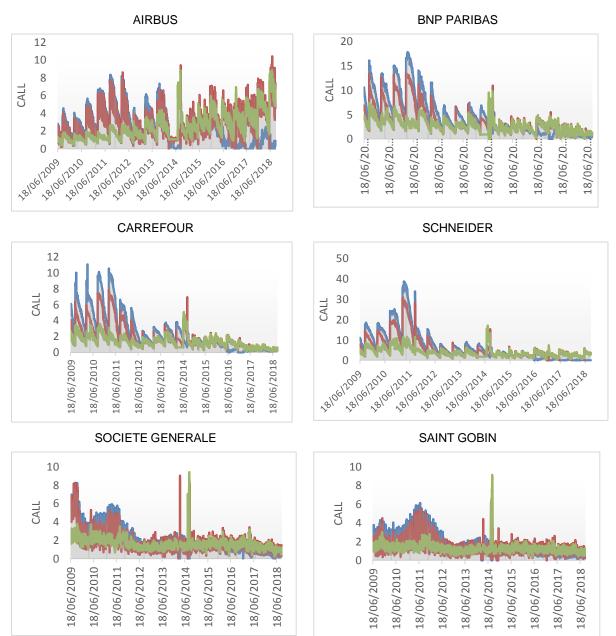
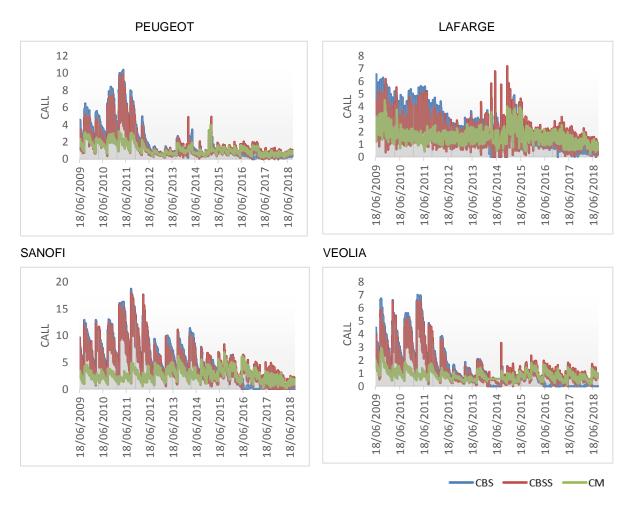


Fig1. Values of the Call according to BS model, BSS model and the market for some French companies.



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From these graphs, presenting the evolution of the estimated values of call calculated from the B-S model with and without the presence of sentiment behavior, we distinguish that the values of the option for most companies in the presence of sentiment C_{BSS} are much closer to the value of the option observed in the market C_M than the values of the options to purchase calculated from standard B-S model (1973) C_{BS} . Therefore, heterogeneity of investor behavior would affect results and give better results.

To verify the success of this novel approach compared to the standard approach based on B-S model (1973), we use a valuation error analysis. We compare results by using the mean square error (MSE) criteria for all selected companies¹ in this research. We calculate MSE, variation of MSE and relative change in MSE between both models. When MSE variation and relative change in MSE values are negative, we conclude that B-S model in presence is better than the basic B-S model and provides estimated option price closer to the option market price. Results are presented in table 2.

	MSE CBS	MSE CBSS	MSE Variation	Relative change in MSE in % ²
Airbus	7.44351894	2.50011279	-4.94340615	-66.4122197%
BNP PARIBAS	15.5649048	5.45265938	-10.1122454	-64.9682446%
Danone	16.7016572	11.0521827	-5.64947457	-33.8258323%

Table2: MS	E and	relative	change	in MSE
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Michelin	28.1229779	16.8028129	-11.320165	-40.2523697%
Carrefour	5.74092298	1.79620414	-3.94471884	-68.712276%
Vivendi	0.46314161	0.3212137	-0.14192791	-30.6446043%
Vinci	1.59303179	0.8075915	-0.78544029	-49.3047469%
Veolia	2.57494379	1.41593753	-1.15900626	-45.0109343%
Valeo	5.71896816	3.31547447	-2.40349369	-42.0267018%
Unibail	68.4763742	32.26038	-36.2159942	-52.8883058%
Total	11.8311683	7.84505725	-3.98611103	-33.6916096%
Suez	1.53460301	1.11922598	-0.41537703	-27.0673931%
St Microelectronics	0.4809566	0.20564506	-0.27531154	-57.2424912%
Société Générale	1.82218109	0.9095062	-0.91267489	-50.0869475%
Schneider	47.9320953	20.2666099	-27.6654854	-57.7180807%
Sanofi	22.3507309	14.5547882	-7.79594267	-34.8800346%
Saint Gobin	1.62863036	0.90796092	-0.72066944	-44.2500313%
Renault	14.1401262	4.9683727	-9.1717535	-64.8633072%
Publicis	1.81722966	1.20227386	-0.6149558	-33.8402908%
Peugeot	3.49287179	1.53229759	-1.96057421	-56.1307234%
Pernod	36.9788197	21.5106005	-15.4682192	-41.8299429%
LVMH	109.84805	57.7812606	-52.0667899	-47.3989203%
L'oréal	59.7971175	34.3851598	-25.4119577	-42.496961%
LAFARGE	1.71096211	0.85608945	-0.85487266	-49.9644414%
Essilor	5.14533369	4.03238623	-1.11294747	-21.6302291%
Crédit Agricole	0.16376886	0.11626079	-0.04750807	-29.0092202%
Cap Gemini	1.73459184	0.8982544	-0.83633745	-48.21523%
Axa	0.38992099	0.27050867	-0.11941232	-30.6247484%
Atos	20.4367171	15.6972817	-4.73943534	-23.1907861%
Air Liquide	54.5244693	35.8100975	-18.7143718	-34.3228867%

We can notice from table2 a net performance of B-S model incorporating sentiment behavior compared to that of standard B-S model in computing Call value estimates. For our analysis, we obtain, in average, a gain of about



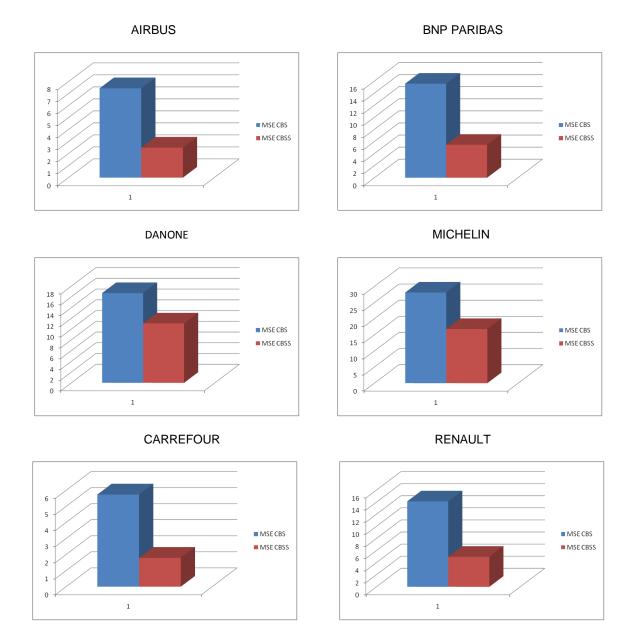
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44% in terms of relative change in MSE. Indeed, we can see for example that the value of MSE decreased from 7.44 to 2.50 for the company Airbus with a change of about 66.5% in terms of relative change in MSE.

The relative change in MSE varies between -21.6302291% and -68.712276% which explains the performance of the B-S model in the presence of sentiment compared to the basic B-S model (1973). One third of our sample represents a variation in MSE which exceeds 50% such as the Renault company -64.86%, Peugeot -56.13%, Airbus -66.41%, BNP PARIBAS -64.97% and Carrefour -68.71%.

In order to prove that the B-S model in the presence of sentiment behavior constitutes a better approximation of the market values of the options than those obtained using the basic B-S model, we illustrate the graphs below which represent the MSE of both models for some companies.





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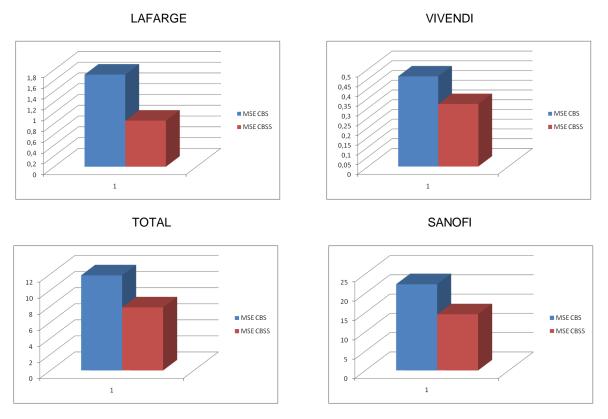


Fig2. Average quadratic deviation of the basic Black and Scholes model and the model in the presence of sentiment for some companies in our sample.

We can notice throughout the period studied from 2009 to 2018 that the B-S model in presence of sentiment presents better estimates of Call compared to the traditional B-S model. Therefore, we can conclude from this study that volatility which is unobservable variable influences the value of the option. By taking into account that investor's behavior affects this unobservable variable by incorporating the sentiment factor in the B-S model to compute Call estimates for 30 companies operating in the French market, we obtain better results than that of the standard B-S model (1973). Sentiment incorporated in B-S model constitutes a better approximation of the market price of a European option.

IV- CONCLUSION

Under the traditional assumptions of the basic B-S model (1973), investors are assumed to be risk-neutral and the expected rate of return is assumed to be equal to the risk free-rate. However, economic and neuro-economic researches show that the price of the call can be affected by subjective opinions, psychological and sentiment factors. Nowadays behavioral finance has become an important field to analyze stock markets in order to better understand investor behavior and include it in medialization to adopt the right strategy. It is therefore important to include the investor sentiment for the evaluation of call options.

In this paper, we have studied the impact of introducing sentiment factor to option evaluation. For doing this, we make a comparison of Call estimates obtained by both B-S model with and without sentiment factor. By considering 30 listed companies of France in the CAC40 index ranging from the period June 18, 2009 to May 09, 2018, results show that that the introduction of the sentiment factor, representing the difference in behaviors among chartists and fundamentalists in the market, to the Black and Scholes option pricing model improves the pricing performance of the model. However, the introduction of sentiment into the option evaluation formula has led to estimated prices closer to that of the market. In average, we found a gain of about 44% in terms of relative change in MSE when we use the B-S sentiment model.



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Endnote

- ¹ MSE(C_{BS}) = $\frac{1}{n}\sum (C_{BS} C_M)^2$ and MSE(C_{BSS}) = $\frac{1}{n}\sum (C_{BSS} C_M)^2$.
- ² Relative change in MSE in percentage is calculated as follows: $\frac{(MSE(CBSS) MSE(CBS))}{MSE(CBS)} \times 100.$