New Decentralized Control of Interconnected Systems

Amal Zouhri, Ismail Boumhidi

Abstract: *In this paper, we present a new decentralized H∞ control for interconnected systems, the interconnected system consists of several subsystems. The proposed approach based on Lyapunov functional and a H∞ criterion, employed to reduce the effect of interconnections between subsystems. In the first, we study the stability of the global system in closed loop using a criterion H∞, the stability conditions are presented in terms of LMI. In the second, to improve this approach, a Finsler's lemma is used for the stability analysis by LMIs. Some sufficient conditions, ensuring all the closed-loop stability are supplied in terms of Linear Matrix Inequalities (LMIs), and the new feedback gain matrix of each local controller is obtained by solving the LMIs. Finally, the practice examples are given to illustrate the efficiency of the present method.*

Keywords : Interconnected system, H∞ control, decentralized state feedback, linear matrix inequality (LMI).

I. INTRODUCTION

In Microelectronics and power electronics, there are several dynamic systems in this field, their systems are classified in the large dimension, also their physical configuration is more complex [1-2], for the control and the modelization for this system, we composite this system in interconnected subsystems, these decompositions facilitate the resolution of the issue of stability [?]. The synthesis of law control and stabilization can be ensuring the global safety of the system and responding in a more secure of the system [4-5], Moreover, in the phase of the modeling, the classical approach can be very hard and complicated to use to these complex systems in the context of the reliability and practical conception, this method of decentralized control is alternative to assert the stabilization of the complex and large systems [6-7-8-9].

In this paper, we focus to develop a new approaches and to extend the conditions of stability for the class of interconnected subsystems based on the Lyapunov approach and on Finsler's lemma. A H∞ criterion is employed to minimize the effects of interconnections between subsystems.

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The stability conditions are presented in terms of LMIs [10-11]. In order to improve this approach, a Finsler's lemma is used for the stability analysis by LMIs, with better results than the quadratic stability of LMIs, but with extra matrices, which allows a certain relaxation in the stability analysis, by obtaining a larger feasibility region. A new set of N decentralized controllers will thus be synthesized to ensure the stability of the global system in a closed loop. The seat of this work is arranged as following: in section 2, an overview of system description have been provided. Main results has been introduced in section 3. In section 4, we present numerical example to show the usefulness of the proposed results. Finally, the paper is concluded by brief conclusion in section 5.

II. SYSTEM DESCRIPTION

We consider the following interconnected systems (Fig. 1), where the ith subsystem is given by:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1, j \neq i}^{N} A_{ij} x_j(t) + B_i u_i(t), \, i, j \in \{1, ..., N\} \quad (1)$$

with $x_i(t) \in \Re^{n_i}$ and $x_j(t) \in \Re^{n_j}$ are the states of the ith and the jth subsystem, $u_i(t) \in \Re^{m_i}$ is the control input. The system matrices A_i, A_{ij} and B_i, are known real constant of appropriate dimensions
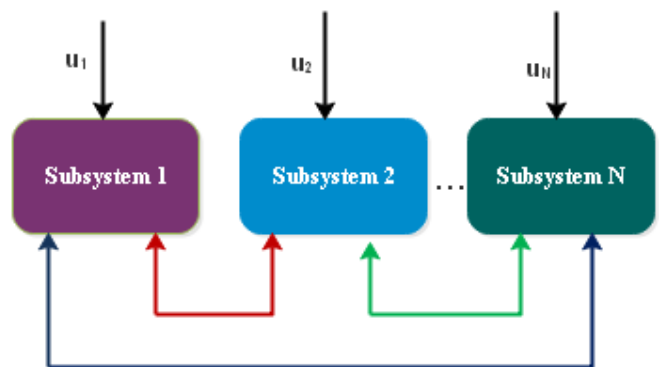

Fig. 1.Flowshart of decentralized control issue of the interconnected system .

The closed-loop subsystem is given by

$$\dot{x}_i(t) = A_{ci} x_i(t) + \sum_{j=1, j \neq i}^{N} A_{ij} x_j(t), \, i, j \in \{1, ..., N\} \quad (2)$$

where $u_i(t) = k_i x_i(t)$,

$A_{ci} = A_i + B_i k_i$,

First, lemma is given which is essential for later evelopment.

Lemma 1. The Schur complement converts a class of convex nonlinear inequalities that appears regularly in control problems to an LMI. We have [12]

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \succ 0$$

is equivalent to

$$R \succ 0, \quad Q - S R^{-1} S^T \succ 0,$$

where Q and R are symmetric.

Lemma 2. [13] For any $n \times n$ matrix Q, any constant $\varepsilon \succ 0$ and any symmetric positive matrix $P \succ 0$,

$$2x^T Q y \le \varepsilon x^T Q P^{-1} Q^T x + \varepsilon^{-1} y^T P y$$

For all $x, y \in \Re^n$

Lemma 3. [14] Consider $\xi \in \Re^n$, $Q = Q^T \in \Re^{n \times n}$ and $\beta \in \Re^{m \times n}$ such that $rank(\beta) \prec n$, and $\beta^\perp$ a basis for the null-space of β (i.e. $\beta^\perp \beta = 0$).

(1) $\xi^T Q \xi \prec 0$, for all $\xi \ne 0$, $\beta \xi = 0$;

(2) $\beta_\perp^T Q \beta_\perp \prec 0$;

(3) $\exists \chi \in \Re^{n \times m} : Q + \chi \beta + \beta^T \chi^T \prec 0$

III. MAIN RESULTS

In this part, the principal objective is to create a new decentralized controller guarantee the stability of interconnected systems represented by (2).

A. Decentralized state feedback H∞ control

In this subsection, our objective is to provide a method of synthesis of decentralized control laws allow both the stability asymptotic of each of the interconnected subsystems and

reduce the effect of interconnections between subsystems. To realize this, we are based on criterion H∞ to minimize the impact of j subsystem $(i = 1, 2, 3, ..., N, \text{ and } i \ne j)$ on the jth subsystem. these criteria are given as for $j = 1, 2, 3..., N$.

$$\int_0^\infty x_i^T(t) x_i(t) dt \prec \gamma_i^2 \int_0^\infty \phi_i^T(t) \phi_i(t) dt \quad (3)$$

where

$$\phi_i(t) = \sum_{j=1, j \ne i}^N A_{ij} x_j(t) \quad (4)$$

with $\gamma_i \succ 0$ is the performance of H∞ level.

Moreover, the vector $\phi_i(t) = \sum_{j=1, j \ne i}^N A_{ij} x_j(t)$ give the impact of a jth subsystem on i^{th}, where γ_i is performance if H∞ is minimize.

This method makes the stabilizing the system in formula (2) with the decentralized controller $u_i(t) = k_i x_i(t)$, and minimize the impact of interconnections between different subsystems.

Theorem 1. The issue of decentralized H∞ control for the jth subsystem number (2) is resolvable if $\exists$ the resolvable positive matrix $P_i = P_i^T \succ 0$, y_i and scalars γ_i positive satisfying the condition LMI :

Where γ_i is minimize.

$$\begin{bmatrix} \dfrac{1}{N-1}\left(P_i A_i^T + A_i P_i + B_i y_i + y_i^T B_i^T\right) & A_{ij} & P_i \\ A_{ij}^T & -\gamma_i^2(2N-3)A_{ij}^T A_{ij} & 0 \\ P_i & 0 & \dfrac{-1}{N-1} \end{bmatrix} \prec 0 \quad (5)$$

Where, the local matrix is
$$k_i = y_i P_i^{-1} \quad (6)$$

Proof

The stability of the system (2) for closed-loop under the criterion H∞ satisfy:

$$\sum_{i=1}^N \left[\dot{V}_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 \phi_i^T(t) \phi_i(t) \right] \prec 0 \quad (7)$$

The H∞ (3) can be reform as:

$$\int_0^\infty x_i^T x_i dt \prec \gamma_i^2 \int_0^\infty \left[\sum_{j=1, j \ne i}^N (N-1)\left(x_j^T A_{ij}^T A_{ij} x_j\right) + \sum_{l=1, l \ne i, l \ne j}^N x_l^T A_{il}^T A_{il} x_l \right] dt \quad (8)$$

Then, the system (2) is stable if:

$$\sum_{i=1}^N \left[x_i^T(t)\left(A_{ci}^T P_i^{-1} + P_i^{-1} A_{ci}\right) x_i(t) + x_i^T(t) x_i(t) - \gamma_i^2 \sum_{j=1, j \ne i}^N \left((N-1)x_j^T A_{ij}^T A_{ij} x_j + \sum_{\substack{l=1, l \ne i, \\ l \ne j}}^N x_l^T A_{il}^T A_{il} x_l \right) \right] \prec 0 \quad (9)$$

Now, the principal objective is to reformulate inequality (9) in terms of LMI.

$$\sum_{j=1,\,j\neq i}^{N}\left(\vartheta_{ij}+\sum_{l=1,l\neq i,l\neq j}^{N}\vartheta_{il}\right)=\left(N-1\right)\sum_{j=1,\,j\neq i}^{N}\vartheta_{ij}$$

the inequality (9) becomes:

$$\sum_{i=1}^{N}\Bigg[x_i^T(t)\left(A_{ci}^T P_i^{-1}+P_i^{-1}A_{ci}\right)x_i(t)+\sum_{j=1,\,j\neq i}^{N}x_j^T(t)A_{ij}^T P_i^{-1}x_i(t)+x_i^T(t)P_i^{-1}\sum_{j=1,\,j\neq i}^{N}A_{ij}x_j(t)+x_i^T(t)x_i(t)$$

$$-\gamma_i^2\left(2N-3\right)\sum_{j=1,\,j\neq i}^{N}x_j^T(t)A_{ij}^T A_{ij}x_j(t)\Bigg]\prec 0 \tag{10}$$

The equation (10) can be reform as:

$$\sum_{i=1}^{N}\sum_{i=1,\,j\neq i}^{N}\begin{bmatrix}x_i(t)\\x_j(t)\end{bmatrix}^T\begin{bmatrix}\pi_i & P_i^{-1}A_{ij}\\A_{ij}^T P_i^{-1} & v_i\end{bmatrix}\begin{bmatrix}x_i(t)\\x_j(t)\end{bmatrix}\prec 0 \tag{11}$$

Where,

$$\pi_i=\frac{1}{N-1}\left(A_{ci}^T P_i^{-1}+P_i^{-1}A_{ci}+I\right)$$
$$v_i=-\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij}$$

The formula (11) can be verified $\forall\,x_i$ and $\forall\,x_j$ if, for $i,j=1,2,3...,N$, with $j\neq i$

$$\begin{bmatrix}\dfrac{1}{N-1}\left(A_{ci}^T P_i^{-1}+P_i^{-1}A_{ci}+I\right) & P_i^{-1}A_{ij}\\A_{ij}^T P_i^{-1} & -\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij}\end{bmatrix}\prec 0 \tag{12}$$

Multiplying (12) by $\begin{bmatrix}P_i^T & 0\\0 & I\end{bmatrix}$ in the left and $\begin{bmatrix}P_i & 0\\0 & I\end{bmatrix}$ in the right and using the Schur formula, the equation (12) satisfy

$$\begin{bmatrix}\dfrac{1}{N-1}\left(A_{ci}P_i+P_i A_{ci}^T\right) & A_{ij} & P_i\\A_{ij}^T & -\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij} & 0\\P_i & 0 & \dfrac{-1}{N-1}\end{bmatrix}\prec 0 \tag{13}$$

Using the change of variables

$$y_i=k_i P_i \tag{14}$$

inequality (13) is equivalent to the LMI (5). This completes the proof. □

B. New decentralized H∞ control

In this section, a new decentralized H∞ control conditions for the interconnected closed-loop system (2) is developed. The main result for interconnected system is stated in the following theorem.

Theorem 2. The resolution of the system (2) for decentralized H∞ if $P_i=P_i^T\succ 0$, y_i, F_i and γ_i,a_i,b_i are positives and satisfy

$$\begin{bmatrix}\Upsilon_{i11} & \Upsilon_{i12} & \Upsilon_{i13} & I\\ * & \dfrac{-a_i}{N-1}\left(F_i+F_i^T\right) & a_i F_i\,A_{ij}-\dfrac{b_i}{N-1}F_i^T & 0\\ * & * & \Upsilon_{i33} & 0\\ * & * & * & \dfrac{-1}{N-1}\end{bmatrix}\prec 0 \tag{15}$$

where

$$\Upsilon_{i11}=\frac{1}{N-1}\left(F_i\,A_i^T+A_i F_i^T+B_i\,y_i+y_i^T B_i^T\right)$$

$$\Upsilon_{i12}=\frac{1}{N-1}\left(P_i^{-1}-F_i-a_i A_i F_i^T+a_i B_i\,y_i\right)$$

$$\Upsilon_{i13}=F_i\,A_{ij}+\frac{b_i}{N-1}\left(A_i F_i^T+B_i\,y_i\right)$$

$$\Upsilon_{i33}=-\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij}+b_i F_i\,A_{ij}+b_i A_{ij}^T F_i^T$$

The new local matrix is

$$k_i=y_i\left(F_i^T\right)^{-1} \tag{16}$$

Proof

The stability of system (2) on closed loop satisfy the criterion H∞

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$$\sum_{i=1}^{N}\left[\dot{V}_i(t)+x_i^T(t)x_i(t)-\gamma_i^2\phi_i^T(t)\phi_i(t)\right]\prec 0$$
(17)

The above inequality can be refurmuled as

$$\sum_{i=1}^{N}\sum_{j=1,\,j\neq i}^{N}[A+B-C]\prec 0$$

$$A=\frac{1}{N-1}\left(\dot{x}_i^T(t)P_i^{-1}x_i(t)+x_i^T(t)P_i^{-1}\dot{x}_i(t)\right)$$
(18)

$$B=x_i^T(t)x_i(t))$$

$$C=\gamma_i^2\left(2N-3\right)x_j^T(t)A_{ij}^T A_{ij}x_j(t)$$

we obtained

$$\sum_{i=1}^{N}\sum_{j=1,\,j\neq i}^{N}\zeta_i^T Q_i\zeta_i\prec 0$$
(19)

where

$$\zeta_i=\left[x_i^T(t)\quad \dot{x}_i^T(t)\quad x_j^T(t)\right]^T,$$

$$Q_i=\begin{bmatrix}\dfrac{1}{N-1}&\dfrac{1}{N-1}P_i^{-1}&0\\ *&0&0\\ *&*&-\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij}\end{bmatrix},$$

Expression (2) can be written as:

$$\sum_{j=1,\,j\neq i}^{N}\left[\frac{1}{N-1}A_{ci}\quad \frac{-1}{N-1}I\quad A_{ij}\right]\begin{bmatrix}x_i(t)\\ \dot{x}_i(t)\\ x_j(t)\end{bmatrix}=0$$
(20)

Considering

$$\chi_i=\begin{bmatrix}F_i\\ a_iF_i\\ b_iF_i\end{bmatrix},$$
(21)

$$\beta_i=\left[\frac{1}{N-1}A_{ci}\quad \frac{-1}{N-1}I\quad A_{ij}\right],$$
(22)

with

$$\beta_{i\perp}=\begin{bmatrix}I&0\\ A_{ci}&(N-1)A_{ij}\\ 0&I\end{bmatrix}$$
(23)

and using condition (ii) of lemma 3, we obtained the equality between $\beta_{i\perp}^T Q_i\beta_{i\perp}$ and the LMIs in (5).

Now, Applying Finsler's Lemma 3 in condition (3), with (19), (21) and (22) writes

$$\begin{bmatrix}\dfrac{1}{N-1}&\dfrac{1}{N-1}P_i^{-1}&0\\ *&0&0\\ *&*&-\gamma_i^2\left(2N-3\right)A_{ij}^T A_{ij}\end{bmatrix}$$

$$+\begin{bmatrix}F_i\\ a_iF_i\\ b_iF_i\end{bmatrix}\left[\frac{1}{N-1}A_{ci}\quad \frac{-1}{N-1}I\quad A_{ij}\right]+(*)\prec 0$$
(24)

Thus, (24) is equivalent to (15). This completes the proof of the lemma 2. □

IV. EXPERIMENTAL RESULT

To prove the efficacy of the proposed method, we give some numerical example extract of the literature

Application 1. We give the decentralized H∞ control issue of interconnected system of equation (2) with

$$A_1=\begin{bmatrix}-7&0\\ -6&0.1\end{bmatrix},\ A_{12}=\begin{bmatrix}0.1&0\\ 0&0.1\end{bmatrix},\ A_{13}=\begin{bmatrix}0.01&0\\ 0&0.1\end{bmatrix}$$

$$A_2=\begin{bmatrix}5&0\\ 1&-1\end{bmatrix},\ A_{21}=\begin{bmatrix}0.1&0\\ 0&0.1\end{bmatrix},\ A_{23}=\begin{bmatrix}0.01&0\\ 0&0.1\end{bmatrix}$$

$$A_3=\begin{bmatrix}-1&0\\ 0.1&-1\end{bmatrix},\ A_{31}=\begin{bmatrix}0.01&0\\ 0.1&0.1\end{bmatrix},\ A_{32}=\begin{bmatrix}0.01&0.1\\ 0&0.1\end{bmatrix}$$

$$B_1=\begin{bmatrix}0.47\\ 1.2\end{bmatrix},\ B_2=\begin{bmatrix}0.47\\ 0.8\end{bmatrix},\ B_3=\begin{bmatrix}0.6\\ 0.4\end{bmatrix}$$

Using the first system theorem below system and minimize γ_i, we observe that for this primary subsystem

$$k_1=\begin{bmatrix}5.5149&-6.1398\end{bmatrix},\ P_1=\begin{bmatrix}1.7981&0.0260\\ 0.0260&1.2381\end{bmatrix},$$

$$\gamma_1=0.2667$$

for the secondary subsystem

$$k_2=\begin{bmatrix}-95.3551&22.7011\end{bmatrix},$$
$$P_2=\begin{bmatrix}0.2035&0.5290\\ 0.5290&1.7732\end{bmatrix},\qquad \gamma_2=0.4888$$

for the third subsystem

$$k_3=\begin{bmatrix}-7.4300&-5.6618\end{bmatrix},\ P_3=\begin{bmatrix}1.1218&0.3183\\ 0.3183&0.7969\end{bmatrix},$$

$$\gamma_3=1.1014$$

Using the second system theorem below system and minimize γ_i, we observe that

For the first subsystem
$$k_1=\begin{bmatrix}10.0746&-1.8061\end{bmatrix},$$

for the second subsystem
$$k_2=\begin{bmatrix}-32.5773&6.7252\end{bmatrix},$$

for the third subsystem

$$k_3 = \begin{bmatrix} -0.2800 & -0.3621 \end{bmatrix}$$

In this numerical result, we observe that the following initial conditions use these vectors

$$x_1(0) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad x_3(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

The evolution of the control system and these dynamics in a closed-loop for every each subsystem are shown in Figures 1 to 4.
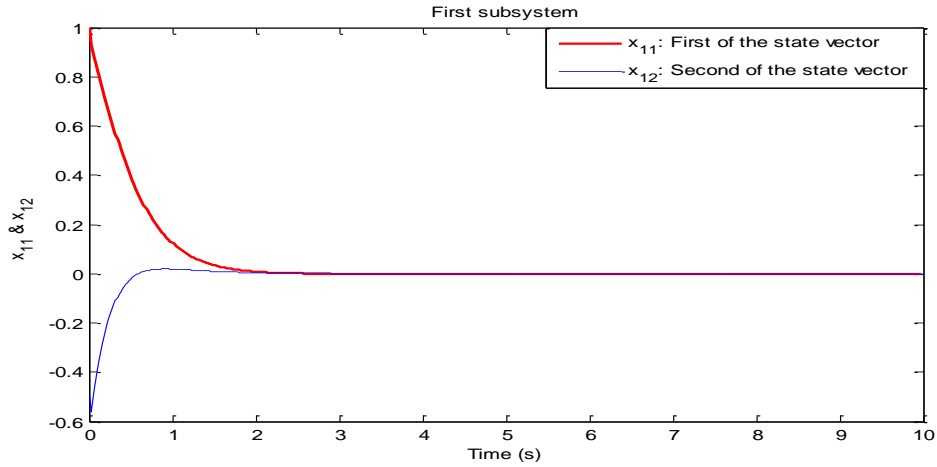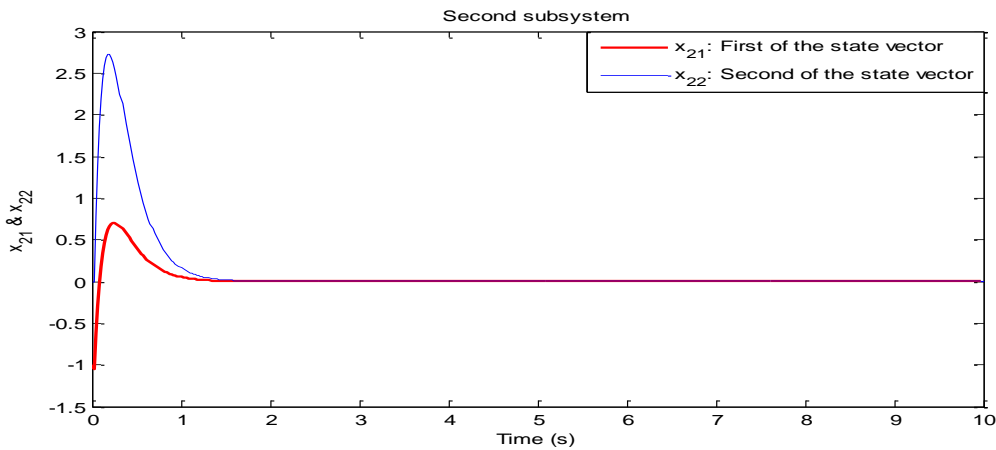

Fig. 2.a-First subsystem .

Fig. 2.b- Second subsystem.
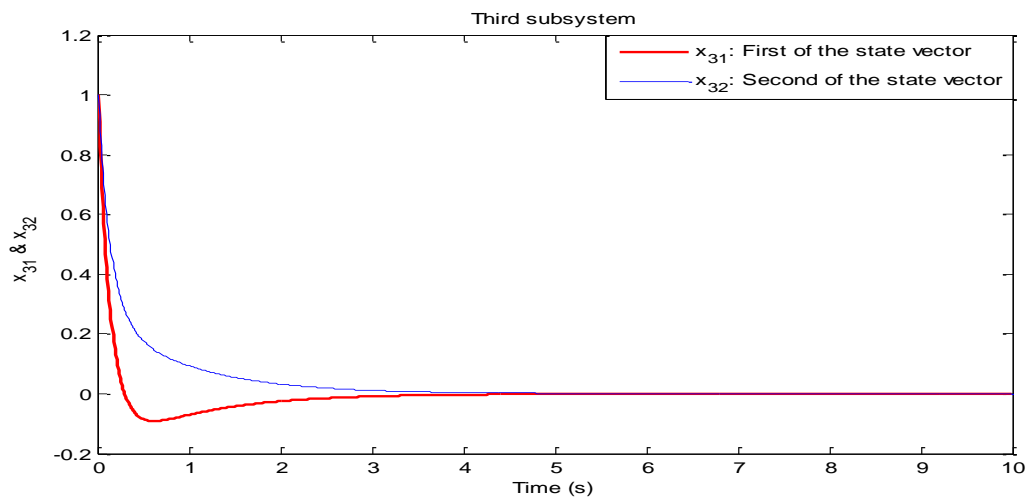
Fig. 2.c- Third subsystem.
Figures 2. Temporal evolution of the state vector of interconnected subsystem with H∞ control.
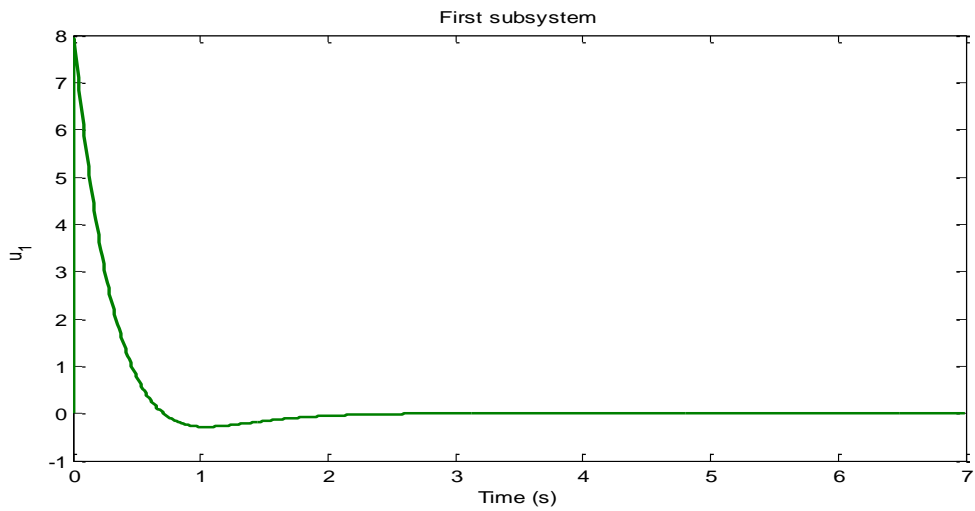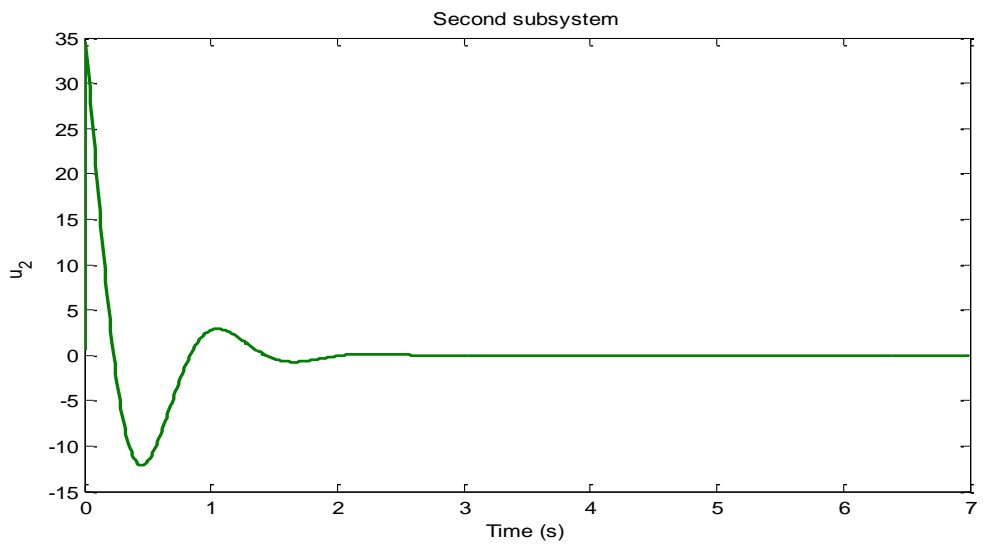
Fig. 3.a- First subsystem.
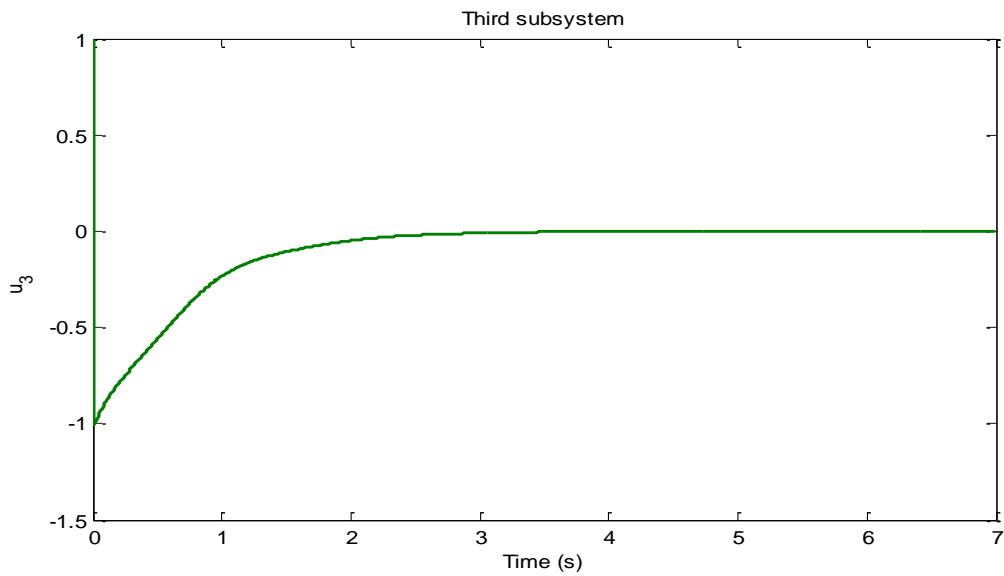

Fig. 3.b. Second subsystem

Fig. 3.c. Third subsystem
Figures 3. Temporal evolution of the control laws (H∞ control)
The resulting controller stabilizes the interconnected system with an average time of convergence of 5s order.
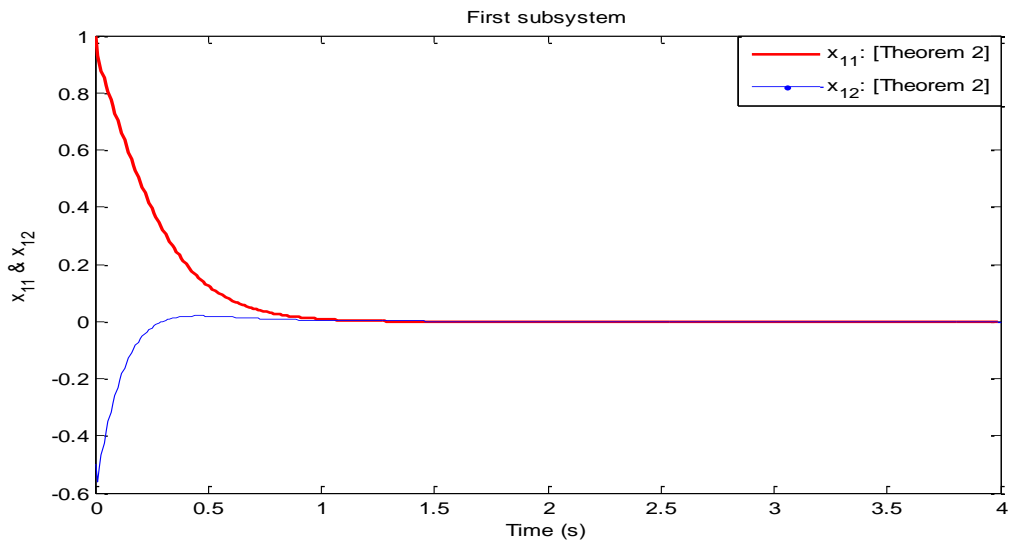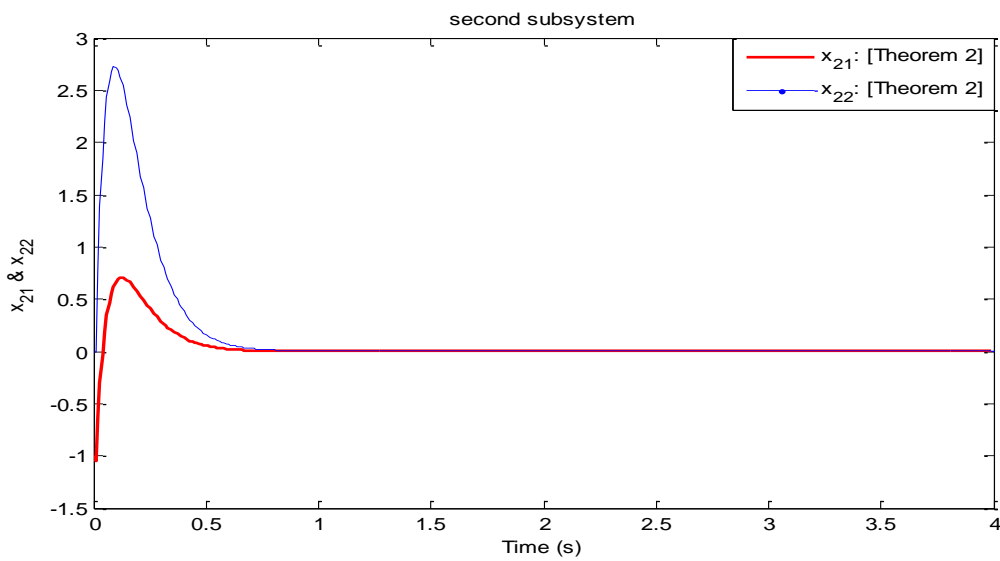
Fig. 4.a. First subsystem
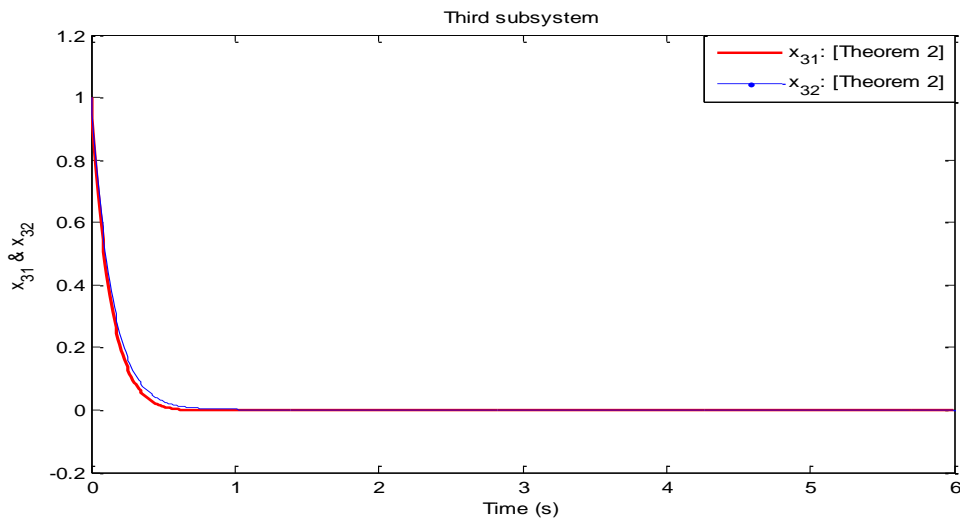

Fig. 4.b. Second subsystem

Fig. 4.c. Third subsystem
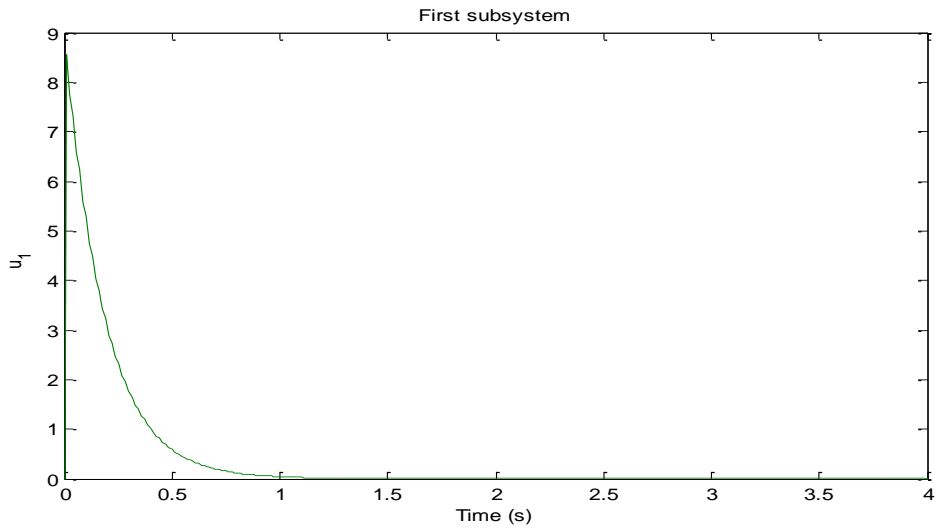**Figures 4. Temporal evolution of the state vector of interconnected subsystem with proposed method.**
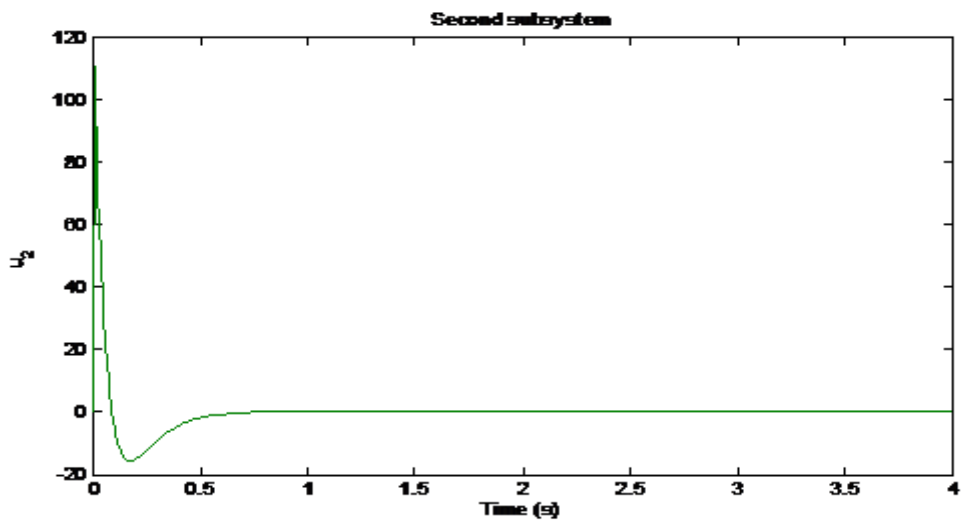
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Fig. 5.a. First subsystem
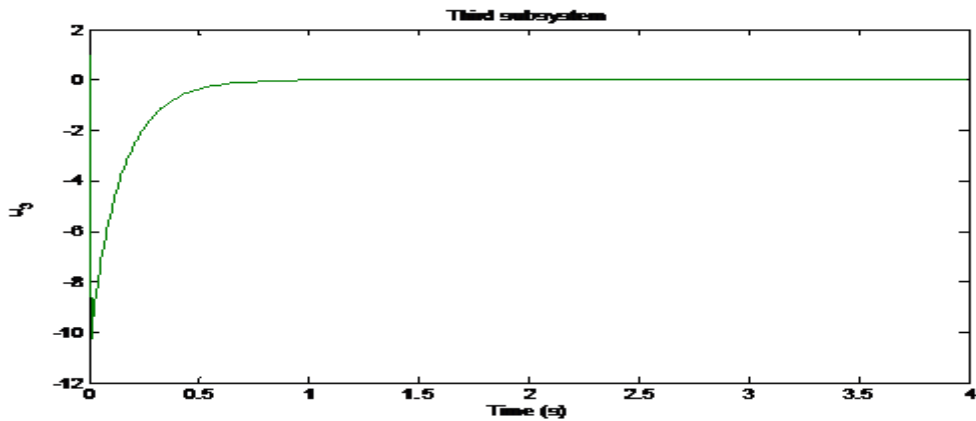

Fig. 5.b. Second subsystem

Fig. 5.c. Second subsystem
Figure 5. Temporal evolution of the control laws (proposed method)

We find that the resulting controller stabilizes the family of subsystems with an average time of convergence of the order of 1s. We note here that each subsystem has its own convergence time. Moreover, we can also note that the dynamics of the second subsystem is faster than the first and third. These results show the effectiveness of the proposed approach in the measure where the family of subsystems are correctly stabilized by the whole of the three laws of new decentralized H∞ control.

V. CONCLUSION

In this work, we proposed a new decentralized H∞ control for interconnected systems such as the closed-loop feedback subsystems are asymptotically stable. We used a Lyapunov function candidate and a H∞ criterion to minimize the impact of interconnections between subsystems, applying Finsler's lemma, the sufficient conditions ensuring stability closed loop were formulated in the terms of LMI. Finally, the obtained result are given to prove the efficiency the proposed method, and the graphics result shows that the system is well stabilized.

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