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# Steiner μ Distance in Fuzzy Graphs with Application

**G.Priscilla Pacifica, J.Jenit Ajitha**

***Abstract** : In this article we define Steiner and upper Steiner μ distances in connected fuzzy graphs by combining the notion of Steiner distance with μ distance and proved that both are metric. Also based on μ length, eccentricity, radius, diameter, diametric vertex, eccentric vertex, centre, convexity, self-centred graphs are introduced for both Steiner and upper Steiner μ distances . Some common characteristic properties are analysed and relation between Steiner and upper Steiner μ distances are discussed with an application. A model result is given for transport network.2010 AMS Classification: 05C72, 05C12*

***Keywords** : Fuzzy Steiner μ-distance, upper Fuzzy Steiner μ-distance, fuzzy Steiner $\mu_k$-eccentricity, upper fuzzy Steiner $\mu_k$-eccentricity.*

## I. INTRODUCTION

We often face unpredictability in many of our real life problems. So we need to consider fuzziness in every field. Rosenfeld developed the postulation of fuzzy graph theory in 1975. Although a fuzzy graph is similar in structure to that of a crisp graph, it better describes a real situation than a crisp graph and has some special characteristics. Steiner distance in crisp graphs and its properties were described in [3] and [10]. The properties of fuzzy graphs and their applications in various fields are studied from [1], [2], [4], [6], [7] and [8]. Some new distance parameters are introduced and examined in [5] and [9]. Here we introduce new parameters Steiner μ distance and upper Steiner μ distance in fuzzy graphs.

## II. PRILIMINARIES

Through out this article we consider only the connected fuzzy graphs G without loops and assume that V is finite and nonempty. Also we use the terms 'nodes' for vertices and 'arcs' for edges.

**Definition 2.1** If G is a connected graph with n nodes and S is a subset of V (G), then the Steiner distance among the nodes of S is defined as the minimum size among all connected minimal sub graphs whose node sets contain S. These sub graphs are called Steiner trees of S.

**Definition 2.2** The Steiner interval, $I_G(S)$ or I(S), of a set S is defined by $I_G(S) = \{w \in V(G) / w$ lies on a Steiner tree for S in G}

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**Definition 2.3** Let G be a connected crisp graph and S, the subset of nodes V (G), if I(S)=V(G) then S is called a Steiner set.

**Definition 2.4** A fuzzy graph is denoted by G (V, σ, μ) where V is a node set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ which satisfies $\mu(u,v) \leq \sigma(u) \wedge \sigma(v) \forall u,v \in V$.

**Definition 2.5** For a connected fuzzy graph, if P is the path: $u_0,u_1,\ldots,u_n$ then the μ−length of P, l(P) is the sum of reciprocals of arc weights . That is $l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1},u_i)}$ . If n = 0, then l(P) = 0.and μ− distance is the smallest μ−length of any u−v path.

## III. FUZZY STEINER μ DISTANCE

**Definition 3.1** Let G (V,σ,μ) be a connected fuzzy graph with n nodes. The Steiner μ distance between any two nodes of a non-empty set S ⊆ V (G) is defined as the minimum sum of reciprocals of arc weights of minimal connected fuzzy sub graphs containing S. These fuzzy sub graphs are called fuzzy Steiner trees for S. The fuzzy Steiner μ distance of S is denoted by $d_{\mu G}(S)$ (or) $d_{\mu S}(u,v)$ where u and v are nodes in S. For k = 2,3,….n we define the following parameters of fuzzy Steiner μ-distance.

**Definition 3.2** The fuzzy Steiner $\mu_k$-eccentricity $e_{\mu_k G}(u)$ of any node u is given below

$$e_{\mu_k G}(u) = \max\{d_{\mu S}(u,v) / S \subseteq V(G), |S| = k \;\&\; u,v \in S\}$$

**Definition 3.3** The fuzzy Steiner $\mu_k$ radius of any node u in G is

$$r_{\mu_k G}(G) = \min\{e_{\mu_k G}(u) / u \in V(G)\}$$

**Definition 3.4** The fuzzy Steiner $\mu_k$ diameter of a node u in V (G) is

$$diam_{\mu_k G}(G) = \max\{e_{\mu_k G}(u) / u \in V(G)\}$$

**Definition 3.5** A node u is a fuzzy Steiner $\mu_k$ diametral node (or) peripheral node if

$$e_{\mu_k G}(u) = diam_{\mu_k G}(G)$$

**Definition 3.6** The fuzzy Steiner $\mu_k$ centre $C_{\mu_k}(G)$ of a fuzzy graph G is the fuzzy subgraph induced by the nodes u of V(G) with $e_{\mu_k G}(u) = r_{\mu_k G}(G)$.These nodes are called fuzzy Steiner $\mu_k$ central nodes (or) fuzzy Steiner $\mu_k$ eccentric nodes.

**Definition 3.7** The fuzzy Steiner $\mu_k$ median of G is the fuzzy subgraph of G induced by the nodes of minimum fuzzy Steiner $\mu_k$ distance in G.

**Definition 3.8** The fuzzy average Steiner $\mu_k$ distance of a graph G, is defined as the average of the fuzzy Steiner μ-distances of all k-subsets of V (G).

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**Definition 3.9** A subset of nodes S is called fuzzy Steiner $\mu_k$ convex if each vertex in any Steiner fuzzy tree of S is contained in S.

**Definition 3.10** A fuzzy graph G(V,σ,μ) is said to be fuzzy Steiner $\mu_k$ self-centred graph if the fuzzy Steiner $\mu_k$ eccentricity of every node of the graph is the same (or) if fuzzy Steiner $\mu_k$ radius and fuzzy Steiner $\mu_k$ diameter of the graph are equal.

**Definition 3.11** The fuzzy Steiner $\mu_k$ interval, $I_{\mu G}(S)$ of a set $S \subseteq V(G)$ with cardinality k is defined by $I_{\mu G}(S)$ = {w ∈ V (G) / w lies on a Steiner fuzzy tree for S in G}

**Definition 3.12** Let G be a connected fuzzy graph and S, a subset of nodes V (G), if $I_{\mu G}(S)$=V(G), then S is called a Steiner fuzzy set.

**Example 1** For example consider the following fuzzy graph

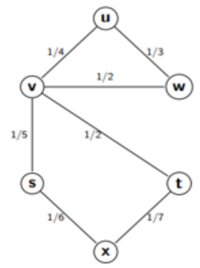
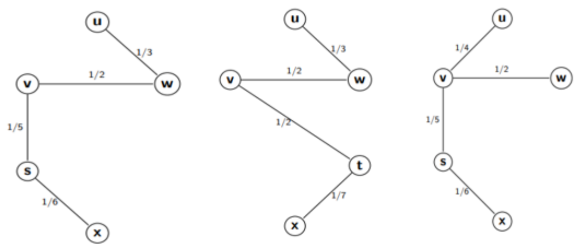



**Figure 1**



T1 T2 T3

**Figure 2**

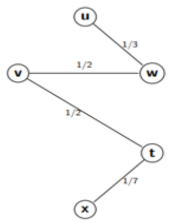


**Figure 3**

Let S={ u, v, w, x}.Then the Steiner fuzzy trees for S are given in Fig 2

Here the set S is a Steiner fuzzy set since every node of the graph is in a Steiner fuzzy tree of S. The fuzzy μ-distances for the above fuzzy trees are given in table 1.

**Table 1**

| Steiner fuzzy trees | fuzzy μ-distance | Total |
|---|---|---|
| $T_1$ | 3+2+5+6 | 14 |
| $T_2$ | 3+2+2+7 | 14 |
| $T_3$ | 4+2+5+6 | 15 |

**Table 2**

| Node set | | Fuzzy Steiner μ distance ($d_{\mu G}$) |
|---|---|---|
| $S_1$ | {u, v, w, x, t} | 14 |
| $S_2$ | { u, v, s, x, t} | 17 |
| $S_3$ | { u, v, s, w, t} | 12 |
| $S_4$ | { u, v, s, w, x} | 16 |
| $S_5$ | { v, w, s, x, t} | 17 |
| $S_6$ | {u, s, w, x, t} | 16 |

**Table 3**

| Nodes | Fuzzy Steiner $\mu_5$−eccentricities | $e_{\mu_5 G}$ |
|---|---|---|
| u | max{ 14,17,12,16,16} | 17 |
| v | max{ 14,17,12,16,17} | 17 |
| w | max{ 14,12,16,16,17} | 17 |
| s | max{17,12,16,16,17 } | 17 |
| t | max{ 14,17,12,16,17} | 17 |
| x | max{ 14,17,16,16,17} | 17 |

The fuzzy Steiner μ distance of S is,

$d_{\mu G}(S) = \min\{14,14,15\}$ =14. Here $T_1$ and $T_2$ are minimum Steiner fuzzy trees for S.

Take k=5. Let $S_1$={u, v, w, x, t} then $d_{\mu G}(S_1) = 14$ .The minimum Steiner fuzzy tree for $S_1$ is given in fig 3.

Similarly the fuzzy Steiner μ distances of $S_1$, $S_2$, $S_3$, $S_4$, $S_5$, $S_6$ are given in table 2.

The fuzzy Steiner $\mu_5$−eccentricities are given in table 3.

The fuzzy Steiner $\mu_5$ eccentricity of every node is same. Hence the above fuzzy graph is fuzzy Steiner $\mu_5$ self centered.

The fuzzy average Steiner $\mu_5$ distance is $\frac{14+17+12+16+16+17}{5} = 18.4$.

## IV. PROPERTIES OF FUZZY STEINER μ DISTANCE

**A. *Theorem*** *The fuzzy Steiner μ -distance of a non-empty set of nodes S on a connected fuzzy graph G, (V, $d_{\mu S}(u,v)$) where $u, v \in S$ is a metric.(i.e)*

i) $d_{\mu S}(u,v) \geq 0$ for all u, v and for all subsets $S \subseteq V(G)$

ii) $d_{\mu S}(u,v) = 0$ iff $u = v$

iii) $d_{\mu S}(u,v) = d_{\mu S}(v,u)$

iv) $d_{\mu S}(u,w) \leq d_{\mu S_1}(u,v) + d_{\mu S_2}(v,w)$ where $u \in S \,\&\, S_1$ , $w \in S \,\&\, S_2$ , $v \in S_1 \,\&\, S_2$ and $S_1$ , $S_2$ and $S$ are subsets of V(G) such that $|S_1|=|S_2|=|S|=k$

***Proof*** **:**The first three results (i),(ii) and (iii) are true from the definition of Steiner μ-distance of S. It remains to show that the triangle inequality holds. Let $S_1$ and $S_2$ be two non-empty subsets of V (G) such that $|S_1|=|S_2|=|S|=k$. Clearly the result is true if any two of the nodes are equal. Now choose distinct nodes u,v and w as in the hypothesis. The Steiner μ-distance $d_{\mu G}(S)$ is the same for any two nodes in S. Also since $|S_1|=|S_2|=|S|=k$, the Steiner μ-distance of S is at most $d_{\mu S_1}(u,v) + d_{\mu S_2}(v,w)$.Hence fuzzy Steiner μ -distance is a metric.

**B. *Remark*** *The triangle inequality holds only if $|S_1|=|S_2|=|S|$.*

**C. *Observation*** *For a fuzzy graph with n nodes, a Steiner fuzzy set which has less than n vertices is not fuzzy Steiner $\mu_k$ convex where k = 2,3,….,n*

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**D. *Observation*** *Obviously* $e_{\mu_k G}(u) > d_{\mu S}(u,v)$ *for all nodes u,v in* $V(G)$ *where* $d_{\mu S}(u,v)$ *is with respect to any non-empty fuzzy subset* $S \subseteq V(G)$ *and k = 2,3,….n where n denotes the number of nodes.*

**E. *Theorem*** *For a connected fuzzy graph G with n nodes where* $n \geq 3$ *and k = 3,4,….,n, if u and v are distinct nodes then* $|e_{\mu_k G}(u) - e_{\mu_k G}(v)| \leq d_{\mu S}(u,v)$ *where* $S \subseteq V(G)$ *with |S|=k and* $u, v \in S$.

***Proof*:** Let v be a node. The fuzzy Steiner $\mu_k$ eccentricity of v is the maximum fuzzy Steiner μ-distance of any non-empty set of nodes with cardinality k that contains v say S. Let w be a node with such maximum fuzzy Steiner μ-distance with respect to the set S with cardinality k.(i.e) $e_{\mu_k G}(v) = d_{\mu S}(w,v)$. Suppose u is a node other than v in S. Since fuzzy Steiner μ-distance is a metric, by triangle inequality we have

$e_{\mu_k G}(v) = d_{\mu S}(w,v) \leq d_{\mu S}(w,u) + d_{\mu S}(u,v)$ . Also $d_{\mu S}(w,u) \leq e_{\mu_k G}(u)$.

Hence $e_{\mu_k G}(v) \leq e_{\mu_k G}(u) + d_{\mu S}(u,v)$.

$\Rightarrow e_{\mu_k G}(v) - e_{\mu_k G}(u) \leq d_{\mu S}(u,v)$ . Similarly if we interchange the nodes u and v, we get $e_{\mu_k G}(u) - e_{\mu_k G}(v) \leq d_{\mu S}(u,v)$.From these two inequalities we obtain the required inequality.

**F. *Theorem*** *For a connected fuzzy graph G,* $r_{\mu_k G}(G) \leq diam_{\mu_k G}(G) \leq 2r_{\mu_k G}(G)$

***Proof:*** It follows from definition that $r_{\mu_k G}(G) \leq diam_{\mu_k G}(G)$ for any k. Let u and w be two nodes with $d_{\mu S}(u,w) = diam_{\mu_k G}(G)$ where S is a non-empty set of nodes with |S|=k. Let v be a node with $r_{\mu_k G}(G) = e_{\mu_k G}(v)$. Since fuzzy Steiner μ-distance is a metric by triangle inequality, we have

$diam_{\mu_k G}(G) = d_{\mu S}(u,w) \leq d_{\mu S_1}(u,v) + d_{\mu S_2}(v,w)$ where $|S_1|=|S_2|=|S|=k$, $S_1$ and $S_2$ are chosen such that $u, v \in S_1$ & $v, w \in S_2$. Now from observation D $d_{\mu S_1}(u,v) \leq e_{\mu_k G}(v)$ and $d_{\mu S_2}(v,w) \leq e_{\mu_k G}(v)$ .Thus $diam_{\mu_k G}(G) \leq e_{\mu_k G}(v) + \leq e_{\mu_k G}(v) = r_{\mu_k G}(G) + r_{\mu_k G}(G) = 2r_{\mu_k G}(G)$

Hence the result.

**G. *Observation*** *For any node u of a connected fuzzy graph G,* $diam_{\mu_k G}(G) - e_{\mu_k G}(u) \geq m$ *where m is any arbitrary non negative real number.*

If G is a connected fuzzy graph then for node u, $r_{\mu_k G}(G) \leq e_{\mu_k G}(u) \leq diam_{\mu_k G}(G)$ .Therefore $diam_{\mu_k G}(G) - e_{\mu_k G}(u) \geq m$ *where* $m \geq 0$

**H. *Theorem*** *For a connected fuzzy graph G, if S,* $S_1$ *and* $S_2$ *are non empty subsets of vertices, then*

$|d_{\mu S_1}(u,x) - d_{\mu S_2}(x,v)| \leq d_{\mu S}(u,v)$ *where* $u \in S$ & $S_1$, $v \in S$ & $S_2$, $x \in S_1$ & $S_2$.

The above theorem follows from triangle inequality in theorem A

**I. *Theorem*** *If a connected fuzzy graph G is Steiner* $\mu_k$ *-self-centred, then every node of G is fuzzy Steiner* $\mu_k$ *eccentric.*

***Proof:*** Suppose G is a connected fuzzy graph which is Steiner $\mu_k$ -self-centred. Then for every node G, the fuzzy Steiner $\mu_k$ eccentricity is the same. Let v be the fuzzy Steiner $\mu_k$ eccentric node of u.(i.e) $e_{\mu_k G}(u) = d_{\mu S}(u,v)$ where S is a non-empty set with cardinality k. Now since G is fuzzy Steiner $\mu_k$ -self-centred we have,

$e_{\mu_k G}(u) = e_{\mu_k G}(v) = d_{\mu S}(u,v)$. Hence u is a fuzzy Steiner $\mu_k$ eccentric node of v. Therefore every node is fuzzy Steiner $\mu_k$ eccentric.

**J. *Remark*** *If a connected fuzzy graph G is Steiner* $\mu_k$*-self-centred and for every node u such that u is a Steiner* $\mu_k$ *eccentric node of another node v, then v should be one of the Steiner* $\mu_k$ *eccentric nodes of u.*

## V. FUZZY UPPER STEINER μ DISTANCE

**Definition 5.1** For a connected fuzzy graph with n nodes the fuzzy upper Steiner μ-distance between any two nodes of a non-empty set S ⊆ V (G) is defined as the maximum sum of reciprocals of arc weights of minimal connected fuzzy sub graphs containing S. The corresponding steiner fuzzy tree is called the maximum steiner fuzzy tree of S. The fuzzy upper Steiner μ distance of S is denoted by $D_{\mu G}(S)$ (or) $D_{\mu S}(u,v)$ where u and v are nodes in S. For k = 2,3,….n, we define the following parameters of fuzzy upper Steiner μ-distance.

**Definition 5.2** The fuzzy upper Steiner $\mu_k$-eccentricity $E_{\mu_k G}(u)$ for a node u in V (G) is

$$E_{\mu_k G}(u) = \max\{D_{\mu S}(u,v)/S \subseteq V(G), |S| = k \ \& \ u, v \in S\}$$

**Definition 5.3** The fuzzy upper Steiner $\mu_k$ radius of any node u in G is given by

$$R_{\mu_k G}(G) = \min\{E_{\mu_k G}(u)/ \ u \in V(G)\}$$

**Definition 5.4** The fuzzy upper Steiner $\mu_k$ diameter of a node u in V (G) is given by

$$DIAM_{\mu_k G}(G) = \max\{E_{\mu_k G}(u)/ \ u \in V(G)\}$$

**Definition 5.5** A node u is a fuzzy upper Steiner $\mu_k$ diametral node (or) peripheral node if

$$E_{\mu_k G}(u) = DIAM_{\mu_k G}(G)$$

**Definition 5.6** The fuzzy upper Steiner $\mu_k$ centre $C_{\mu_k}(G)$ of a connected fuzzy graph G is the fuzzy subgraph induced by the nodes u of V(G) with $E_{\mu_k G}(u) = R_{\mu_k G}(G)$.The node u is called fuzzy upper Steiner $\mu_k$ central node (or) fuzzy upper Steiner $\mu_k$ eccentric node.

**Definition 5.7** The fuzzy upper Steiner $\mu_k$ median of a connected fuzzy graph G is the fuzzy subgraph of G induced by the nodes of minimum fuzzy upper Steiner μ distance of a k-subset in G.

**Definition 5.8** The fuzzy upper average Steiner $\mu_k$ distance of a graph G, is defined as the average of the fuzzy upper Steiner μ-distances of all k-subsets of V (G).

**Definition 5.9** A fuzzy graph G is said to be fuzzy upper Steiner $\mu_k$ self-centred graph if the fuzzy upper Steiner $\mu_k$ eccentricity of every node of the graph is the same (or) if

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fuzzy upper Steiner $\mu_k$ radius and fuzzy upper Steiner $\mu_k$ diameter of the graph are equal.

**Example 2** Let us consider the following fuzzy graph

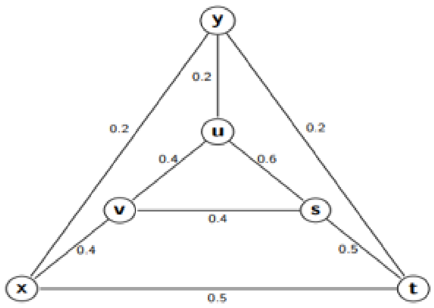


**Figure 4**

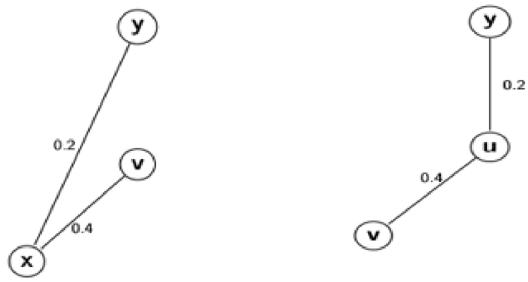

**Figure 5**

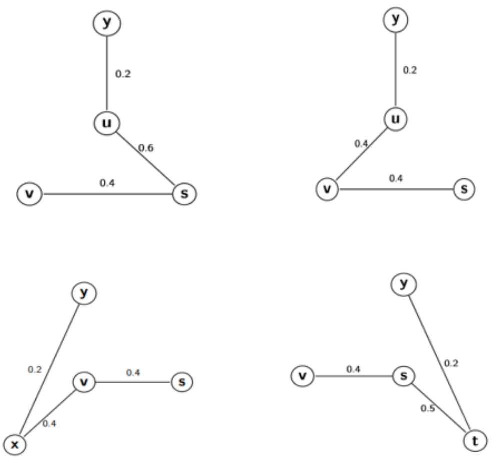

**Figure 6**

| Nodes | Fuzzy upper Steiner $\mu_2$−eccentricities | $E_{\mu_2 G}$ |
|---|---|---|
| u | max{1.6,5,2.5,10,10} | 10 |
| s | max{1.6,2.5,7,2.5} | 7 |
| v | max{2.5,2.5,5,4.5,7.5} | 7.5 |
| y | max{5,7.5,5,7.5} | 7.5 |
| t | max{2,10,2.5,4.5} | 10 |
| x | max{2,5,5,10} | 10 |

**Table 4**

| Nodes | Fuzzy upper Steiner $\mu_3$−eccentricities | $E_{\mu_3 G}$ |
|---|---|---|
| u | max{4.1,7.5,7.6,7.5,12,10,3.6,12.5,5,10} | 12.5 |
| s | max{4.1,10,7.6,7,4.5,7.5,4,10,3.6,5} | 10 |
| v | max{4.1,10,9.5,7.5,4.5,4.5,7.5,5,12.5,5} | 12.5 |
| y | max{10,10,7.5,10,7,7.6,7.5,10,9.5,10} | 10 |
| t | max{9.5,10,7,4.5,12,4,4.5,3.6,12.5,10} | 12.5 |
| x | max{10,7.5,12,4,4.5,10,7.5,10,5,5} | 12 |

**Table 5**

In the fuzzy graph given in fig 4, for a 2-subset S={v,y} the Steiner fuzzy trees given in fig 5 are both minimum and maximum Steiner fuzzy trees. The fuzzy Steiner µ -distance and fuzzy upper Steiner µ -distance of S are same.

$$d_{\mu G}(S) = D_{\mu G}(S) = \frac{1}{0.2} + \frac{1}{0.4} = 7.5$$

The fuzzy upper Steiner $\mu_2$ eccentricities for 2-subsets are given in table 4.

The fuzzy upper Steiner $\mu_2$ radius is

$$R_{\mu_k G}(G) = \min\{10,7,7.5,7.5,10,10\} = 7$$

The fuzzy upper Steiner $\mu_2$ diameter is

$$DIAM_{\mu_k G}(G) = \max\{10,7,7.5,7.5,10,10\} = 10$$

The only fuzzy upper Steiner $\mu_2$ eccentric node is s.

Fuzzy upper Steiner $\mu_2$ diametral nodes are u, x and t.

For a 3-subset, T={s,v,y} the Steiner fuzzy trees are given in fig 6. The fuzzy Steiner µ distance and fuzzy upper Steiner µ -distance of T are

$$d_{\mu G}(T) = \frac{1}{0.2} + \frac{1}{0.4} + \frac{1}{0.6} = 9.1,$$

$$D_{\mu G}(S) = \frac{1}{0.2} + \frac{1}{0.4} + \frac{1}{0.4} = 10$$

The fuzzy upper Steiner $\mu_3$ eccentricities for k = 3 subsets are given in table 5.

The fuzzy upper Steiner $\mu_3$ radius is

$$R_{\mu_3 G}(G) = \min\{10,12.5,12\} = 10$$

The fuzzy upper Steiner $\mu_3$ diameter is

$$DIAM_{\mu_3 G}(G) = \max\{10,12.5,12\} = 12.5$$

Fuzzy upper Steiner $\mu_3$ eccentric nodes are s and y.

Fuzzy upper Steiner $\mu_3$ diametral nodes are u,v and t.

## VI. PROPERTIES OF FUZZY UPPER STEINER µ DISTANCE

The fuzzy upper Steiner µ distance has the same characteristic properties as that of fuzzy Steiner µ distance.The fuzzy upper Steiner µ -distance of a non-empty set of nodes $S \subseteq V(G)$ on a connected fuzzy graph G(V,σ,µ), (V, $D_{\mu S}(u,v)$) where $u, v \in S$ is a metric.(i.e).The triangle inequality holds only if $|S_1|=|S_2|=|S|$.

**A.** *For* $n \geq 3$ *and k = 3,4,….,n, if u and v are distinct nodes then* $|E_{\mu_k G}(u) - E_{\mu_k G}(v)| \leq D_{\mu S}(u,v)$ *where* $S \subseteq V(G)$ *with |S|=k and* $u, v \in S$.

**B.** $E_{\mu_k G}(u) > D_{\mu S}(u,v)$ *for all nodes u,v in* $V(G)$ *where* $D_{\mu S}(u,v)$ *is with respect to any non-empty fuzzy subset* $S \subseteq V(G)$ *and k = 2,3,….n and n is the number of nodes.*

**C.** *If G is a connected fuzzy graph*

$$R_{\mu_k G}(G) \leq DIAM_{\mu_k G}(G) \leq 2R_{\mu_k G}(G)$$

**D.** *For every node u of a connected fuzzy graph G(V,σ,µ),*

$$DIAM_{\mu_k G}(G) - E_{\mu_k G}(u) \geq m$$

where m is any arbitrary non negative real number.

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**E.** *If a connected fuzzy graph G is upper Steiner $\mu_k$-self-centred, then every node of G is fuzzy upper Steiner $\mu_k$ eccentric.*

**F.** *If a connected fuzzy graph G is upper Steiner $\mu_k$-self-centred and for every node u such that u is a upper Steiner $\mu_k$ eccentric node of another node v, then v should be one of the upper Steiner $\mu_k$ eccentric nodes of u.*

## VII. RELATION BETWEEN FUZZY STEINER μ DISTANCE AND FUZZY UPPER STEINER μ DISTANCE

**A.** ***Observation:*** *The following inequality is obvious from the definition of upper Steiner μ -distance for any connected fuzzy graph G and any non empty subset of nodes S of V(G).*

$1 \leq d_{\mu G}(S) \leq D_{\mu G}(S) < \infty.$

**B.** ***Theorem***

For every connected fuzzy graph G for which $r_{\mu_k G}(G) = a$ and $R_{\mu_k G}(G) = b$ where a,b are any real numbers, then $1 \leq a \leq b < \infty$.

***Proof :*** Let S be a non-empty subset of nodes with cardinality k. From Observation A (VII), $d_{\mu G}(S) \leq D_{\mu G}(S)$. $1 \leq a$ and $b < \infty$ are obvious.

Let $r_{\mu_k G}(G) = a$ and $R_{\mu_k G}(G) = b$

$$a = r_{\mu_k G}(G) = \min\{e_{\mu_k G}(u)\backslash u \in V(G)\}$$
$$= min_{u \in V(G)}\{\max\{d_{\mu S}(u,v)\backslash S \subseteq V(G), |S| = k \,\&\, u,v \in S\}$$
$$\leq min_{u \in V(G)}\{\max\{D_{\mu S}(u,v)\backslash S \subseteq V(G), |S| = k \,\&\, u,v \in S\}$$
$$= \min\{E_{\mu_k G}(u)\ u \in V(G)\}$$
$$= R_{\mu_k G}(G) = b.$$

**C.** ***Theorem***

For every connected fuzzy graph G for which $diam_{\mu_k G}(G) = a$ and $DIAM_{\mu_k G}(G) = b$ where a,b are any real numbers, then $1 \leq a \leq b < \infty$.

***Proof:*** Let S be a non-empty subset of nodes with cardinality k. From Observation A(VII), $d_{\mu G}(S) \leq D_{\mu G}(S)$. Let $diam_{\mu_k G}(G) = a$ and $DIAM_{\mu_k G}(G) = b$. Clearly $1 \leq a$ and $b < \infty$ are true.

$$a = diam_{\mu_k G}(G) = \max\{e_{\mu_k G}(u)\backslash u \in V(G)\}$$
$$= max_{u \in V(G)}\{\max\{d_{\mu S}(u,v)\backslash S \subseteq V(G), |S| = k \,\&\, u,v \in S\}$$
$$\leq max_{u \in V(G)}\{\max\{D_{\mu S}(u,v)\backslash S \subseteq V(G), |S| = k \,\&\, u,v \in S\}$$
$$= \max\{E_{\mu_k G}(u)\ u \in V(G)\}$$
$$= DIAM_{\mu_k G}(G) = b$$

## VIII. APPLICATION

The fuzzy Steiner μ distance has application in many communication networks.

**Table 6**

| Roadways Between Two Locations | | | Dist. in km | Vehicles Passing in an Hour | Arc Weight |
|---|---|---|---|---|---|
| $R_1$ | Market | Bustand | 0.5km | 100 | 0.33 |
| $R_2$ | Bustand | Hospital | 3.5km | 250 | 0.83 |
| $R_3$ | Market | Hospital | 3.8km | 230 | 0.77 |
| $R_4$ | Police station | Market | 0.3km | 67 | 0.22 |
| $R_5$ | Police station | Fun Mall | 1.8km | 112 | 0.37 |
| $R_6$ | Mall | Harbour | 1km | 98 | 0.32 |
| $R_7$ | Harbour | College | 5km | 80 | 0.27 |
| $R_8$ | Hospital | College | 2.3km | 300 | 1 |
| $R_9$ | School | College | 2.6km | 280 | 0.93 |
| $R_{10}$ | Bustand | Harbour | 1.2km | 200 | 0.67 |

We can consider a traffic network problem.There are 8 major locations in an area such as bustand,hospital, mall, school, college, harbour,market and police station.There are roadways connecting these facilities.A sample data is given in table 6.This can be modelled into a fuzzy graph with locations as nodes and the roadways connecting the facility locations as arcs given in fig 7. Arc weight for roadway $R_8$ with maximum number of vehicles(300) is given the maximum membership value 1.For the other roadways,

$$Arc\ weight = \frac{Number\ of\ vehicles}{300}$$

Let S={bustand,hospital,college,harbour}.The minimum Steiner fuzzy tree of S which gives the Steiner μ -distance is the shortest route that connecting all the locations in S with more traffic shown in fig 8.The maximum Steiner fuzzy tree of S which gives the upper Steiner μ -distance is the shortest route that connecting the locations in S with less traffic given in fig 9.

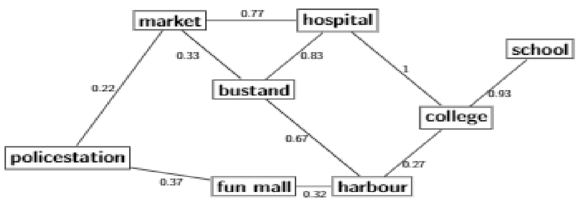


**Figure 7**

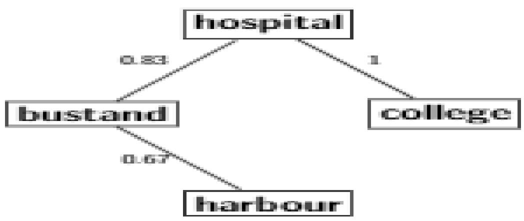


**Figure 8**

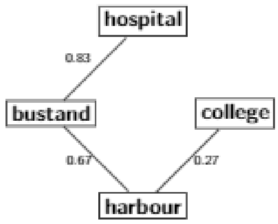


**Figure 9**

Steiner µ -distance of S=3.69
Upper Steiner µ -distance of S=6.39.

## IX. RESULT AND DISCUSSION

Another real life application of Steiner and upper Steiner µ distances is a message transmitting network. Suppose a message transmitting network with processors, can be modelled into a fuzzy graph with processors as nodes and link between two processors as arcs and message transmitting speed between the processors as arc weights. The reciprocal of arc weight denotes the message flow rate between the processors. Here the Steiner µ distance of some k processors gives the net message flow rate of minimal fast communicating link containing these processors and fuzzy upper Steiner µ distance gives the net message flow rate of minimal slow communicating link containing these k processors.

## X. CONCLUSION

In this article we have introduced and scrutinized the similar properties of Steiner µ distance and upper Steiner µ distances in fuzzy graphs by merging the Steiner distance and µ distance with a real life application. These distances also have application background in various communication networks like telecommunication networks, radio stations message transmitting networks, channel networks, radio stations, transports, wireless mobile Ad-hoc networks..etc. and are used in network analysis.

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