Yale

Mixed Modes and the Asteroseismic Surface Term

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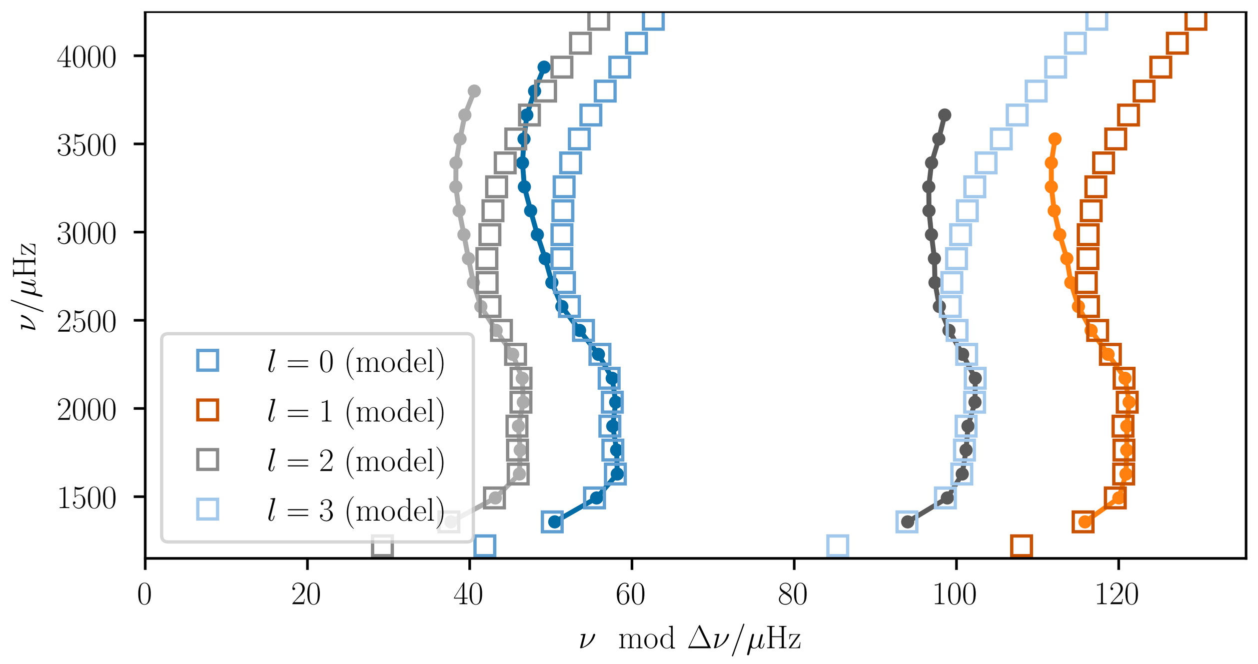
Introduction

- Normal modes of stellar oscillations computed from stellar models exhibit systematic deviations compared to those obtained observationally (fig. 1). These deviations must be corrected when mode frequencies are used in stellar modelling.

Figure 1: Echelle diagram showing observational oscillation frequencies of the Sun, along with those of a standard solar model with matching interior structure.

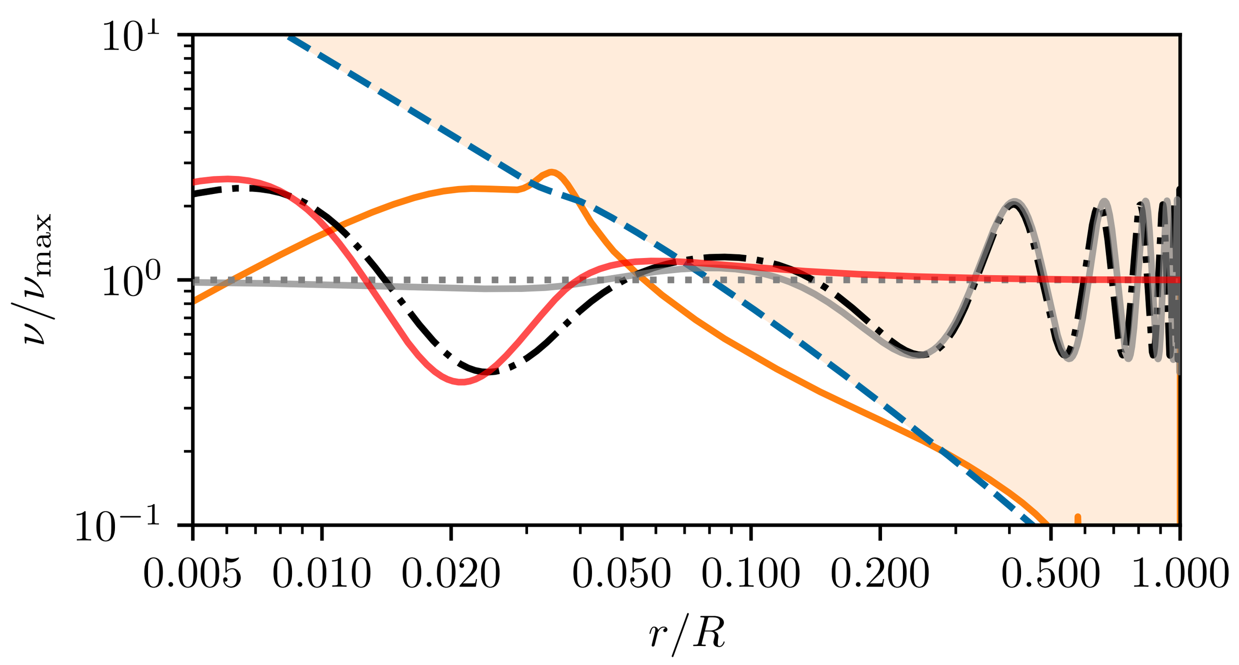


- Evolved stars exhibit **mode mixing** between the solar-like p-modes and an interior g-mode cavity, confined to the radiative interior. This mode mixing interferes with surface-term corrections proposed in the literature, and has not hitherto been investigated analytically in this context.

Coupling Matrices for Mixed Modes

- Ong & Basu (2020) propose a decomposition of mixed modes into pure p-like π modes, and pure g-like γ-modes (fig. 2), with associated coupling matrices.

Figure 2: Schematic representation of a mixed mode wavefunction (black) in a subgiant MESA model, as a linear combination of those of a π-mode (grey) and a γ-mode (red)



- Under the action of a perturbation to the stellar structure, the perturbed mixed-mode frequencies satisfy the Generalised Hermitian Eigenvalue Problem (Ong et al., 2021):

$$\left(\begin{bmatrix} -\Omega_\pi^2 & R_{\pi\gamma} \\ R_{\pi\gamma}^\dagger & -\Omega_\gamma^2 \end{bmatrix} + \lambda \mathbf{V}\right)\xi = -\omega^2 \begin{bmatrix} 1 & D_{\pi\gamma} \\ D_{\pi\gamma}^\dagger & 1 \end{bmatrix}\xi, \quad (1)$$

where $\lambda \in [0, 1]$ is a small parameter describing the strength of the perturbation.

- If the perturbation is localised to the surface, as is the case with the surface term, then $\mathbf{V}$ vanishes on the γ-mode subspace.

Perturbation Analysis for Mixed Modes

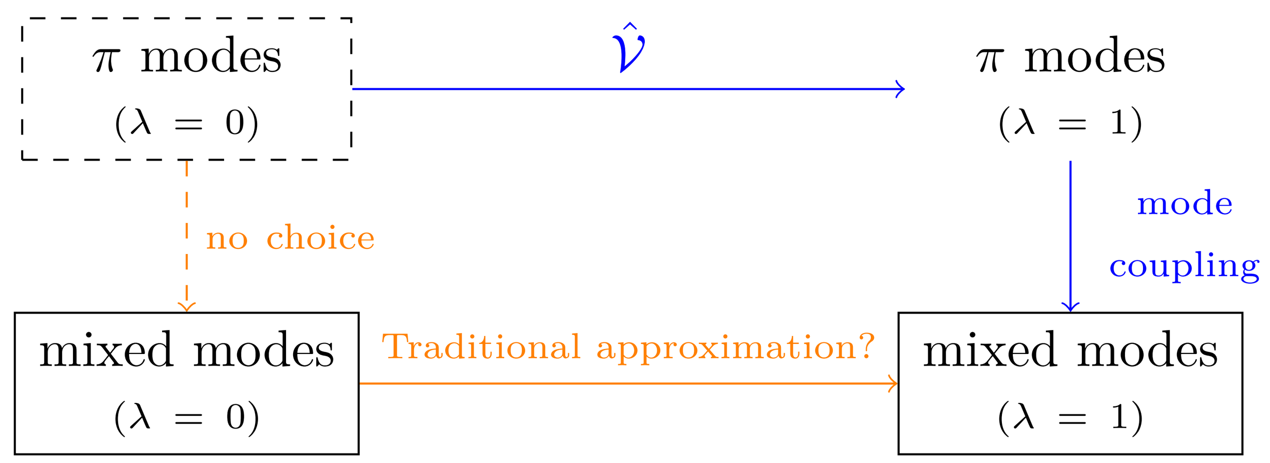
- In general, the perturbed eigenvalues can be written in powers of λ as

$$\omega_{nlm}^{\mathrm{surf}} = \omega_{nlm}^{\mathrm{model}} + \lambda\omega_{nlm}^{(1)} + \lambda^2\omega_{nlm}^{(2)} + \lambda^3\omega_{nlm}^{(3)} + \ldots \quad (2)$$

Traditional treatments of mode coupling, of the form $\delta\omega_{nlm} \sim f(\omega_{nlm})/I_{nlm}$, can be shown to **truncate this series expansion to leading order**.

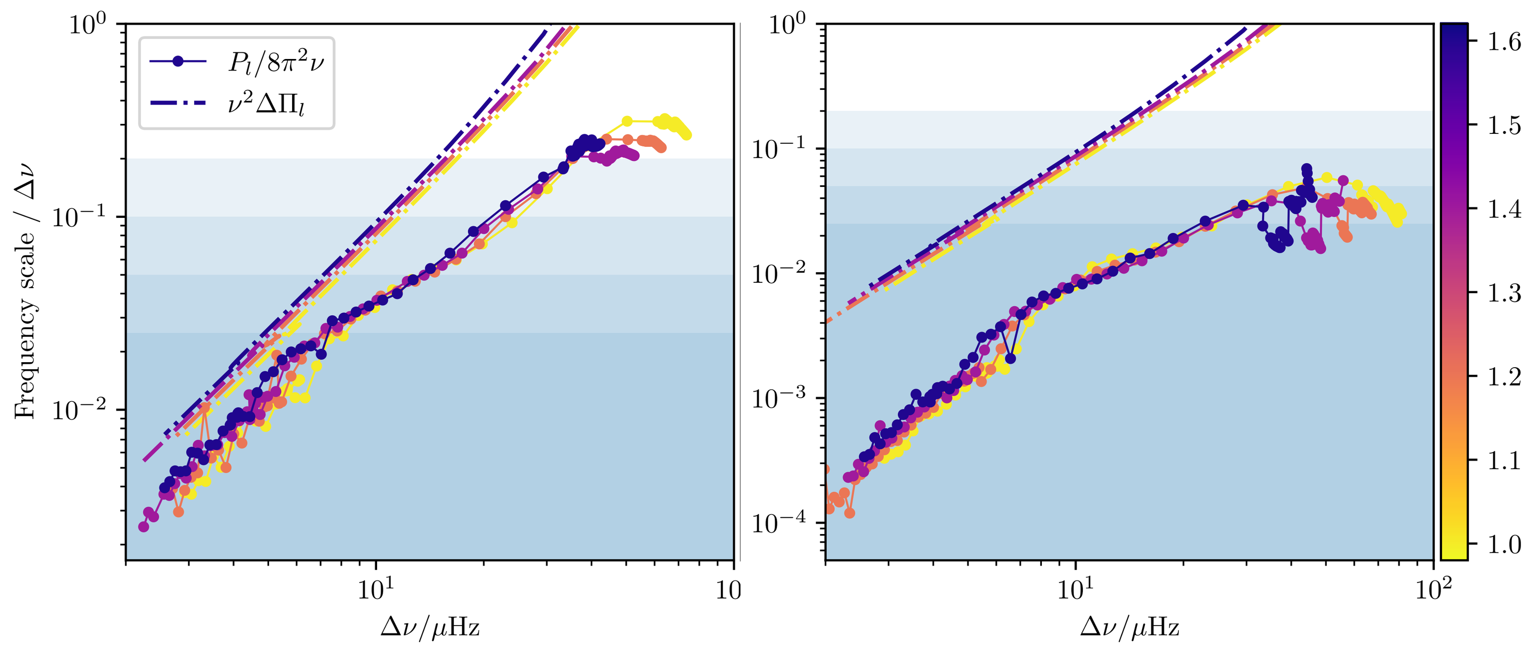
- Traditional treatments of mode coupling in the surface term also apply **two noncommuting operations in the incorrect order** (fig. 3).

Figure 3: Schematic comparison of coupling-matrix surface-term corrections (blue) vs. traditional ones operating directly on model mixed modes (orange). This diagram does not commute.



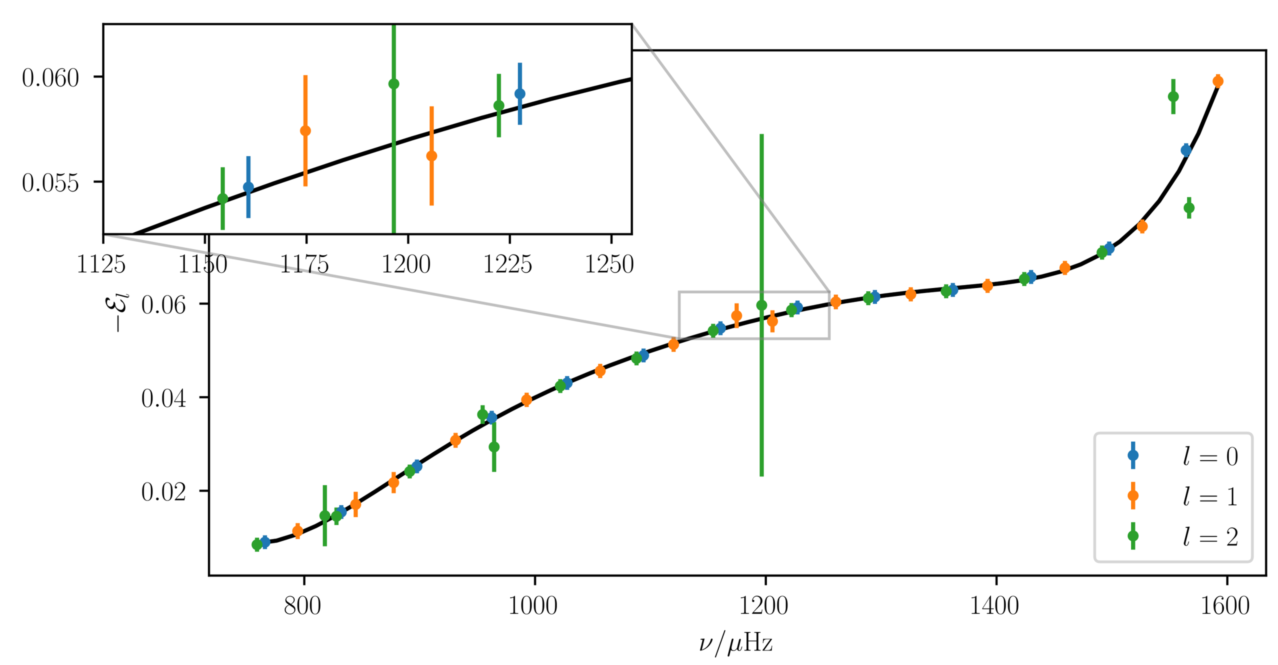
- We show that these operations approximately commute **only to leading order in λ**
- **Traditional approximations to the surface term may only be applied if the series expansion in powers of λ converges.** In Ong et al. (2021), we derive criteria under which power series of the form of eq. (2) may converge (fig. 4).

Figure 4: Local g-mode spacing (dot-dashed lines) and coupling strengths (solid lines with filled points) near ν_{max} for a sequence of solar-composition MESA evolutionary tracks at various stellar masses. Traditional approximations may only be applied when the surface term is much smaller than both of these quantities.



- We also derive generalised surface term corrections for mixed modes (e.g. fig. 5).

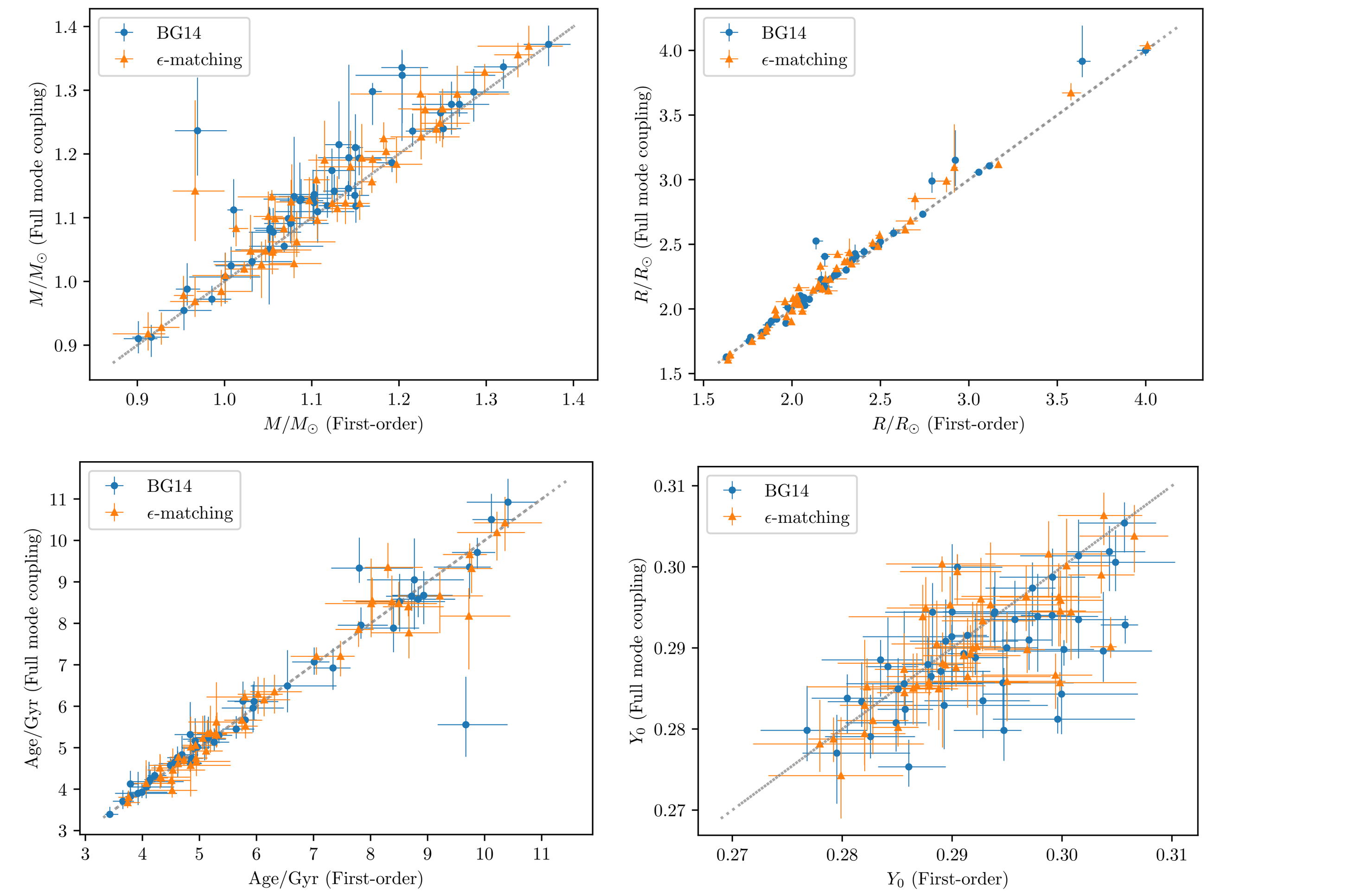
Figure 5: Generalised ϵ_l-matching algorithm, based on the prescription of Roxburgh (2016), applied to an artificial surface term perturbing mixed modes in a subgiant model. From Ong et al. (2021).



Population-level systematic biases

- We derived estimates of global properties (e.g. masses, radii, ages, Y_0) with respect to different treatments of the surface term, for a sample of subgiants observed with *Kepler*, *K2*, and *TESS*.

Figure 6: Comparison of inferred values using different approaches to mode coupling, under two different prescriptions for surface-term correction

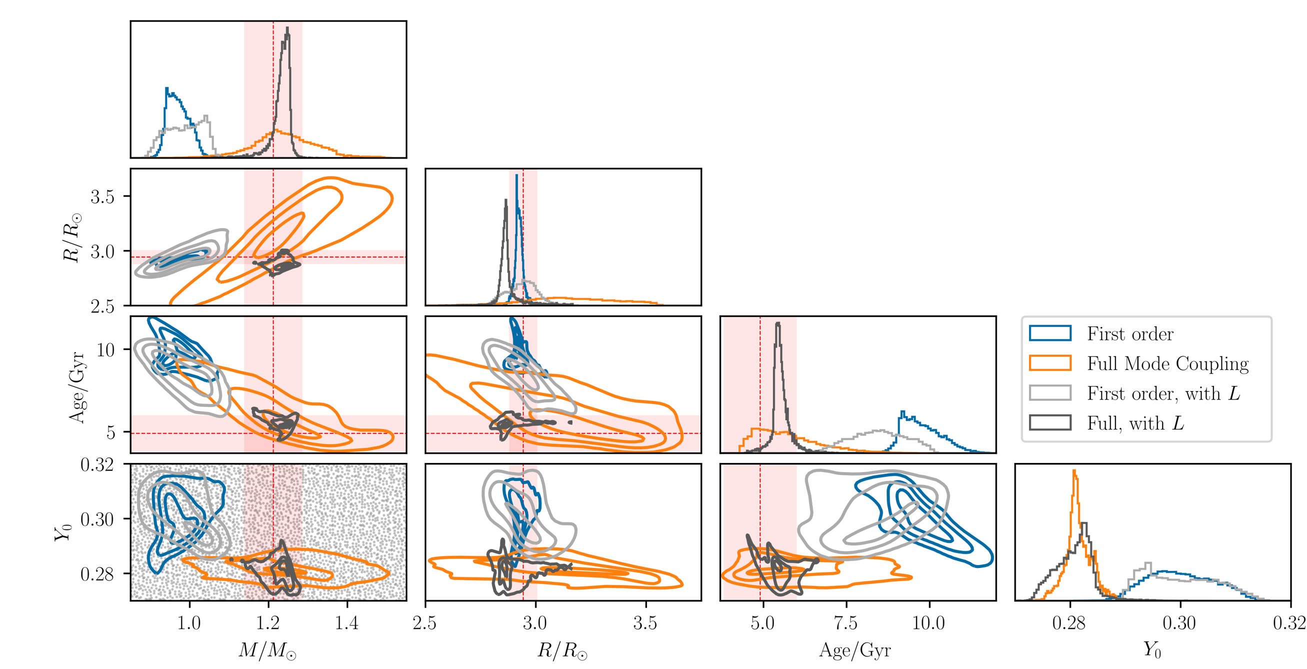


- We find that using traditional first-order techniques yields signficant **systematic biases in the inferred properties** at the 3σ level when considering the sample as a whole.

Single-Target Systematics

- We also consider in detail the interaction between the surface term and mode coupling (fig. 7) for the most extreme outlier in our sample (TOI 197; Huber et al., 2019)

Figure 7: Joint posterior distributions in various inferred quantities for TOI-197, for different combinations of surface-term prescriptions and spectroscopic constraints.



- Inappropriate treatment of mode coupling when correcting for the surface term potentially yields **significant single-target measurement error** in the global properties inferred from asteroseismology.

References

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