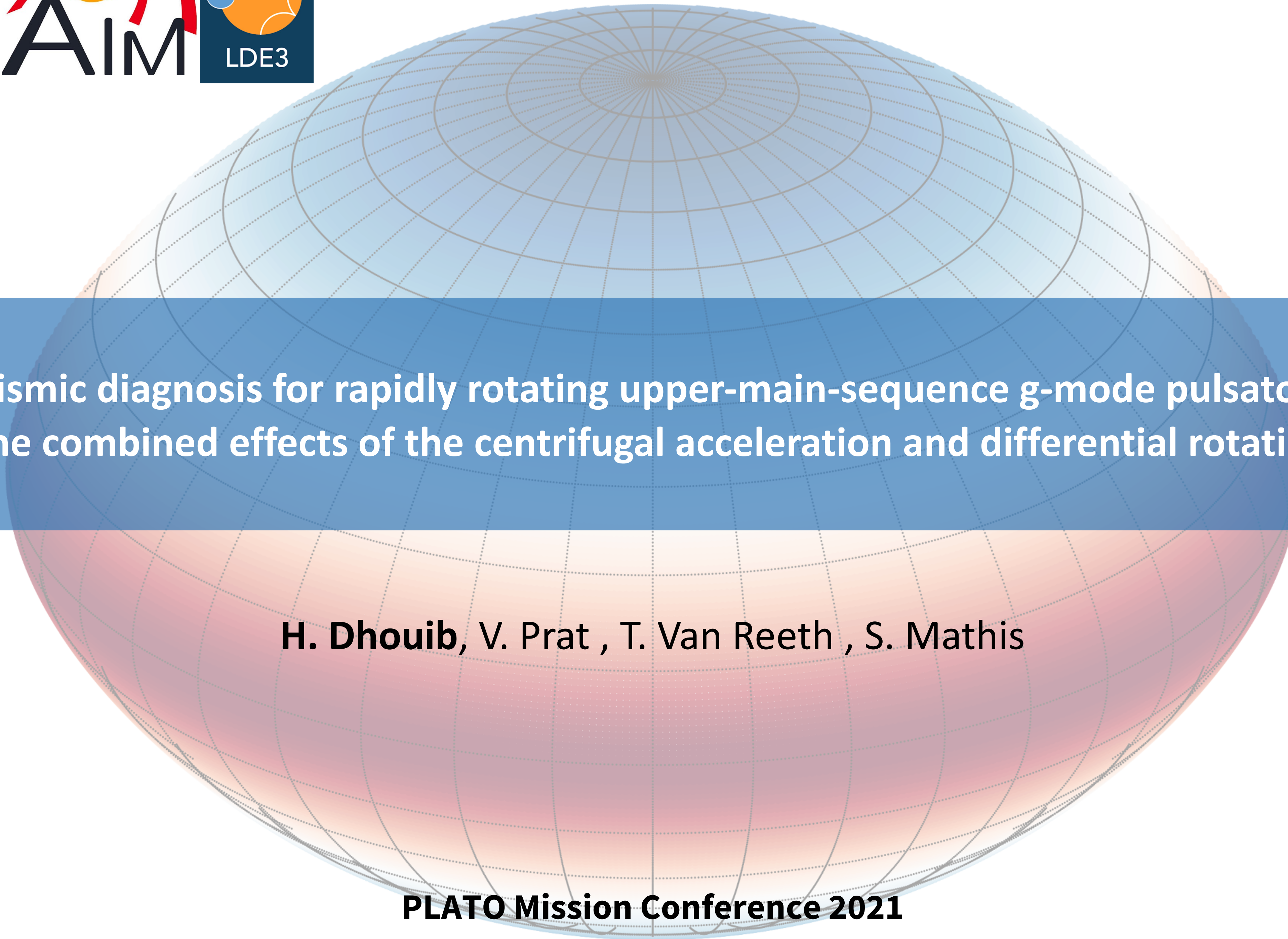

Seismic diagnosis for rapidly rotating upper-main-sequence g-mode pulsators: the combined effects of the centrifugal acceleration and differential rotation

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PLATO Mission Conference 2021

11-15 October 2021

image credit: Y.Frémat

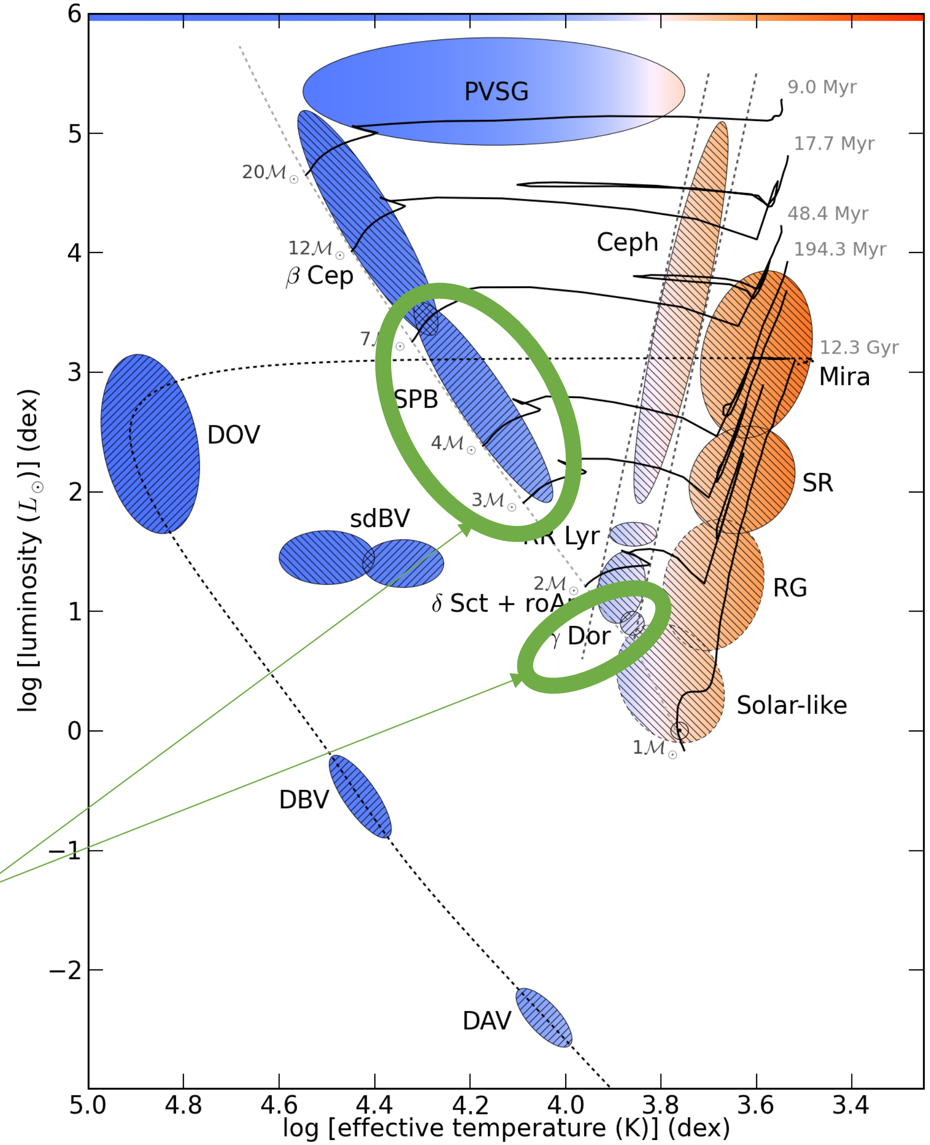
Space-based Asteroseismology

- **Asteroseismology:** only way to probe the **internal** structural, chemical, rotational and magnetic properties of stars

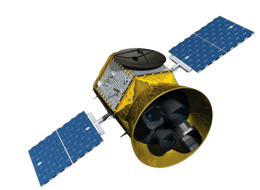
- Study pulsating stars **in the whole HR diagram**

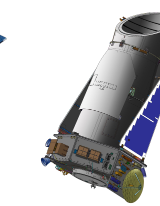
Predicting & searching signatures of rotation in **rapid rotators where it is not a perturbation**

Christensen-Dalsgaard (1997); Van Reeth PhD Thesis (2017)

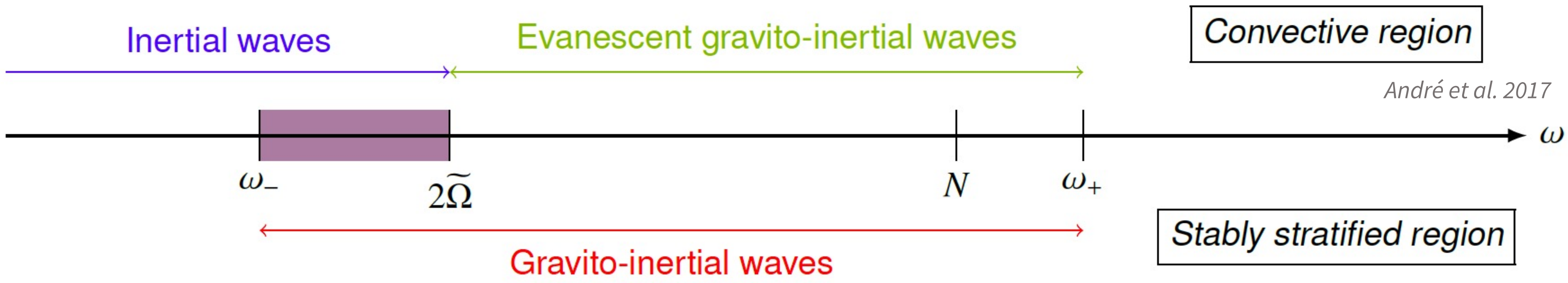


Asteroseismic HR Diagram





Rapidly rotating g mode pulsators



- Gravito-inertial waves (GIWs) driven by:
  - **buoyancy**: chemical and thermal stratifications
  - **rotation**: Coriolis and centrifugal accelerations

- Observing their frequencies ⇒ direct probe of internal structure, rotation and mixing

(e.g. Neiner et al. 2012; Van Reeth et al. 2016; Li et al. 2020; Ouazzani et al. 2020; Saio et al. 2021)

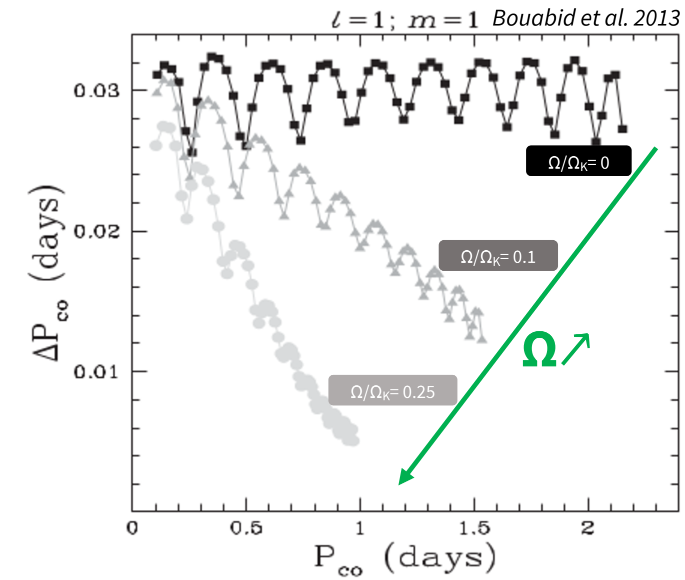
➞ Needs the more realistic modelling of these modes in rotating stars

ΔP versus P diagram

Period spacing: $\Delta P = P_{n+1} - P_n = f(P)$

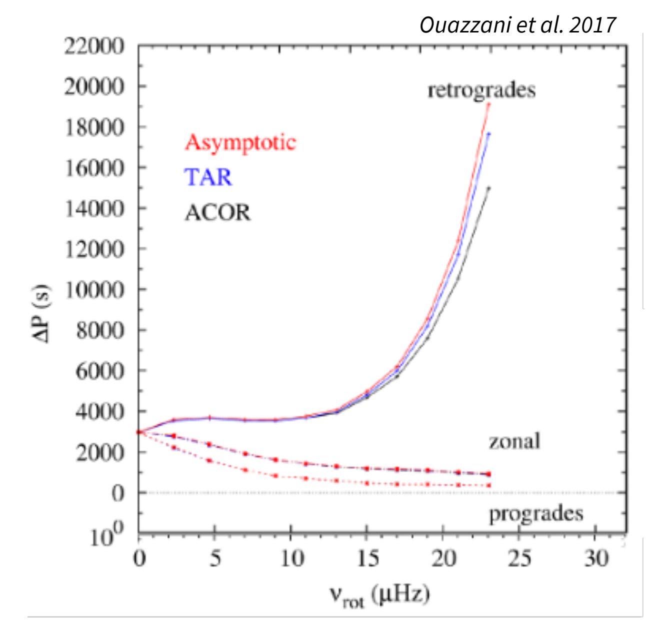
❖ ***Why this diagram?***

- Slope in ΔP–P diagram ⇒ measure of Ω of the inner radiative regions of early type stars *(Bouabid et al. 2013; Ouazzani et al. 2017)*
- Dips ⇒ signature for chemical mixing & resonant coupling between inertial modes in the convective core and g modes (*Ouazzani et al. 2020, Pedersen et al. 2021, Saio et al. 2021)*
  - ➢ a probe of the convective core rotation



❖ ***How to compute this diagram?***

- Direct computations using 2D stellar oscillation codes TOP *(Reese et al. 2021)* and ACOR *(Ouazzani et al. 2017)*
  - ➢ Difficult to perform intensive detailed seismic modelling
- Traditional approximation of rotation (TAR) *(e.g. Eckart 1960)*:
  - ➢ Flexible and robust; allows us to derive powerful seismic diagnostics
  - ✓ Applicable only in stably stratified regions
  - ✓ Standard version: spherical uniformly rotating stars



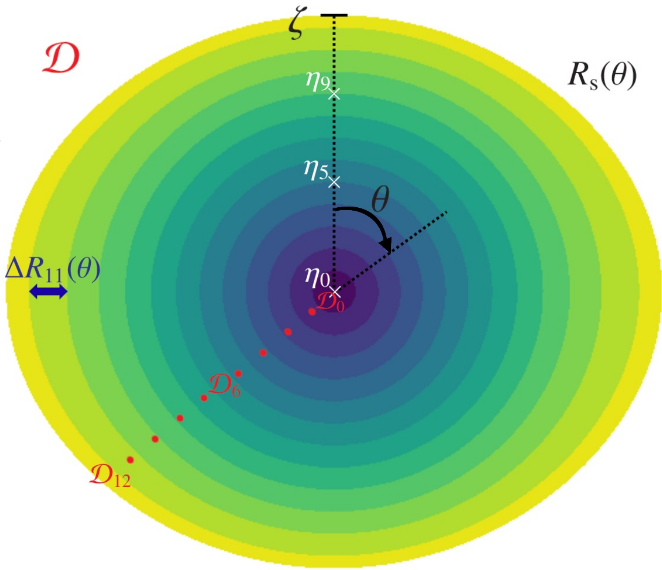
Generalisation of the TAR

Early type stars can be strongly deformed and differentially rotating

- First steps:
  - ✓ **Differential rotation**: Mathis (2009) & Van Reeth et al. (2018) in spherical stars
  - ✓ **Centrifugal distortion**: Mathis & Prat (2019) & Henneco et al. (2021) when perturbative
- Our work:
  - ✓ **Generalise the TAR to take into account simultaneously the centrifugal deformation and differential rotation**

Spheroidal geometry

Dhouib et al. 2021



3M$_\odot$ ESTER model with X$_c$=0.7 and Ω/Ω_K=60%

- New coordinate system (ζ, θ, ϕ) linked to spherical coordinates (r, θ,ϕ) (*Bonazzola et al. 1998*)

$\zeta = 0 \longrightarrow r = 0$: the centre of the star

$\zeta = 1 \longrightarrow r = R_s(\theta)$: the deformed surface of the star

Generalised Laplace tidal equation (GLTE)

General linearised hydrodynamic system

+

Frequency hierarchy		Approximation on the stratification profile
$\omega \ll N$	$2\Omega \ll N$	$N^2(\zeta,\theta) \approx N^2(\zeta)$

→ Simplified system

The vertical and horizontal dynamics are partially decoupled as in the non-rotating case

+

2D JWKB Approximation (*Mathis 2009*)	Approximation on the coefficient A	Approximation on the angular velocity
$X(\zeta,\theta) = \sum_k \left\{ x_{\nu k m}(\zeta,\theta) \frac{A_{\nu k m}}{k_{V;\nu k m}^{1/2}} \exp\left[i \int^{\zeta} k_{V;\nu k m} \mathrm{d}\zeta \right] \right\}, \quad X = \{\widetilde{W}, w^j\}.$	$\mathcal{A}(\zeta,\theta) \approx \mathcal{A}(\zeta)$	$\Omega(\zeta,\theta) \approx \Omega(\zeta)$

↓

$$\mathcal{L}_{\omega^{\rm in} m}[w_{\omega^{\rm in} k m}] = \omega \partial_x \left[\frac{1}{\omega} \frac{1}{\mathcal{B}(\mathcal{E}-\mathcal{F})} \left(\mathcal{E} + \frac{\nu^2 \mathcal{C}^2}{\mathcal{B}} \right) (1-x^2) \partial_x \right] - \frac{m\mathcal{F}}{\nu \mathcal{C}(\mathcal{E}-\mathcal{F})} \partial_x w_{\omega^{\rm in} k m} + \left(m\omega \partial_x \left(\frac{\nu \mathcal{C}}{\omega \mathcal{B}(\mathcal{E}-\mathcal{F})} \right) - \frac{m^2}{(\mathcal{E}-\mathcal{F})(1-x^2)} \right) w_{\omega^{\rm in} k m}$$

$$= -\Lambda_{\omega^{\rm in} k m}(\zeta) w_{\omega^{\rm in} k m}$$

Eigenvalues (for $\Lambda_{\omega^{\rm in} k m}(\zeta)$) · Hough functions (for $w_{\omega^{\rm in} k m}$)

- $\mathcal{A} \rightarrow \mathcal{E}$: centrifugal deformation | $\mathcal{F}$: differential rotation

Asymptotic frequency of low-frequency GIWs

- Asymptotic eigenfrequencies (quantisation in the pseudo-radial direction):

$$\omega_{nkm} = \frac{\int_{\zeta_1}^{\zeta_2} N(\zeta)\sqrt{\mathcal{A}(\zeta)\Lambda_{\omega_n^{\rm in}km}(\zeta)}\,{\rm d}\zeta}{(n+1/2)\pi}$$

Stratification ($N(\zeta)$) — Rotation ($\Lambda_{\omega_n^{\rm in}km}(\zeta)$)

Period spacing:

$$\Delta P_{km} \approx \frac{2\pi^2}{\int_{\zeta_1}^{\zeta_2} \underline{N}\sqrt{\mathcal{A}\underline{\Lambda}_{\omega_{n+1}^{\rm in}km}}\,{\rm d}\zeta \left(1+\frac{1}{2}\frac{\int_{\zeta_1}^{\zeta_2} \underline{N}\sqrt{\mathcal{A}\underline{\Lambda}_{\omega_n^{\rm in}km}}\frac{d\ln\underline{\Lambda}_{\omega_n^{\rm in}km}}{d\ln\omega^{\rm in}}{\rm d}\zeta}{\int_{\zeta_1}^{\zeta_2} \underline{N}\sqrt{\mathcal{A}\underline{\Lambda}_{\omega_n^{\rm in}km}}\,{\rm d}\zeta}\right)}$$

- Easily applicable for a large number of stars
- Great advantage when large grids of stellar models need to be calculated for detailed seismic modelling

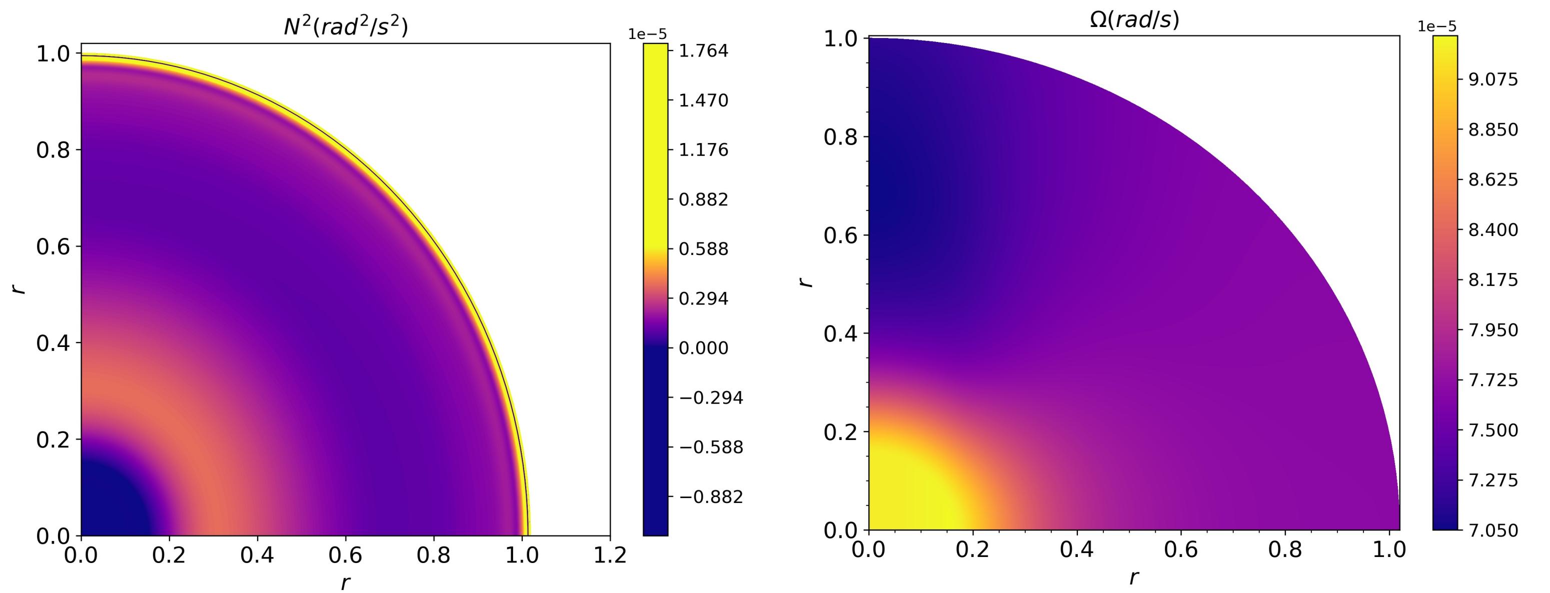
Validity domain of the generalised TAR

Dhouib et al. 2021

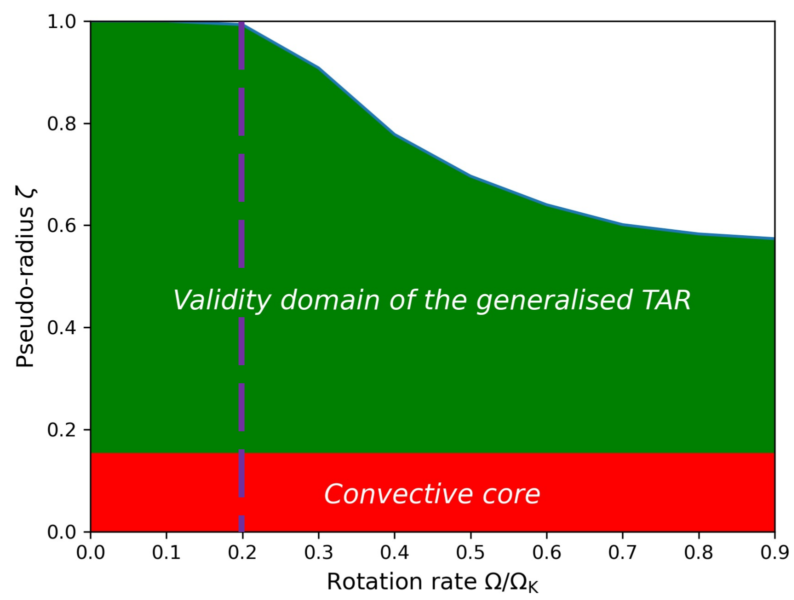
- **Application to rapidly rotating early-type stars**
- **ESTER code** (*Espinosa Lara & Rieutord 2013*):
  - 2D stellar structure
  - stationary hydrodynamic model
  - non-perturbative centrifugal acceleration and differential rotation

- **The generalised TAR is applicable to early-type stars rotating up to Ω/Ω_K=20%.**
- This limit proposed by Mathis & Prat (2019) using a perturbative approach was 40%

➢ **20% more realistic : derived from a coherent 2D model**

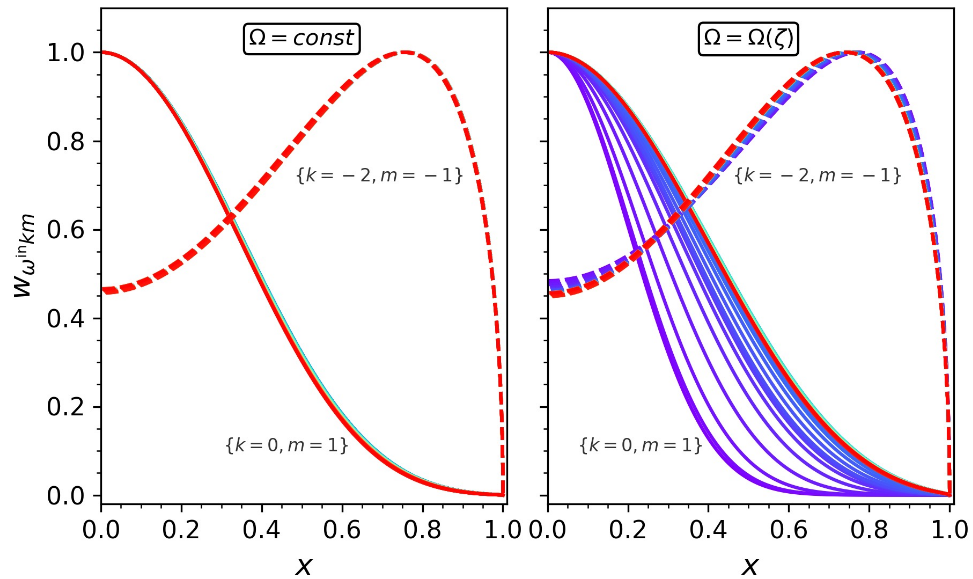


ESTER model ($3M_\odot$, X_c = 0.7, Ω/Ω_K=20%)



Hough functions, Asymptotic period spacing pattern and Detectability

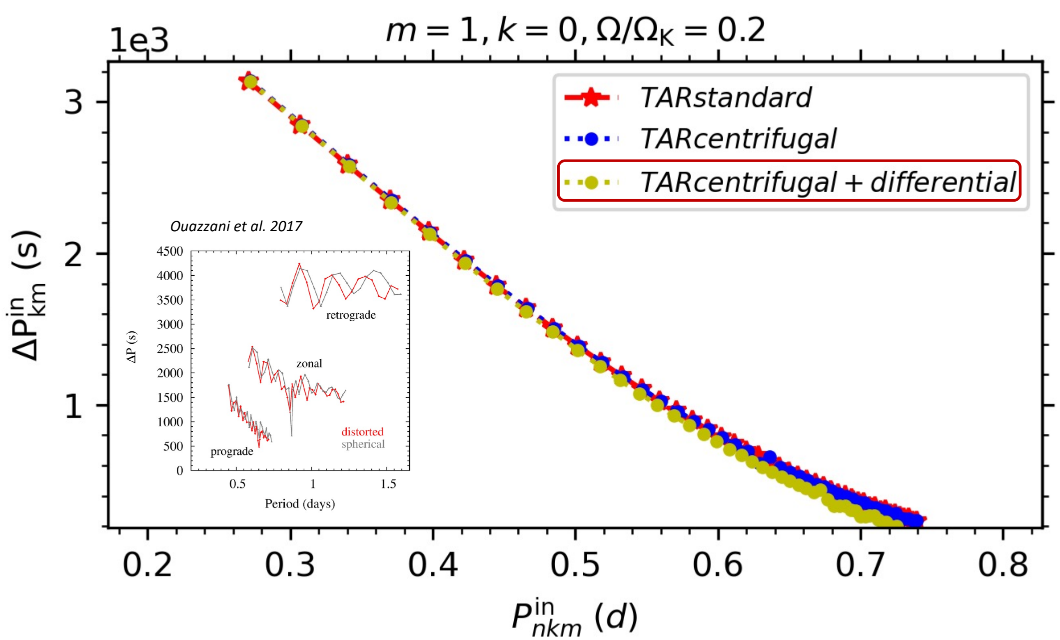
- ***Latitudinal eigenfunctions:***



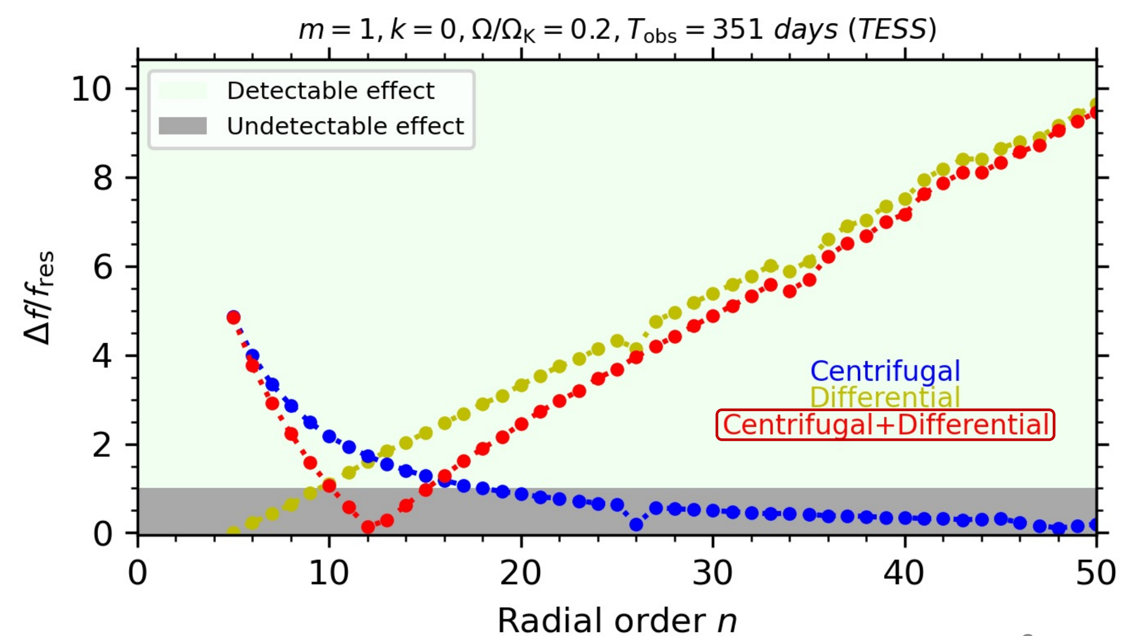
➢ A dependence not only on $x = cos\ \theta$ but also on ζ
(purple: edge radiation zone/ red: surface)

Dhouib et al. accepted

- ***Period spacing pattern:***



- ***Detectability:***



Take home messages and future work

- New generalisation of the TAR with **simultaneously the differential rotation and the centrifugal acceleration in a non-perturbative way**. **Designed for 2D stellar models like ESTER**

- Study the **detectability** and the **signature** of the centrifugal effects on GIWs in differentially rotating deformed stars **until 20% of the critical angular velocity**

- **Theoretically detectable in early-type stars using *Kepler* and TESS**

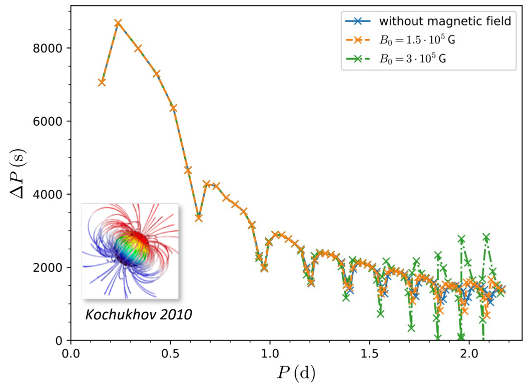
- **Early-type stars can host magnetic fields**

- Prat et al. (2019, 2020) and Van Beeck et al. (2020) studied the effects of a magnetic field on the periods of g modes using a perturbative treatment

- Mathis & de Brye (2011, 2012) have taken into account the toroidal magnetic fields in a non-perturbative way but with a very simple topology

⇒ Generalise the TAR to account for general magnetic field in differentially rotating stars

Prat et al. (2019)



References

Bonazzola et al.1998, Phys. Rev. D, 58, 104020
Bouabid et al. 2013, MNRAS, 429, 2500
Christensen-Dalsgaard 1999, Lecture Notes on Stellar Oscillations
Dhouib et al. 2021, A&A, 652, A154
Eckart, C. 1960, Hydrodynamics of Oceans and Atmospheres (Pergamon Press)
Espinosa Lara & Rieutord 2013, A&A, 552, A35
Henneco et al. 2021, A&A, 648, A97
Lee & Saio 1997, ApJ, 491, 839
Li et al. 2020, MNRAS, 491, 3586
Mathis 2009, A&A, 506, 811
Mathis & Prat 2019, A&A, 631, A26
Neiner et al. 2012b, A&A, 539, A90
Ouazzani et al. 2017, MNRAS, 465, 2294
Ouazzani et al. 2020, A&A, 640, A49
Pedersen et al. 2021, Nature Astronomy, 5, 715
Reese et al. 2021, A&A, 645, A46
Saio et al. 2021, MNRAS, 502, 5856
Van Reeth et al. 2016, A&A, 593, A120
Van Reeth 2017, PhD Thesis
Van Reeth et al. 2018, A&A, 618, A24