

Rough interval Max Plus Algebra for Transportation Problems

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Abstract: The traffic problem is an important problem which has been broadly learnt in Operations Research domain. This paper presents a new Rough Interval Max Algebra Approach (RIMAA) for solving the traffic problem with Rough Interval data. The proposed approach is simple and able to give a suitable solution to this problem. Finally, a descriptive example is given to evaluate performance of the proposed approach.

Keywords : Max-Plus Algebra, Rough Interval, Traffic Problems, Transportation.

I. INTRODUCTION

Traffic as a part of public transport develops the economy, compacts with the protection of energy and resources, downgrades jamming, affords critical support in danger situations and disasters, grows the progress and usefulness of real estate, grows mobility in small urban and countryside communities, and decreases health costs. All of this donates to a better quality of life. The traffic accessibility affecting different suburban communities is a complicated process. Problems of discovering an optimal solution, that is, tasks of optimization, are found and solved in real life. They are found almost everywhere, in technical and economic systems, in the family, companies, sports clubs etc.

Max-plus algebra is a mathematical system in which the arithmetic operation of addition is replaced by determining the maximum, and the multiplication operation is replaced by the addition. This mathematical approach provides an interesting way suitable for modeling discrete event systems (DES) and optimization problems in production and transport. Moreover, there is a strong similarity with the classical linear algebra[1]-[7].

The rough sets, first presented (Pawlak, 1982). In the concept of rough sets, is determined by the approximation which is the basic concept of the theory of rough sets. The theory of rough sets uses only internal knowledge, that is, operational data, and there is no need to rely on modeling assumptions. In rough sets, measurement of uncertainty is based on the uncertainty already controlled in the data (Khoo & Zhai, 2011). This leads to objective values that are contained in the data. In addition, the theory of rough sets is fit for applications that are typified by a scarcity of data, and for which statistical techniques are not appropriate (Pawlak,1991, 1993; Stević, 2018).

In this paper using arithmetic operations of the Rough

Interval and max plus algebra, for the treatment of uncertainty in traffic problems.[9]-[13]

II. TRAFFIC PROBLEM

Traffic problem is the mainly problems in many urban towns round the world. This traffic problem can touch the economy, slow down the growth, reduce the manufacture, growth cost, and hamper people's daily life. There are many effects that can create traffic problem in a large town. Among them growth sum of vehicles, shortage of enough paths and highways, and traditional traffic light system. All of these factors are helping the traffic security at the junction, increasing the capacity at the intersection Fuzzy Max Plus Algebra Algorithm for Traffic Problems and minimizing the delays[6].

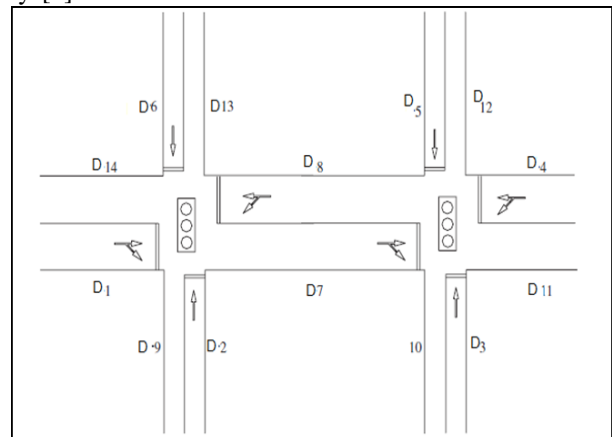


Fig .1 traffic area intersections.

III. ROUGH SET THEORY

The theory of rough sets can be viewed as an effective mathematical vehicle for dealing with inaccurate and uncertain data analyses, which can be subsequently useful to pattern recognition, machine learning, and intelligence discovery. Rough interval using similarity classes, an arbitrary subset can be estimated by two subsets called the lower approximation and the upper approximation.

Definition 1 Let R^R denote a set of rough intervals. A rough interval A^R is tuple of rough intervals, and a rough interval matrix A^R_{ij} matrix of rough interval whose part are rough intervals:

$$A^R = \{a_i^R [[a_i^{-U}, a_i^{+U}] : [a_i^{-L}, a_i^{+L}]], \forall i \}, A^R \in R^{R(1 \times n)}$$

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$$A^R_{ij} = \{a_{ij}^R : [a_{ij}^{-U}, a_{ij}^{+U}] : [a_{ij}^{-L}, a_{ij}^{+L}], \forall i, j\}, A^R \in R^{(m \times n)}$$

Definition 2 Assume that

$$a^R = [[a^{-U}, a^{+U}] : [a^{-L}, a^{+L}]], b^R = [[b^{-U}, b^{+U}] : [b^{-L}, b^{+L}]]$$

We define

$$a^R + b^R = [[a^{-U} + b^{-U}, a^{+U} + b^{+U}] : [a^{-L} + b^{-L}, a^{+L} + b^{+L}]],$$

$$a^R - b^R = [[a^{-U} - b^{+U}, a^{+U} - b^{-U}] : [a^{-L} - b^{+L}, a^{+L} - b^{-L}]],$$

$$a^R \times b^R = [[a^{-U} \times b^{-U}] : [a^{+U} \times b^{+U}]] : [[a^{-L} \times b^{-L}] : [a^{+L} \times b^{+L}]],$$

IV. THE PROPOSED APPROACH: ROUGH INTERVAL MAX ALGEBRA APPROACH (RIMAA)

A new approach called Rough Interval Max Algebra (RIMAA) in order to solve the traffic problem with Rough Interval data.

Step 1: Determine a set of Rough Interval traffic problem: Vehicles network is collected of a set of lines, which are linked by swap points called connection stops. In a Rough Interval case, the estimate of a travel performed by any customer consists in determining its connection times at each vehicle change and to add them to the moving times on each line from an origin point to a destination one.

Step 2 : Constructing the Rough Interval matrix for duration : Construct the Rough Interval matrix of each row determine the durations of the of the selected area

$$A^{RI} = \begin{bmatrix} a_{11}^{RI} & \dots & a_{12}^{RI} & \dots & a_{1n}^{RI} \\ \vdots & & \vdots & & \vdots \\ a_{21}^{RI} & \dots & a_{22}^{RI} & \dots & a_{2n}^{RI} \\ \vdots & & \vdots & & \vdots \\ a_{m1}^{RI} & \dots & a_{m2}^{RI} & \dots & a_{mn}^{RI} \end{bmatrix} ; i=1,2, \dots, m; j=1,2, \dots, n,$$

Where m – number of rows, n – number of columns, a_{ij}^{RI} representing the performance rough interval value of the i rows in terms of the j criterion, $a_{ij}^{RI} = [a_{ij}^{UAI} : a_{ij}^{LAI}]$ is defined as an interval with lower bound $a_{ij}^{LAI} = [a_{ij}^{-LAI}, a_{ij}^{+LAI}]$ and upper bound $a_{ij}^{UAI} = [a_{ij}^{-UAI}, a_{ij}^{+UAI}]$ respectively.

Step 3:Apply Rough Interval Max Algebra Approach (RIMAA):

a) Determining the Rough Interval duration using max-plus addition \oplus in traffic problem

$$a_{ij}^{RI} \oplus b_{ij}^{RI} = \begin{bmatrix} [a_{11}^{UAI} : a_{11}^{LAI}] & [a_{12}^{UAI} : a_{12}^{LAI}] \\ [a_{21}^{UAI} : a_{21}^{LAI}] & [a_{22}^{UAI} : a_{22}^{LAI}] \end{bmatrix} \oplus \begin{bmatrix} [b_{11}^{UAI} : b_{11}^{LAI}] & [b_{12}^{UAI} : b_{12}^{LAI}] \\ [b_{21}^{UAI} : b_{21}^{LAI}] & [b_{22}^{UAI} : b_{22}^{LAI}] \end{bmatrix}$$

$$[[[a_{11}^{-UAI}, a_{11}^{+UAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}]] & [[a_{12}^{-UAI}, a_{12}^{+UAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}]] \\ [[a_{21}^{-UAI}, a_{21}^{+UAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}]] & [[a_{22}^{-UAI}, a_{22}^{+UAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}]]]$$

$$\begin{bmatrix} \max([a_{11}^{-UAI}, a_{11}^{+UAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [b_{11}^{-UAI}, b_{11}^{+UAI}], [b_{11}^{-LAI}, b_{11}^{+LAI}]) & \max([a_{12}^{-UAI}, a_{12}^{+UAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [b_{12}^{-UAI}, b_{12}^{+UAI}], [b_{12}^{-LAI}, b_{12}^{+LAI}]) \\ \max([a_{21}^{-UAI}, a_{21}^{+UAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [b_{21}^{-UAI}, b_{21}^{+UAI}], [b_{21}^{-LAI}, b_{21}^{+LAI}]) & \max([a_{22}^{-UAI}, a_{22}^{+UAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [b_{22}^{-UAI}, b_{22}^{+UAI}], [b_{22}^{-LAI}, b_{22}^{+LAI}]) \end{bmatrix}$$

$$\begin{bmatrix} \max([a_{11}^{-UAI}, a_{11}^{+UAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}]) & \max([a_{12}^{-UAI}, a_{12}^{+UAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}]) \\ \max([a_{21}^{-UAI}, a_{21}^{+UAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}]) & \max([a_{22}^{-UAI}, a_{22}^{+UAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}]) \end{bmatrix}$$

$$\begin{bmatrix} (\max(a_{11}^{-U}, b_{11}^{-U}), \max(a_{11}^{+U}, b_{11}^{+U}), \max(a_{11}^{-L}, b_{11}^{-L}), \dots) & \dots & (\max(a_{1j}^{-U}, b_{1j}^{-U}), \max(a_{1j}^{+U}, b_{1j}^{+U}), \max(a_{1j}^{-L}, b_{1j}^{-L}), \dots) & \dots & (\max(a_{1n}^{-U}, b_{1n}^{-U}), \max(a_{1n}^{+U}, b_{1n}^{+U}), \max(a_{1n}^{-L}, b_{1n}^{-L}), \dots) \\ \vdots & & \vdots & & \vdots \\ (\max(a_{21}^{-U}, b_{21}^{-U}), \max(a_{21}^{+U}, b_{21}^{+U}), \max(a_{21}^{-L}, b_{21}^{-L}), \dots) & \dots & (\max(a_{2j}^{-U}, b_{2j}^{-U}), \max(a_{2j}^{+U}, b_{2j}^{+U}), \max(a_{2j}^{-L}, b_{2j}^{-L}), \dots) & \dots & (\max(a_{2n}^{-U}, b_{2n}^{-U}), \max(a_{2n}^{+U}, b_{2n}^{+U}), \max(a_{2n}^{-L}, b_{2n}^{-L}), \dots) \\ \vdots & & \vdots & & \vdots \\ (\max(a_{m1}^{-U}, b_{m1}^{-U}), \max(a_{m1}^{+U}, b_{m1}^{+U}), \max(a_{m1}^{-L}, b_{m1}^{-L}), \dots) & \dots & (\max(a_{mj}^{-U}, b_{mj}^{-U}), \max(a_{mj}^{+U}, b_{mj}^{+U}), \max(a_{mj}^{-L}, b_{mj}^{-L}), \dots) & \dots & (\max(a_{mn}^{-U}, b_{mn}^{-U}), \max(a_{mn}^{+U}, b_{mn}^{+U}), \max(a_{mn}^{-L}, b_{mn}^{-L}), \dots) \end{bmatrix}$$

b) Determining the Rough Interval duration using max-plus multiplication, \otimes in traffic problem:

$$a_{ij}^{RI} \otimes b_{ij}^{RI} = \begin{bmatrix} [a_{11}^{UAI} : a_{11}^{LAI}] & [a_{12}^{UAI} : a_{12}^{LAI}] \\ [a_{21}^{UAI} : a_{21}^{LAI}] & [a_{22}^{UAI} : a_{22}^{LAI}] \end{bmatrix} \otimes \begin{bmatrix} [b_{11}^{UAI} : b_{11}^{LAI}] & [b_{12}^{UAI} : b_{12}^{LAI}] \\ [b_{21}^{UAI} : b_{21}^{LAI}] & [b_{22}^{UAI} : b_{22}^{LAI}] \end{bmatrix}$$

$$= \begin{bmatrix} [a_{11}^{-UAI}, a_{11}^{+UAI}] : [a_{11}^{-LAI}, a_{11}^{+LAI}] \otimes [b_{11}^{-UAI}, b_{11}^{+UAI}] : [b_{11}^{-LAI}, b_{11}^{+LAI}] & [a_{12}^{-UAI}, a_{12}^{+UAI}] : [a_{12}^{-LAI}, a_{12}^{+LAI}] \otimes [b_{12}^{-UAI}, b_{12}^{+UAI}] : [b_{12}^{-LAI}, b_{12}^{+LAI}] \\ [a_{21}^{-UAI}, a_{21}^{+UAI}] : [a_{21}^{-LAI}, a_{21}^{+LAI}] \otimes [b_{21}^{-UAI}, b_{21}^{+UAI}] : [b_{21}^{-LAI}, b_{21}^{+LAI}] & [a_{22}^{-UAI}, a_{22}^{+UAI}] : [a_{22}^{-LAI}, a_{22}^{+LAI}] \otimes [b_{22}^{-UAI}, b_{22}^{+UAI}] : [b_{22}^{-LAI}, b_{22}^{+LAI}] \end{bmatrix}$$

$$\begin{bmatrix} [a_{11}^{-UAI} \otimes b_{11}^{-UAI}] \oplus [a_{11}^{-LAI} \otimes b_{11}^{-LAI}] : [a_{11}^{-UAI} \otimes b_{11}^{+UAI}] \oplus [a_{11}^{-LAI} \otimes b_{11}^{+LAI}] & [a_{12}^{-UAI} \otimes b_{12}^{-UAI}] \oplus [a_{12}^{-LAI} \otimes b_{12}^{-LAI}] : [a_{12}^{-UAI} \otimes b_{12}^{+UAI}] \oplus [a_{12}^{-LAI} \otimes b_{12}^{+LAI}] \\ [a_{21}^{-UAI} \otimes b_{21}^{-UAI}] \oplus [a_{21}^{-LAI} \otimes b_{21}^{-LAI}] : [a_{21}^{-UAI} \otimes b_{21}^{+UAI}] \oplus [a_{21}^{-LAI} \otimes b_{21}^{+LAI}] & [a_{22}^{-UAI} \otimes b_{22}^{-UAI}] \oplus [a_{22}^{-LAI} \otimes b_{22}^{-LAI}] : [a_{22}^{-UAI} \otimes b_{22}^{+UAI}] \oplus [a_{22}^{-LAI} \otimes b_{22}^{+LAI}] \end{bmatrix}$$

$$\begin{bmatrix} \max([a_{11}^{-UAI}, a_{11}^{+UAI}], [a_{11}^{-LAI}, a_{11}^{+LAI}], [b_{11}^{-UAI}, b_{11}^{+UAI}], [b_{11}^{-LAI}, b_{11}^{+LAI}]) & \max([a_{12}^{-UAI}, a_{12}^{+UAI}], [a_{12}^{-LAI}, a_{12}^{+LAI}], [b_{12}^{-UAI}, b_{12}^{+UAI}], [b_{12}^{-LAI}, b_{12}^{+LAI}]) \\ \max([a_{21}^{-UAI}, a_{21}^{+UAI}], [a_{21}^{-LAI}, a_{21}^{+LAI}], [b_{21}^{-UAI}, b_{21}^{+UAI}], [b_{21}^{-LAI}, b_{21}^{+LAI}]) & \max([a_{22}^{-UAI}, a_{22}^{+UAI}], [a_{22}^{-LAI}, a_{22}^{+LAI}], [b_{22}^{-UAI}, b_{22}^{+UAI}], [b_{22}^{-LAI}, b_{22}^{+LAI}]) \end{bmatrix}$$

Step 4 : Convert the solution of Rough Interval traffic problem output solution into suitable, real crisp variable.

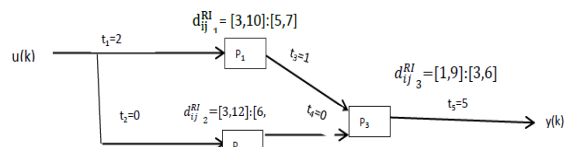
$$y_{ij}^{RI} = (y_1, y_2, y_3, y_4)$$

$$y_{out} = \frac{y_1 + y_2 + y_3 + y_4}{4}$$

Where y_{out} the final crisp results of RIMAA.

Numerical Example

This road consists of 3 streets: P₁, P₂, and P₃. The times for P₁, P₂, and P₃ are respectively. The Rough Interval duration (d₁, d₂, d₃ and d₄), we suppose that it takes t₁ = 2 time to get from the source to P₁ and that it takes t₃ = 1 time to reach P₁ to P₃. The other transportation times (t₂, t₄, and t₅) are supposed to be small. A processing can only start working on new streets if it has finished the previous one. We Define.



- u(k): time instant at which streets is fed to the system for the kth time,
- xi(k): time instant at which the ith starts working for the kth time,
- y(k): time instant at which the kth finished vehicle leaves the system.

$$A^{RI} = \begin{bmatrix} [3, 10] : [5, 7] & [-\infty, -\infty] : [-\infty, -\infty] & [-\infty, -\infty] : [-\infty, -\infty] \\ [-\infty, -\infty] : [-\infty, -\infty] & [3, 12] : [6, 8] & [-\infty, -\infty] : [-\infty, -\infty] \\ [9, 15] : [11, 13] & [10, 19] : [12, 15] & [1, 9] : [3, 6] \end{bmatrix}$$



V. RESULT AND DISCUSSION

By applying the steps of RIMAA, we obtain the results in table 1, by Eqs. and Defuzzification for obtained RIMAA as listed in table 2 are calculated by Eqs.
$$Y_{out} = \frac{y_1 + y_2 + y_3 + y_4}{4}$$

Table 1: results of RIMAA

	x ₁	x ₂	x ₃
a₁	2	0	8
	2	0	8
	2	0	8
	2	0	8
a₂	5	4	12
	8	7	14
	10	9	16
	13	13	20
a₃	9	8	16
	14	14	20
	18	18	25
	24	26	33
a₄	15	14	22
	22	23	29
	28	29	36
	37	41	48
a₅	16	15	23
	25	27	33
	33	35	42
	45	51	58

Table 2 Convert the solution of RIMAA

	y	Y _{out}
a₁	11	11
	11	
	11	
	11	
a₂	15	1.850000e+01
	17	
	19	
	23	
a₃	19	2.650000e+01
	23	
	28	
	36	
a₄	25	3.675000e+01
	32	
	39	
	51	
a₅	26	42
	36	
	45	
	61	

From above table 2 gives the results obtained by the proposed approach, the proposed approach has the ability to handle Rough Interval max algebra, while the other method can deal with the crisp data.

VI. CONCLUSION

The major goal of this paper is to solve the traffic problem by using a new Rough Interval max algebra. The problem has

been described as a max algebra method under uncertainty. Anew Rough Interval of the max algebra is applied to calculate the result.

The Rough Interval uncertainty, which is typical of real decision problems, with Rough Interval in the time matrix. Here, the Max-Plus Algebra technique using Rough Interval as the basis for evaluating values of The Traffic Problems

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