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The Discrete Rational Cubic Spline Interpolator

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***Abstract:** The problem of Discrete C^2 Rational Cubic Spline has been proposed and Error bound obtained. The Discrete Rational Method have unique representation.*

***Keywords:** Rational, Discrete, Cubic, Shape Preserving, Error bounds, Interpolation Parameter.*

I. INTRODUCTION

Rational Cubic spline with one free parameter useful to obtained positive and convex curve from positive and convex data respectfully. discrete spline obtained by using differences in place of derivative . Rana and Dubey [3] constructed discrete cubic spline. Duen [4] study of rational interpolation with functional values .To find design and shape of curve rational and discrete rational cubic spline are applicable So many author studies for shape preservation of curve (see [6], [7], [8] ,[9] [10],). Rational discrete cubic spline preserve convexity and positivity but simple spline not preserve both. Duan et.al. [4]have found shape of curves by using rational spline and obtained condition that curves lie to above, blow or between the straight lines . In this paper authors assumed suitable values of parameters to obtain C^2 Conversion Curve and the scheme work for uniform mesh. Hussain et.al. [5] investigate a rational cubic function which was used to achieve designs for shape data. They found relation on free parameters in the description of C^1 rational cubic function to obtain desire shapes of the data. In this paper we have using different values of free parameter to obtained curves for uniform and non uniform case. We have developed discrete rational cubic spline with one free parameter

Remark 1.1

(i) When $h_i \to 0$, we may obtained C^2 rational cubic spline with two parameters this gives particular case. M.Z. Hussain et.al. [5].

(ii) Sarfaraz [6], Abbas [7], Duan [4] and Bao [9] error bound obtained by sub interval in our paper error bounds obtain one time full interval [0, 1].

(iii) Error bound obtained in our paper is very accurate and minimum but other paper error bound get maximum inaccurate.

(iv) When differences is given then our scheme is beneficial but Broadlieand Butt[11] Abbas et.al [7].Sarfaraze et.al [6].and so many paper methods are not useful.

II. C^2 DISCRETE RATIONAL CUBIC FUNCTION

Now s(x) is defined as

$$s(x) = s(x_i) = \frac{P_i(\theta)}{q_i(\theta)} = \frac{\sum_{i=0}^{3} w_i(1-\theta)^{3-i}\theta^i}{q(\theta)}$$

with..(2.1) $h_i = x_{i+1} - x_i$

$$\theta = \frac{(x - x_i)}{h_i}, \quad i = 1,2,\ldots..n-1$$

and $w_0 = f_i$

$$w_1 = \left[h_i d_i\left(h_i^2 - (v_i - 1)\right)^2 h_i^2 - f_i\{2(h^2 - h_i^2) - (h_i^2 + 3h^2)v_i\}\right.$$

$$+\frac{w_2 h^2 h_i v_i - v_i h_{i+1} h^2]}{(h^2 + h^2) + 2h^2 v_i}$$

$$w_2 = [-d_{i+1} h_i\{(h_i + v_i h_i)^2 - v_i^2 h^2\} - h_i h^2 v_i h_i$$

$$+v_i h_{i+1}\{h_i^2 + 3h_i^2 h\}$$

$$\frac{2h^2 v_i + 3h_i^2 h + 2v_i h_i^2\}]}{[2h^2 h_i - h^2 h_i v_i - v_i h_i^3]}$$

$$w_3 = v_i f_{i+1}$$

$$q_i(\theta) = (1-\theta)^2 + (1 + v_i)\theta(1-\theta) + v_i\theta^2 \qquad (2.2)$$

Where v_i is the shape parameter used to control the shape of the interpolation and Let $D_h^{(2)} s(x)$ the second differences with respect to x and $d_i = D_h^{(1)} s(x_i)$ denote first differences value at knots x, then C^2 splining constrains :

$$s(x_i) = f_i; s(x_{i+1}) = f_{i+1}$$

$$D_h^{(1)} f(x_i) = d_i, D_h^{(1)} f(x_{i+1}) = d_{i+1}$$

$$D_h^{(2)}(x_i +) = D_n^{(2)}(x_i -), i = 1,2,\ldots..n-1 \quad (2.3)$$

Where $D_h^{(1)}(x) = \{f(x+h) - f(x-h)\}/2h$, $D_h^{(2)}f(x) = [f(x+h) - 2f(x) + f(x-h)]/h^2$.

Using continuity of second derivative we get

$$\frac{h_i, d_{i-1} v_{i-1}[h^2(1 - v_{i-1}) + h_{i-1}^2 v_{i-1}^2]}{\{-(1 - v_{i-i})^2 h^2 + v_{i-1}^2 h_{i-1}^2\}}$$

$$+\frac{d_i[\{h_i v_{i-1}\{h^2(1 - v_{i-1})v_{i-1} + h_{i-1}^2 v_{i-1} + h_{i-1}^2 v_{i-1}^2\}}{[-(1 - v_{i-1})^2 h^2 + v_{i-1}^1 h_{i-1}^2]}$$

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$$+h_{i-1}d_i\{-2(1-v_i)^2h^2+\left(h_i^2+h^2\right)\}]$$

$$\frac{+v_ih_{i-1}d_{i+1}[h^2(1-v_i)+h_i^2]}{[-(1-v_i)^2h^2+h^2]}$$

$$=F_i*i=1,2,.......n-1.$$

Where

$$F_i^*$$
$$=h_{i-1}(1+2v_i)$$
$$[(1-v_i)(h^2+h_i^2)+2h_i^2+2h_i^2]\Delta_i$$
$$+h_iv_{i-1}(v_{i-1}+2)$$
$$[(h_{i-1}^2+h^2)(1-v_{i-1})v_{i-1}+2h_{i-1}^2v_{i-1}^2]\Delta_{i-1}$$
$$[(1-v_{i-1})^2h^2+v_{i-1}^2h_{i-1}^2] \qquad (......2.4)$$

Where $\Delta_i=(f_{i+1-}\ \ f_i)/h_i$. Different parameter d_i and d_{i-1} determinate by end conditions.

Since the system of linear equation is diagonally dominant for all $v_i>0$, it has a unique solution for the difference parameters d_is. We can find solution of above equation.

Remark 2.1: When $h\to 0$,the Rational Discrete cubic spline reduce to rational cubic spline.

Remark 2.2: Without shape parameter when $v_i\ =1$ so the rational discrete cubic spline reduce to discrete cubic spline.

III. POSITIVITY OF RATIONAL DISCRETE CUBIC SPLINE FUNCTION

Let $\{x_i,f_i\}, i=1,2,.....n,\ x_1<x_2<......<x_n$ and $f_1>0, f_2>0.......f_n>0$ it is required to construct a positive interpolant $s(x)$. The rational discrete cubic function (1) is positive if $p_i(\theta)$ and $q_i(\theta)$ both positive, since$q_i(\theta)>0, \forall\ \ v_i>0$.

Now $p_i(\theta)$ can be rewritten as follows:

$$p_i(\theta)=a_1\theta^3+b_i\theta^2+c_i\theta+d_i\,.$$

Where

$$a_i=w_1-w_0-w_2+w_3$$
$$b_i=3w_0-2w_1$$
$$c_i=-3w_0+w_1+w_2$$
$$d_i=w_0\,.$$

So $a_i+b_i+c_i+d_i$ (....3.1)

$=w_3=v_if_{i+1}>0$. (...3.4)

Theorem 3.1:

The C^2 Rational Discrete Cubic function (1.1) is positive in each interval $[x_i,x_{i+1}]$ if the shape parameter v_i satisfy following constraints

$v_i>0$

and$w_3=v_if_{i+1}>0$.

IV. ERROR BOUNDS

For a given$h>0$, we introduce the set $R_{ha}=\{a+jh{:}\,j\ is\ an\ \ integer\}$and define a discrete interval as follows:

$$[a,b]_h=[a,b]\cap R_{ha}$$

For a function f and three disjoint points x_1, x_2, x_3 in its domain. The first and second divided differences are defined by

$$[x_1,x_2]_f=\frac{\{f(x_1)-f(x_2)\}}{(x_1-x_2)}$$

and $\{x_1,x_2,x_3\}_f=\dfrac{\{[x_1,x_2]f-[x_2,x_3]f\}}{(x_1-x_3)}$ respectively.

Now we write $f^{(2)}$ for $D_h^{(2)}f$ and the modulus of continuity of f is $w(f,p)$.

And $\|f\|=\max\limits_{x\in[a,b]_h}|f(x)|$ discrete norms of a function f over the interval $[a,b]_h$

To obtained error bound we have to state following

Theorem 4.1:

Let s(x, h)be the unique periodic discrete Rational cubic spline interpolant f under the assumption of condition (2.3) . Then over the discrete interval $[a,b]_h$.

$$\|e(x)\|\le p\,k(h)w\left(D_h^{(1)}f,p\right)$$

(.....4.1)

Where $k(h)$ is some function of h defined earlier and w (f, p) is the discrete modules of the continuity of f.

In order to show the convergence of the discrete rational spline. We shall need the following Lemma due to Lyche [12].

Lemma 4.1 : Let $\{a_j\}_{j=1}^m$ and $\{b_j\}_{j=1}^n$ .be given sequence of non negative real number such $\sum\limits_{j=1}^m a_j=\sum\limits_{j=1}^n b_j$. Then for any real valued function f, defined on a discrete interval [0, 1] we have

$$\left|\sum_{j=1}^m a_j\left[x_j,x_{j,1}-x_{j,k}\right]_f-\sum_{j=1}^n b_j\left[y_{j0},y_{j,1}.....y_{jn}\right]_f\right|$$

$$\le w\{D_h^{(k)}f,|1-kh|\}\sum a_j/k! \qquad (.......4.2)$$

Where x_{jk},$y_{jk}\epsilon[0,1]_h$for rational values of j, k. Replacing m_i by $D_h^{(1)}e(x_i)$ in Equation (2.4).

We have

$$e(x)=v_i,h_{i-1}e_{i-1}\frac{[h_i^2(1-v_{i-1})+h_{i-1}^2v_{i-1}^2]}{[-(1-v_{i-1})^2h^2+h_{i-1}^2v_i^2]}$$

$$+\{e_i[h_i\{h^2(1-v_{i-1})+h_{i-1}^2(1+v_{i-1})\ v_{i-1}\}$$

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$$+\frac{h_{i-1}\{-(1-v_i)(h^2+h^2)+2h_i^2\}}{\{-(1-v_i)^2h^2+h_i^2\}}]$$

$$+\frac{e_{i+1}[v_i h_{i-1}[(h^2)\,(1-v_i)+h_i^2]}{[-(1-v_i)^2h^2+h_i^2]}$$

+

$$\frac{h_{i-1}(1+2v_i)[-(1-v_i)(h^2+h_i^2)+2h_i^2]}{[-(1-v_i)^2+h_i^2]}$$

$$\frac{\Delta_i + h_i v_{i-1}(v_{i-1}+2)\left[(h_{i-1}^2+h^2)(1-v_{i-1})v_{i-1}+2h_{i-1}^2v_{i-1}^2\right]}{[-(1-v_{i-1})^2h^2+v_{i-1}^2h_{i-1}^2]}\Delta_{i-1}$$

$$\frac{-v_{i-1}h_i f_{i-1}^1[h^2(1-v_{i-1})v_{i-1}+h_{i-1}^2v_{i-1}^2]}{[-(1-v_{i-1})^2h^2+h_{i-1}^2v_{i-1}^2]}$$

$$f_i^{\{1\}}[h_i\{h^2(1-v_{i-1})+h_{i-1}^2(1+v_{i-1})\,v_{i-1}\}$$

$$\frac{+h_{i-1}\{-(1-v_i)h^2+2h_i^2\}]}{[-(1-v_i)^2h^2+h_i^2]}$$

$$\frac{v_i h_i f_{i+1}^{\{1\}}[-h^2(1-v_i)+2h_i^2]}{\{-(1-v_i)^2h^2+h_i^2\}}$$

After using from Lyche [12] formula we get,

$$\frac{\sum a_i = h_{i-1}\,(1+2v_i)[-(1-v_i)^2(h^2+h_i^2)+2h_i^2]}{[-(1-v_i)^2h^2+h_i^2]}$$

$$\frac{+h_i v_{i-1}(v_{i-1}+2)[(h_{i-1}^2+h^2)(1-v_{i-\ \ 1})v_{i-1}+2h_{i-1}^2v_{i-1}^2]}{[-(1-v_{i-1})^2h^2+v_{i-1}^2h_{i-1}^2]}$$

$$=\sum b_j$$

Where $x_{10}=x_i+h$

$x_{20}=x_i$

$x_{30}=x\ \ +h$

Where $x_{10}=y_{30}=x_{i+1}+h$

$x_{11}=y_{31}=x_i-h$

$y_{20}=x_{20}=x_i+h$

$y_{21}=x_{21}=x_i-h$

$x_{30}=y_{1-0}=x_{i-1}+h$

$x_{31}=y_{11}=x_{i-1}h$

and

$$a_1=\frac{h_{i-1}(1+2v_i)(-(1+v_i(h^2+h_i^2)+2h_i^2)}{[-(1-v_i)^2+1]}$$

$$a_2=\frac{h_i v_{i-1}(v_{i-1}+2)[(h_{i-1}^2+h^2)(1-v_{i-1})+2h_{i-1}^2v_{i-1}^2]}{[-(1-v_{i-1})^2+h^2+v_{i-1}^2h_{i-1}^2]}$$

$$b_1=\frac{v_i-h_i(h^2(1-v_{i-1})v_{i-1}h_{i-1}^2v_{i-1}^2)}{[-(1-v_{i-1})^2h^2+h_{i-1}^2\,v_{i-1}^2]}$$

$$b_2=\frac{\{h_i(h^2(1-v_{i-1})v_{i-1}+h_{i-1}^2(1+v_{i-1})v_{i-1}+h_{i-}1\{-(1-v_i)\mu_i h^2+2h_i^2\}}{[-(1-v_{i-1})^2h^2+h_{i-1}^2v_{i-1}^2]}$$

$$b_3=\frac{v_i h_{i-1}\{-h^2(1-v_i)+2h_i^2\}}{\{-(1-v_i)^2h^2+h_i^2\}}.$$

Clearly $k(h)=\sum a_j=\sum b_j$. (.....4.4)

This theorem (4.1) is completed.

V. CONCLUSION

we have investigate method of Discrete Rational cubic Spline Interpolations to obtain construction, Shape, error bound and convergence.

Table 1. Given Data

i	1	2	3	4	5	6	7	8	9	10	11
x_i	−1	4	8	16	20	24	28	32	36	40	44
y_i	18	3	0.05	1	4	8	15	24	34	40	43

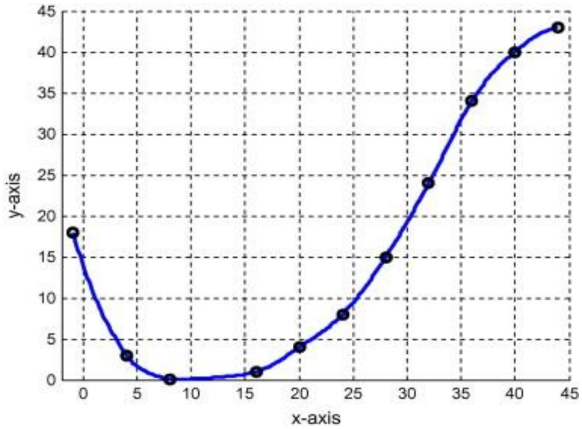

Figure 2: Rational cubic function

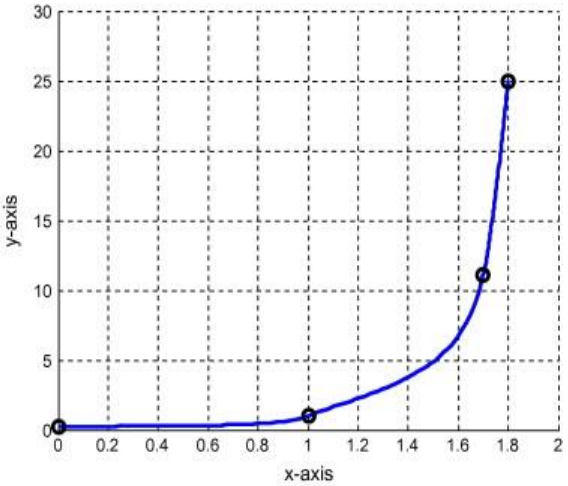

Figure 3. C^2 rational cubic curve with constant parameter

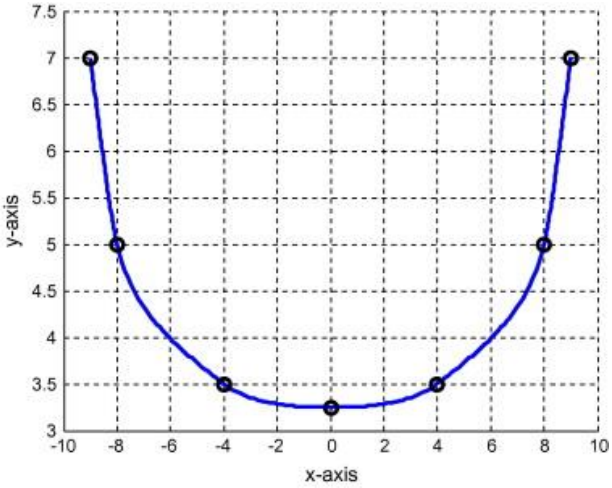

Figure 4. C^2 convex rational cubic curve

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