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Abstract

We develop a tractable model of price dynamics in a general equilibrium economy with cash-in-advance constraints. The dynamics emerge from local interactions between firms that are governed by the production network underlying the economy. We analytically characterise the influence of network structure on the propagation of monetary shocks. In the long run, the model converges to general equilibrium and the quantity theory of money holds. In the short run, monetary shocks propagate upstream via nominal changes in demand and downstream via real changes in supply. Lags in the evolution of supply and demand at the micro level can give rise to arbitrary dynamics of the distribution of prices. Our model provides an explanation of the price puzzle: a temporary rise in the price level in response to monetary contractions. In our setting, the puzzle emerges under two assumptions about downstream firms: they are disproportionally affected by monetary contractions and they account for a sufficiently small share of the wage bill. Empirical evidence supports the two assumptions for the US economy. Our model calibrated to the US economy using a data set of more than fifty thousand firms generates the empirically observed magnitude of the price level rise after monetary contractions.

JEL Codes C63, C67, D80, E31, E52

Key Words Price Puzzle, Production Network, Money, Monetary Non-Neutrality, Out-of-Equilibrium Dynamics.

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1 Introduction

A basic implication of the quantitative theory of money is that monetary contractions induce a decrease in the price level. However, numerous empirical studies report monetary contractions generate a temporary increase in the price-level. This "price puzzle" is sizeable: a monetary contraction that generates a 0.1% decrease in the price-level in three years is capable of generating a 0.1% increase in three months (Rusnak et al., 2013, Table 4). The sizeable wrong directional movement in the price-level is significant because the price-level mediates the relation between money and output in theoretical models of monetary neutrality (Lucas, 1972; Ball and Mankiw, 1994). Within sticky-price and sticky-information models of monetary neutrality, a wrong directional change in the price-level implies a counterfactual relation between money and output (Mankiw and Reis, 2002). The Price Puzzle is therefore empirically sizeable and theoretically significant.

In this paper, we show that the price puzzle can be explained by the propagation of monetary shocks through the production network of the economy. The propagation of monetary shocks generate differential and time-varying responses in the supply and demand for consumer goods. The price puzzle emerges if monetary shocks have a *strong* short-term impact on the supply and a *weak* short-term impact on the demand of consumption goods. Two conditions prove sufficient to induce these dynamics: downstream firms must be disproportionally affected by monetary contractions and they must account for a sufficiently small share of the wage bill, or more precisely sufficiently small to ensure that demand for final goods does not decline relative to supply following a monetary shock. Both conditions find empirical support. Also, our model calibrated to the US economy generates the empirically observed magnitude of the wrong directional movement in the price-level following a monetary contraction.

Our approach builds on Acemoglu et al.'s (2012) model of the network origins of aggregate fluctuations. We consider an economy with a representative consumer and a finite number of firms with Cobb-Douglas production functions. The production network of the economy is identified with the weights of these Cobb-Douglas functions. We introduce two novel elements in this framework. First, we consider prices are set locally by firms. Second,we assume firms face cash-in-advance constraints.

These assumptions yield an analytically tractable model of price and monetary dynamics .

Building on the existing literature emphasizing the heterogeneous impact of monetary shocks on firms (Greenwald and Stiglitz, 1993; Holmstrom and Tirole, 1997; Clementi and Hopenhayn, 2006), we model negative monetary shocks as (non-uniform) decrease of the working capital of firms. A monetary contraction then affects both the demand and the supply of consumer goods but through different mechanisms and with different time lags. On the demand side, firms hurt by the initial impact and subsequent propagation of a monetary contraction decrease wages. The decrease in wages decreases the household's income and therefore the demand for consumer goods. The time necessary for a monetary contraction to generate the long run decrease in nominal demand for consumer goods depends on the topology of the production network and the distribution of labor among firms. On the supply side, firms hurt by a monetary contraction and its propagation temporarily decrease their output as their inputs are bid away by competing users. This decrease in output propagates downstream through the production network[1].

We then investigate how the propagation of monetary shocks affect price dynamics. In the long run, the model converges to general equilibrium and the quantity theory of money holds. In the short run, monetary disequilibrium propagates upstream via changes in nominal demand and downstream via changes in real supply. In this setting, we analytically characterise two conditions under which the price puzzle emerges. The first condition is firms hurt by monetary contractions are sufficiently downstream. This condition guarantees that monetary contractions generate a decrease in the supply of consumer goods. The second condition is firms hurt by monetary contractions bear a sufficiently small share of the economy's wage bill. This condition guarantees that monetary contractions take sufficient time to generate the long run decrease in the nominal demand for consumer goods.

Theory and data support the two aforementioned conditions. The first condition follows from the joint observation that downstream firms are disproportionately small *and* small firms are disproportionately hurt by monetary contractions (Gertler and Gilchrist, 1994; Gaiotti and Generale, 2002; Iyer et al., 2013; Carbó-Valverde et al., 2016).

Our second condition emphasizes that nominal demand decreases slowly because small firms which bear the brunt of the initial impact of monetary contractions account for a limited portion of the economy's wage bill. Firms with annual receipts of less than one million account for less than 8% of the wage bill in the US economy. Therefore the initial impact of a monetary contraction generates a limited decrease in wages.

We further investigate the empirical validity of the model by calibrating it on the US production network using a novel data set with more than 100,000 buyer-seller relations between more than 50,000 US firms including all major publicly listed firms. We use Monte Carlo methods to study the dynamics of the calibrated synthetic economy with computational experiments. In the experiments, the initial impact of monetary shock scales sublinearly with firm size: small firms are disproportionately hurt by contractions. The model calibrated to the US production network robustly reproduces the empirically observed magnitude of the increase in price level after monetary contractions.

1.1 Related literature

The existence of the Price Puzzle has been documented in a number of economies including Australia, France, Germany, India, Italy, Japan, United Kingdom, and the United States (Sims, 1992; Gaiotti and Secchi, 2006; Mishra and Mishra, 2012; Bishop and Tulip, 2017). Following the seminal work by Sims (1992) and Eichenbaum (1992), a large number of contributions have attempted to identify variables whose inclusion in VAR models would eliminate the puzzle.

Rusnak et al. (2013) present a meta analysis of the empirical literature on the subject. They find that about 50% of modern studies find a temporary rise in the price level following a monetary contraction after controlling for a variety of variables. Similarly, Ramey (2016, p.

[1] A corollary of the argument is that a monetary contraction can increase the supply of consumer goods if firms most hurt by the shock are sufficiently upstream. When firms far upstream are hurt by a monetary contraction, downstream firms purchase more of the inputs used by both sets of firms by outbidding upstream firms. This allows downstream firms to expand the output of consumer goods.

36) finds most models of monetary policy shocks "are plagued by the Price Puzzle to greater or lesser degree".

Against this background, three main types of explanations have been put forward in the literature. the "common factors hypothesis", the"reverse causation hypothesis", and the "cost channel hypothesis". The common factors hypothesis states that the price level increase and monetary contraction are driven by common factors (Brown and Santoni, 1987). The empirical performance of the common factors hypothesis is mixed. In a meta analysis of empirical studies on the Price Puzzle, Rusnak et al. (2013) find that eight of eleven estimates using factor augmented VAR exhibit the Price Puzzle.

The reverse causation hypothesis states that the present monetary contraction is caused by future increase in the price level (Brissimis and Magginas, 2006; Barakchian and Crowe, 2013; Cloyne and Hürtgen, 2016). More specifically, present monetary contraction is a consequence of a policy decision based on an expectation of a future increase in the price level. Thapar (2008) however finds the Price Puzzle to be present even after conditioning for the information set of central banks. And Hanson (2004) finds no correlation between a variable's ability to forecast inflation and its ability to resolve the Price Puzzle. Ramey (2016) re-estimates several models to control for all information available to the Federal Reserve. Her estimates show that the wrong directional movement in the price level does not dampen due to the expansion of the information set, in fact it increases the price puzzle.

The cost channel hypothesis argues that an increase in the interest rate increases the cost of production and therefore the price level (Barth III and Ramey, 2001)

However, since a negative monetary shocks entails a decrease in the quantity of money, the rise in prices of some goods must be more than compensated for by the fall in prices of other goods. Our model presents an explanation for why those goods whose prices rise happen to be disproportionately represented in basket of consumer goods.

Our approach also emphasizes the role of the production network in monetary transmission mechanisms. In this respect, our contribution is related to recent work on 'micro to macro via production networks' (see e.g. Acemoglu et al., 2012; Di Giovanni et al., 2014; Carvalho and Tahbaz-Salehi, 2018; Baqaee and Farhi, 2019; Moran and Bouchaud, 2019).

However, few of these contributions have investigated the role of production networks in monetary transmission mechanisms. Notable "historical" exceptions are Cantillon (1755), Mises (1953), and Friedman (1961). Among contemporary contributions, Nakamura and Steinsson (2010) and Paston et al. (2018) show intermediate inputs amplify the impact of monetary shocks. Anthonisen (2010) formalises monetary dynamics on production networks using an overlapping generations model. In a previous contribution, Mandel et al. (2019), we study price dynamics on production networks using an agent-based computational model and show, by means of simulations that there can be wrong-directional price movements following a monetary shock. However the analysis in Mandel et al. (2019) (i) focuses only on the micro level: it shows that certain prices move in the wrong direction but gives no evidence that this effect can scale up to the macro level, i.e. that the price level per se can move in the wrong direction, (ii) does not provide analytical results (it relies entirely on simulations), (iii) considers a "toy" simulation model rather than one calibrated on empirical data and (iv) more broadly does not discuss the empirical conditions under which the prize puzzle can materialize. Ò Ozdagli and Weber (2017) present the only notable empirical measure of the role of production network in transmitting monetary shocks. They attribute 50-85% of the real effect of monetary shocks to propagation through the production network. Preliminary empirical evidence therefore suggests the production network is a significant transmission mechanism.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analytically characterizes the propagation of monetary shocks in the model and provides sufficient conditions for the price level to increase after a monetary contraction. Section 4 presents empirical evidence on the sensitivity of small firms to monetary shocks as well as their contribution to the price level and to the wage bill in the US. Section 5 presents the results of simulations performed with the model calibrated to granular data on more than 100,000 buyer-seller relations between more than 50,000 US firms. Section 6 concludes the paper. All proofs are in the Appendix.

2 Model

2.1 The general equilibrium framework

We consider a general equilibrium economy as in Acemoglu et al. (2012) with a finite number of firms and a representative household. The firms and the differentiated goods they produce are indexed by $M = \{1, \cdots, n\}$. The household has index 0 and $N = \{0, \cdots, n\}$ denotes the set of agents. The representative household inelastically supplies λ_0 units of labor and has a Cobb-Douglas utility function of the form:

$$u(x_{0,1}, \cdots, x_{0,n}) := \prod_{j \in M} x_{0,j}^{a_{j,0}} \tag{1}$$

where for all $j \in M$, $a_{j,0} \in \mathbb{R}_+$ is the share of good j in the household's consumption expenditures, thus $\sum_{j \in M} a_{j,0} = 1$.

Each firm $i \in M$ has a Cobb-Douglas production function of the form

$$f_i(x_{i,0}, \cdots, x_{i,n}) := \lambda_i \prod_{j \in N} x_{i,j}^{a_{j,i}} \tag{2}$$

where $\lambda_i \in \mathbb{R}_{++}$ is a productivity parameter. For all $j \in M$, $a_{j,i} \in \mathbb{R}_+$ is the share of good j in firm i's intermediary consumption expenditures, thus $\sum_{j \in N} a_{j,i} = 1$ and there are constant return to scale. We assume each firm uses a non-zero quantity of labor in its production process, i.e. for all $i \in M$, $a_{0,i} > 0$. The production structure is a network with adjacency matrix $A = (a_{i,j})_{i,j \in N}$ where $a_{i,j} > 0$ if and only if agent i is a supplier of agent j and $|a_{i,j}|$ measures the share of good i in the production costs of j. The network economy is denoted by $\mathcal{E}(A, \lambda)$ and satisfies the following assumption.

Assumption 1. *The adjacency matrix of the production network A is irreducible and aperiodic.*

The adjacency matrix A is irreducible if there exists a path between every two nodes in the network, i.e. for every $i, j \in N$, there exists $t \in \mathbb{N}$ such that $A^t_{i,j} > 0$. An irreducible matrix is aperiodic if there is no period in the length of cycles around any node, i.e. the greatest common divisor of $\{k \in \mathbb{N} \mid \exists i \in N A^k_{i,i} > 0\}$ is 1. In our setting, mild sufficient conditions for irreducibility and aperiodicity can be expressed in terms of the relationship between firms and the representative household. Given that each firm uses some labor in its production process, the matrix is irreducible if each good enters the supply chain of at least one good consumed by the household (the production of a good not satisfying this condition would be zero at equilibrium). The matrix is aperiodic if at least one firm uses another good as an input in its production process, i.e. if it is not the case that all firms produce using labor only. To sum up, Assumption 1 holds in all but degenerate cases. The standard notion of general equilibrium can then be defined as follows for the network economy $\mathcal{E}(A, \lambda)$.

Definition 1. *A collection* $(\overline{p}_i, \overline{x}_i, \overline{q}_i)_{i \in N} \in \mathbb{R}^N_+ \times (\mathbb{R}^N_+)^N \times \mathbb{R}^N_+$ *of prices, intermediary consumption vectors, and outputs is a general equilibrium of* $\mathcal{E}(A, \lambda)$ *if and only if:*

$$\forall i \in M,\ \overline{q}_i = \lambda_i \prod_{j \in N} \overline{x}_{i,j}^{a_{j,i}} \text{ and } \overline{q}_0 = \lambda_0 \qquad \textit{feasibility} \quad (3)$$

$$\forall i, j \in N,\ \overline{x}_{i,j} := a_{j,i} \frac{\overline{p}_i \overline{q}_i}{\overline{p}_j} \qquad \textit{profit and utility maximization} \quad (4)$$

$$\forall i \in N,\ \sum_{j \in N} \overline{x}_{j,i} = \overline{q}_i \qquad \textit{market clearing} \quad (5)$$

This definition is standard but for the fact that the condition for profit and utility maximization have been expressed directly through first order conditions. It also accounts for the fact that profit at equilibrium is zero given there are constant returns to scale. Under Assumption 1, there exists a unique general equilibrium in the economy.

Proposition 1. *Up to price normalization, the economy* $\mathcal{E}(A, \lambda)$ *has a unique equilibrium* $(\overline{p}_i, \overline{x}_i, \overline{q}_i)_{i \in N} \in \mathbb{R}^N_+ \times (\mathbb{R}^N_+)^N \times \mathbb{R}^N_+$, *which moreover satisfies:*

$$\forall i \in M,\ \overline{p}_i = \frac{1}{\lambda_i} \prod_{j \in N} \left(\frac{\overline{p}_j}{a_{j,i}}\right)^{a_{j,i}}. \quad (6)$$

Using the normalization $\overline{p}_0 = 1$, *this can also be written as:*

$$\log(\overline{p}) = (I - A'_{|M})^{-1} \gamma \quad (7)$$

where A' *denotes the transpose of* A, $A'_{|M}$ *its restriction to* M *and for all* $i \in M$, $\gamma_i := -\log(\lambda_i) - \sum_{j \in \mathcal{M}} a_{j,i} \log(a_{j,i})$.

2.2 Dynamics

Building on Gualdi and Mandel (2016), we introduce a simple discrete-time model of out-of-equilibrium dynamics in the network economy $\mathcal{E}(A, \lambda)$ in order to study the short-term effects of monetary shocks on prices.

The defining characteristic of our approach is that we model local decision-making of firms without a central signal about prices or quantities. This implies agents must use a decentralized mechanism to make decisions about prices they set, combinations of inputs they demand, and quantities of output they produce.

Agents must hold money balances in order to engage in decentralized trades. We assume each agent $i \in N$ initially holds a certain money balance $m_i^0 \in \mathbb{R}_+$ and denote by m_i^t the money balance of agent i in period $t \in \mathbb{N}$. The money balances of the household correspond to its consumption budget. The money balances of a firm can be thought of as its working capital or as a credit line extended by funders. Each firm $i \in M$ initially holds a certain stock of output $q_i^0 \in \mathbb{R}_+$. More generally, we denote by $q_i^t \in \mathbb{R}_+$ the stock of output of firm i at the beginning of period t. The household supplies a constant quantity of labor $q_0^t = \lambda^0$ every period. We study the dynamics of prices, quantities, and money balances which emerge from such a setting.

With Cobb-Douglas production technologies the optimal allocation of the working capital of a firm among intermediary inputs is independent of the prices of inputs. More specifically, given working capital m_i^t firm i's nominal demand to agent $j \in N$ is equal to $a_{j,i}m_i^t$. Similarly, the representative household's optimal nominal demand to agent j is $a_{j,0}m_0^t$ because preferences are Cobb Douglas. Thus total nominal demand faced by agent j is $\sum_{i\in N} a_{j,i}m_i^t$. For sake of parsimony and to remain as close as possible to the conventional equilibrium framework, as a first approximation we assume prices are fully flexible and thus each agent $j \in N$ sets its price in period t to:

$$p_j^t = \frac{\sum_{i\in N} a_{j,i}m_i^t}{q_j^t} \tag{8}$$

This has two implications. First, firms sell their complete stock of output every period and thus do not carry inventories. Second, the intermediary consumption of good j by firm i in period t, $x_{i,j}^t \in \mathbb{R}_+$ is given by:

$$x_{i,j}^t = \frac{a_{j,i}m_i^t}{p_j^t} \tag{9}$$

Hence, the production and the stock of output available next period is:

$$q_i^{t+1} = f_i(x_i^t) \tag{10}$$

Finally, each firm i must determine the share of revenues to carry over as working capital to the next period. In this paper, we consider a stylized setting in which all revenues are carried over. Namely, we assume:

$$m_i^{t+1} = \sum_{j\in N} a_{i,j}m_j^t \tag{11}$$

Remark 1. *Equation 11 implicitly conveys assumptions about the target profit rate of the firm on the one hand and its demand expectations on the other hand.*

- *First, the choice of the share of revenues carried over as working capital for the next period implicitly corresponds to the choice of the profit rate at a stationary state. In our setting as all revenues are reinvested, the profit rate at a stationary state ought to* *be zero. This ensures that the stationary state is consistent with general equilibrium (see Proposition 2 below). From a behavioural perspective, it amounts to consider that competitive pressure, or threat to entry, is such that firms consider zero profit rate as the benchmark.*

The model can be extended to a setting where firms pay a dividend, or an interest, to its funders by assuming that a firm with revenue $m \in \mathbb{R}_+$ only uses a share $\mu \in [0,1]$ of the revenue as working capital for next period. Then, at a stationary state, the firm makes a profit of $m - \mu m$ and the profit rate is $(1-\mu)$. This profit can be distributed to the funders of the firm as dividends in case of equity funding, or interests in case of debt funding. Our zero profit assumption corresponds to the case where $\mu = 1$. Considering an arbitrary $\mu \in [0,1]$ would not qualitatively change the linear structure of the model nor our results about price dynamics. The notion of equilibrium would however slightly deviate from the conventional one (see Gualdi and Mandel, 2016) and the model would have an additional free parameter. We thus focus on the case $\mu = 1$ below.

- *Second, the fact that the working capital for next period is determined as a function of the revenues of the current period only, amounts to assuming that the firms have myopic expectations and assuming that nominal demand is stationary. This assumption is consistent with the fact that all preferences and technologies in the economy are Cobb-Douglas and thus that nominal demand is à priori inelastic to price changes. However, this assumption neglects potential changes in the revenues, and thus in nominal demands, of customers. Furthermore, myopic expectations discard the possibility for the firm to save cash for precautionary motives in the face of the variability of its own revenues (note however that such precautionary motives would be unnecessary at a stationary state).*

Equations (8) to (11) define a dynamical system on $\mathbb{R}_+^N \times \mathbb{R}_+^N \times (\mathbb{R}_+^N)^N \times \mathbb{R}_+^N$, which define the trajectories of prices $(p_i^t)_{i \in N}^{t \in \mathbb{N}}$, intermediary and final consumptions $(x_i^t)_{i \in N}^{t \in \mathbb{N}}$, output $(q_i^t)_{i \in N}^{t \in \mathbb{N}}$ and money balances $(m_i^t)_{i \in N}^{t \in \mathbb{N}}$ in the network economy $\mathcal{E}(A, \lambda)$. In particular, this dynamical system provides an analytically tractable model of price dynamics with good properties of convergence towards equilibrium. Namely, the dynamics of prices can be characterized by the following lemma:

Lemma 1. *In the dynamical system defined by Equations* (8) *to* (11), *for all $t \in \mathbb{N}$ and for all $i \in M$:*

$$p_i^{t+1} = \frac{1}{\lambda_i} \frac{m_i^{t+2}}{m_i^t} \prod_{i \in N} \left(\frac{p_j^t}{a_{j,i}} \right)^{a_{j,i}} \tag{12}$$

Furthermore, if p^0 is an equilibrium price:

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t} \sum_{j \in N} A_{j,i}^{\tau} \log \left(\frac{m_j^{t+2-\tau}}{m_j^{t-\tau}} \right) \tag{13}$$

This lemma highlights the fact that price dynamics are eventually determined by monetary dynamics. In particular, the lemma underlines, through the term $m_j^{t+2-\tau}/m_j^{t-\tau}$, the role of a two-period long budgetary cycle. Although production is instantaneous, inputs are purchased using revenues from the previous period ($m_i^{t-\tau}$) and output is sold in the following period (yielding revenues $m_i^{t-\tau+2}$). This 2-period cycle impacts the price dynamics as the firm sets its price by confronting the nominal demand in the period following the production with output produced using the revenues from the period preceding the production.

Remark 2. *The existence of this 2-period cycle reflects lags in the accounting/budgetary process rather than in the production process. This lag is somehow minimal. Indeed, in our setting, production is instantaneous. However, goods cannot be sold more than once per period and thus there must be a one period lag between purchase of inputs and sale of output. Moreover, if there is a cash-in-advance constraint, or more generally if the financing conditions of firms depend on observables (e.g. in case of bank credit), the purchase of inputs depend on the revenues of the preceding period. Hence the two periods lag. If there is delay in production, the lag can be greater than two periods. More precisely, it is straightforward to extend the model to a setting where the production process of firm $i \in M$ lasts for $\nu_i \in \mathbb{N}$ periods, Equation 13 is then extended to*

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t} \sum_{j \in N} A_{j,i}^{\tau} \log\left(\frac{m_j^{t+2-\tau}}{m_j^{t-\tau-\nu_j}}\right) \tag{14}$$

where price dynamics depend on monetary dynamics through the heterogenous distribution of lags $(2+\nu_j)_{j\in M}$.

As monetary dynamics are linear according to Equation (11), we can use standard results from the theory of Markov chains to show the convergence of monetary holdings to an invariant distribution and therefrom, using Lemma 1, the convergence of the system to general equilibrium. Namely, one has:

Proposition 2. *Assume $m^0 > 0$, $q^0 > 0$ and $p^0 > 0$. The only globally asymptotically stable state of the dynamical system defined by equations (8) to (11) is the element $(\overline{m}_i, \overline{p}_i, \overline{x}_i, \overline{q}_i)_{i\in N} \in \mathbb{R}_+^N \times \mathbb{R}_+^N \times (\mathbb{R}_+^N)^N \times \mathbb{R}_+^N$ such that:*

$$\overline{m} = A\overline{m} \tag{15}$$

$$(\overline{p}, \overline{x}, \overline{q}) \text{ is a general equilibrium of } \mathcal{E}(A,\lambda) \tag{16}$$

$$\sum_{j\in M} \overline{p}_j \overline{q}_j = \sum_{j \in M} \overline{m}_j. \tag{17}$$

Moreover, there exists $\kappa \in \mathbb{R}_+$ such that for all $m^0 \in \mathbb{R}^N$, one has:

$$\|m_t - \overline{m}\| \leq \kappa \hat{\rho}^t \tag{18}$$

where $\hat{\rho}$ is the second largest eigenvalue of A.

In the absence of exogenous shocks, the out-of-equilibrium dynamics introduced above converges to the general equilibrium of the network economy $\mathcal{E}(A,\lambda)$. The level of prices is completely determined by the monetary mass in circulation. And in equilibrium the revenue of a firm is proportional to its eigenvector centrality (Equation 15). The speed of convergence and thus the very notion of the long-run depends on the structure of the network through the second largest eigenvalue.

3 Monetary shocks and price dynamics

In this section, we aim to analyze the impact of exogenous monetary shocks on price dynamics. As hinted at by Lemma 1, a potential channel for this impact is through changes in the working capital of firms. Monetary policy cannot impact directly cash holdings of firms but if these holdings are associated to a form of external funding, e.g. a credit line offered by a bank, the reduction of the latter mechanically induces a reduction of the former.

Accordingly, in the following, we represent a negative monetary shock as the reduction of the working capital available to firms[2]. More precisely, we assume (although we do not model it explicitly) that the working capital of firms is financed by some form of external funding, e.g. a credit line. The economy is initially in an equilibrium state, which is in particular such that firms are not credit constrained. The monetary shock is assumed to reduce the credit line of firms, and thus their working capital, below the level required to sustain equilibrium. Hence, following the monetary shock, firms temporarily become credit constrained (as in e.g. Greenwald and Stiglitz, 1993; Clementi and Hopenhayn, 2006). As emphasized below, the monetary shock and the induced credit constraints are eventually resorbed by the adjustment of the price level as the economy reverts back to equilibrium. We shall however mostly be concerned with the rich set of transient dynamics that emerge from the heterogeneous initial impact of monetary shock among firms.

More formally, we consider an economy $\mathcal{E}(\lambda, A)$ that is initially at an equilibrium $(m_i^0, p_i^0, x_i^0, q_i^0) := (\overline{m}_i, \overline{p}_i, \overline{x}_i, \overline{q}_i)$. The economy faces a monetary shock at the end of period 0. This shock is represented by a vector $\epsilon \in \mathbb{R}^N$ such that for all $i \in N$ the working capital of agent i is shifted to $\overline{m}_i + \epsilon_i$. Shocks can have heterogenous impacts at the micro-economic level. The long-term behavior of the economy following the shock is completely determined by Proposition 2. Namely:

Corollary 1. *An economy at the equilibrium $(\overline{m}_i, \overline{p}_i, \overline{x}_i, \overline{q}_i) \in \mathbb{R}_+^N \times \mathbb{R}_+^N \times (\mathbb{R}_+^N)^N \times \mathbb{R}_+^N$ subject to a monetary shock $\epsilon \in \mathbb{R}^N$ converges, for the dynamical system defined by equations* (8) *to* (11) *to the new equilibrium $(\hat{m}_i, \hat{p}_i, \overline{x}_i, \overline{q}_i) \in \mathbb{R}_+^N \times \mathbb{R}_+^N \times (\mathbb{R}_+^N)^N \times \mathbb{R}_+^N$ such that for all $i \in M$:*

$$\hat{m}_i = (1+\iota)\overline{m}_i \text{ and } \hat{p}_i = (1+\iota)\overline{p}_i \tag{19}$$

where $\iota = \dfrac{\sum_{i\in N} \epsilon_i}{\sum_{i \in N} \overline{m}_i}$.

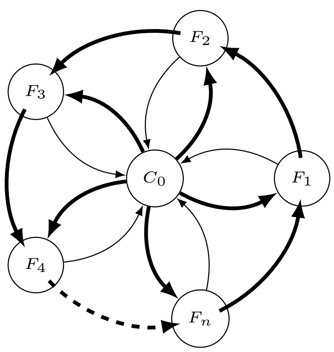
Hence in the long-run the the monetary shock and the induced credit constraints are resorbed by the adjustment of the price level and the economy reverts back to equilibrium. Moreover, the quantitative theory of money holds: prices adjust exactly to the quantity of money in circulation. Our principle question is however the short run dynamics of prices following a monetary shock. What drives the short run dynamics of prices is the heterogeneity of shocks at the micro-level. It is straightforward to check using Equation 8 that if shocks are homogeneous, i.e. if for all $i, j \in N$ $\epsilon_i/\epsilon_j = m_i/m_j$, prices adjust instantaneously to their new

[2]We neglect potential short-term credit constraints faced by households and thus assume their consumption budget is only indirectly impacted by the monetary shock, i.e. through changes in wages as the shock propagates. Indeed, short-term credit to household is much less available than to firms, except in certain countries such as the US.

equilibrium values. Non-trivial dynamics materialize only if the initial impact of monetary shocks is heterogeneous., i.e. if ${}^{\epsilon_i}/_{\epsilon_j} \neq {}^{m_i}/_{m_j}$ where either $\epsilon \in \mathbb{R}_+^N$ (positive monetary shock) or $\epsilon \in \mathbb{R}_-^N$ (negative monetary shock) .

3.1 Introductory Example

We consider the circular economy introduced in Figure 1 to highlight the effects at play when the initial impact of monetary shocks is heterogeneous. Within the circular economy the consumer assigns the same consumption budget to each firm and each firm uses a share $1/2$ of labor in its production process. Furthermore, each firm has a single supplier and a single buyer among firms. We assume the productivity parameter is such that $\lambda_i = 2$ for each firm. The household supplies n units of labor.



$$A = \begin{pmatrix} 0 & 1/2 & \dots & \dots & \dots & 1/2 \\ 1/n & 0 & \dots & \dots & \dots & 1/2 \\ \vdots & 1/2 & \ddots & & & 0 \\ \vdots & 0 & \ddots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 1/n & 0 & 0 & 0 & 1/2 & 0 \end{pmatrix}$$

Figure 1: A circular economy with n firms, $F_1, \cdots F_n$ and a consumer C_0. The consumer assigns the same consumption budget to each firm, each firm uses a share $1/2$ of labor in its production process, has a single supplier and a single buyer among firms. Links are directed from suppliers towards buyers. Thick lines correspond to weights of $1/2$ thin lines correspond to weights of $1/n$. The matrix A is the adjacency matrix of the network where the first line (resp. column) corresponds to the revenues (resp. expenses) of the household and the subsequent lines (resp. columns) to these of the firms.

In this framework, assuming total monetary mass is $3n$, it is clear that at equilibrium all prices are equal to 1, each firm holds 2 units of working capital, produces 2 units of output using 1 unit of labor and 1 unit of input from its supplier. The household holds n units of wealth and consumes 1 unit of every good. The equilibrium distribution of wealth, output, and prices are given by the following table.

agent	0	1	2	3	$\cdots$	n
$\overline{m}$	n	2	2	2	2	2
$\overline{q}$	n	2	2	2	2	2
$\overline{p}$	1	1	1	1	1	1

Suppose firm 1 receives a positive monetary shock of $2\epsilon > 0$ at the end of period 0. The induced change in demand for firm 1 leads to the following state in stage 1.

agent	0	1	2	3	$\cdots$	n
m^1	n	$2+2\epsilon$	2	2	2	2
q^1	n	2	2	2	2	2
p^1	$1+\epsilon/n$	1	$1+\epsilon/2$	1	1	1
m^2	$n+\epsilon$	2	$2+\epsilon$	2	2	2

Following the shock, firm 1 increases its demand to firm 2 and the household. This raises the price of labor and of firm 2's output. For firm 1 the rise in labor price is more than compensated by its increase in working capital. Firm 1 thus increases its output. For the other firms the rise in the price of labor induces a decrease in output. The state at $t=2$ is given by:

agent	0	1	2	3	$\cdots$	n
m^2	$n+\epsilon$	2	$2+\epsilon$	2	2	2
q^2	n	$2\sqrt{\frac{(1+\epsilon)^2}{(1+\epsilon/n)(1+\epsilon/2)}}$	$2\sqrt{\frac{1}{1+\epsilon/n}}$	$2\sqrt{\frac{1}{1+\epsilon/n}}$	$2\sqrt{\frac{1}{1+\epsilon/n}}$	$2\sqrt{\frac{1}{1+\epsilon/n}}$
p^2	$1+\epsilon/2n$	$\frac{2+\epsilon/n}{2\sqrt{\frac{(1+\epsilon)^2}{(1+\epsilon/n)(1+\epsilon/2)}}}$	$\frac{2+\epsilon/n}{2\sqrt{\frac{1}{1+\epsilon/n}}}$	$\frac{2+\epsilon/2+\epsilon/n}{2\sqrt{\frac{1}{1+\epsilon/n}}}$	$\frac{2+\epsilon/n}{2\sqrt{\frac{1}{1+\epsilon/n}}}$	$\frac{2+\epsilon/n}{2\sqrt{\frac{1}{1+\epsilon/n}}}$
m^3	$n+\epsilon/2$	$2+\epsilon/n$	$2+\epsilon/n$	$2+\epsilon/2+\epsilon/n$	$2+\epsilon/n$	$2+\epsilon/n$

The increase in nominal demand triggered by the initial shock at this stage is mainly concentrated on firm 3 and on the household. This implies firm 3 has to increase price given output has not increased. Furthermore, the nominal demand faced by firm 1 is decoupled from the increase in its output. Therefore, firm 1 has to decrease its price. Thus a positive monetary shock can induce both increase and decrease of prices in the short-run.

To simplify analysis, we assume n is large and discard the terms ϵ/n. This simplifies the expression of the state in stage 2 to:

agent	0	1	2	3	$\cdots$	n
m^2	$n+\epsilon$	2	$2+\epsilon$	2	2	2
q^2	n	$2\sqrt{\frac{(1+\epsilon)^2}{(1+\epsilon/2)}}$	2	2	2	2
p^2	1	$\sqrt{\frac{1+\epsilon/2}{(1+\epsilon)^2}}$	1	$1+\epsilon/4$	1	1
m^3	$n+\epsilon/2$	2	2	$2+\epsilon/2$	2	2

The shock propagates further through two effects on production. Firm 2 increases its output because of the increase in nominal demand it faced. Firm n increases its output because of the decrease in the price of the input it buys from firm 1. This leads to the following state in stage 3 (neglecting terms of order ϵ/n):

agent	0	1	2	3	4	$\cdots$	n
m^3	$n+\epsilon/2$	2	2	$2+\epsilon/2$	2	2	2
q^3	n	2	$2\sqrt{\frac{(1+\epsilon/2)^2}{(1+\epsilon/4)}}$	2	2	2	$2(\frac{(1+\epsilon)^2}{1+\epsilon/2})^{1/4}$
p^3	1	1	$\sqrt{\frac{1+\epsilon/4}{(1+\epsilon/2)^2}}$	1	$1+\epsilon/8$	1	$(\frac{1+\epsilon/2}{(1+\epsilon)^2})^{1/4}$
m^4	$n+\epsilon/4$	2	2	2	$2+\epsilon/4$	2	2

As for firm 1 in stage 2, the increase of output by firms 2 and n is not matched by corresponding increase in demand. Both firms thus decrease prices.

In the following periods, the sequence of events "positive demand shock, increase of output, decrease in price, increase in output" propagates further. However, the size of the shock decays progressively each period as a part of the shock is absorbed by the household which redistributes demand uniformly among firms (through the term ϵ/n neglected in our analysis). Eventually, the shock is evenly distributed among firms generating a general rise in prices by a factor of $1+2\epsilon/3n$.

3.2 Network structure and transmission of shocks

The above example shows that the impact of a monetary shock can be ambiguous in the short run. For example, a positive monetary shock can induce both increase and decrease in prices in the short run. The temporary volatility in prices generated by a monetary shock propagates across the network through two channels. One, a direct channel whereby the monetary shock induces a change in demand for inputs. Two, an indirect channel whereby firms respond to changes in the prices of inputs.

More generally, the short-term dynamics of prices is related to the network structure as follows. From equation 11, we have $m^t = A^t(m^0+\epsilon) = A^t m^0 + A^t\epsilon = m^0 + A^t\epsilon$. Using Equation (13) we can completely characterize the dynamics of prices following a monetary shock by the structure of the supply network. First, for $t=2$, for all $i \in N$:

$$\log(p_i^2) = \log(p_i^0) + \log\left(\frac{m_i^3}{m_i^1}\right) + \sum_{k\in N} A_{k,i}\log\left(\frac{m_k^2}{m_k^0}\right) \tag{20}$$

$$= \log(p_i^0) + \log\left(\frac{m_i^0 + A^3_{i,\cdot}\epsilon}{m_i^0 + A_{i,\cdot}\epsilon}\right) + \sum_{k\in N} A_{k,i}\log\left(\frac{m_k^0 + A^2_{k,\cdot}\epsilon}{m_k^0}\right) \tag{21}$$

Equation (21) suffices to illustrate the main drivers of price dynamics. The primitive cause of the variation of prices is the non-uniform speed of propagation of monetary shocks across the network. The shocks get propagated through walks of different lengths in the network and thus reach a given node with different lags. This implies firms might experience changes in the nominal demand they face as highlighted by terms of the form m_k^{t+2}/m_k^t. The changes in nominal demand has a direct impact on prices corresponding to the term $\log\left(m_i^3/m_i^1\right)$ in

Equation (20). The direct channel induces a rate of change of prices that is proportional to the rate of change of nominal demands. This direct impact propagates upstream in the network: from buyers to sellers. The changes in nominal demand also has an indirect impact represented by the term $\sum_{k\in N} A_{k,i} \log\left(m_k^2/m_k^0\right)$, which corresponds to the feedback effect on a firm from the changes in the prices of inputs induced by the direct demand effect. This indirect effect is mediated by the fact that changes in prices of the suppliers affects the production level of the firm and thus its price. The indirect effect propagates downstream in the network: from sellers to buyers.

More generally, the relation between price dynamics, network structure, and shocks is given by:

$$\log(p_i^{t+1}) - \log(p_i^0) = \sum_{\tau=0}^{t-1} \sum_{k\in N} A_{k,i}^{\tau} \log\left(\frac{m_k^0 + A_{k,\cdot}^{t+2-\tau}\epsilon}{m_k^0 + \xi_{\{\tau<t\}} A_{k,\cdot}^{t-\tau}\epsilon}\right) \tag{22}$$

where ξ denotes the indicator function.

A linear expansion of this expression highlights the relative role of upstream and downstream propagation mechanisms. Namely, denoting by $log(p^t) \in \mathbb{R}^N$ the column vector formed by the $\log(p_i^t)$ and by A' the transpose of A, one has:

Proposition 3. *For small enough monetary shocks ϵ, the dynamics of prices can be approximated by:*

$$\log(p^{t+1}) - \log(p^0) = \sum_{\tau=0}^{t} (A')^{\tau} \Delta_m A^{t-\tau}(A^2 - I)\epsilon + (A')^t \Delta_m \epsilon \tag{23}$$

where Δ_m is the diagonal matrix whose coefficients are the $(m_k^0)_{k\in N}$.

In line with the preceding discussion, Equation (23) highlights the fact that the primitive driver of the dynamics is the volatility in nominal demand induced by the initial shock. This is represented by the term $(A^2 - I)\epsilon$ corresponding to the differences between the direct impact of the shock ϵ and its impact after a two period lag $A^2\epsilon$. As emphasized in Remark 2, the two period lag embodies the fact that inputs are purchased using the revenues from the previous period and yield new revenues in the following period. This lag in the propagation of shocks creates a mismatch between supply and demand which is resorbed by a deviation of prices from their equilibrium values. The size of this initial deviation $(A^2 - I)\epsilon$ is determined by the local structure of the network. More precisely, it depends on the incoming degree of order two i.e. the weights of incoming paths of length two. The initial price deviation is likely to be small if the network is homogeneous and the distribution of two-degrees is centred around average (equal to one). If a node has a large incoming degree of order two i.e. it is a large indirect supplier of the economy, it is likely to initially react in an unconventional manner to a monetary shock. For example, the firm may decrease its price following a positive monetary shock (indeed $A_{i,\cdot}^2\epsilon - \epsilon_i$ is likely to be positive if $\epsilon \in \mathbb{R}_+^N$). Conversely, a node with small incoming degree of order two is likely to initially exacerbate the effect of a monetary shock. This role of second order interconnectivity is reminiscent of the results in Acemoglu et al. (2012).

This initial volatility is then propagated in the network. The propagation is represented by the linear operator $\sum_{\tau=0}^{t}(A')^{\tau}\Delta_m A^{t-\tau}$. The operator corresponds to the combination of upstream/forward and downstream/backward propagation channels identified above. The term $A^{t-\tau}$ corresponds to the upstream propagation of the variation of demand and of the correlated variation of price from a firm to its suppliers. The term $(A')^{\tau}$ corresponds to the propagation of the feedback effect induced by the price variation of a firm on the price of its buyers through changes in production costs. Hence the term $(A')^{\tau}A^{t-\tau}(A^2-I)\epsilon$ can be interpreted as the propagation of the shock $(A^2-I)\epsilon$ for $t-\tau$ steps in the upstream/forward direction and then for τ steps in the downstream/backward direction. In other words, the shock propagates through demand effects for $t-\tau$ steps via $A^{t-\tau}$ and through price effects for τ steps via $(A')^{\tau}$. More formally, $(A')^{\tau}A^{t-\tau}$ is the matrix of weights of walks of length n whose $t-\tau$ first steps are in the direction of the edges of the network and whose τ last steps are in the opposite direction. The factor Δ_m corresponds to the scaling of the shock proportionally to the initial monetary holdings. The operator $\sum_{\tau=0}^{t}(A')^{\tau}\Delta_m A^{t-\tau}$ corresponds (up to the rescaling factor Δ_m) to the total weights of walks of length t in the network when allowing for both forward and backward propagation. To sum up, the log-variation of price at the end of period $t+1$, $\log(p^{t+1})-\log(p^0)$, corresponds to the propagation of the initial shock $(A^2-I)\epsilon$ through the network in both forward and backward directions. The term $(A')^{t}\Delta_m\epsilon$ is a corrective term accounting for the fact that there is no variation of prices in period 0 to be propagated backwards.

Remark 3. *The propagation operator is very similar to the matrix of weights of walks of length t upon which the computation of feedback centrality measures is based (see e.g. Bonacich, 1987; Jackson, 2010). Explicitly, if one discards the rescaling factor Δ_m, one has $\sum_{\tau=0}^{t}(A')^{\tau}A^{t-\tau}=(A+A')^{t}$, which corresponds to the matrix with the walks of length t for the adjacency matrix $\tilde{A}:=A+A'$ of the undirected network obtained by discarding the direction of links in A. Thus, up to the rescaling factor Δ_m, the cumulative logarithmic variation of prices at time T, $V_T:=\sum_{t=0}^{T}\log(p^{t+1})-\log(p^0)$, is proportional to the sum of the Bonacich centrality, truncated at time T, of the undirected network $\tilde{A}$ with weights $(A^2-I)\epsilon$ and of the original network A with weights ϵ (see Ballester et al., 2006).*

It is worth noting that the transient variations in nominal demand at a node i following a shock $m_i^{t+2}/m_i^{t}=(m_i^0+A_{k,\cdot}^{t+2}\epsilon)/(m_i^0+A_{k,\cdot}^{t}\epsilon)$ are to a certain extend arbitrary. The transient changes in demand are determined by the ratio between the weights of walks of different length leading from the sources of the shocks to the node under consideration. This ratio is determined by the structure of the network and can assume a wide range of value as discussed below (see Proposition 4). A foremost consequence of this remark is that a given monetary shock can induce heterogeneous transient price response in different parts of the production network. In particular, a positive monetary shock could induce negative price change in the short-run. In what follows, we derive the behavior of the distribution of prices and the price-level in the course of the out-of-equilibrium propagation of monetary shocks.

3.3 The dynamics of the distribution of prices

At the micro-economic level, empirical evidence suggests monetary shocks generate wrong directional movement in some prices. For instance, Lastrapes (2006) and Balke and Wynne (2007) find that about 50% of commodity prices change in the wrong direction in response to monetary shocks[3]. Our model provides an explanation of the wrong directional movement in some prices.

Suppose a local positive shock induces an increase in the working capital of a firm i and consequently its demand for inputs from firm j. Firm j is likely to increase its price. However in so far as other buyers of firm j are not yet affected by the shock, the increase in price may be less than the increase in demand by firm i. This allows firm i to purchase more inputs and increase output. Given the local nature of the shock, the increase in output may not be matched by a corresponding rise in demand faced by firm i. And thus firm i may decrease its price next period. The price decrease induced by the intertemporal gap between supply and demand faced by firms propagates upstream through the supply chain, i.e. it affects the suppliers of the firm with a delay. Firm i's price decrease creates a second source of potential price anomalies: following the price decrease, the buyers of firm i can increase their output. The process can lead to further decrease in prices if the increase in output is not matched by a corresponding increase in demand. This effect propagates downstream through the supply chain: from a firm to its buyers. These price anomalies progressively disappear as the initial monetary shock diffuses in the network. Over time the increase in nominal demand becomes uniform and is matched by a corresponding general rise in prices. In the long-run the quantity theory of money holds.

As emphasized in Propositions 1 and 3, the magnitude and the duration of the price anomalies are determined by the structure of the network and the characteristics of the initial shock. At the micro-economic level, the variations induced by the shock can be arbitrary. Namely, one has:

Proposition 4. *For every finite sequence of prices* $\mathbf{p} = (p_0, p_1, \cdots, p_T) \in \mathbb{R}_+^{T+1}$ *such that* $p_1 \geq p_0$, *there exists a network economy* $\mathcal{E}(A, \lambda)$, *a monetary shock* $\epsilon \in \mathbb{R}^N$, *and a firm* i *such that the sequence of prices of firm* i *in the* T *first periods is* $\mathbf{p}$.

More generally, according to Proposition 3, the dynamics of the price distribution is completely determined by the structure of the network.

3.4 The Price Puzzle

Numerous econometricians have reported a short run increase in the price level in response to monetary contractions (see e.g. Hanson, 2004; Rusnak et al., 2013, and references therein). Our model generates a rise in the price level if monetary contractions disproportionately affect the suppliers of consumer goods but are transmitted to consumer's final demand with a lag. Under these two conditions, in the short run the supply of consumer goods decreases more than the final demand thereby generating an increase in the prices of consumer goods. In what follows, we characterize a general class of "supply-chain" economies which generate an increase in the price level following a contractionary monetary shock.

We define a supply-chain economy as a network economy $\mathcal{E}(A, \lambda)$ together with a set of L layers: $\{\mathcal{L}_1, \cdots, \mathcal{L}_L\}$. The set of layers form a partition of the set of firms M. A firm in layer ℓ

[3]Balke and Wynne (2007) use data on monthly changes in the components of commodities Producer Price Index (PPI). At the highest level of disaggregation (the eight digit level) the data contains 130 time series in 1959 to 1228 time series in 2001. Lastrapes (2006) uses data of 69 commodity price indexes with complete monthly observations from 1959:1 to 2003:10.

has all its suppliers in layer $\ell - 1$ and all its consumers in layers $\ell + 1$ (among firms). In other words, the adjacency matrix (restricted to firms) of the economy is upper triangular by block. This structure is very similar to the one considered in early contributions investigating the propagation of shocks in production networks (Bak et al., 1993; Scheinkman and Woodford, 1994; Battiston et al., 2007). We assume the household consumes goods produced by firms in layer $\mathcal{L}_L$. The household's supply of labor is normalized to 1. Let k denote the share of labor supplied to layer $\mathcal{L}_L$ at equilibrium. Assuming without loss of generality that the total wealth is such that the price of labor is normalized to 1, one has:

$$k = \sum_{i \in \mathcal{L}_L} a_{0,i} m_i^0 \text{ and } (1-k) = \sum_{j \in \mathcal{L}_{-L}} a_{0,j} m_j^0 \tag{24}$$

where $\mathcal{L}_{-L}$ denote the layers others than L.

In this setting, suppose all firms in the $\mathcal{L}_L$ are affected by an homogeneous negative monetary shock of magnitude ρ: the working capital of each firm i in layer $\mathcal{L}_L$ decreases to $(1-\rho)m_i^0$. One has:

Proposition 5. *Assume the share of labor supplied to layer $\mathcal{L}_L$ at equilibrium is k and that there is an homogeneous negative monetary shock of magnitude ρ on the firms of layer $\mathcal{L}_L$, then the price of firms of layer $\mathcal{L}_L$ in period 2 is given by:*

$$p_i^2 = \frac{(1-\rho k)^{a_{0,i}+1}}{(1-\rho)^{a_{0,i}}} p_{i,0}. \tag{25}$$

Corollary 2. *Under the conditions of Proposition* 5, *for k sufficiently small, following a negative monetary shock, all consumer prices (i.e. all prices in layer $\mathcal{L}_L$) increase in the short run (i.e. in period* 2*).*

Under Corollary 2 a negative monetary shock induces a short-term increase (at least in period 2) of the price level. Two conditions are sufficient for a rise in the price level in response to monetary contractions to emerge in the network economy. First, the shock must be sufficiently downstream (in layer $\mathcal{L}_L$). This condition ensures a monetary contraction generates a decrease in the supply of consumer goods. Second, firms initially affected by the monetary contraction must bear a sufficiently low proportion of the economy's wage bill (k must be sufficiently small). This condition ensures that the monetary contraction takes sufficiently long time to generate the full long run decrease in final demand. The two conditions jointly guarantee that in the short run supply decreases more than final demand, thereby generating an increase in the price-level in response to monetary contractions. The lower the share k of downstream firms in the wage bill, the larger the discrepancy between the variations of final supply and demand for consumer goods. And thus the more pronounced the rise in the price level after a monetary contraction as highlighted by Equation 25.

4 Empirical evidence

Our explanation of the price puzzle relies on three empirical assertions: (i) the fact that small firms are disproportionally affected by monetary shocks, (ii) the fact that there is a disproportionately high presence of small firms downstream and (iii) the share of small firms in the wage bill is small. Subsection 4.1 summarises the substantial evidence in the literature that small firms are disproportionally affected by monetary shocks. Subsection 4.2 presents empirical evidence supporting the claim that small firms are disproportionately present downstream. Subsection 4.3 analyzes the share of small firms in the wage bill.

4.1 Empirical evidence on the impact of monetary contraction on small firms

There is substantial empirical evidence published in the literature supporting the claim that small firms are disproportionally affected by monetary shocks.

- First, there is a direct empirical evidence that small firm are more affected by monetary contractions in the US (Gertler and Gilchrist, 1994; Thorbecke and Coppock, 1996; Basistha and Kurov, 2008), Germany (Ehrmann, 2005), Japan (Masuda, 2015), and the UK (Bougheas et al., 2006). Overall, there is a reallocation of all kinds of credit from small to large firms after a monetary contraction (Oliner and Rudebusch, 1996).

- Second, there is indirect evidence stemming from the fact firm size appears to have a significant impact on the response to monetary policy at the sectoral and regional levels. Peersman and Smets (2005) and Dedola and Lippi (2005) find that variation in the impact of monetary contraction across sectors is partly due to variation in firm size across the sectors. Similarly, the variation in the impact of monetary shocks across regions in the US is partly accounted for by variation in firm size across regions (Carlino and DeFina, 1998). Overall the impact of monetary policy is not neutral to firm size.

- Finally, there is evidence that small firms face binding financial and liquidity constraints (Froot, 1993; Carpenter and Petersen, 2002) and that these constraints affect their production decisions (BaÃśos-Caballero et al., 2002; Carreira and Silva, 2010; Chakraborty and Mallick, 2012). Accordingly, small firms are disproportionately affected by monetary contractions both through the bank-lending channel (Kashyap et al., 1996; Morgan, 1998) and the balance-sheet channel (Gaiotti and Generale, 2002; Ashcraft and Campello, 2007).

4.2 Small firms and price indices

4.2.1 Variation in firm sizes across sectors

In what follows, unless otherwise explicitly stated 'small firms' mean firms with annual sales of less an a million USD. Figure 2 presents the share of small firms across sectors in the US economy. The figure uses data from the Small Business Administration on the size distribution of the population of firms in the US economy pertaining to the year 2007. Figure 2 shows that there is sizeable heterogeneity in the share of small firms across different sectors of the US economy at the three digit NAICS-level. Sizeable heterogeneity at the three digit NAICS level exists with alternate definitions of small firms such as those with annual sales of less than ten million USD. Furthermore, the heterogeneity exists both at higher and lower levels of granularity. At the lowest level of granularity, we consider 1095 sectors[4]. At this level of granularity, firms with annual sales of less than ten million USD account for less than 1% of the total sales in 105 sectors, however they account for more than 10% of the total sales in 656 sectors. At the highest level of granularity, we consider nine major sector classifications[5]. Firms with annual sales of less than a million USD account about 20% the output of Services, while their share in Manufacturing, Mining, and Utilities is less than 2%.

[4]Of the 1095 sectors, 644 are at the NAICS six digit level, 378 at the five digit level, 72 at the four digit level, and 1 at the three digit level. All sectors are considered at the most granular level for which firm size distribution data is available.

[5]The major sectors are created by aggregating two digit NAICS sectors as follows: Agriculture: 11, Services: 61-62, 71-72, 81, Construction: 23, Retail: 44-45, Wholesale: 42, Manufacturing: 31-33, Mining: 21, Utilities: 22, Others: 48-56, 61-62.

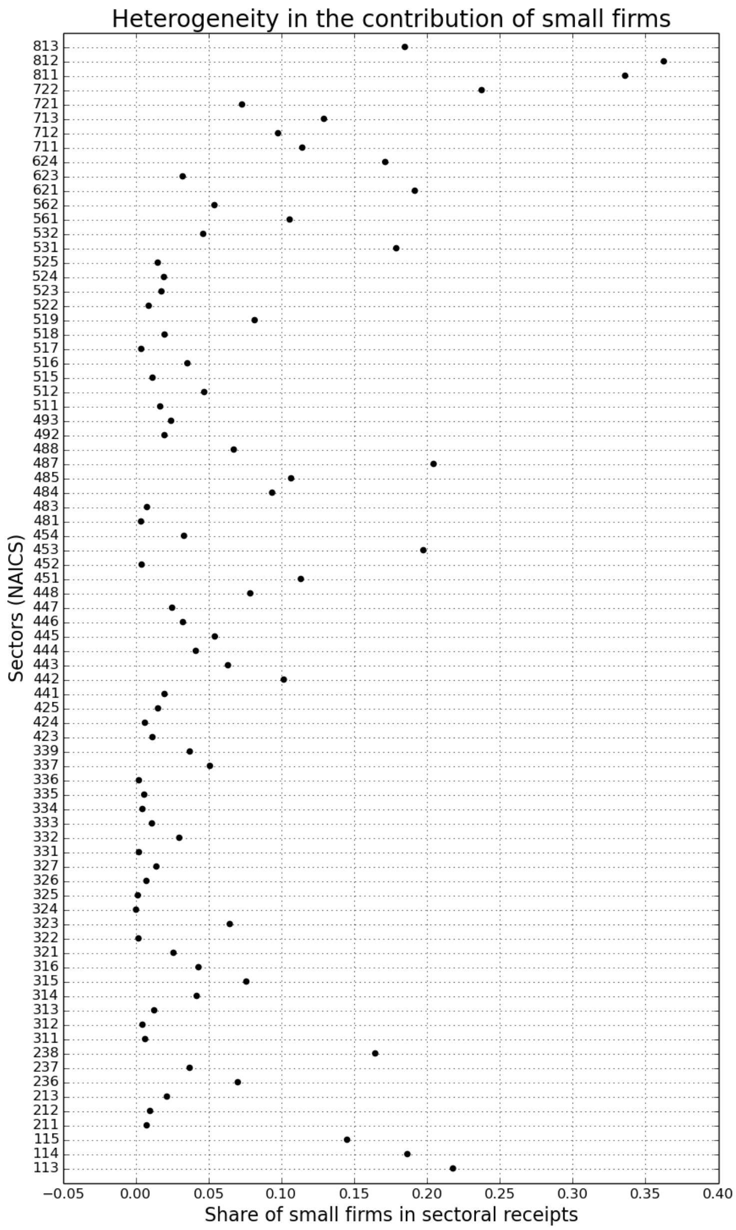


Figure 2: The contribution of small firms to total sectoral sales. The figure takes "small firms" to mean firms with annual sales of less than a million USD.

4.2.2 Small firms and the price-level

The sizeable difference in the share of small firms suggests that in so far as small firms respond differently to monetary shocks (see Section 4.1), the response of a price index to monetary shocks will depend on the sectoral composition of the index. We now present data to show the disproportionately high contribution of small firms to the basket of goods that make up the price-level. We consider two definitions of the price-level: Personal Consumption Expenditure (PCE) deflator and Consumer Price Index (CPI) Urban (The Supplementary Appendix presents details on the construction of the CPI). We combine data from the BEA on consumer expenditure across sectors with the data from SBA on size distribution of firms across sectors to determine the relation between the role of small firms in sector and the weight of the sector in the price-level. We find that sectors in which small firms account for more than 10% of the annual sales make up 14.7% of the economy, 34.8% of the PCE, and 48.8% of the CPI. In other words, sectors with a significant share of small firms have a higher representation in the price-level than in the economy, with these sectors playing a sizeably greater role in the CPI than the PCE[6]. A more direct measure is the share of small firms in PCE rather than the share of sectors with a significant share of small firms. This measure too suggests that small firms play a disproportionately high role in determining the price-level[7]. More specifically, small firms account for 8.6% (resp. 9.4%) of the sales of basket of goods used to compute the PCE (resp. CPI) while their share in the economy is 4.4%.

Figure 3 compares the share of small firms in a sector to the relative importance of the sector in CPI, which is measured by the ratio between the share of the sector in the economy and its share in the CPI. More specifically, the figure considers the 53 sectors plotted in Figure 2. The x-axis of Figure 3 marks the share of small firms in the total sales of a sector. The y-axis marks the ratio of the share of a sector in CPI to its share in the economy, i.e. the disproportionally in a sector's share in the price index. Pearson's correlation coefficient between the variables on the x and y axis of Figure 2 is 0.22. The share of small firms in a sector is therefore positively correlated with its importance in the CPI. The correlation coefficient declines to 0.09 when we consider PCE instead of CPI. This is consistent with the previous observation about the higher presence of small firm sectors in CPI than in PCE.

[6]Note the basic mechanism of our model along with this empirical observation is consistent with the finding that CPI exhibits greater wrong directional change than the PCE.

[7]We match sectors using the NAICS-BEA codes mapping provided by the BEA. Not all the sectoral classification of the Input-Output are at the same NAICS digit level as the SBA size distribution. Exact matching of NAICS codes accounts for about 78% of PCE, whereas fuzzing matching accounts for more than 99% of PCE. Fuzzy matching of NAICS follows the simple rule of matching unmatched NAICS codes from the Input-Output table with up to 1-3 digit higher level codes.

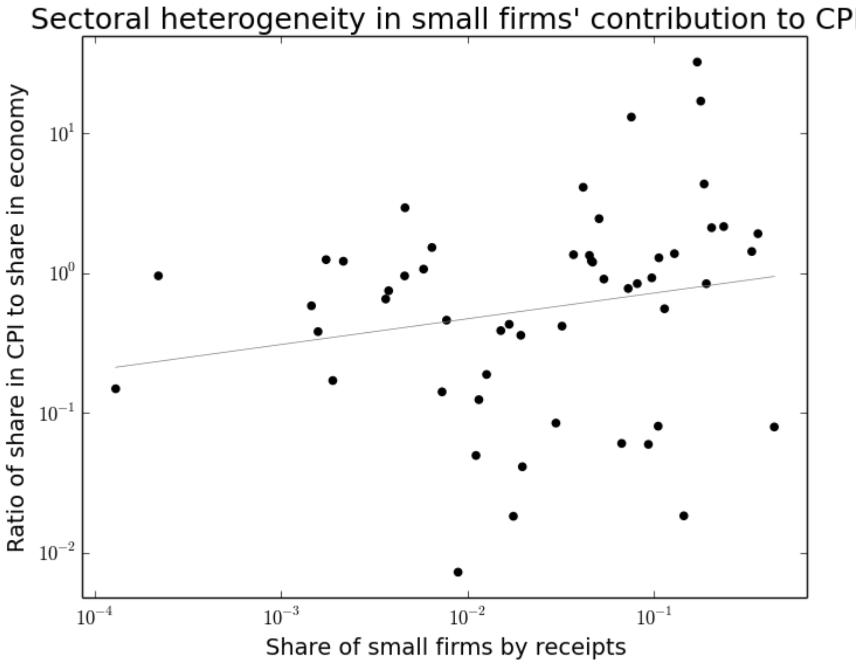


Figure 3: Contribution of small firms to CPI. Slope and intercept of OLS regression line are 0.13 and -0.07 respectively.

4.3 Share of small firms in the wage bill

We measure the share of small firms in the wage bill of the economy using Small Business Administration's data on sectoral contribution to wages for the year 2007. Figure 4 presents the relation between size of firms and their share in the economy's wage bill. The x-axis present annual receipts. The y-axis presents cumulative share in wages. Figure 4 shows that small firms do not account for the majority of the wage bill in the US economy. More specifically, firms with annual revenues of less than a million USD account for 8.6% of the economy's wage bill, and firms with annual revenues of less than ten million US account for less than a third of the economy's wage bill. Note that the share of small firms in the wage bill determines the initial impact of monetary contraction on nominal demand. The lower the share of small firms in the wage bill, the lower the initial decline in nominal demand, and therefore the lower the initial decline in real supply necessary for the realization of the Price Puzzle.

A direct comparison between the 8.6% percent share of small firms in the wage bill and their 9.4% share in the PCE might suggest this difference is far from sufficient to generate the Price Puzzle. Let us however first remark that the difference is more substantial when the CPI is taken into consideration as the share of small firms in the CPI is 10.7%. This difference of more than 20% scales up when an alternative definition of small firms is used. Furthermore, the direct impact of the monetary shock on nominal demand and real supply mediated by small firms is more than proportional to their share in the wage bill and the price index respectively because of the θ parameter. Finally, and more importantly, a major amplification of the shock on real supply occurs through network effects: if small firms are sufficiently downstream, upstream sectors will bid resources away from downstream sectors, thereby generating a decline in final output.

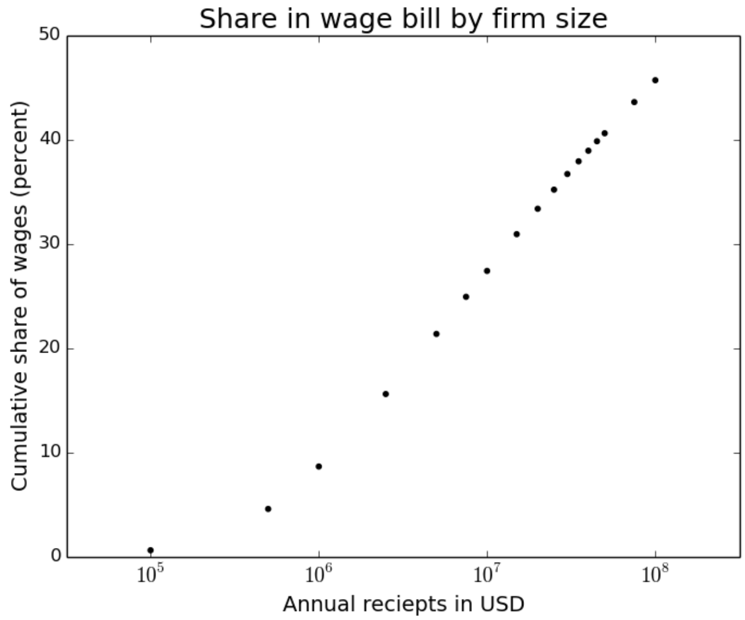


Figure 4: The plot marks the share of firms with different sizes in the wage bill of the economy. The y-axis is the cumulative wage bill. The x-axis is the log of the annual receipts with 17 different sizes.

5 Computational experiments on the U.S. production network

Section 3 illustrates the emergence of an increase in the price level in response to a monetary contraction within a stylised setting. In this section, we investigate the empirical relevance of our model by calibrating it to granular data on the US production network. Furthermore, we extend the model to study the consequences of a more general distribution of monetary shocks. Rather than assuming the initial impact of monetary shock falls upon firms in specific parts of the network, we allow all firms to bear the initial impact of monetary shocks albeit to different degrees. The magnitude of the initial impact of a monetary shock is inversely correlated to the size of the firm.

5.1 US firm network data

We use firm-level data from S&P Capital IQ and Orbis BvD datasets to calibrate our model. Our data set includes 105,940 buyer-seller relationships between 51,913 firms for whom we have revenue information and NAICS codes[8]. While the coverage of our data set is sparse

[8]S&P Capital IQ has a rich database about supply relationships between US firms from 2005 to 2017. We restricted the universe of firms to private firms and publicly listed firms. We excluded banks, non-

compared to the universe of US firms, it is nevertheless the most comprehensive dataset on supply relationships between US firms currently available. Our data set contains both private and publicly listed firms whereas datasets used in previous studies on production networks are limited to only contain listed firms (Atalay et al., 2011; Wu and Birge, 2014) or aggregated input-output network (Acemoglu et al., 2012). The inclusion of a subset of non-listed firms is relevant given the role of small firms in our explanation of the price puzzle. Figure 5 shows our data set matches the stylized fact that the degree distribution of the production network follows a power-law (Atalay et al., 2011; Konno, 2009). And Figure 6 shows the revenue distribution of the firms in our data set follows a power-law like the revenue distribution of the universe of firms in the US (Axtell, 2001).

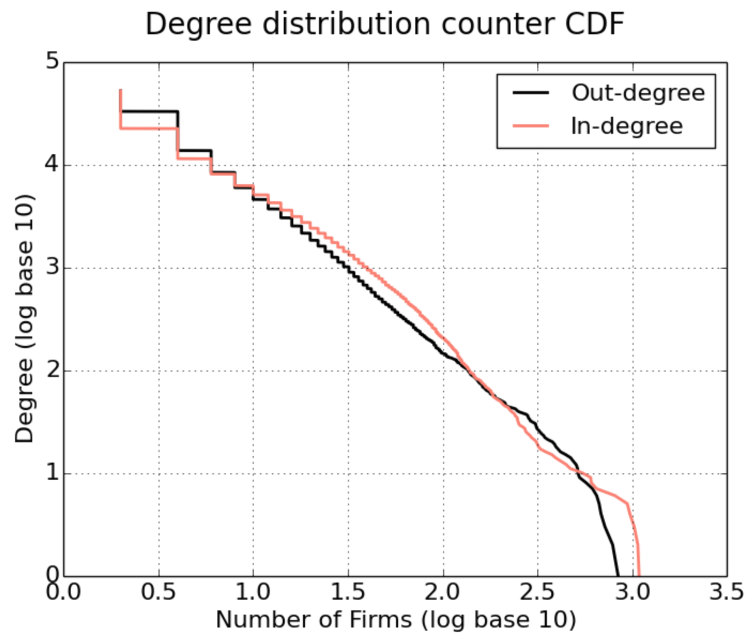


Figure 5: Degree distribution counter CDF.

banking financial corporations, and government entities. These restrictions left us with data on the business relationships between 80,092 US firms. Of the 80,092 firms Capital IQ provided data on revenues and NAICS sector codes for 43,321 firms. We used Orbis BvD to gather data on revenues or NAICS codes for 8,592 of the 36,771 firms with missing data. Completing the Capital IQ data set with information from Orbis BvD yielded a total of 105,940 supply relations between 51,913 firms for whom we know revenues and NAICS sectoral codes.

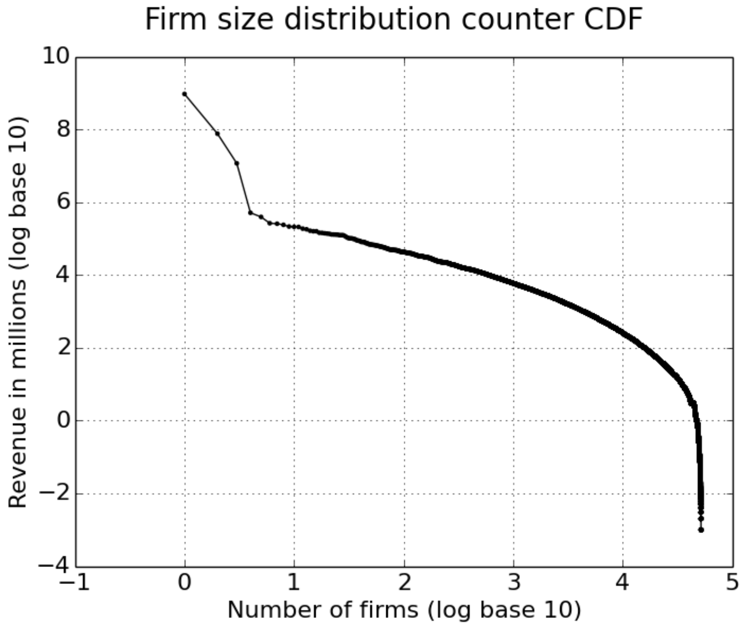


Figure 6: Annual revenues.

5.2 Baseline computational experiments

We calibrate the model to US firms production network by assigning the matrix of observed supply relationships as the adjacency matrix of the network. This matrix is however unweighted (as Capital IQ does not collect the corresponding information). We use a tailored algorithm to calibrate the weights in such a way that the empirical distribution of revenues for the firms is an invariant distribution of Equation 15[9]. We also calibrate the network economy so that the households spending on different firms in our data set matches the sectoral distribution of household personal consumption expenditure (PCE) in the Input-Output table. We then set the initial value of prices, money holdings, and stocks of output to their equilibrium values. The system is initialized in the steady state—in the sense of Proposition 2—corresponding to empirical observations about revenues and supply relationships.

We then run a series of computational experiments to investigate the dynamic response of prices following a monetary shock. In these experiments, each time step of the model is interpreted as a quarter on the basis of empirical observations about the frequency of price changes. Blinder (1991) for instance finds firms change prices in response to changes in cost and demand with the delay of a quarter. The monetary shocks considered in our experiments are constructed as follows. A first parameter $s \in [0,1]$ measures the aggregate size of the shock. A second parameter $\theta \in \mathbb{R}_+$ measures the heterogeneity of the shock with respect to firms' sizes. Let $M = \sum_{i \in N} m_i$ denote the total money in circulation in the economy. A monetary contraction of size s removes sM units of money from circulation. A monetary contraction is implemented through a reduction in the working capital of each firm i proportional to m_i^θ. We focus on the case where $\theta < 1$ when small firms are disproportionally

[9]Estimating the network weights so that given firm sizes are the invariant distribution is a large matrix quadratic programming problem. We solved the problem using IBM's CPlex software.

affected by monetary contractions. The case $\theta > 1$ would correspond to large firms being disproportionally affected by the contraction. While the case $\theta = 1$ corresponds to the case where firms are homogeneously affected by the shock and prices instantaneously adjust to their new equilibrium value (see Corollary 1).

We run series of computational experiments with variation in the size s of the monetary shock and the heterogeneity θ of the initial impact of the shock. We focus on the dynamics of the price level measured using weights of the Personal Consumption Expenditure (PCE) from the Input-Output Table[10]. The figures below summarise data from about 500 computational experiments on the calibrated US economy with more than a million firm interactions.

Figure 7 highlights a typical response of the system following a shock in that case. The x-axis represents quarters, the y-axis represents the price-level. Since one time step of our model represents one month, we average three month prices to compute the quarterly price level. Figure 7 shows that the wrong directional change in the price level is of the same order of magnitude as the long run right directional change. Furthermore, the wrong directional change in the price level generated by our model is on the same order of magnitude as that which is reported by several empirical studies (see Rusnak et al., 2013).

An increase in the price level after monetary contractions systematically occurs when small firms are disproportionally affected by monetary contractions for sufficiently low value of θ. The presence of the price puzzle is robust to changes in the magnitude s and the heterogeneity of the shock θ (for a sufficiently low value of θ). Figure 8 illustrates the sensitivity of the model with respect to the strength of the monetary shock by showing the relation between the size of the monetary contraction (that is measured indirectly by the long-run change in the price level) and the size of the short-run change of the price level in the opposite direction. The y-axis in Figures 8 and 9 measure the percent change between the pre-shock price level and the maximum price level in six quarters after a monetary contraction. Figure 8 shows that the wrong directional change in the price level is of the same order of magnitude as the long-term effect of the monetary contraction. Figure 9 illustrates the sensitivity of the price level response with regards to the heterogeneity of the impacts. The response is non-linear[11]. The size of the price puzzle increases rapidly with the heterogeneity in initial impact of monetary shocks ($|1-\theta|$). This highlights the importance of the correlation between network structure and firm size in the propagation of monetary shocks. Taking the magnitude of the price puzzle as an indirect indicator of the welfare losses during a monetary contraction, this sensitivity analysis implies that improving the financial conditions of small firms can have positive welfare implications. Yet the non-linear response of the economy to a decrease in θ imply that a sizeable positive impact will materialise only if the financial conditions of small firms become qualitatively similar to that of large firms. Outside of the range where

[10]The price level is computed in the following manner. The sectoral weights of the Personal Consumption Expenditure Index (PCE) are obtained from the Input-Output Table. The total weights of firms of a given sector in the price level is set equal to the share of that sector in the PCE. The total weight of a given sector is equally divided among firms in that sector within our data set.

[11]Note that in Figure 9 a monetary contraction of size $s = -0.001$ does not cause a temporary increase in the price level in the calibrated economy when $\theta > 0.97$. This is simply because when θ is sufficiently high the decrease is supply of consumer goods is overwhelmed by the short run decrease in the nominal demand for consumer goods for the given size of monetary contraction. The exact value of θ beyond which monetary contractions will not generate an increase in the price level will depend on the size of the contraction and the size distribution of firms within the production network.

this qualitative equivalence holds, i.e. in the neighborhood of $\theta = 1$, changes in then financial conditions of small firms would only have marginal impact on welfare.

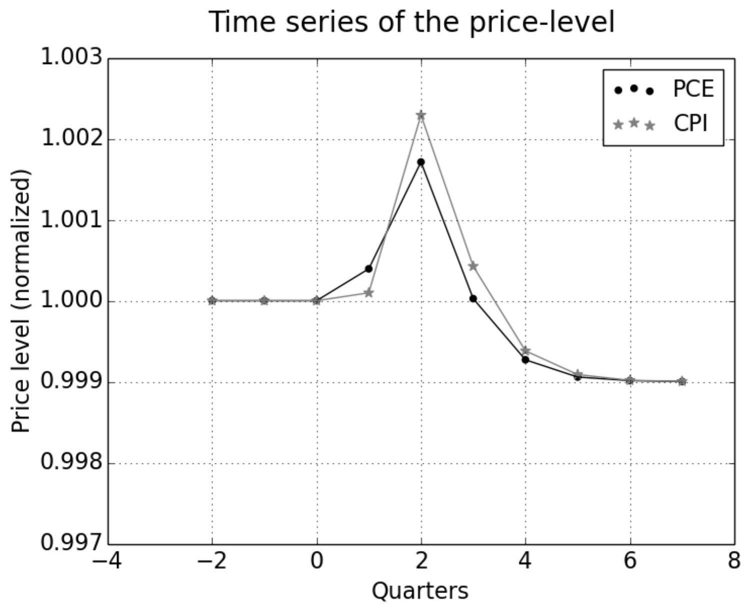


Figure 7: Price response to a negative monetary shock of size $s = -0.001.$, $\theta = 0.9$.

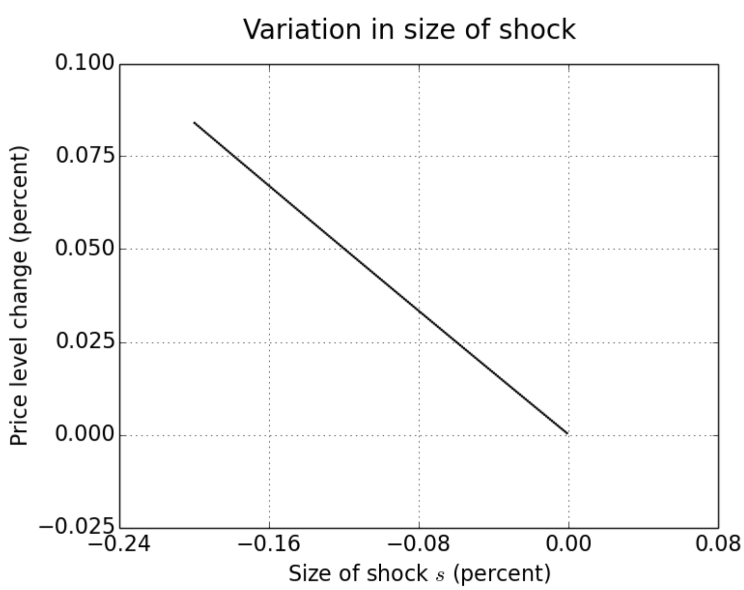


Figure 8: Long-run versus short-run changes in the price level for $\theta = 0.95$. The x-axis corresponds to the size of the long-run (negative) price level change or equivalently to the size s of the monetary shock. The y-axis measured the (positive) change in prices the first quarter after a monetary contraction.

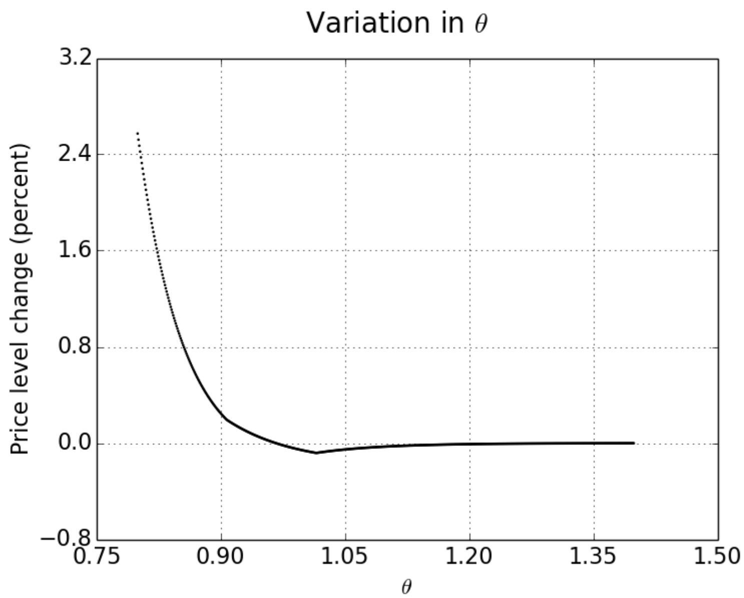


Figure 9: Short-run price level change as a function of the heterogeneity parameter θ for a fixed size of monetary shock $s = -0.001$.

5.3 Impact of price rigidity

We investigate the sensitivity of our results to price rigidity by considering a variant of the probabilistic price change model a la Calvo (1983). Namely, instead of considering that each firm i updates its price every period, we consider that it updates its price at time t with a probability ϕ_i^t given by:

$$\phi_i^t = \Big(\frac{\rho_i^t}{1+\rho_i^t}\Big)^{\psi} \tag{26}$$

where $\psi \geq 0$ is a scaling parameter and $\rho_i^t = \frac{|p_i^{t-1} - \bar{p}_i^t|}{p_i^{t-1}}$ with p^{t-1} the current price and $\bar{p}_i^t$ the clearing price given by Equation 8. Note that the source of heterogeneity in the probability of price change is intrinsically related to the model dynamics we have empahized in explaining the Price Puzzle. More specifically, after a monetary shock, firms that experience greater changes in their excess demands are more likely to change prices. And in so far as the network positions of firms determine the temporal sequence of the changes in money balances (and therefore in excess demands), the network positions are intricately related to the probability of price change. More specifically, downstream firms are disproportionally hurt by the initial impact of monetary contraction, they and their suppliers are more likely to change prices in the early time steps after a monetary shock. As the shock propagates and dissipates through the production network, all firms become less likely to change prices and therefore less heterogeneous with regards to probability of price change.

We ran 100 computational experiments for each value of ψ ranging from 0.05 to 0.50 with increments of 0.05. Figure 10 shows a time series of the price level from four of these experiments. The price-level increases in the transition from the pre-shock to the post-shock equilibrium. Figure 11 presents boxplots of the maximum price-level for different values of ψ.

Figure 11 therefore summarizes results from 1000 experiments (100 for each of the 10 values of ψ). The figure shows that a wrong directional change in the price-level is realized in all experiments. Furthermore, the wrong directional response of the price-level declines with an increases in the scaling parameter ψ. This is because as ψ increase, the probability of price change decreases. The wrong directional change in the price-level is less pronounced when fewer firms respond to monetary dynamics by changing prices. In other words, the price puzzle is more likely to emerge if there is overshooting in the price adjustment process.

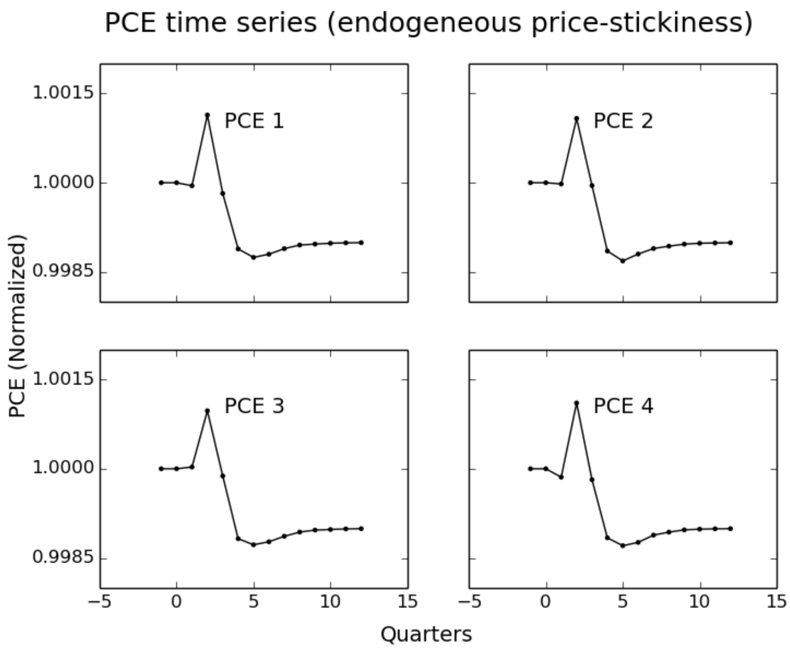


Figure 10: Time series of PCE after a monetary contraction with endogenous price-stickiness. Parameters: $\psi = 0.1$, $s = -0.001$.

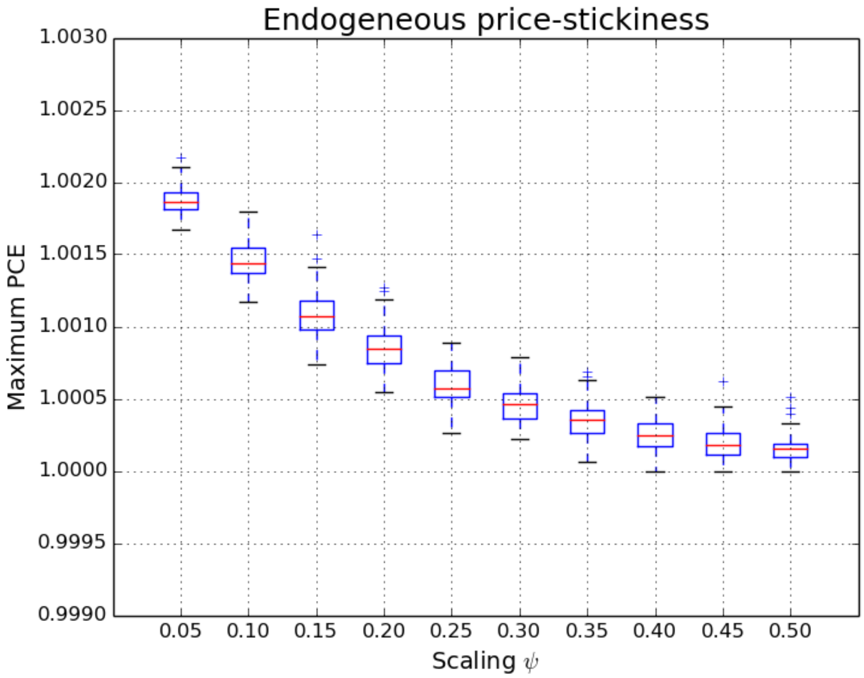


Figure 11: Boxplot of maximum PCE after a monetary contraction with endogenous price-stickiness. Parameters: $s = -0.001$.

5.4 Structural change and the price puzzle

As we have access to a single dataset on the U.S. network, we can not empirically investigate impacts of the network structure on price dynamics and the price puzzle in particular. However, Equation 18 in Proposition 2 highlights that the relaxation time of the economy to equilibrium depends on the second eigenvalue of the network. Thus, the duration, if not the magnitude, of the price puzzle increases with the second eigenvalue of the network. The second eigenvalue is a well-known measure of connectivity in graph theory (see e.g. Lovász, 2007). Notably, one can derive from an upper bound on the second eigenvalue, a lower bound on the edge and vertex connectivity of a graph[12] (Abiad et al., 2017). Hence, production networks with large second eigenvalues correspond to less diversified/integrated economies, whose sectoral components can easily be disconnected. This implies in particular, in line with the findings in subsection 3.4, that the price puzzle ought to be more pronounced in economies with a clearcut separation between upstream and downstream sectors. In this respect, empirical evidence (Carvalho and Voigtländer, 2014) suggests that producers direct their search for new inputs along vertical linkages, thus increasing the connectivity between upstream and downstream sectors and, incidentally, dampening the price puzzle.

Another structural limitation of our analysis is that we focus on Cobb-Douglas production functions. Atalay (2017) finds that complementarities between inputs across sectors can amplify sector-specific productivity shocks. The basic dynamics of our model suggests that complementarities between inputs across firms can amplify the wrong directional change in

[12] The edge (resp. vertex) connectivity is the minimal number of edges (resp. vertex) that must be removed for the graph to become disconnected.

the price-level. Indeed, monetary contractions percolate through the production network by inducing changes in relative prices or equivalently by inducing changes in the relative supply/demand of different intermediate inputs. Thus, the lower the substitutability between inputs, the greater shall be the decline in output due to the decline in the supply of any given input (for any given increase in other inputs). The impact of monetary contractions on the real supply of final goods shall therefore increase with an increase in input-complementarity. In so far as the wrong directional change in the price-level arises because the real supply of final goods decreases more than the nominal demand for these goods, greater input-complementarity will exacerbate the price-puzzle by generating a greater decline in real supply while leaving nominal demand for final goods unchanged.

Entry and exit processes might also interact with the propagation of monetary shocks to the extent that they modify the financial conditions of firms (or the average financial conditions within a sector). More specifically, in our setting, an increase in the liquidity available to small firms contributes to dampening the price puzzle. If, following a monetary shock, small firms exit because of insolvency and are replaced by firms with a more sustainable financial structure but the same liquidity, there is no impact on the dynamics. If entering firms have better (worse) access to liquidity than exiting ones, entry might milden (resp. exacerbate) the price puzzle. This might in particular be the case if small exiting firms are replaced or taken over by larger firms.

The role of the financial conditions of small firms on the transmission of monetary policy is well illustrated by the case of Germany. On the one hand, the strength of relationship lending in Germany, notably via the hausbank model, leads to improved financial conditions for small firms (see e.g. Agarwal and Elston, 2001; Chatelain et al., 2003). On the other hand, the Price Puzzle is much less pronounced in Germany than in countries where commercial lending is predominant (see Sims, 1992; Rusnak et al., 2013).

6 Conclusion

More than 150 years ago, Thomas Tooke (1844, p. 85) observed that much of the data on the relation between money and the price-level shows conventional wisdom “is not only not true, but the reverse of the truth”. By conventional wisdom he meant the quantity theory of money relation. Generations of economists since Tooke rediscovered the empirical phenomena under various guise including the ‘Price Puzzle’ and the ‘Gibson Paradox’ (Sargent, 1973). Despite the empirical findings, most economist did not abandoned conventional wisdom (Laidler, 1991). There are good reasons for this conservative attitude, not the least of which is the remarkable ability of the quantity theory to explain the long run co-movements in money and the price level (Lucas, 1996). Our paper reconciles the apparent contradiction between ‘a positive relation between money and the price-level in the long run’ with ‘a negative relation in the short run’. In constructing such a theory we did not resort to treating the price-level as the independent variable and money as the dependent variable, nor did we let money and the price-level be driven by common factors. Rather, we constructed a model in which monetary shocks causally generate temporary wrong directional change in the price-level.

Unlike most models of monetary non-neutrality, within our model firms’ prices are fully flexible. We introduced the production network as a mechanism for the transmission of monetary shocks and let firm size distribution within the network influence magnitudes and time-lags of changes in demand and supply of consumer goods. Under certain empirically plausible conditions, a monetary contraction causes supply of consumer goods to decrease more than demand in the short-run, thereby generating an increase in the price-level. In the long-run, real supply of consumer goods remains unchanged and nominal demand decreases as the economy reaches a new equilibrium with a lower price-level.

We substantiated our theoretical results with computational experiments on the US economy. There is great scope to extend the empirical analysis. The network used in this paper contains more firms and buyer-seller relations than hitherto available on the US economy. But our network data is two orders of magnitude short of the empirical reality. The US economy has more than six millions firm with employees related to each other on a production network. Our data set therefore contains fewer than one in every hundred firms in the US economy (Axtell, 2001). Some of the structure of production must have been lost in the truncation. The problem is particularly pronounced in the context of arguments developed in this paper because small firms which drive the dynamics of the price-level are typically underrepresented in non-universal data sets of an economy's production network[13]. We hope to have presented some motivations for the collecting detailed production network data to understand the dynamics of the price level: a variable of non-trivial significance for the formulation of monetary policy.

[13] Within our setting the Input-Output table is no substitute for firm level network data. The Input-Output Table does not contain the long supply chain linkages through which products pass from upstream producers to retail firms. More specifically, beginning from the final consumer any sector of the Input-Output Table can be reached within two steps. This is certainly not true of the production network between firms. Within our data set of US firms network, some firms are up to seven steps away from the final consumer. The long chains of flow of goods and money matter because they influence the time sequence and magnitudes of the responses of supply and demand of consumer goods to monetary shocks, thereby shaping the behavior of the price-level.

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A Proofs

Proof: (proof of Proposition 1). *By substituting first-order conditions for profit maximization in the production technology of firms, one obtains for all $i \in M$:*

$$\overline{q}_i = \lambda_i \prod_{j \hat{A}\breve{a} \in N} \left(a_{j,i} \frac{\overline{p}_i \overline{q}_i}{\overline{p}_j} \right)^{a_{j,i}}$$

Using the fact that $\sum_{j \in \mathbb{N}} a_{j,i} = 1$, one can then factor out the term $\overline{p}_i \overline{q}_i$ to obtain equation 6 :

$$\forall i \in M, \ p_i = \frac{1}{\lambda_i} \prod_{j \in N} (\frac{p_j}{a_{j,i}})^{a_{j,i}}$$

Under the assumption that $p_0 = 1$ and thus $\log(p_0) = 0$, this can be written in matricial forms as:

$$\log(\ p) = A^t_{|M} \log(\overline{p}) + \gamma \tag{27}$$

where $\log(p)$ denotes the vector $(\log(p_1), \cdots, \log(p_n)) \in R^n$, $A^t_{|M}$ denotes the restriction to M of A^t and for all $i \in M$, $\gamma_i := -\log(\lambda_i) - \sum_{j \in \mathcal{M}} a_{j,i} \log(a_{j,i})$.

Furthermore by combining the market clearing and the first-order conditions for profit and utility maximization, one obtains for all $i \in n$, the equation

$$\overline{p}_j \overline{q}_j = \sum_{i \in N} a_{j,i} \overline{p}_i \overline{q}_i$$

This can be written in matricial form as

$$\overline{pq} = A \overline{pq} \tag{28}$$

where $\overline{pq} := (\overline{p}_0 \overline{q}_0, \cdots, \overline{p}_n \overline{q}_n)$

Given that A is aperiodic and irreducible and $\overline{p}_0 \overline{q}_0$ is fixed, it is a standard result from the theory of non-negative matrices that there exist a unique solution to Equation 28(see e.g. Seneta, 2006) .

Moreover, given that for all $i \in M$, $a_{0,i} > 0$ and $\sum_{j \in N} a_{j,i} = 1$, it is clear that the sum of lines of $A^t_{|M}$ are all strictly less than one. It is then a standard result from the theory of non-negative matrices that the spectral radius of $A^t_{|M}$ is strictly less than one (see e.g. Minc, 1988). This implies that $(I - A^t_{|M})$ is invertible and hence that Equation 27 has a unique solution given by:

$$\log(p) = (I - A^t_{|M})^{-1} \gamma$$

Equations 27 and 28 hence determine a unique vector of outputs $q \in \mathbb{R}^N$ and normalized prices $\overline{p} \in \mathbb{R}^N$. It is then straightforward to check that $(\overline{p}, \overline{q})$ induces a unique general equilibrium in the sense of definition 1.

Proof: (proof of Lemma 1). *Let us first show Equation 12 holds. Using Equations (8) to*

(11) sequentially yields:

$$
\begin{aligned}
p_i^{t+1} &= \frac{m_i^{t+2}}{q_i^{t+1}} \\
&= \frac{m_i^{t+2}}{f_i(x_i^t)} \\
&= \frac{m_i^{t+2}}{f_i\left(\frac{a_{0,i}m_i^t}{p_0^t}, \cdots, \frac{a_{n,i}m_i^t}{p_n^t}\right)} \\
&= \frac{m_i^{t+2}}{\lambda_i \prod_{j\in N}\left(\frac{a_{j,i}m_i^t}{p_j^t}\right)^{a_{j,i}}}
\end{aligned}
$$

Then using the fact that $\sum_{j\in N} a_{j,i} = 1$, *one gets:*

$$p_i^{t+1} = \frac{1}{\lambda_i}\frac{m_i^{t+2}}{m_i^t}\prod_{j\in N}\left(\frac{p_j^t}{a_{j,i}}\right)^{a_{j,i}} \tag{29}$$

As for 13, let us show by recursion it holds if p^0 *is an equilibrium price and thus satisfies Equation 6:*

- *For* $t = 1$, *using the preceding condition one has*

  $$p_i^1 = \frac{m_i^2}{m_i^0}\frac{1}{\lambda_i}\prod_{j\in N}\left(\frac{p_j^0}{a_{j,i}}\right)^{a_{j,i}}$$

  Using equation 6 then yields:

  $$p_i^1 = \frac{m_i^2}{m_i^0}p_i^0$$

  or equivalently:

  $$\log(p_i^1) = \log(p_i^0) + \log\left(\frac{m_i^2}{m_i^0}\right)$$

- *Let us then assume the property holds up to rank* t.

  One has according to Equation 29:

  $$\log(p_i^{t+1}) = -\log(\lambda_i) + \log\left(\frac{m_i^{t+2}}{m_i^t}\right) + \sum_{j\in N}\left(a_{j,i}\log(p_j^t) - a_{j,i}\log(a_{j,i})\right)$$

Hence, by recursion:

$$\log(p_i^{t+1}) = -\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}) + \log\left(\frac{m_i^{t+2}}{m_i^t}\right) + \sum_{j\in N} a_{j,i}\left[\log(p_j^0) + \sum_{\tau=0}^{t-1}\sum_{k\in N}(A^\tau)_{k,j}\log\left(\frac{m_k^{t+1-\tau}}{m_k^{t-\tau-1}}\right)\right]$$

which can equivalently be written as:

$$\log(p_i^{t+1}) = -\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}) + \sum_{j\in N} a_{j,i}\log(p_j^0) + \log\left(\frac{m_i^{t+2}}{m_i^t}\right) + \sum_{\tau=0}^{t-1}\sum_{k\in N}(A^{\tau+1})_{k,i}\log\left(\frac{m_k^{t+1-\tau}}{m_k^{t-\tau-1}}\right)$$

$$\log(p_i^{t+1}) = -\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}) + \sum_{j\in N} a_{j,i}\log(p_j^0) + + \sum_{\tau=0}^{t}\sum_{k\in N}(A^{\tau})_{k,i}\log\left(\frac{m_k^{t+2-\tau}}{m_k^{t-\tau}}\right)$$

which eventually yields using Equation 6:

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t}\sum_{k\in N}(A^{\tau})_{k,i}\log\left(\frac{m_k^{t+2-\tau}}{m_k^{t-\tau}}\right) \tag{30}$$

Proof: (Proposition 2)**.** *Given Assumption 1, the convergence of $(m^t)^{t\in\mathbb{N}}$ to $\overline{m}$ and the bound on convergence speed given by Equation (18) are straightforward consequences of existing results about Markov chains (see e.g. Seneta, 2006; Rosenthal, 1995). It is then straightforward using Equation 8 that $p_0^t = m_0^{t+1}$ and thus that p_0^t converges towards $\overline{m}_0$.*

To show the convergence of $(p_i^t)_{t\in\mathbb{N}}$, we shall first show it is bounded above and bounded away from 0*.*

- *To show that p_i^t is bounded away from* 0*, it suffices, using Equation* 8*, to show that (i) production of each firm is uniformly bounded above and that (ii) monetary holdings for each firm are bounded away from* 0*. The former follows from the fact that labor resources are bounded and for each $i \in M$, $a_{0,i} > 0$. The latter follows from the fact that m_0^t is bounded away from* 0 *(using the fact that for each $i \in M$, $a_{0,i} > 0$) and from the irreducibility of A.*

- *To show that p_i^t is bounded above, it suffices, using Equation* 8*, to show that (i) the monetary holdings of each firm are bounded above and (ii) the output of each firm is bounded below. The former follows directly from the fact that total monetary holdings is bounded above. The latter follows from the fact that given monetary holdings of each firm are bounded below and above, there is a minimal share $\mu > 0$ of the output of its* *supplier that the firm can purchase at each time step. Then, if $\epsilon > 0$ denotes the minimal level of output among firms in period t, one has for all $i \in M$ $q_i^{t+1} \geq \mu^{a_{0,i}}(\mu\epsilon)^{1-a_{0,i}}$ and thus $q_i^{t+1} \geq \epsilon$ as soon as $\epsilon \leq \mu^{1/a_{0,i}}$. In other words, one has for all $i \in M$, and all $t \in \mathbb{N}$, $q_i^t \geq \min(\mu^{1/a_{0,i}}, \min_{\tau<t} q_i^\tau)$ and thus q_i^t is bounded away from 0 as soon as for all $i \in M$, $q_i^0 > 0$.*

The fact that for all $i \in M$, $(p_i^t)_{t\in\mathbb{N}}$ is bounded above and bounded away from 0 implies that for all $i \in M$, $(\log(p_i^t)_{t\in\mathbb{N}}$ is bounded. Furthermore using Equation 12, one has for all $i \in M$ and all $t \in \mathbb{N}$

$$\log(p_i t+1) = -\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}) + \log(\frac{m_i^{t+2}}{m_i^t}) + a_{0,i}\log(p_0^t) + \sum_{j\in M} a_{j,i}\log(p_j^t)$$

$$\log(p_i^{t+1}) = -\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}) + \log(\frac{m_i^{t+2}}{m_i^t}) + a_{0,i}\log(m_0^{t+1}) + \sum_{j\in M} a_{j,i}\log(p_j^t) \quad (31)$$

Let us then denote by $X^t := (\log(p_i^t))_{i\in M}$, $Y_t = (\log(\frac{m_i^{t+2}}{m_i^t}) + a_{0,i}\log(m_0^{t+1}))_{i\in M}$ and $Z^t = (-\log(\lambda_i) - \sum_{j\in N} a_{j,i}\log(a_{j,i}))_{i\in M}$. One can then write Equation 31 in matricial form as:

$$X^{t+1} = Z + Y^t + A'_{|M} X^t \quad (32)$$

where Y^t converges to some $\bar{Y} \in \mathbb{R}^M$ because of the convergence of $(m^t)^{t\in\mathbb{N}}$ and $\|A'_{|M}\| < 1$ because A is column stochastic and for all $i \in M, a_{0,i} > 0$.

Thus, one has

$$X^{t+1} - X^t = Y^t - Y^{t-1} + A'_{|M}(X^t - X^{t-1}).$$

It is then straightforward to show that the sequence $(X^{t+1} - X^t)_{t\in\mathbb{N}}$ can not have other adherence value than 0. As it is bounded, it must then be that $X^{t+1} - X^t$ tends towards 0.

Now, as $(X^t)_{t\in\mathbb{N}}$ is bounded, it has at least an adherence value. Let us then consider such a value $\bar{X}$. Equation 32 implies that one must have $\bar{X} = (I - A'_{|M})^{-1}(Z + \bar{Y})$. This implies that $(X^t)_{t\in\mathbb{N}}$ has at most one adherence value/ As it is bounded, it must converge. In other words, one has shown the convergence of $(\log(p_t^i))_{t\in\mathbb{N}}$ for all $i \in M$. One can deduce, by continuity, the convergence of $(p^t)^{t\in\mathbb{N}}$. Equation 12 then implies that the limit of $(p^t)^{t\in\mathbb{N}}$ indeed is the equilibrium price $\overline{p}$. The convergence of $(x^t)^{t\in\mathbb{N}}$ and $(q^t)^{t\in\mathbb{N}}$ to their equilibrium values as well as Equation 17 are then obtained by substitution in the Equations (9) and (10).

Proof: (Proposition 3)**.** *For all $i \in N$, one can write equation 22 as:*

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t}\sum_{k\in N}(A^\tau)_{k,i}\log\left(1 + \frac{(A_{k,\cdot}^{t+2-\tau} - \xi_{\{\tau<t\}}A_{k,\cdot}^{t-\tau})\epsilon}{m_k^0 + \xi_{\{\tau<t\}}A_{k,\cdot}^{t-\tau}\epsilon}\right)$$

A linear expansion in ϵ then yields:

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t} \sum_{k\in N} (A^\tau)_{k,i} \left(\frac{A_{k,\cdot}^{t+2-\tau} - \xi_{\{\tau<t\}} A_{k,\cdot}^{t-\tau}}{m_k^0} \epsilon + o(\epsilon) \right)$$

where $\lim_{\|\epsilon\|\to 0} o(\epsilon) = 0$. *This yields*

$$\log(p_i^{t+1}) = \log(p_i^0) + \sum_{\tau=0}^{t} \sum_{k\in N} (A')^\tau_{i,k} \frac{1}{m_k^0} \left(A_{k,\cdot}^{t+2-\tau} - \xi_{\{\tau<t\}} A_{k,\cdot}^{t-\tau} \right) \epsilon + o(\epsilon)$$

This can be written in matricial terms as

$$\log(p^{t+1}) = \log(p^0) + \sum_{\tau=0}^{t} (A')^\tau \Delta_{m^0} (A^{t+2-\tau} - \xi_{\{\tau<t\}} A^{t-\tau}) \epsilon + o(\epsilon)$$

where Δ_m is the diagonal matrix whose coefficients are the m_k^0. This eventually yields:

$$\log(p^{t+1}) = \log(p^0) + \left(\sum_{\tau=0}^{t} (A')^\tau \Delta_m A^{t-\tau} (A^2 - I)\epsilon + (A')^t \Delta_m \epsilon \right) + o(\epsilon)$$

Proof: (Proposition 4). *We consider an economy with $2S$ firms, where $S > T+1$, such that firm 1 uses only labor in its production process, for every $i \geq 2$ firm i uses only the output of firm $i-1$ as input and the household only consumes the output of firm $2S$. Formally, it corresponds to the adjacency matrix $A = (A_{i,j})_{i,j\in N}$ such that $a_{j,i} = \delta_{(j,i-1\ [2S+1])}$ where δ is the Kronecker symbol and $[2S+1]$ indicates that the sum is modulo $2S+1$. Explicitly:*

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \ddots & 1 \\ 1 & 0 & \cdots & \cdots & 0 \end{pmatrix}$$

We shall further assume that the productivity parameter is such that $\lambda_i = 1$ for each firm and that the household supplies 1 unit of labor. In this framework, assuming total monetary mass is Np_0 it is clear that at the initial equilibrium all prices are equal to p_0, each agent initially holds $m_i^0 = p_0$ units of wealth, produces 1 unit of output using 1 unit of input and the household provides one unit of labor and consumes one unit of good $2S$.

Let us then restrict attention to vectors of shocks ϵ that affect only the S last firms, i.e such that for all $i \leq S, \epsilon_i = 0$. We shall show that this vector of shocks can be chosen such that the induced sequence of prices for firm S is $\mathbf{p}$. Indeed, in this simple setting, the price variation of firm S are completely determined by the monetary shocks received downstream in the supply chain whose variations induce a volatility of demand and thus of price for firm S. The simple structure of then economy allows to derive analytically the expression of this volatility and then to choose the sequence of shocks generating the desired price sequence.

Let us first remark that Lemma 1 implies that, in our framework, one has for all $i \geq 1$

and all $t \in \mathbb{N}$:

$$p_i^{t+1} = \frac{m_i^{t+2}}{m_i^t} p_{i-1}^t$$

and for all $t \leq T$

$$m_i^t = m_i^0 + A_{i,\cdot}^t \epsilon = p_0 + \epsilon_{i+t}.$$

Thus, one has for all $i \geq 1$ and all $t \leq T$:

$$p_i^{t+1} = \frac{p_0 + \epsilon_{i+t+2}}{p_0 + \xi_{\{t>0\}} \epsilon_{i+t}} p_{i-1}^t$$

where ξ_X denotes the indicator function of the set X. Noting that $\epsilon_i = 0$ for all $i \leq S$, a straightforward recursion then shows that, one has for all $1 \leq t \leq T$:

$$p_{S-1}^t = p_S^{t-1}$$

Therefore, one has for all $t \leq T$:

$$p_S^{t+1} = \frac{p_0 + \epsilon_{i+t+2}}{p_0 + \xi_{\{t>0\}} \epsilon_{i+t}} p_S^{t-1}$$

It is then straightforward to choose $(\epsilon_{S+2}, \cdots, \cdots \epsilon_{S+T+1})$ such that

$$(p_S^0, \cdots, p_S^T) = \mathbf{p}.$$

This ends the proof.

Proof: (proof of Proposition 5 and of the corollary)**.** *We consider that all firms in layer L are affected by an homogeneous negative monetary shock of magnitude ρ, i.e. the working capital of each firm i in layer L is decreased to $(1-\rho)m_i^0$. Then, the nominal demand of labor of firms in layer L is reduced to $\sum_{i \in \mathcal{L}_L} a_{0,i}(1-\rho)m_i^0 = (1-\rho)k$ while the demand of labor of firms in other layers is maintained at $= \sum_{j \in \mathcal{L}_{-L}} a_{0,j} m_j^0 = (1-k)$. Thus the supply of labor to each firm in layer L is reduced by a factor ${}^{(1-\rho)}/{}_{1-\rho k} < 1$ in period 1.*

As firms in layer L are the only consumers of the firms of layer $L-1$, their supply of inputs is not modified by the negative shock on their nominal demand. Overall, the production of firm i in layer L in period 1, and thus its supply in period 2 is reduced by a factor $\left(\frac{1-\rho}{1-\rho k}\right)^{a_{0,i}}$ In parallel, the revenues of the household in period 1, and thus her demand in period 2, are reduced to $1-\rho k$

The price of a firm i in layer L in period 2 is therefore given by:

$$p_i^2 = \frac{(1-\rho k)a_{i,0}}{\left(\frac{1-\rho}{1-\rho k}\right)^{a_{0,i}} q_i^0} = \frac{(1-\rho k)p_{i,0}}{\left(\frac{1-\rho}{1-\rho k}\right)^{a_{0,i}}} = \frac{(1-\rho k)^{a_{0,i}+1}}{(1-\rho)^{a_{0,i}}} p_{i,0}$$

where q_i^0 is the initial production level and p_i^0 the initial, equilibrium, price.

One then has $p_i^2 \geq p_i^0$ *if and only if* $\frac{(1-\rho k)^{a_{0,i}+1}}{(1-\rho)^{a_{0,i}}} \geq 1$ *or equivalently:*

$$\frac{a_{0,i}+1}{a_{0,i}} < \frac{\log(1-\rho)}{\log(1-\rho k)} \tag{33}$$

It is then clear that Equation 33 *holds as long as* k *is sufficiently small.*